A framework for second order parton showers

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Based on the work arXiv:1611.00013
In collaboration with Peter Skands

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Precision prediction from parton shower

The formal accuracy of these showers remains governed by leading-order splitting functions

- Combine parton shower with fixed-order matrix elements (improve hard emissions)
- Match the shower evolution to higher-order analytical resummation

Our proposal (For now, we concentrate on final-state showers.)

- Construct a framework to include NLO corrections into the Sudakov form factor
- Full evolution kernels are accurate to $\mathcal{O}(\alpha_s^2)$ at leading color
- More subleading logarithmic terms will be resummed automatically

We work in the framework of dipole-antenna showers which combines dipole showers with antenna subtraction formalism embodied in VINCIA [1].

1. a plugin to the PYTHIA 8, see Peter's talk, http ://vincia.hepforge.org/

Sudakov form factor

Sudakov form factor represents the no-branching probability. The differential branching probability per phase-space element is given by the derivative of the Sudakov factor,

- Leading order Sudakov form factor ($q\bar{q}$ dipole as an example)

$$\frac{d}{dQ^2} \underbrace{\left(1 - \Delta(Q_0^2, Q^2)\right)}_{\text{branching probability}} = - \underbrace{\int \frac{d\Phi_3}{d\Phi_2}\, \delta(Q^2 - Q^2(\Phi_3))\, a_3^0}_{q\bar{q} \to qg\bar{q} \text{ phase space and antenna function}} \quad \Delta(Q_0^2, Q^2)$$

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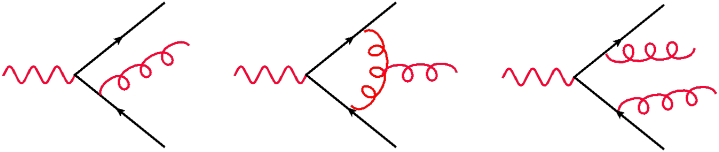
- Next-to-leading order Sudakov form factor



LO Virtual Real

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$$\frac{d}{dQ^2}\underbrace{\left(1-\Delta(Q_0^2,Q^2)\right)}_{\text{branching probability}} = -\underbrace{\int \frac{d\Phi_3}{d\Phi_2}\,\delta(Q^2-Q^2(\Phi_3))\left(a_3^0+a_3^1\right)}_{\text{born and virtual correction}}\Delta(Q_0^2,Q^2)$$
$$-\underbrace{\int \frac{d\Phi_4}{d\Phi_2}\,\delta(Q^2-Q^2(\Phi_4))\,a_4^0}_{\text{real correction}}\;\Delta(Q_0^2,Q^2)$$

where $Q(\Phi_4)$ is the resolution scale for real corrections.

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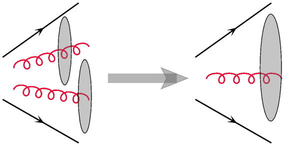
where $Q(\Phi_4)$ is the resolution scale for real corrections. How to define $Q(\Phi_4)$?

Sudakov form factor

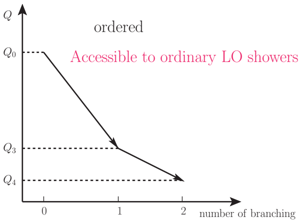
Ordered and unordered Real correction can be written as two iterated $2 \to 3$ shower paths.

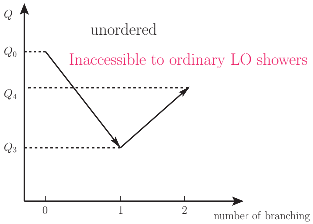
$$\int \frac{d\Phi_4}{d\Phi_2}\, \delta(Q^2 - Q^2(\Phi_4))\, a_4^0 \to \sum_{s \in a,b} \int \frac{d\Phi_4}{d\Phi_2}\, \delta(Q^2 - Q^2(\Phi_4))\, R_{2\to 4}\, s_3\, s_3'$$

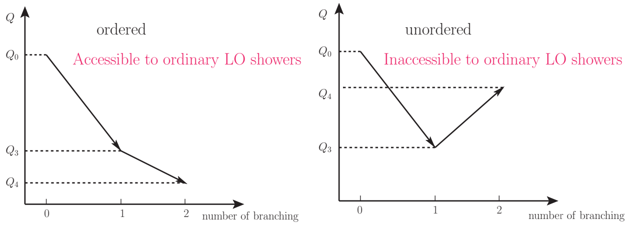


choose smaller one as Q_4 cluster $4 \to 3$ and get Q_3

The branching is in ordered region if $Q_4 < Q_3$ and it is in unordered region if $Q_4 > Q_3$.







$Q(\Phi_4) \equiv \max(Q_4, Q_3)$. In ordered region the second emission is considered as unresolved. We use Q_3 as the evolution scale (2→3 showers). In unordered region we use the scale of four-parton state as the evolution scale (direct 2→4 showers).

Using antenna phase space factorization we have $\frac{d\Phi_4}{d\Phi_2} = d\Phi_{\rm ant}^{2\to3} d\Phi_{\rm ant}^{3\to4}$

$$\int d\Phi_{\rm ant}^{2\to3} d\Phi_{\rm ant}^{3\to4} = \left(\int_{\rm ord} + \int_{\rm unord}\right) d\Phi_{\rm ant}^{2\to3} d\Phi_{\rm ant}^{3\to4}$$

Expression of Sudakov factor

- The NLO Sudakov factor is written as the product of $2 \to 3$ and $2 \to 4$ ones :

$$\Delta(Q_0^2, Q^2) = \Delta_{2\to3}(Q_0^2, Q^2)\Delta_{2\to4}(Q_0^2, Q^2)$$

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- $2 \to 3$ Sudakov factor

$$\Delta_{2\to3}(Q_0^2, Q^2) = \exp\left[-\int_{Q^2}^{Q_0^2} dQ_3^2 \int_{\zeta_-(Q_3)}^{\zeta_+(Q_3)} d\zeta \,\frac{|J|}{16\pi^2 m^2} a_3^0 \left(1 + \frac{a_3^1}{a_3^0} + \sum_{s\in a,b} \int_{\mathrm{ord}} d\Phi^s_{\mathrm{ant}}\, R_{2\to4}\, s_3' + \underbrace{\int_{Q_3^2}^{Q_0^2} d\tilde{Q}_3^2 \int_{\zeta_-(\tilde{Q}_3)}^{\zeta_+(\tilde{Q}_3)} d\tilde{\zeta}\, \frac{|\tilde{J}|}{16\pi^2 m^2}\, a_{\tilde{3}}^0}_{\text{from expansion of Sudakov factor}} \right)\right]$$

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- $2 \to 4$ Sudakov factor

$$\Delta_{2\to4}(Q_0^2, Q^2) = \exp\Bigg[-\sum_{s\in a,b} \int_{Q^2}^{Q_0^2} dQ_4^2 \int_0^{Q_4^2} dQ_3^2 \int_{\zeta_{4-}}^{\zeta_{4+}} d\zeta_4 \int_{\zeta_{3-}}^{\zeta_{3+}} d\zeta_3\, \frac{|J_3 J_4|}{(16\pi^2)^2 m^2 m_s^2} \int_0^{2\pi} \frac{d\phi_4}{2\pi} R_{2\to4}\, s_3 s_3'\Bigg]$$

2→4 shower framework

For a direct branching $1\ 2 \to 3\ 4\ 5\ 6$ the resolution scale is $Q_4 = 2\min(p_\perp^{345}, p_\perp^{456})$.
Partition the direct $2 \to 4$ phase space into two sectors

- Sector A, with condition $p_\perp^{345} < p_\perp^{456}$
- Sector B, with condition $p_\perp^{345} > p_\perp^{456}$.

The Sudakov factor can be written as

$$\Delta_{2\to4}(Q_0^2, Q^2) = \Delta_{2\to4}^{A}(Q_0^2, Q^2)\Delta_{2\to4}^{B}(Q_0^2, Q^2)$$

How to work with these two sectors

- Use the same $2 \to 4$ sub-antenna function(with full singularities) for sectors A and B, such as a_4^0.
- Use different path to parametrise phase space according to the choice of Q_4
- Veto configurations which do not fall in the appropriate sector

2→4 shower framework

It is not possible to calculate the $2 \to 4$ Sudakov form factor analytically. This can be done numerically via the veto algorithm. The trick is to find a simple function which is larger than the integrand in any phase space points.

Shower algorithm

- Choose a trial function motivated by smoothly ordering showers : $2a_{\text{trial}}^{2\to3}(Q_3^2)P_{\mathrm{imp}}a_{\text{trial}}^{2\to3}(Q_4^2)$
- Generate a new scale Q form a random number $R \in [0,1]$ according to $R = \Delta_{2\to4}(Q_0^2, Q^2) = \exp[-\mathcal{A}_{\text{trial}}(Q_0^2, Q^2)]$
- Generate other kinematic variables according to the trial integral $\mathcal{A}_{\text{trial}}$
- Check the sector condition
- Accept this trial with a probability $P_{\mathrm{trial}}^{2\to4} = \frac{\alpha_s^2}{\hat{\alpha}_s^2}\frac{a_4}{a_{\mathrm{trial}}^{2\to4}}$

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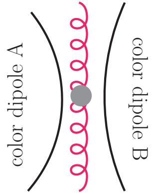
- Generate other kinematic variables according to the trial integral $\mathcal{A}_{\text{trial}}$
- Check the sector condition
- Accept this trial with a probability $P_{\rm trial}^{2\to 4} = \frac{\alpha_s^2}{\hat{\alpha}_s^2}\frac{a_4}{a_{\rm trial}^{2\to 4}}$

What we need is the sub-antenna functions.

Sub-antenna functions

For a branching 1 2 → 3 4 5 6 we consider partons 1 and 2 (3 and 6) as the hard radiators (recoilers) and partons 4 and 5 as the radiated soft and/or collinear partons. For a $q\bar{q}$ parent antenna, the sub-antenna functions are equal to the full ones and we use a_4^0 from paper [Gehrmann-De Ridder et al., 2005].

For a qg or gg dipole it is non-trivial to define sub-antenna functions.



There are two color dipoles for Higgs decaying into gluon-gluon which can generate two color-unconnected 2 → 3 emissions.

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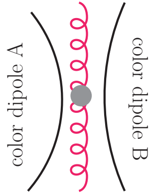

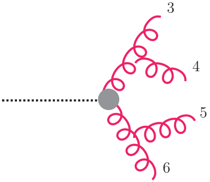

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ME is symmetric under cyclic interchanges of the momenta. More than one sub-antenna functions contribute

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sub-antenna functions for gg dipole

$$f_4^0 = \begin{cases} a_4^0(3,4,5,6)\,\frac{f_3^0(\widehat{34},\widehat{45},6)}{a_3^0(\widehat{34},\widehat{45},6)}\,\frac{f_3^0(3,4,5)}{d_3^0(3,4,5)} & \text{Sec. A} \\ a_4^0(3,4,5,6)\,\frac{f_3^0(3,\widehat{45},\widehat{56})}{a_3^0(3,\widehat{45},\widehat{56})}\,\frac{f_3^0(4,5,6)}{d_3^0(4,5,6)} & \text{Sec. B} \end{cases}$$

a_3^0 : $q\bar{q} \rightarrow qg\bar{q}$

d_3^0 : $qg \rightarrow qgg$

f_3^0 : $gg \rightarrow ggg$

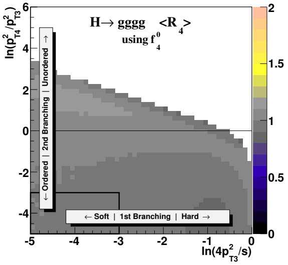
a_4^0 : $q\bar{q} \rightarrow qgg\bar{q}$

f_4^0 : $gg \rightarrow gggg$

Sub-antenna functions

As numerical validation we compare the leading-color matrix element squared for $H \to g_1 g_2 g_3 g_4$ with our sub-antenna function.

$$R_4 = \frac{|M(h \to gg)|^2}{|M(1,2,3,4)|^2} \Big(f_4^0(1,2,3,4) + f_4^0(2,3,4,1) + f_4^0(3,4,1,2) + f_4^0(4,1,2,3) \\ + f_3^0(\widehat{23},1,\widehat{34}) f_3^0(2,3,4) + f_3^0(\widehat{34},2,\widehat{41}) f_3^0(3,4,1) \\ + f_3^0(\widehat{41},3,\widehat{12}) f_3^0(4,1,2) + f_3^0(\widehat{12},4,\widehat{23}) f_3^0(1,2,3) \Big)$$



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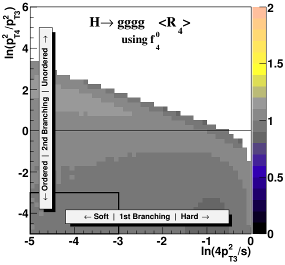
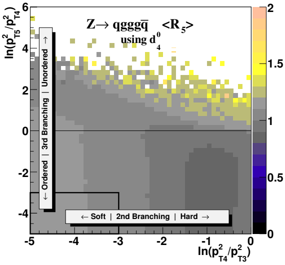
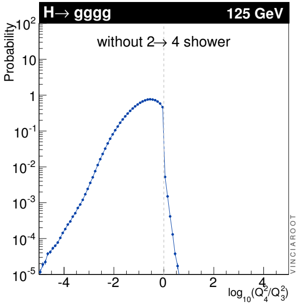
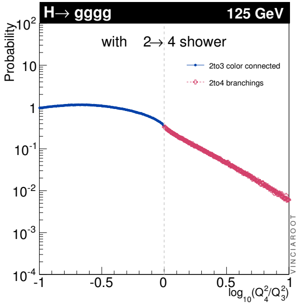
$$+ f_3^0(\widehat{23},1,\widehat{34}) f_3^0(2,3,4) + f_3^0(\widehat{34},2,\widehat{41}) f_3^0(3,4,1)$$
$$+ f_3^0(\widehat{41},3,\widehat{12}) f_3^0(4,1,2) + f_3^0(\widehat{12},4,\widehat{23}) f_3^0(1,2,3) \Big)$$





Numerical results





Due to momentum recoil effects, the local definition of the evolution scale may not be the smallest global scale in a given event. The pure $2 \to 3$ shower can generate some contributions classified as $Q_4 > Q_3$.

In the $2 \to 4$ shower, the iterated $2 \to 3$ branchings are matched by sub-antenna functions f_4^0 and the direct $2 \to 4$ branchers are used to populate the unordered phase space.

As shown in the right-hand plots, the $2 \to 4$ shower fills in the unordered phase space, and, in the limit $Q_4 \sim Q_3$, consistently matches onto the $2 \to 3$ result.

Conclusion and outlook

Conclusion

- We presente a framework for including the NLO corrections into Sudakov form-factor
- A crucial new ingredient development is the direct 2→4 branchings
- We define sub-antenna functions for double gluon emissions
- As a validation, we compare $2 \to 4$ and $2 \to 3$ showers at phase-space boundary

Outlook

- In near future extend $2 \to 4$ showers to include $g \to q\bar{q}$ splittings
- Include the second-order correction for $2 \to 3$ showers
- In long term we will turn our attention to the initial state

Thank you for your attention !

backup

Phase space integral

Using the antenna phase-space factorization

$$d\Phi_{m+1}(p_1, \ldots, p_{m+1}) = d\Phi_m(p_1, \ldots, p_I, p_K, \ldots, p_{m+1}) \times d\Phi_{\mathrm{ant}}(i, j, k)$$

$2 \to 4$ phase space integration can be written as

$$\frac{d\Phi_4(3,4,5,6)}{d\Phi_2(1,2)} = \begin{cases} \text{path a :} & d\Phi_{\mathrm{ant}}(\widehat{34}, \widehat{45}, 6)\, d\Phi_{\mathrm{ant}}(3,4,5) \\ \text{path b :} & d\Phi_{\mathrm{ant}}(3, \widehat{45}, \widehat{56})\, d\Phi_{\mathrm{ant}}(4,5,6) \end{cases}$$

where $d\Phi_{\mathrm{ant}}(i,j,k) = \frac{1}{16\pi^2} \frac{|J|}{s_{ijk}} \frac{d\phi}{2\pi} dQ^2 d\zeta$

2→4 shower framework

2→4 trial function

$$\frac{1}{(16\pi^2)^2} a_{\text{trial}}^{2\to4} = \mathcal{C} \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{128}{(Q_3^2 + Q_4^2)Q_4^2}$$

Technically, we generate these phase spaces by oversampling. And we set the lower and upper limit for ζ(3→4 one) independent on the evolution scale and calculate integral I_ζ.

Solution for Q

- with α_s fixed, $Q^2 = m^2 \exp\left(-\sqrt{\ln^2(Q_0^2/m^2) + 2f_R/\hat{\alpha}_s^2}\right)$ where $f_R = -4\pi^2 \ln R/(\ln(2)\mathcal{C} I_\zeta)$
- with one-loop running α_s, $Q^2 = \frac{4\Lambda^2}{k_\mu^2}\left(\frac{k_\mu^2 m^2}{4\Lambda^2}\right)^{-1/W_{-1}(-y)}$ where $y = \frac{\ln k_\mu^2 m^2/4\Lambda^2}{\ln k_\mu^2 Q_0^2/4\Lambda^2} \exp\left[-f_R b_0^2 - \frac{\ln k_\mu^2 m^2/4\Lambda^2}{\ln k_\mu^2 Q_0^2/4\Lambda^2}\right]$ and $W_{-1}(z)$ is the Lambert W function

check singularities of d_4^0 (1 2->ijkl)

infrared limits	$\tilde{d}_4^0(i,j,k,l)$ limits	valid sector
$j_g\ ,k_g \to 0$	S_{ijkl}	A and B
$j_g \to 0,\ k_g\|\|l_g$	$S_{i;jkl}(z)\frac{1}{s_{kl}}P'_{gg\to G}(z)$	A and B
$i_q\|\|j_g\ ,k_g \to 0$	$S_{l;kji}(z)\frac{1}{s_{ij}}P_{qg\to Q}(z)$	A and B
$i_q\|\|j_g\|\|k_g$	$P_{ijk\to Q}(w,x,y)$	A and B
$j_g\|\|k_g\|\|l_g$	unknown	unknown
$i_q\|\|j_g\ ,k_g\|\|l_g$	$\frac{1}{s_{ij}s_{kl}}P_{qg\to Q}(z)P'_{gg\to G}(y)$	A and B
$j_g \to 0$	$S_{ijk}d_3^0(\hat{ij},\hat{jk},l)$	A (B with factor $\frac{18s_{lj}+15s_{jk}}{18s_{lj}+16s_{jk}}$)
$k_g \to 0$	$S_{jkl}d_3^0(i,\hat{jk},\hat{kl})$	A and B
$i_q\|\|j_g$	$\frac{1}{s_{ij}}P_{qg\to Q}(z)d_3^0(i+j,k,l)$	A
$j_g\|\|k_g$	$\frac{1}{s_{jk}}P_{gg\to G}(z)d_3^0(i,j+k,l)$	A and B
$k_g\|\|l_g$	$\frac{1}{s_{kl}}P'_{gg\to G}(z)d_3^0(i,j,k+l)$	B

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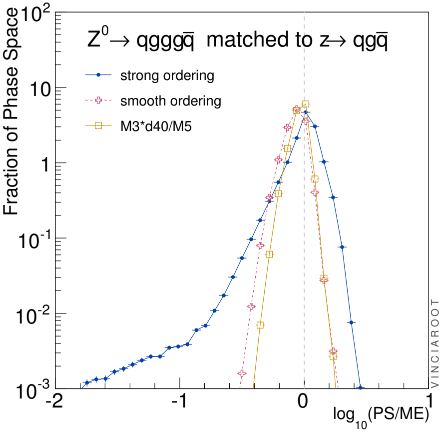

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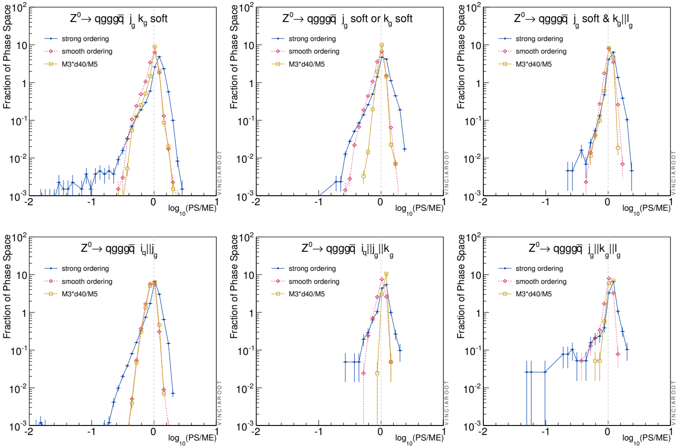
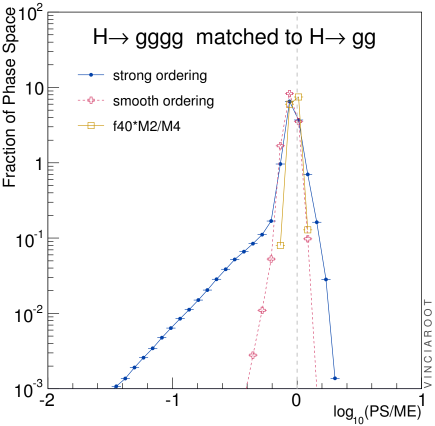


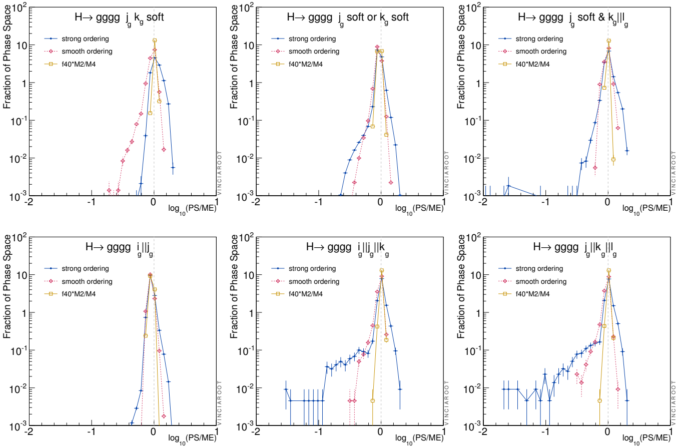
check singularities of f_4^0 (1 2->ijkl)

infrared limits	$\tilde{f}_4^0(i,j,k,l)$ expression	valid sector
j_g , $k_g \to 0$	S_{ijkl}	A and B
$j_g \to 0,\ k_g\|\|l_g$	$S_{i;jkl}(z)\frac{1}{s_{kl}}P'_{gg\to G}(z)$	A and B
$i_g\|\|j_g$, $k_g \to 0$	$S_{l;kji}(z)\frac{1}{s_{ij}}P'_{gg\to G}(z)$	A and B
$i_g\|\|j_g\|\|k_g$	unknown	unknown
$j_g\|\|k_g\|\|l_g$	unknown	unknown
$i_g\|\|j_g$, $k_g\|\|l_g$	$\frac{1}{s_{ij}s_{kl}}P'_{gg\to G}(z)P'_{gg\to G}(y)$	A and B
$j_g \to 0$	$S_{ijk}F_3^0(\hat{ij},\hat{jk},l)$	A (B with factor $\frac{18s_{lj}+15s_{jk}}{18s_{lj}+16s_{jk}}$)
$k_g \to 0$	$S_{jkl}F_3^0(i,\hat{jk},\hat{kl})$	B (A with factor $\frac{18s_{lj}+15s_{jk}}{18s_{lj}+16s_{jk}}$)
$i_g\|\|j_g$	$\frac{1}{s_{ij}}P'_{gg\to G}(z)F_3^0(i+j,k,l)$	A
$j_g\|\|k_g$	$\frac{1}{s_{jk}}P_{gg\to G}(z)F_3^0(i,j+k,l)$	A and B
$k_g\|\|l_g$	$\frac{1}{s_{kl}}P'_{gg\to G}(z)F_3^0(i,j,k+l)$	B

check singularities of f_4^0 (1 2->ijkl)



check singularities of f_4^0 (1 2->ijkl)



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