

Introduction to Accelerator Physics

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A Real Introduction ...



the beta function is usually obtained via the matrix element „m12“, which is in Twiss form for the undistorted case

$$m_{12} = \beta_0 \sin 2\pi Q$$

$$(1) \quad m_{12}^* = \beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta kds$$

and including the error:

$$\begin{aligned} m_{12}^* &= b_{11} a_{12} + b_{12} a_{22} - \cancel{b_{12} a_{12} \Delta kds} \\ m_{12} &= \beta_0 \sin 2\pi Q \end{aligned}$$

As M^* is still a matrix for one complete turn we still can express the element m_{12} in twiss form:

$$(2) \quad m_{12}^* = (\beta_0 + d\beta)^* \sin 2\pi(Q + dQ) - a_{12} b_{12} \Delta kds = \beta_0 2\pi dQ \cos 2\pi Q + d\beta_0 \sin 2\pi Q$$

Equalising (1) and (2) and assuming a small error

$$dQ = \frac{\Delta k \beta_1 ds}{4\pi}$$

$$\beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta kds = (\beta_0 + d\beta)^* \sin 2\pi(Q + dQ) \dots$$

$$\beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta kds = (\beta_0 + d\beta)^* \sin 2\pi Q \cos 2\pi dQ + \cos 2\pi Q \sin 2\pi dQ$$

What we will NOT do

$$\approx 1$$

$$\approx 2\pi dQ$$

$$- a_{12} b_{12} \Delta kds = \frac{\beta_0 \Delta k \beta_1 ds}{2} \cos 2\pi Q + d\beta_0 \sin 2\pi Q$$

$$\beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta kds = \beta_0 \sin 2\pi Q + \beta_0 2\pi dQ \cos 2\pi Q + d\beta_0 \sin 2\pi Q + d\beta_0 2\pi dQ \cos 2\pi Q$$

$$d\beta_0 = \frac{-1}{2 \sin 2\pi Q} \{2a_{12}b_{12} + \beta_0 \beta_1 \cos 2\pi Q\} \Delta kds$$



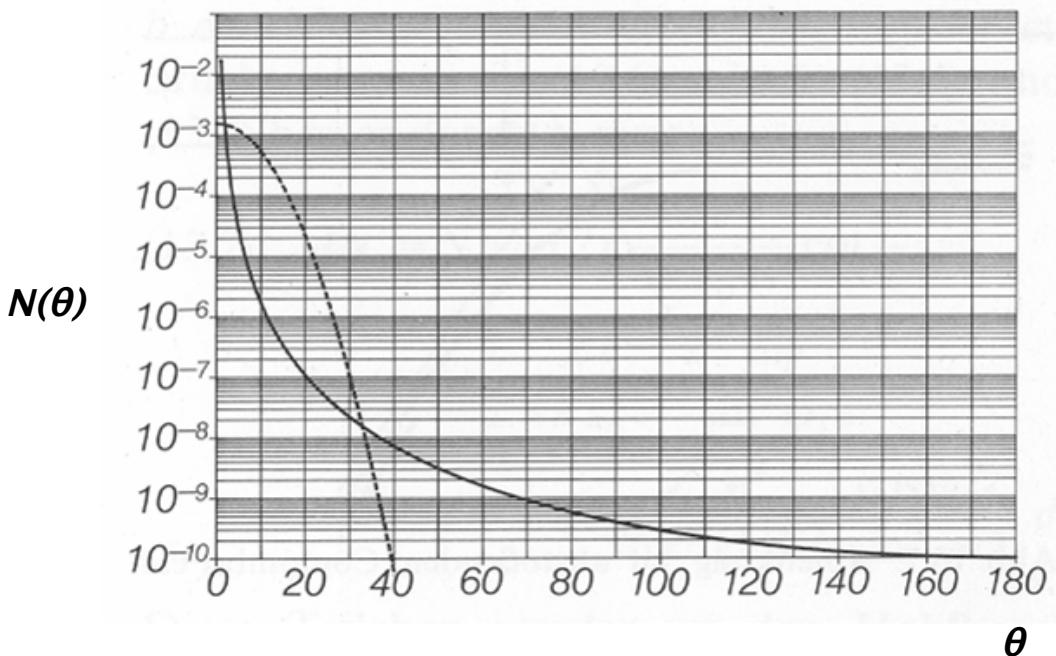
$$M = \begin{pmatrix} \sqrt{\frac{\beta_s}{\beta_0}} (\cos \psi_s + \alpha_0 \sin \psi_s) & \sqrt{\beta_s \beta_0} \sin \psi_s \\ \frac{(\alpha_0 - \alpha_s) \cos \psi_s - (1 + \alpha_0 \alpha_s) \sin \psi_s}{\sqrt{\beta_s \beta_0}} & \sqrt{\frac{\beta_0}{\beta_s}} (\cos \psi_s - \alpha_s \sin \psi_s) \end{pmatrix}$$

I.) A Bit of History



Rutherford Scattering, 1911
Using radioactive particle sources:
 α -particles of some MeV energy

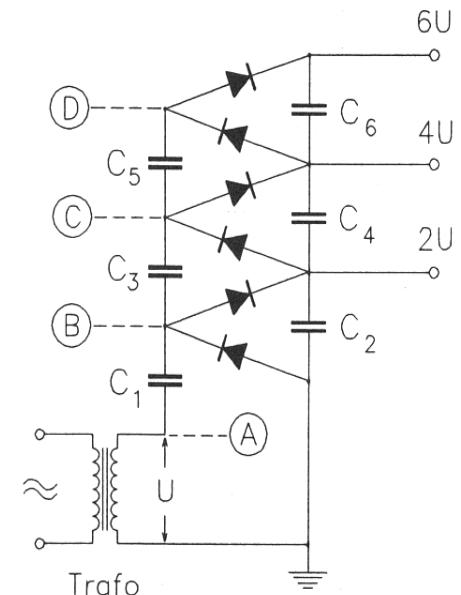
$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta/2)}$$



1.) Electrostatic Machines: The Cockcroft-Walton Generator

1928: Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam

1932: First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV



Particle source: Hydrogen discharge tube on 400 kV level

Accelerator: evacuated glas tube

Target: Li-Foil on earth potential

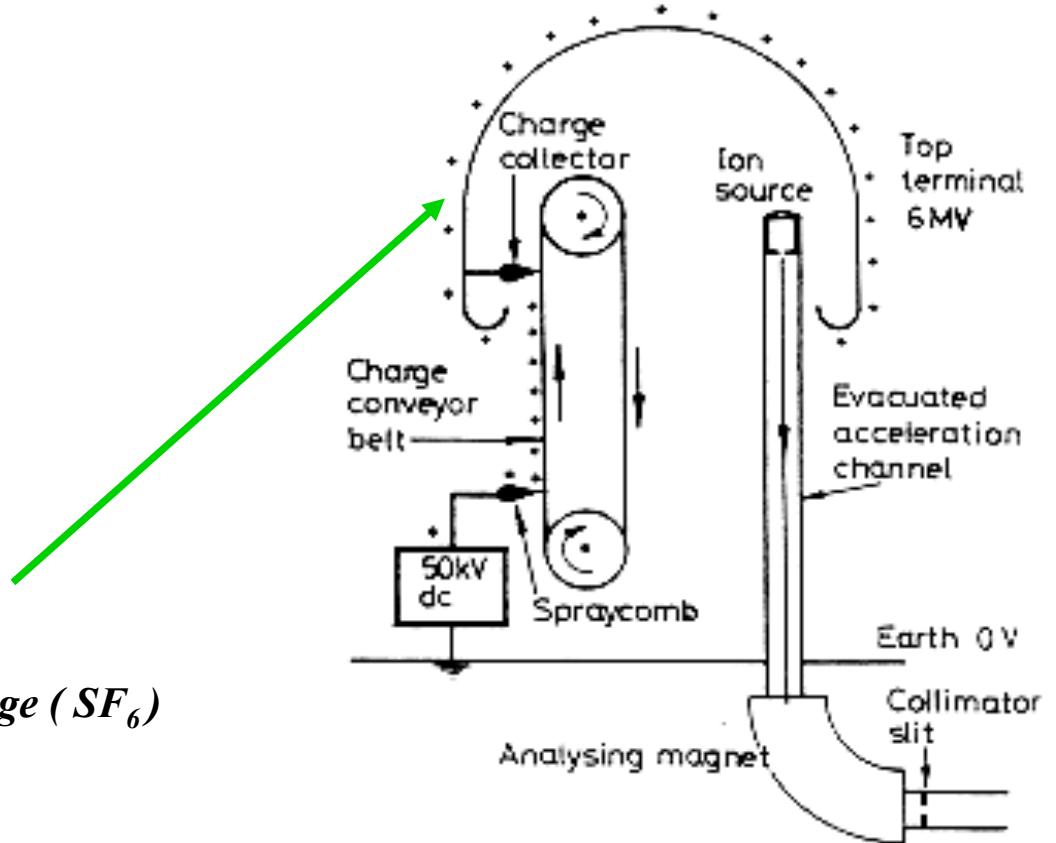
Technically: rectifier circuit, built of capacitors and diodes (Greinacher)

Problem:

DC Voltage can only be used once

2.) Electrostatic Machines: (Tandem -) van de Graaff Accelerator (1930 ...)

*creating high voltages by mechanical
transport of charges*



* Terminal Potential: $U \approx 12 \dots 28 \text{ MV}$
using high pressure gas to suppress discharge (SF_6)

Problems:

- * Particle energy limited by high voltage discharges
- * high voltage **can only be applied once per particle ...**
... or twice ?

The „Tandem principle“: Apply the accelerating voltage twice ...

*... by working with negative ions (e.g. H^-) and
stripping the electrons in the centre of the
structure*

Example for such a „steam engine“: 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg



Gretchen Frage (J.W. Goethe, Faust)

Fallen die Dinger eigentlich runter ?

Antwort: JA !!

Gretchen Frage (J.W. Goethe, Faust)

Do they actually drop ?

Yes, they do !!

$$l_{VdG} = 30m$$

$$v \approx 10\% c \approx 3 * 10^7 m/s$$

$$\Delta t = 1\mu s$$

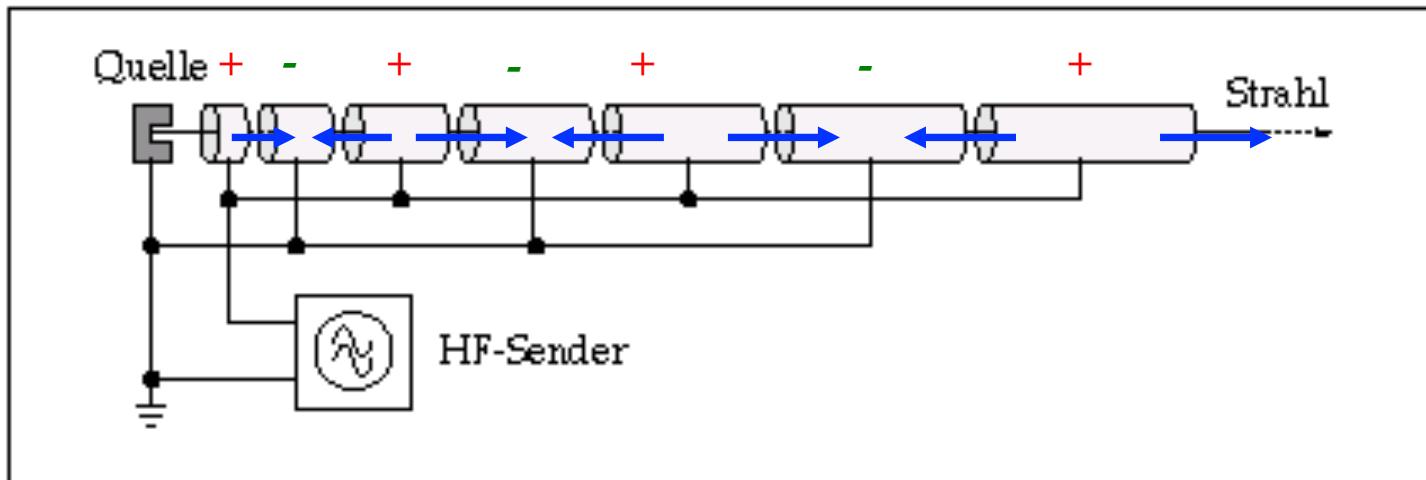
Free Fall in Vacuum:

$$\begin{aligned}s &= \frac{1}{2} g t^2 \\&= \frac{1}{2} 10 \frac{m}{s^2} * (1\mu s)^2 \\&= 5 * 10^{-12} m = 5 pm\end{aligned}$$

3.) The first RF-Accelerator: „Linac“

1928, Wideroe: how can the acceleration voltage be applied several times to the particle beam

schematic Layout:



Energy gained after n acceleration gaps

$$E_n = n * q * U_0 * \sin \psi_s$$

n number of gaps between the drift tubes

q charge of the particle

U_0 Peak voltage of the RF System

Ψ_s synchronous phase of the particle

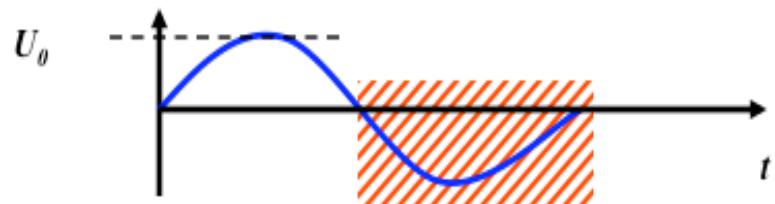
* acceleration of the proton in the first gap

* voltage has to be „flipped“ to get the right sign in the second gap → RF voltage

→ shield the particle in drift tubes during the negative half wave of the RF voltage

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



Time span of the negative half wave: $\tau_{RF}/2$

Length of the Drift Tube:

Kinetic Energy of the Particles

$$l_i = v_i * \frac{\tau_{rf}}{2}$$

$$E_i = \frac{1}{2}mv^2$$

$$\rightarrow v_i = \sqrt{2E_i/m}$$

$$l_i = \frac{1}{v_{rf}} * \sqrt{\frac{i * q * U_0 * \sin \psi_s}{2m}}$$

valid for non relativistic particles ...

Alvarez-Structure: 1946, surround the whole structure by a rf vessel

Energy: ≈ 20 MeV per Nucleon $\beta \approx 0.04 \dots 0.6$, Particles: Protons/Ions

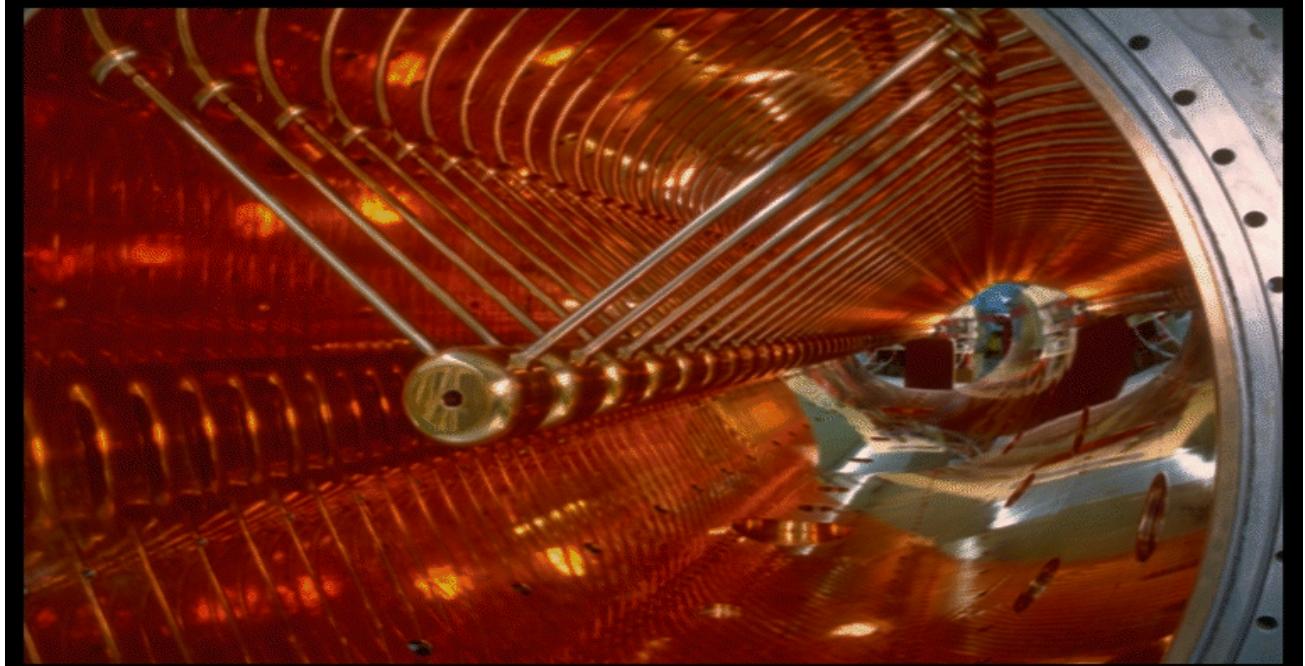
Accelerating structure of a Proton Linac (DESY Linac III)

$$E_{total} = 988 \text{ MeV}$$

$$m_0 c^2 = 938 \text{ MeV}$$

$$p = 310 \text{ MeV} / c$$

$$E_{kin} = 50 \text{ MeV}$$



Beam energies

1.) reminder of some relativistic formula

$$\text{rest energy} \quad E_0 = m_0 c^2$$

$$\text{total energy} \quad E = \gamma * E_0 = \gamma * m_0 c^2$$

$$\text{kinetic energy} \quad E_{kin} = E_{total} - m_0 c^2$$

Energy Gain per „Gap“:

$$W = q U_0 \sin \omega_{RF} t$$

$$\text{momentum} \quad E^2 = c^2 p^2 + m_0^2 c^4$$

3.) The Cyclotron: (Livingston / Lawrence ~1930)

Idea: $B = \text{const}$, $RF = \text{const}$

Synchronisation particle / RF via orbit

Lorentzforce

$$\vec{F} = q * (\vec{v} \times \vec{B}) = q * v * B$$

circular orbit

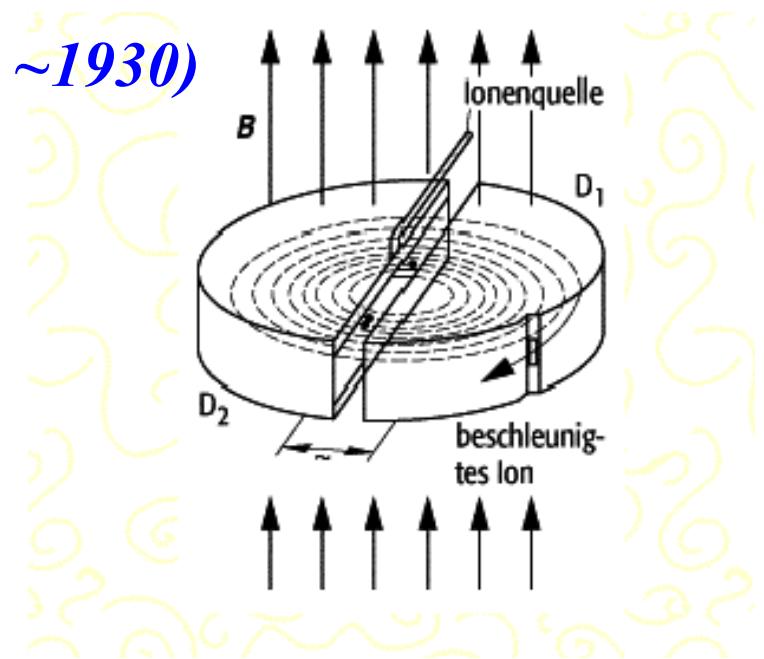
$$q * v * B = \frac{m * v^2}{R} \rightarrow B * R = p/q$$

revolution frequency

$$\omega_z = \frac{v}{R} = \frac{q}{m} * B_z$$

*the cyclotron (rf-) frequency
is independent of the momentum*

rf-frequency = $h * \text{revolution frequency}$, $h = \text{"harmonic number"}$



increasing radius for
increasing momentum
→ Spiral Trajectory

Cyclotron:

exact equation for revolution frequency:

$$\omega_z = \frac{v}{R} = \frac{q}{\gamma * m} * B_z$$

1.) if $v \ll c \Rightarrow \gamma \approx 1$

2.) γ increases with the energy
 \Rightarrow no exact synchronism

Syn

"synchronisation" with the spiraling orbit length



Cyclotron SPIRAL at GANIL

$B = \text{constant}$

$\gamma \omega_{RF} = \text{constant}$

ω_{RF} decreases with time

$$\omega_s(t) = \omega_{rf}(t) = \frac{q}{\gamma(t) * m_0} * B$$

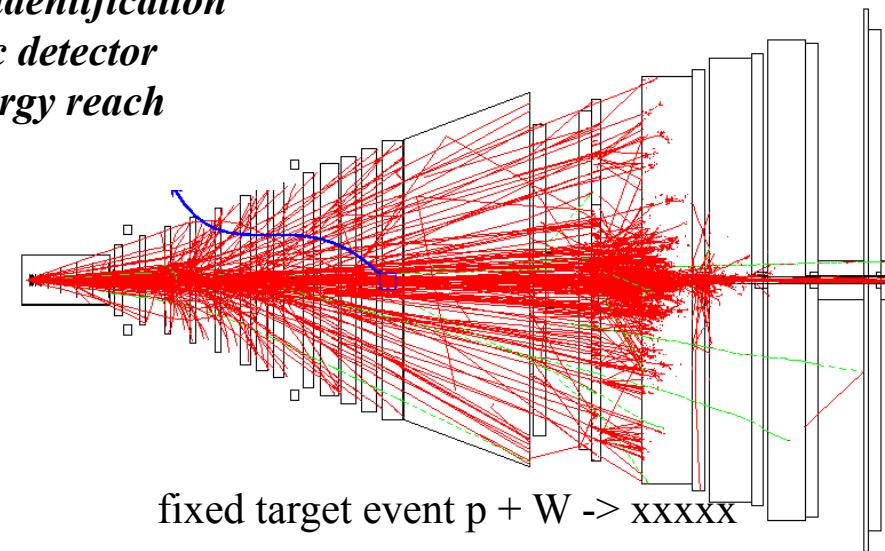
keep the synchronisation condition by varying the rf frequency

Fixed target experiments:



HARP Detector, CERN

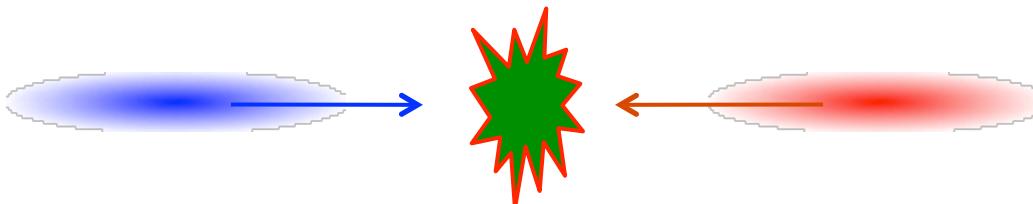
*high event rate
easy track identification
asymmetric detector
limited energy reach*



fixed target event $p + W \rightarrow \text{xxxxx}$

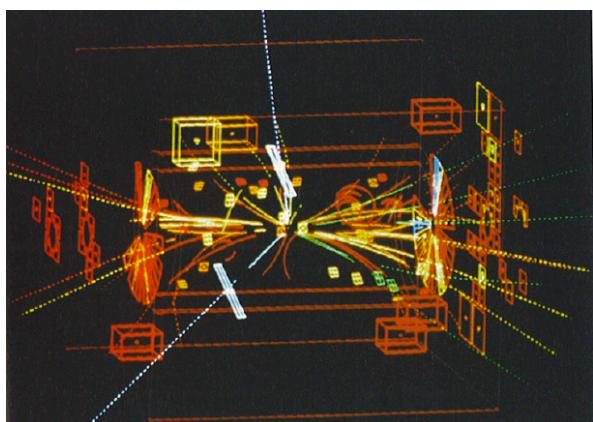
Collider experiments:

$$E=mc^2$$



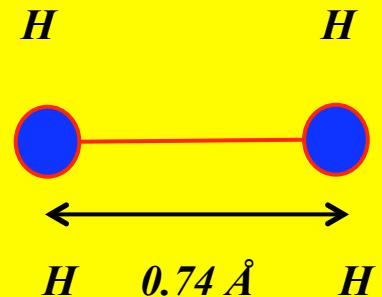
*low event rate (luminosity)
challenging track identification
symmetric detector*

$$E_{lab} = E_{cm}$$



Z_0 boson discovery at the UA2 experiment (CERN).
The Z_0 boson decays
into a e^+e^- pair, shown as white dashed lines.

Particle Density in matter

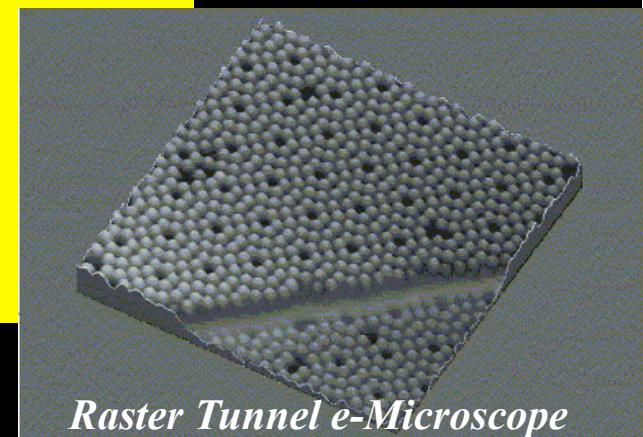


Atomic Distance in Hydrogen Molecule

$$R_B \approx 0.5 \text{ \AA}$$

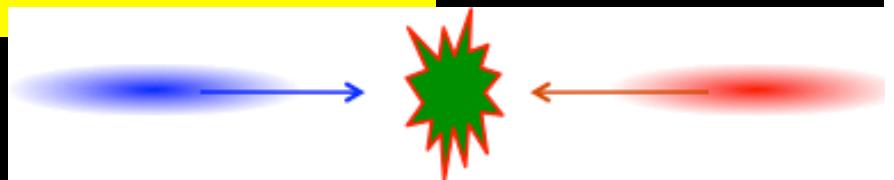
in solids / fluids $\lambda \approx 1 \dots 3 \text{ \AA}$

in gases $\lambda \approx 35 \text{ \AA} = 3.5 \text{ nm}$



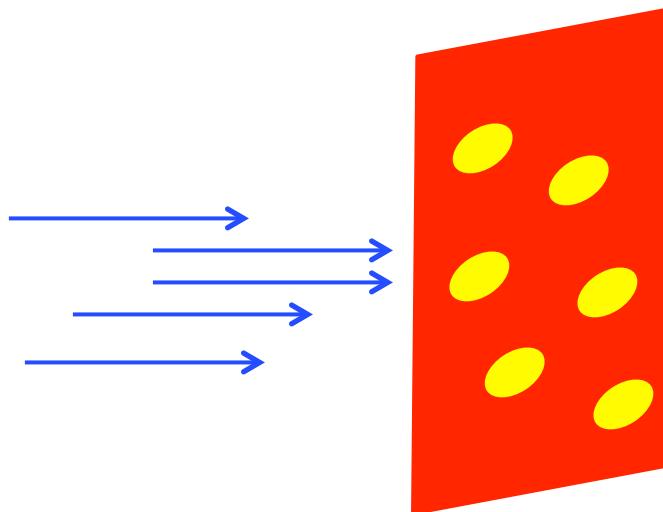
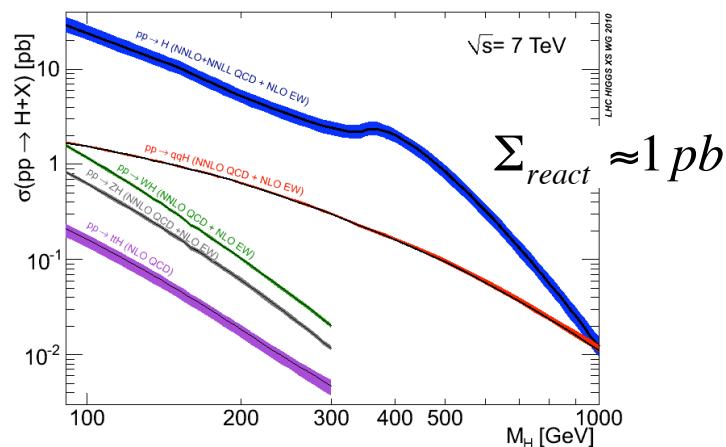
Raster Tunnel e-Microscope

Particle Distance in Accelerators: $\lambda \approx 600 \text{ nm (Arc)} \dots 300 \text{ nm (IP LEP)}$
 $= 3000 \text{ \AA}$



*Problem: Our particles are *VERY* small !!*

Overall cross section of the Higgs:

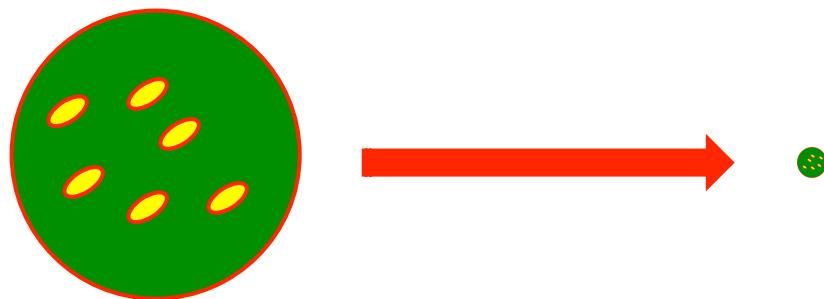


$$1b = 10^{-24} \text{ cm}^2$$

$$1pb = 10^{-12} * 10^{-24} \text{ cm}^2 = 1/\text{mio} * 1/\text{mio} * 1/\text{mio} * 1/\text{mio} * 1/\text{mio} * 1/10000 \text{ mm}^2$$

*The only chance we have:
compress the transverse beam size ... at the IP*

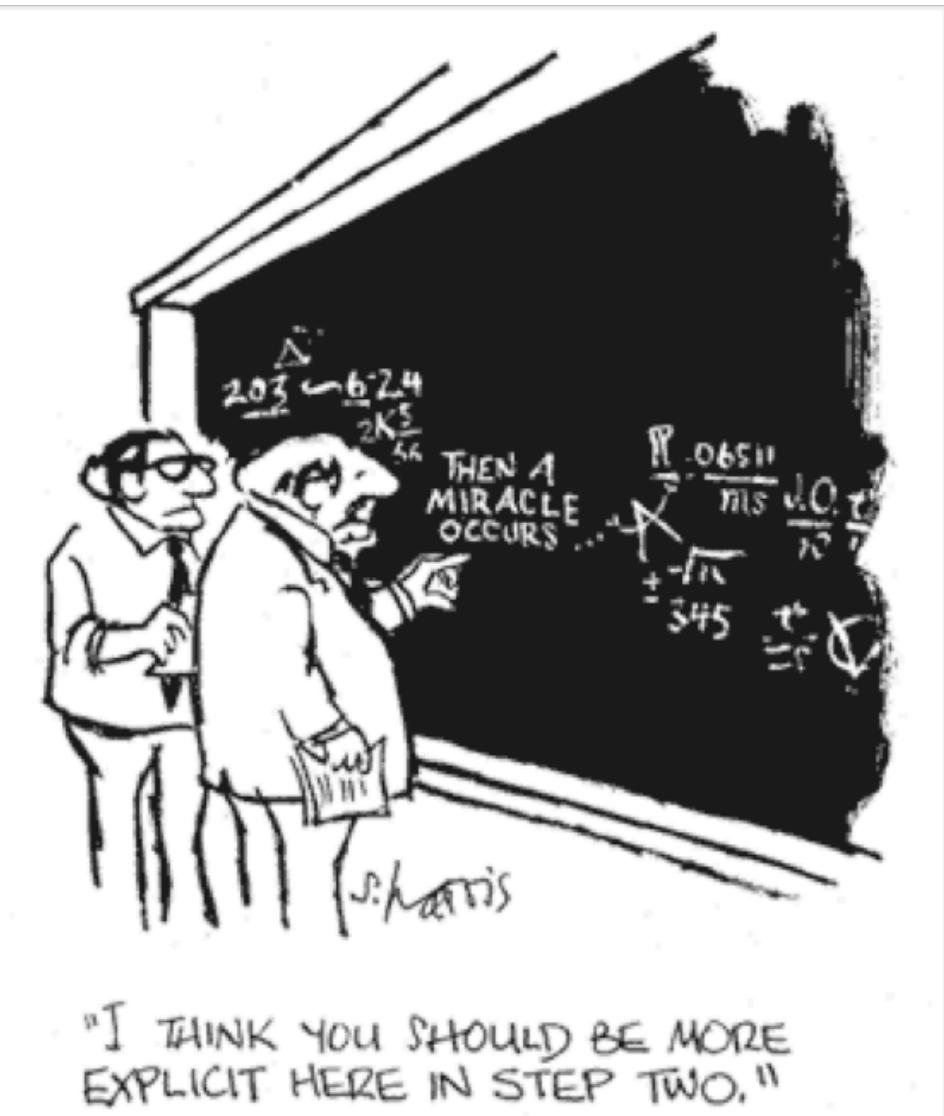
The particles are “very small”



*LHC typical:
 $\sigma = 0.1 \text{ mm} \rightarrow 16 \mu\text{m}$*

II.) A Bit of Theory

The big storage rings: „Synchrotrons“

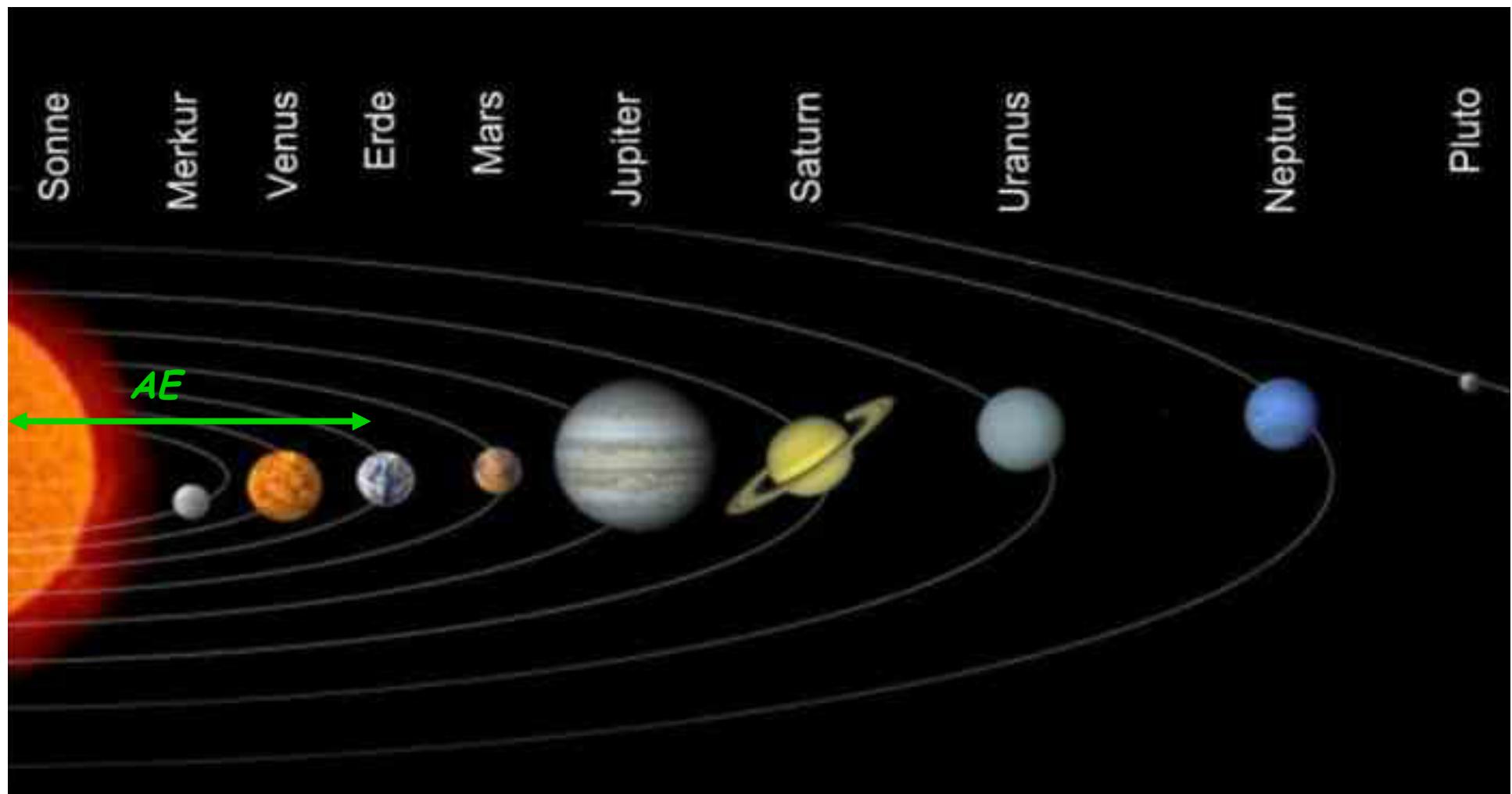


Largest storage ring: The Solar System

astronomical unit: average distance earth-sun

$$1\text{AE} \approx 150 * 10^6 \text{ km}$$

$$\text{Distance Pluto-Sun} \approx 40 \text{ AE}$$



1.) Introduction and Basic Ideas

,, ... in the end and after all it should be a kind of circular machine“
→ need transverse deflecting force

Lorentz force

$$\vec{F} = q * (\vec{E} + \vec{v} \times \vec{B})$$

typical velocity in high energy machines:

$$v \approx c \approx 3 * 10^8 \text{ m/s}$$

Example:

$$B = 1 \text{ T} \quad \rightarrow \quad F = q * 3 * 10^8 \frac{\text{m}}{\text{s}} * 1 \frac{\text{Vs}}{\text{m}^2}$$

$$F = q * 300 \underbrace{\frac{\text{MV}}{\text{m}}}_{\text{equivalent electrical field:}}$$

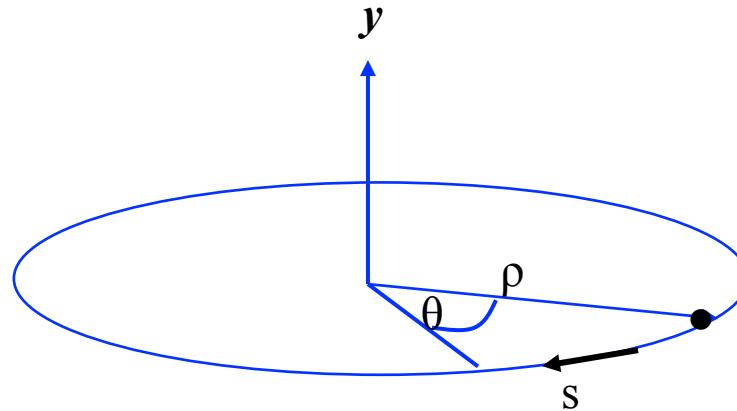
Technical limit for electrical fields:

$$E \leq 1 \frac{\text{MV}}{\text{m}}$$

Pearl of Wisdom:

if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit



circular coordinate system

condition for circular orbit:

Lorentz force

$$F_L = evB$$

centrifugal force

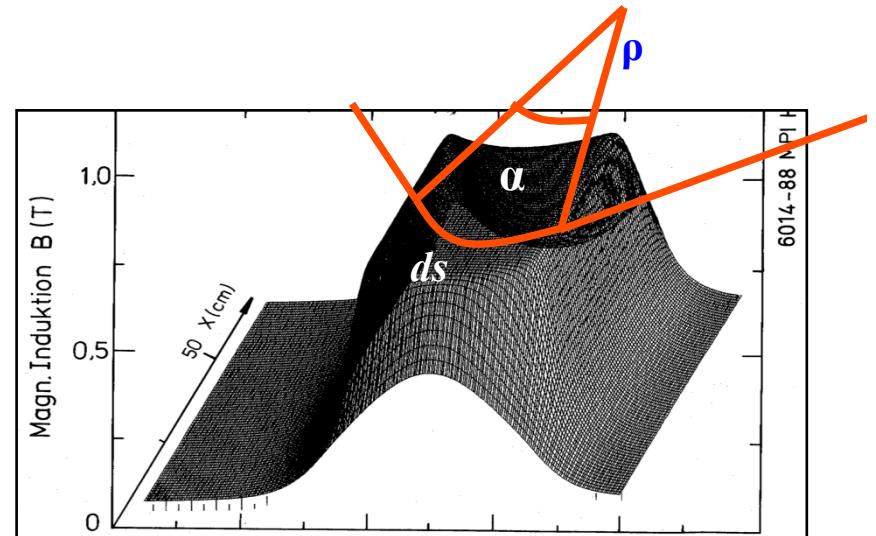
$$F_{centr} = \frac{\gamma m_0 v^2}{\rho}$$

$$\frac{\gamma m_0 v^2}{\rho} = evB$$

$$\frac{p}{e} = B\rho$$

Bρ = "beam rigidity"

The Magnetic Guide Field



field map of a storage ring dipole magnet

$$\rho = 2.8 \text{ km} \quad \longrightarrow \quad 2\pi\rho = 17.6 \text{ km}$$
$$\approx 66\%$$

$$B \approx 1 \dots 8 \text{ T}$$

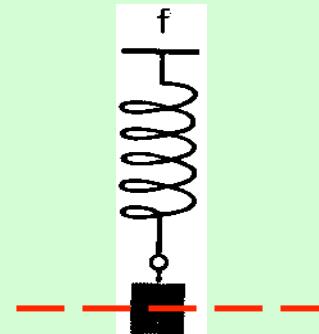
rule of thumb:

$$\frac{1}{\rho} \approx 0.3 \frac{B[T]}{p[GeV/c]}$$

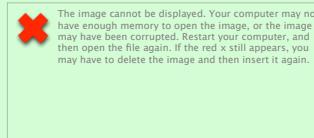
„normalised bending strength“

Focusing Properties and Quadrupole Magnets

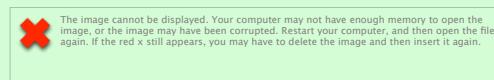
classical mechanics:
pendulum



there is a **restoring force**, proportional
to the elongation x :



general solution: free harmonic oszillation



this is how grandma's Kuckuck's clock is working!!!

Storage Rings: **linear increasing Lorentz force** to keep trajectories in vicinity of
the ideal orbit
linear increasing magnetic field

$$B_y = g \cdot x \quad B_x = g \cdot y$$



LHC main quadrupole magnet $g \approx 25 \dots 220 \text{ T/m}$

Focusing forces and particle trajectories:

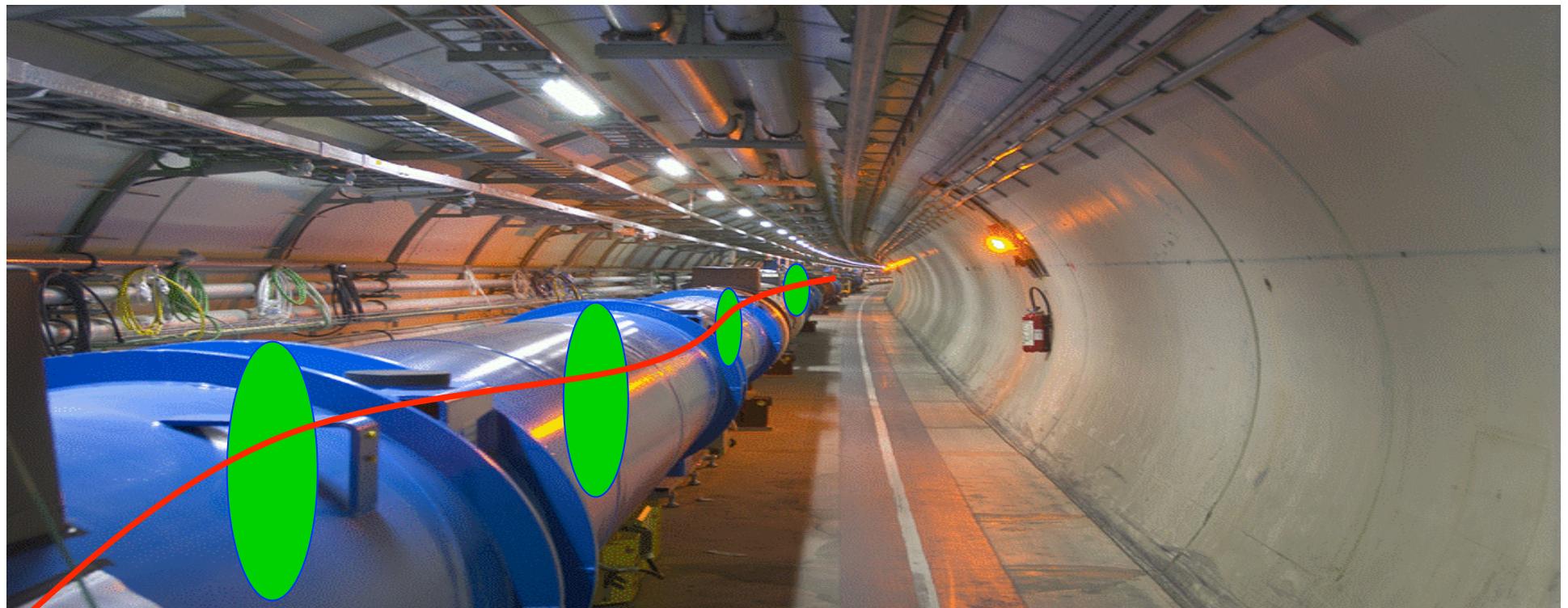
*normalise magnet fields to momentum
(remember: $B^*p = p / q$)*

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

Quadrupole Magnet

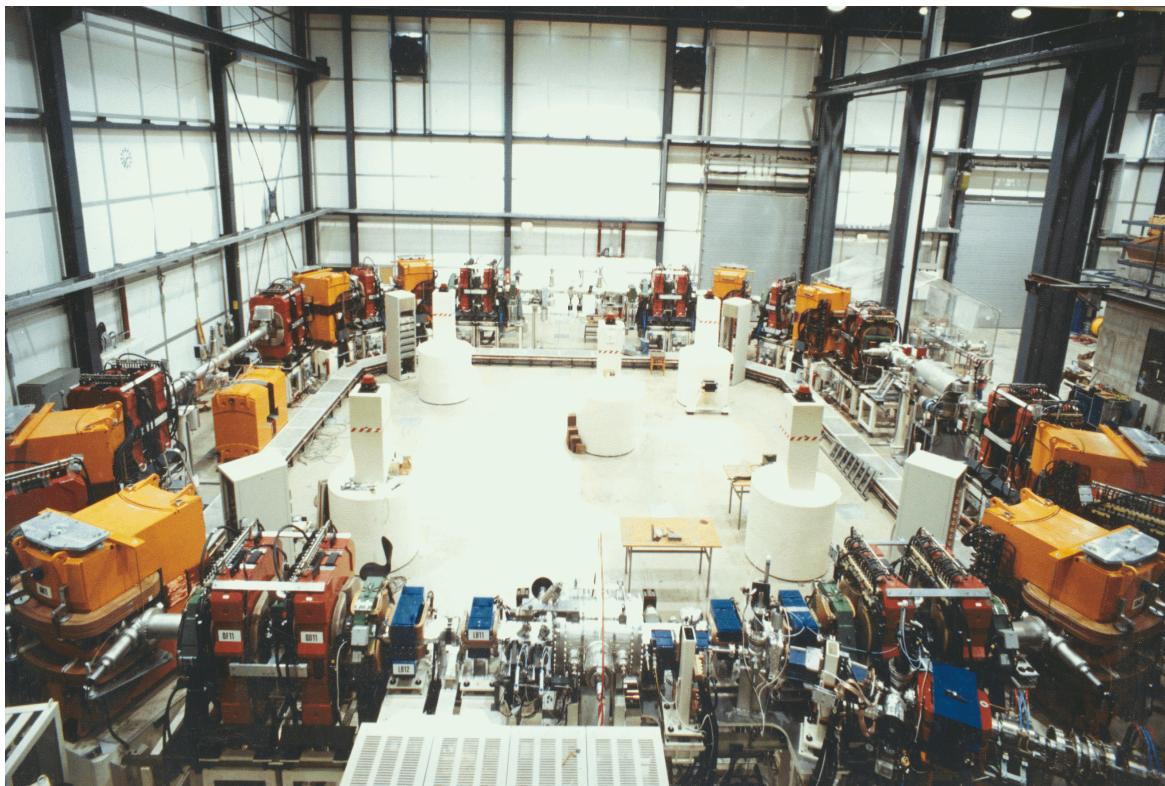
$$k := \frac{g}{p/q}$$



3.) The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k x + \cancel{\frac{1}{2!} m x^2} + \cancel{\frac{1}{3!} n x^3} + \dots$$

*only terms linear in x, y taken into account dipole fields
quadrupole fields*



Separate Function Machines:

Split the magnets and optimise them according to their job:

bending, focusing etc

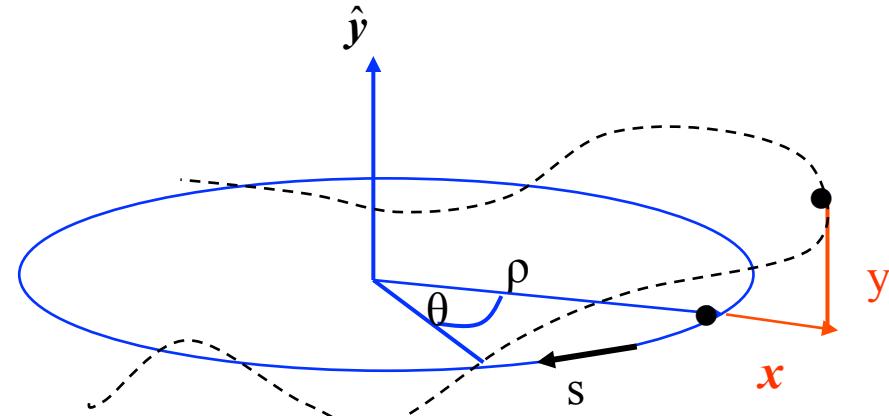
Example:
heavy ion storage ring TSR

 *man sieht nur
dipoles und quads → linear*

The Equation of Motion:

- * Equation for the horizontal motion:

$$x'' + x \left(\frac{1}{\rho^2} + k \right) = 0$$



x = particle amplitude

x' = angle of particle trajectory (wrt ideal path line)

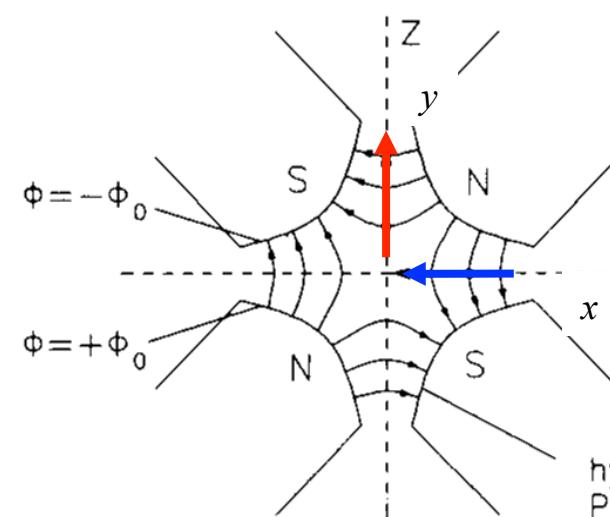
- * Equation for the vertical motion:

$$\frac{1}{\rho^2} = 0$$

no dipoles ... in general ...

$k \leftrightarrow -k$ quadrupole field changes sign

$$y'' - k y = 0$$



4.) Solution of Trajectory Equations

Define ... hor. plane: $K = 1/\rho^2 + k$
 ... vert. Plane: $K = -k$

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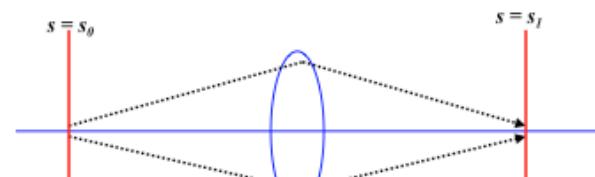
$$x'' + K x = 0$$

Differential Equation of harmonic oscillator ... with **spring constant K**

Ansatz: **Hor. Focusing Quadrupole $K > 0$:**

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x'_0 \cdot \cos(\sqrt{|K|}s)$$



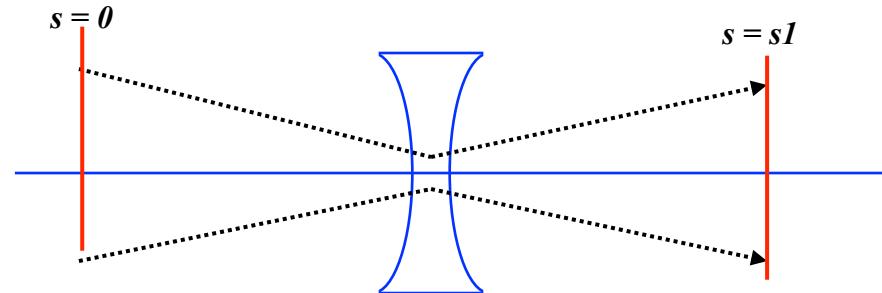
For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|}l) & \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}l) \\ -\sqrt{|K|} \sin(\sqrt{|K|}l) & \cos(\sqrt{|K|}l) \end{pmatrix}$$

hor. defocusing quadrupole:

$$x'' - K x = 0$$



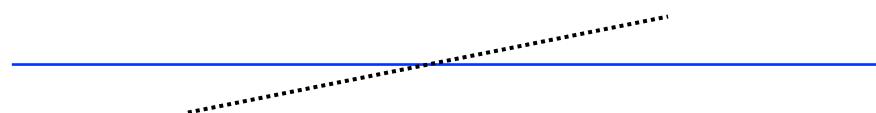
Ansatz: Remember from school

$$x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$$

$$M_{defoc} = \begin{pmatrix} \cosh \sqrt{|K|}l & \frac{1}{\sqrt{|K|}} \sinh \sqrt{|K|}l \\ \sqrt{|K|} \sinh \sqrt{|K|}l & \cosh \sqrt{|K|}l \end{pmatrix}$$

drift space:

$$K = 0$$



$$x(s) = x'_0 * s$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

! *with the assumptions made, the motion in the horizontal and vertical planes are independent „... the particle motion in x & y is uncoupled“*

*Ok ... ok ... it's a bit complicated and cosh and sinh and all that is a pain.
BUT ... compare ...*

Weak Focusing / Strong Focusing

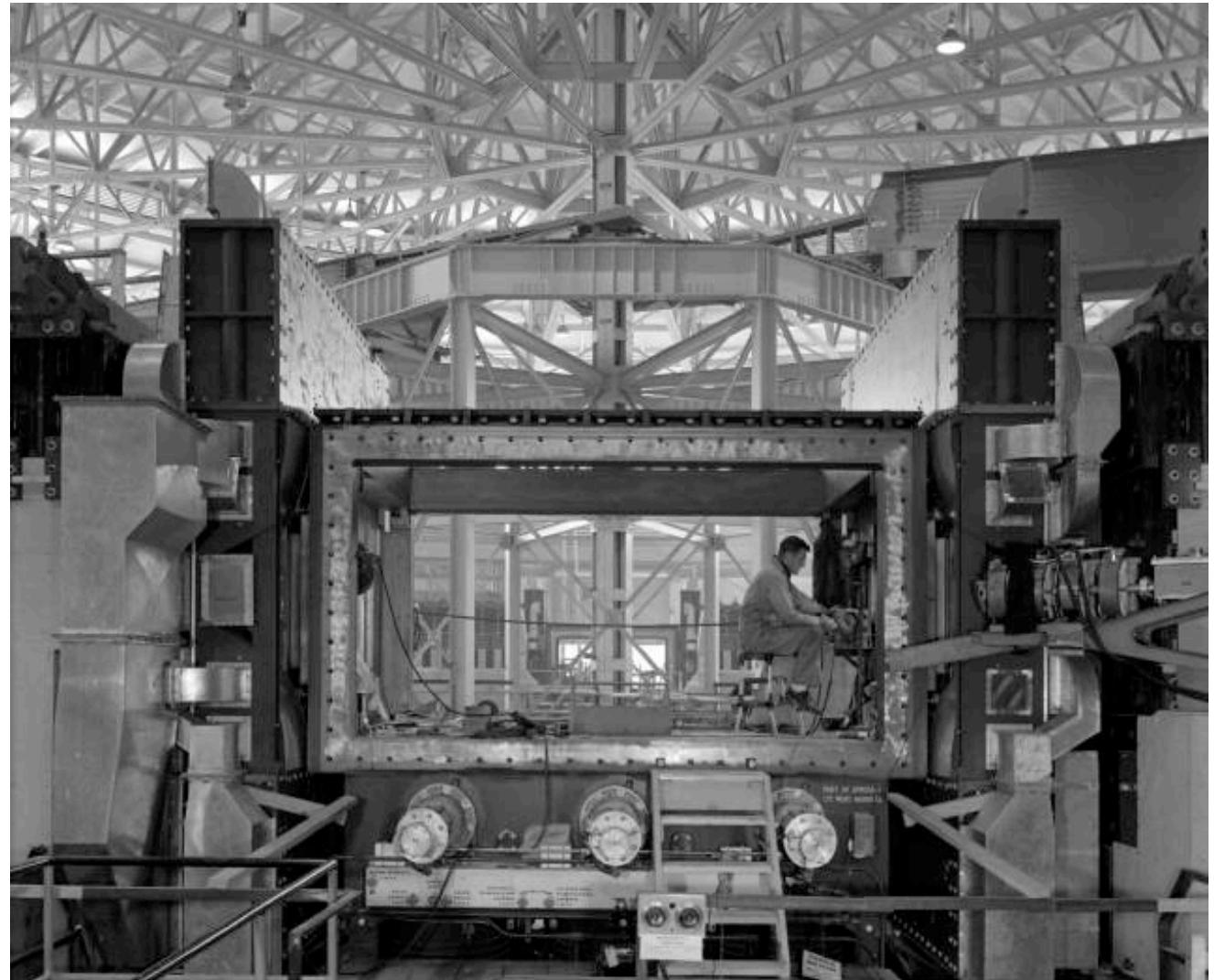
weak focusing term = $1/\rho^2$

$$x'' + x \left(\frac{1}{\rho^2} + k \right) = 0$$

*Problem: the higher the energy,
the larger the machine*

*The last weak focusing
high energy machine ...
BEVATRON*

- large apertures needed
- very expensive magnets

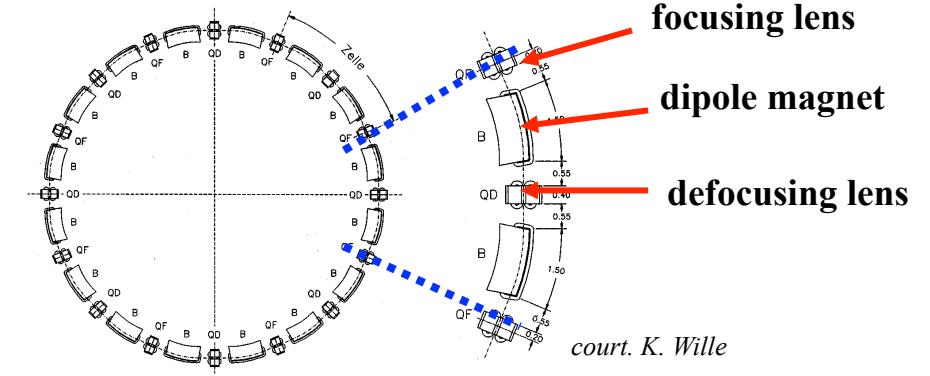


Transformation through a system of lattice elements

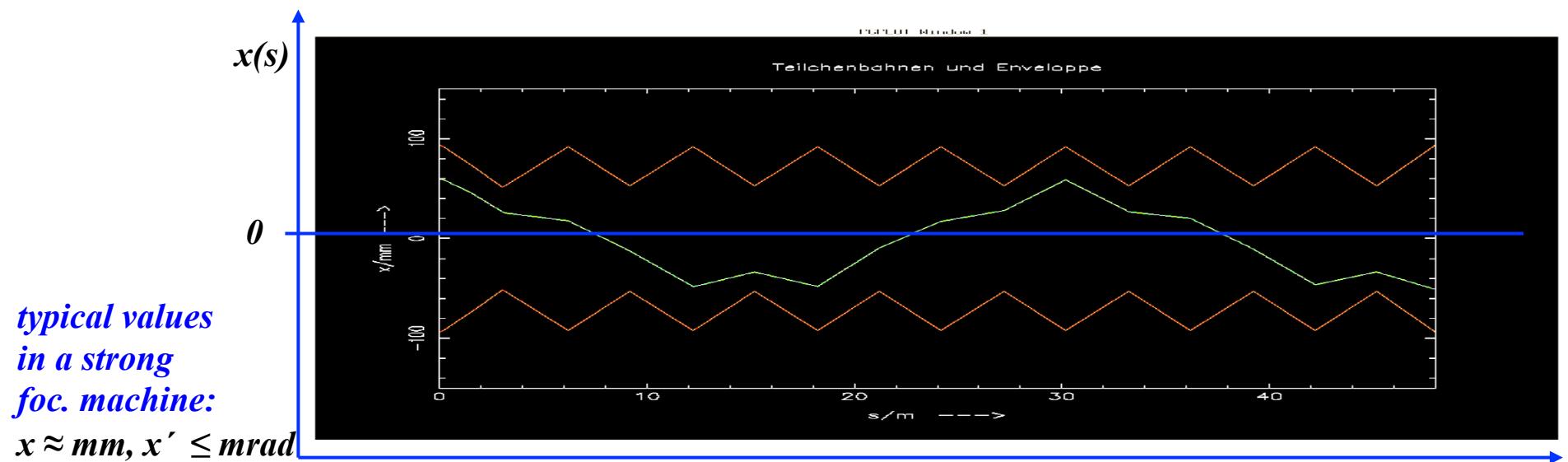
combine the single element solutions by multiplication of the matrices

$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_D * \dots$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



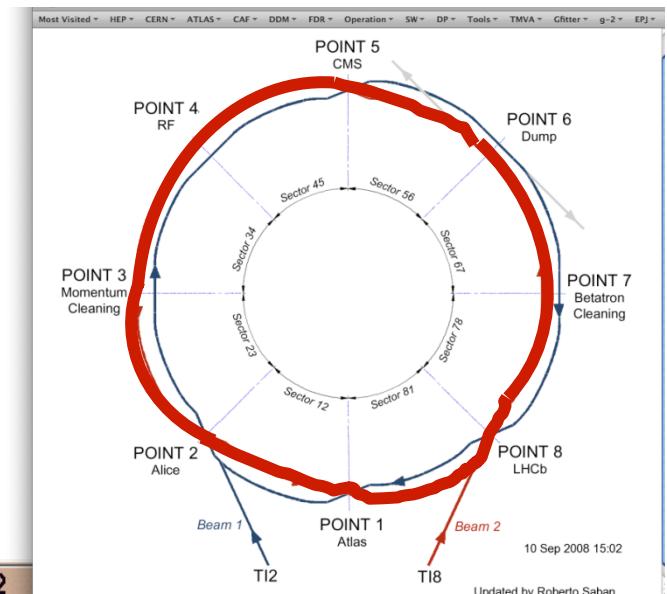
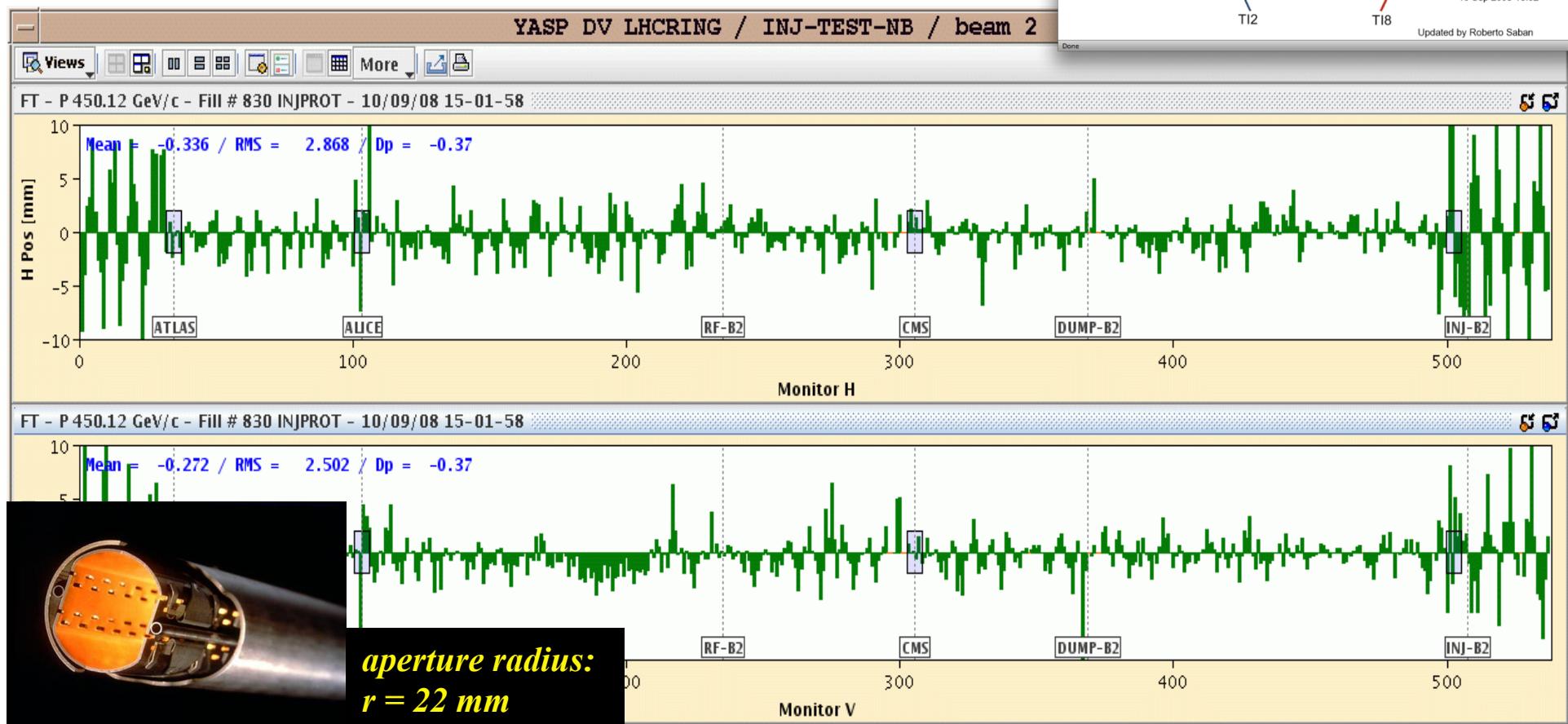
in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „,



LHC Operation: Beam Commissioning

The transverse focusing fields create a harmonic oscillation of the particles with a well defined “Eigenfrequency” which is called **tune**

First turn steering "by sector:"



“Once more unto the breach, dear friends, once more”
(W. Shakespeare, Henry 5)

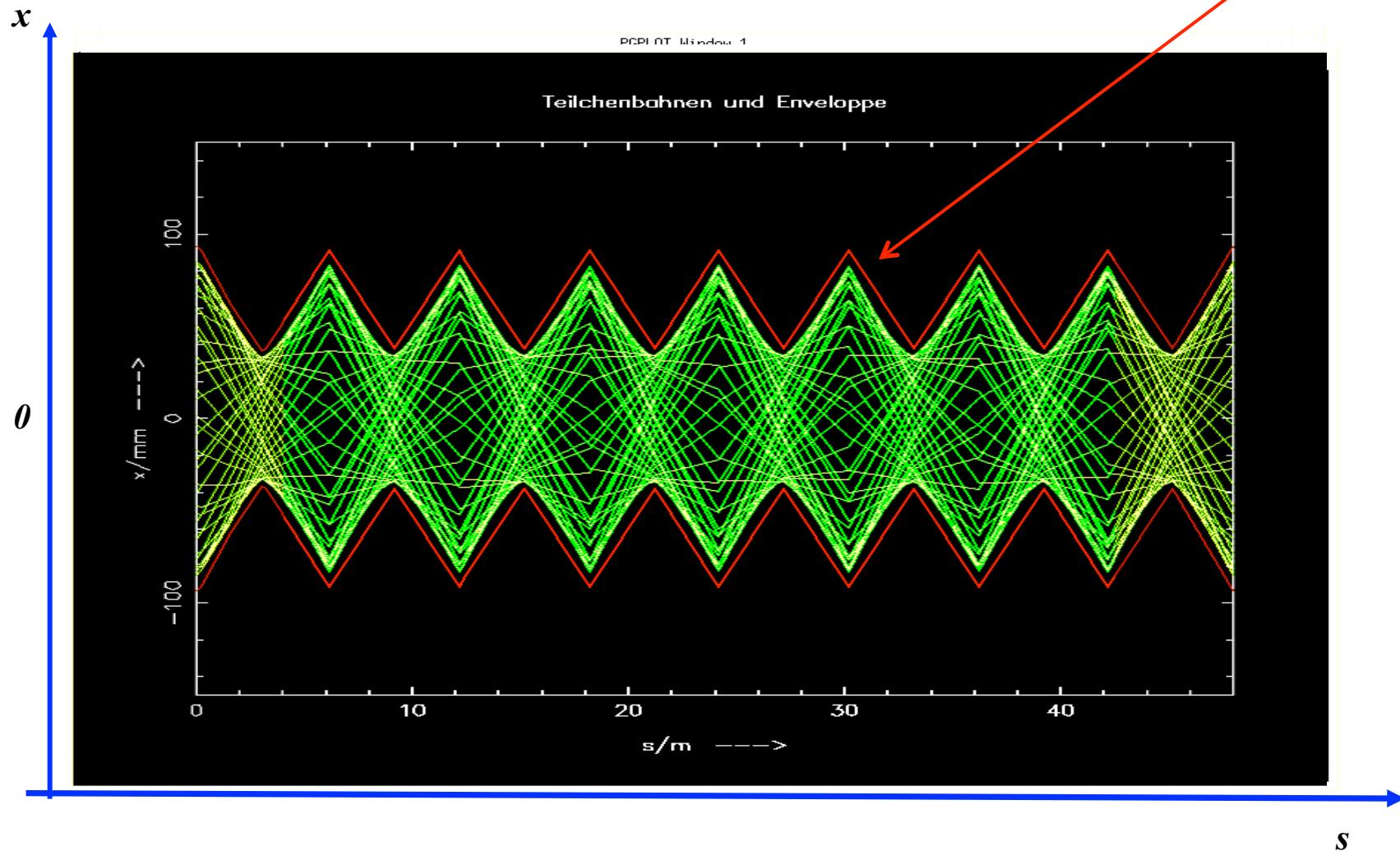
“Do they actually drop ?”

Answer: No

Question: what will happen, if the particle performs a second turn ?

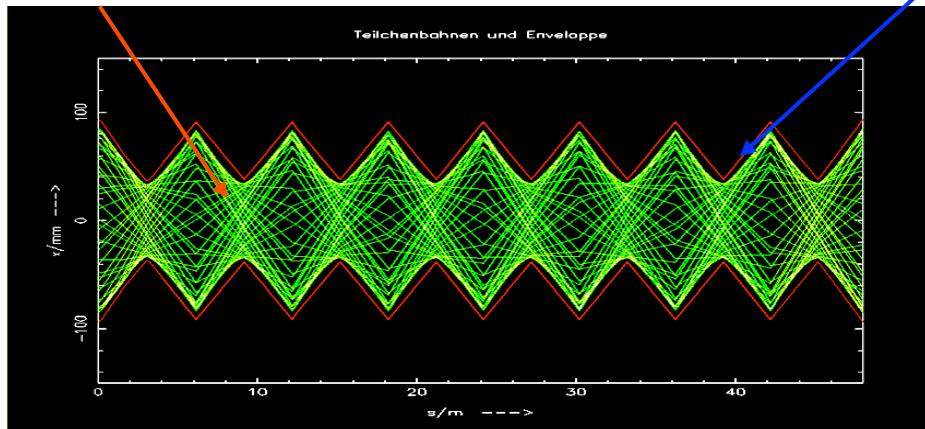
... or a third one or ... 10^{10} turns

$$\sigma = \sqrt{\varepsilon \beta}$$



Emittance of the Particle Ensemble:

$$x(s) = \sqrt{\varepsilon} \sqrt{\beta(s)} \cdot \cos(\Psi(s) + \phi) \quad \hat{x}(s) = \sqrt{\varepsilon} \sqrt{\beta(s)}$$



single particle trajectories, $N \approx 10^{11}$ per bunch

Gauß
Particle Distribution:

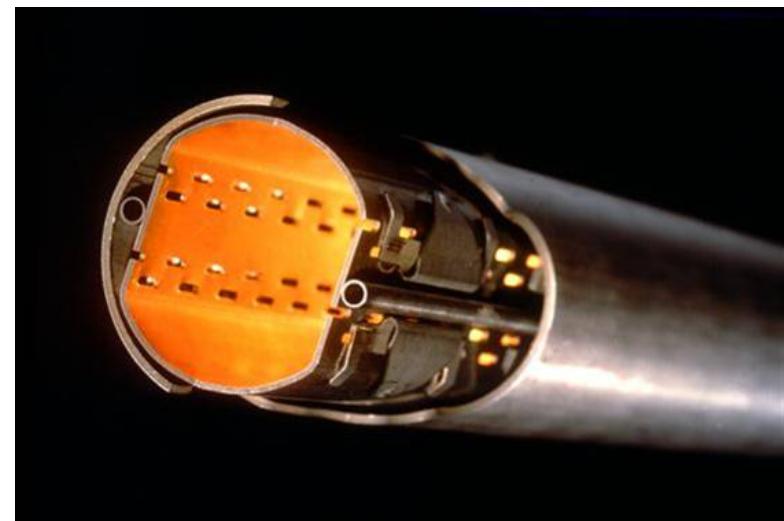
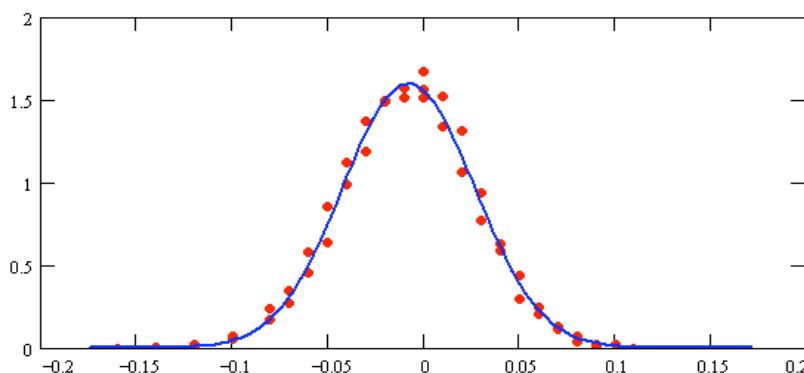
$$\rho(x) = \frac{N \cdot e}{\sqrt{2\pi}\sigma_x} \cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$$

particle at distance 1σ from centre
 ↔ 68.3 % of all beam particles

LHC: $\beta = 180\text{ m}$

$$\varepsilon = 5 * 10^{-10} \text{ m rad}$$

$$\sigma = \sqrt{\varepsilon * \beta} = \sqrt{5 * 10^{-10} \text{ m} * 180 \text{ m}} = 0.3 \text{ mm}$$

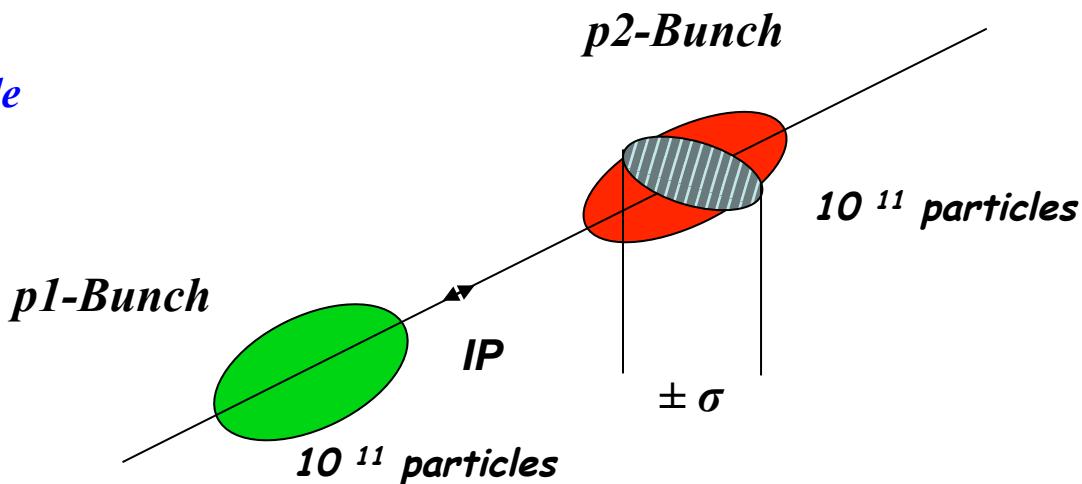


aperture requirements: $r_0 = 17 * \sigma$

5.) Luminosity

Ereignis Rate: "Physik" pro Sekunde

$$R = L * \Sigma_{react}$$



Example: Luminosity run at LHC

$$\beta_{x,y} = 0.55 \text{ m}$$

$$f_0 = 11.245 \text{ kHz}$$

$$\varepsilon_{x,y} = 5 * 10^{-10} \text{ rad m}$$

$$n_b = 2808$$

$$\sigma_{x,y} = 17 \mu\text{m}$$

$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

$$I_p = 584 \text{ mA}$$

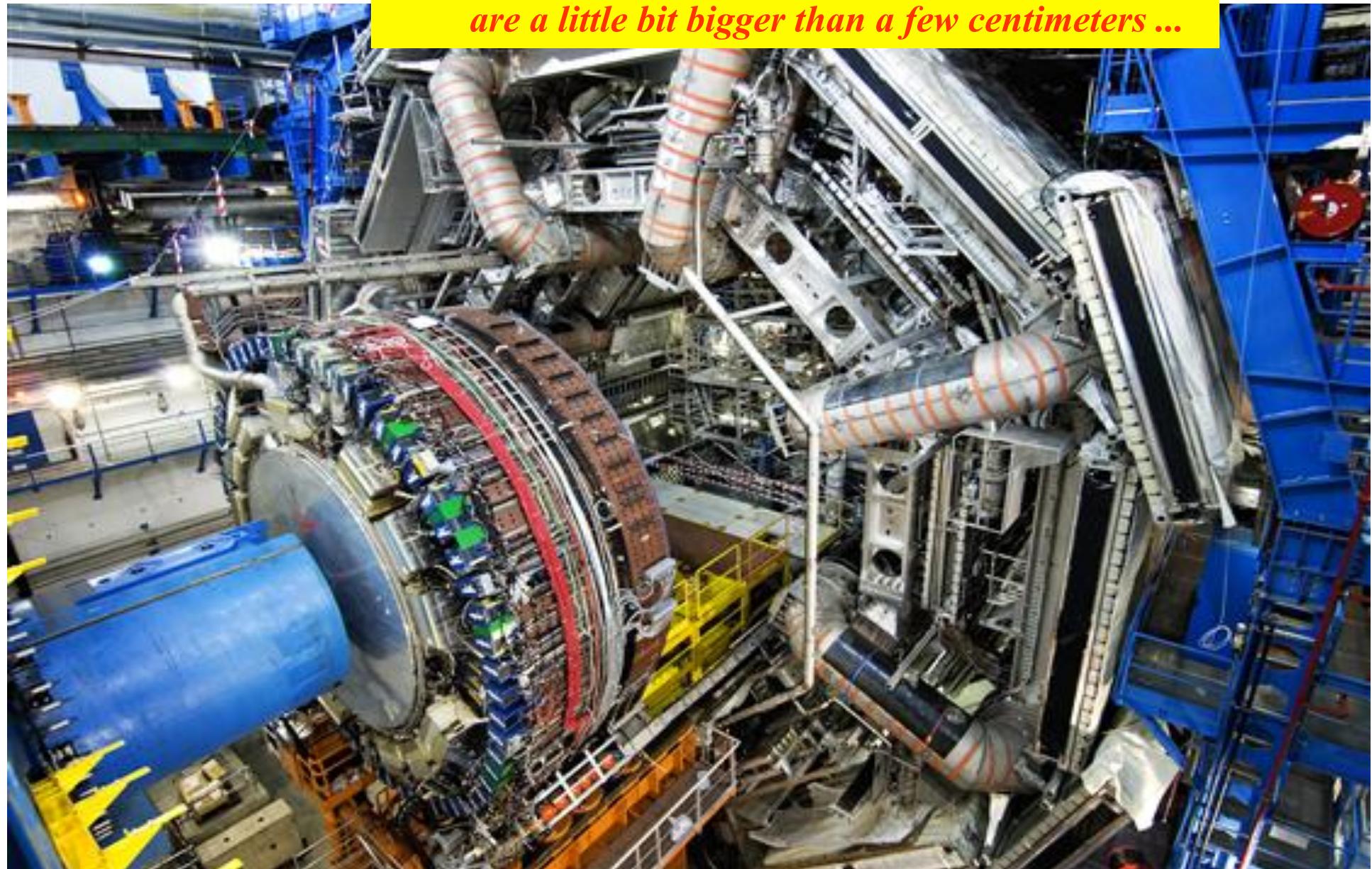
$$L = 1.0 * 10^{34} \frac{1}{\text{cm}^2 \text{s}}$$



beam sizes in the order of my cat's hair !!

... clearly there is an

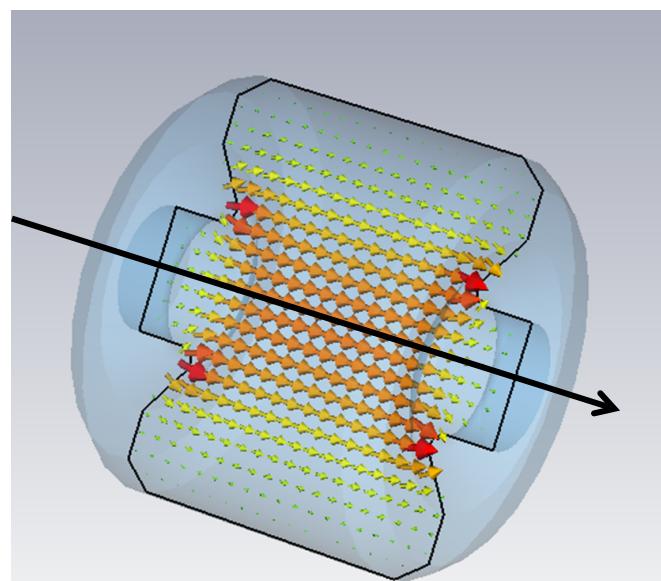
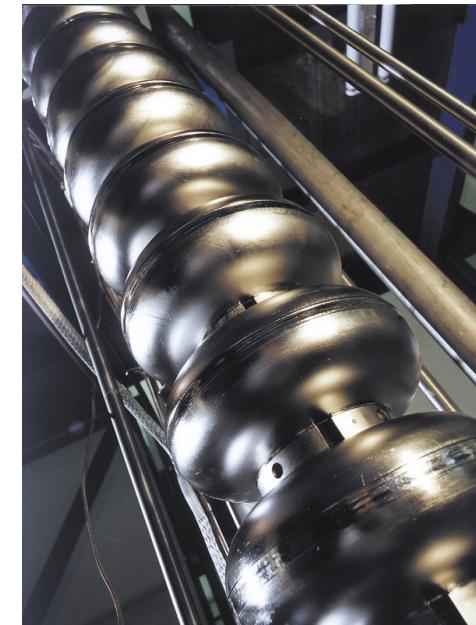
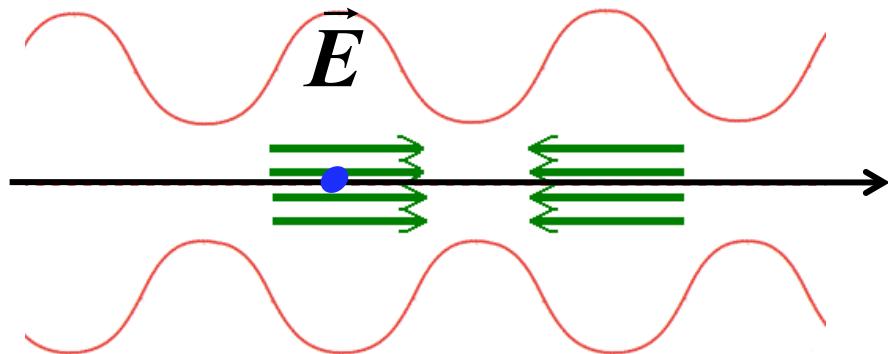
*... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...*



III. The Acceleration

Where is the acceleration?

Install an RF accelerating structure in the ring:



*B. Salvant
N. Biancacci*

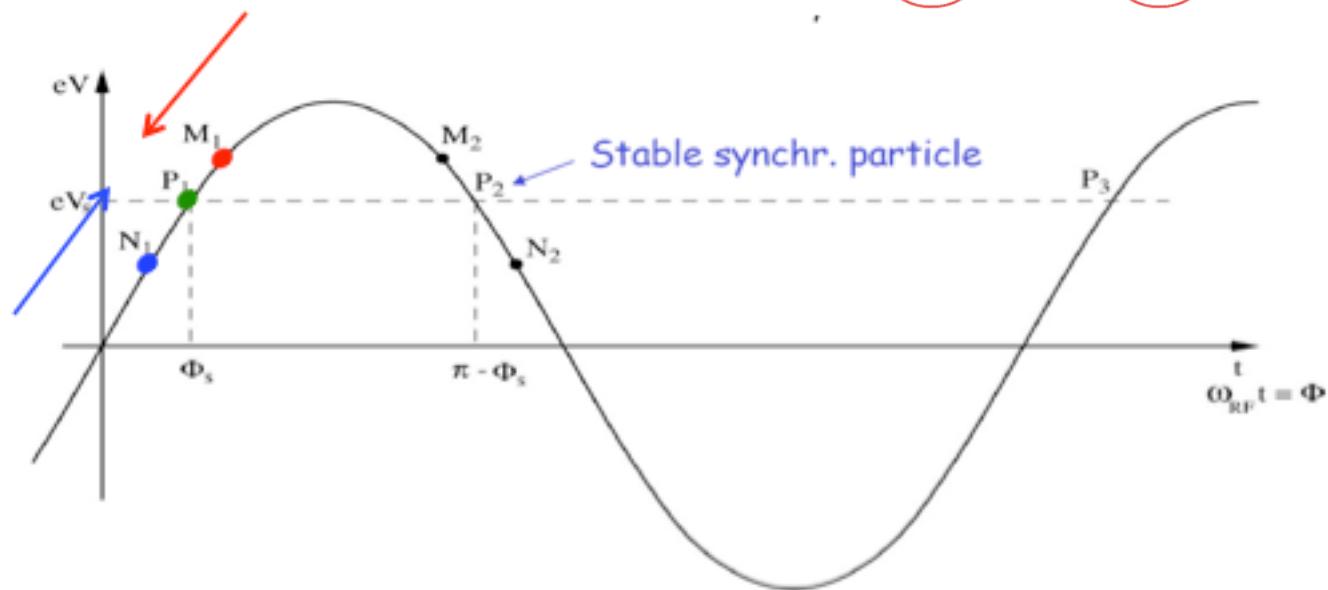
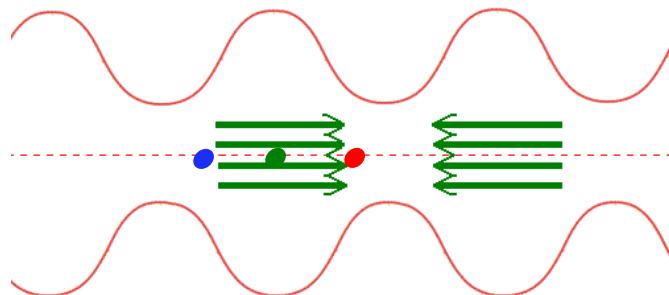
The Acceleration & "Phase Focusing"

$\Delta p/p \neq 0$ below transition

ideal particle •

particle with $\Delta p/p > 0$ • faster

particle with $\Delta p/p < 0$ • slower



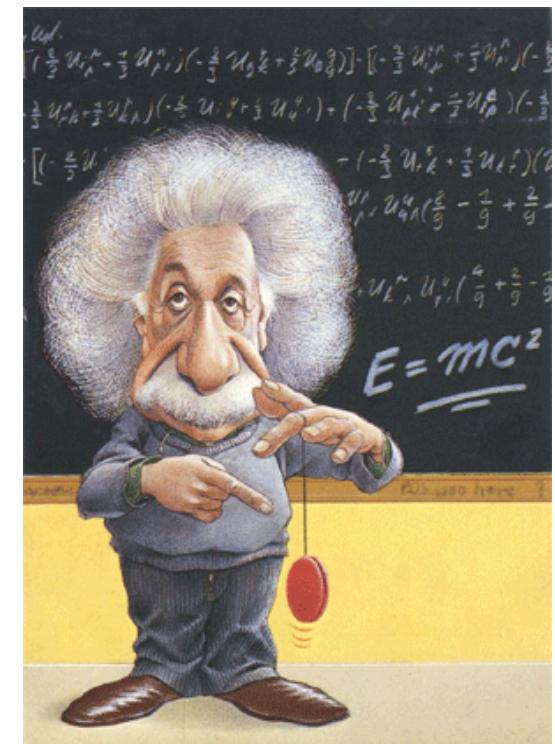
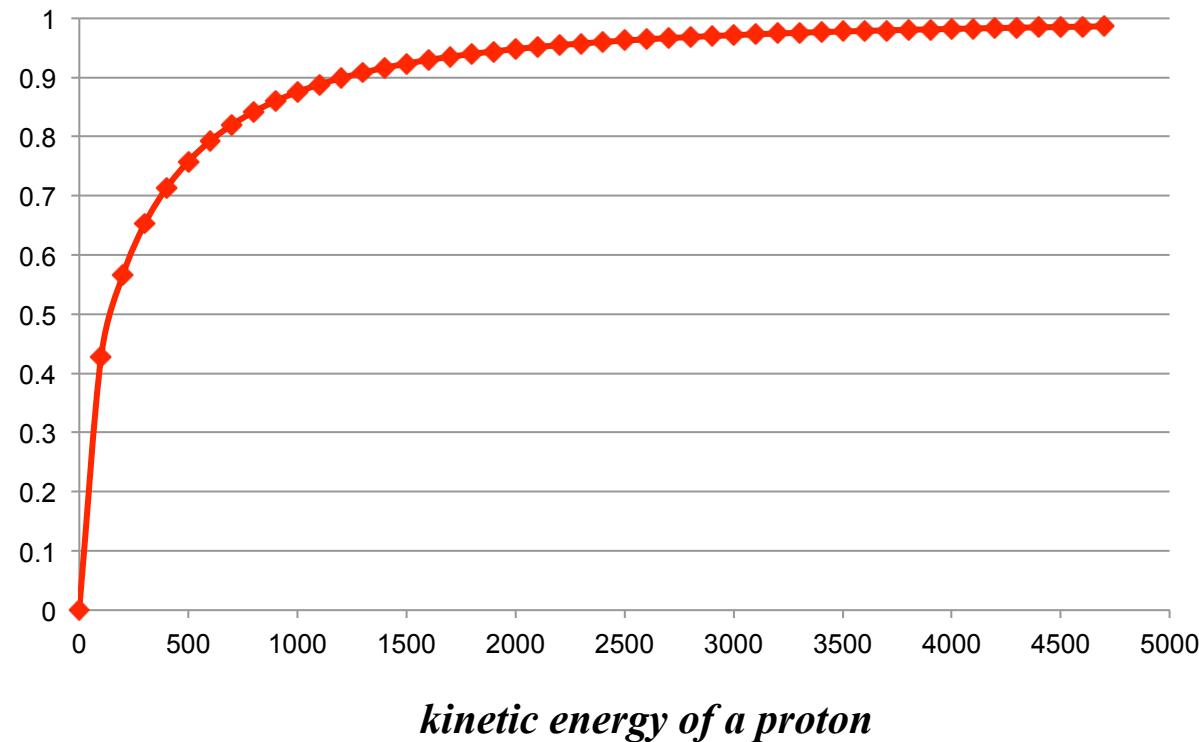
Focussing effect in the longitudinal direction
keeping the particles close together
... forming a “**bunch**”

oscillation frequency: $f_s = f_{rev} \sqrt{-\frac{h\alpha_s}{2\pi} * \frac{qU_0 \cos \phi_s}{E_s}}$ $\approx \text{some Hz}$

... so sorry, here we need help from Albert:

$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \longrightarrow \quad \frac{v}{c} = \sqrt{1 - \frac{mc^2}{E^2}}$$

v/c



*... some when the particles
do not get faster anymore*

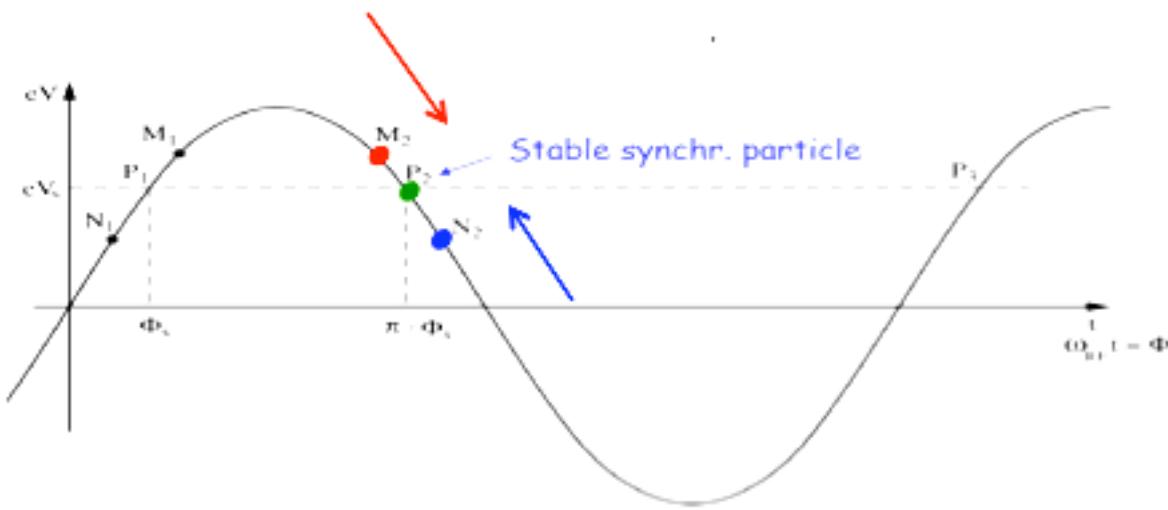
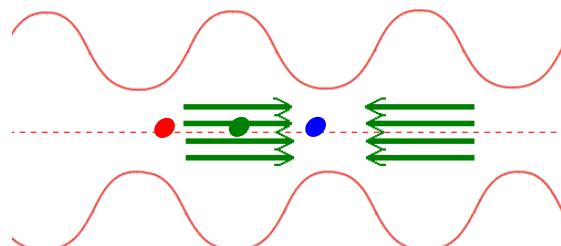
.... but heavier !

The Acceleration above transition

ideal particle •

particle with $\Delta p/p > 0$ • *heavier*

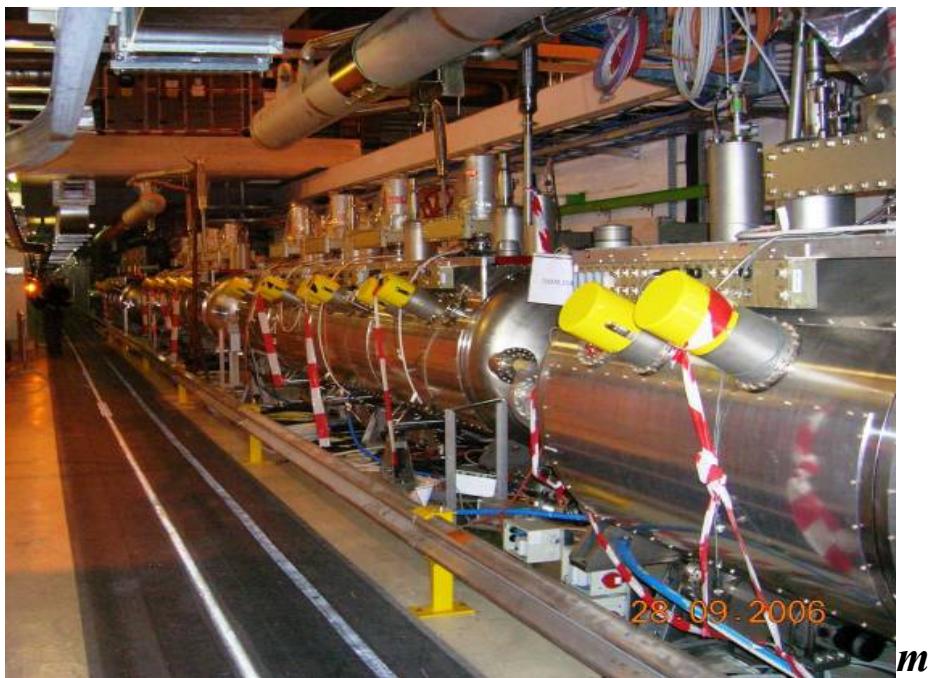
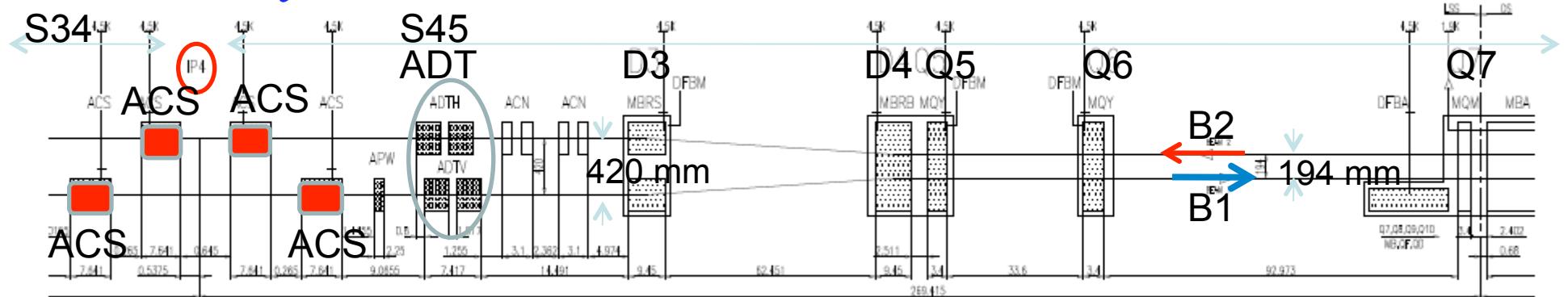
particle with $\Delta p/p < 0$ • *lighter*



*Focussing effect in the longitudinal direction
keeping the particles close together ... forming a “**bunch**”*

*... and how do we accelerate now ???
with the dipole magnets !*

The RF system: IR4



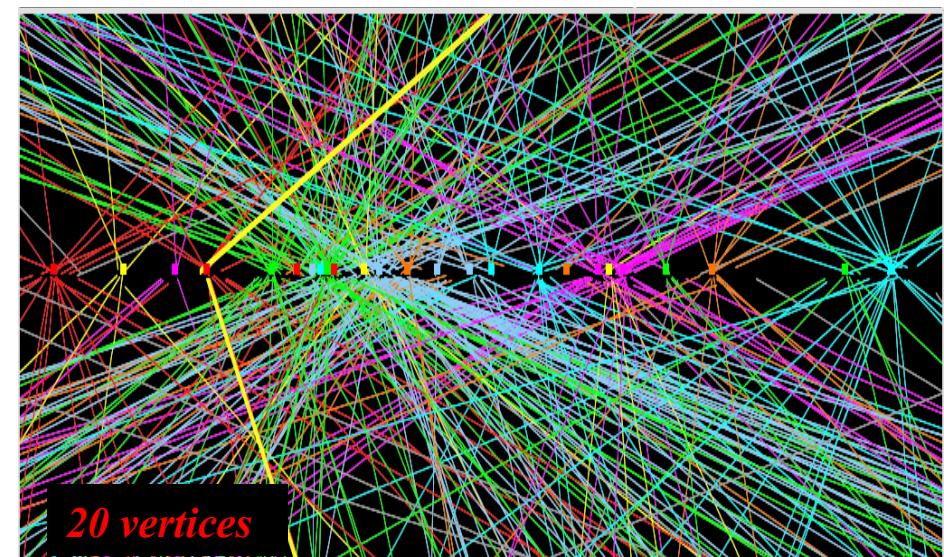
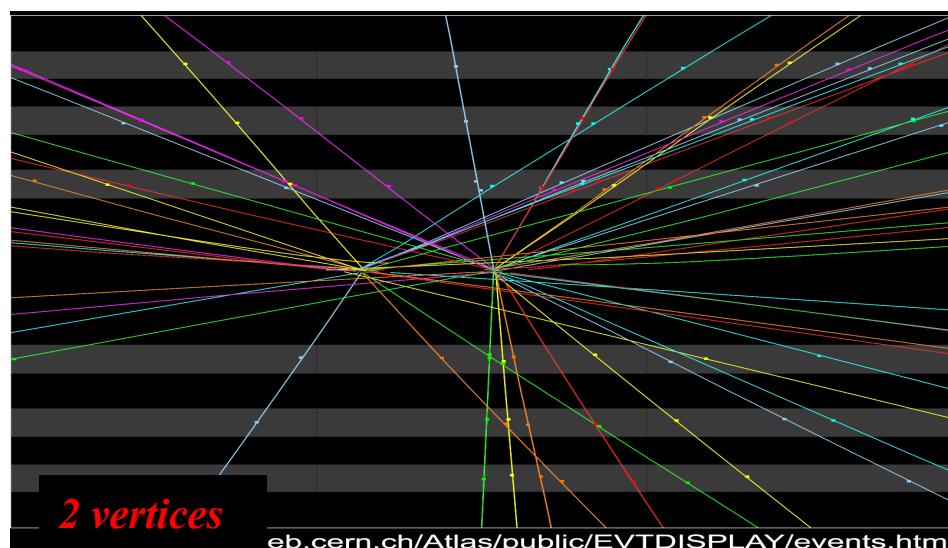
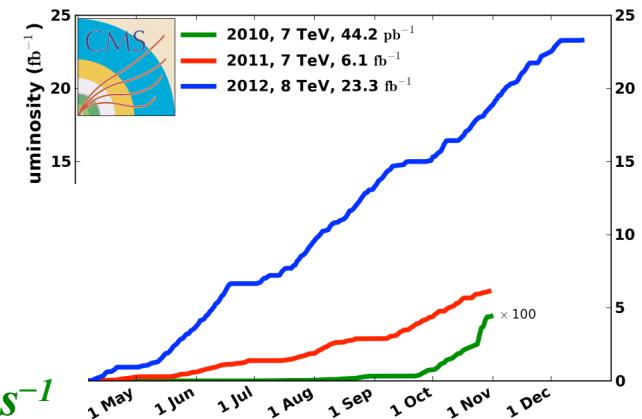
*Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm*

<i>Bunch length</i> (4σ)	<i>ns</i>	1.06
<i>Energy spread</i> (2σ)	10^{-3}	0.22
<i>Synchr. rad. loss/turn</i>	<i>keV</i>	7
<i>Synchr. rad. power</i>	<i>kW</i>	3.6
<i>RF frequency</i>	<i>M</i> <i>Hz</i>	400
<i>Harmonic number</i>		35640
<i>RF voltage/beam</i>	<i>MV</i>	16
<i>Energy gain/turn</i>	<i>keV</i>	485
<i>Synchrotron frequency</i>	<i>Hz</i>	23.0

And still...

The LHC Performance in Run 1

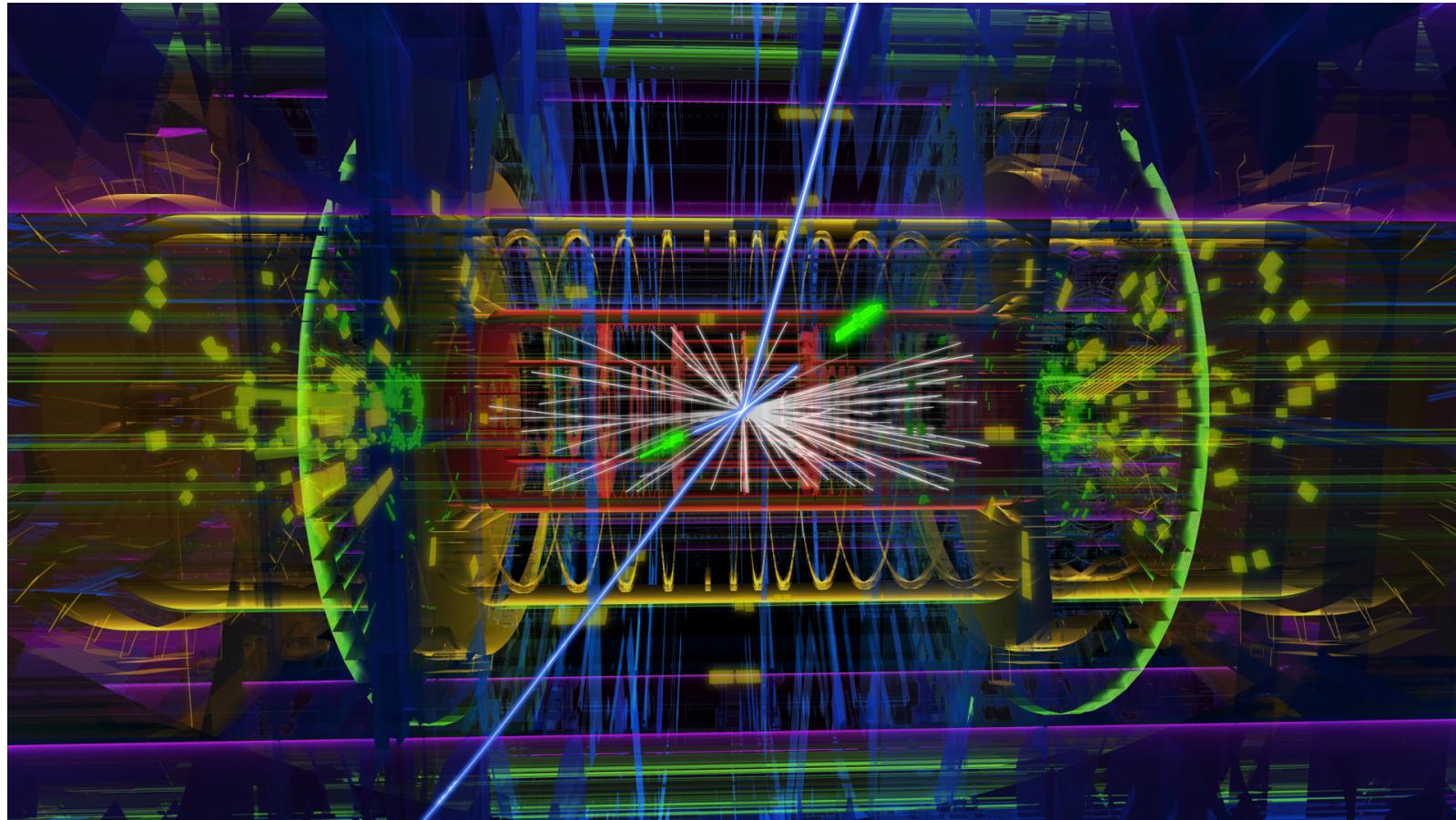
	<i>Design</i>	<i>2012</i>
<i>Momentum at collision</i>	$7 \text{ TeV}/c$	$4 \text{ TeV}/c$
<i>Luminosity</i>	$10^{34} \text{ cm}^{-2} \text{ s}^{-1}$	$7.7 * 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$
<i>Protons per bunch</i>	1.15×10^{11}	1.50×10^{11}
<i>Number of bunches/beam</i>	2808	1380
<i>Nominal bunch spacing</i>	25 ns	50 ns
<i>beta *</i>	55 cm	60 cm
<i>rms beam size IP</i>	$17 \mu\text{m}$	$20 \mu\text{m}$



1.) Where are we ?

- * *Standard Model of HEP*
- * *Higgs discovery*

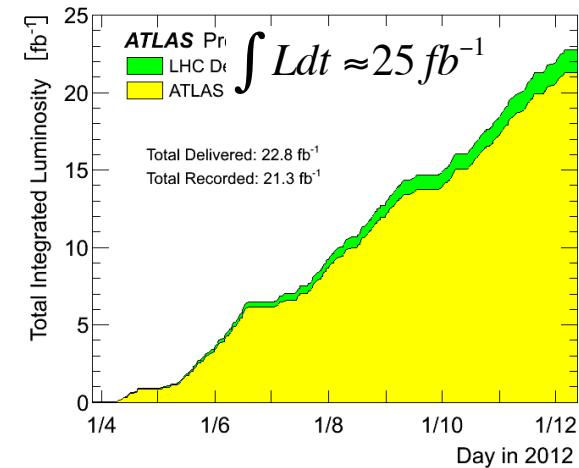
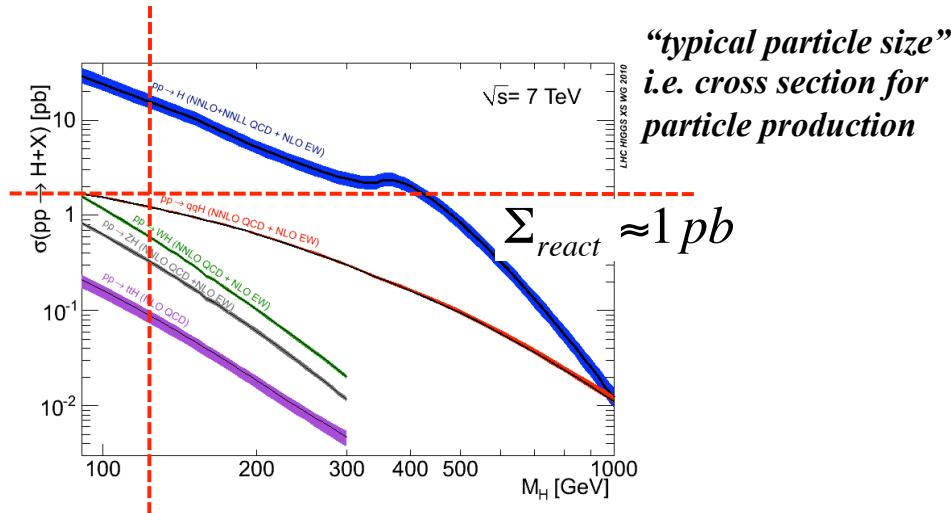
High Light of the HEP-Year 2012 / 13 naturally the HIGGS



ATLAS event display: Higgs => two electrons & two muons

The High light of the year

*production rate of events is determined by the cross section Σ_{react} and a parameter L that is given by the design of the accelerator:
... the luminosity*



$$1b = 10^{-24} \text{ cm}^2 = 1/\text{mio} * 1/\text{mio} * 1/\text{mio} * \frac{1}{100} \text{ mm}^2$$

The particles are “very small”

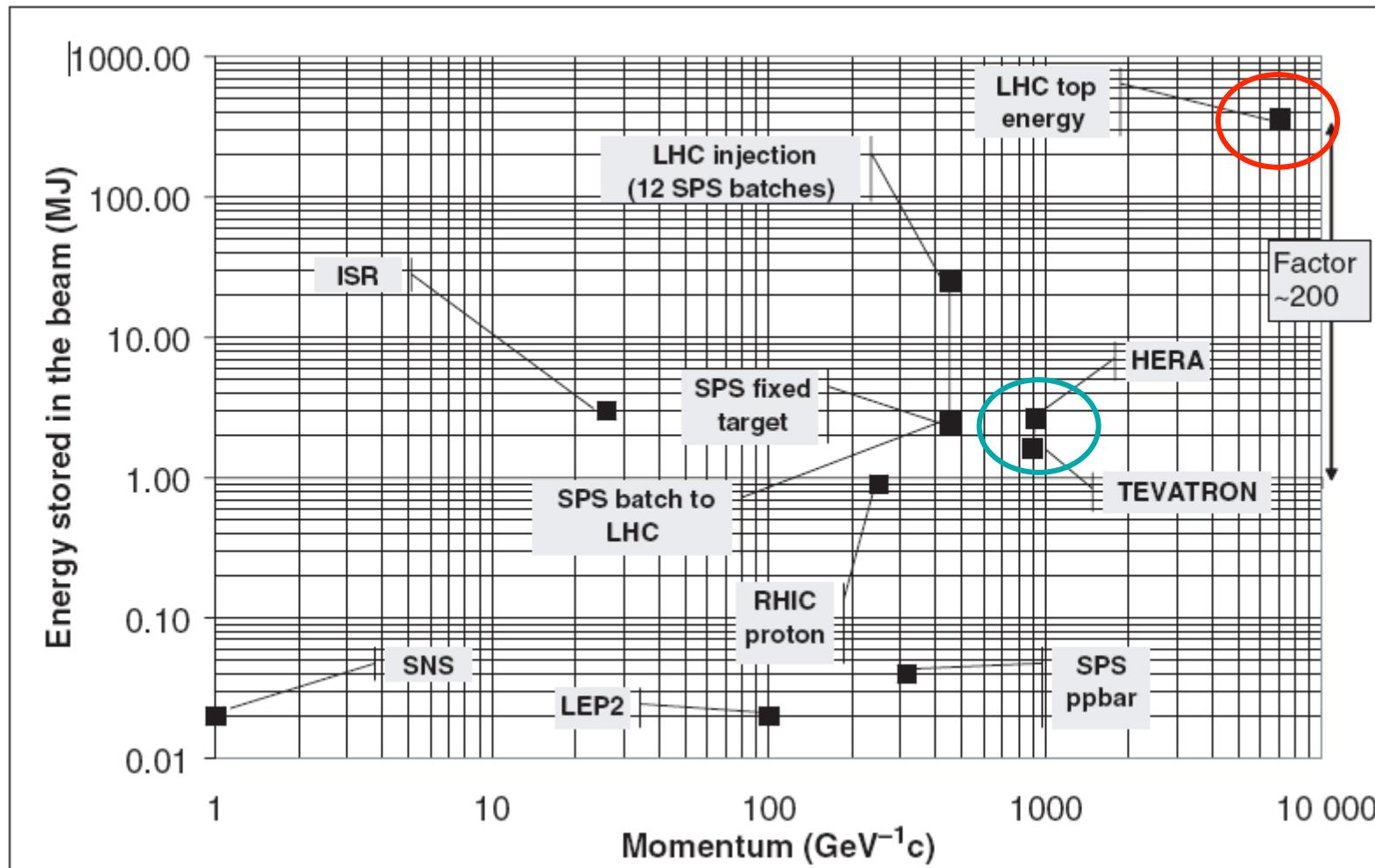
$$R = L * \Sigma_{react} \approx 10^{-12} b \cdot 25 \frac{1}{10^{-15} b} = \text{some} 1000 H$$

During collider run we had in Run 1 ...

*1400 bunches circulating,
with 800 Mio proton collisions per second in the experiments
and collected only 450 Higgs particles in three years.*

LHC Operation: Machine Protection & Safety

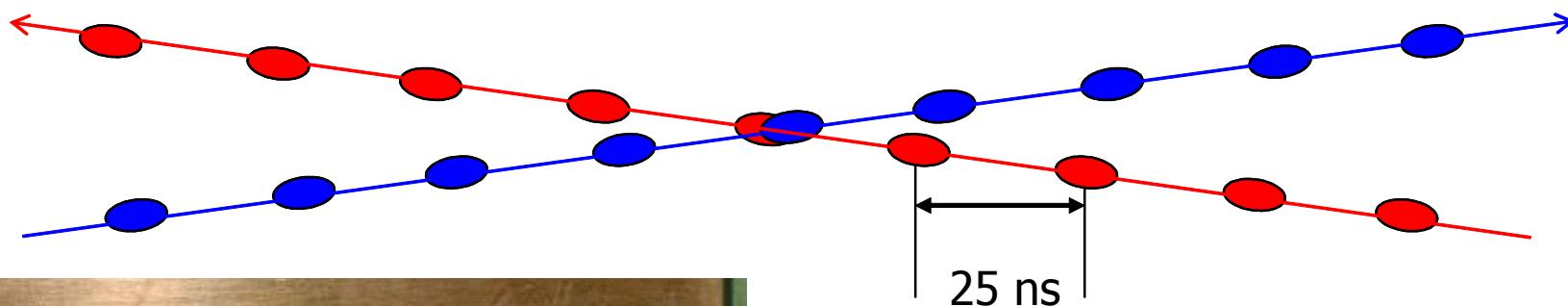
Energy Stored in the Beam of different Storage Rings



LHC Operation: Machine Protection & Safety

Energy stored in magnet system	10	GJ
Energy stored in one main dipole circuit	1.1	GJ
Energy stored in one beam	362	MJ

Enough to melt 500 kg of copper

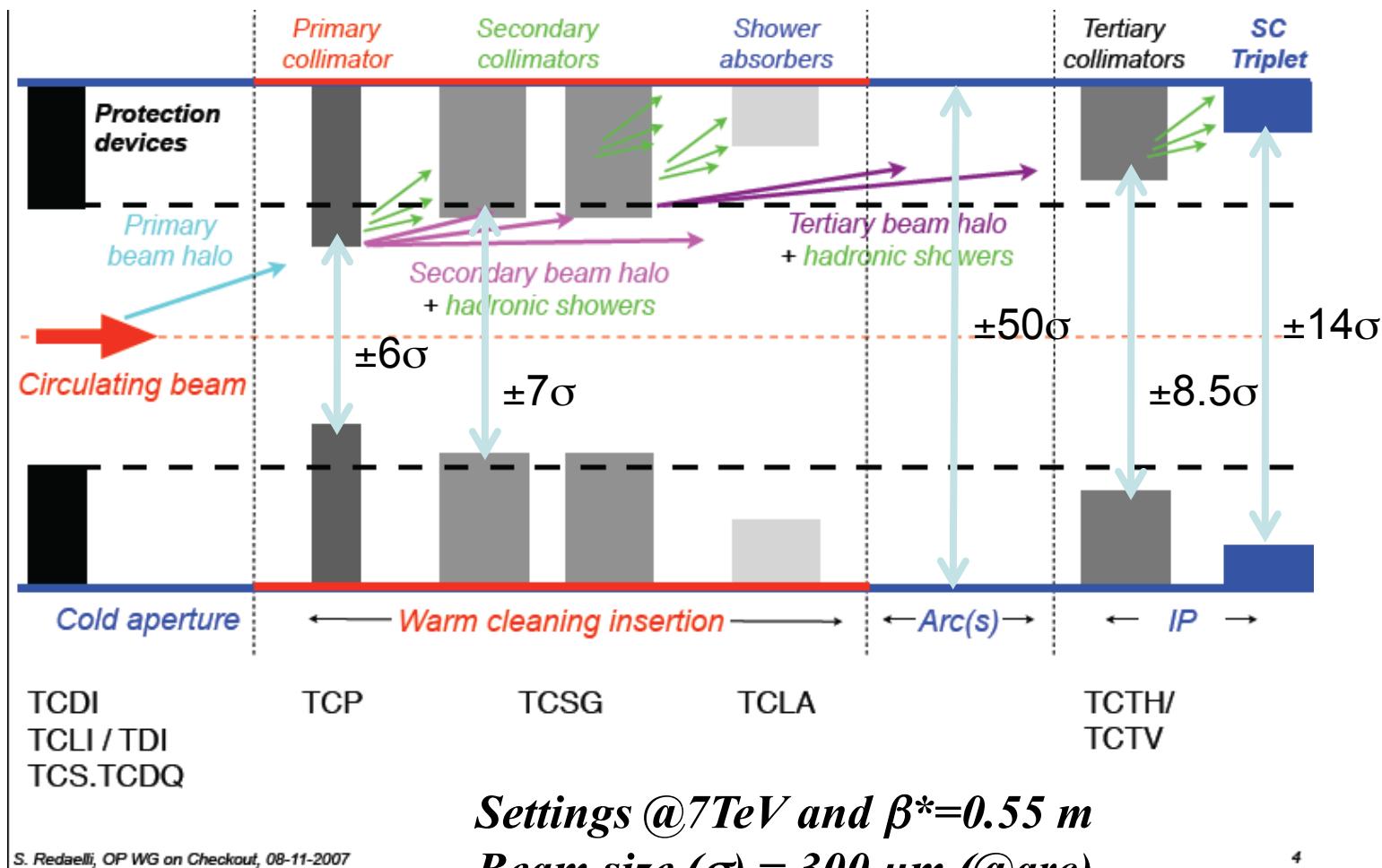


$2 \cdot 10^{12}$ $4 \cdot 10^{12}$ $8 \cdot 10^{12}$ $6 \cdot 10^{12}$

450 GeV p Strahl



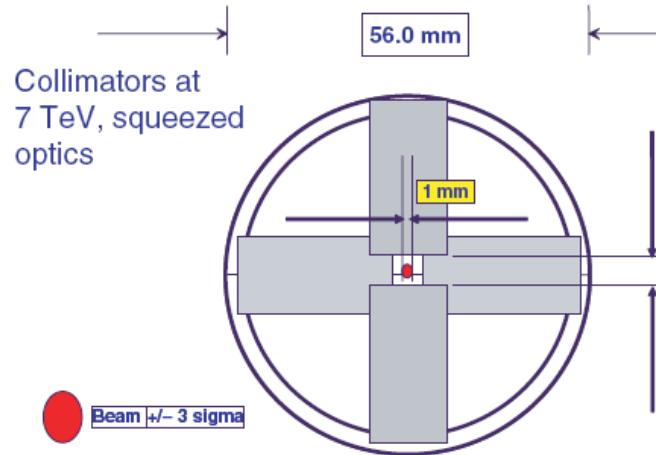
LHC Aperture and Collimation



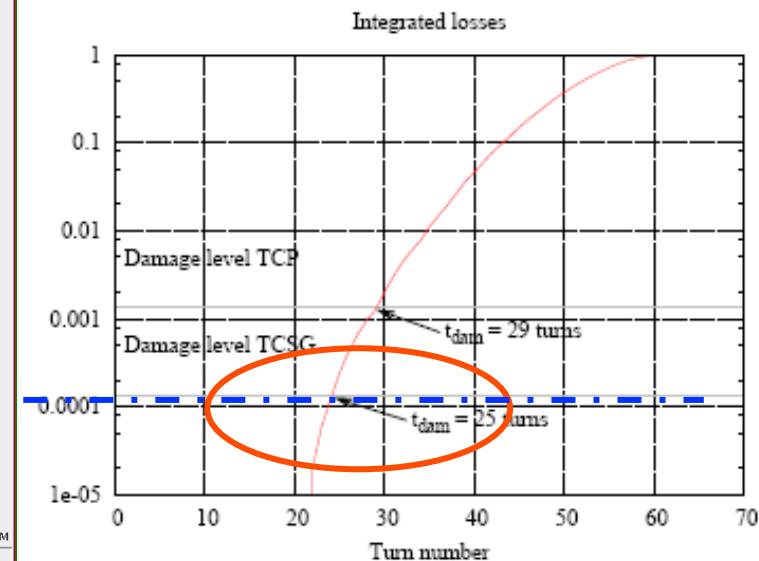
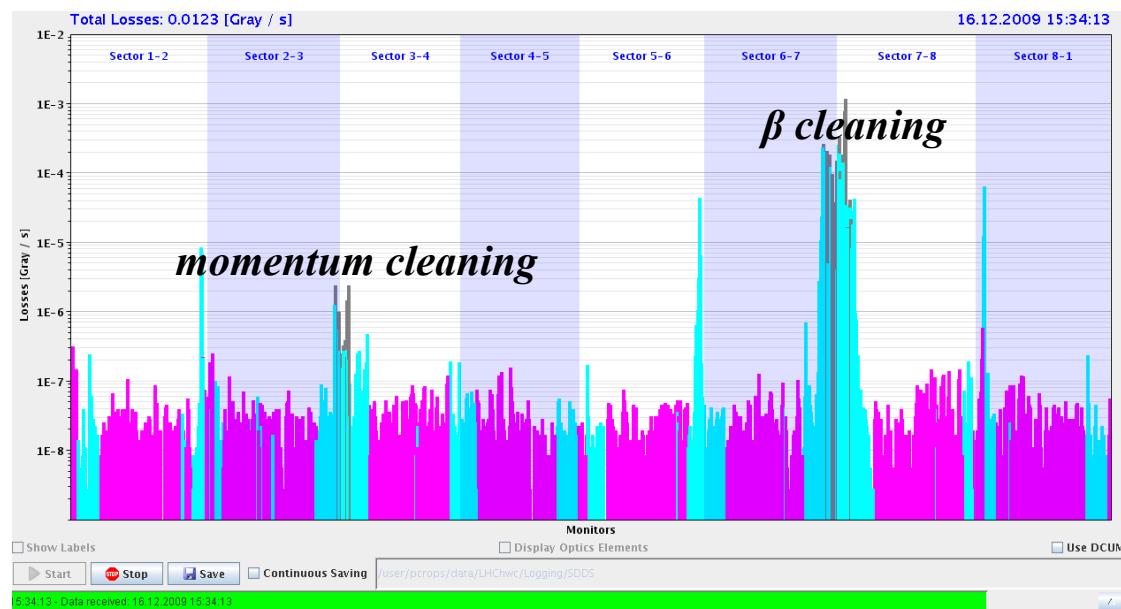
Settings @7TeV and $\beta^*=0.55$ m
 Beam size (σ) = 300 μ m (@arc)
 Beam size (σ) = 17 μ m (@IR1, IR5)

LHC Operation: Machine Protection & Safety

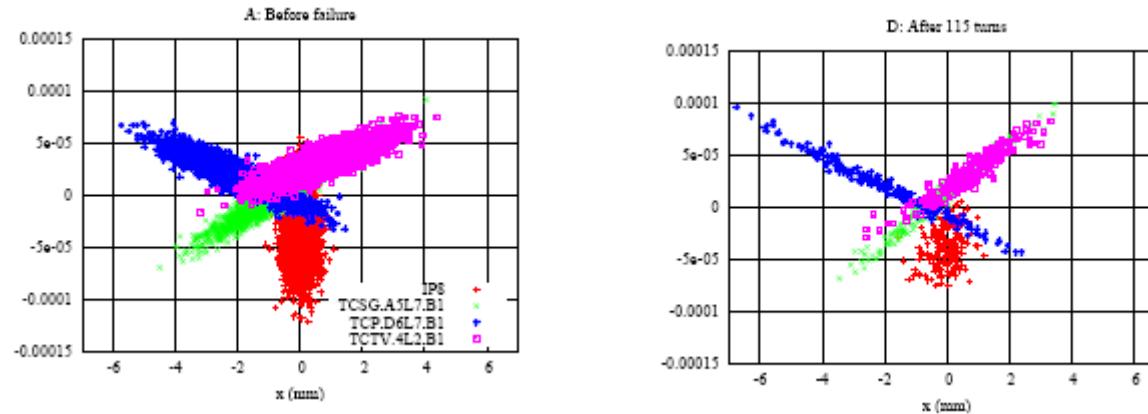
... Komponenten des Machine Protection Systems :



*beam loss monitors
QPS
permit server
orbit control
power supply control
collimators
online on beam check of all (?)
hardware components
a fast dump
the gaussian beam profile*



LHC Operation: Machine Protection & Safety



*Phase space deformation in case of failure of RQ4.LR7
(A. Gómez)*

Short Summary of the studies:

quench in sc. arc dipoles: $\tau_{loss} = 20 - 30 \text{ ms}$

BLM system reacts in time, QPS is not fast enough

quench in sc. arc quadrupoles: $\tau_{loss} = 200 \text{ ms}$

BLM & QPS react in time

failure of nc. quadrupoles: $\tau_{det} = 6 \text{ ms}$

$\tau_{damage} = 6.4 \text{ ms}$

failure of nc. dipole:

$\tau_{damage} = 2 \text{ ms}$

$\rightarrow \text{FMCM installed}$

*What will happen in
case of Hardware Failure*

Energy stored in the magnets

~ 10 Gjoule* (only in the main dipoles) corresponds to ...



***The energy of ~3 Tons TNT
The energy of 370 kg dark chocolate***

***More important than the amount of energy is ...
How fast (and safe) can this energy be released?***

$$*E = \frac{1}{2}LI^2$$

L: inductance ~0.1 Henry for LHC dipoles

Energy stored in the magnets: quench

If not fast and safe ...

Quench in a magnet



P. Pugnat

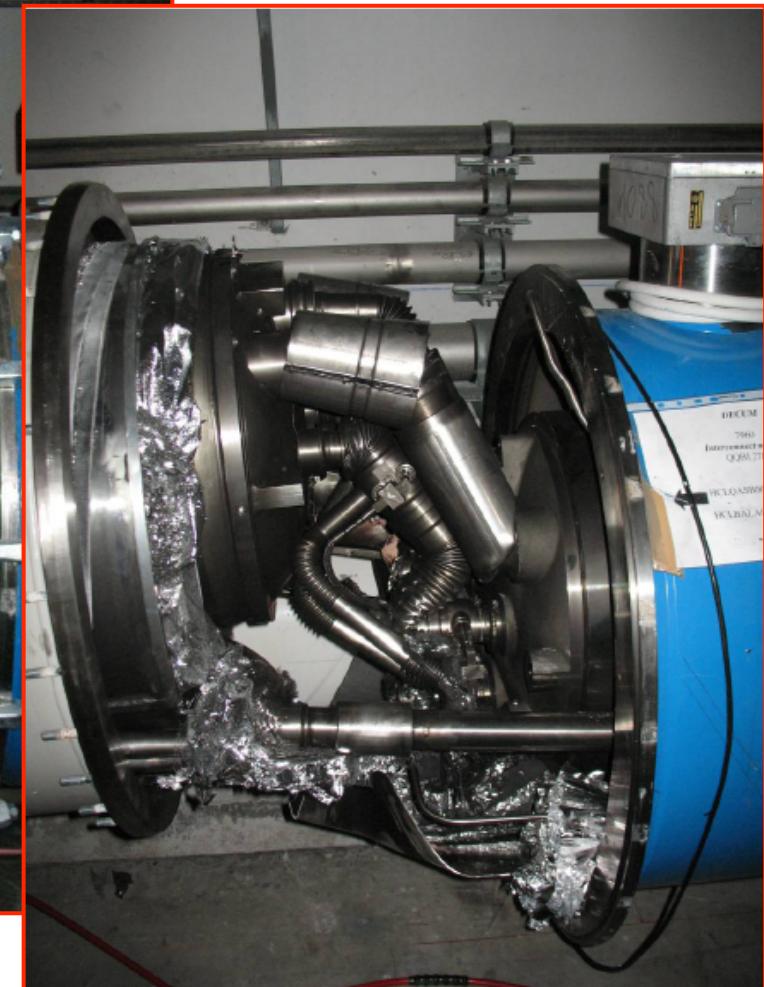
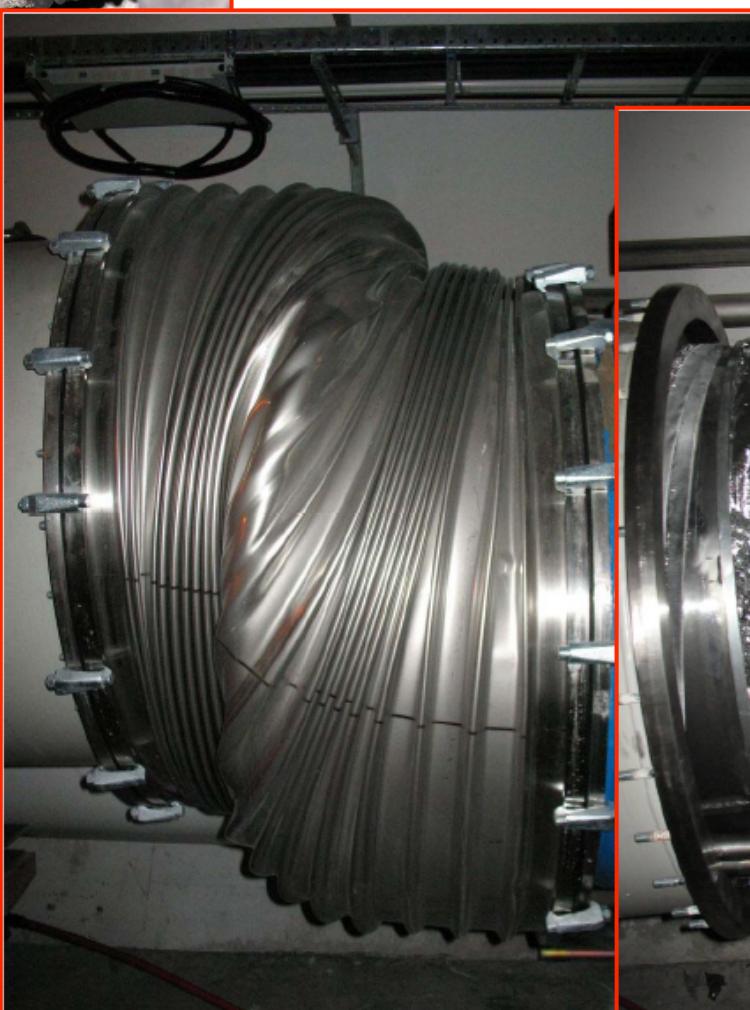
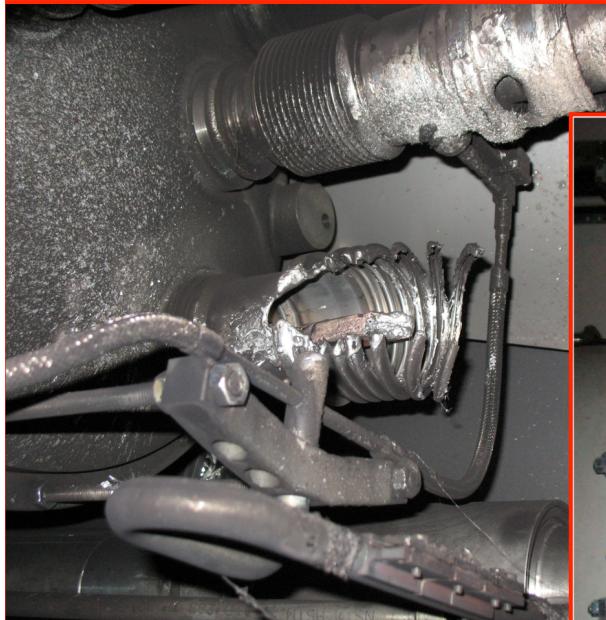
LHC Interconnects *(... splices / ... Loetverbindingen / ... silent killers)*



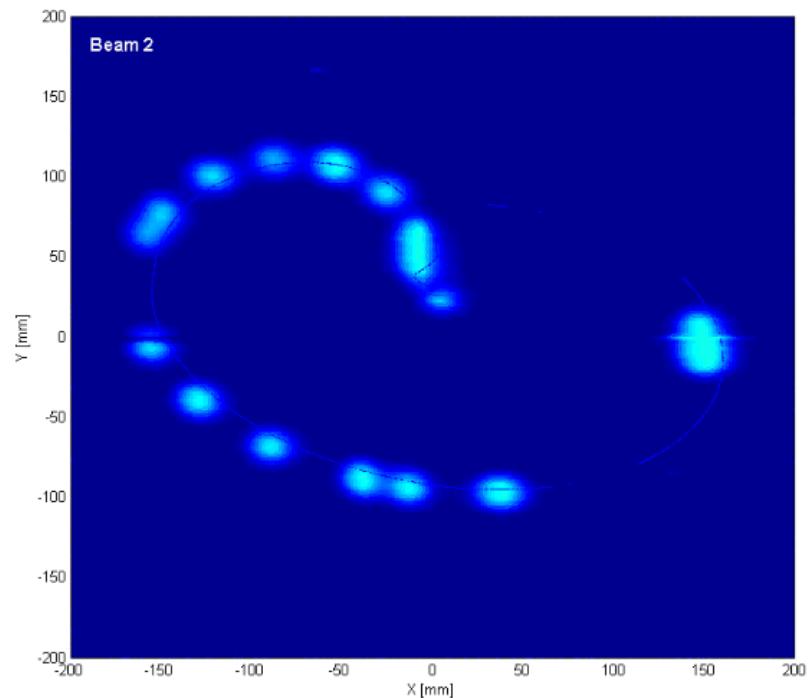
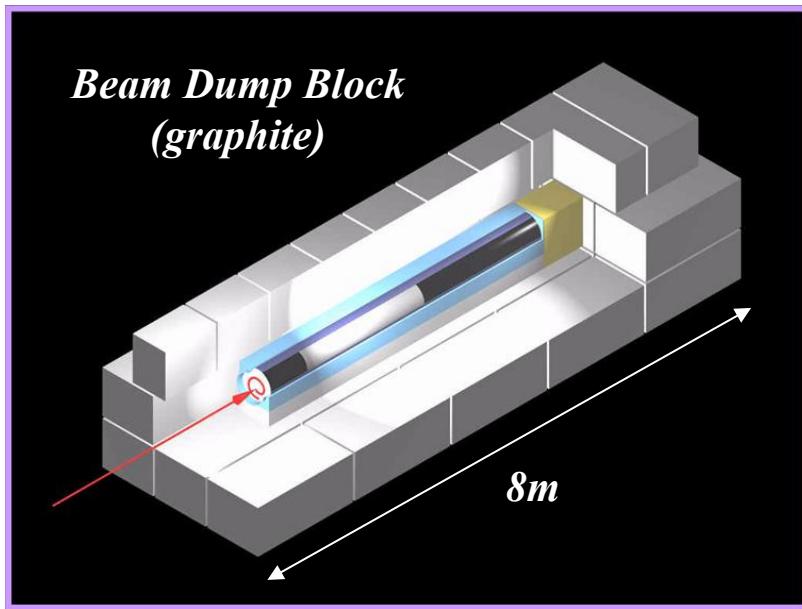
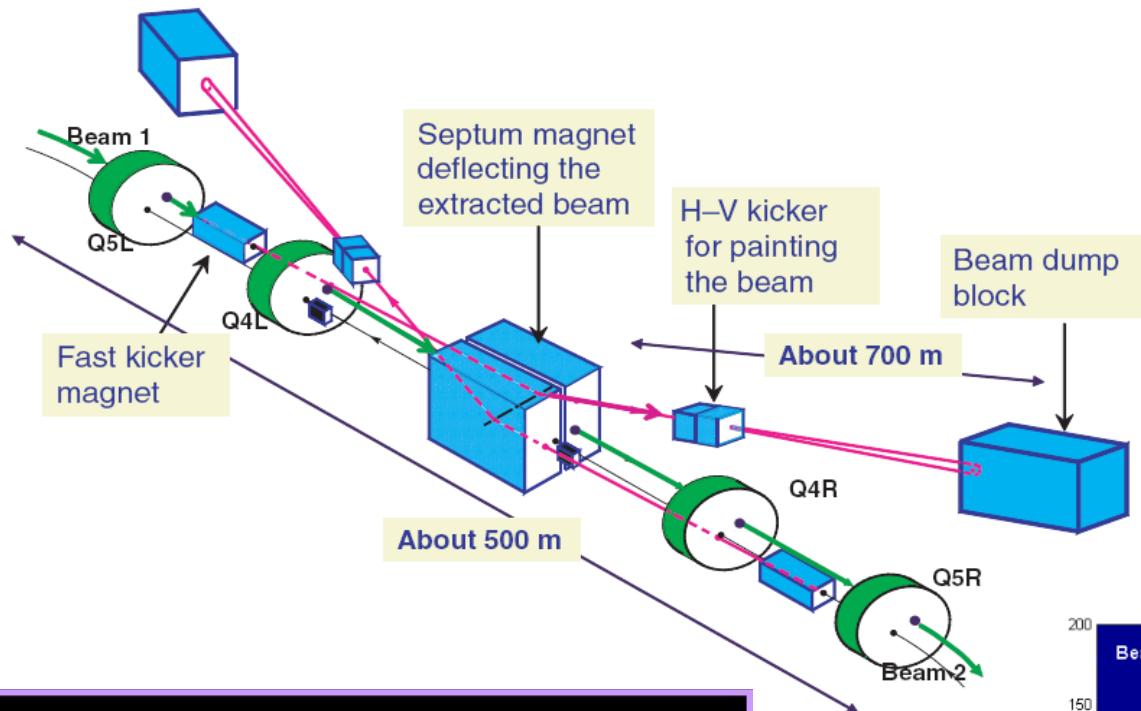
13/01/2010

Quench in a bus bar (19 Sep 2008)

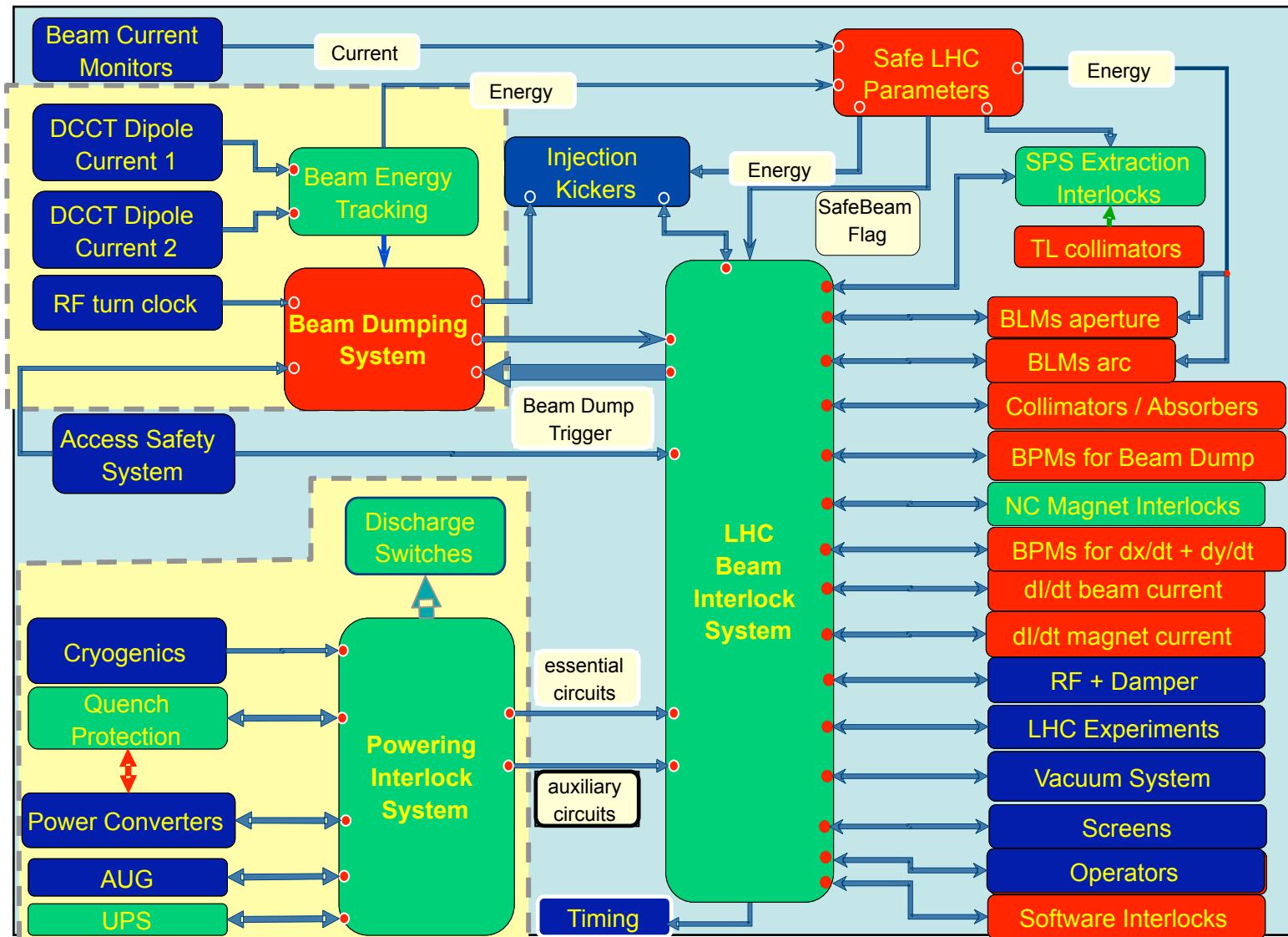
Electrical arc between C24 and Q24



LHC Operation: Dump System



LHC Operation: Machine Protection & Safety



... no comment

Booooooom

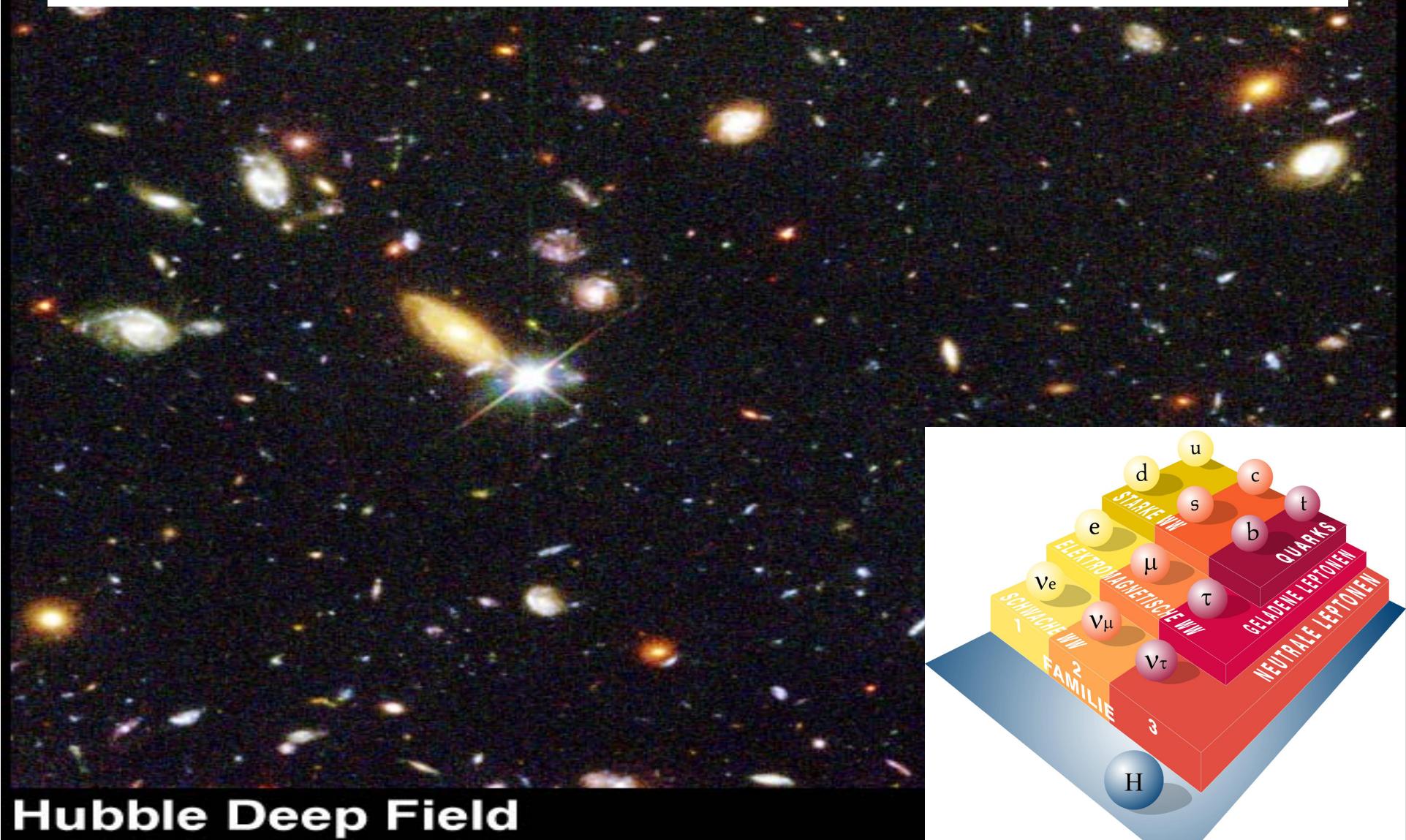


2.) Where do we go ?

- * *Physics beyond the Standard Model*
- * *Dark Matter / Dark Energy*

What 's next ???

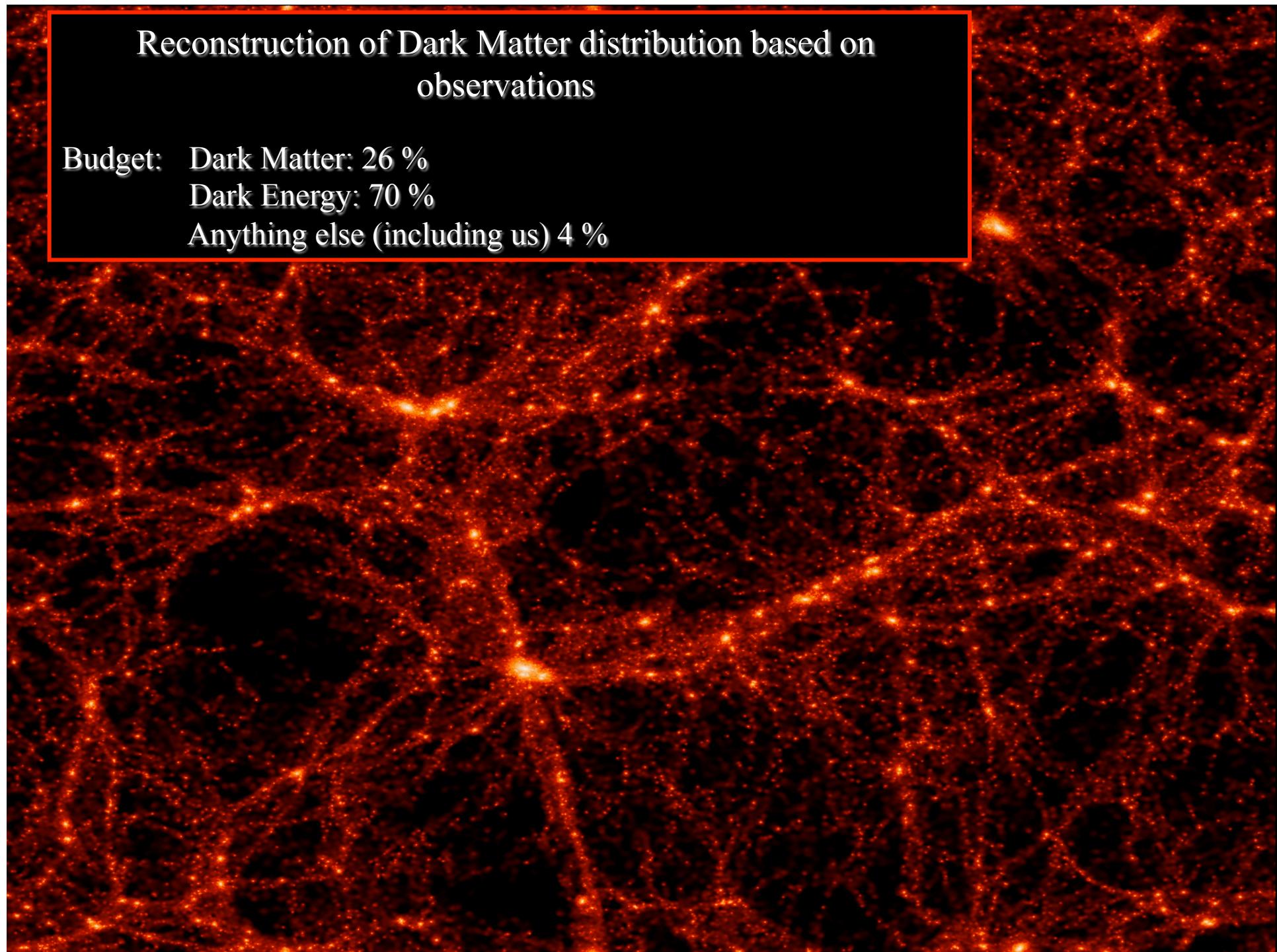
*Dark Matter & Dark Energy
Physics beyond the Standard Model*



Reconstruction of Dark Matter distribution based on observations

Budget:

- Dark Matter: 26 %
- Dark Energy: 70 %
- Anything else (including us) 4 %

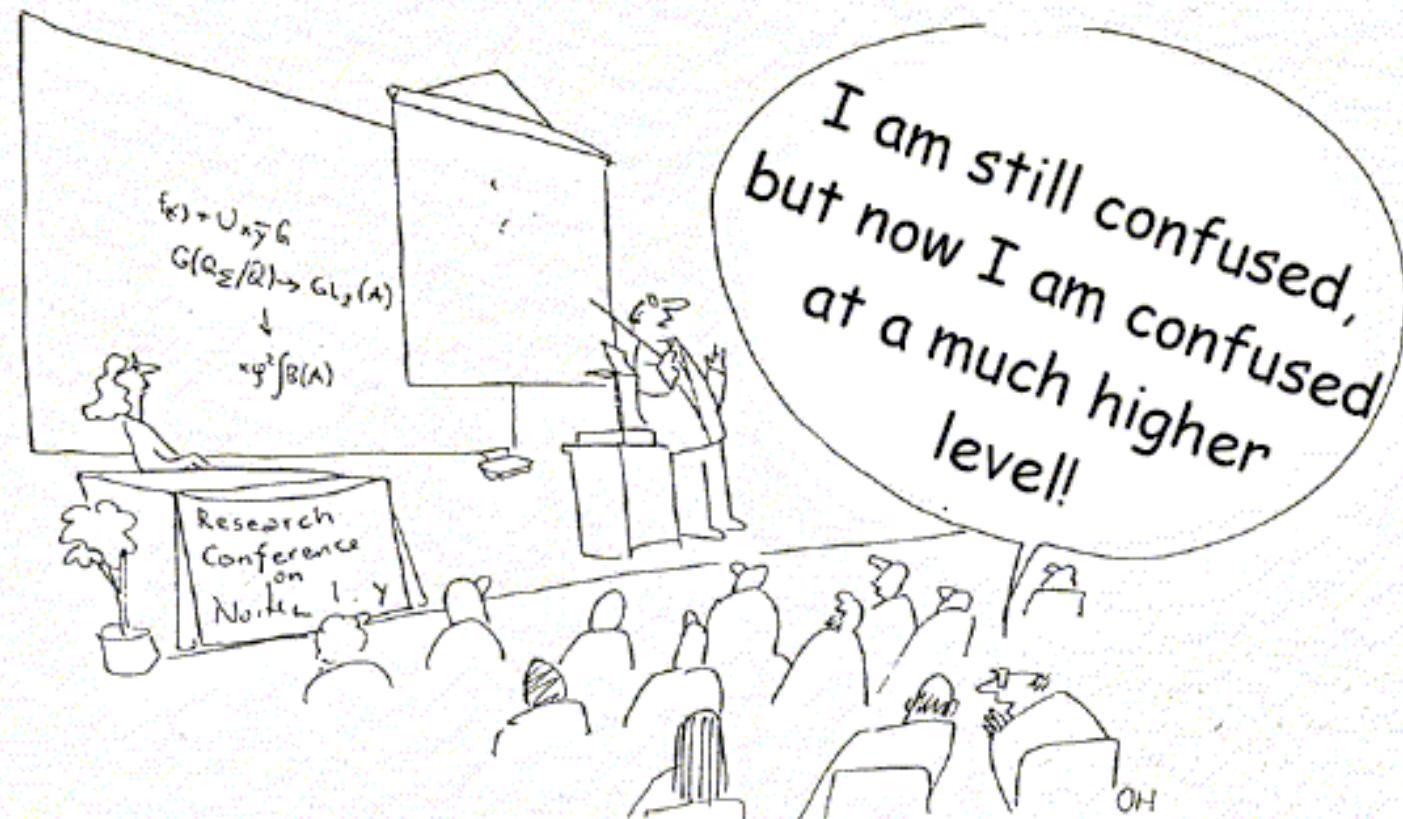




FCC-pp - Collider

The Next Generation Ring Collider





Future Projects

Recommendations from European Strategy Group

- #1 c) The discovery of the Higgs boson is the start of a major programme of work to measure this particle's properties with the highest possible precision for testing the validity of the Standard Model and to search for further new physics at the energy frontier. The LHC is in a unique position to pursue this programme. *Europe's top priority should be the exploitation of the full potential of the LHC, including the high-luminosity upgrade of the machine and detectors with a view to collecting ten times more data than in the initial design, by around 2030. This upgrade programme will also provide*

#2 d) To stay at the forefront of particle physics, Europe needs to be in a position to propose an ambitious post-LHC accelerator project at CERN by the time of the next Strategy update, when physics results from the LHC running at 14 TeV will be available. *CERN should undertake design studies for accelerator projects in a global context, with emphasis on proton-proton and electron-positron high-energy frontier machines. These design studies should be coupled to a vigorous accelerator R&D programme, including high-field magnets and high-gradient accelerating structures, in collaboration with national institutes, laboratories and universities worldwide.*

\rightarrow **Proton –Proton Colliders** $\Rightarrow e^+/e^-$ **colliders**

LHC / HL-LHC, HE-LHC

TLEP, CLIC

4.) Push for higher energy: FCC

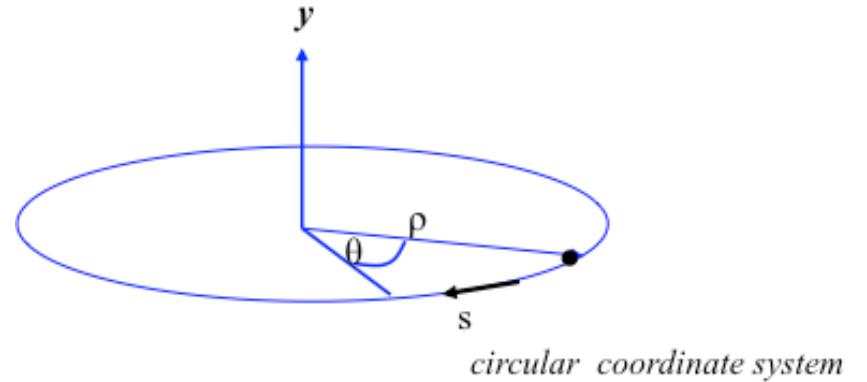
- * *increasing the ring size*
- * *stronger magnets*

Maximum Beam Energy in a Storage Ring:

*For a given magnet technology it is the size of the machine that defines the maximum particle momentum
... and so the energy*

$$\cancel{E = mc^2}$$

$$E^2 = (pc)^2 + m^2 c^4$$



Condition for an ideal circular orbit:

Lorentz force

$$F_L = evB$$

centrifugal force

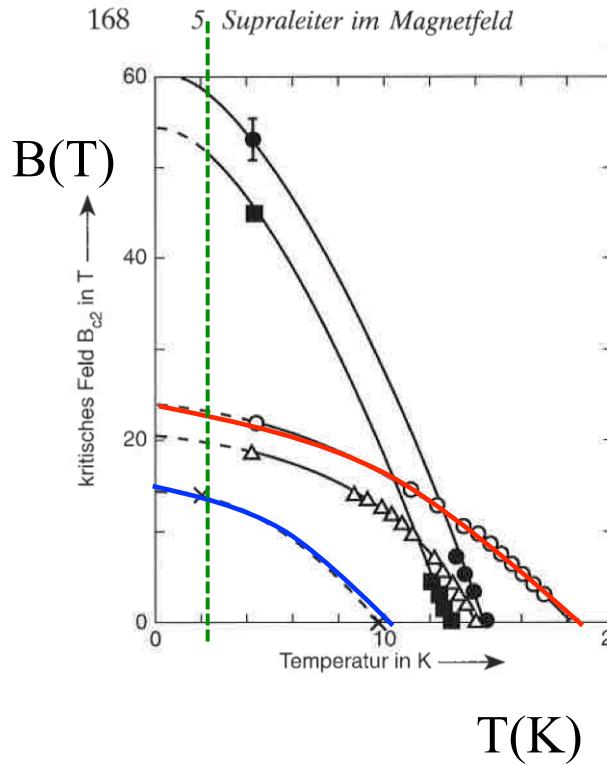
$$F_{centr} = \frac{\gamma m_0 v^2}{\rho}$$

$$\frac{\gamma m_0 v^2}{\rho} = evB$$

$$\boxed{\frac{p}{e} = B\rho}$$

$B\rho$ = "beam rigidity"

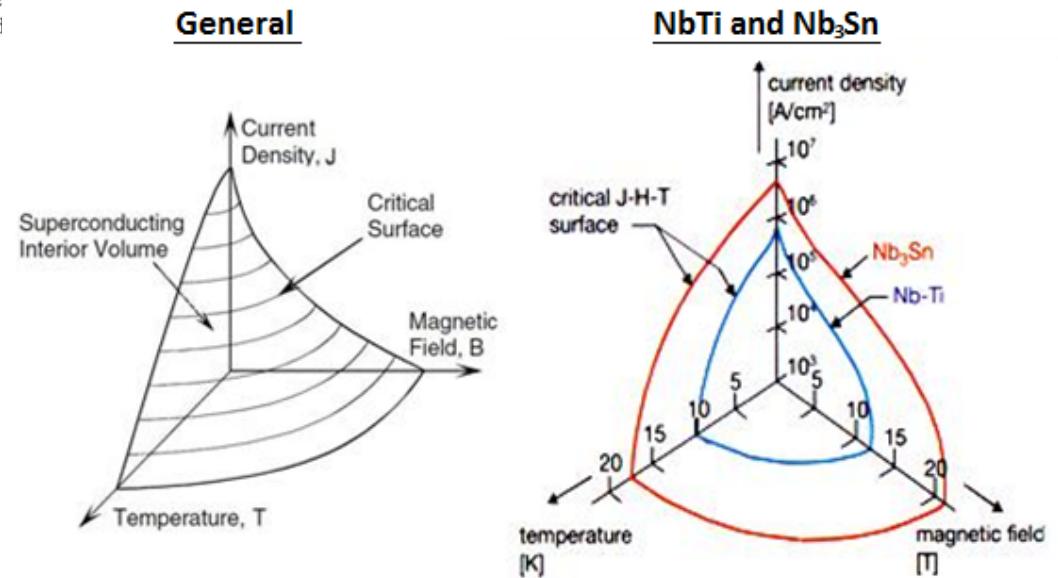
Two key players in sc magnet technology: NbTi and Nb₃Sn



critical field in NbTi and Nb₃Sn

Abb. 77 Oberes kritisches Feld einiger Hochfeldsupraleiter.

- Nb₃Sn, Drahtdurchmesser 0,5 mm [127]
 - △-△-△- V₃Ga, Sinterprobe [127]
 - ×-×-×- Nb₅₀ Ti₅₀ [128]
 - PbMo_{6,35}S₈ [130]
 - PbGd_{0,3}Mo₆S₈ [130]
- (siehe auch Ø. Fische
Otaniemi 1975, Band
Publ. Comp. 1975).



The Push for Higher Beam Energy



NbTi LHC standard dipoles,
8.3 T

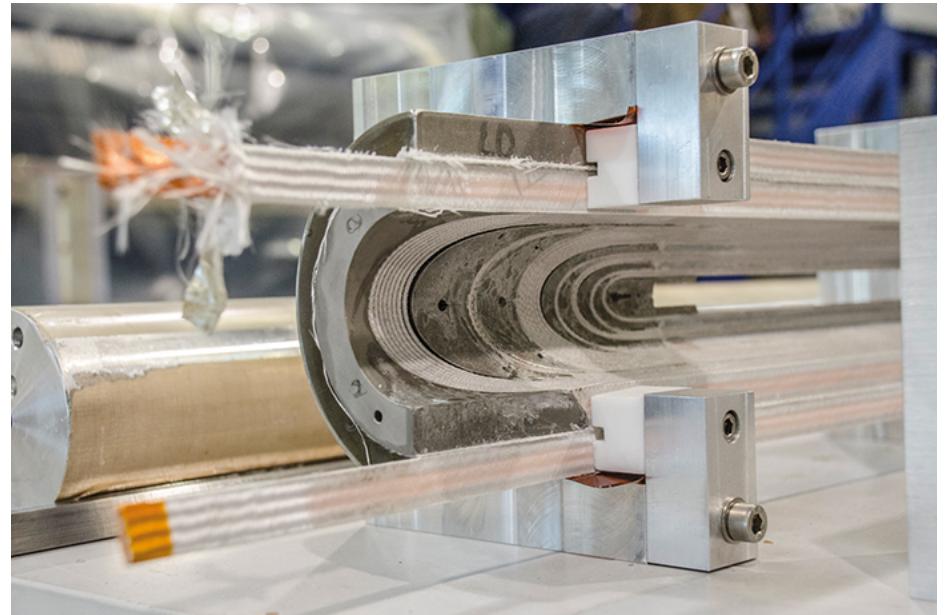
*it is a simple scaling wrt LHC:
circumference 100km / 27km
→ Factor 3.7*

*dipole field: 16 T / 8.3 T
→ Factor 1.93*

*LHC energy $E_{cm} = 2 * 7 \text{ TeV} * 7.1$*

FCC energy $E_{cm} = 100 \text{ TeV}$ centre of mass

Nb₃Sn FCC type dipole coils,
11 T – 16 T



5.) High Energy Lepton Colliders

- * *Limited by Synchrotron Radiation*
- * *and RF Power*



FCC-ee Collider

The next Generation e^+e^- Ring Collider



Synchrotron Radiation



ca 400 000 v. Chr.: Mankind discovers the Fire

Synchrotron Radiation

In a circular accelerator charged particles lose energy via emission of intense light.

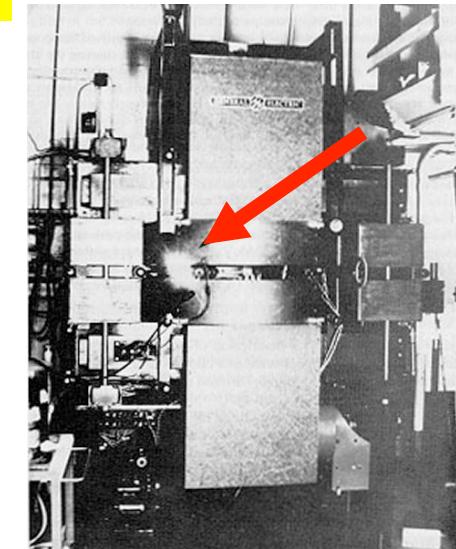
$$P_s = \frac{2}{3} \alpha \hbar c^2 \frac{\gamma^4}{\rho^2} \quad \text{radiation power}$$

$$\Delta E = \frac{4}{3} \pi \alpha \hbar c \frac{\gamma^4}{\rho} \quad \text{energy loss}$$

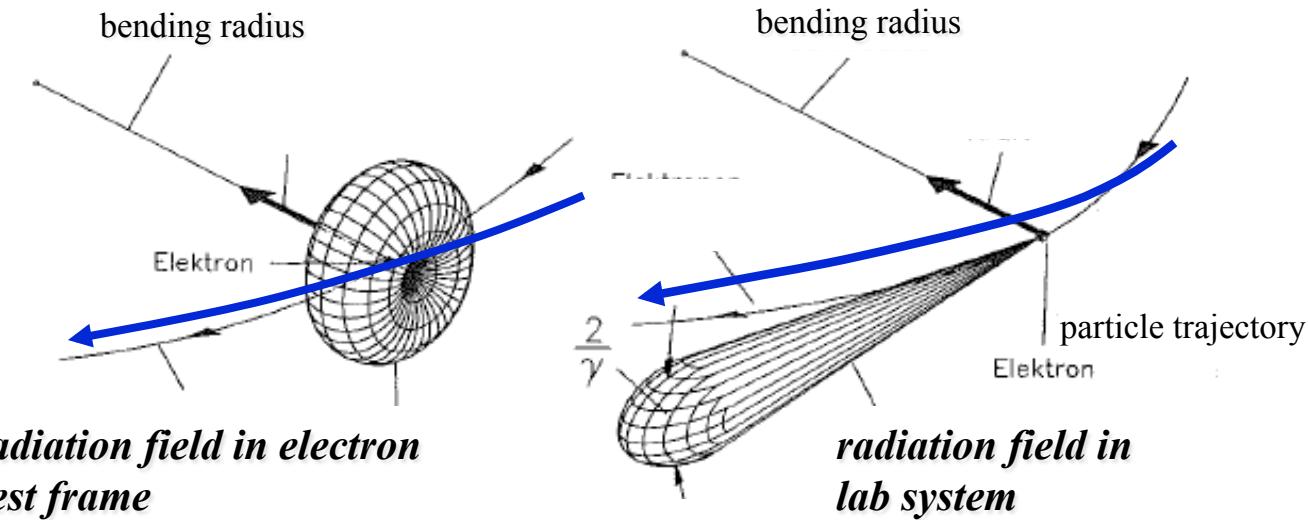
$$\omega_c = \frac{3}{2} \frac{c \gamma^3}{\rho} \quad \text{critical frequency}$$

$$\alpha \approx \frac{1}{137}$$

$$\hbar c \approx 197 \text{ MeV fm}$$

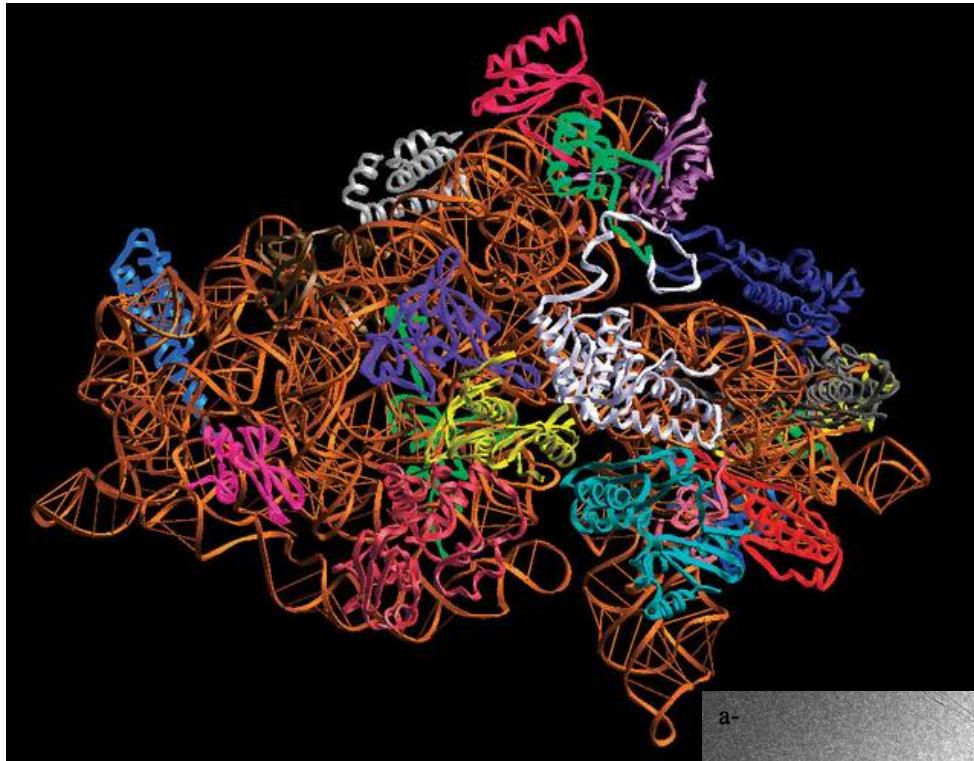


1946 observed for the first time in the General Electric Synchrotron



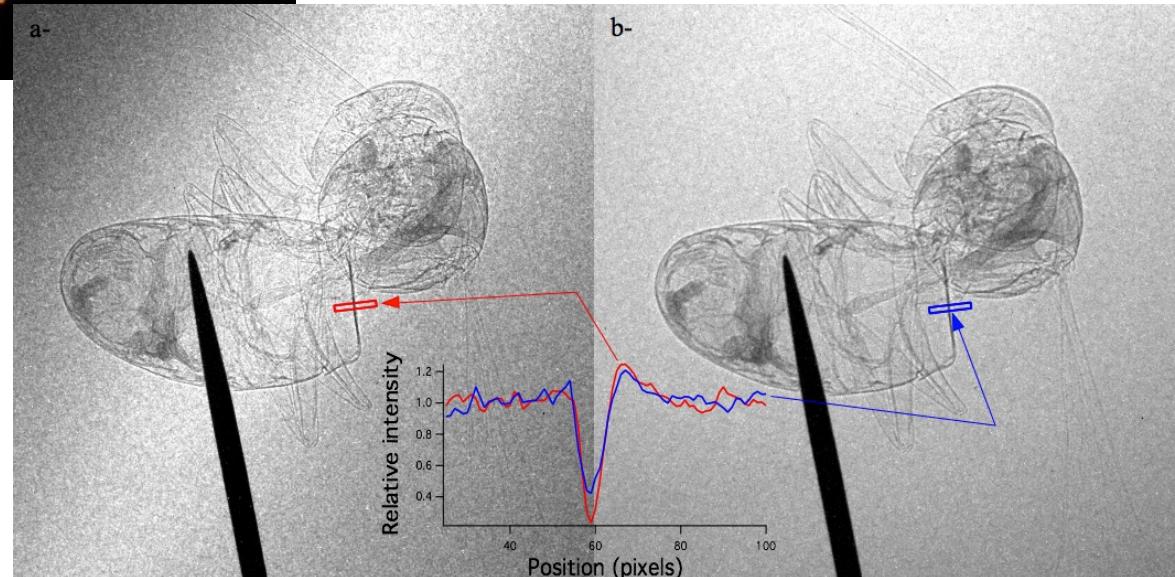
court. K. Wille

Synchrotron Radiation as useful tool



Absorption Line Radiographie

*structure analysis with
highest resolution
Ribosome molecule*



Planning the next generation e+ / e- Ring Colliders

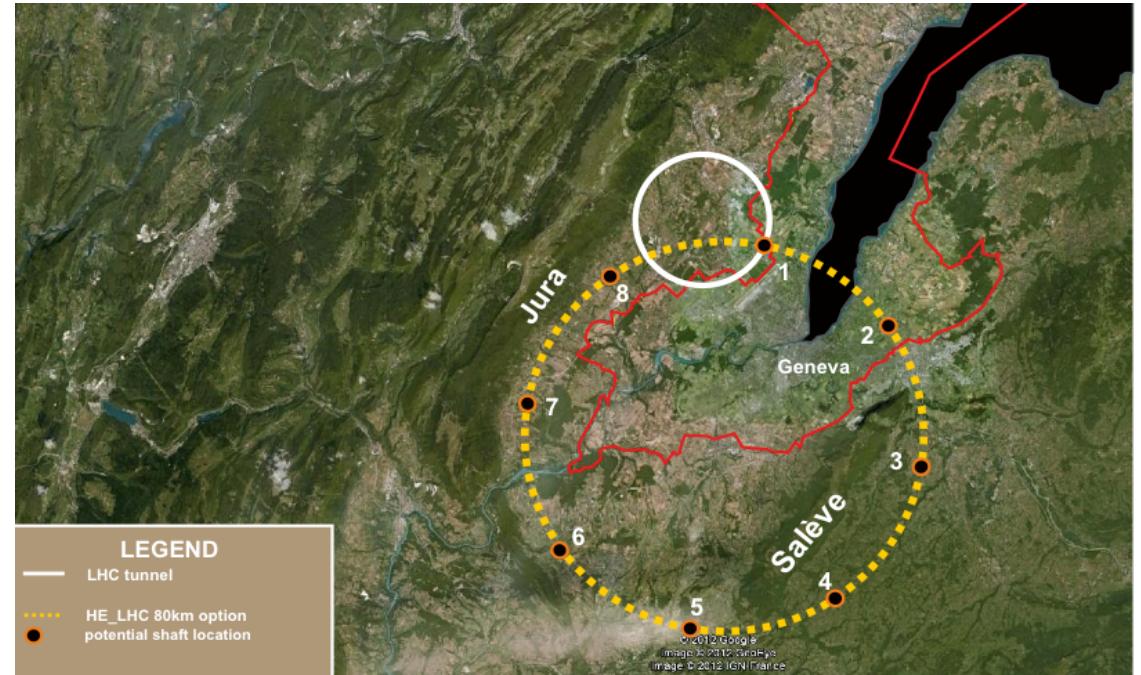
Design Parameters FCC-ee

$$E = 175 \text{ GeV/beam}$$

$$L = 100 \text{ km}$$

$$\Delta U_0(\text{keV}) \approx \frac{89 * E^4(\text{GeV})}{\rho}$$

$$\Delta U_0 \approx 8.62 \text{ GeV}$$



$$\Delta P_{sy} \approx \frac{\Delta U_0 * N_p}{T_0} = \frac{10.4 * 10^6 \text{ eV} * 1.6 * 10^{-19} \text{ Cb}}{263 * 10^{-6} \text{ s}} * 9 * 10^{12}$$

$$\Delta P_{sy} \approx 47 \text{ MW}$$

Circular e+ / e- colliders are severely limited by synchrotron radiation losses and have to be replaced for higher energies by linear accelerators

6.) Push for higher energy

- * *go linear*
- * *higher acceleration gradients*

Lepton Colliders: Linear / Storage Rings

Avoid bending forces → go linear

Storage Ring: *dipole magnets*

synchrotron radiation

energy loss per turn

high RF power to compensate losses

very efficient,

turn by turn acceleration

$$P_\gamma = \frac{c}{2\pi} \frac{C_\gamma E^4}{\rho^2}, \quad C_\gamma = 8.9 * 10^{-5} \text{ m/GeV}^3$$

Linear Collider: *no synchr. Radiation*

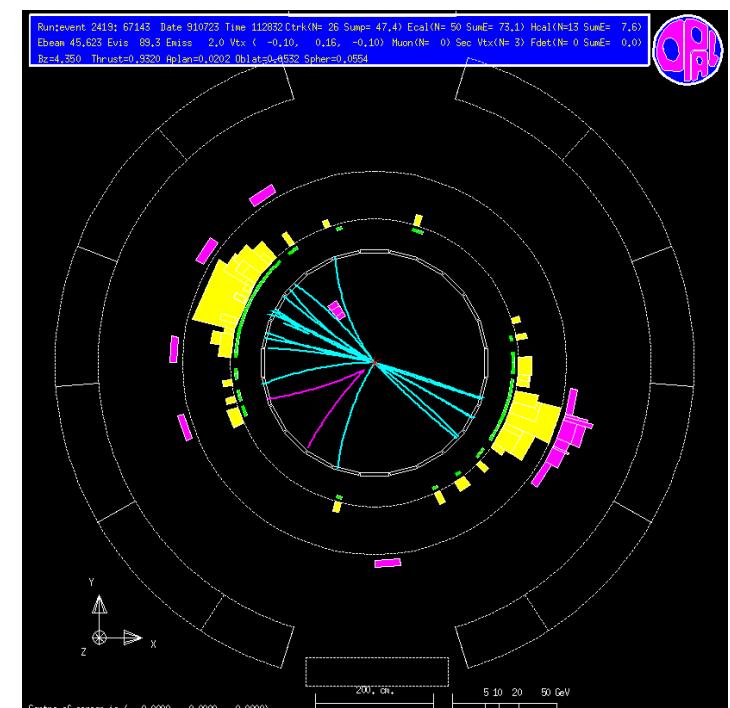
limited efficiency:

$N^{10}-1$ particles are lost after the collision

need highest acceleration gradient

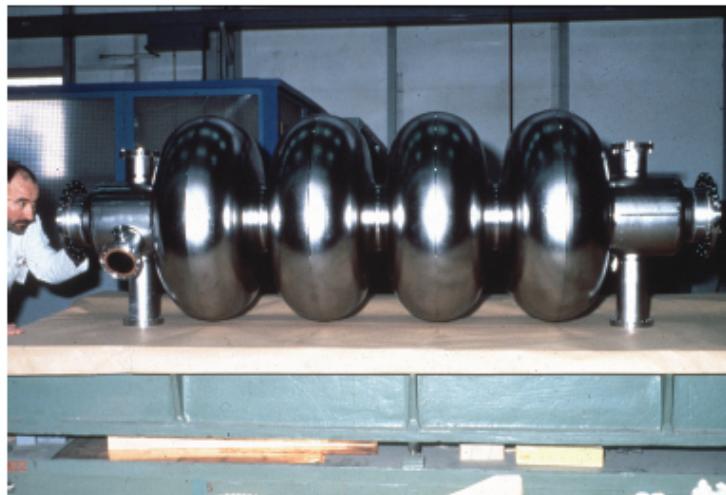
“one turn” machines”

lepton collisions are “clean”



Plasma Wake Acceleration

RF Cavity

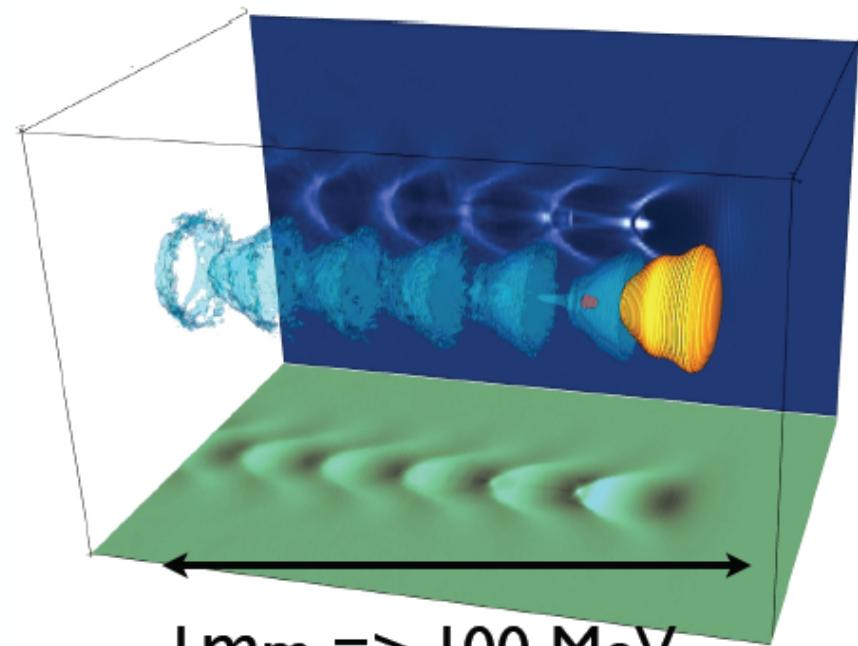


↔

1 m => 50 MeV Gain

Electric field < 100 MV/m

Plasma Cavity



↔

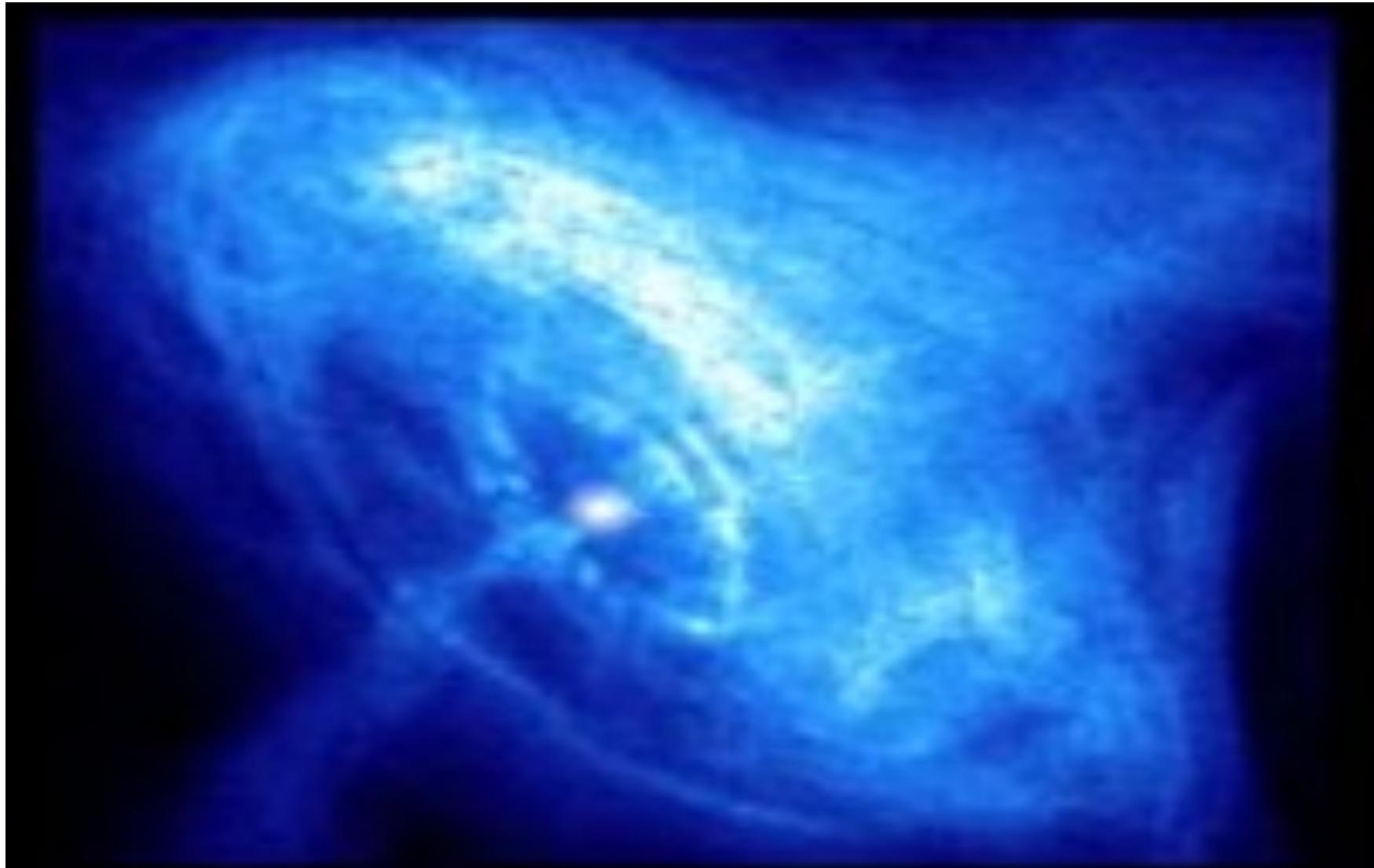
1 mm => 100 MeV

Electric field > 100 GV/m

The power of Plasma Wake Acceleration

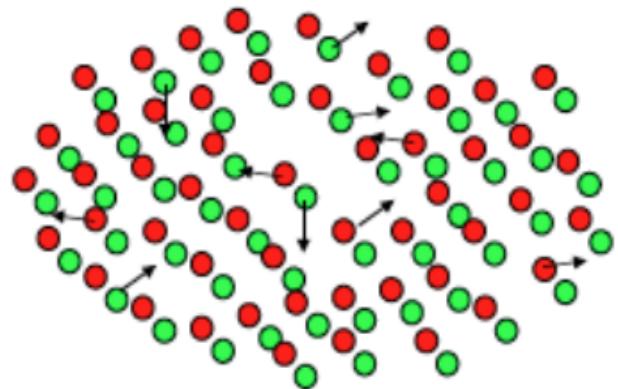
$E=10^{20} \text{ eV}$

$\rightarrow 100*1\text{Mio}*\text{LHC}$



crab nebula, burst of charged particles

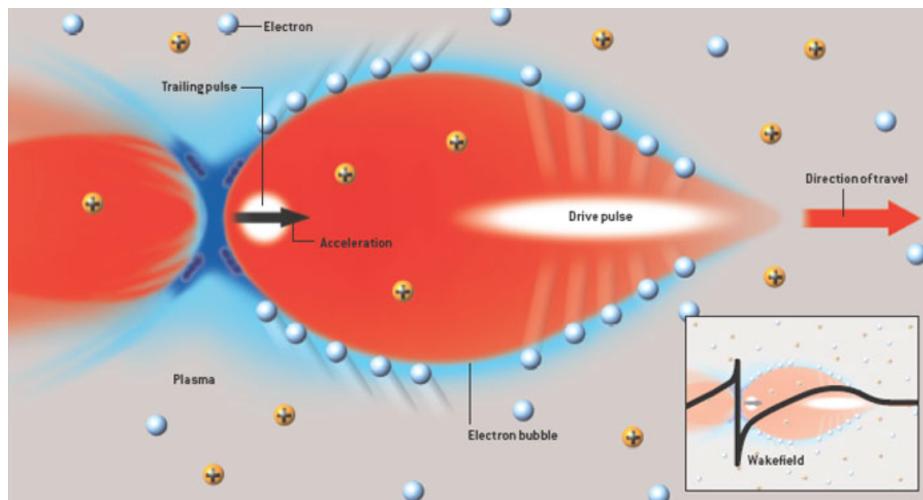
Study of High Gradient Acceleration Techniques



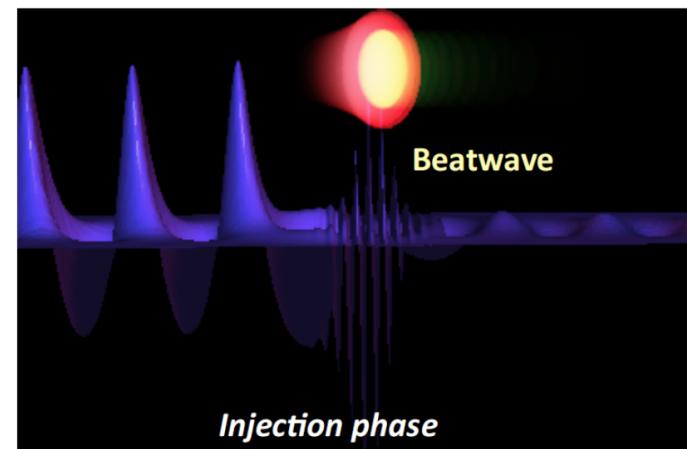
Plasma oscillation frequency:

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{m^* \epsilon_0}},$$

Intense Laser light creates a plasma beat wave, that separates the electrons from the heavy (and so much slower) ions. A quasi electron free region (bubble) is created and as consequence a large electric field that can be used to accelerate particles.



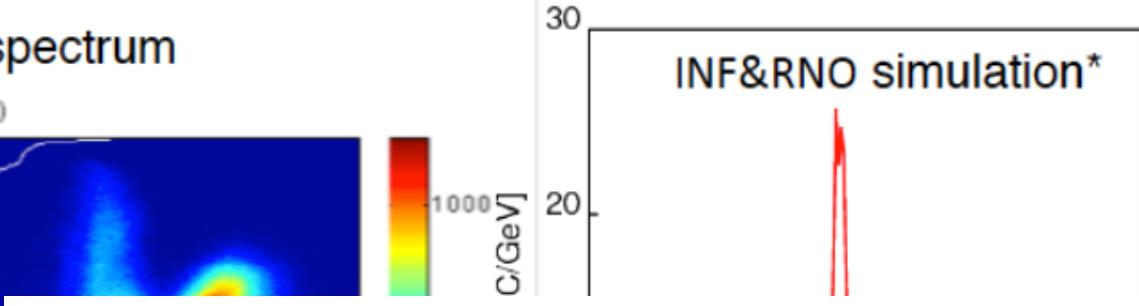
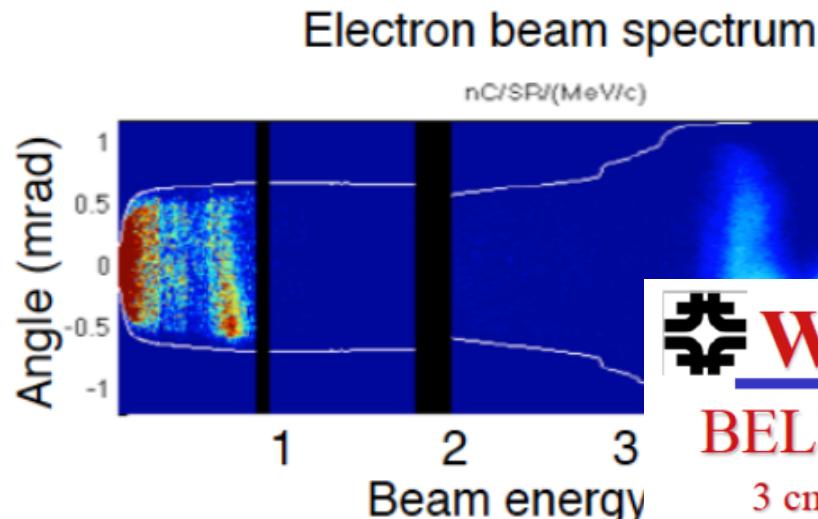
The first laser creates the accelerating structure, a second laser beam is used to heat electrons



Theory : E. Esarey et al., PRL **79**, 2682 (1997), H. Kotaki et al., PoP **11** (2004)
Experiments : J. Faure et al., Nature **444**, 737 (2006)

4.25 GeV beams have been obtained from 9 cm plasma channel powered by 310 TW laser pulses (15 J)

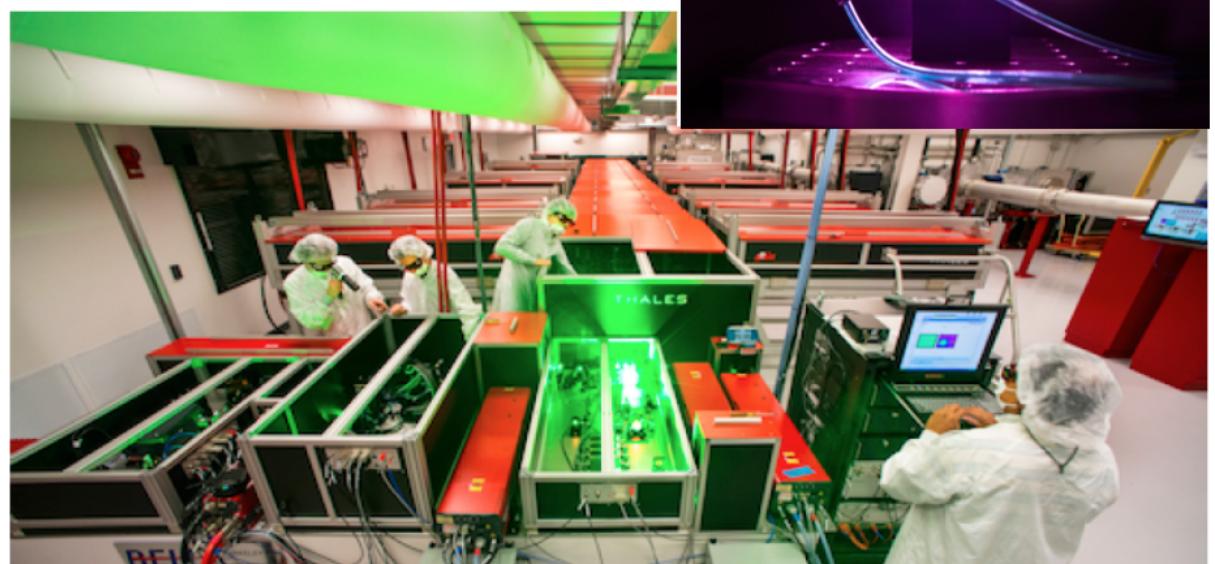
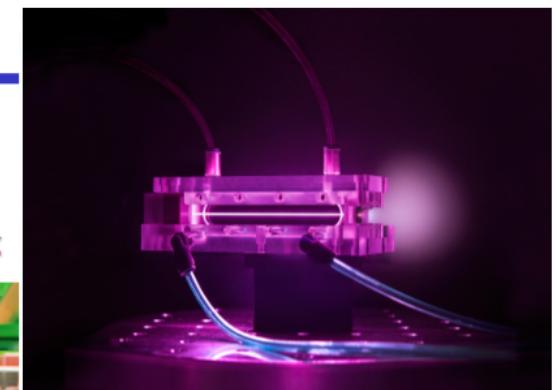
*C. Benedetti et al., proceedings of AAC2010, proceedings of ICAP2012



 **World Leader**

BELLA LPWA facility:

3 cm 1 GeV 40 TW laser ~1Hz
10-30 cm 5-10 GeV PW laser, ~1 Hz



*There is no such thing as a free lunch !!
High power Lasers are not small and not efficient.*

→ work in progress

Study of High Gradient Acceleration Techniques

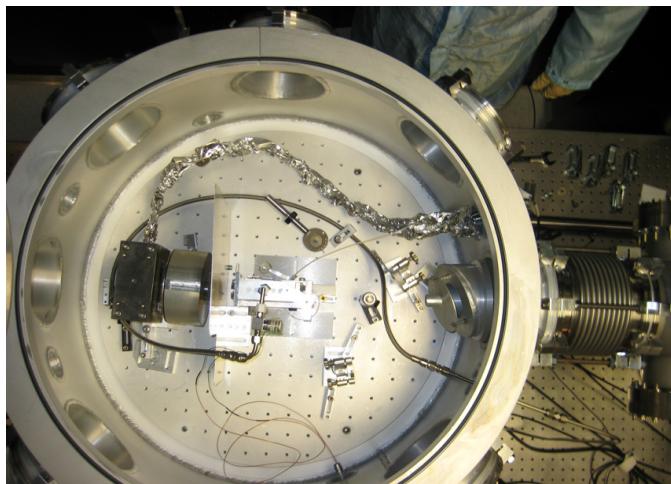
Plasma Wake Acceleration

particle beam driven / LASER driven

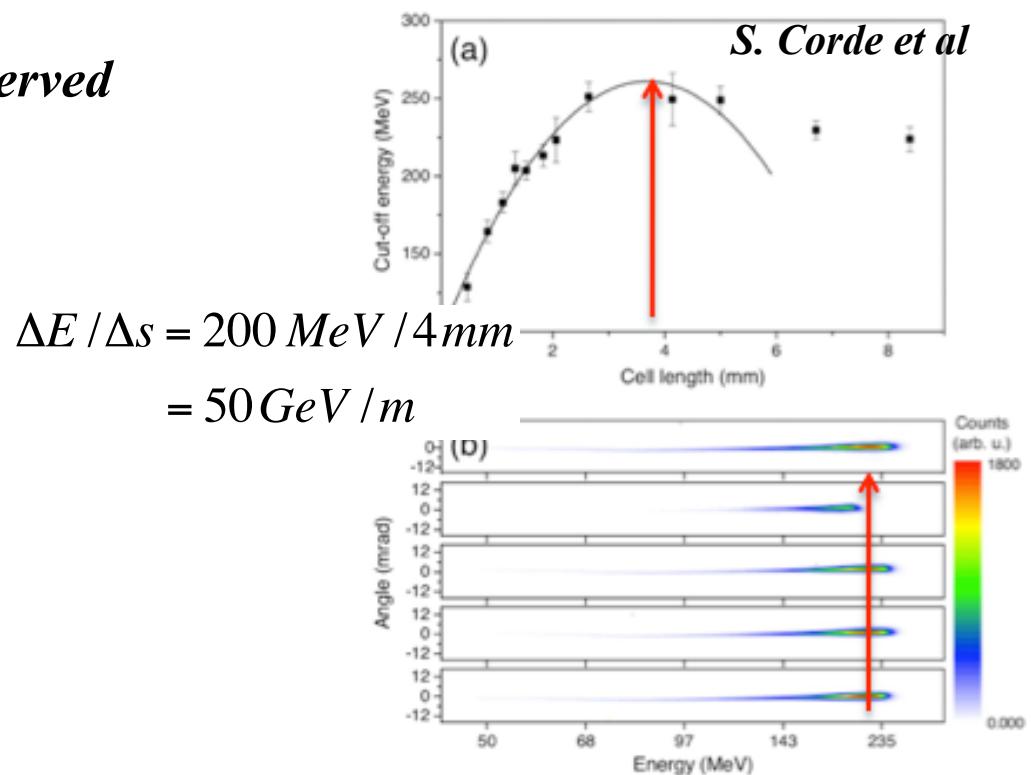
Incoming laser pulse (or pulse of particles) creates a travelling plasma wave in a low-pressure gas

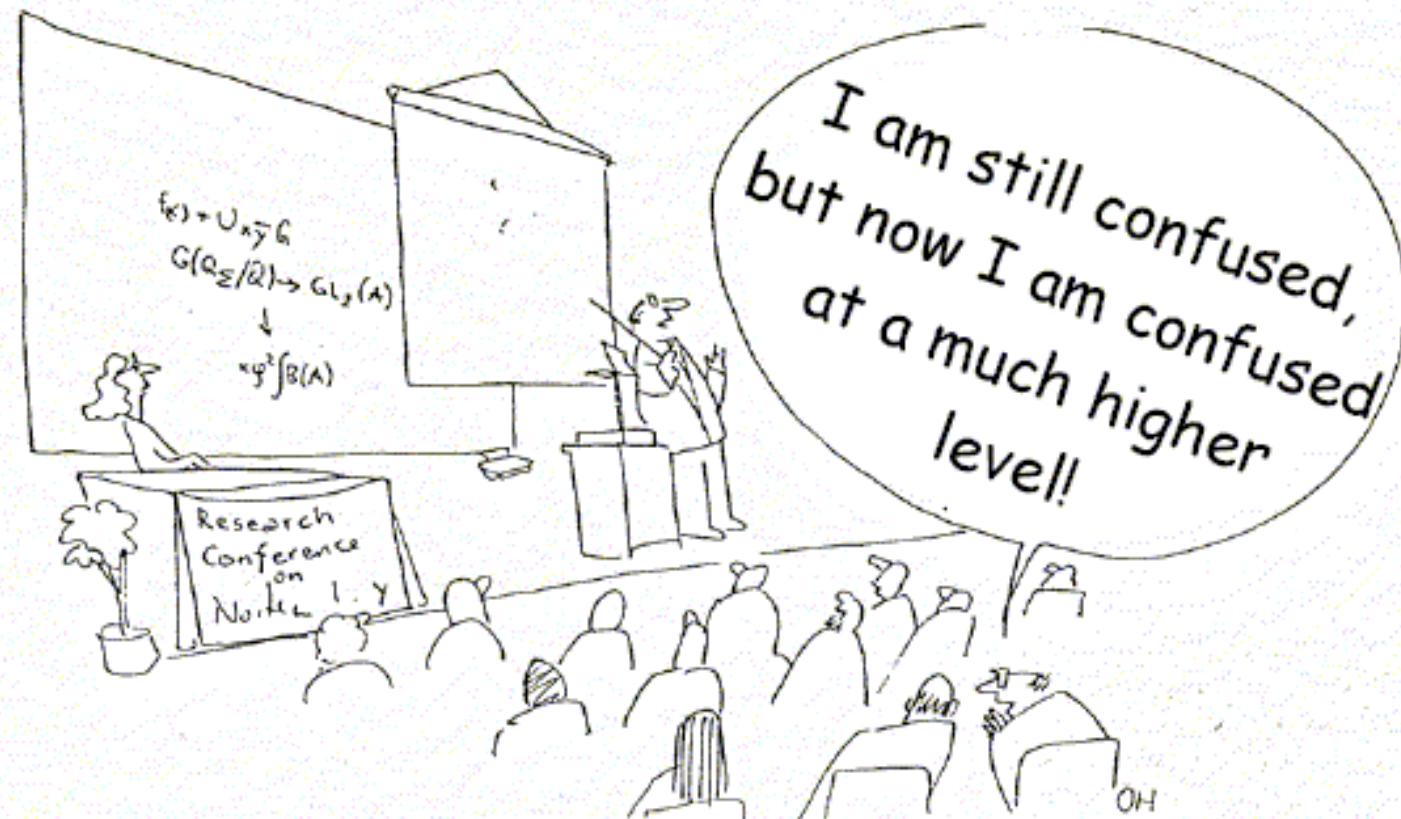
Plasma wake field gradient accelerates electrons that ‘surf’ on the plasma wave

Field Gradients up to 100 GeV/m observed



*Plasma cell Univ. Texas, Austin
 $E_e = 2 \text{ GeV}$*



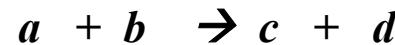
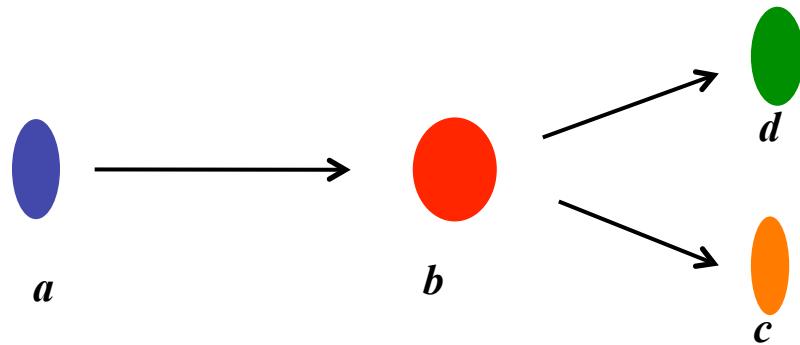


Fixed Target Machines

The (Problem of the) Centre of Mass Energy

Fixed Target experiments

accelerated particle beam hits a target at rest



Lab system: $p_b^{lab} = 0, E_b^{lab} = m_b c^2$

Centre of mass system: $p_b^{cm} + p_b^{cm} = 0$

$$\text{relativistic total energy} \quad E^2 = p^2 c^2 + (mc^2)^2$$

and for a single particle as well as for system of particles the overall rest energy is constant

... invariance of the 4momentum scalar product

$$\sum_i E_i^2 - \sum_i p_i^2 c^2 = (Mc^2)^2 = \text{const}$$

$$(E_a^{cm} + E_b^{cm})^2 - (p_a^{cm} + p_b^{cm})^2 c^2 = (E_a^{lab} + E_b^{lab})^2 - (p_a^{lab} + p_b^{lab})^2 c^2$$

The (Problem of the) Centre of Mass Energy

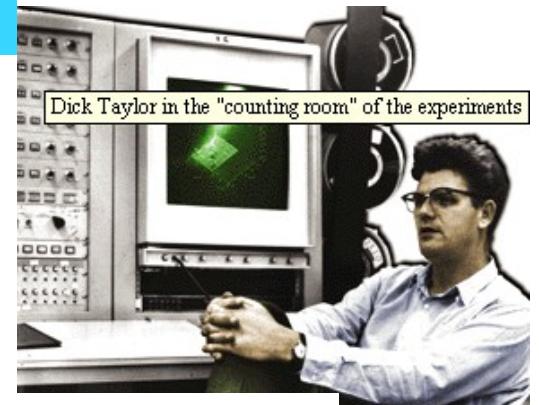
Fixed Target experiments:

$$W^2 = \left(E_a^{cm} + E_b^{cm} \right)^2 = \left(E_a^{lab} + m_b c^2 \right)^2 - \left(p_a^{lab} c \right)^2 = 2 E_a^{lab} m_b c^2 + \left(m_a^2 + m_b^2 \right) c^4$$

for $E_a^{lab} \gg m_a c^2, m_b c^2$

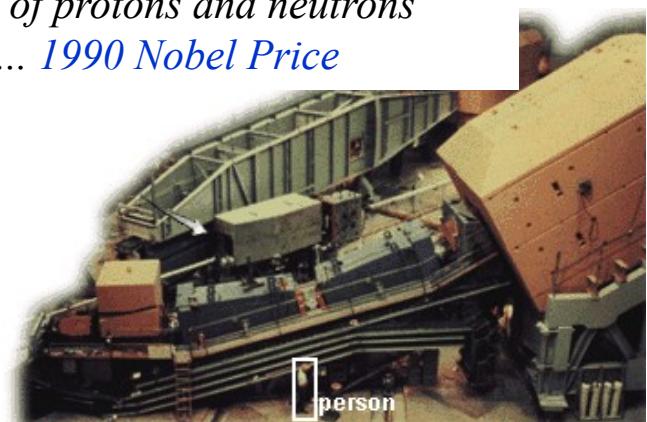
$$\Rightarrow W \approx \sqrt{2E_a^{lab} m_b c^2}$$

*For high energies in the centre of mass system,
fixed target machines are not effective.
... → need for colliding beams*



Taylor/Kendall/Friedman: Discovery of the quark structure of protons and neutrons

1966-1978 1990 Nobel Price



→ go for particle colliders

The (Problem of the) Centre of Mass Energy

Colliding Beams experiments:

$$\left(E_a^{cm} + E_b^{cm} \right)^2 - \left(p_a^{cm} + p_b^{cm} \right)^2 c^2 = \left(E_a^{lab} + E_b^{lab} \right)^2 - \left(p_a^{lab} + p_b^{lab} \right)^2 c^2$$

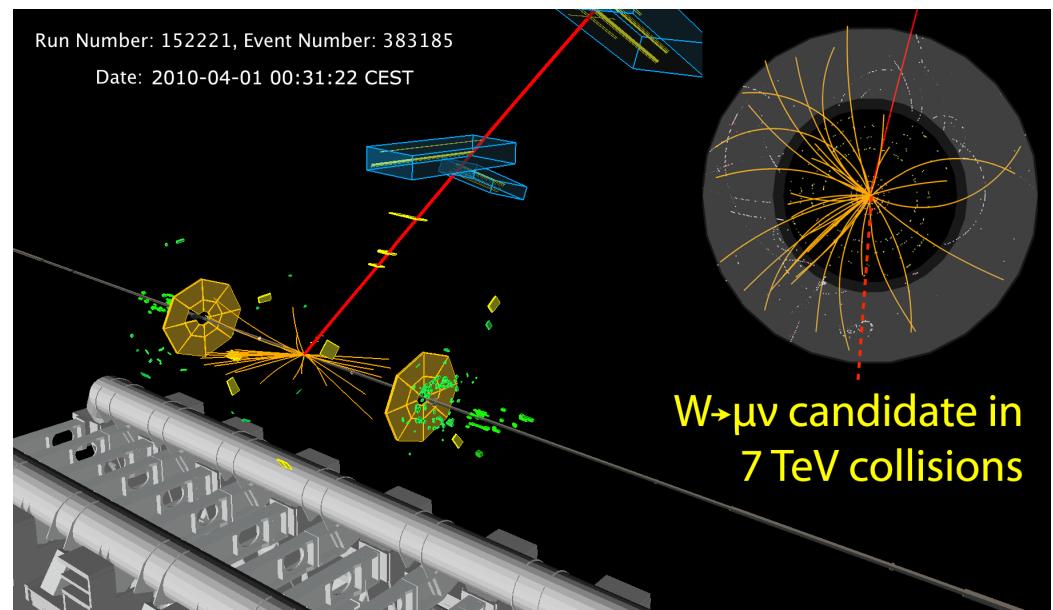
$= 0$

$$p_a^{lab} = -p_b^{lab} = 0$$

$$W^2 = \left(E_a^{cm} + E_b^{cm} \right)^2$$

$$\Rightarrow W = 2E_a^{lab}$$

*The full lab energy is available
in the center of mass system.
Prize to pay: we have to build colliders
... beam sizes = μm*



19th century:

Ludwig van Beethoven: „Mondschein Sonate“



Sonate Nr. 14 in cis-Moll (op. 27/II, 1801)

A musical score for piano, featuring two staves. The top staff is in G major (one sharp) and the bottom staff is in C major (no sharps or flats). The key signature changes to one sharp (C-sharp minor) at the beginning of the piece. The score consists of two measures of music, each starting with a forte dynamic (indicated by a large 'F'). The notes are eighth notes, and the time signature is common time (indicated by a 'C').

Cis-Moll op. 27 Nr. 2

Astronomer Hill:

differential equation for motions with periodic focusing properties
„Hill ‘s equation“

*Example: particle motion with
periodic coefficient*



equation of motion: $x''(s) - k(s)x(s) = 0$

*restoring force $\neq \text{const}$,
 $k(s)$ = depending on the position s
 $k(s+L) = k(s)$, periodic function*

}

*we expect a kind of quasi harmonic
oscillation: amplitude & phase will depend
on the position s in the ring.*

Amplitude of a particle trajectory:

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \varphi)$$

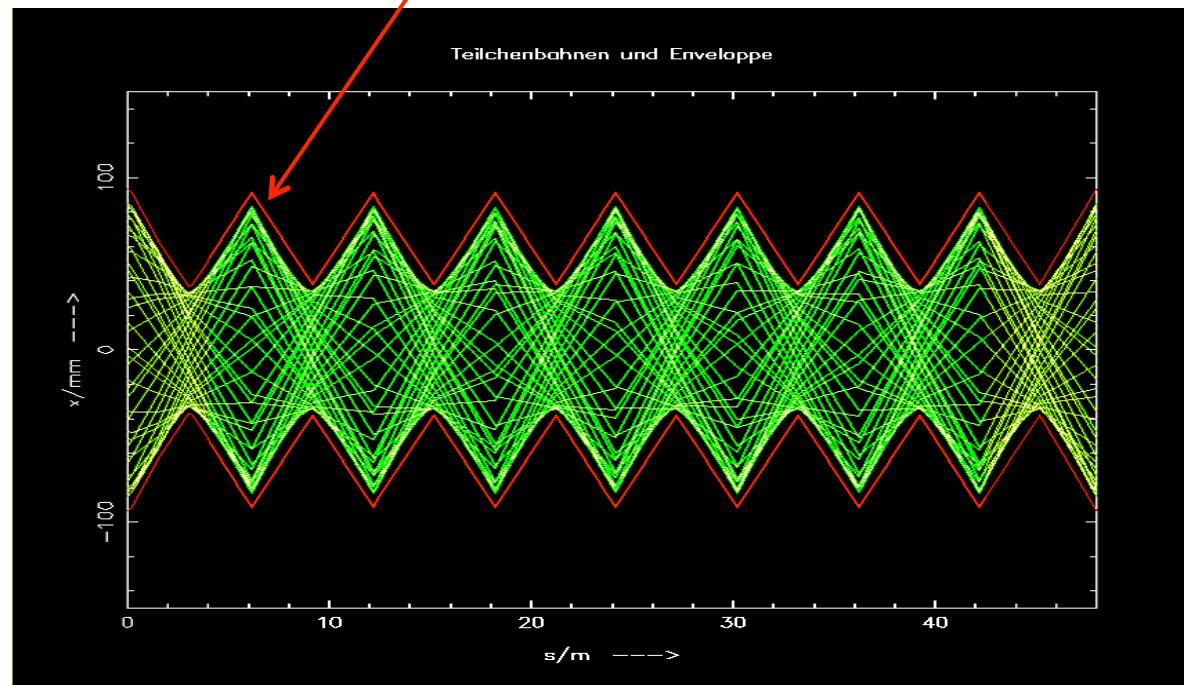
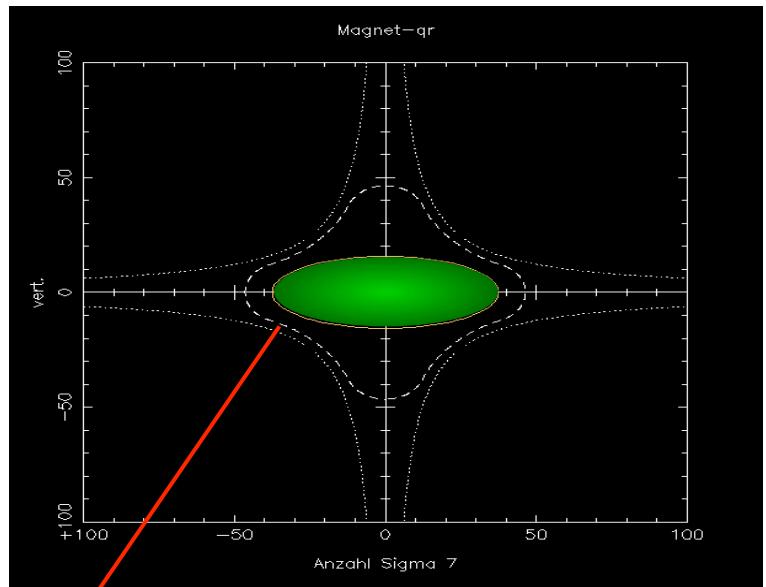
Maximum size of a particle amplitude

$$\hat{x}(s) = \sqrt{\varepsilon} \sqrt{\beta(s)}$$

The Beta Function

β determines the beam size
... the envelope of all particle
trajectories at a given position
“s” in the storage ring under
the influence of all (!) focusing
fields.

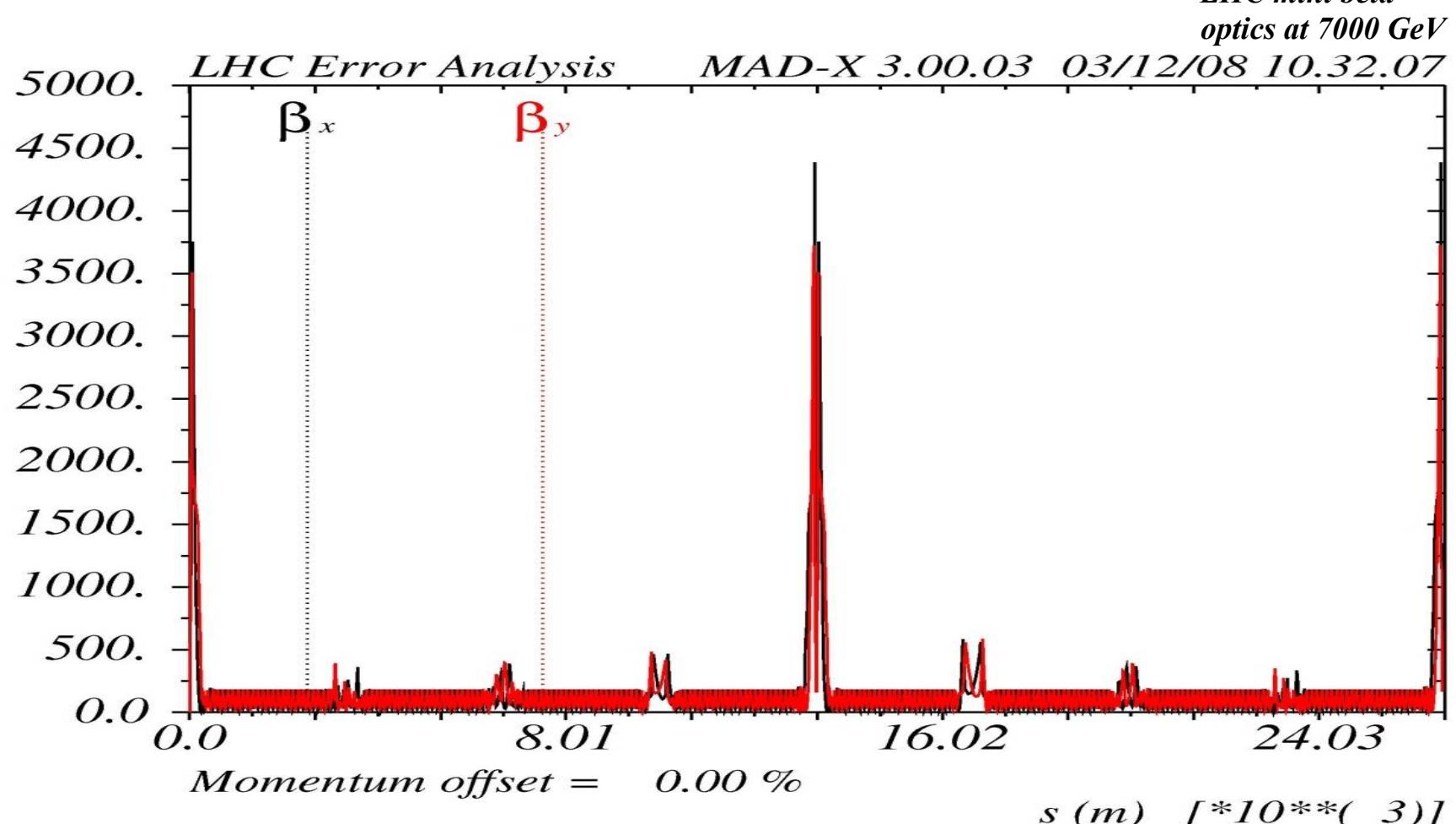
It reflects the periodicity of the
magnet structure.



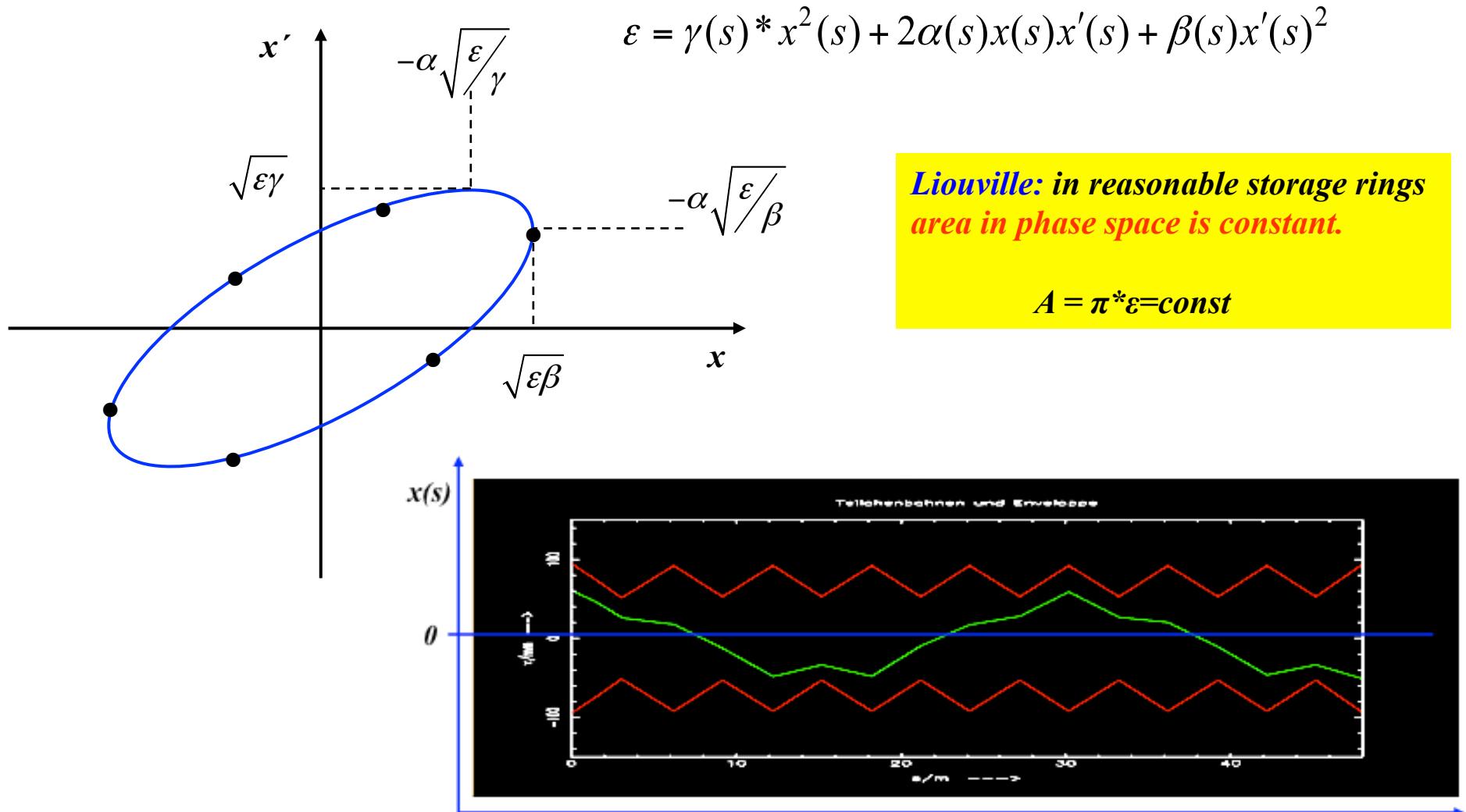
The Beta Function: Lattice Design & Beam Optics

The beta function determines the maximum amplitude a single particle trajectory can reach at a given position in the ring.

It is determined by the focusing properties of the lattice and follows the periodicity of the machine.



Beam Emittance and Phase Space Ellipse



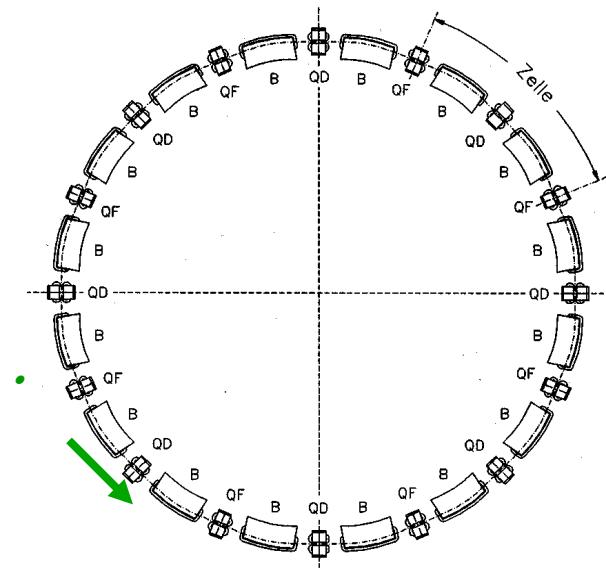
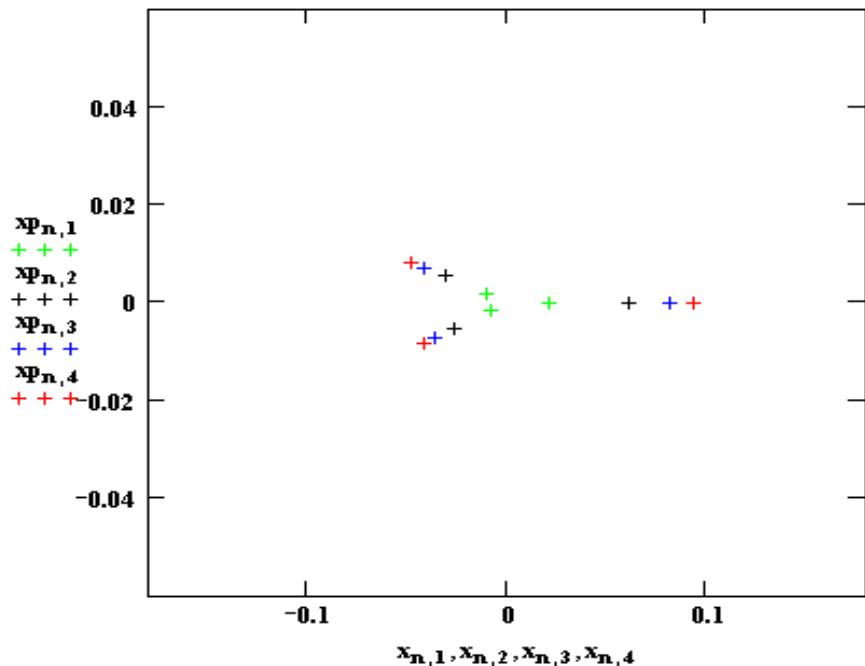
ε beam emittance = *woozility* of the particle ensemble, *intrinsic beam parameter*, cannot be changed by the foc. properties.

Scientifiquely spoken: area covered in transverse x, x' phase space ... and it is constant !!!

Particle Tracking in a Storage Ring

Calculate x, x' for each accelerator element according to matrix formalism and plot x, x' at a given position „ s “ in the phase space diagram

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$



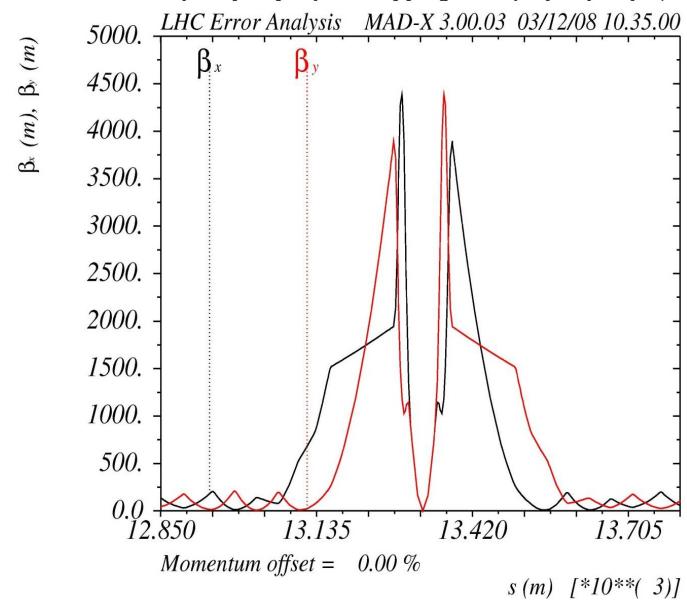
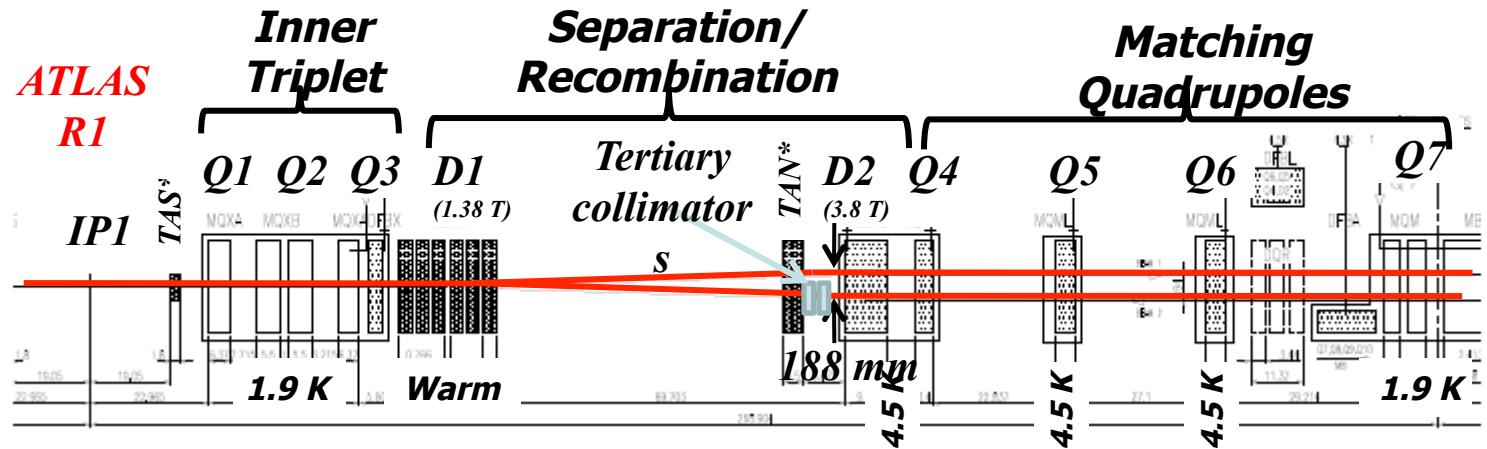
A beam of 4 particles
– each having a slightly
different emittance:

... just as Big Ben

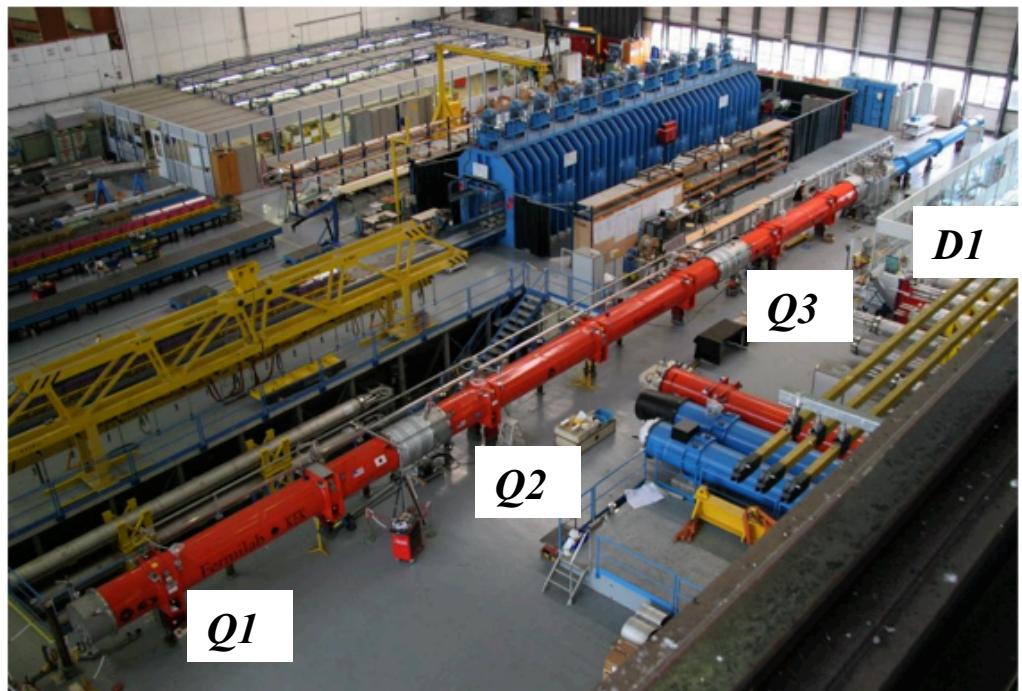


... and just as any harmonic pendulum

The LHC Mini-Beta-Insertions



mini β optics



Mini-Beta-Insertions in phase space

*A mini- β insertion is always a kind of
special symmetric drift space.*

→ greetings from Liouville

*the smaller the beam size
the larger the beam divergence*

$$\sqrt{\varepsilon / \beta}$$

