Cold QCD and
Compact Stars:
EOS, Cooling
and Axions

A Sedrakian

Introduction

Cold QCD and
compact stars

Phase
transitions in
QCD and
cooling
transient in Cas
A

Axion cooling
of NS

Thank you

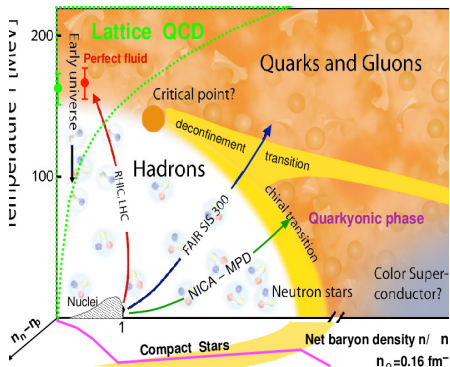# Cold QCD and Compact Stars: EOS, Cooling and Axions

Armen Sedrakian

From quarks to gravitational waves: Neutron stars as a laboratory for fundamental physics

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Cold QCD and
Compact Stars:
EOS, Cooling
and Axions

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1. Equation of state of hypernuclear and color superconducting matter
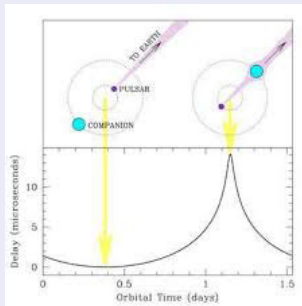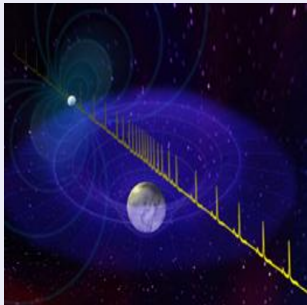2. Phase transitions, neutrino radiation and thermal cooling of neutron stars
3. Axion limits from astrophysics

I. Hypernuclear matter, cold QCD and compact stars

A two-solar-mass neutron star measured

The largest pulsating star yet observed doubts on exotic matter theories

The binary millisecond pulsar J1614-223010+11 Shapiro delay signature:

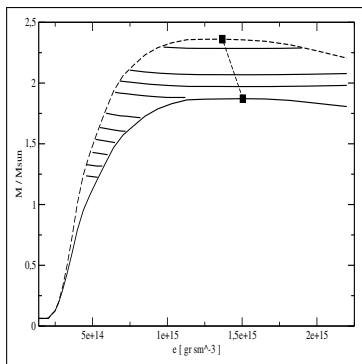$$\Delta t = -\frac{2GM}{c^3}\log(1 - \vec{R} \cdot \vec{R}').$$ (1)

The pulsars mass 1.97 ± 0.04 solar masses which rules out almost all currently proposed hyperon or boson condensate equations of state. (Demorest et al, 2010, Nature 467, 1081)

- Einstein's field equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -8\pi T_{\mu\nu},$$

- Energy-momentum tensor:

$$T_{\mu\nu} = -P(r)g_{\mu\nu} + [P(r) + \epsilon(r)]\, u_\mu u_\nu$$

TOV equations:

$$
\begin{aligned}
\frac{dP(r)}{dr} &= -\frac{G\epsilon(r)M(r)}{c^2 r^2}\left(1 + \frac{P(r)}{\epsilon(r)}\right)\left(1 + \frac{4\pi r^3 P(r)}{M(r)c^2}\right)\left(1 - \frac{2GM(r)}{c^2 r}\right)^{-1}. \\
M(r) &= 4\pi \int_0^r r^2 \epsilon(r)\, dr.
\end{aligned}
$$

To generate the DFT start with the Lagrangian :

$$
\begin{aligned}
\mathcal{L} &= \sum_B \bar{\psi}_B \left[\gamma^\mu \left(i\partial_\mu - g_{\omega BB}\omega_\mu - \frac{1}{2}g_{\rho BB}\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu \right) - (m_B - g_{\sigma BB}\sigma) \right] \psi_B \\
&+ \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\boldsymbol{\rho}^{\mu\nu}\boldsymbol{\rho}_{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}^\mu\cdot\boldsymbol{\rho}_\mu \\
&+ \sum_\lambda \bar{\psi}_\lambda (i\gamma^\mu\partial_\mu - m_\lambda)\psi_\lambda - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad\quad\quad (2)
\end{aligned}
$$

- B-sum is over the baryonic octet $B \equiv p, n, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}$
- Meson fields include σ meson, $\boldsymbol{\rho}_\mu$-meson and ω_μ-meson
- Leptons include electrons, muons and neutrinos for $T \neq 0$

Pure phenomenology: the mesonic mean fields generate the most general form of Lorentz couplings consistent with the symmetries, but the parameters such as masses and couplings do not have a stict physical interpretation in terms of observable mesons and are not fitted to the scattering or bound state data. DFT fits the bulk properties of matter and nuclear binding and rms radii.

$$
\begin{aligned}
g_{iN}(\rho_B) &= g_{iN}(\rho_0)h_i(x), \qquad i = \sigma, \omega, \qquad h_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2} \quad (3) \\
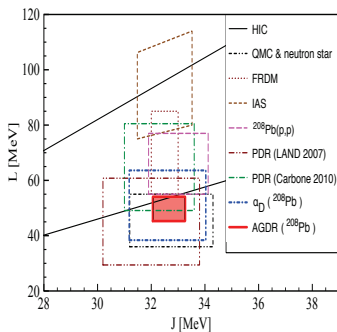g_{\rho N}(\rho_B) &= g_{\rho N}(\rho_0)\exp[-a_\rho(x - 1)]. \quad (4)
\end{aligned}
$$

DD-ME2 parametrization of D. Vretenar, P. Ring et al. Phys. Rev. C 71, 024312 (2005).

	σ	ω	ρ
m_i [MeV]	550.1238	783.0000	763.0000
$g_{Ni}(\rho_0)$	10.5396	13.0189	3.6836
a_i	1.3881	1.3892	0.5647
b_i	1.0943	0.9240	—
c_i	1.7057	1.4620	—
d_i	0.4421	0.4775	—

Total number of parameters 8: boundary conditions on $h(x)$ at $x = 1$.

Consistency between the density functional and experiment

- saturation density
  $\rho_0 = 0.152 \text{ fm}^{-3}$

- binding energy per nucleon
  $E/A = -16.14 \text{ MeV}$,

- incompressibility
  $K_0 = 250.90 \text{ MeV}$,

- symmetry energy $J = 32.30$
  MeV,

- symmetry energy slope
  $L = 51.24 \text{ MeV}$,

- symmetry incompressibility
  $K_{sym} = -87.19 \text{ MeV}$

$$K_0 = k_F^2 \frac{\partial E/A}{\partial k^2}\Big|_{k=k_F} = 9\rho_0^2 \frac{\partial^2 E/A}{\partial \rho^2}\Big|_{\rho=\rho_0}, \qquad S(\rho) = \frac{1}{2}\frac{\partial^2 \epsilon/\rho}{\partial \delta^2}\Big|_{\delta=0}. \qquad (5)$$

$$S(\rho) = J + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{1}{2}K_{\text{sym}}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2. \qquad (6)$$

Hyperon-vector mesons couplings: $SU(3)$-flavor symmetry and vector dominance model

$$
\begin{aligned}
g_{\Xi\omega} &= \frac{1}{3}g_{N\omega}, \quad g_{\Sigma\omega} = g_{\Lambda\omega} = \frac{2}{3}g_{N\omega}. \quad (7)\\
g_{\Xi\rho} &= g_{\rho N}, \qquad g_{\Sigma\rho} = 2g_{\rho N}, \qquad g_{\Lambda\rho} = 0. \quad (8)
\end{aligned}
$$

And no strange mesons (for example ϕ) in the theory; see Schaffner et al, Bednarek et al

Scalar σ-meson – hyperon couplings are varied:

$$
1 \leq \frac{1}{2}(3x_{\Lambda\sigma} + x_{\Sigma\sigma}) \leq 2x_{N\sigma}, \quad x_{i,\sigma} = \frac{g_{i\Sigma}}{g_{N\sigma}} \quad (9)
$$

Nijmegen soft-core potential implies that

$$
x_{\Lambda\sigma} = 0.58, \qquad x_{\Sigma\sigma} = 0.448. \quad (10)
$$

	Λ $1s_{1/2}$ state [MeV]	E/A [MeV]	r_p [fm]	r_n [fm]	r_Λ [fm]
$^{17}_{\Lambda}$O					
a	0.846	-7.443	2.609	2.579	8.313
b	-4.564	-7.760	2.606	2.576	3.203
c	-27.279	-9.035	2.563	2.534	1.977
$^{41}_{\Lambda}$C					
a	0.934	-8.336	3.372	3.319	8.710
b	-8.519	-8.565	3.370	3.317	3.168
c	-35.224	-9.199	3.347	3.294	2.298
$^{49}_{\Lambda}$C					
a	0.973	-8.442	3.389	3.576	8.825
b	-9.882	-8.662	3.387	3.571	3.140
c	-37.257	-9.207	3.365	3.548	2.419

Properties of Λ-hypernuclei $^{17}_{\Lambda}$O, $^{41}_{\Lambda}$C, and $^{49}_{\Lambda}$C for the models a, b, and c. The columns list the single-particle energy of the Λ $1s_{1/2}$ state, the binding energy and the rms radii for neutrons, protons and Λ-hyperon. model a with $x_{\sigma\Lambda} = 0.52$, model b with $x_{\sigma\Lambda} = 0.59$, and model c with $x_{\sigma\Lambda} = 0.66$; all three models have $x_{\omega\Lambda} = 2/3$ and $x_{\rho\Lambda} = 0$.

	$E_{Mass}[\Lambda\ 1s_{1/2}]$ [MeV]	$E[\Lambda\ 1s_{1/2}]$ [MeV]	E/A [MeV]	r_p [fm]	r_n [fm]	r_Λ [fm]
$^{17}_{\Lambda}$O	-12.109	-11.716	-8.168	2.592	2.562	2.458
^{16}O	$-$	$-$	-8.001	2.609	2.579	$-$
$^{41}_{\Lambda}$C	-17.930	-17.821	-8.788	3.362	3.309	2.652
^{40}Ca	$-$	$-$	-8.573	3.372	3.320	$-$
$^{49}_{\Lambda}$Ca	-19.215	-19.618	-8.858	3.379	3.562	2.715
^{48}Ca	$-$	$-$	-8.641	3.389	3.576	$-$

Single-particle energies of the Λ $1s_{1/2}$ states, binding energies, and rms radii of the Λ-hyperon, neutron, and proton of $^{17}_{\Lambda}$O, $^{41}_{\Lambda}$C, and $^{49}_{\Lambda}$Ca are presented for optimal model. In addition, single-particle energies of the Λ $1s_{1/2}$ states, i.e. separation energies of the Λ-particle, obtained from the mass formula of Levai et al (1998) are given for these Λ-hypernuclei. Furthermore, the properties of ^{16}O, ^{40}Ca, and ^{48}Ca are given for the optimal model. The optimal model obtained in this way has $x_{\sigma\Lambda} = 0.6164$.
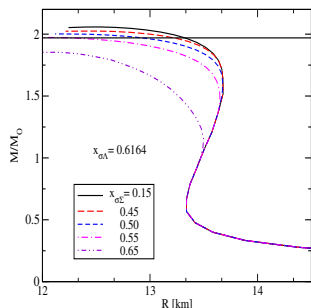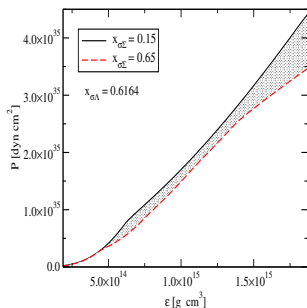
Zero temperature equations of state of hypernuclear matter for fixed $x_{\sigma\Lambda} = 0.6164$ and a range of values $0.15 \leq x_{\sigma\Sigma} \leq 0.65$. These values generate the shaded area, which is bound from below by the softest EoS (dashed red line) corresponding to $x_{\sigma\Sigma} = 0.65$ and from above by the hardest EoS (solid line) corresponding to $x_{\sigma\Sigma} = 0.15$.
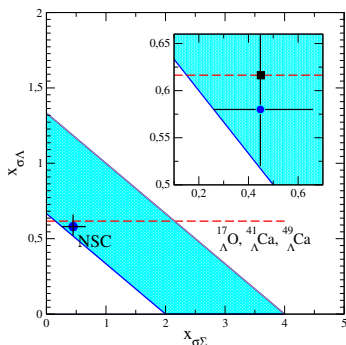
- Blue region - values allowed by the inequality
- NSC values are shown by dot, red line - Λ-hypernuclei, cross - our parameter space hypernuclear couplings, square - limit set by $2M_\odot$.

- We used a relativistic density functional theory of hypernuclear matter to extract bounds on the density-dependent couplings of a hypernuclear DFT

- Did simultaneous fits to the medium-heavy Λ-hypernuclei and used the requirement that the maximum mass of a hyperonic compact star is at least two-solar masses.

- Narrowed down significantly the parameter space of couplings of DFT - the range of optimal values of parameters is given by

$$x_{\sigma\Lambda} = 0.6164, \qquad 0.15 \le x_{\sigma\Sigma} \le 0.5. \qquad (11)$$

Our work provides a proof-of-principle of the method for constraining any theoretical framework that describes hypernuclear systems using laboratory and astrophysical data.

E. N. E. van Dalen, G. Colucci and A. Sedrakian,
Constraining hypernuclear density functional with Λ-hypernuclei and compact stars.
Physics Letters B **734**, 383 (2014).
G. Colucci and A. Sedrakian,
Equation of state of hypernuclear matter: impact of hyperon–scalar-meson couplings.
Phys. Rev. C **87**, 055806 (2013).

Color-superconductivity within the NJL model

Lagrangian of the model:

$$
\begin{aligned}
\mathcal{L}_Q &= \bar{\psi}(i\gamma^\mu\partial_\mu - \hat{m})\psi + G_V(\bar{\psi}i\gamma^\mu\psi)^2 + G_S\sum_{a=0}^{8}[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma_5\lambda_a\psi)^2] \\
&+ G_D\sum_{\gamma,c}[\bar{\psi}_\alpha^a i\gamma_5\epsilon^{\alpha\beta\gamma}\epsilon_{abc}(\psi_C)_\beta^b][(\bar{\psi}_C)_\rho^r i\gamma_5\epsilon^{\rho\sigma\gamma}\epsilon_{rsc}\psi_\sigma^s] \\
&- K\left\{\det_f[\bar{\psi}(1+\gamma_5)\psi] + \det_f[\bar{\psi}(1-\gamma_5)\psi]\right\},
\end{aligned}
$$

- quarks: ψ_α^a, color $a = r, g, b$, flavor ($\alpha = u, d, s$);
- mass matrix: $\hat{m} = \mathrm{diag}_f(m_u, m_d, m_s)$;
- other notations: λ_a, $a = 1, ..., 8$, $\psi_C = C\bar{\psi}^T$ and $\bar{\psi}_C = \psi^T C$, $C = i\gamma^2\gamma^0$.

Parameters of the model:

- G_S the scalar coupling and cut-off Λ are fixed from vacuum physics
- G_D is the di-quark coupling $\simeq 0.75 G_S$ (via Fierz) but free to change
- G_V and ρ_{tr} are treated as free parameters

QCD interactions are most attractive in color anti-triplet channel. For order parameter:

$$\Delta \propto \langle 0|\psi_{\alpha\sigma}^a \psi_{\beta\tau}^b|0\rangle$$

- Antisymmetry in colors a, b for attraction
- Antisymmetry in spins σ, τ for the BCS mechanism to work
- Antisymmetry in flavors α, β to avoid Pauli blocking

Low densities and temperatures: 2SC phase

$$\Delta(2SC) \propto \Delta\epsilon^{ab3}\epsilon_{\alpha\beta} \qquad \delta\mu \ll \Delta, \qquad m_s \simeq 0.$$

Low-temperatures, large mismatch *inhomogeneous (crystalline)* 2SC phase

$$\Delta(CSC) \propto \Delta\epsilon^{ab3}\epsilon_{\alpha\beta}, \qquad \delta\mu \sim \Delta, \qquad m_s \neq 0.$$

High densities nearly massless u, d, s quarks: CFL phase

$$\Delta(CFL) \propto \langle 0|\psi_{\alpha L}^a \psi_{\beta L}^b|0\rangle = -\langle 0|\psi_{\alpha R}^a \psi_{\beta R}^b|0\rangle = \Delta\epsilon^{abC}\Delta\epsilon_{\alpha\beta C}.$$

Other options are less favorable energetically.

Combining the equations of state:

- Assume the surface tension between nuclear and quark matter is high enough - no mixed phases and Maxwell construction can be applied
- Transition (deconfinement) occurs at some baryon chemical potential

$$P_N(\mu_B) = P_Q(\mu_B)$$

If surface tension is small, mixed phases will appear (N. Glendenning 90', more recently F. Weber et al developed this line, inluding transport and neutrino radiation in mixed phases.)

EOS with equilibrium between nuclear, hypernuclear, 2SC and CFL phases of matter
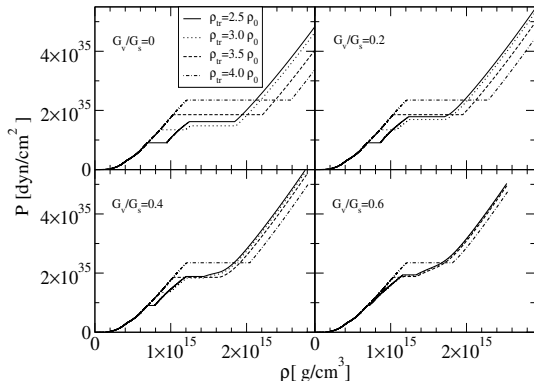
- Phase equilibrium is constructed via Maxwell prescription
- Sequential phase transition $NM \rightarrow 2SC \rightarrow CFL$

L. Bonanno, A. Sedrakian,
Composition and stability of hybrid stars with hyperons and quark color-superconductivity.
Astronomy and Astrophysics **539**, A16 (2012).

Stability (*M-R* diagram) and Internal structure of hybrid massive $\sim 1.9 M_\odot$ star with color superconducting phases
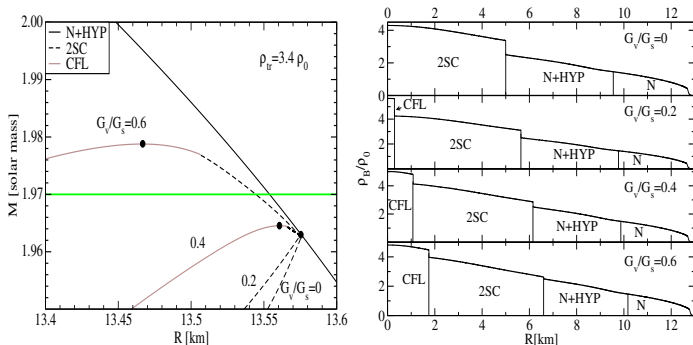
- Small G_V 2SC only. For lagre G_V CFL phase appears.
- Stability is achieved for $G_V > 0.2$ and transition densities few ρ_0

The parameter space for the stability of compact stars
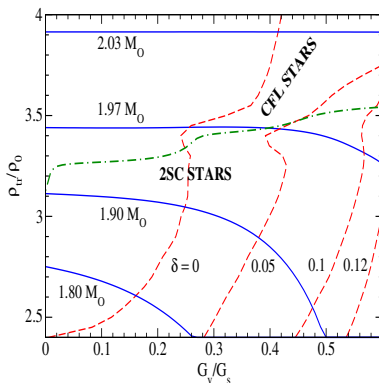
- 2SC stars are stable below green dash-dotted line
- CFL stars are stable to the right of red dashed curves
- δ indicates the fraction of the CFL phase in the hybrid star

- To produce heavy and exotics featuring neutron stars it is sufficient to generate a stiff NM equation state above saturation.
- Vector interactions are mandatory to stabilize color superconducting quark stars.
- A $2M_\odot$ mass star does not exclude exotic matter in the cores of NS
- Hyperonic sector requires also repulsive vector interactions or weakening of the σ exchange compared to SU(3) predictions
- Modifications of the gravity are also a possibility

- improvements in the range $0.5 \leq \rho/\rho_0 \leq 2$ should be possible with the use of microscopically motivated models
- Quark matter EoS can be constrained at high densities by perturbative QCD results

L. Bonanno, A. Sedrakian,
Composition and stability of hybrid stars with hyperons and quark color-superconductivity.
Astronomy and Astrophysics **539**, A16 (2012).

G. Colucci, D. H. Rischke and A. Sedrakian,
Rotating hybrid compact stars.
Astron. and Astrophys. **559**, A118 (2013).

II. Phase transitions in QCD and cooling transient in Cas A

Chandra X-ray image of Cas A - the youngest (320 yr) SNR in the Milky Way.
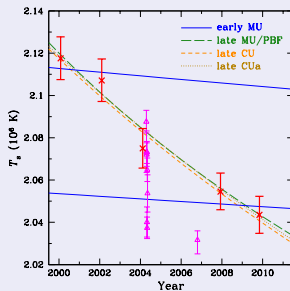
NASA's Chandra X-ray Observatory has discovered the first direct evidence for a superfluid. Conclusions drawn from cooling simulations of the neutron stars.

The data has been doubted since. Possible instrumental contamination effects, obscuring of the star by passing cloud, geometrical effects
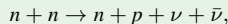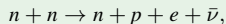
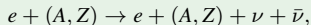## Thermal history of Compact Stars (Cosmology in a single compact star)
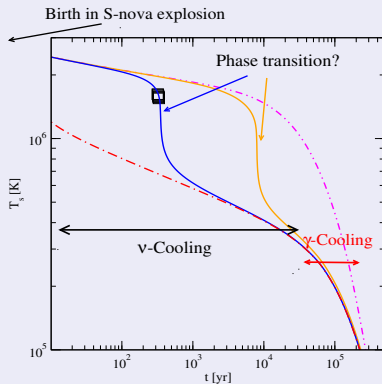
$$\boxed{\frac{\partial S}{\partial t} + \vec{\nabla} \cdot \vec{q} = \mathcal{R} - \mathcal{L}_\nu - \mathcal{L}_\gamma} \quad dS = c_V dT, \quad \vec{q} = \kappa \vec{\nabla} T$$

- Dissipative function $\mathcal{R} \propto \sigma(\nabla\phi)^2 + \kappa(\nabla T)^2/T + \dots$
- Neutrino losses $\mathcal{L}_\nu$ from the bulk
- Photon losses $\mathcal{L}_\gamma$ from the surface
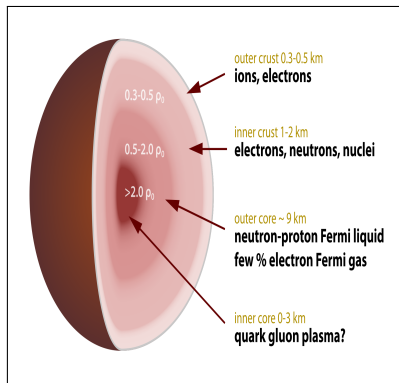
- Modified Urca/brems process

$$n + n \rightarrow n + p + e + \bar{\nu},$$
$$n + n \rightarrow n + p + \nu + \bar{\nu},$$

- Crustal bremsstrahlung

$$e + (A, Z) \rightarrow e + (A, Z) + \nu + \bar{\nu},$$

- Cooper pair-breaking-formation

$$[NN] \rightarrow [NN] + \nu + \bar{\nu},$$

- Surface photo-emission

$$L_\gamma = 4\pi\sigma R^2 T^4$$

- Quark Urca process

$$d \rightarrow u + e + \bar{\nu}, \quad u + e \rightarrow d + \nu.$$

outer crust 0.3-0.5 km
ions, electrons

0.3-0.5 ρ_0

0.5-2.0 ρ_0

inner crust 1-2 km
electrons, neutrons, nuclei

>2.0 ρ_0

outer core ~ 9 km
**neutron-proton Fermi liquid
few % electron Fermi gas**

inner core 0-3 km
quark gluon plasma?

All axial vector neutral current process lead to axion emission

- ν and $\bar{\nu}$ - Boltzmann equations with KB collision integrals

$$\left[\partial_t + \vec{\partial}_q\, \omega_\nu(\vec{q})\vec{\partial}_x\right] f_\nu(\vec{q},x) = \int_0^\infty \frac{dq_0}{2\pi} \mathrm{Tr}\left[\Omega^<(q,x)S_0^>(q,x) - \Omega^>(q,x)S_0^<(q,x)\right],$$

- ν-quasiparticle propagators:

$$S_0^<(q,x) = \frac{i\pi\slashed{q}}{\omega_\nu(\vec{q})}\left[\delta\left(q_0 - \omega_\nu(\vec{q})\right)f_\nu(q,x) - \delta\left(q_0 + \omega_\nu(\vec{q})\right)\left(1 - f_{\bar{\nu}}(-q,x)\right)\right].$$

- self-energies of neutrinos

$$-i\Omega^{>,<}(q_1,x) = \int \frac{d^4q}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}(2\pi)^4\delta^4(q)i\Gamma_{Lq}^\mu\, iS_0^<(q_2,x)i\Gamma_{Lq}^{\dagger\,\lambda}\, i\Pi_{\mu\lambda}^{>,<}(q,x),$$
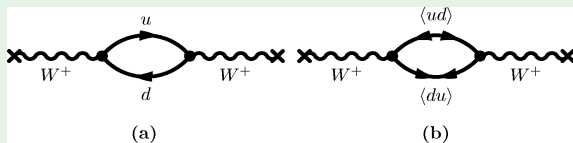
- energy loss per unit time and volume

$$\epsilon_{\nu\bar{\nu}} = \frac{d}{dt}\int \frac{d^3q}{(2\pi)^3}\left[f_\nu(\vec{q}) + f_{\bar{\nu}}(\vec{q})\right]\omega_\nu(\vec{q}) \tag{12}$$

- expressed through the collision integrals

$$\epsilon_{\nu\bar{\nu}} = -2\left(\frac{G}{2\sqrt{2}}\right)^2\sum_f \int \frac{d^3q_2}{(2\pi)^3 2\omega_\nu(\vec{q}_2)}\int \frac{d^3q_1}{(2\pi)^3 2\omega_\nu(\vec{q}_1)}\int \frac{d^4q}{(2\pi)^4}$$
$$(2\pi)^4\delta^3(\vec{q}_1 + \vec{q}_2 - \vec{q})\delta(\omega_\nu(\vec{q}_1) + \omega_\nu(\vec{q}_2) - q_0)\left[\omega_\nu(\vec{q}_1) + \omega_\nu(\vec{q}_2)\right]$$
$$g_B(q_0)\left[1 - f_\nu(\omega_\nu(\vec{q}_1))\right]\left[1 - f_{\bar{\nu}}(\omega_\nu(\vec{q}_2))\right]\Lambda^{\mu\lambda}(q_1,q_2)\mathrm{Im}\,\Pi_{\mu\lambda}^R(q).$$

Polarization tensors at one loop

(a)   (b)

$$\Pi_{\mu\lambda}(q) = -i \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left[(\Gamma_-)_\mu S(p)(\Gamma_+)_\lambda S(p+q) \right]$$

with the input

$$
\begin{aligned}
\Gamma_{\pm}(q) &= \gamma_\mu (1 - \gamma_5) \otimes \tau_{\pm} \\
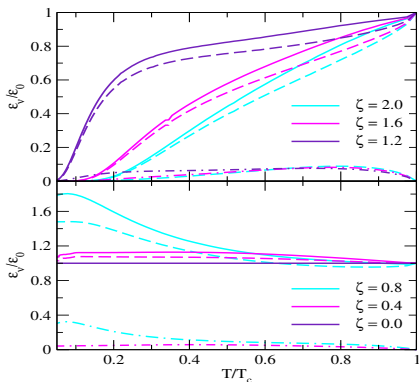S_{f=u,d} &= i\delta_{ab} \frac{\Lambda^+(p)}{p_0^2 - \epsilon_p^2} (\slashed{p} - \mu_f \gamma_0), \\
F(p) &= -i\epsilon_{ab3}\epsilon_{fg}\Delta \frac{\Lambda^+(p)}{p_0^2 - \epsilon_p^2} \gamma_5 C
\end{aligned}
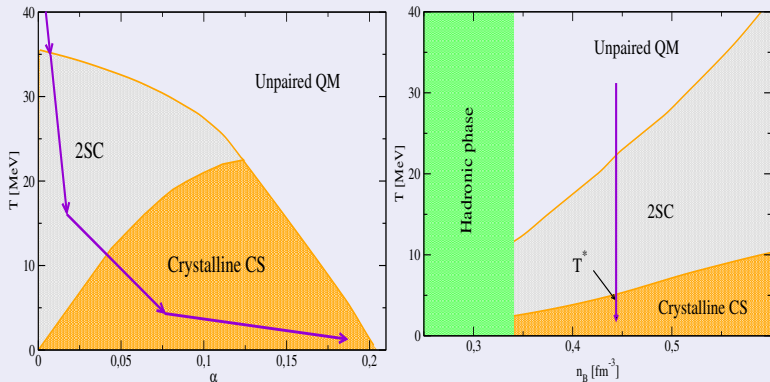\tag{13}
$$

In the red-green color sector

$$\zeta = \Delta_{rd}/\delta\mu, \qquad \delta\mu = (\mu_d - \mu_u)/2$$

and in the blue color sector

$$\Delta_b = 0 \qquad \Delta_b \neq 0.$$

Upper panel: perfect 2SC phase $\zeta > 1$, Lower panel: crystalline phase $\zeta < 1$

Phase transition within the QCD phase diagram can take place from one pairing pattern to the other (e.g. 2SC to Crystalline)

Left: Generic phase diagram of imbalanced fermi-systems; Right: β-equilibrated stellar matter

- 2SC phase - fully gapped, no excitations, ν-emission strongly suppressed
- gapless (or crystalline) phase ν-emission enhanced
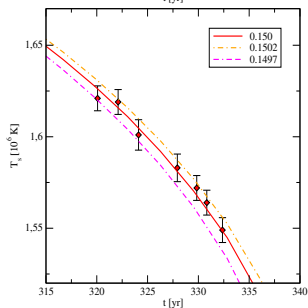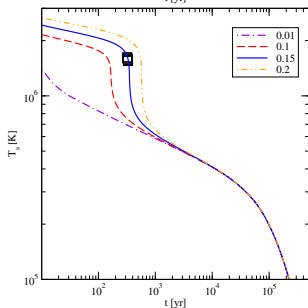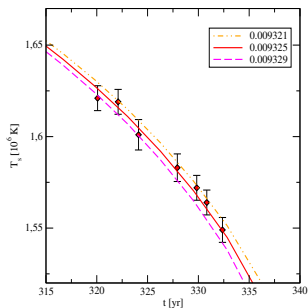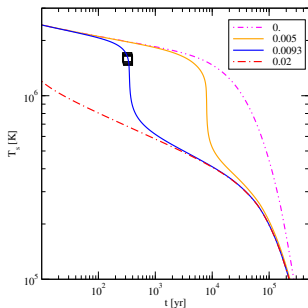
Fine tuned to the Cas A data using the phase transition temperature T^* and Δ_b

Conclusions on Cas A transient

- At lower temperatures a transition from the BCS to a generic gapless phase must take place in the QCD phase diagram.
- We have modelled the emissivities of both phase in terms of a simple parameterization, which however takes into account the fact that as the temperature is lowered there is a phase transition from the 2SC to FF phase.
- The rapid cooling of the CS in Cas A can be accounted for via the phase transition described above. The Cas A data can be fitted by varying the phase transition temperature T^* at fixed value of the remaining physical parameters.
- Independent of the quality and correctness of the observational data, provided proof of principle that phase transitions may show up in transients in neutron star cooling evolution.

A. Sedrakian,
Rapid cooling of Cassiopea A as a phase transition in dense QCD.
Astron. and Astrophys. **555**, L10 (2013).

A. Sedrakian,
Cooling compact stars and phase transitions in dense QCD.
Eur. Phys. J. A **52**, 44 (2016).

III. Axion cooling of NS

The origin of axions (Wilczek, Weinberg, 1978)

$$\mathcal{L}_{QCD} = \sum_q \bar{\psi}_q (iD - m_q e^{i\theta_q}) \psi_q - \frac{1}{4} G_{\mu\nu a} G_a^{\mu\nu} - \theta \frac{\alpha_s}{8\pi} G_{\mu\nu a} \tilde{G}_a^{\mu\nu} \tag{14}$$

The phase can be pushed in the last term by chiral rotation $\psi_q' = e^{-i\gamma_5 \theta_q / 2} \psi_q$

$$\mathcal{L}_{QCD} = \sum_q \bar{\psi}_q (iD - m_q) \psi_q - \frac{1}{4} G_{\mu\nu a} G_a^{\mu\nu} - \underbrace{(\theta - \arg \det M_q)}_{\bar{\theta}} \frac{\alpha_s}{8\pi} G_{\mu\nu a} \tilde{G}_a^{\mu\nu} \tag{15}$$
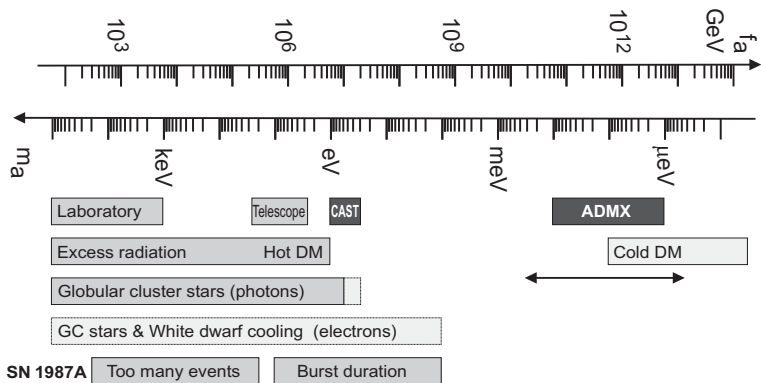
Experimental value: $\bar{\theta} \leq 10^{-11}$ comes from neutron electric dipole moment

$$|d| = 0.63 \times 10^{-25} \, e \, cm \tag{16}$$

The strong CP problem - smallness of $\bar{\theta}$ (unnatural). Reinterpretation of $\bar{\theta}$ term as a dynamical field pseudo-scalar axion field $a(x)$ with decay constant f_a

$$\mathcal{L}_{CP} = -\frac{\alpha_S}{8\pi} \bar{\theta} \text{Tr} G \tilde{G} \quad \rightarrow \quad \mathcal{L}_{CP} = -\frac{\alpha_S}{8\pi} \frac{a(x)}{f_a} \text{Tr} G \tilde{G} \tag{17}$$

The potential (mass term) forces the field to its minimum where $\bar{\theta} = 0$.

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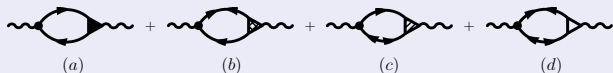cooling
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Thank you

- Among the population of NS with measured surface temperature those that are warm are expected to have canonical masses $M \sim 1.4 M_\odot$. (No exotic process are operating in their interiors). Thus, the physics of cooling is dominated by well-constrained nuclear physics of NS.

- Use a benchmark code (NSCool, D. Page) with standard input (APR EOS and gaps) to make results easy to check. (Note that in QCD part we use our own code tailored to treat quark matter phases, see arXiv:1509.06986).

- One single data point is sufficient to put a tight constraint on axion properties via cooling. We keep only data that is considered reliable (Cas A + several NS with age 10^5 yr).

- Among the continuum of models define by the coupling constants choose the most conservative ones $C_e = 0$ (hadronic model of axions KVSZ).

- Most of the axion process relevant for NS cooling were computed by Iwamoto, Burrows 80'a, Raffelt et al., Reddy et al in 90's

- First cooling simulations with axions in conference proceedings by Umeda, Tsuruta, Iwamoto in 1998 and then complete silence until 2015 [arXiv:astro-ph/9806337]

- Pair-breaking processes - J. Keller and A. S. (NPA, 2013) [arXiv:1205.6940]

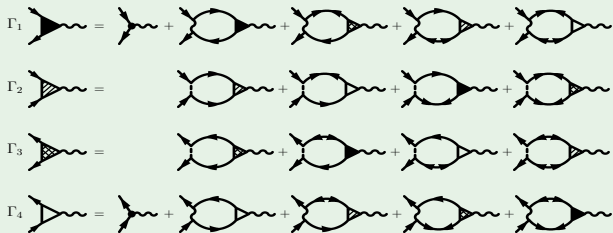- Full scale cooling simulations - A. S. (PRD, 2016) [arXiv:1512.07828]

The neutrino emissivity is expressed in terms of the polarization tensor of baryonic matter

$$\varepsilon_{\nu\bar{\nu}} = -2\left(\frac{G_F}{2\sqrt{2}}\right)^2 \int d^4q\, g(\omega)\omega \sum_{i=1,2} \int \frac{d^3q_i}{(2\pi)^3 2\omega_i} \text{Im}[L^{\mu\lambda}(q_i)\,\Pi_{\mu\lambda}(q)]\delta^{(4)}(q - \sum_i q_i),$$

Four polarization tensors in Nambu-Gorkov space:

(a) (b) (c) (d)

Effective Bethe-Salpeter equations for vertices:

Γ_1

Γ_2

Γ_3

Γ_4

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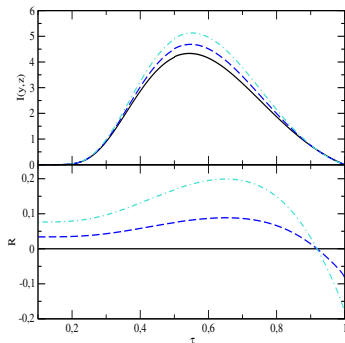cooling
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Thank you

The vector current emissivity is given by

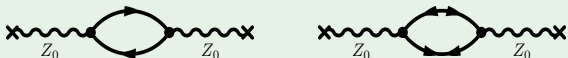$$\epsilon = \frac{16 G^2 c_V^2 \nu(0) v_F^4}{1215 \pi^3} I(z) T^7,$$

$$I(z) = z^7 \int_1^\infty \frac{dy\, y^5}{\sqrt{y^2 - 1}} f(zy)^2$$
$$\times \left[1 + \left(\frac{7}{33} + \frac{41}{77} \gamma \right) v_F^2 \right].$$

$z \equiv \Delta/T.$

- Systematic expansion in v_F^2, leading order at v_F^4
- The next-to-leading correction in v_F^2 which is $O(v_F^6)$ contributes about 10%.
- The emissivity has a maximum at $\tau = T/T_c = 0.6$, i.e., affects cooling close to the phase transition

Results due to Leinson-Perez (2006) Kolomeitsev-Voskresensky (2008) Reddy-Steiner (2008) A. S. et al (2007,2012)

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The two diagrams contributing to the polarization tensor of baryonic matter, which defines the axial vector emissivity. The "normal" baryon propagators for particles (holes) are shown by single-arrowed lines directed from left to right (right to left). The double arrowed lines correspond to the "anomalous" propagators F (two incoming arrows) and F^+ (two outgoing arrows).

The emissivity of this processes is given by

$$\epsilon_\nu = \frac{4G_F^2 g_A^2}{15\pi^3}\zeta_A \nu(0) v_F^2 T^7 I_\nu, \quad I_\nu = z^7 \int_1^\infty dy \frac{y^5}{\sqrt{y^2-1}} f_F (zy)^2 . \tag{18}$$

Note the v_F^2 scaling of the axial neutrino emissivity compared to the v_F^4 scaling.

Axial neutrino emissivity dominates the vector current emissivity because of v^2 scaling instead of v^4 scaling.
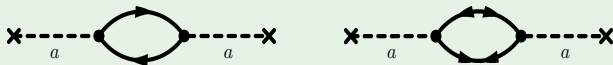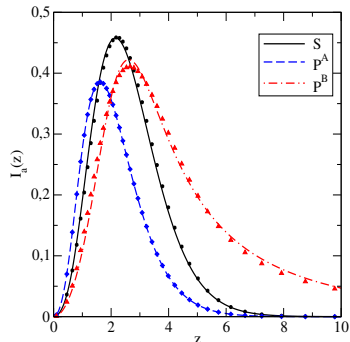
Key diagrams contributing to the axion pair-breaking emissivities are at one-loop level.

Axion emissivity for S-wave condensate

$$\epsilon_{aN}^{S} = \frac{2C_N^2}{3\pi} f_a^{-2} \nu_N(0) v_{FN}^2 T^5 I_{aN}^S,$$

$$I_{aN}^{S} = z_N^5 \int_1^\infty dy \, \frac{y^3}{\sqrt{y^2 - 1}} f_F(z_N y)^2.$$

J. Keller and A. S. (NPA, 2013) [arXiv:1205.6940] for S-wave condensate, A. S. (PRD, 2016) [arXiv:1512.07828] for P-wave condensates

The C_N charges are generally given by generalized Goldberger-Treiman relations

$$C_p = (C_u - \eta)\Delta_u + (C_d - \eta z)\Delta_d + (C_s - \eta w)\Delta_s, \qquad (19)$$

$$C_n = (C_u - \eta)\Delta_d + (C_d - \eta z)\Delta_u + (C_s - \eta w)\Delta_s, \qquad (20)$$

where $\eta = (1 + z + w)^{-1}$, with $z = m_u/m_d$, $w = m_u/m_s$, and $\Delta_u = 0.84 \pm 0.02$, $\Delta_d = -0.43 \pm 0.02$ and $\Delta_s = -0.09 \pm 0.02$, $z = m_u/m_d = 0.35$–0.6.

For *hadronic axions*, $C_{u,d,s} = 0$, and the nucleonic charges vary in the range

$$\boxed{-0.51 \leq C_p \leq -0.36, \quad -0.05 \leq C_n \leq 0.1.} \qquad (21)$$

- neutrons may not couple to axions ($C_n = 0$)
- protons always couple to axions $C_p \neq 0$
- invisible axion DFSZ model

$$C_e = \cos^2 \beta/3, \quad C_u = \sin^2 \beta/3, \quad C_d = \cos^2 \beta/3, \quad \text{arbitrary } \beta$$

- KVSZ model $C_e = 0$ and C_p and C_n from above inequalities
The axion mass is related to f_a via the relation

$$m_a = \frac{z^{1/2}}{1+z} \frac{f_\pi m_\pi}{f_a} = \frac{0.6 \text{ eV}}{f_a/10^7 \text{ GeV}} \quad m_\pi = 135 \text{ MeV}, \ f_\pi = 92 \text{ MeV}, \ z = 0.56. \qquad (22)$$

We now require that the axion luminosity does not exceed the neutrino luminosity

$$\frac{\epsilon_a}{\epsilon_\nu} = \frac{\alpha}{f_a^2 G_F^2 \Delta(T)^2} < 1, \tag{23}$$
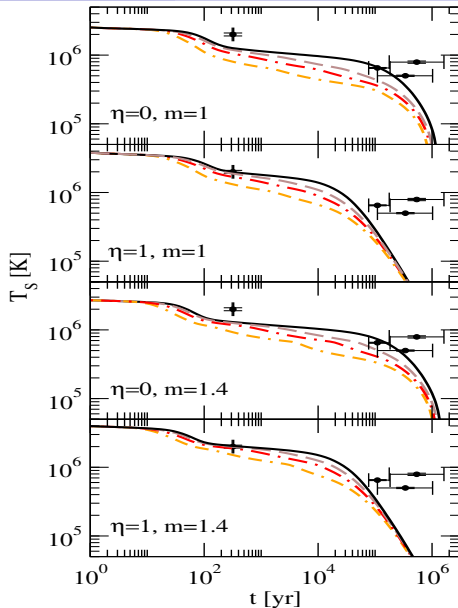
where we set the ratio $I_a/I_\nu \simeq 1$ and find that $\alpha \simeq 73.7$ ($g_A = 1.25$). Our bound is given by (J. Keller quand A. Sedrakian, Nucl. Phys. A, 897, 62, 2013)

$$\boxed{f_a > 7.4 \times 10^9 \text{ GeV} \left[\frac{0.1 \text{ MeV}}{\Delta(T)} \right],} \tag{24}$$
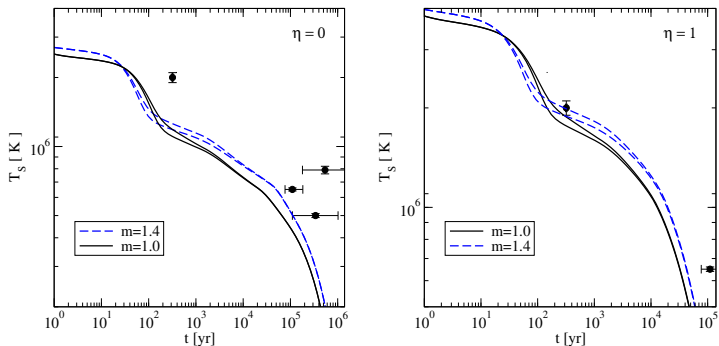
which translates into an upper bound on the axion mass

$$m_a = 0.62 \times 10^{-3} \text{ eV} \left(\frac{10^{10} \text{GeV}}{f_a} \right) \leq 0.84 \times 10^{-3} \text{ eV} \left[\frac{\Delta(T)}{0.1 \text{ MeV}} \right]. \tag{25}$$
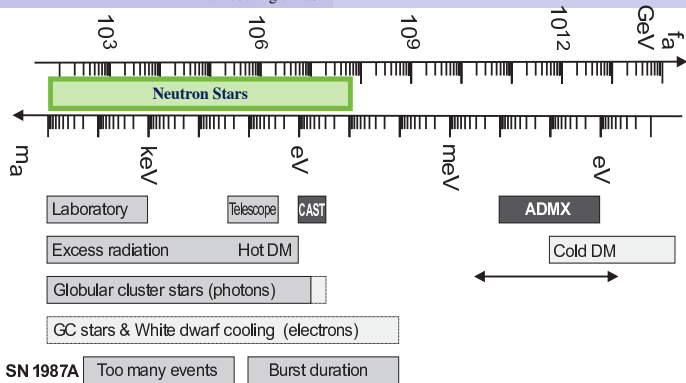
The bound on f_a can be written in terms of the critical temperature by noting that $\Delta(T) \simeq 3.06 T_c \sqrt{1 - T/T_c} \simeq T_c$ in the temperature range $0.5 \leq T/T_c < 1$ of most interest.

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$f_{a7} = \infty$ (solid line) 10 (dashed), 5 (dash-dotted), and 2 (double-dash-dotted).

Cooling tracks of neutron star models with masses $m = 1$ (solid) and 1.4 (dashed) for the case of a nonaccreted iron envelope ($\eta = 0$) and accreted envelope ($\eta = 1$) and for $f_{a7} = 10$. For each mass, the two tracks differ by the value of the neutron PQ charge. The upper curves correspond to our standard choice $|C_n| = 0.04$, while the lower curves correspond to the case of enhanced axion emission with $|C_n| = |C_p| = 0.4$.

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The NS constraint are based on highly conservative physics of hadronic matter and on benchmarked code for cooling simulations of NS.

A. Sedrakian,
Axion cooling of neutron stars.
Phys. Rev. D **93**, 065044 (2016).

J. Keller and A. Sedrakian,
Axions from cooling compact stars: pair-breaking processes.
Nuclear Physics A **897**, 62 (2013).

Thanks to collaborators:

- L. Bonanno, G. Colluci, J. Keller