CERN, Tuesday, 6th December 2016.

Gravitational waves from binary neutron star systems: from the equation of state to the properties of the signal using general relativistic numerical simulations.

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Abstract

Binary Neutron Star systems are a potential source of gravitational wave signals (that would be likely be detected in the coming months by the LIGO/VIRGO detectors) and the associate signal will carry important information about the equation of state (EOS) of matter at high density.

In this talks I will discuss the various steps that one need to perform, starting from the EOS of matter at high density, to obtained the properties of the Gravitational Wave signal emitted during the merger.

I would like to remark that it is now possible to investigate the physics of Binary Neutron Star System merger using only **publicly available open source software**, the Einstein Toolkit for the dynamical evolution and the LORENE code for the generation of the initial models.

Abstract …….

I will also present three-dimensional simulations of the dynamics of binary neutron star (BNS) mergers from the late inspiral stage up to ~20 ms after the system has merged, either to form a hyper-massive neutron star (HMNS) or a rotating black hole (BH). In particular we show the fate of the six galactic systems (J0453+1559, J1756-2251, J0737-3039A, B1913+16, J1906+0746*, B1534+12) when they will finally merge and the gravitational wave signal that will be emitted.

I also report results for equal and un-equal-mass models and on the strength of the Gravitational Signal and its dependence on the EOS, the mass ratio of the two stars, the radiated energy and angular momentum. We use a semi-realistic description of the equation of state (EOS) where the EOS is described by a seven-segment piece-wise polytropic with a thermal component given by Γ_{th}=1.8

GR NS-NS simulations: State of the Art

- One of the main and hottest research topic in Numerical Astrophysics.
- A comprensive discussion of the subject can be found in (www.livingreviews.org): J.A. Faber & F.A. Rasio, "Binary neutron star mergers", Living Reviews in Relativity (2012). This review contains 338 references.
- New review by **Rezzolla** and Baiotti (arXiv:1607.03540), "Binary neutron-star mergers: a review of Einstein's richest laboratory"
- Impossible to give a comprensive list of all the individual contributor and their roles.
- Among them is worth citing:
  - The people that start it back in '99: Shibata&Uryu: **Phys. Rev. D 61 064001** (gr-qc/9911058)
  - and (in alphabetic order): Alic, Anderson, Baiotti , Bauswein, Bernuzzi , Bruegmann , Ciolfi, Dietrich , Duez , Etienne , Foucart, Giacomazzo , Gold, Haas , Hotokezaka, Janka, Kastaun , Kawaguchi, Kidder , Kiuchi, **Kokotas**, Kyutoku, Lehner , Liebling , Liu, Nielsen , Ott , O'Connor , **Pachalidis**, Palenzuela , Pfeiffer, **Rezzolla**, Scheel , Sekiguchi , Shapiro , Shibata, Stergioulas, Taniguchi, Uryu, …

Mostly based on:

- A. Feo, R. De Pietri, F. Maione and F. Loeffler, arXiv:1608.02810.
  Classical and Quantum, to appear.
  Modeling Mergers of Known Galactic Systems of Binary Neutron Stars.

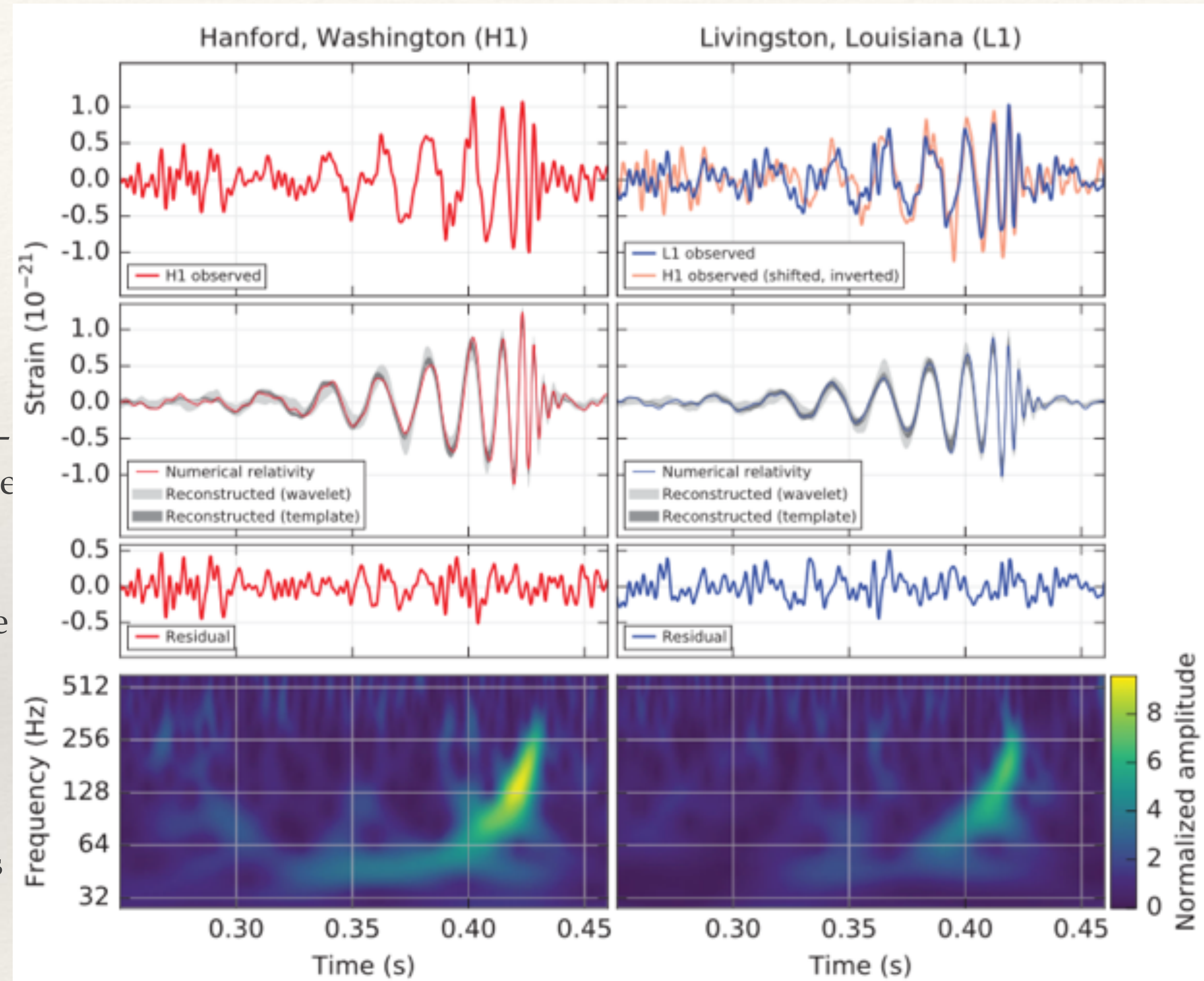
- F. Maione, R. De Pietri, A. Feo and F. Loeffler, arXiv:1605.03424.
  Classical and Quantum Gravity, 33, no. 17, 175009 (2016).
  Binary neutron star merger simulations with different initial orbital frequency and equation of state.

- R. De Pietri, A. Feo, F. Maione and F. Loeffler, arXiv:1509.08804.
  Phys. Rev. D 93, 064047 (2016).
  Modeling Equal and Unequal Mass Binary Neutron Star Mergers Using Public Codes

work in collaboration with :
A.Feo, F. Maione, F. Loeffler

Gravitational Wave detected!

- **The gravitational waves were detected on September 14, 2015 at 5:51 a.m. Eastern Daylight Time (09:51 UTC)** by both of the twin Laser Interferometer Gravitational-wave Observatory (LIGO) detectors, located in Livingston, Louisiana, and Hanford, Washington, USA.
- The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than **5.1σ**. The source lies at a luminosity distance of **410(**18) **Mpc** corresponding to a redshift z=0.09(4). In the source frame, the initial black hole masses are **36**(5)$\mathbf{M_\odot}$ and **29**(4)$\mathbf{M_\odot}$, and the final black hole mass is **62**(4)$\mathbf{M_\odot}$, with 3.0(5) $\mathbf{M_\odot c^2}$ radiated in gravitational waves. ***All uncertainties define 90% credible intervals***.

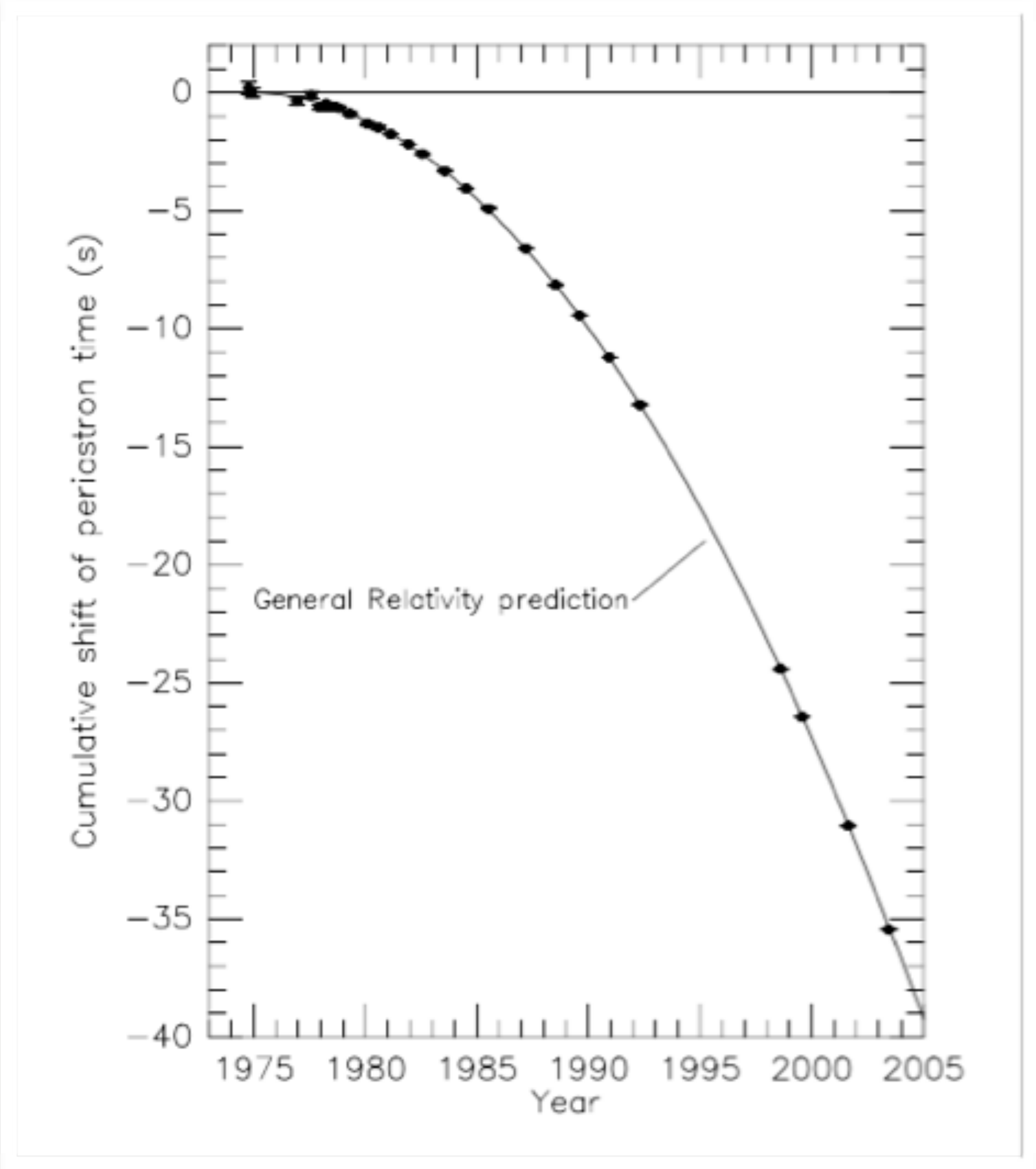


GW150914

Observation of Gravitational Waves from a Binary Black Hole Merger B. P. Abbott et al. (LIGO Scientific Collaboration and **Virgo Collaboration**)
Phys. Rev. Lett. 116, 061102 – Published 11 February 2016

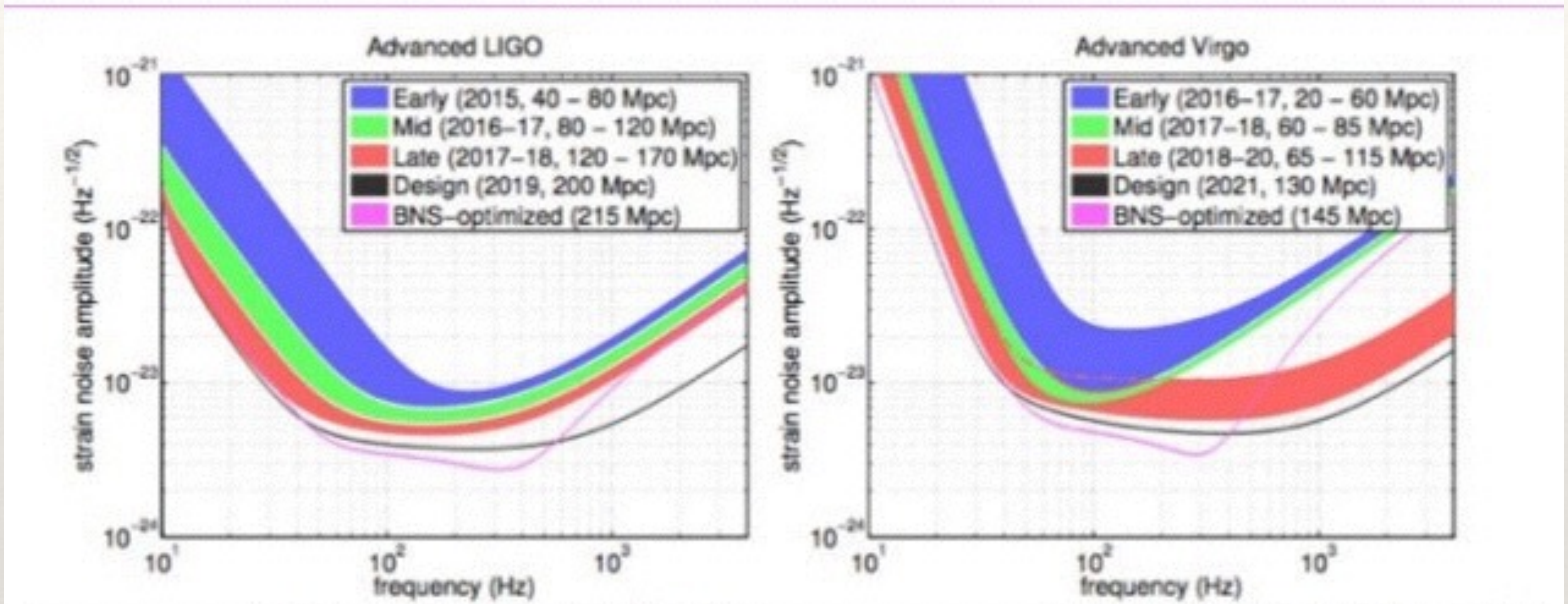
We already knew they (GW) exists!

- PSR B1913+16 (also known as J1915+1606) is a pulsar in a binary star system, in orbit with another star around a common center of mass. In 1974 it was discovered by Russell **Alan Hulse and Joseph Hooton Taylor, Jr., of Princeton University, a discovery for which they were awarded the 1993 Nobel Prize in Physics**
- Nature 277, 437 - 440 (08 February **1979**), J. H. TAYLOR, L. A. FOWLER & P. M. McCULLOCH: Measurements of second- and third-order relativistic effects in the orbit of binary pulsar PSR1913 + 16 have yielded self-consistent estimates of the masses of the pulsar and its companion, **quantitative confirmation of the existence of gravitational radiation at the level predicted by general relativity**, and detection of geodetic precession of the pulsar spin axis.



Why we do want to study BNS mergers?

- First: the LIGO/Virgo collaboration will see the signal from BNS system. They are among the most powerful sources of GWs



	Epoch	Estimated Run Duration	$E_{GW} = 10^{-2} M_\odot c^2$ Burst Range (Mpc) LIGO	Virgo	BNS Range (Mpc) LIGO	Virgo	Number of BNS Detections	% BNS Localized within 5 deg^2	20 deg^2
O1 →	2015	3 months	40 – 60	–	40 – 80	–	0.0004 – 3	–	–
O2 →	2016–17	6 months	60 – 75	20 – 40	80 – 120	20 – 60	0.006 – 20	2	5 – 12
	2017–18	9 months	75 – 90	40 – 50	120 – 170	60 – 85	0.04 – 100	1 – 2	10 – 12
	2019+	(per year)	105	40 – 80	200	65 – 130	0.2 – 200	3 – 8	8 – 28
	2022+ (India)	(per year)	105	80	200	130	0.4 – 400	17	48

UKZN: Jan 20, 2014 [Aasi+ (LSC+Virgo), arXiv:1304.0670] 25

Gravitational Waves sources: compact objects

- MAIN TARGET LIGO/Virgo coll.:
  NS-NS merger
  sensitive frequency band approx. (40-2000) Hz
  Expected to rate ≈ 0.2 – 200 events per year events between 2016 – 19
  [J. Abadie et al. (VIRGO, LIGO Scientific), Class. Quant. Grav. 27, 173001 (2010)]

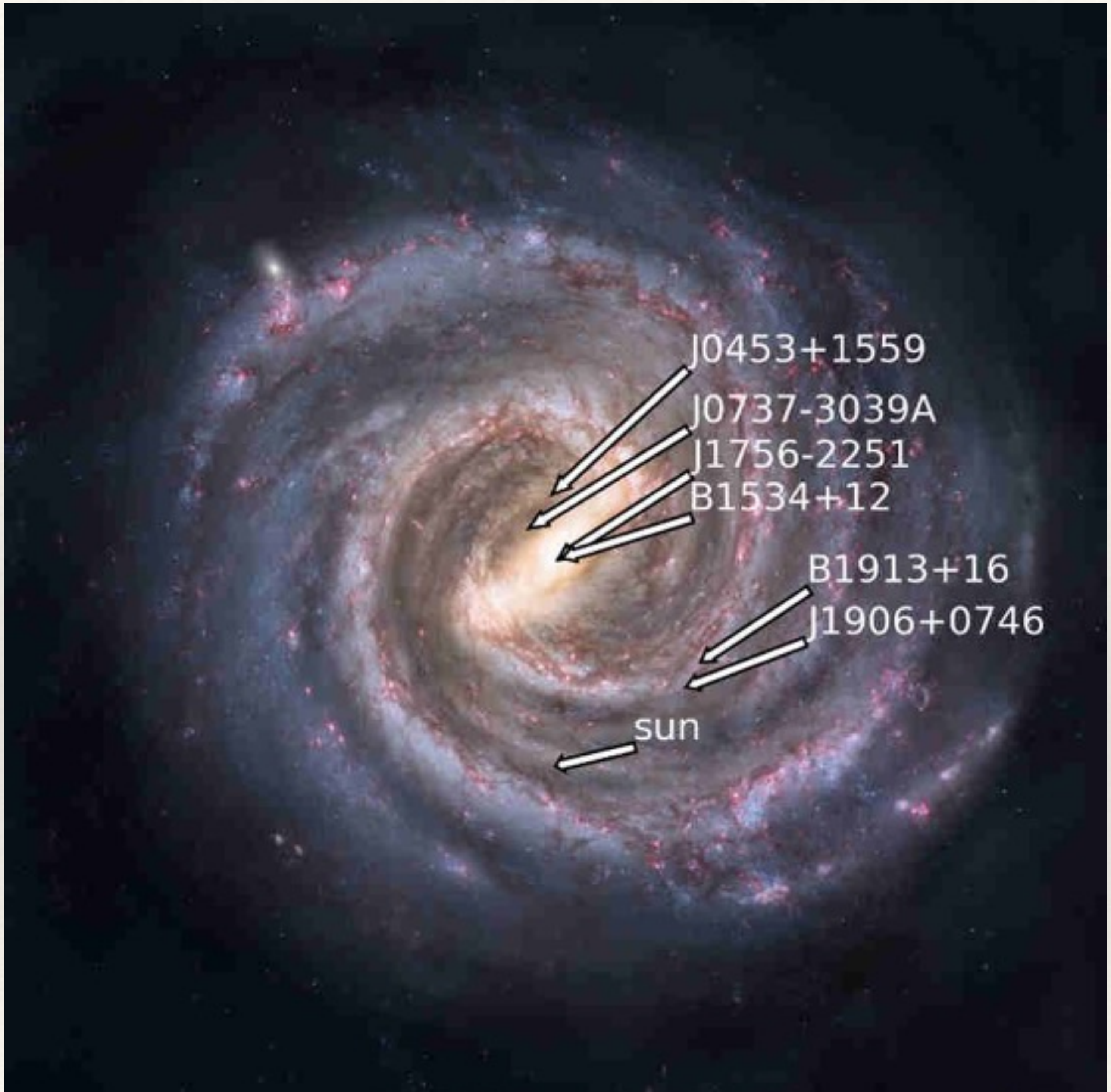
- **Core collapse in supernova**
- **BH-BH merger** —— **(FOUND!)**
- **BH-NS merger**
- **"*Mountains*" (deformation) on the crust of Neutron Stars**
- **Secular instability of Neutron stars**
- **Dynamical instability of Neutron star**

Table 1: Double neutron star systems known in the Galaxy

Pulsar	Period (ms)	P_b (days)	x (lt-sec)	e	M ($M_\odot$)	M_p ($M_\odot$)	M_c ($M_\odot$)	References
J0737−3039A	22.699	0.102	1.415	0.0877775(9)	2.58708(16)	1.3381(7)	1.2489(7)	1
J0737−3039B	2773.461		1.516					
J1518+4904	40.935	8.634	20.044	0.24948451(3)	2.7183(7)	-	-	2
B1534+12	37.904	0.421	3.729	0.27367740(4)	2.678463(4)	1.3330(2)	1.3454(2)	3
J1753−2240	95.138	13.638	18.115	0.303582(10)	-	-	-	4
J1756−2251	28.462	0.320	2.756	0.1805694(2)	2.56999(6)	1.341(7)	1.230(7)	5
J1811−1736	104.1	18.779	34.783	0.82802(2)	2.57(10)	-	-	6
J1829+2456	41.009	1.760	7.236	0.13914(4)	2.59(2)	-	-	7
J1906+0746*	144.073	0.166	1.420	0.0852996(6)	2.6134(3)	1.291(11)	1.322(11)	8
B1913+16	59.031	0.323	2.342	0.6171334(5)	2.8284(1)	1.4398(2)	1.3886(2)	9
J1930−1852	185.520	45.060	86.890	0.39886340(17)	2.59(4)	-	-	10
J0453+1559	45.782	4.072	14.467	0.11251832(4)	2.734(3)	1.559(5)	1.174(4)	This Letter
Globular cluster systems								
J1807−2500B*	4.186	9.957	28.920	0.747033198(40)	2.57190(73)	1.3655(21)	1.2064(20)	12
B2127+11C	30.529	0.335	2.518	0.681395(2)	2.71279(13)	1.358(10)	1.354(10)	13

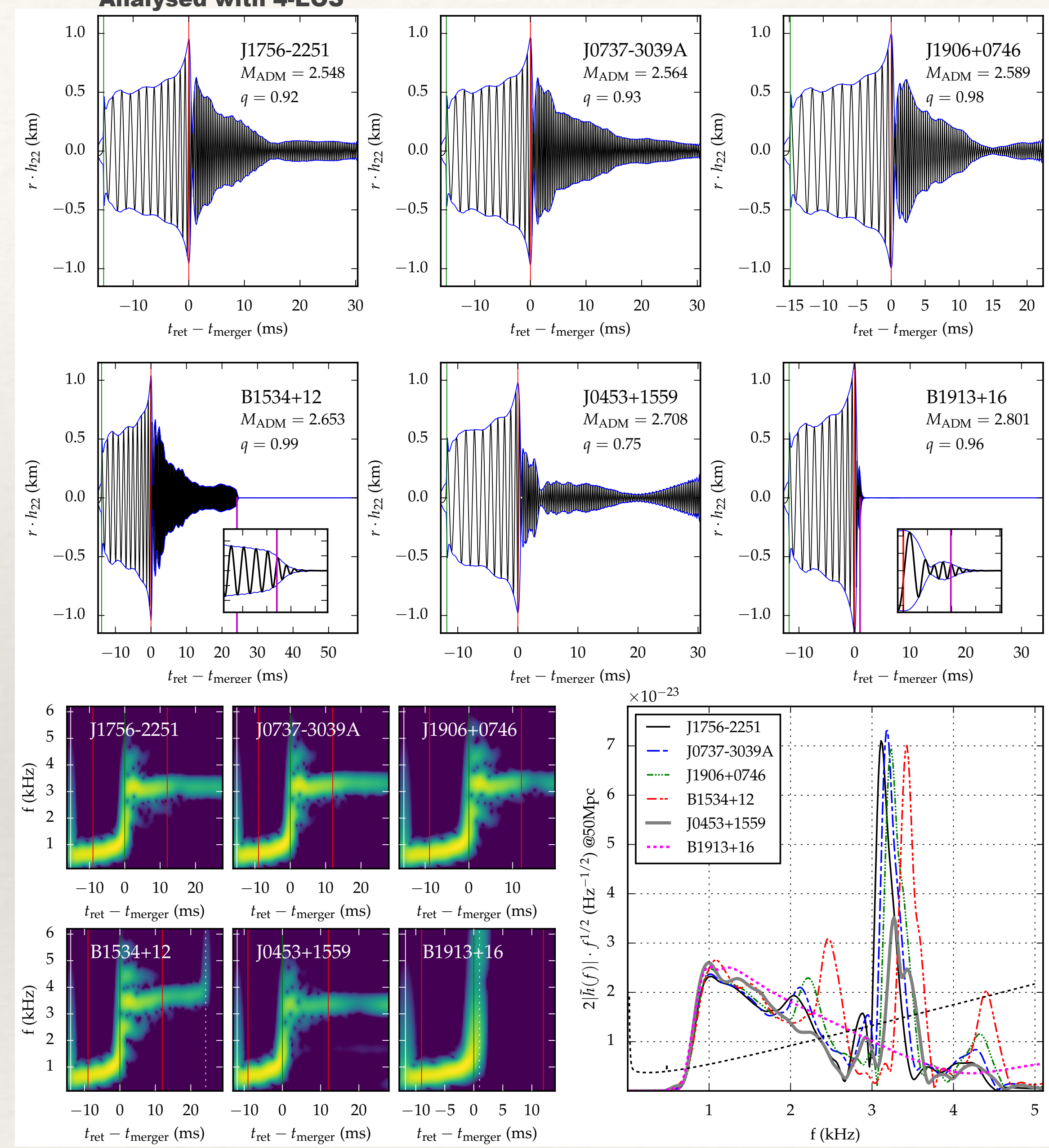
Table from: Martinez et al.: "Pulsar J0453+1559: A Double Neutron Star System with a Large Mass Asymmetry" arXiv:1509.08805v1

Artistic view of the location of the six galactic system.



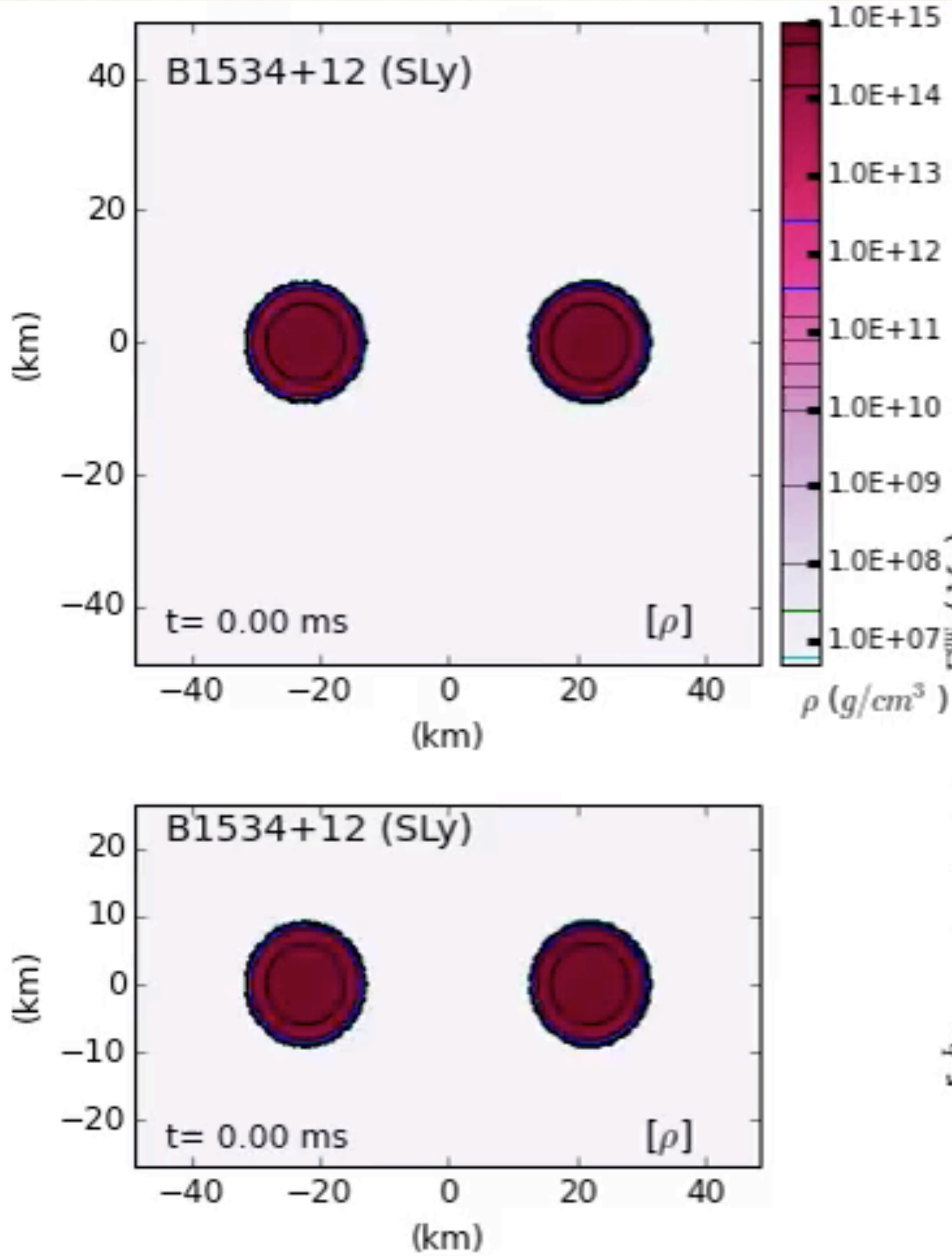
The simulated GW signal

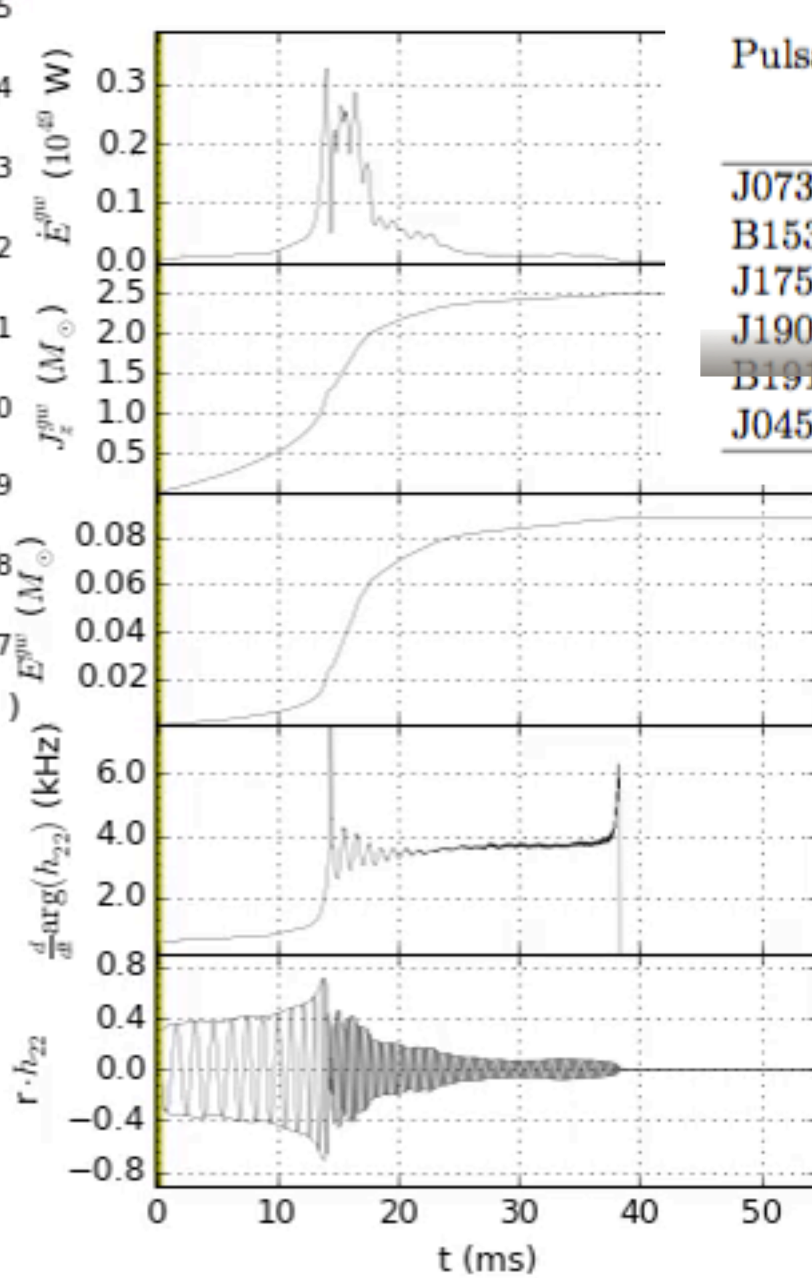
Analysed with 4-EOS



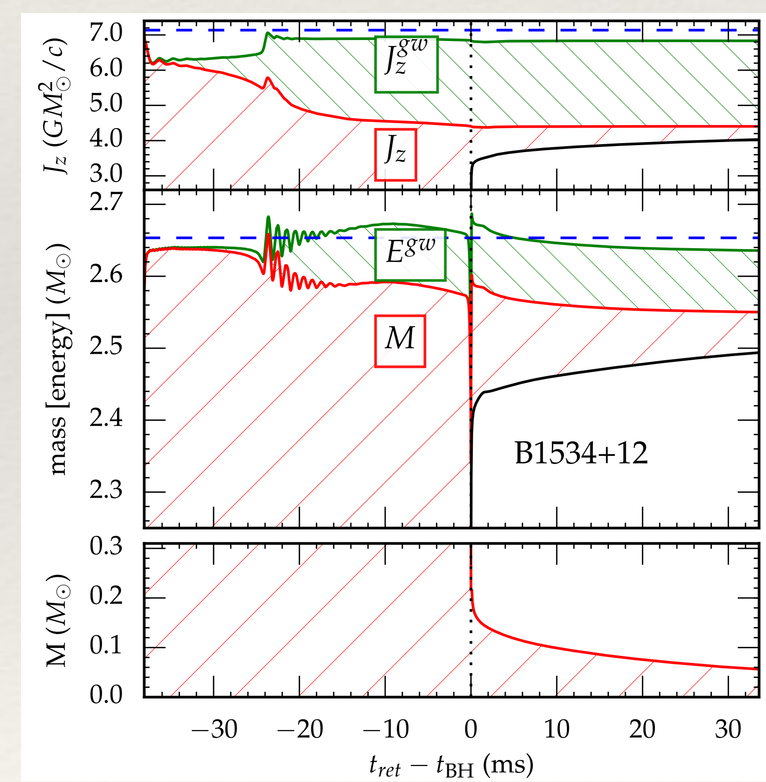
A. Feo, R. De Pietri, F. Maione and F. Loeffler,
Modeling Mergers of known Galactic Binary Neutron Stars,
Class and Quantum Gravity (to appear) arXiv 1608.02810(2016)

The evolution of the B1534+12 system.



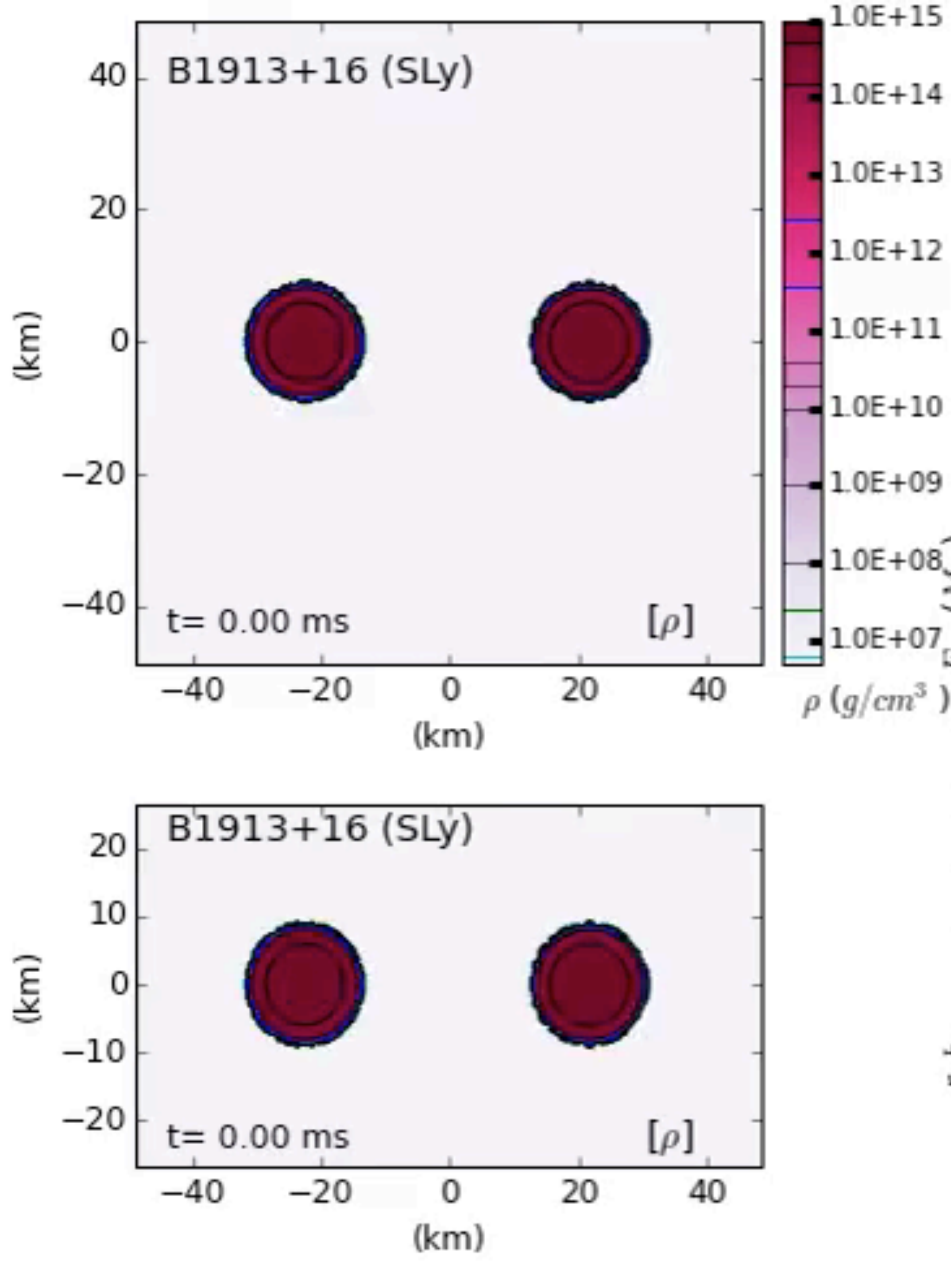


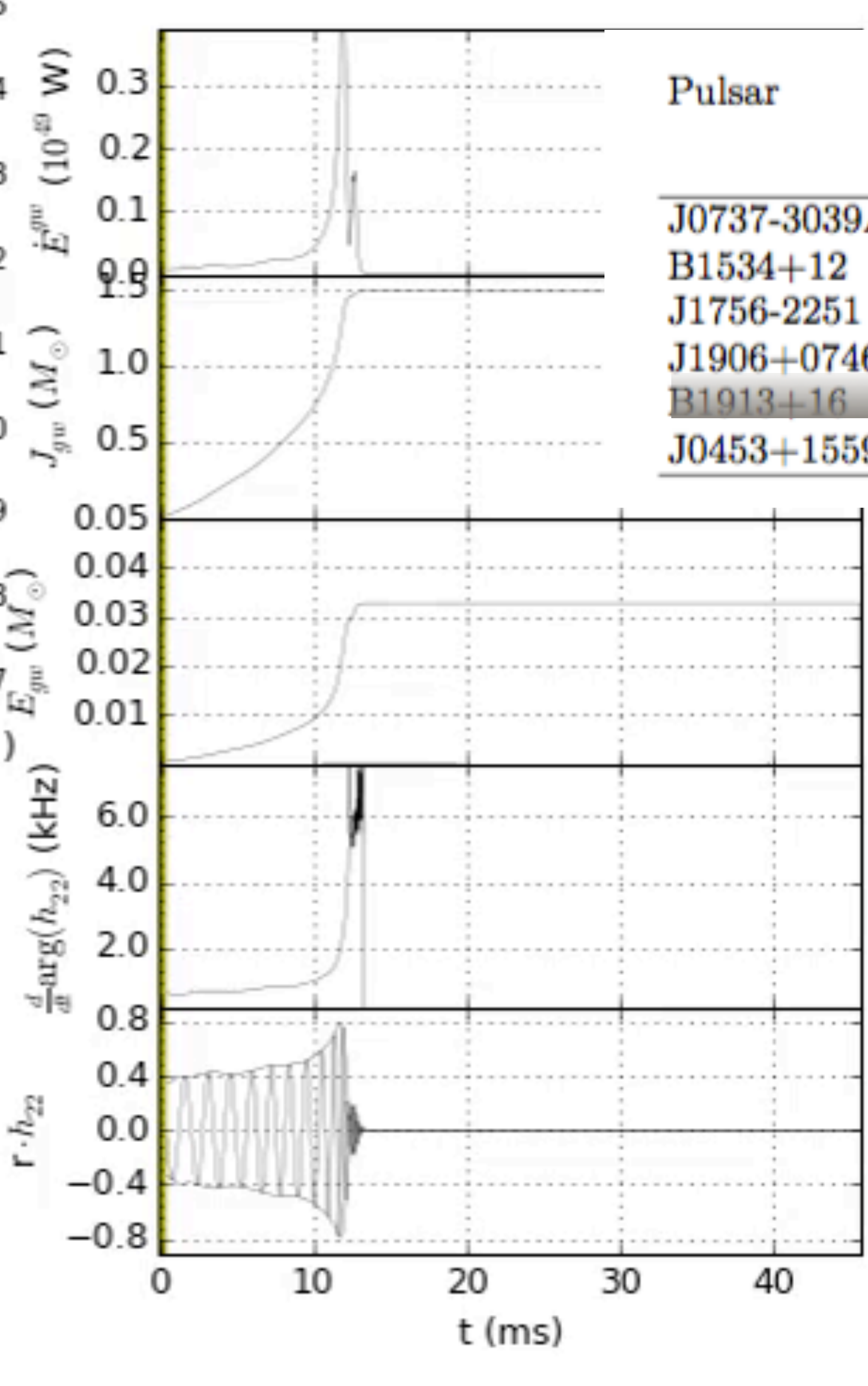
Pulsar	M_p $[M_\odot]$	M_c $[M_\odot]$	q	$M^{(1)}$ $[M_\odot]$	$M^{(1)}$ $[M_\odot]$
J0737-3039A	1.3381(7)	1.2489(7)	0.93	1.36	1.47
B1534+12	1.3330(2)	1.3454(2)	0.99	1.46	1.47
J1756-2251	1.341(7)	1.230(7)	0.92	1.33	1.47
J1906+0746*	1.291(11)	1.322(11)	0.98	1.41	1.45
B1913+16	1.4398(2)	1.3886(2)	0.96	1.53	1.59
J0453+1559	1.559(5)	1.174(4)	0.75	1.27	1.74



and the SOUND!

Hulse and Taylor pulsar (B1913+16)





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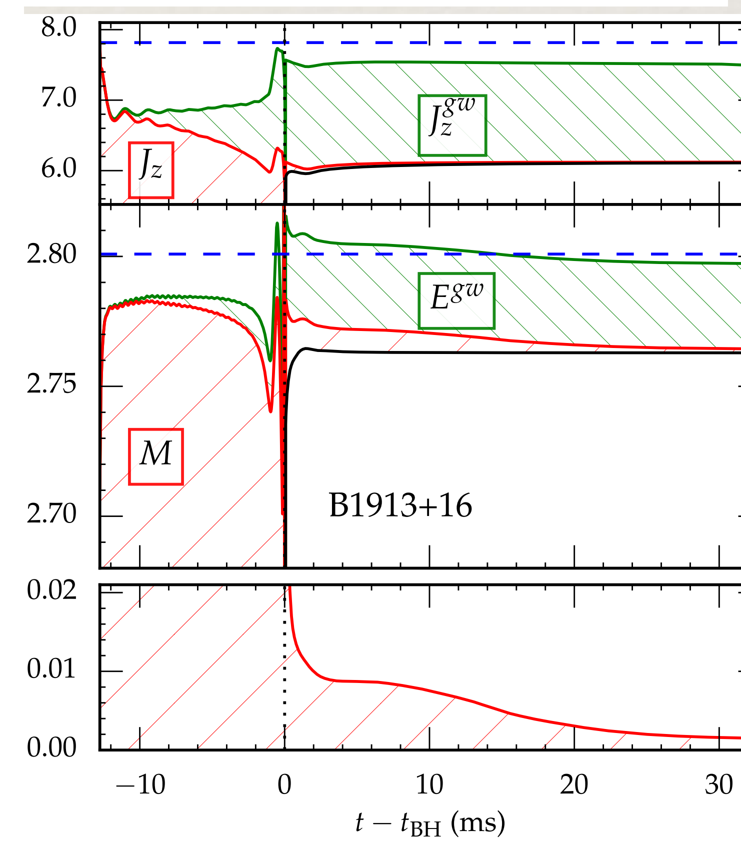
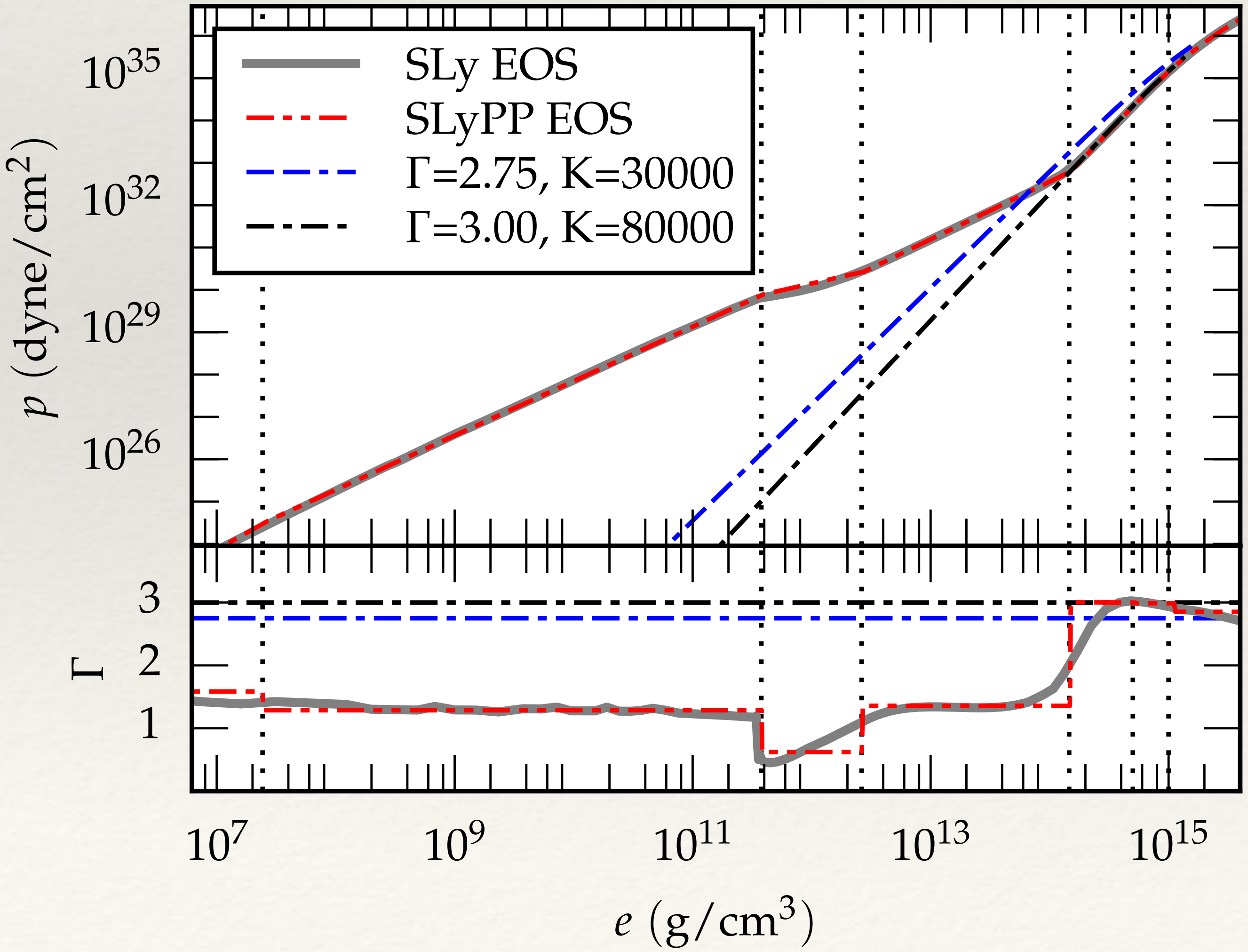


EOS in numerical relativity

- Usually just a pice-wise politropic approximant.
- More works using tabulated EOS, realistic thermal effect and Neutrinos dynamics.
- The real challenge is to introduce more Nuclear-Physics in simulations.



The true EOS for nuclear matter in a system similar to a NS is still unknown, not even assuming a small effect on the temperature, i.e., cold neutron star, as expected here for initial data.

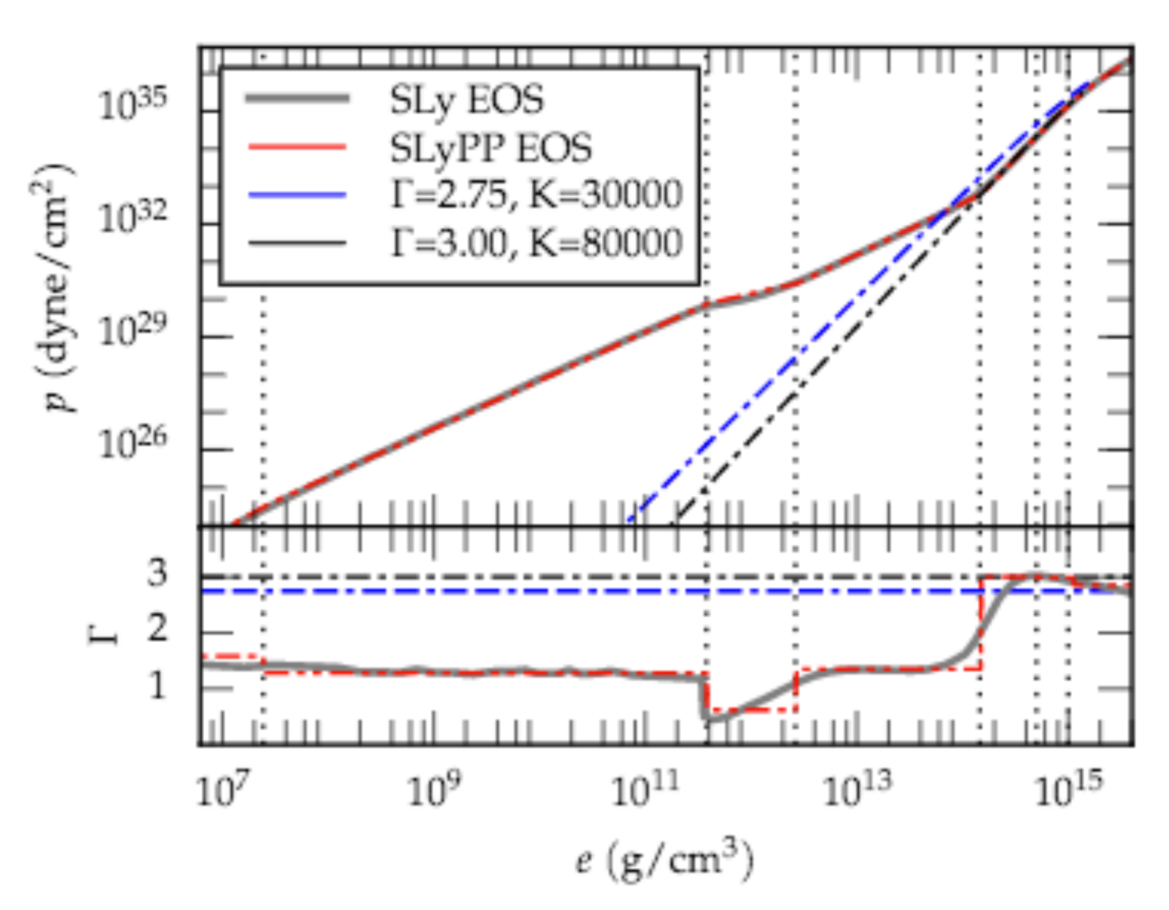
EOS used in our simulations

- Piecewise polytropic representation of SLy EOS + thermal component:
- 7 pieces EOS => realistic treatment of the NS crust and the BH accretion disk eventually produced
- High density region similar to Gamma = 3.00 polytropic.
- Still only approximate treatment of thermal component.

$$P(\rho, \epsilon) = P_{\rm cold}(\rho) + P_{\rm th}(\rho, \epsilon),$$

$$P_{\rm cold} = K_i \rho^{\Gamma_i}$$
$$\epsilon_{\rm cold} = \epsilon_i + \frac{K_i}{\Gamma_i - 1}\rho^{\Gamma_i - 1}$$
$$P_{\rm th} = \Gamma_{\rm th}\rho(\epsilon - \epsilon_{\rm cold}),$$

Read et al. 2009

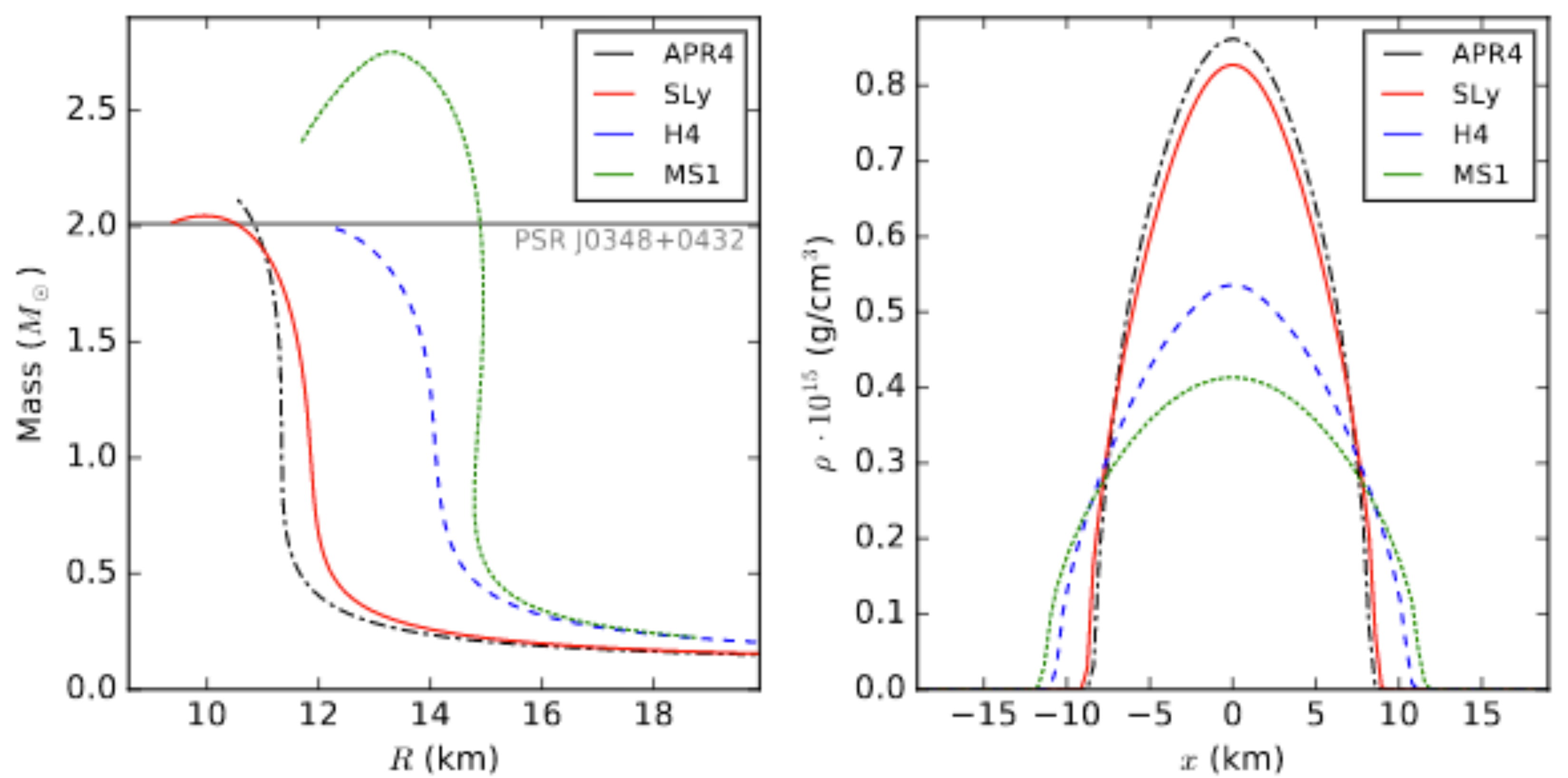


Douchin and Haensel 2000,2001

FIG. 1. Plot of the pressure (P) and of the adiabatic index ($\Gamma = d\log(P)/d\log(\rho)$) as a function of the energy density ($e = \rho(1+\epsilon)$) for the SLy EOS, its piece-wise polytropic approximation (the one used in the present work) and two isentropic polytropic EOS $P = K\rho^\Gamma$.

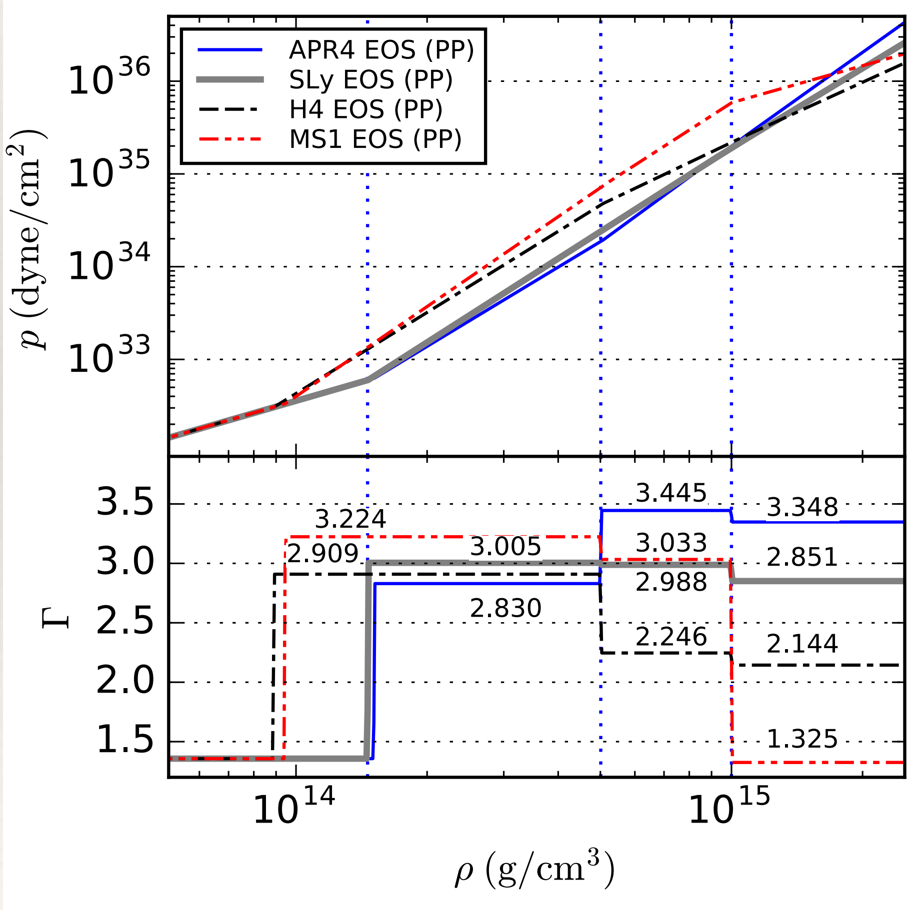
Effect of the EOS (four different EOS)

- APR4 EOS obtained using variational chain summation methods with the Argonne two-nucleon interaction and including also boost corrections and three-nucleon interactions
- The SLy EOS based on the Skyrme Lyon effective nuclear interaction
- The H4 EOS constructed in a relativistic mean field framework including also Hyperons contributions and tuning the parameters to have the stiffest possible EOS compatible with astrophysical data
- The MS1 EOS constructed with relativistic mean field theory considering only standard nuclear matter.



(a) Mass-Radius relations

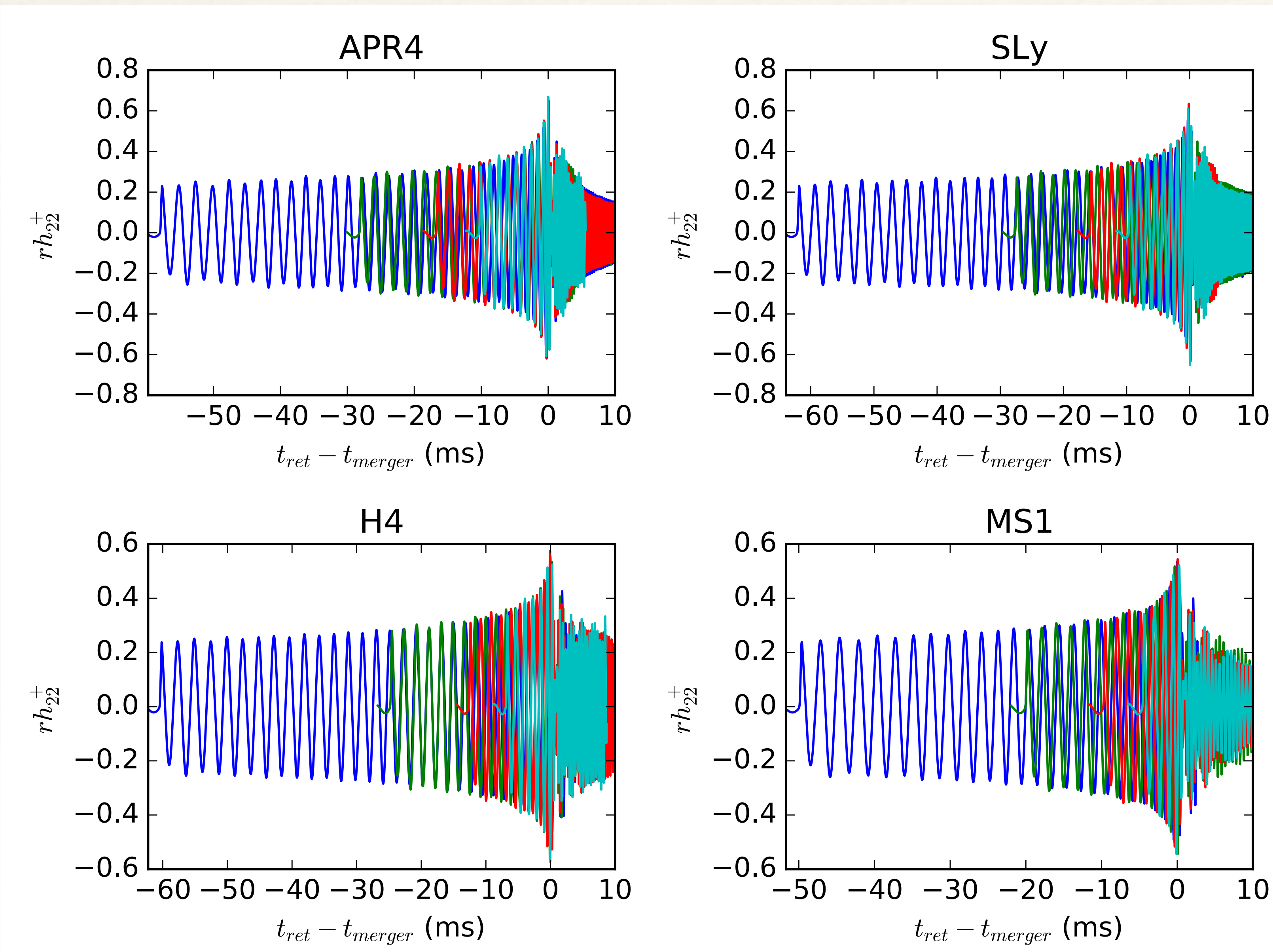
(b) Initial density profiles

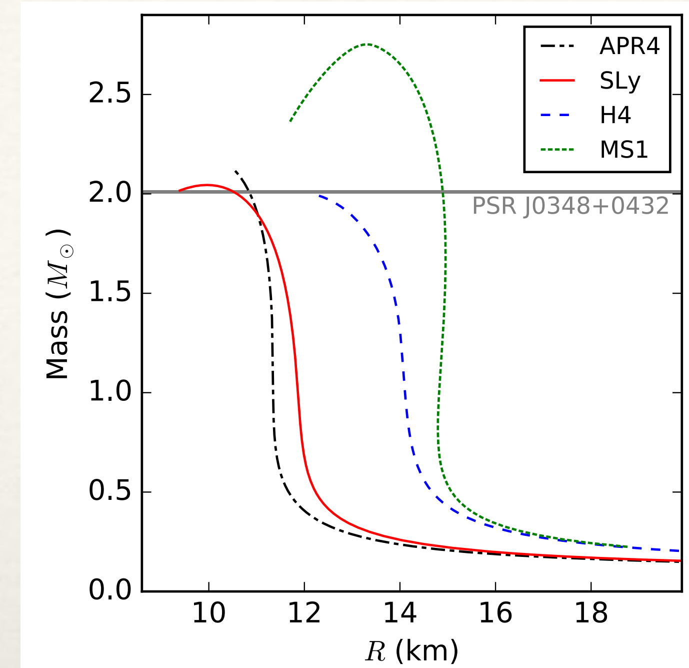


Binary neutron star merger simulations with different initial orbital frequency and equation of state, F. Maione, R. De Pietri, A. Feo and F. Loeffler, Classical and Quantum Gravity, 33, no. 17, 175009 (2016). arXiv:1605.03424

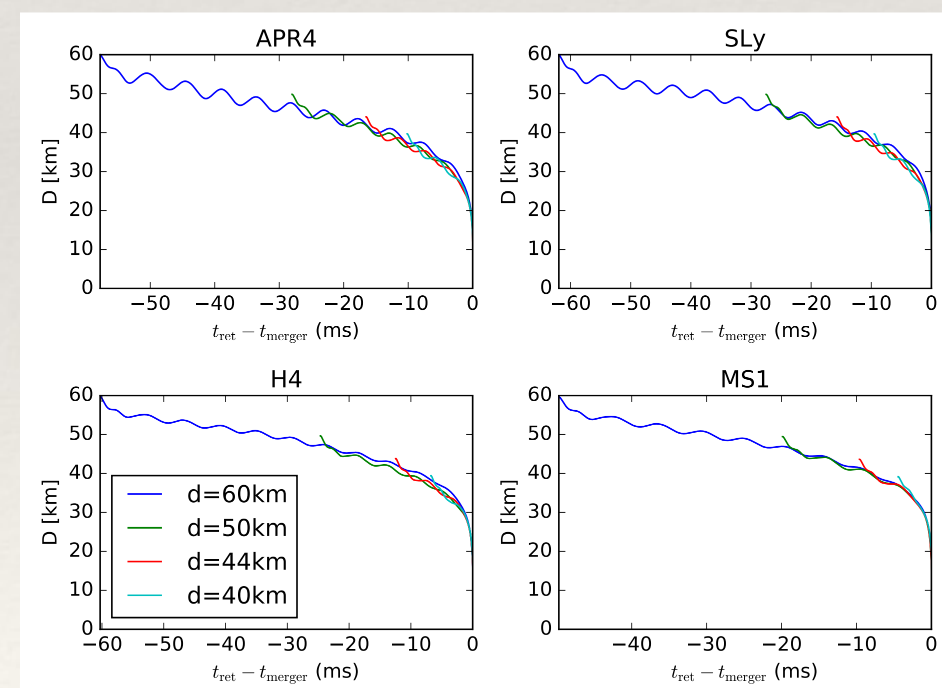
Multi Orbits simulations (four different EOS)

- Long term (up to 16 orbits) equal mass BNS simulations with four different EOS, starting with four different values of the star center d=(40,44.3,50,60)Km





Separation as function of time (eccentricity)



The pulsar J0453+1559 (q=0.75) with four different EOS

A. Feo, R. De Pietri, F. Maione and F. Loeffler,
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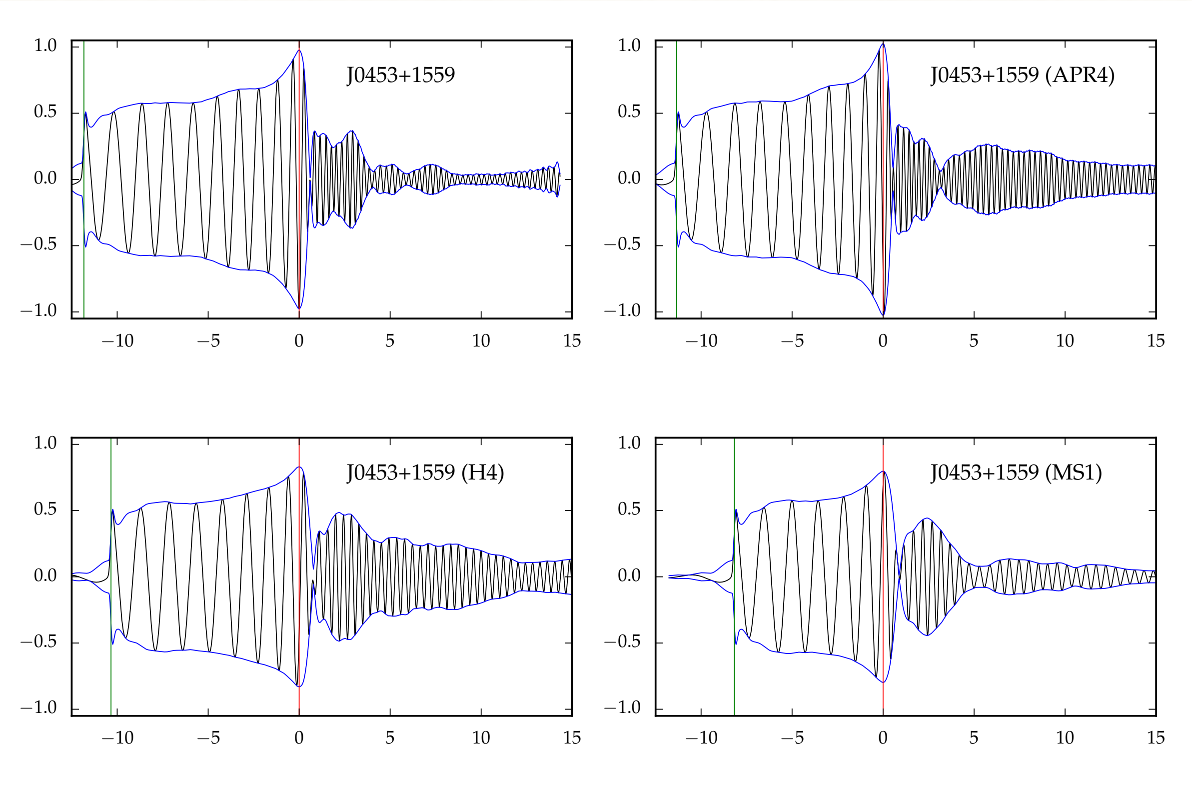

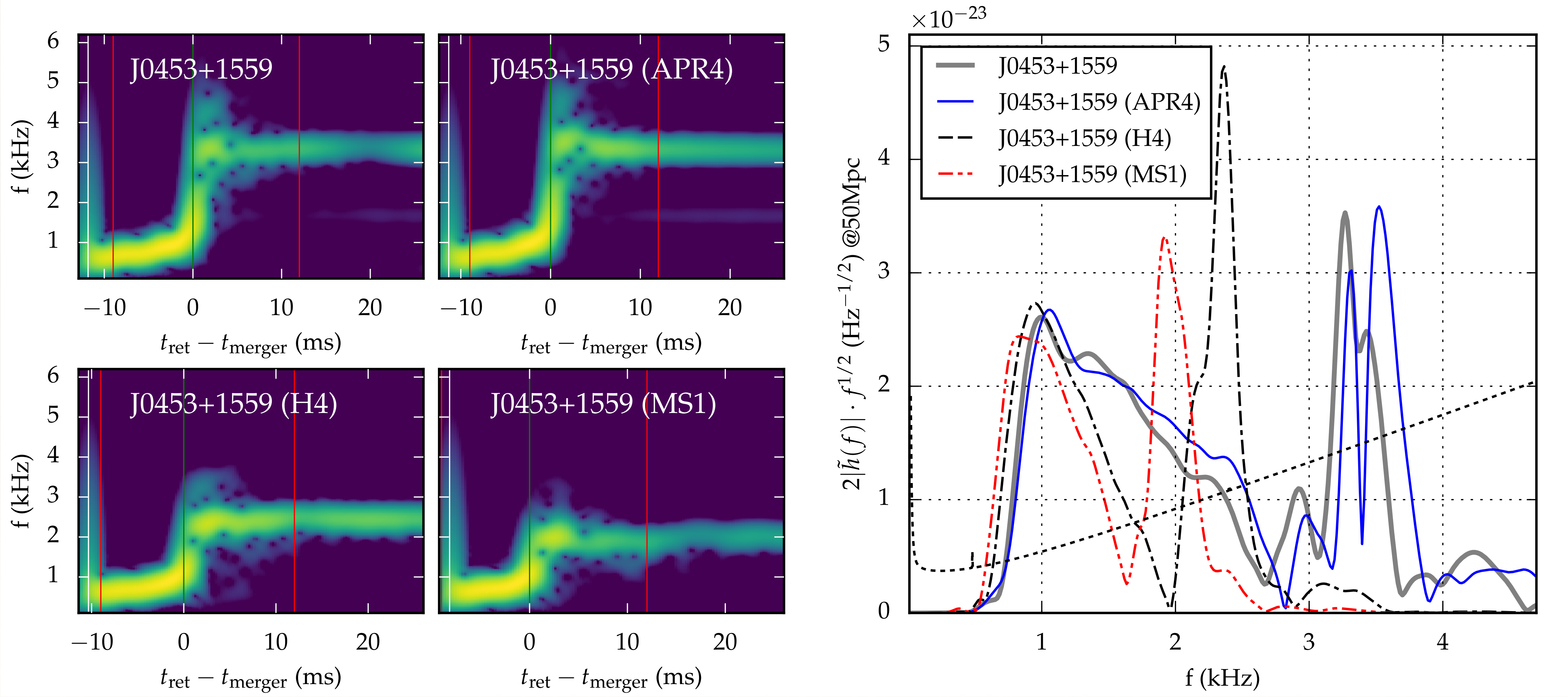

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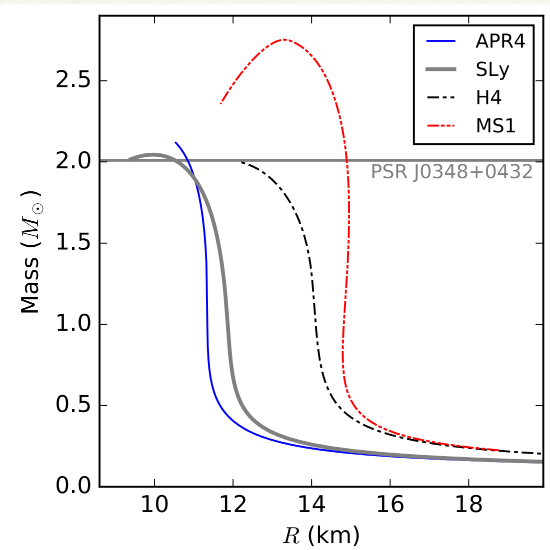
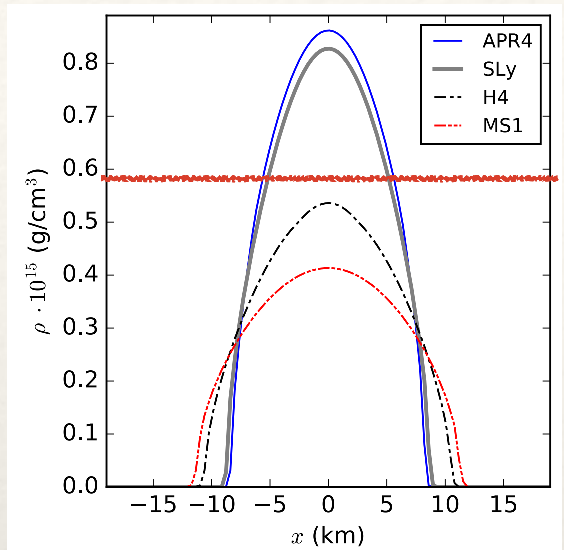
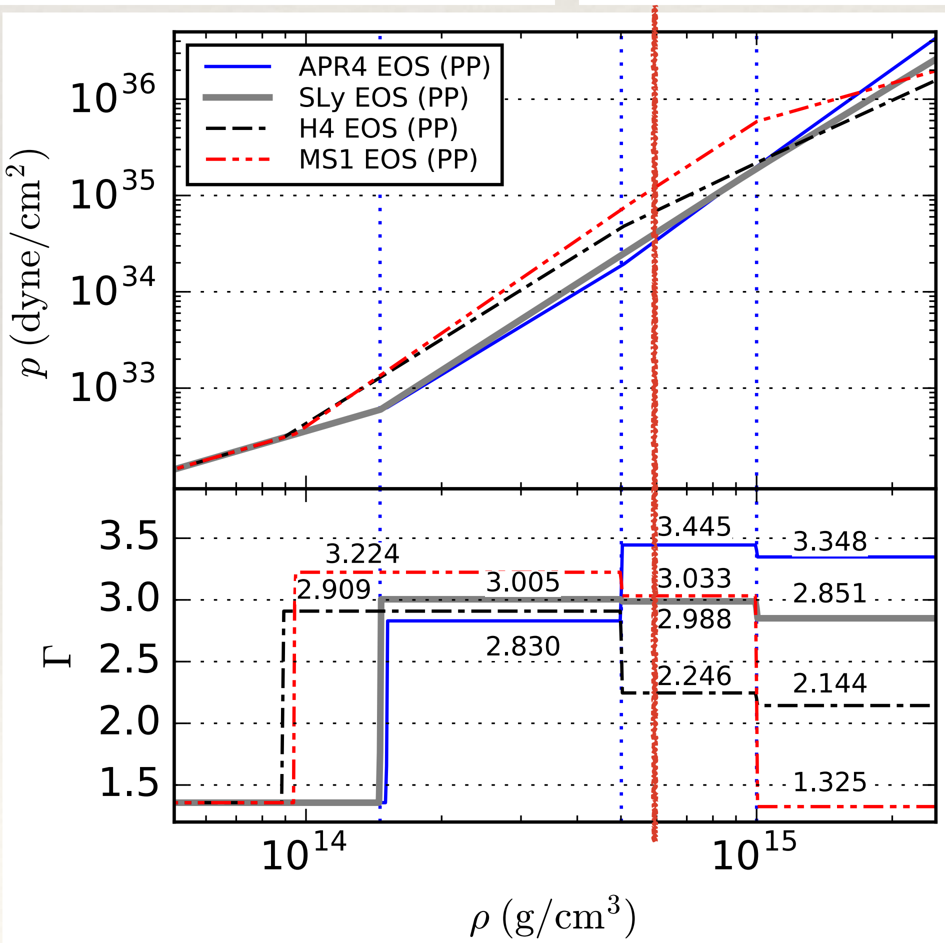
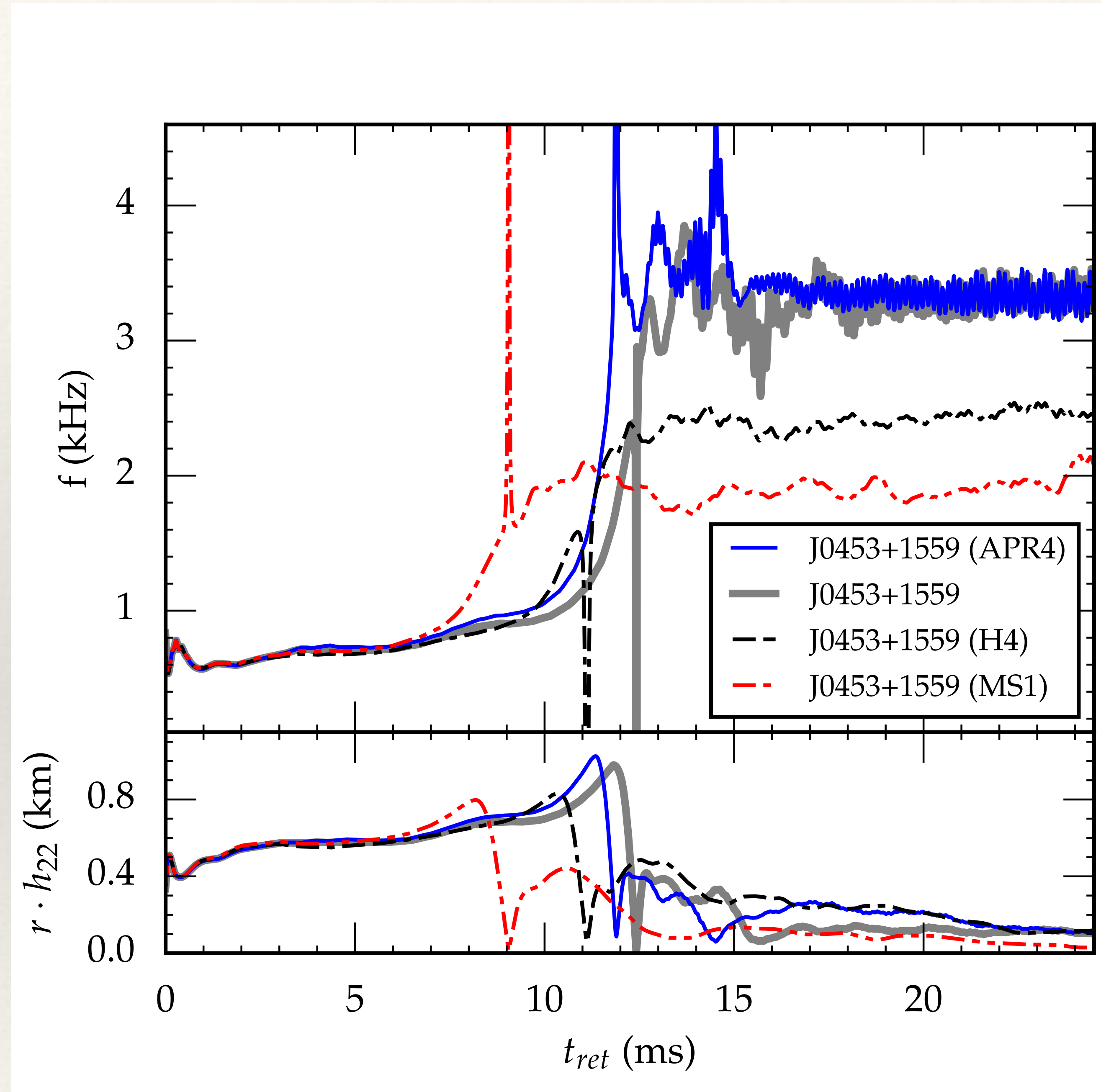








Simulation of NS merger as a key to get insight on the EOS

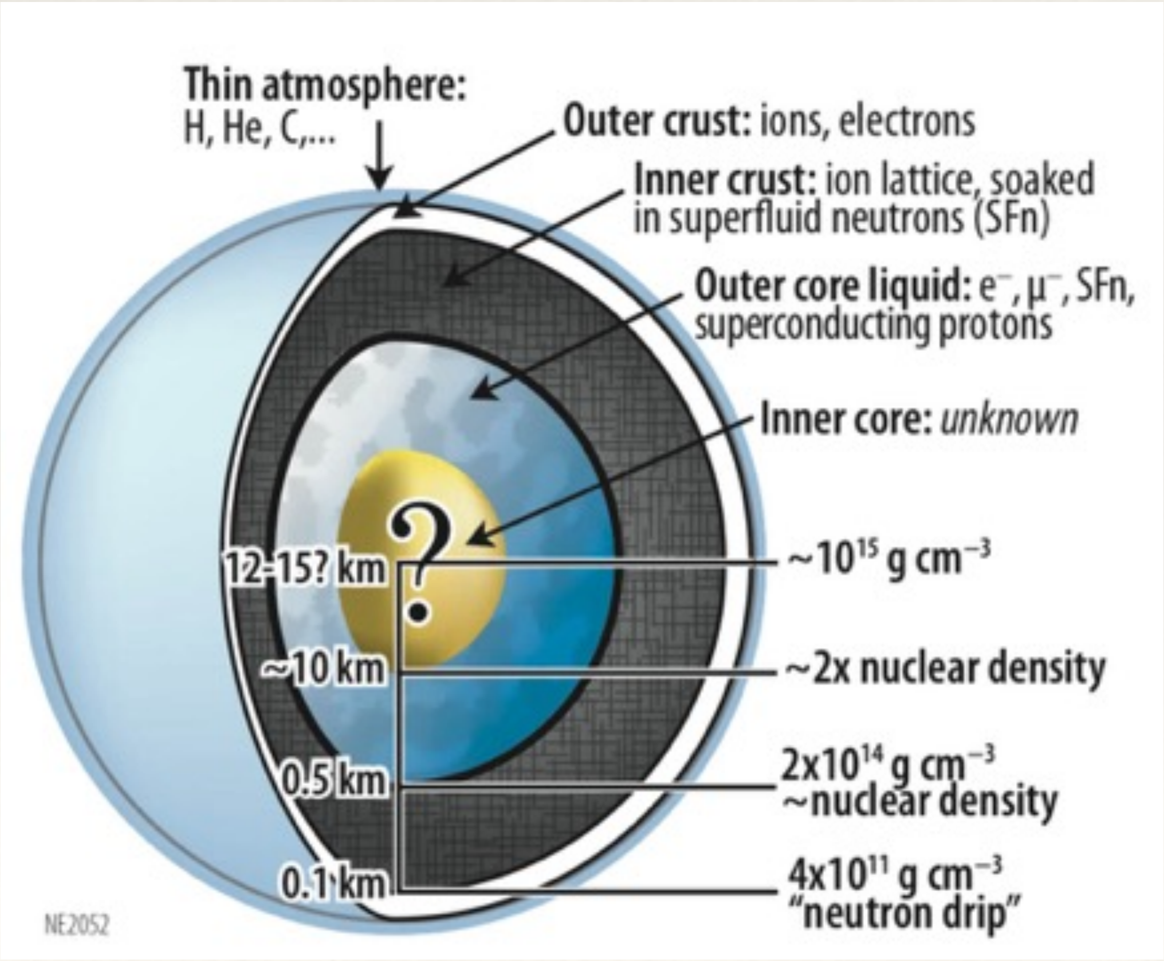
Neutron Stars

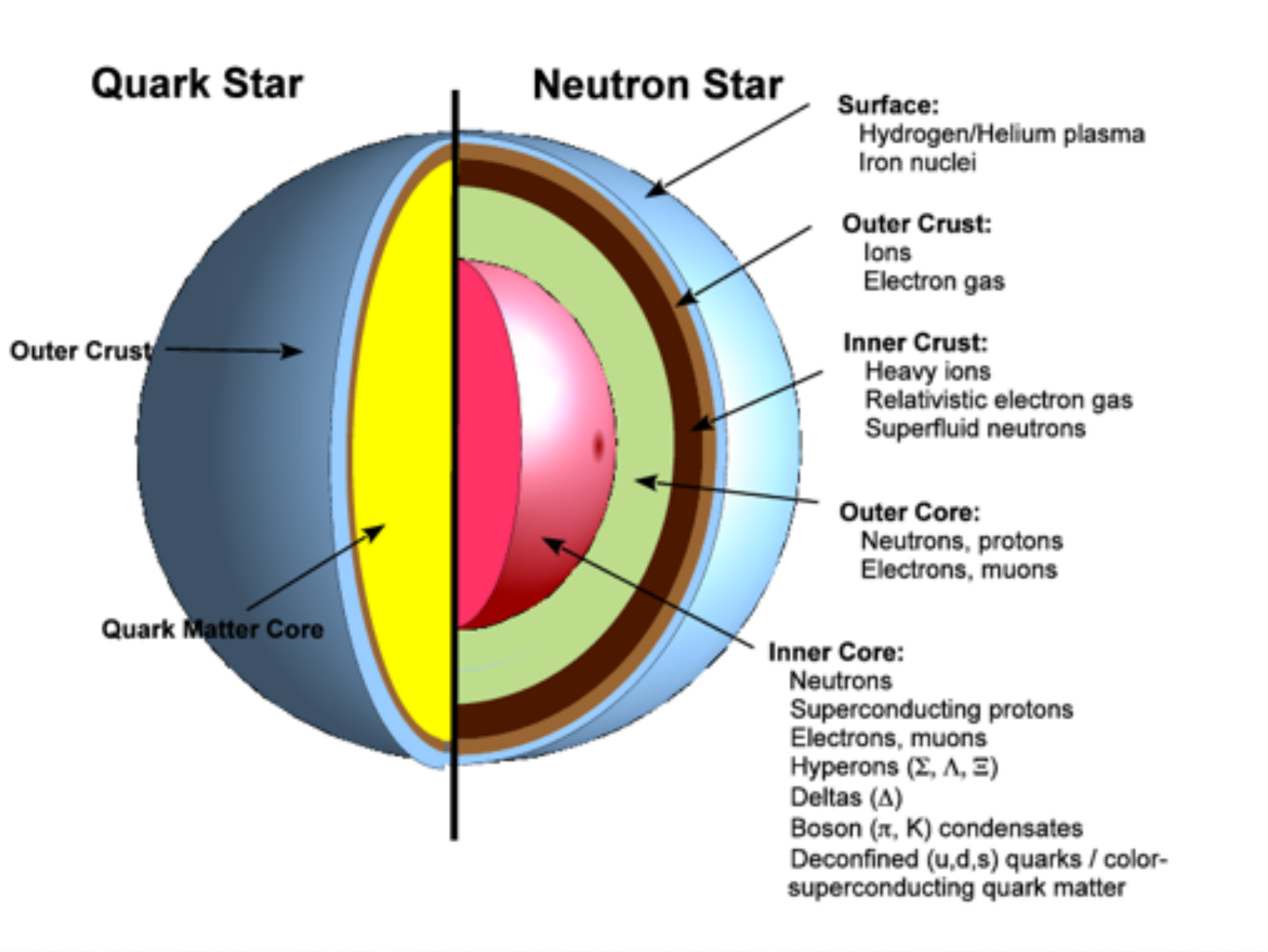
- Neutron Stars are a degenerate state of matter that is formed after the core collapse in a supernova event (where the electrons fall into nuclear matter and get captured by protons forming neutrons).
- Excellent laboratory to study high-density nuclear physics and EOS.
- Neutron star composition still unknown (neutron, resonance, hyperons,…)
- The extreme condition inside a NS cannot be reproduced in a laboratory.
- Typical properties of NS:

$$R \simeq 10Km$$
$$M \simeq 1.4M_\odot$$
$$T \in [1.4ms, 8.5s]$$
$$B \in [10^8, 10^{14}]Gauss$$

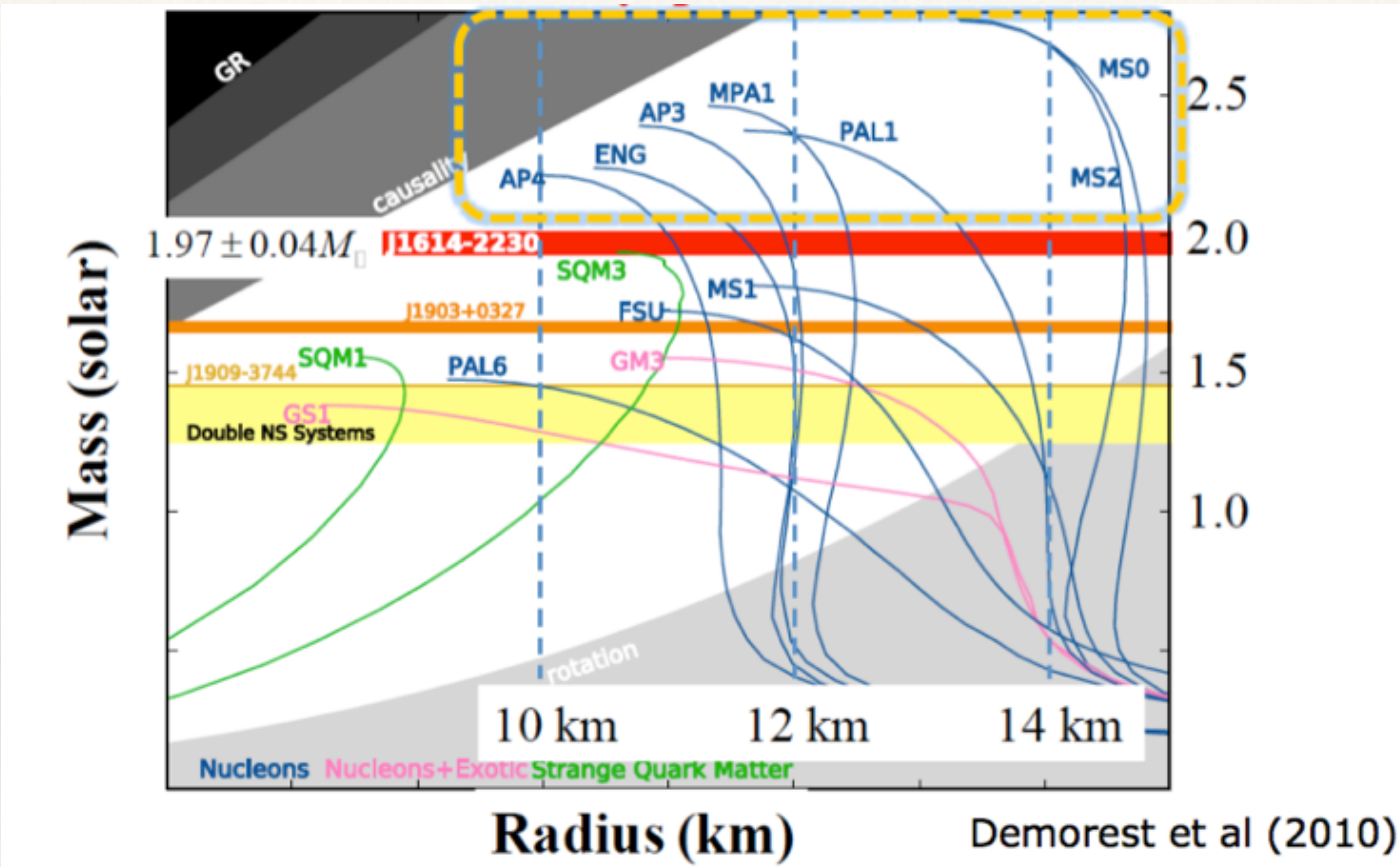
BNS as a probe for Nuclear Matter EOS

- Gravitational wave detection by BNS system will give us information on the EOS that cover matter at extreme conditions.
- Different possibilities:





Many different possibilities depending on the EOS



Many different possibilities depending on the EOS. **GWs** in the late inspiral and merger phases could constrain NS EOS. **Many GW templates from Numerical Relativity are necessary**

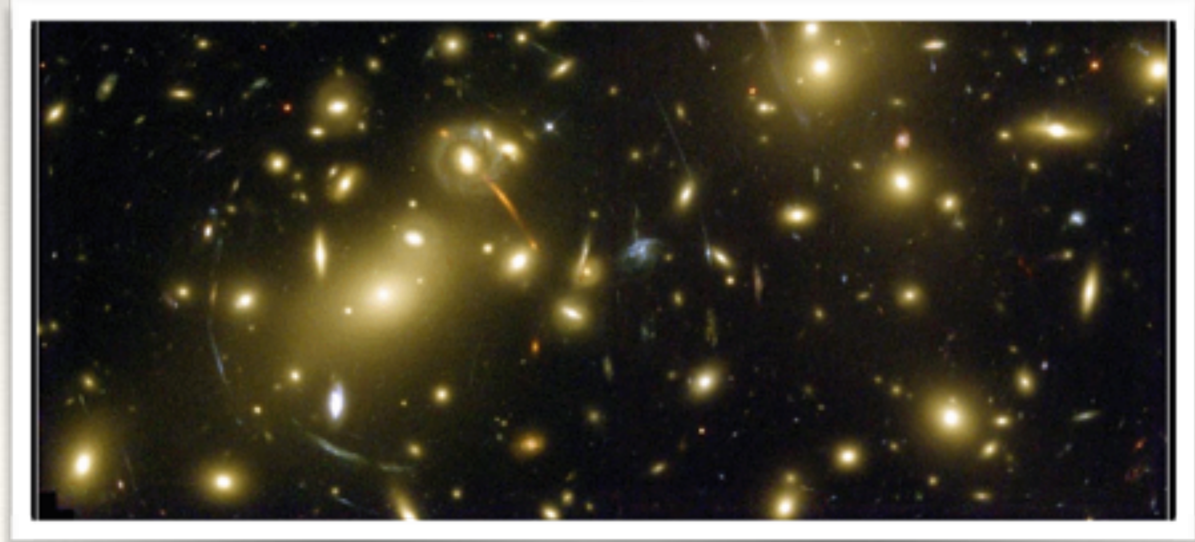
How we do simulate such system.

General Relativity (in short)

- The gravity is shown as a result of the fact that the space-time is curved!
    - Each mass-energy curved the space-time
    - Freely falling objects follow the geodetic (straight line) of a curved space-time.
    - Einstein's fields equation are:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

- There is a real space-time but we are free to choose any reference systems (atlas) to describe physical laws.



John Archibald Wheeler:
spacetime tells
matter how to move;
matter tells
spacetime how to curve

Numerical Relativity in a nutshell

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}$$ **Einstein Equations**

$$\nabla_\mu T^{\mu\nu} = 0$$ **Conservation of energy momentum**

$$\nabla_\mu (\rho\, u^\mu) = 0$$ **Conservation of baryon density**

$$p = p(\rho, \epsilon)$$ **Equation of state**

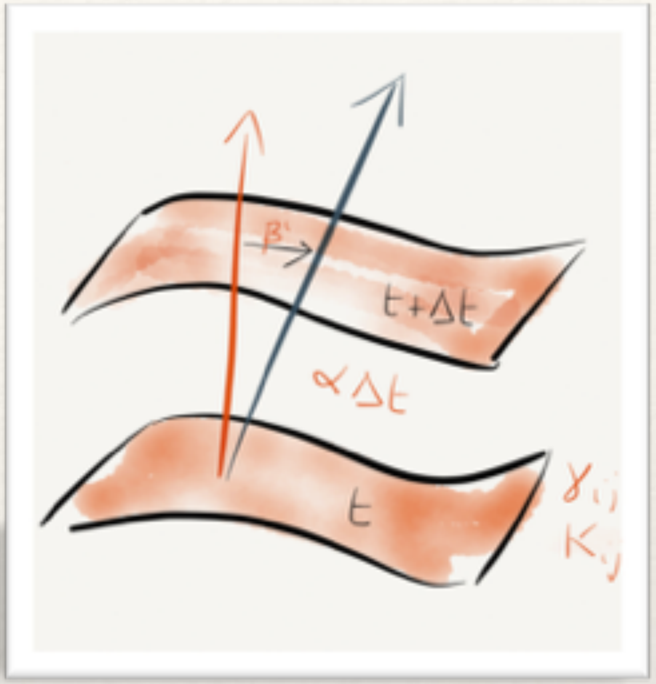
Ideal Fluid Matter

$$T^{\mu\nu} = (\rho(1+\epsilon) + p) u^\mu u^\nu + p g^{\mu\nu}$$

- But these are 4D equations! Need to write as 3+1 evolution equations.
- Spacetime get foliated into 3D spacelike surfaces, in which we define our variables. We evolve them along a time direction normal to those surfaces.
- (Magneto)Hydrodynamics is written in terms of conservative form and special numerical techniques are used for the fluxes calculations.
- All physical variables and equations are discretized on a 3D Cartesian mesh and solved by a computer. Uses finite differences for derivative computations and standard Runge-Kutta method for time integrations.
- Different formulation of the Einstein Eqs have been developed in the last 20 years. BSSN-NOK version of the Einstein's Eqs.

The base formalism (ADM)

1. **Choose initial spacelike surface and provide initial data (3-metric, extrinsic curvature)**



2. **Choose coordinates:**
   - Construct timelike unit normal to surface, choose lapse function
   - Choose time axis at each point on next surface (shift vector)
   - Evolve 3-metric, extrinsic curvature

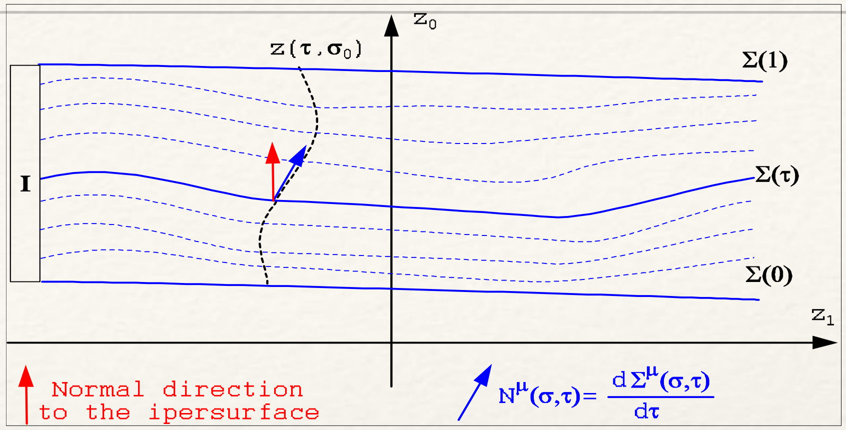
Use usual numerical methods:

1. **Structured meshes (including multi-patch), finite differences (finite volumes for matter), adaptive mesh refinement (since ~2003). High order methods.**
2. **Some groups use high accuracy spectral methods for vacuum space times**

Unfortunately Einstein Equation must be rewritten !

$$ds^2 = -\alpha^2 dt^2 + g_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$



- BSSN version of the Einstein's equations that introduce additional conformal variables:

$$R_{ij} = -\tfrac{1}{2}\tilde{g}^{lm}\tilde{g}_{ij,lm} - \tilde{g}_{k(i}\partial_{j)}\tilde{\Gamma}^k + \tilde{\Gamma}^k\tilde{\Gamma}_{(ij)k} + \tilde{g}^{lm}(2\tilde{\Gamma}^k_{l(i}\tilde{\Gamma}_{j)km} + \tilde{\Gamma}^k_{im}\tilde{\Gamma}_{klj})$$

$$\partial_t\varphi = -\tfrac{1}{6}\alpha K + \beta^i\partial_i\varphi + \tfrac{1}{6}\partial_i\beta^i$$

$$\partial_t K = -g^{ij}\nabla_i\nabla_j\alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + \tfrac{1}{3}K) + \beta^i\partial_i K$$

$$\partial_t\tilde{g}_{ij} = -2\alpha K_{ij} + \tilde{g}_{jk}\partial_i\beta^k + \tilde{g}_{ik}\partial_j\beta^k - \tfrac{2}{3}\tilde{g}_{ij}\partial_k\beta^k$$

$$R^{TF}_{ij} = R_{ij} - \tfrac{1}{3}g_{ij}R$$

$$\partial_t\tilde{\Gamma}^i = -2\tilde{A}^{ij}\partial_j\alpha + 2\alpha(\Gamma^i_{jk}\tilde{A}^{jk} - \tfrac{2}{3}\tilde{g}^{ij}\partial_j K + 6\tilde{A}^{ij}\partial_j\varphi) + $$
$$+ \beta^k\partial_k\tilde{\Gamma}^i - \tilde{\Gamma}^k\partial_k\beta^i + \tfrac{2}{3}\tilde{\Gamma}^i\partial_k\beta^k + \tfrac{1}{3}\tilde{g}^{ij}\partial_j\partial_k\beta^k + \tilde{g}^{jk}\partial_j\partial_k\beta^i$$

$$\partial_t\tilde{A}_{ij} = e^{-4\varphi}(-(\nabla_i\nabla_j\alpha)^{TF} + \alpha R^{TF}_{ij}) + \alpha\left(\tilde{A}_{ij}K - 2\tilde{A}_{ik}\tilde{A}^k{}_j\right) - \partial_i\partial_j\alpha + $$
$$+ \beta^k\partial_k\tilde{A}_{ij} + \left(\tilde{A}_{ik}\partial_j + \tilde{A}_{jk}\partial_i\right)\beta^\kappa - \tfrac{2}{3}\tilde{A}_{ij}\partial_k\beta^k$$

- Matter evolution (B set to zero) using shock capturing methods based on the GRHydro code

[4] M. Shibata, T. Nakamura: "Evolution of three dimensional gravitational ..", Phys. Rev. D52(1995)5429
[5] T.W. Baumgarte, S.L. Shapiro: "On the numerical integration of Einstein..", Phys. Rev. D59(1999)024007

Other formulation with the same good properties and constrain dumping are used: namely Z4, Z4c,…..

Matter evolution need HRSC Methods

$$\nabla_\mu T^{\mu\nu} = 0 \qquad p = p(\rho, \epsilon)$$

Ideal Fluid Matter

$$T^{\mu\nu} = (\rho(1+\epsilon) + p)u^\mu u^\nu + p g^{\mu\nu}$$

- **The equation of a perfect fluid are a non linear hyperbolic system.**

$$\frac{\partial \vec{u}}{\partial t} + \frac{\partial \vec{f}^i}{\partial x^i} = \vec{s}(\vec{u})$$

- **Wilson (1972) wrote the system as a set of advection equation within the 3+1 formalism.**
- **Non-conservative. Conservative formulations well-adapted to numerical methodology:**
  - Martí, Ibáñez & Miralles (1991): 1+1, general EOS
  - Eulderink & Mellema (1995): covariant, perfect fluid • Banyuls et al (1997): 3+1, general EOS
  - Papadopoulos & Font (2000): covariant, general EOS

The numerical challenge

A challenging numerical problem

- The accurate simulation of a BNS merger is among the most challenging tasks in numerical relativity.
- Involve strong gravitational fields, matter motion with relativistic speeds, relativistic shock waves, (and strong magnetic fields).
- Increasing difficulty due to the multidimensional character of the PDE and by the complexity of the Einstein's equations such as coordinates degrees of freedom and formation of black holes (curvature singularity).
- Despite the problems, major progress achieved during the last decade in numerical simulations of BNS mergers (since the seminal work by Shibata and Uryu, 2000) due to: improved numerical methods (high resolutions methods and adaptive mesh refinements), improved physics (nuclear physics EOS, thermal effects) and increased computational resources!!

A challenging numerical problem (2)

- In the description of BNS mergers are involved three stages, the inspiral, the merger and the evolution to its final state (post-merger stage) that would quite likely be a BH surrounded by an accretion disk.

- The inspiral stage can be modeled with good accuracy by analytical techniques (PN calculations and EOB). Produce accurate waveforms up to a time very close to the merger. Useful to quickly computing waveform templates to matched filtering searches in GW detector data analysis. The role of NR in this regime is mainly to test and help improve these techniques.

- For the merger and post-merger stage, NR is the only available investigation tool to compare the experimental results that would be obtained by LIGO/Virgo detection with the underlying physics of the NS.

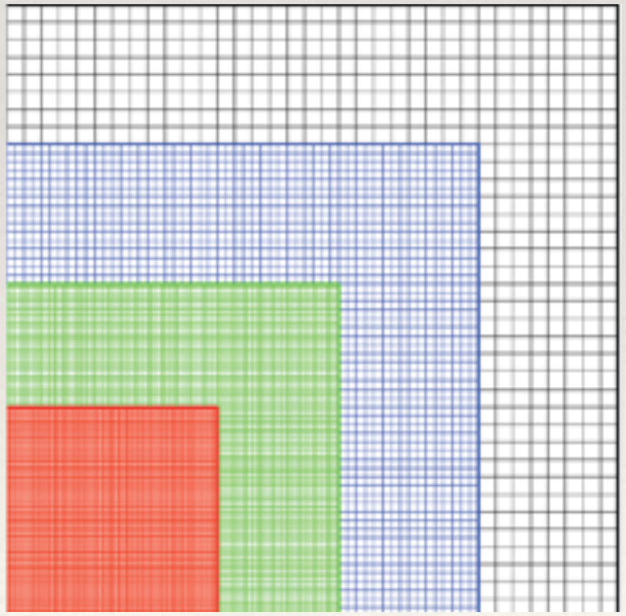
- An accurate description of GW emission of different model sources (different choice of the underlying NS physics through different choices of EOS) are useful for developing empirical relations to be able to infer NS parameter from future GW detections, as well as, to get information on the correct EOS that describe matter at this extreme conditions.

The code: Einstein TOOLKIT + LORENE

- **Einstein Toolkit** open set of over 100 Cactus thorns for computational relativity along with associated tools for simulation management and visualization
- **Cactus** framework for parallel high performance computing (Grid computing, parallel I/O)
- Data are evolved on a Cartesian Mesh with 6 levels of refinement with **Carpet**
- Matter Evolution with the module **GRHydro:**
  (Magnetic+**CT evolution** of Magnetic Field)
  HLLE Riemann Solver
  WENO Reconstruction method (*)
  PPM Reconstruction methods
- Spacetime Metric evolution is performed with the module MacLachlan implementing a 3+1 dimensional split of the Einstein Eqs.
  BSSN-NOK Gravitational Evolution scheme (*)
  CCZ4 gravitational evolutions
- Initial data computed using the **LORENE CODE**

einsteintoolkit.org

cactus code

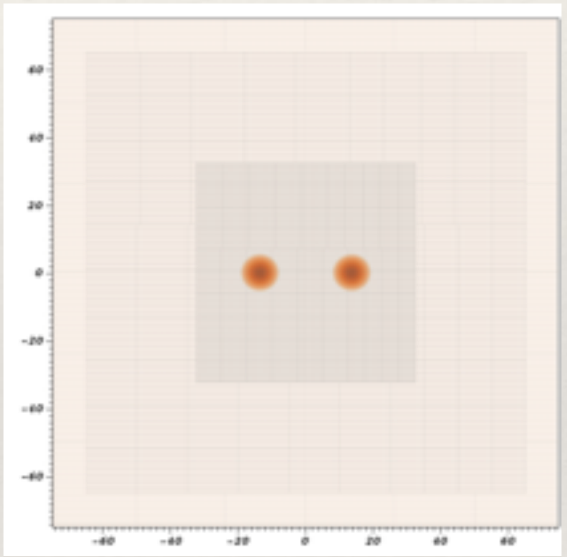


The computational challenge

- Cartesian grid with 6 refinement levels (7 when we get a BH).
- Standard Resolution in the finest grid 0.25 CU and up to 0.125 CU. => from 5,337,100 points up to 42,696,800 per grid.
- Outer grid extends to (1063Km) to extract gravitational waves far from the source.
- One extra refinement level added just before collapse to black hole.
- 12 spacetime variables + 4 gauge variables + 5 hydrodynamical variables evolved in each point.
- MPI+OpenMP code parallelization.

Level	$\min(x/y)$ (CU)	$\max(x/y)$ (CU)	$\min(z)$ (CU)	$\max(z)$ (CU)	(N_x, N_y, N_z) $dx = 0.25$
1	-720	720	0	720	(185,185,96)
2	-360	360	0	360	(205,205,106)
3	-180	180	0	180	(205,205,106)
4	-90	90	0	90	(205,205,106)
5	-60	60	0	30	(265,265,76)
6	-30	30	0	15	(265,265,76)
(7	-15	15	0	7.5)	(265,265,76)

1 CU = 1.4 km



Δx (CU)	0.75	0.50	0.375	0.25	0.185	0.125
# threads	16	64	128	256	512	2048
# MPI	2	8	16	32	64	256
Memory (GBytes)	3.8	19	40	108	237	768
speed (CU/h)	252	160	124	53	36	16
speed (ms/h)	1.24	0.78	0.61	0.26	0.18	0.08
cost (SU/ms)	13	81	209	974	2915	26053
total cost (kSU, 50 ms)	0.65	4	10.5	49	146	1300

Scaling on real world simulations

- Scaling of the the Einstein Toolkit on the CINECA "Galielo" system.
- Performance on a real world simulation!

$\triangle x$ (CU)	0.75	0.50	0.375	0.25	0.185	0.125
# threads	16	64	128	256	512	2048
# MPI	2	8	16	32	64	256
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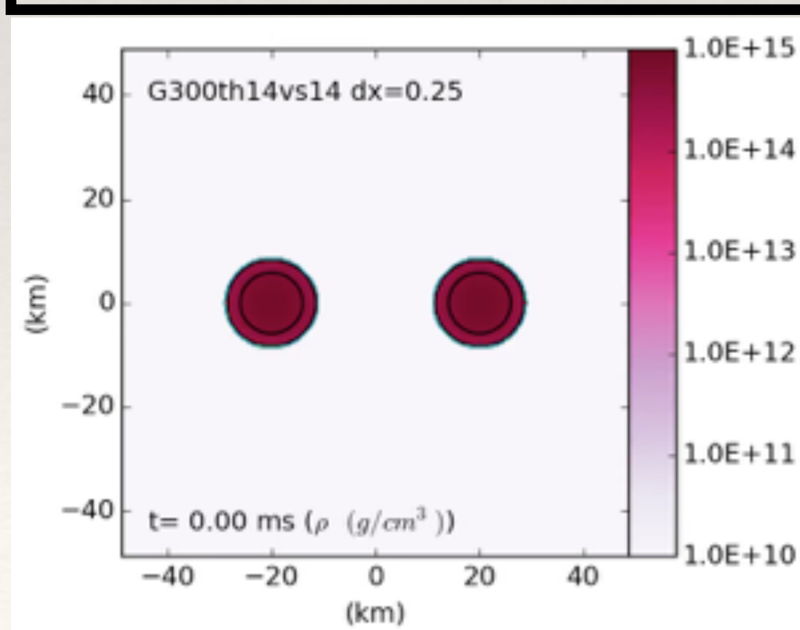
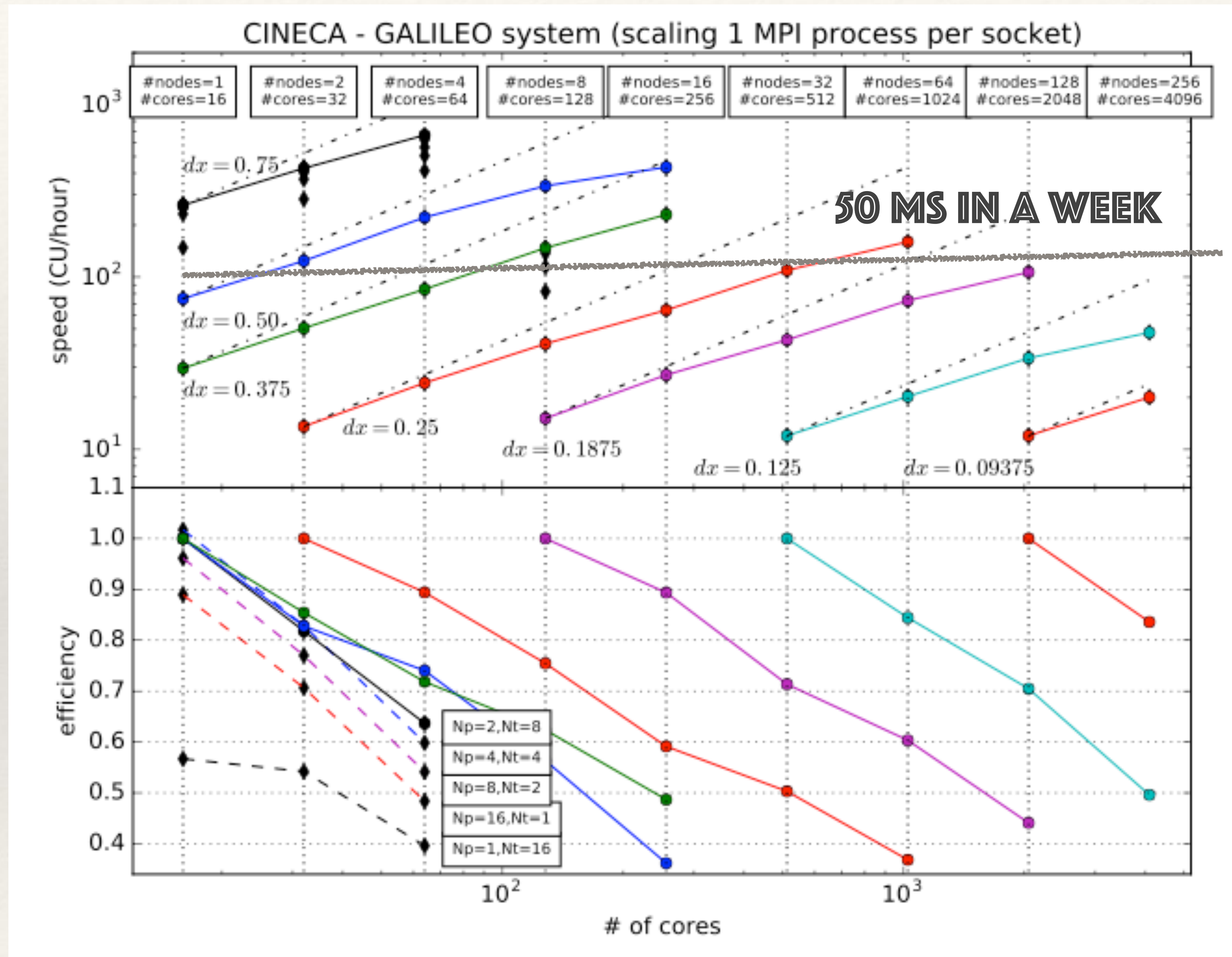
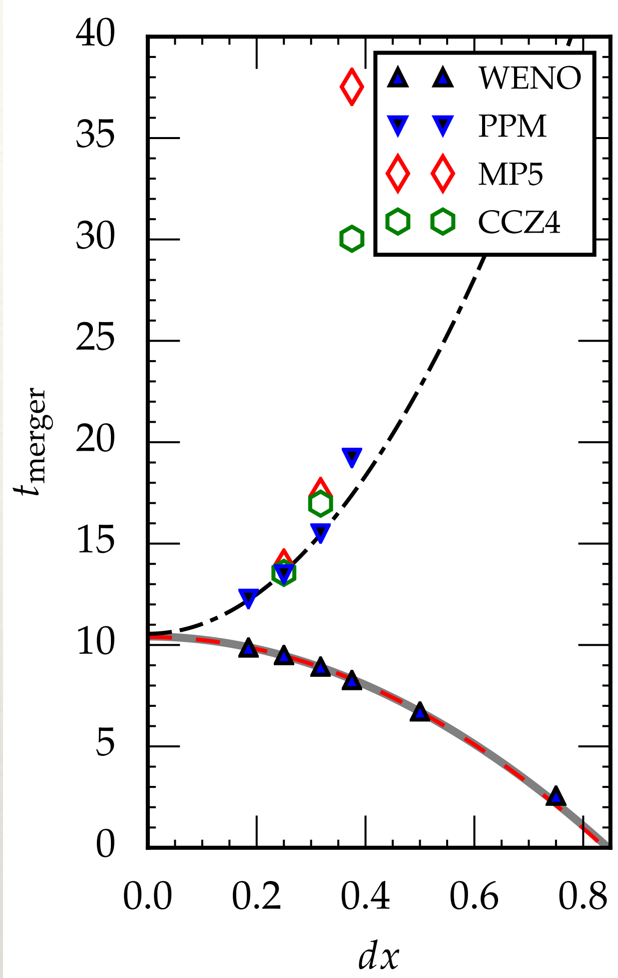




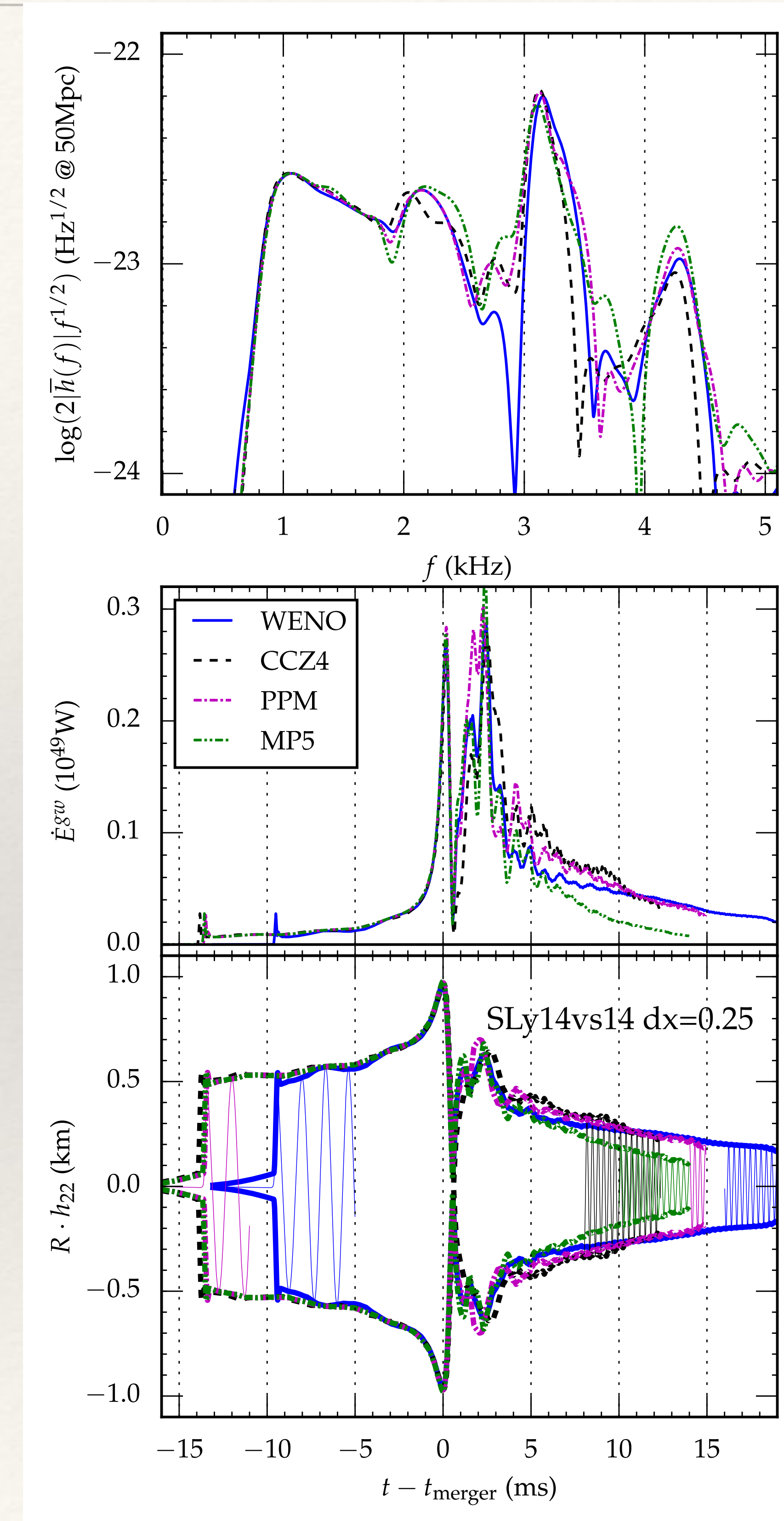
Results on Numerical Methods comparisons

Comparison between three different reconstruction methods for hydrodynamics (**WENO**,**PPM**,**MP5**)

and two gravity (metric) evolution schemes (**BSSN**,**CCZ4**).



	dx (CU)	t_{merger} (ms)
WENO	0.75	2.56
WENO	0.50	6.72
WENO	0.375	8.27
WENO	0.3125	8.93
WENO	0.25	9.50
WENO	0.185	9.87
PPM	0.375	19.23
PPM	0.3125	15.50
PPM	0.25	13.49
PPM	0.185	12.28
MP5	0.375	37.53
MP5	0.3125	17.34
MP5	0.25	13.81
CCZ4	0.375	30.03
CCZ4	0.3125	16.97
CCZ4	0.25	13.56



- The combination BSSN + WENO is the best for running sensible simulations at low resolution.
- With those methods you can run a qualitatively correct BNS simulation on your laptop!

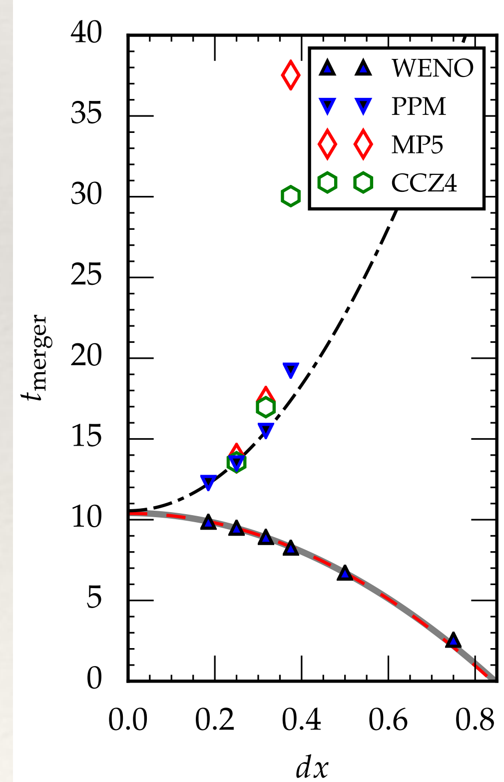
Data Analysis: Convergence

- Merger time measured from at least three different resolution simulations for each model.
- Convergence order and extrapolated "infinite" resolution merger time obtained with a fit to:

$$t_{\rm merger}(dx) = t^{dx=0}_{\rm merger} + A \cdot dx^{\gamma},$$

- Despite all observed differences it is important to make sure that all tested method lead to the same determination of the "true" merger time t_merger(dx=0).

name	$t^{dx=0.50}_{\rm merger}$ [ms]	$t^{dx=0.375}_{\rm merger}$ [ms]	$t^{dx=0.25}_{\rm merger}$ [ms]	$t^{dx=0\ (ext)}_{\rm merger}$ [ms]	$t^{\rm EOB}_{\rm merger}$ [ms]
SLy12vs12	9.22	11.76	13.61	15.07±0.03	21.55
SLy13vs13	8.21	10.02	11.25	12.28±0.04	17.25
SLy14vs14	6.72	8.27	9.50	10.39±0.08	14.08
SLy15vs15	5.93	6.99	7.71	8.31±0.02	11.64
SLy16vs16	5.00	6.13	6.81	7.44±0.08	9.78
SLy135vs145	6.66	8.19	9.45	10.34±0.10	14.09
SLy13vs15	6.52	7.91	9.31	10.14±0.25	14.12
SLy125vs155	6.19	7.60	9.09	9.93±0.29	14.21
SLy12vs16	5.52	7.26	8.73	9.75±0.13	14.33
G275th14vs14	4.22	4.81	5.52	5.88±0.17	13.63
G300th14vs14	7.63	9.69	10.55	11.67±0.37	14.78



	dx (CU)	t_{merger} (ms)
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PPM	0.3125	15.50
PPM	0.25	13.49
PPM	0.185	12.28
MP5	0.375	37.53
MP5	0.3125	17.34
MP5	0.25	13.81
CCZ4	0.375	30.03
CCZ4	0.3125	16.97
CCZ4	0.25	13.56

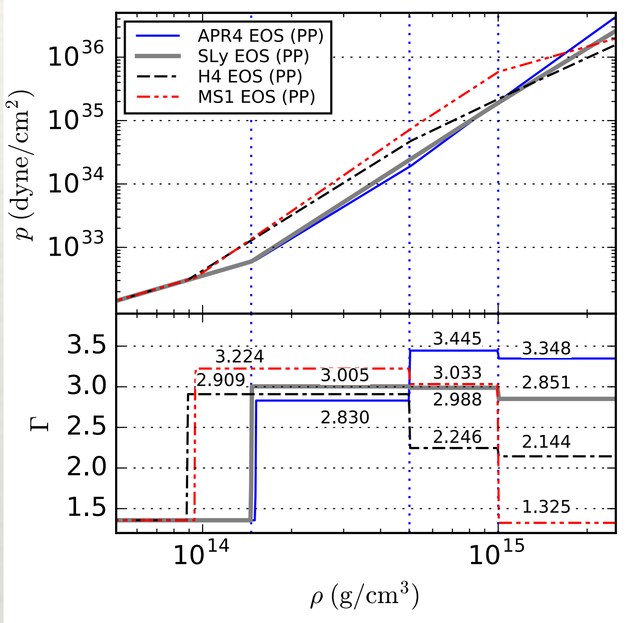
Initial Models

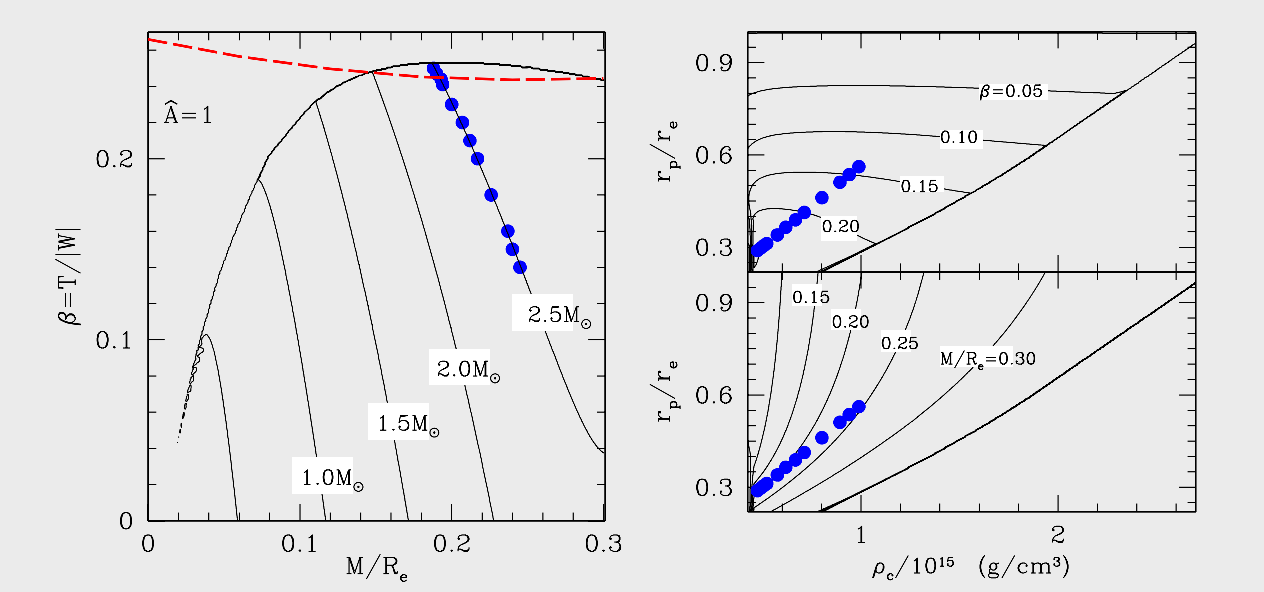
From EOS to initial data

- An EOS is a table that connect pressure and energy to barion density (possible also to temperature, electron fraction,…)
- Given the EOS is possible to solve Einstein Equations + Matter imposing stationarity and axial symmetry => Models for Isolated Stars. Various codes allow to get such solutions: LORENE and RNSID for uniformly and differentially rotating Stars.
- It is possible to calculate the maximum mass for non-rotating or uniformly rotating stars.

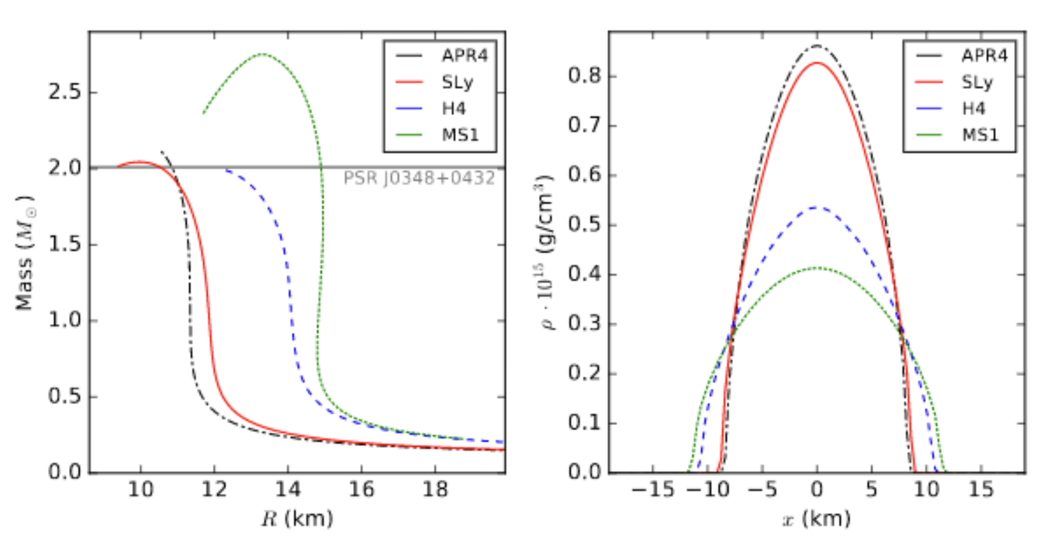
EOS	SMNS ($M_\odot$)	HMNS ($M_\odot$)
SLy	2.04 (2.42)	2.41 (2.82)
H4	2.01 (2.30)	2.37 (2.70)
APR4	2.19 (2.66)	2.60 (3.09)
MS1	2.75 (3.30)	3.29 (3.90)

TOV **UNIF.**



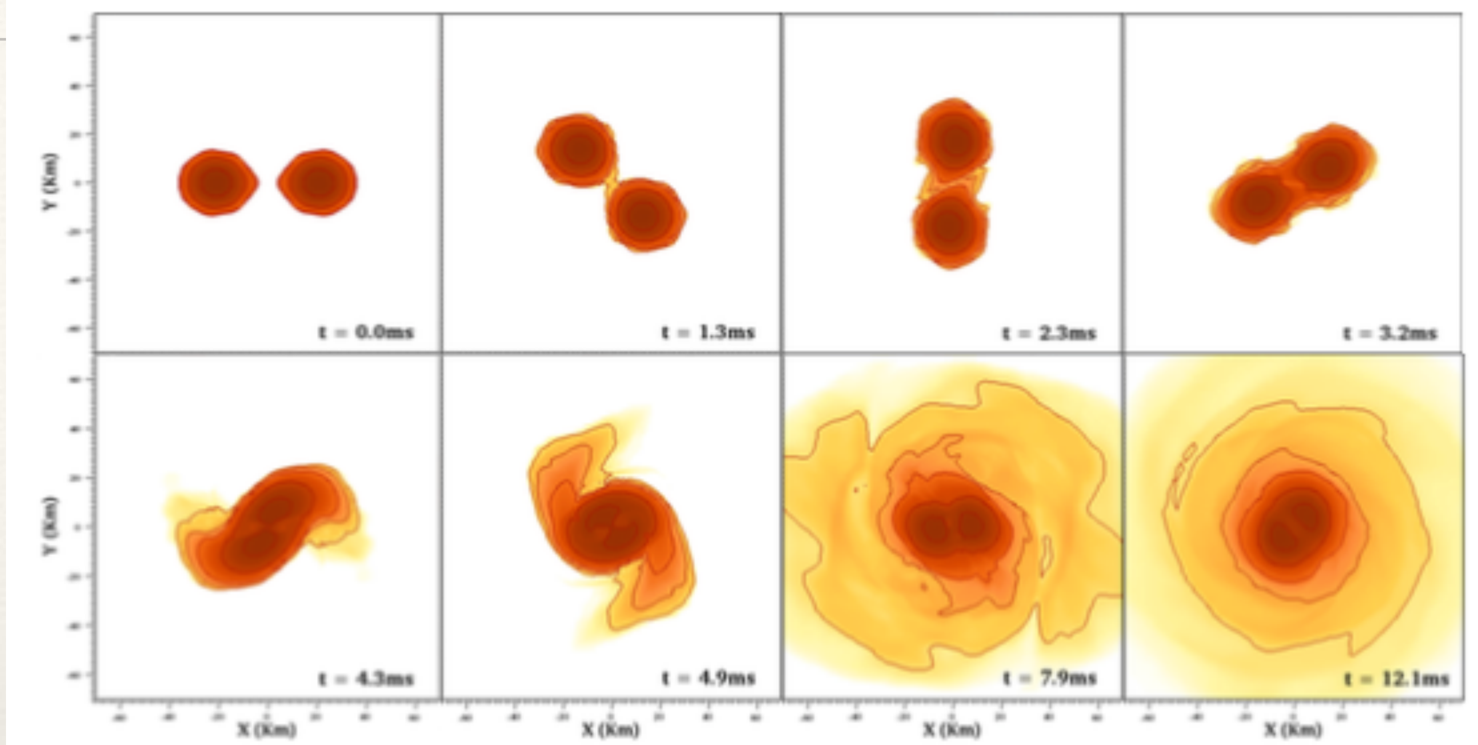


SLy - differential rotation



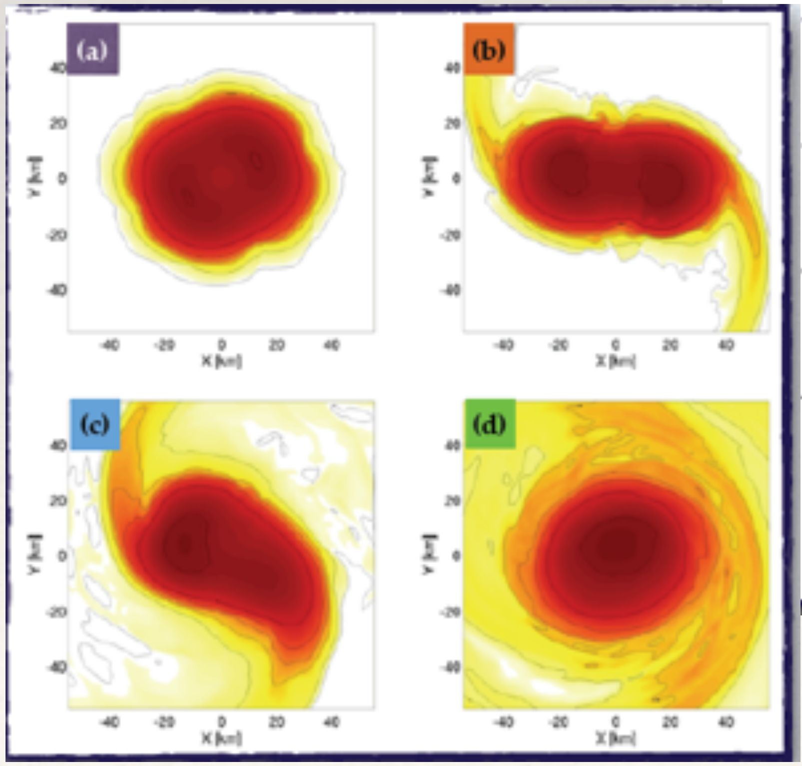
(a) Mass-Radius relations (b) Initial density profiles

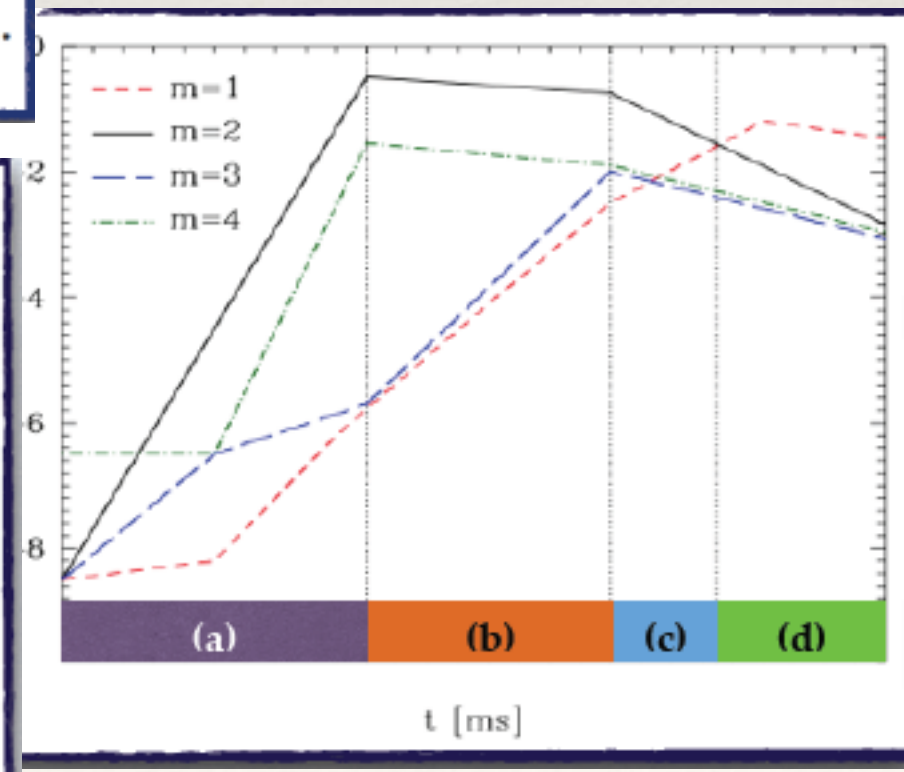
Dynamical bar-mode instability



- Dynamics of a Binary Neutron Merger.. … just after the formation of an HyperMassive Neutron Star there is a BAR-deformed stage
- BAR-MODE unstable stars show a stage that have a similar stage
- NICE PLAYGROUND to study magnetic DYNAMICS in NSs

$$P_m \equiv \int d^3x \, \rho \, e^{im\phi}.$$





Instability types in rotating stars

- Secular (m = 2): $\beta \geq \beta_{sec} \approx 0.14$
  - growth time determined by dissipative time scale (tens of seconds for neutron stars)

  e.g., see Chandrasekhar (1970), Ou, Tohline and Lindblom (2004)

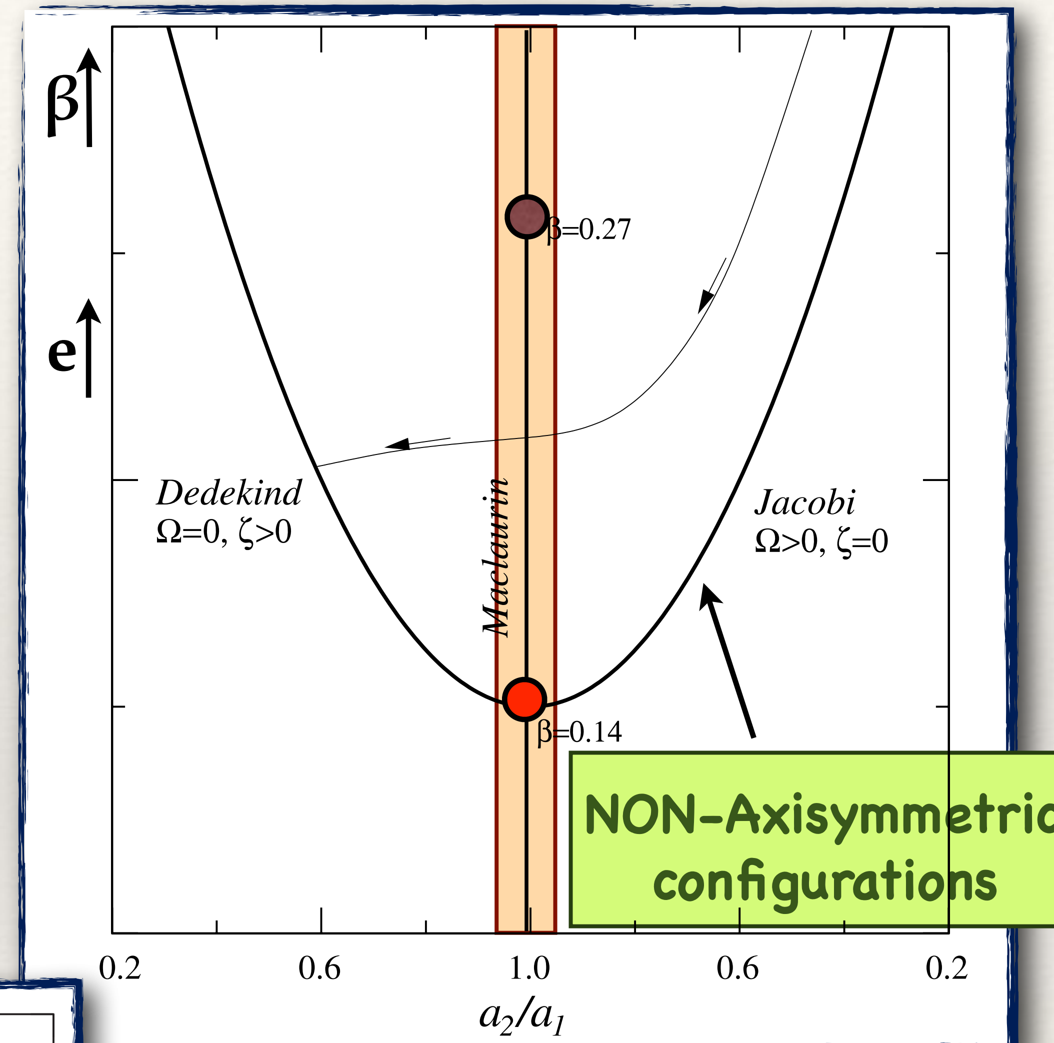
- Dynamical (m = 2): $\beta \geq \beta_{dyn} \approx 0.27$
  - grows on dynamical timescale (tens of milliseconds for neutron stars)

  e.g., see Shibata, Tohline, Baiotti, Manca, ...

- "Low T/W instability" - Shear?
  - first "observed" numerically (grows on dynamical time scale)

  e.g., see Centrella et al (2001), Corvino (2010), ...

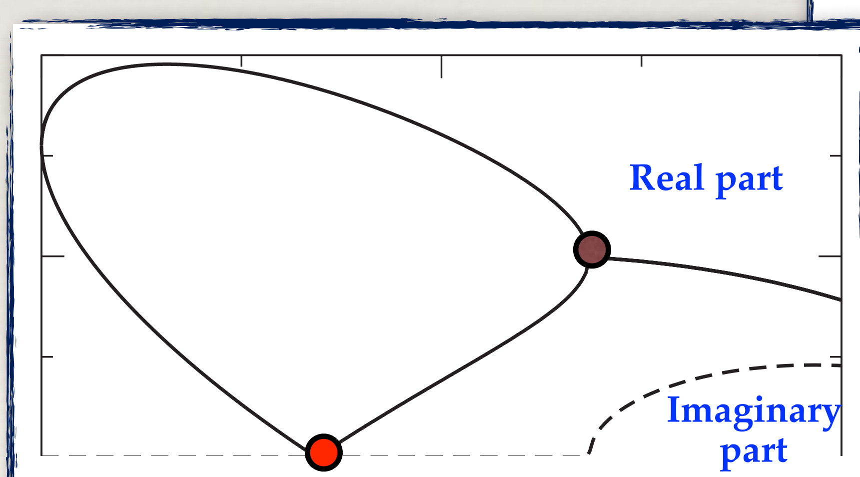
Axisymmetric configurations



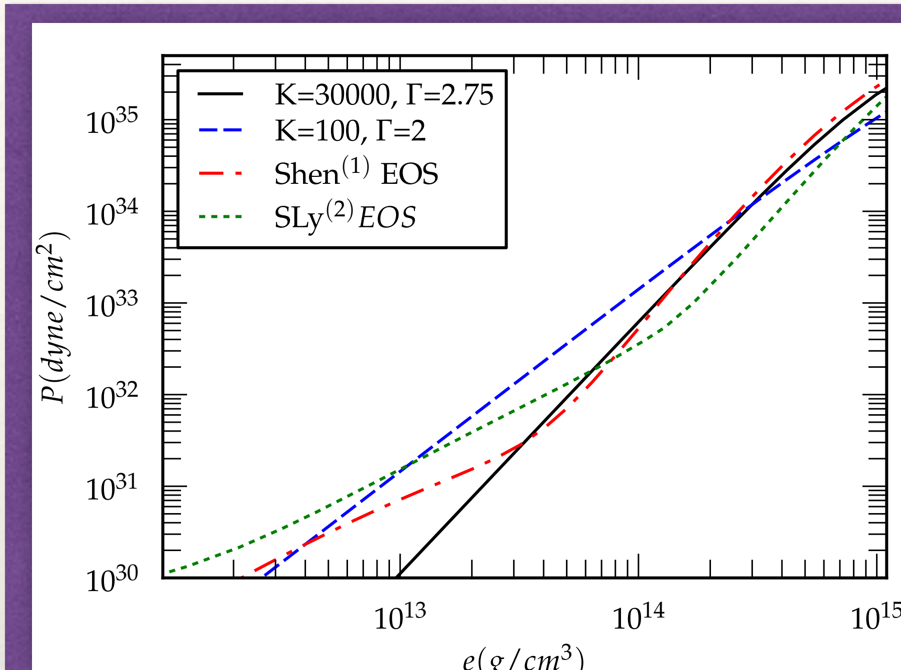
axes ratio in the x-y plane

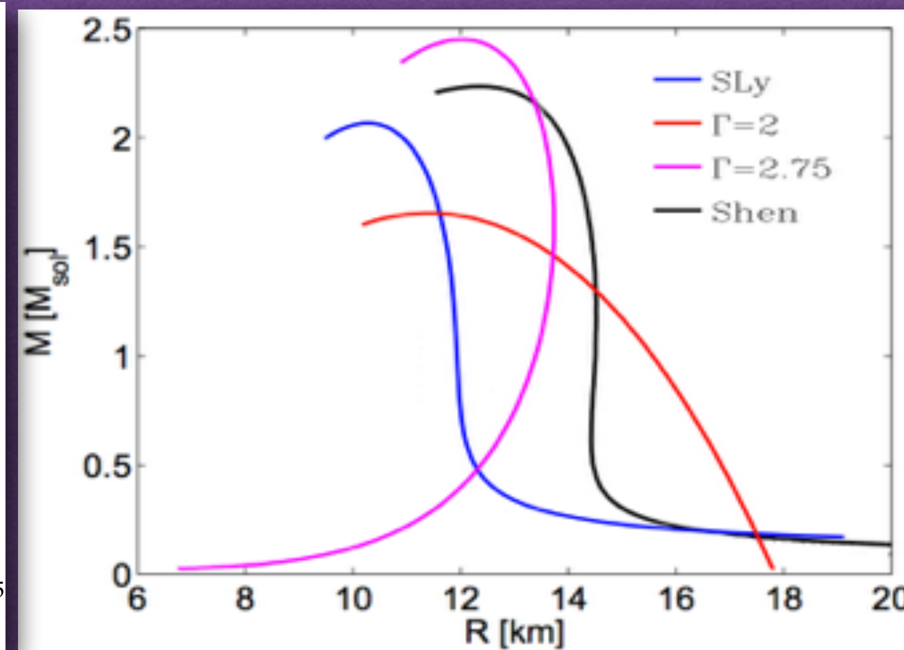
[*] Chandrasekhar, "*Elipsoidal figures of Equilibrium*" (Yale Univ. Press, 1969)

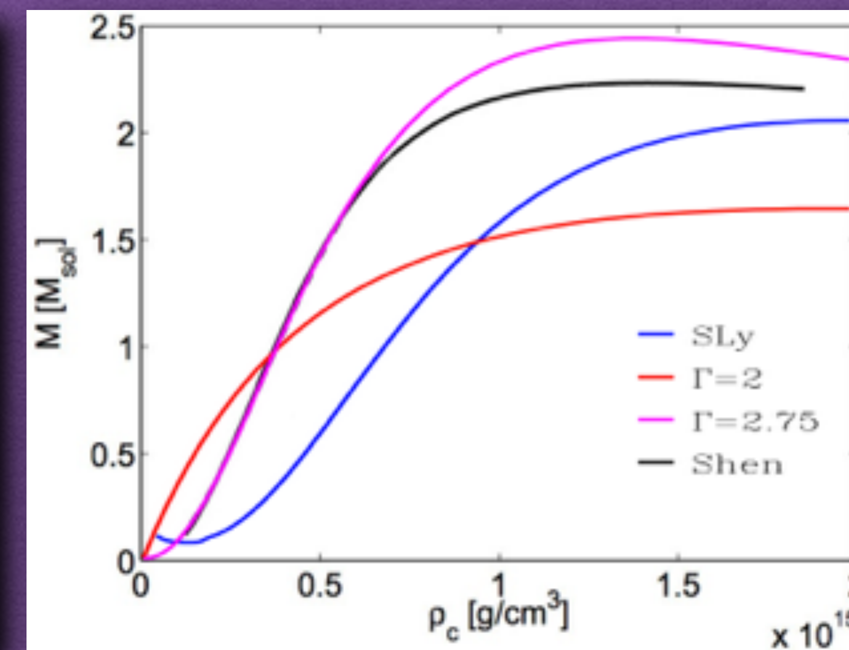
eigenvalue of the m=2 mode



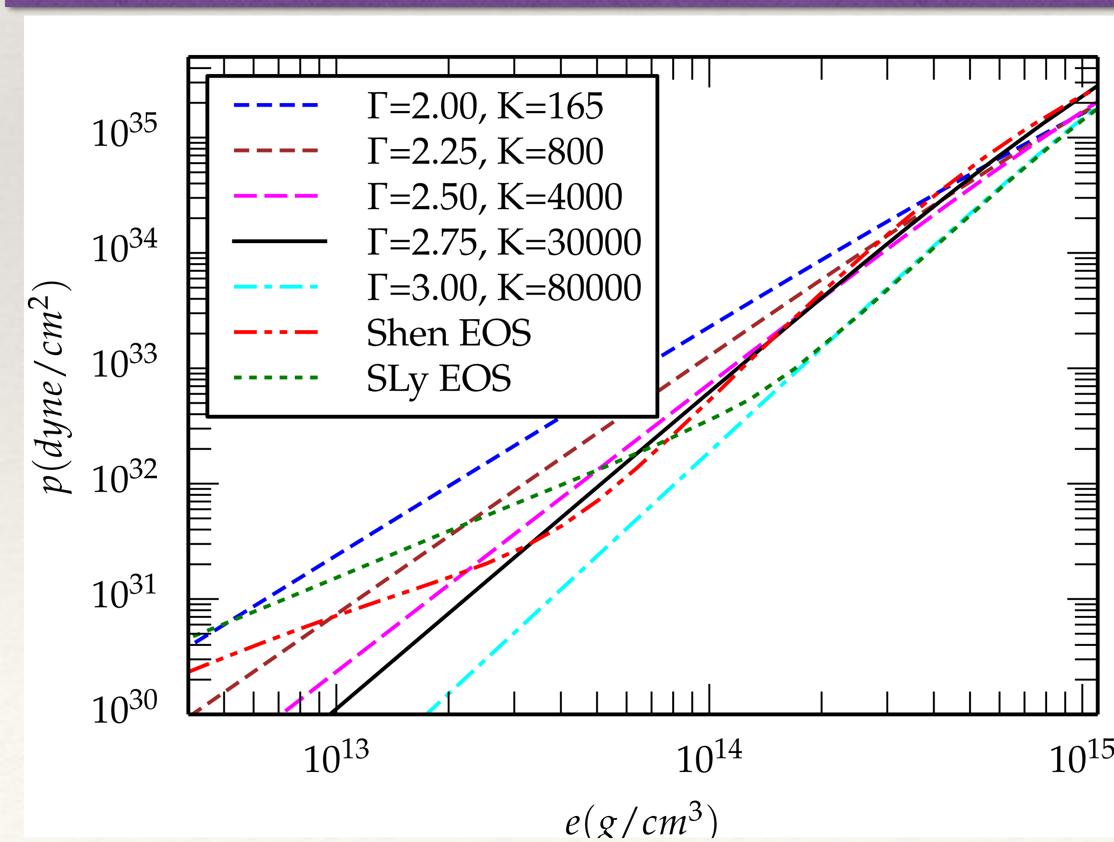
Effect of the EOS







[1] R. De Pietri, A. Feo, L. Franci and F. Loeffler "*Neutron star instabilities in full general relativity using a Γ=2.75 ideal fluid*" Phys. Rev. **D 90**, 024034 arXiv:1403.8066.



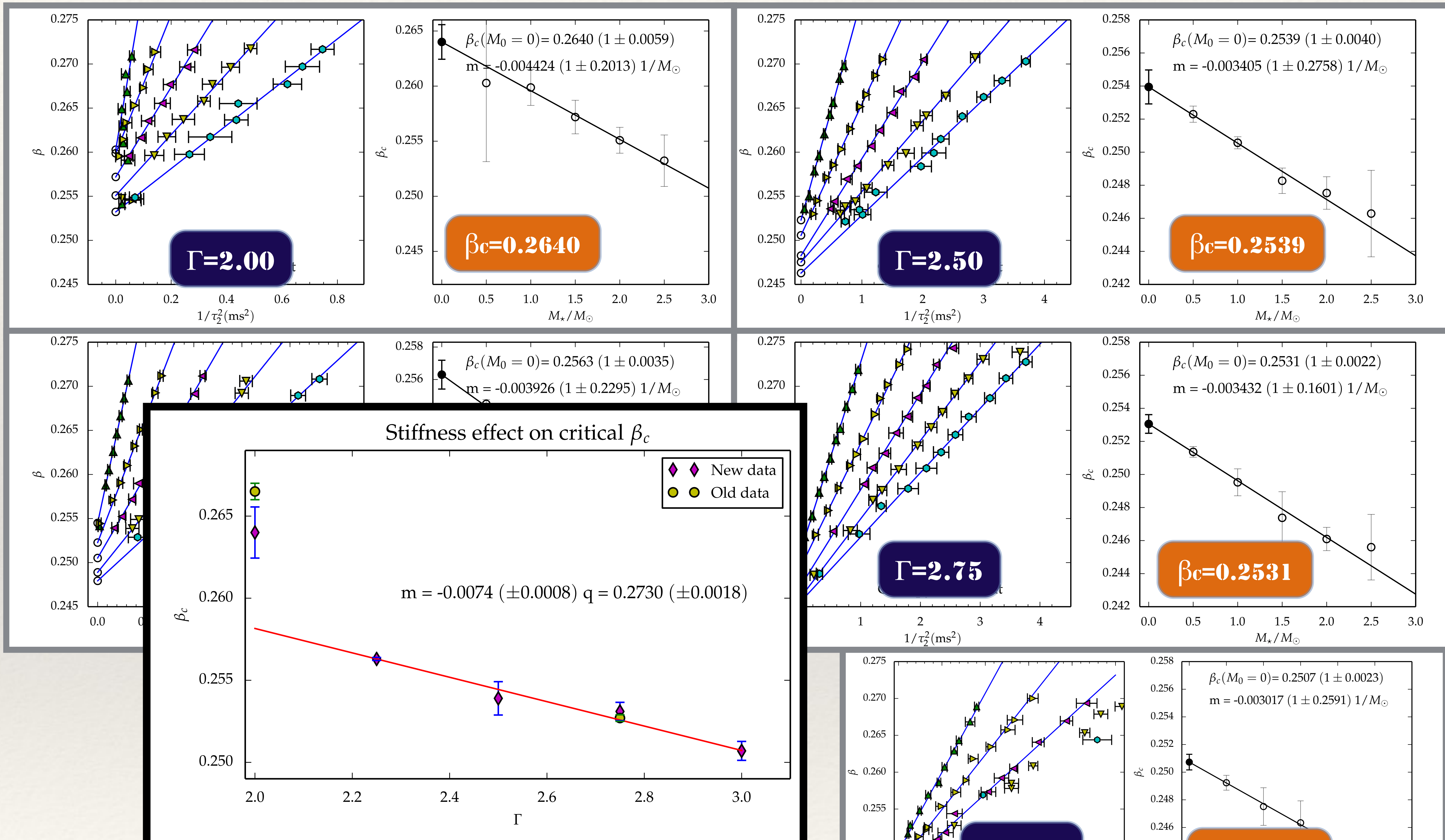
SLy: unified Sly EOS models high-density and cold (i.e. zero temperature) matter via a Skyrme effective potential for the nucleon-nucleon interactions

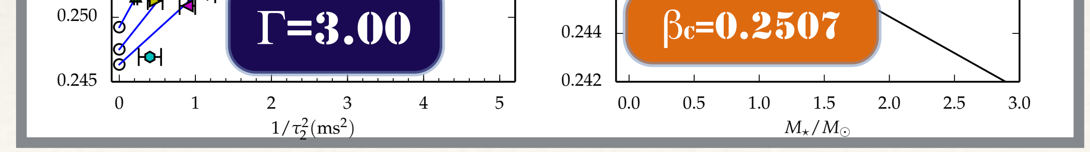
Shen: relativistic mean-field (RMF) framework

polytropic EoS $p = K\rho^{\Gamma}$

Γ = 2 -> 2.25, 2.50, 2.75, 3.00

Bar mode instabilities (Effect of EOS)

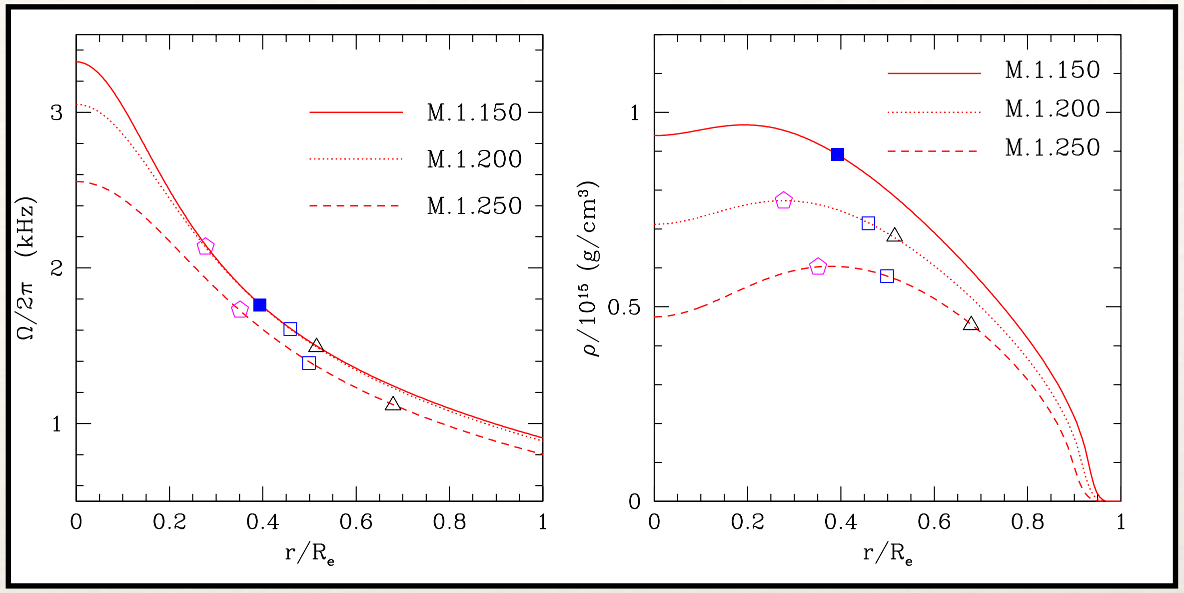


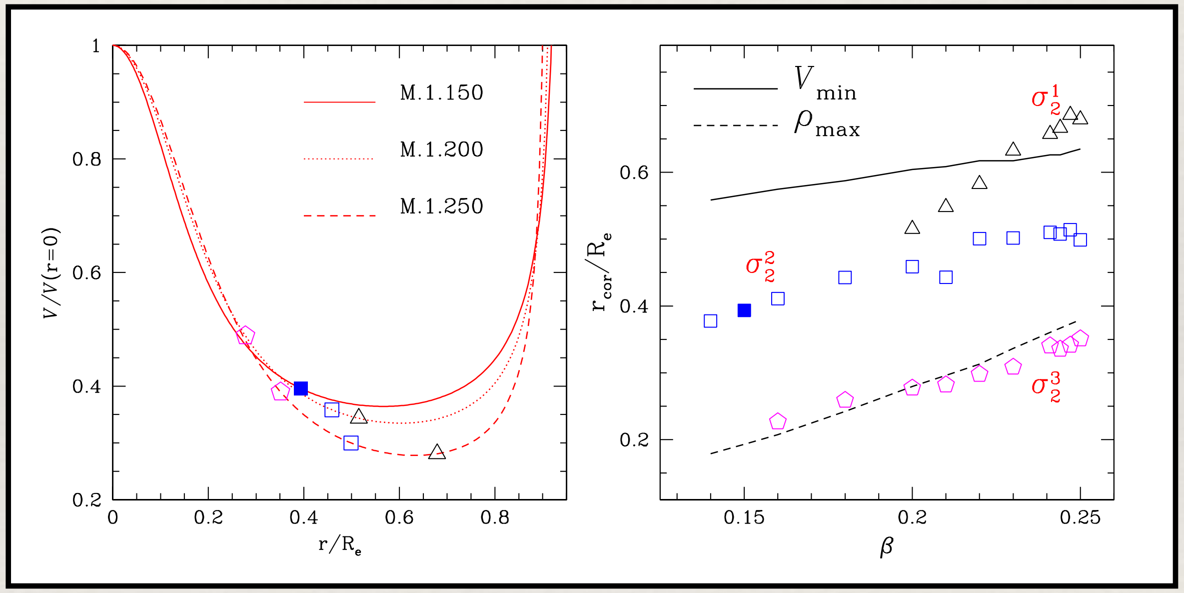


Stiffness effects on the dynamics of the bar-mode instability of Neutron Stars in full General Relativity F Löffler, R De Pietri, A Feo, L Franci, F Maione - arXiv preprint arXiv:**1411.1963, 2014**

Shear instability

- Possibility of other instabilities in rapidly and differentially rotating neutron stars.
- Evidence of instabilities below the expected threshold for the dynamical bar-mode instability, $\beta c \equiv T/|W| \simeq 0.25$
- Shear Instability on a dynamical timescale and for a wide range of values of β.
- This class of instability support the phenomenological predictions made by Watts et al (2005 Astrophys. J. 618 L37) on the nature of the low-T / | W | instability.
- Manifestation of a shear instability in a region where the latter is possible only for small values of β.





Corvino et al., “*On the shear instability in relativistic neutron stars*” , Class. Quantum Grav. 27 (2010) 114104

A short note on Magnetic Fields

Magnetic fields of realistic strengths imprint correction on matter dynamics to small to be significative but:

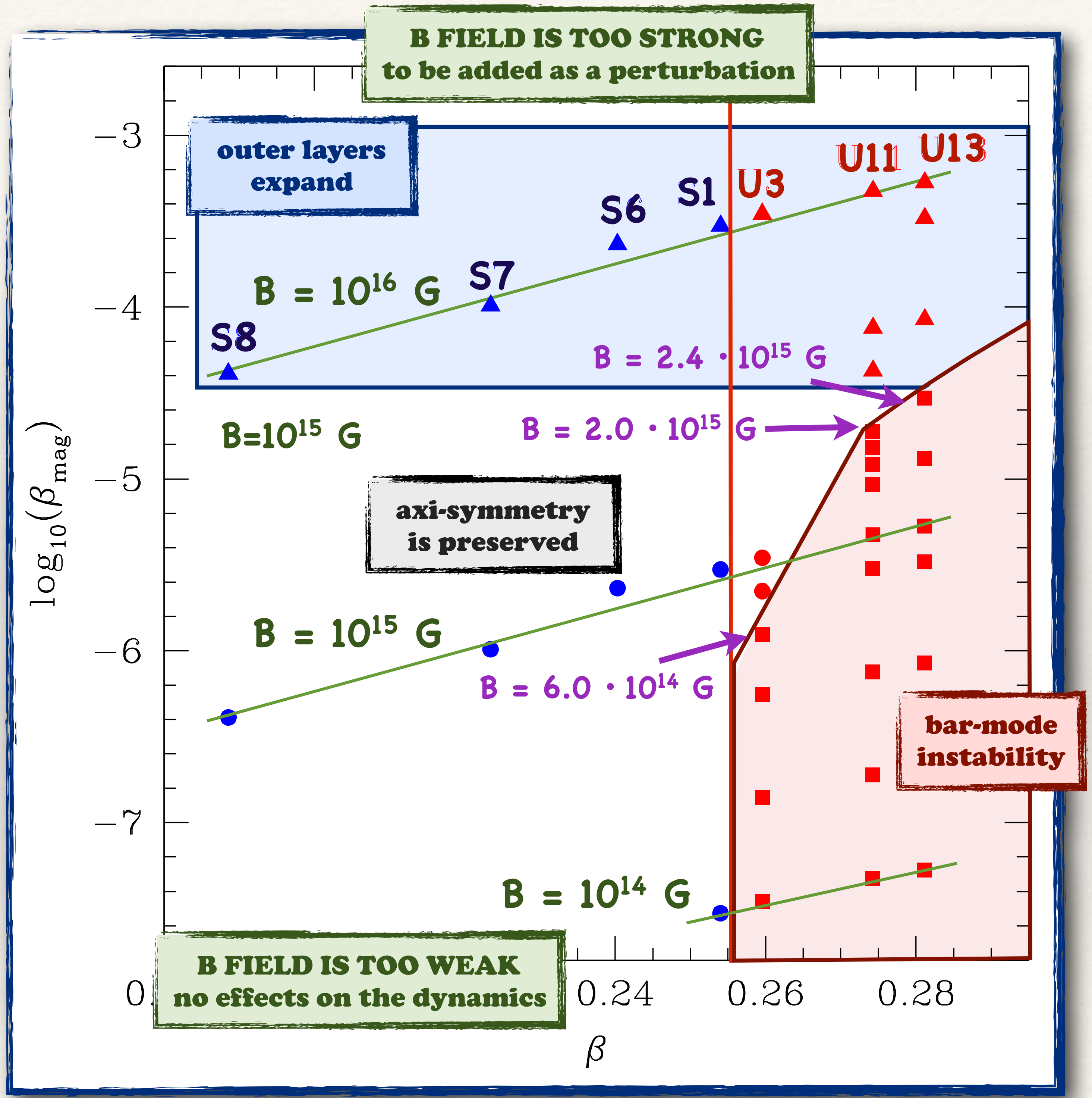
Magnetic fields are expected to be amplified:

* At the merger (via Kelvin-Helmholtz instability),
* After the merger (via a magneto-rotational instability or a dynamo action converting small-scale fields into large-scale ones).
* The final and effective amplification of the resulting magnetic fields is still uncertain, although it should be of at least two-three orders of magnitude.

Effect of adding a poloidal magnetic on BAR-mode rotating stars.

- **for all values of B**: a very strong growth of the toroidal component due to the winding of the magnetic field lines
- **B < 10^{14} G:**: the matter dynamics is quite unaffected for all models
- **10^{14} G < B < 10^{15} G:**: onset and development of the bar-mode instability are affected by the presence of the B field
- **10^{15} G < B < 10^{16} G:** the bar instability is completely suppressed (the threshold depends on the model) and the outer layers expand
- **B > 10^{16} G:** the initial configuration is no longer an equilibium model

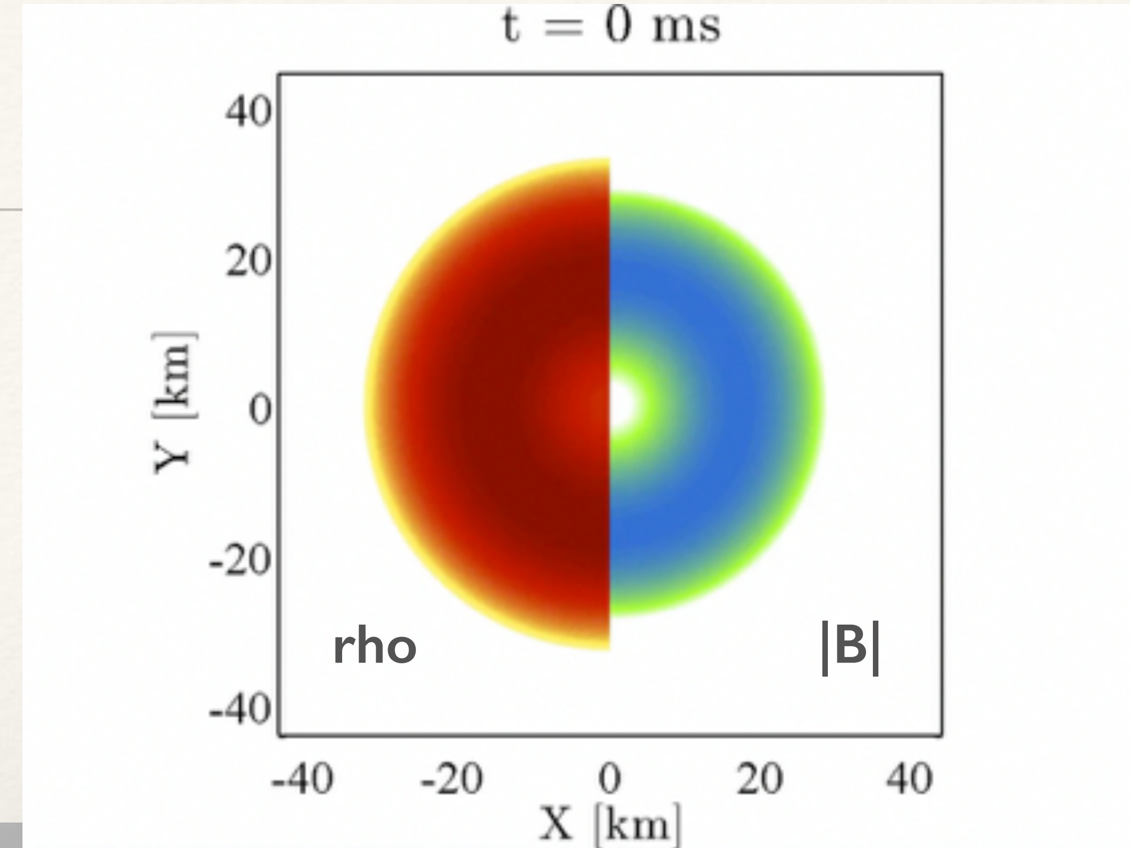
$\beta = T/|W|$ $\beta_{mag} = E_{mag}/(T + |W|)$

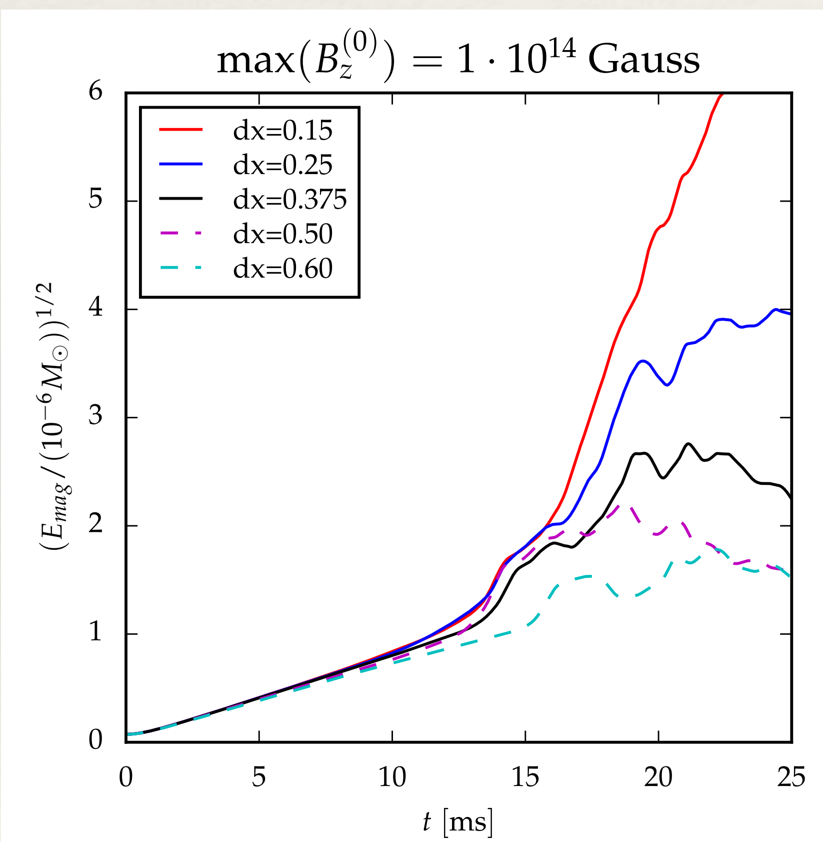


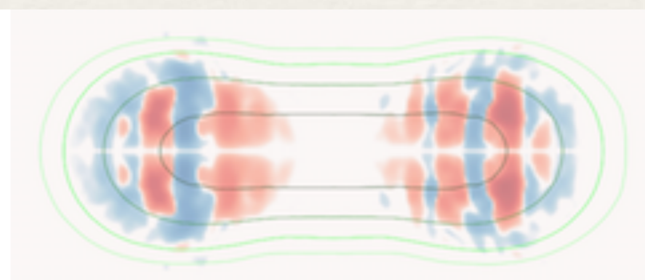
L. Franci, R. De Pietri, K. Dionysopoulou, L. Rezzolla,
Dynamical bar-mode instability in rotating and magnetized relativistic stars,
Phys. Rev. D 88, 104028 (2013). arXiv:1308.3989

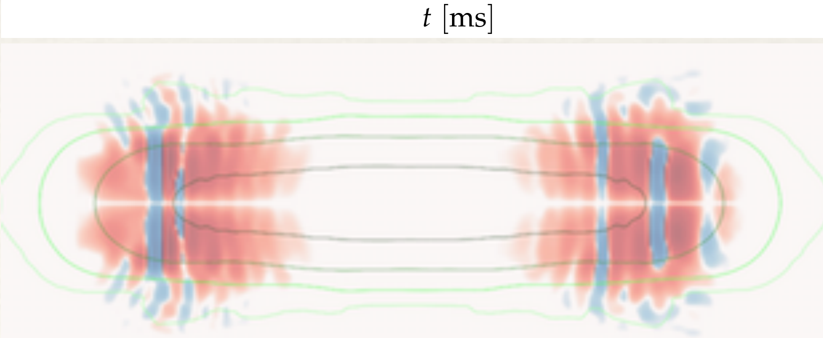
Magnetic Fields

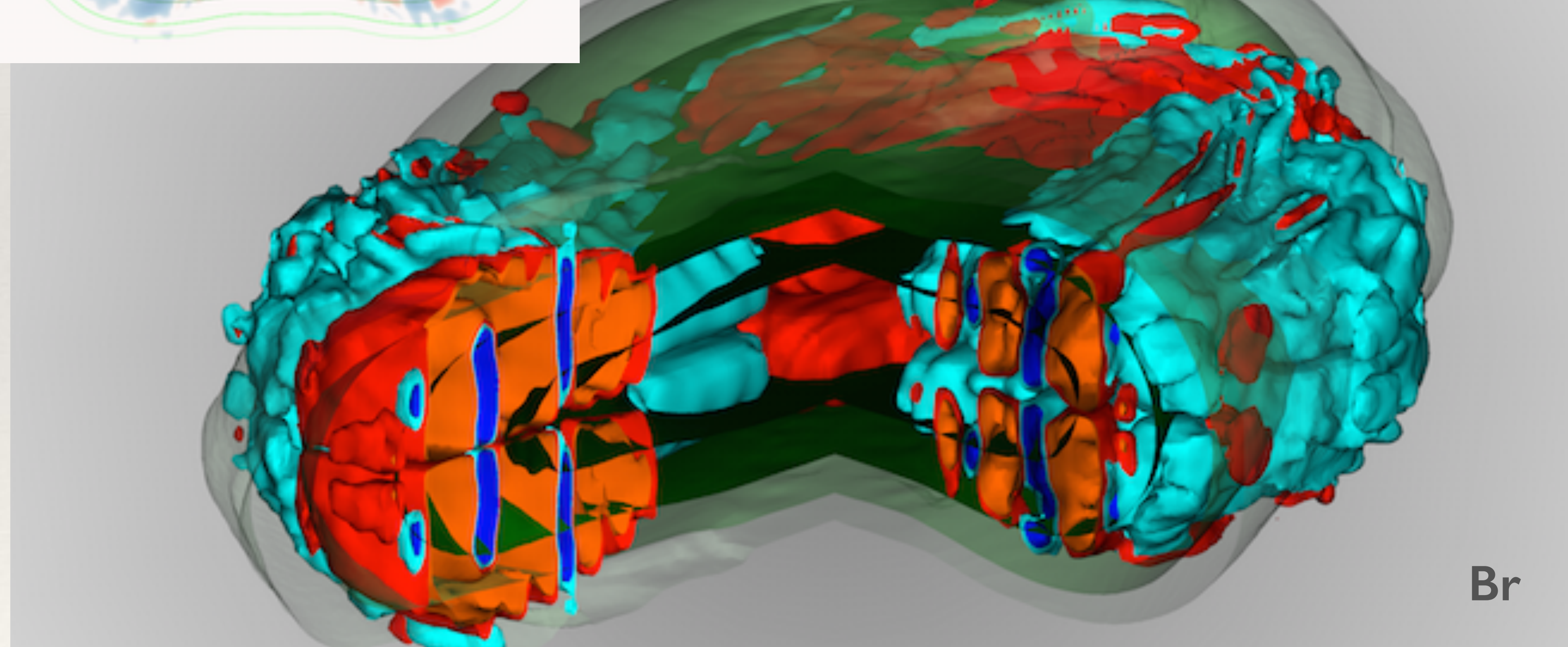
- Dynamics of the evolution of a model with a seed magnetic field of 10^14 Gauss.
- Left: matter density
- Right: modulus of the magnetic fields [1012-1016,5 Gauss] in the xy plane a z=1.5
- grid = [207x407x407], i.e more the 300 points inside the stars











Snapshot of the amplitude of the radial component of magnetic fields at time **t=13.5 ms** of the evolution of the stellar model U13-1.0e14. Please note, in the two sections on the xz-plane and yz-plane, the typical "wave" structure expected in the presence of MRIs.

Binary Neutron Stars System

- EOS … initial data for binary neutron star system … waveform … detection … validate the proposed form for the EOS.
- Question: **Is it possible to discriminate between different EOS**. Answer: **Yes, it is**.
- Main problem are:
- It is not easy to generate (consistent) initial data with complete control of the spin, orbital parameter, initial magnetic fields,… Recent progress by Rezzolla,Tichy, Kyutoku groups.
- HOWEVER: exist a PUBLIC CODE that allows to generate ID for non-rotating stars starting from a tabulated EOS at T=0. Need to extend the availability of PUBLIC initial data.

Initial models we studied and how we computed them .

- The initial data of our simulations is calculated using the **LORENE** code ["LORENE: Langage Objet pour la RElativité NumériquE," **http://www.lorene.obspm.fr/**] that provides the possibility to generate arbitrary initial data for irrotational BNS.
- The code is GPL free and can be freely and easily used to generate the initial data for the simulations.
- The initial data generated by LORENE show a residual eccentricity and we will show how this can be seen in numerical simulation **(SLy14vs14)**

Pulsar	M_p $[M_\odot]$	M_c $[M_\odot]$	q	$M^{(1)}$ $[M_\odot]$	$M^{(1)}$ $[M_\odot]$
J0737-3039A	1.3381(7)	1.2489(7)	0.93	1.36	1.47
B1534+12	1.3330(2)	1.3454(2)	0.99	1.46	1.47
J1756-2251	1.341(7)	1.230(7)	0.92	1.33	1.47
J1906+0746*	1.291(11)	1.322(11)	0.98	1.41	1.45
B1913+16	1.4398(2)	1.3886(2)	0.96	1.53	1.59
J0453+1559	1.559(5)	1.174(4)	0.75	1.27	1.74

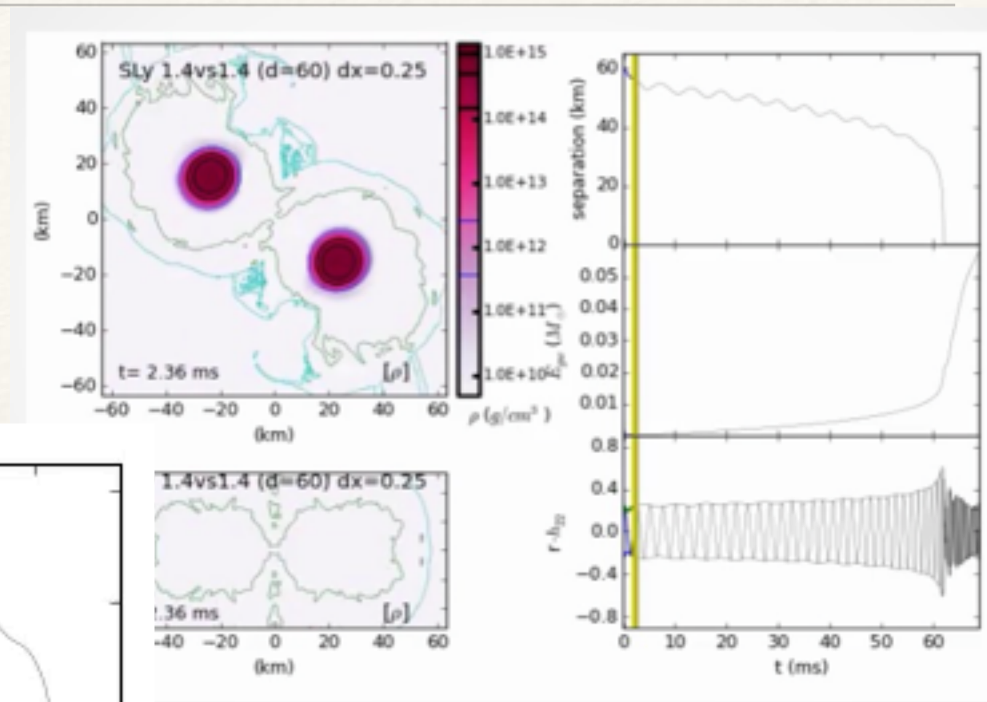
Table from: Martinez et al.: "Pulsar J0453+1559: A Double Neutron Star System with a Large Mass Asymmetry" arXiv:1509.08805v1

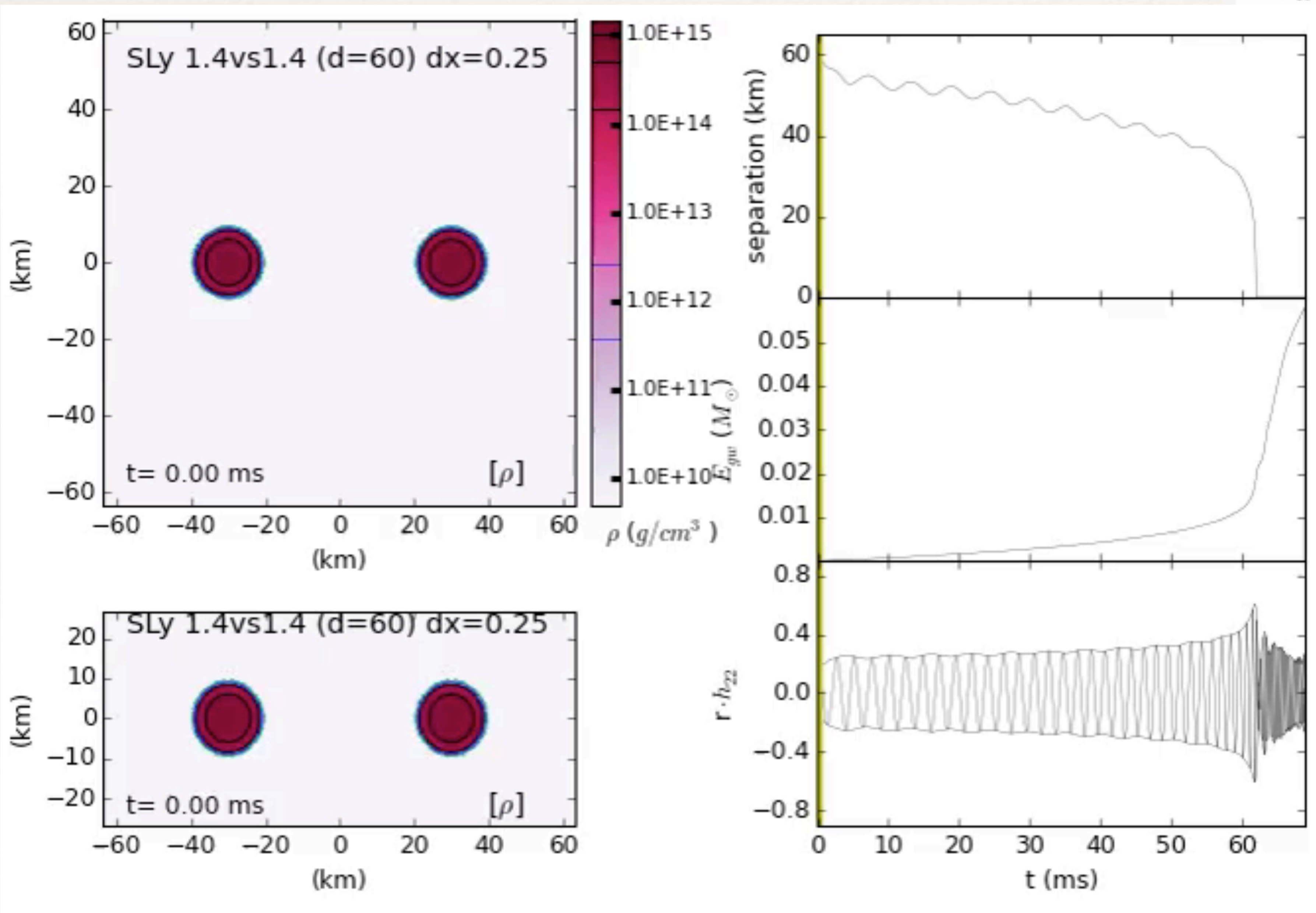
name	$M_0^{(1)}$ $[M_\odot]$	$M_0^{(2)}$ $[M_\odot]$	$M^{(1)}$ $[M_\odot]$	$M^{(2)}$ $[M_\odot]$	Ω $\left[\frac{\text{krad}}{s}\right]$	M_{ADM} $[M_\odot]$	J $\left[\frac{GM_\odot^2}{c}\right]$
SLy12vs12	1.20	1.20	1.11	1.11	1.932	2.207	5.076
SLy13vs13	1.30	1.30	1.20	1.20	1.989	2.373	5.730
SLy14vs14	1.40	1.40	1.28	1.28	2.040	2.536	6.405
SLy15vs15	1.50	1.50	1.36	1.36	2.089	2.697	7.108
SLy16vs16	1.60	1.60	1.44	1.44	2.134	2.854	7.832
SLy135vs145	1.35	1.45	1.24	1.32	2.040	2.536	6.397
SLy13vs15	1.30	1.50	1.20	1.36	2.040	2.535	6.376
SLy125vs15	1.25	1.55	1.16	1.40	2.040	2.533	6.337
SLy12vs16	1.20	1.60	1.11	1.44	2.039	2.531	6.281
G275th14vs14	1.40	1.40	1.29	1.29	2.053	2.554	6.513
G300th14vs14	1.40	1.40	1.26	1.26	2.028	2.498	6.243

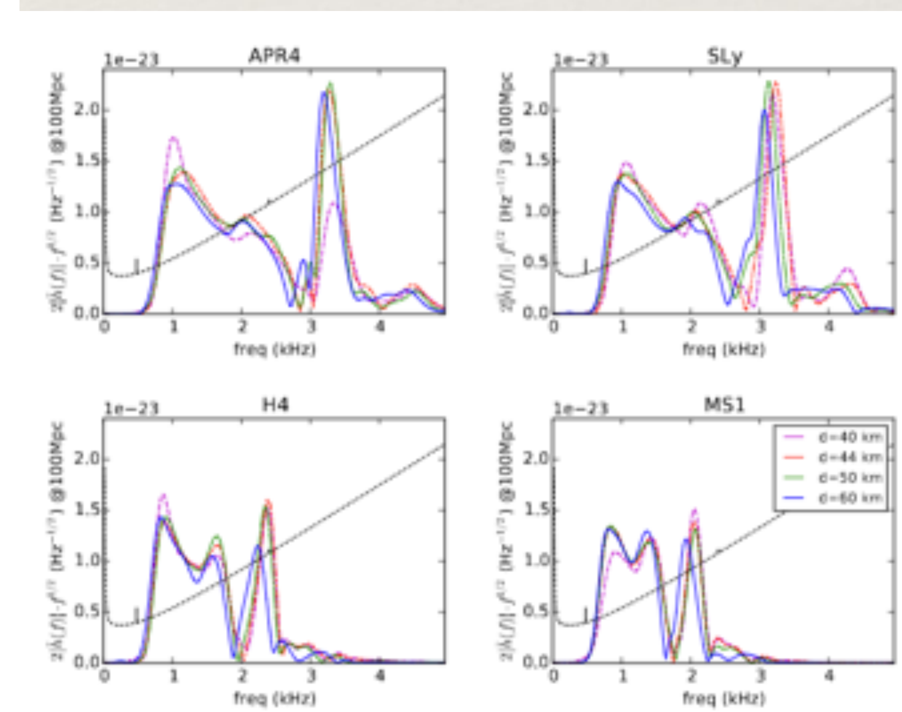
Analysed with 4-EOS and different initial separation

Gravitational Waves from our BNS merger simulations (or…what we do … the problems we have … and what we found)

- GW signal from BNS merger simulations using different EOS.
- We look at the EOS signature in the GW signal. Different EOS give different signal.



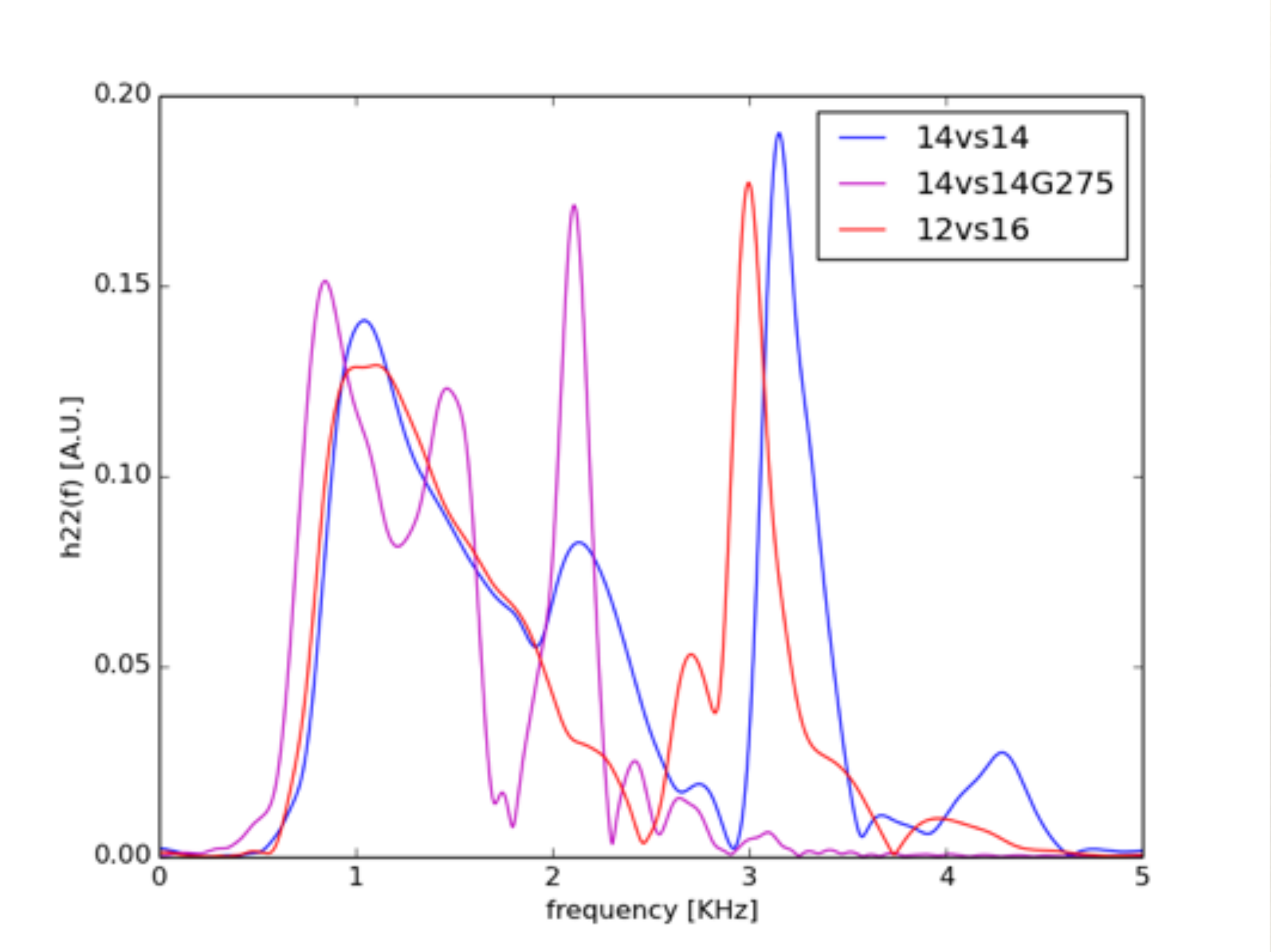




16 orbits before merger

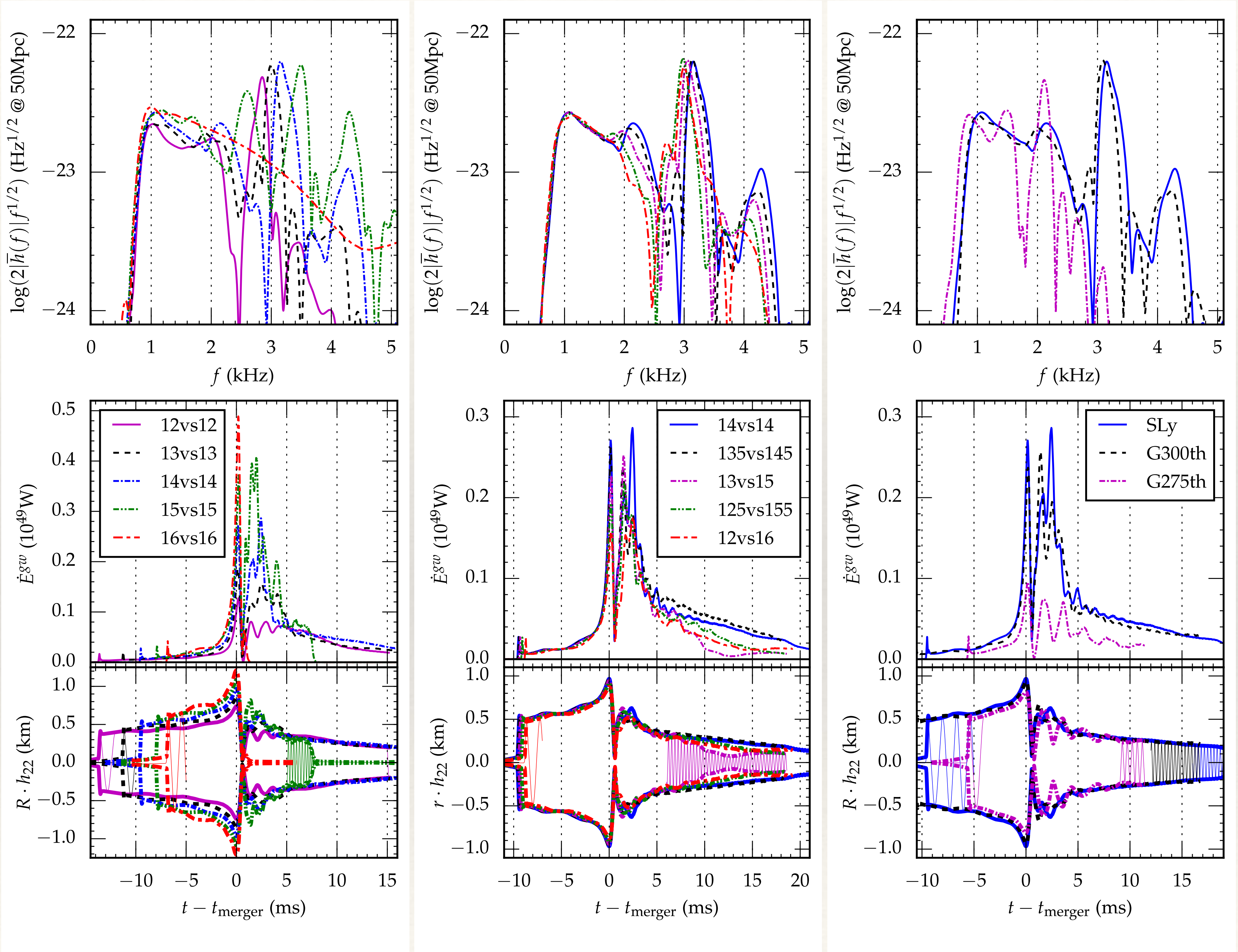
Results: gravitational waves signal properties

- Example of the FFT of the gravitational wave signals and the oscillation of the maximum density for three simulations: an equal mass and an unequal-mass one and the one with a significant softer EOS.
- Only the equal mass one show the two side peaks
- The softer one show a clear effect of its greater deformability.

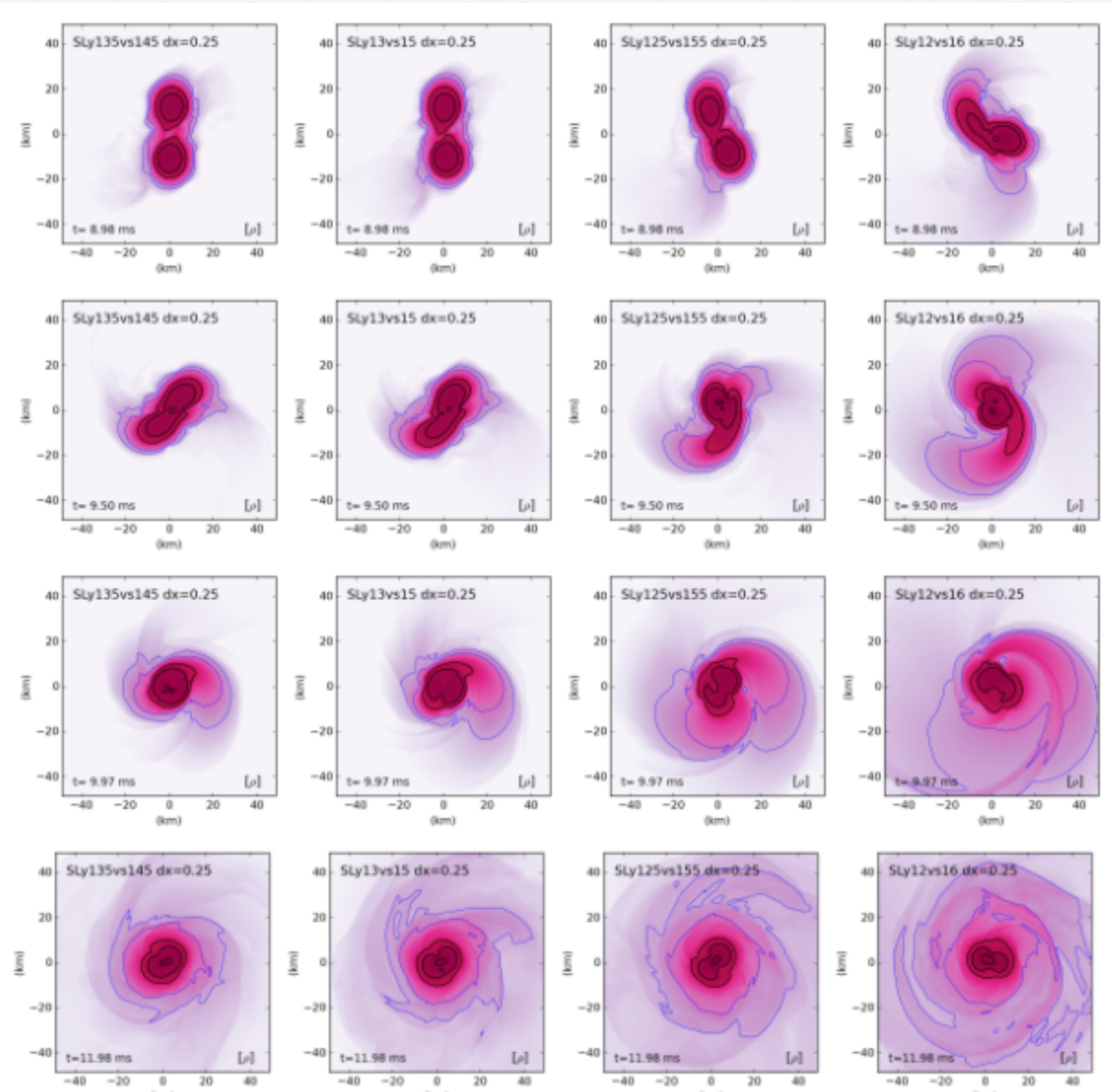


Result for the post merger spectrum

- Position of the peaks depends on the EOS and on the fact that the two masses are equal or unequal.
- Spectroscopic data are a direct route to the investigation of the true-EOS governing matter at extreme conditions.



Unequal mass models

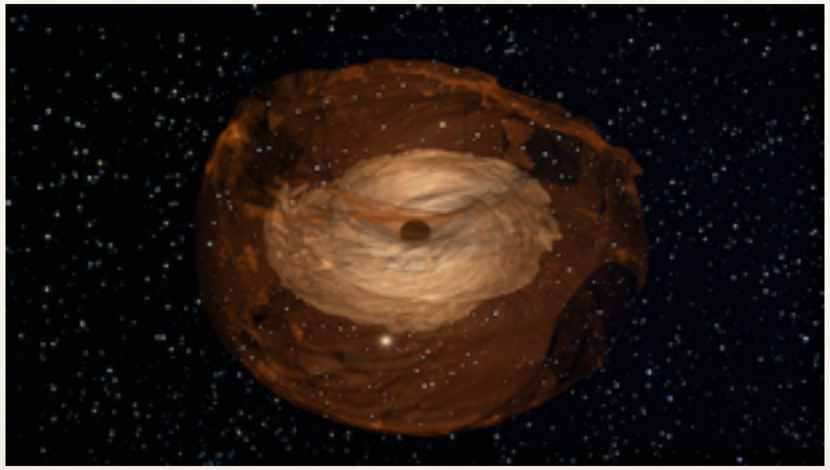


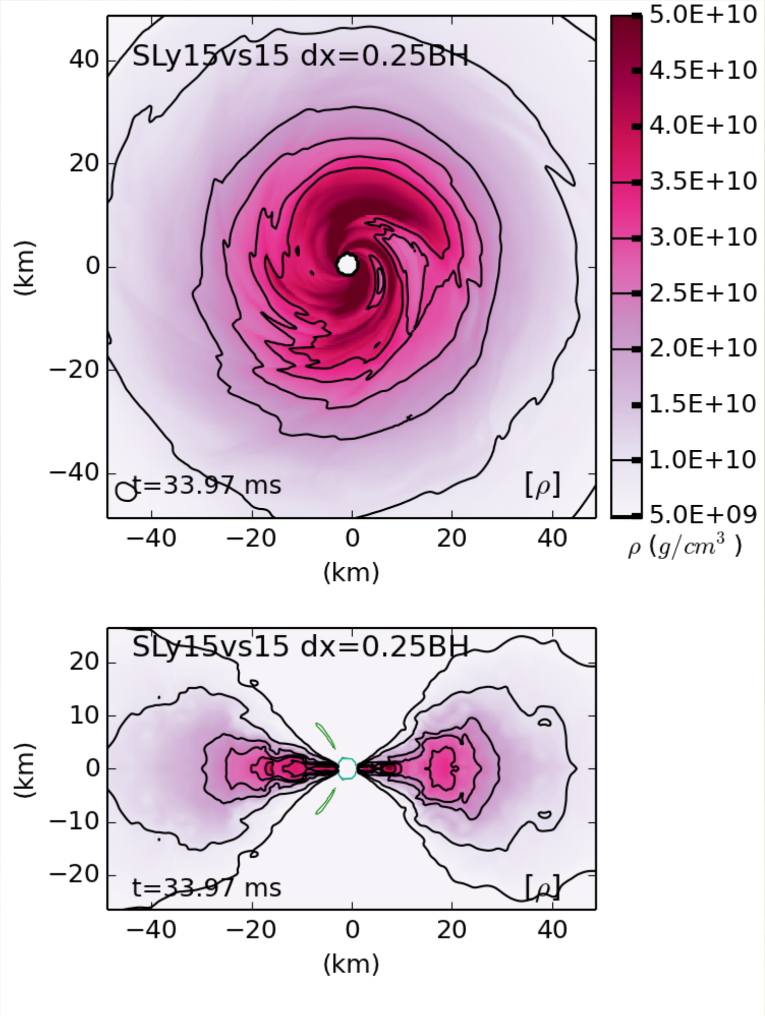
In the case of unequal mass models the remnant neutron star do not shows a single bar deformation.

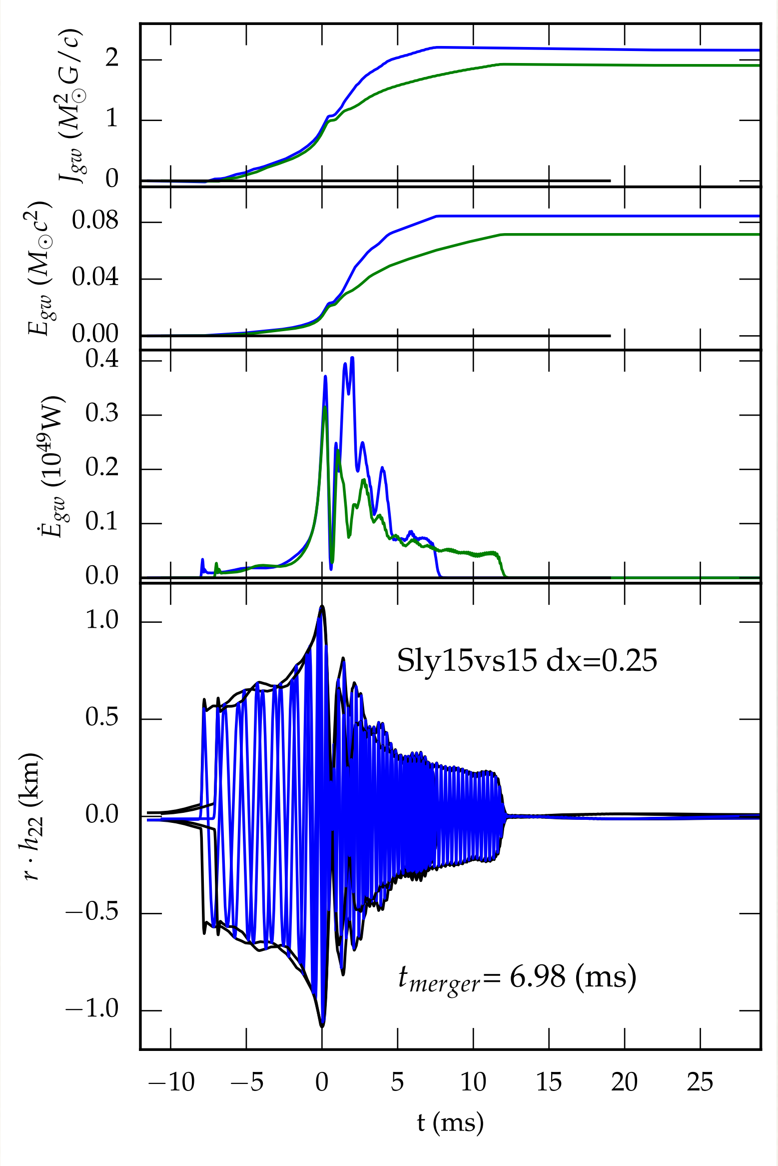
In the merger phase the two arms structure present in the case of equal mass systems is transformed into a single arm structure as the mass ratio increases.

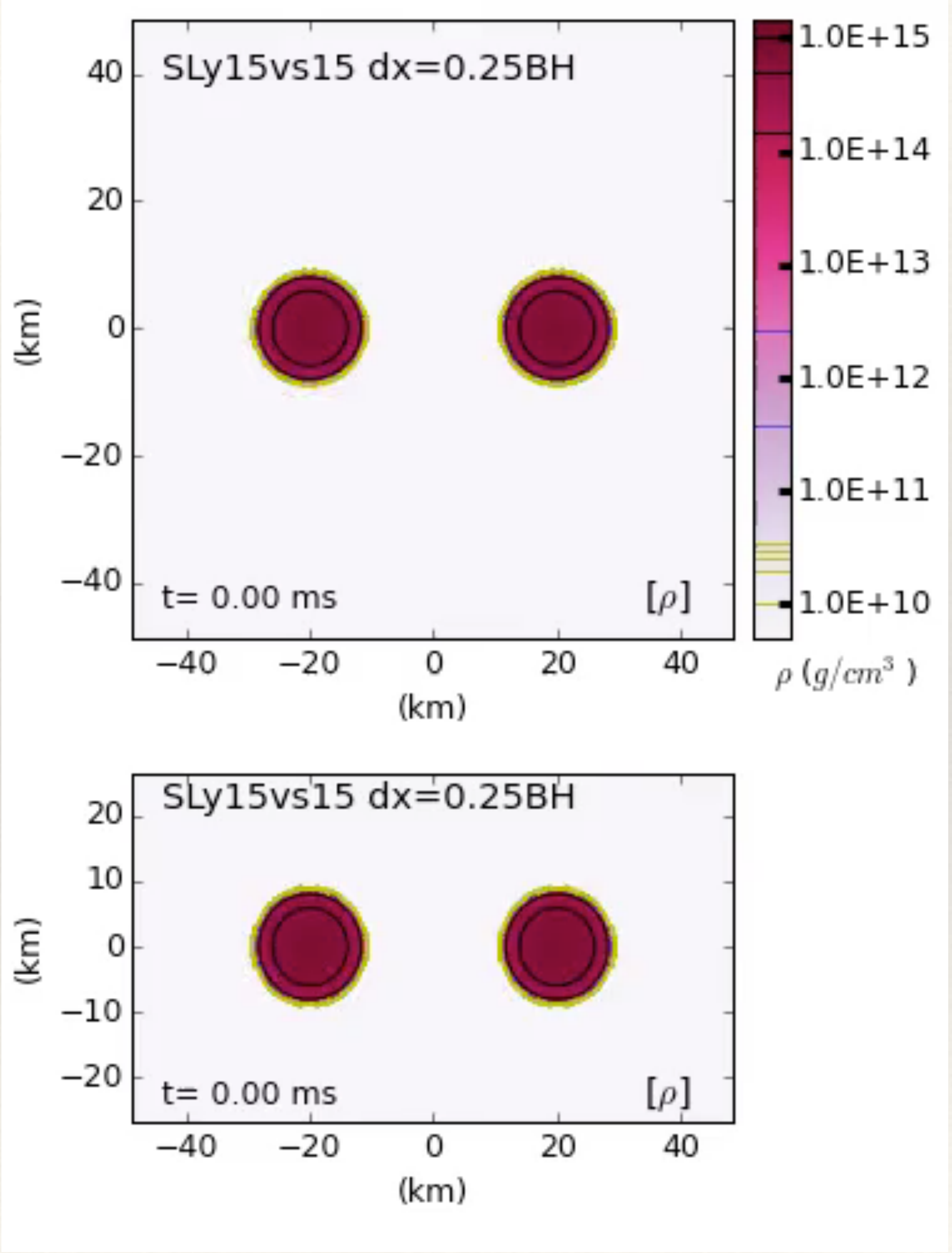
SLy 1.5 vs1.5 (Barionic Mass)

Model	dx=0.50	dx=0.375	dx=0.25
Sly15vs15	6.11 ms	11.81 ms	7.36 ms

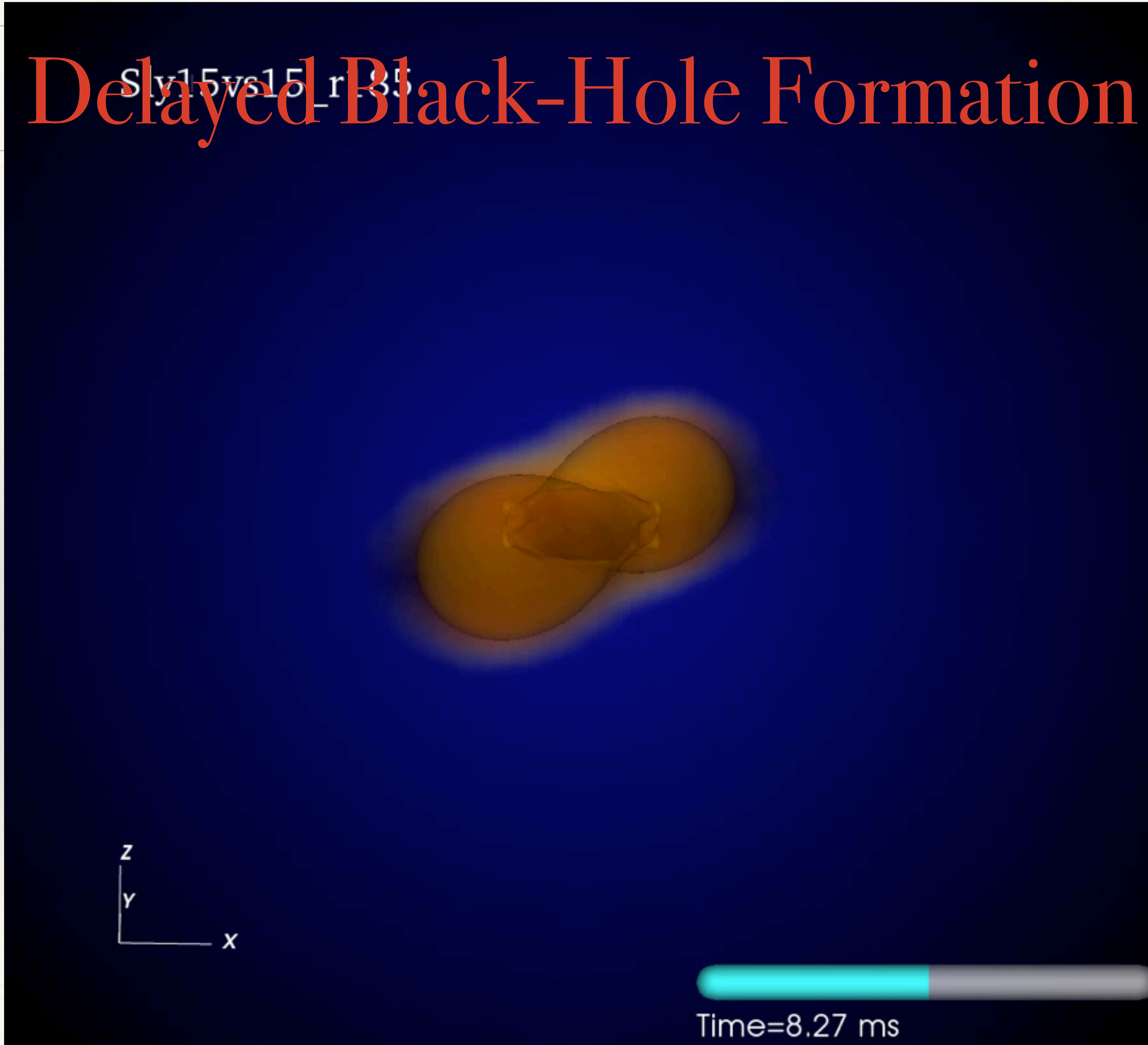






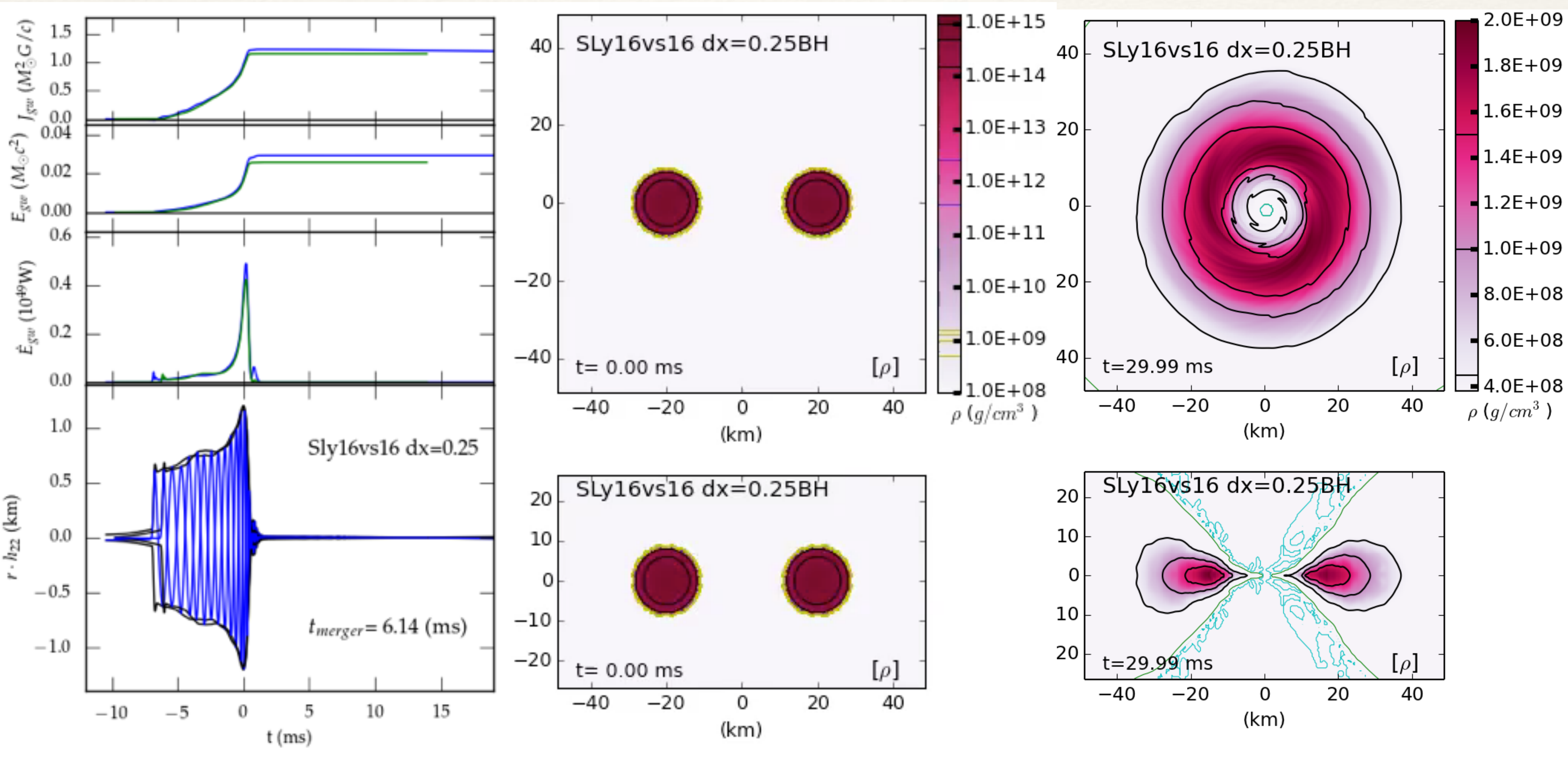


Delayed Black-Hole Formation



SLy 1.6 vs1.6 (Baryonic Mass)

Model	dx=0.50	dx=0.375	dx=0.25
Sly16vs16	0.83 ms	0.81 ms	0.79 ms

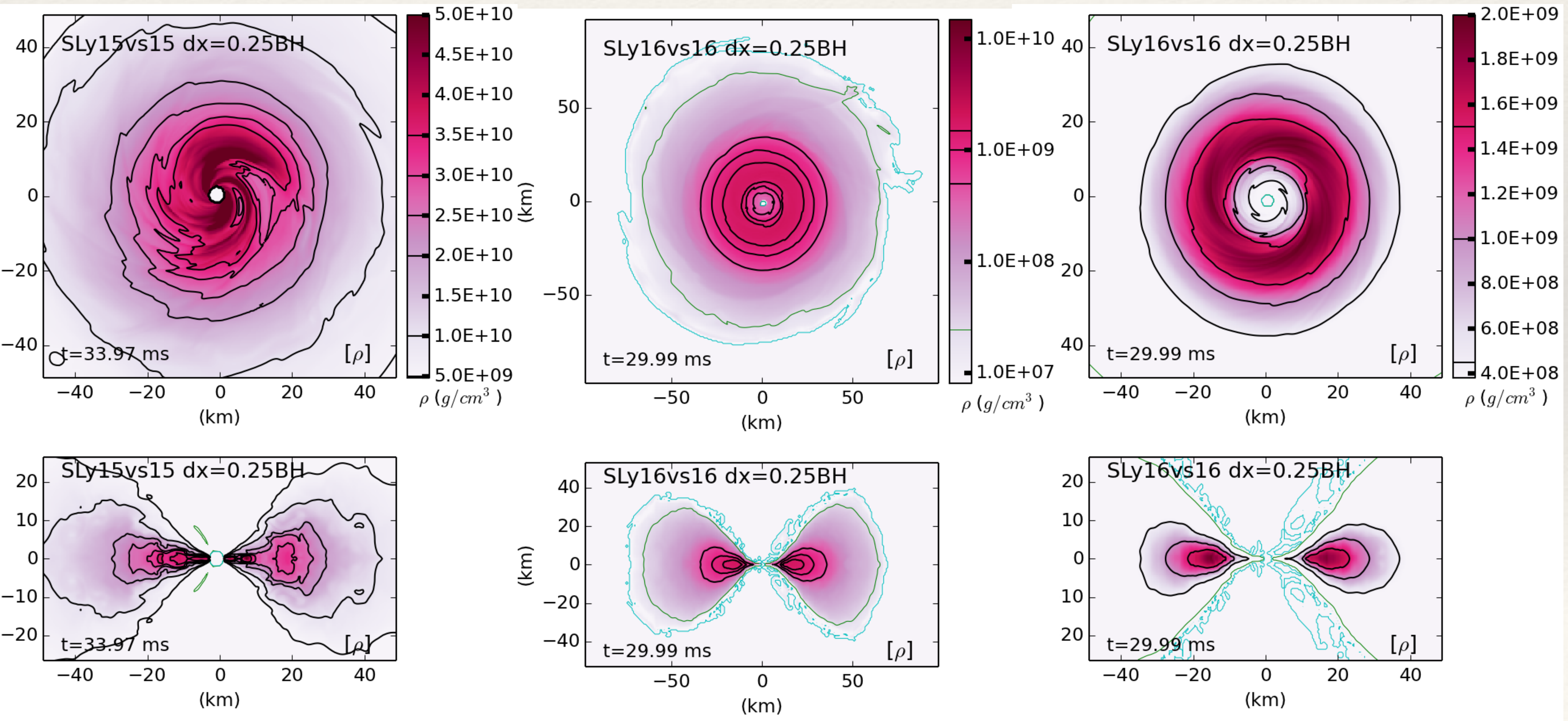


Collapse time to Black Hole (after Merger)

Not to easy to get physical predictions.

Presence of matter instabilities in the after merger dynamics (like Kelvin Helmotz)

Model	dx=0.50	dx=0.375	dx=0.25
Sly15vs15	6.11 ms	11.81 ms	7.36 ms
Sly16vs16	0.83 ms	0.81 ms	0.79 ms

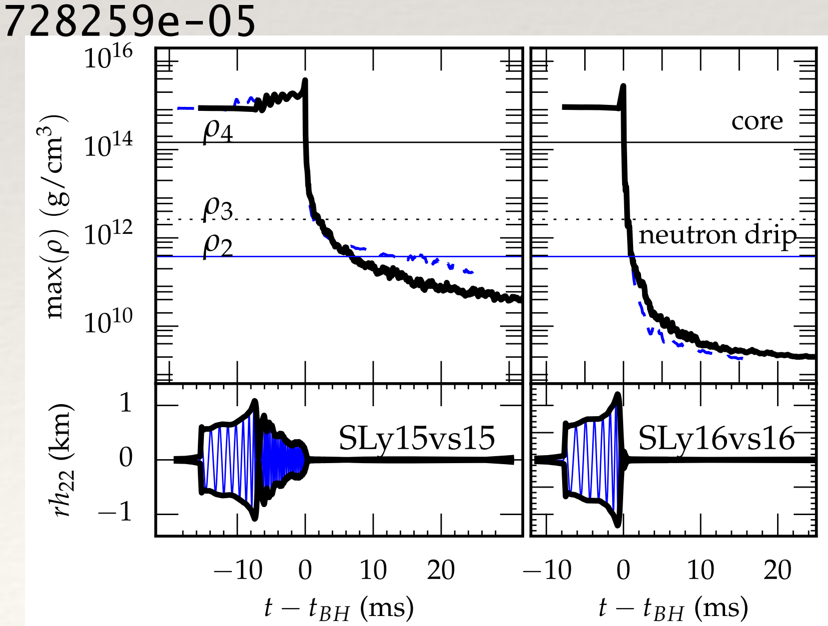


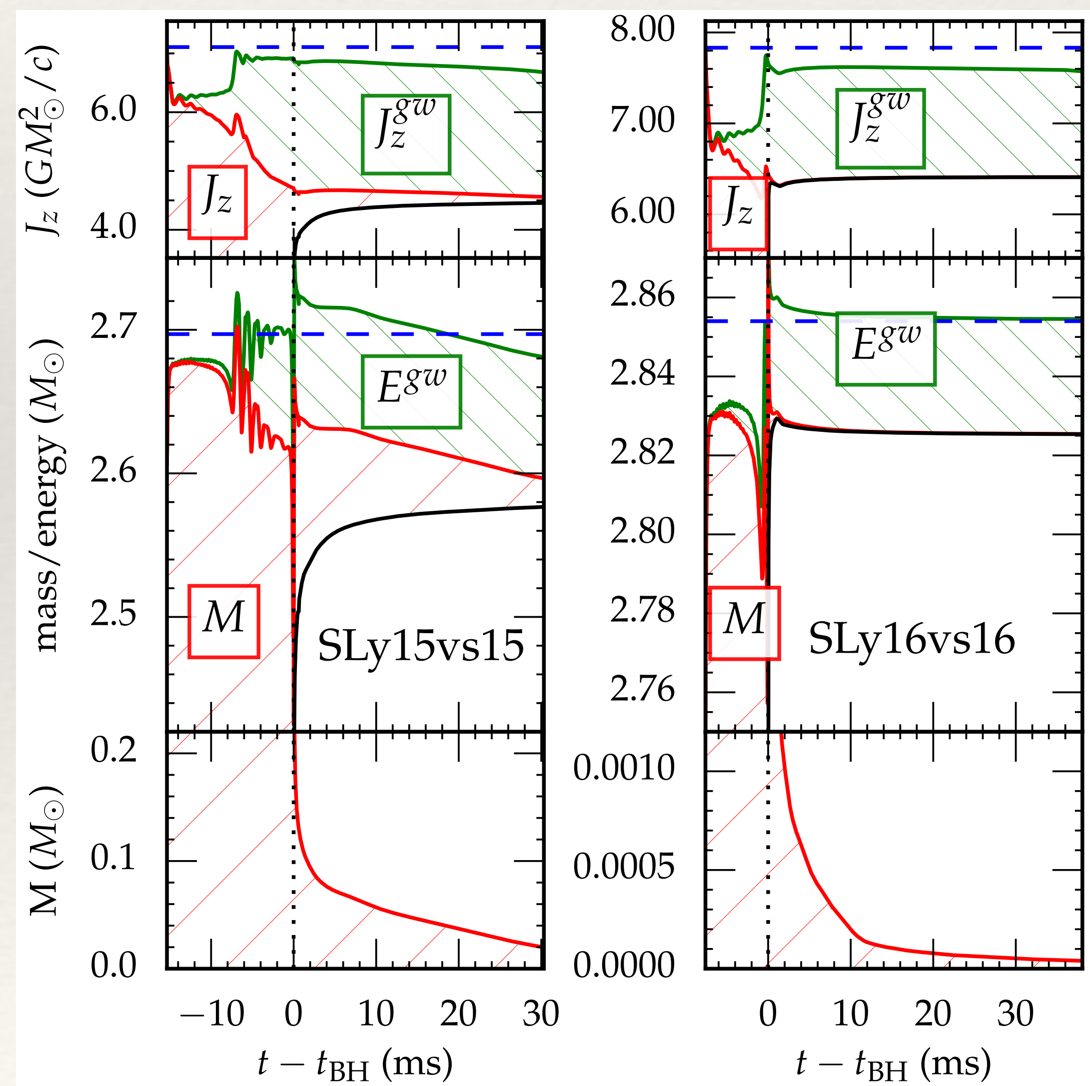
Collapse time to Black Hole (after Merger)

```
BUDGET for model 15vs15
    Final mass BH np.max(Mbh[0]) is: 2.57733745845
    Final J    BH np.max(Lbh[0]) is: 4.46170131712
    Mass in disk  np.min(MATTER) is: 0.0162523005579
    Total simulated time after BH  : 33.9091725788
    Values 25 ms after BH formation
           BH M     =  2.57531472352
           BH J     =  4.44682924429
           BH J/M^2 =  0.670486181372
           Mass =  0.0278916700256
```

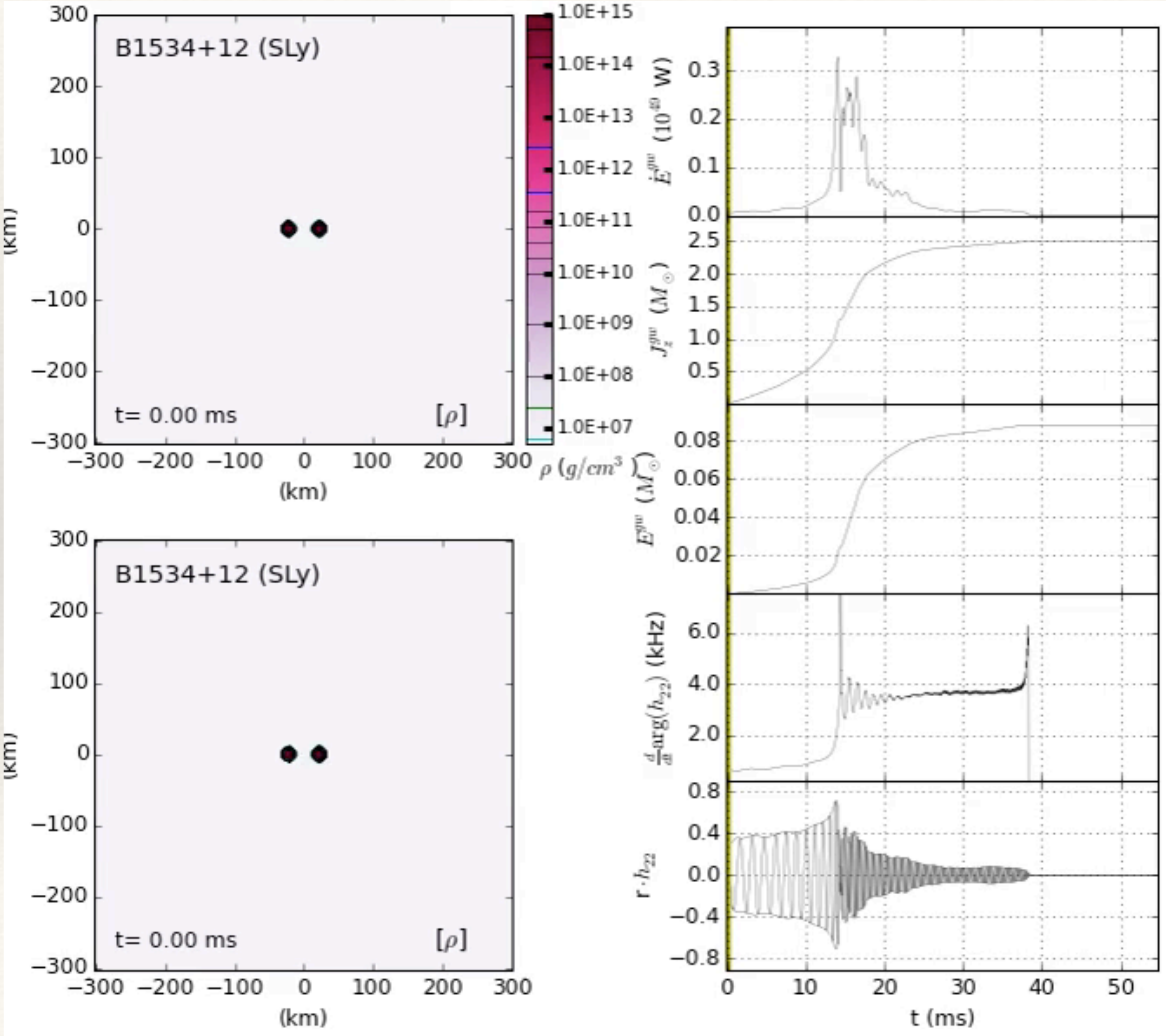
Model	dx=0.50	dx=0.375	dx=0.25
Sly15vs15	6.11 ms	11.81 ms	7.36 ms
Sly16vs16	0.83 ms	0.81 ms	0.79 ms

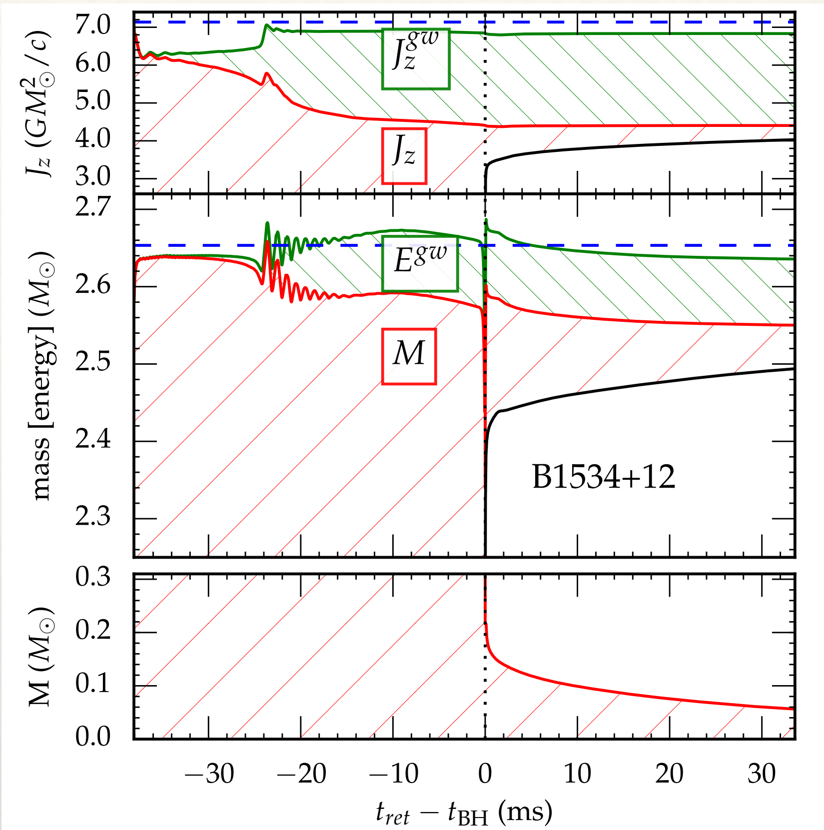
```
BUDGET for model 16vs16
    Final mass BH np.max(Mbh[1]) is: 2.82937414009
    Final J    BH np.max(Lbh[1]) is: 6.40877814116
    Mass in disk  np.min(MATTER) is: 3.60524241442e-05
    Total simulated time after BH  : 41.6623738628
    Values 25 ms after BH formation
           BH M     =  2.82551213935
           BH J     =  6.40772796317
           BH J/M^2 =  0.802619506998
           Mass =  6.1875728259e-05
```





The pulsar B1534+12 – The disk…

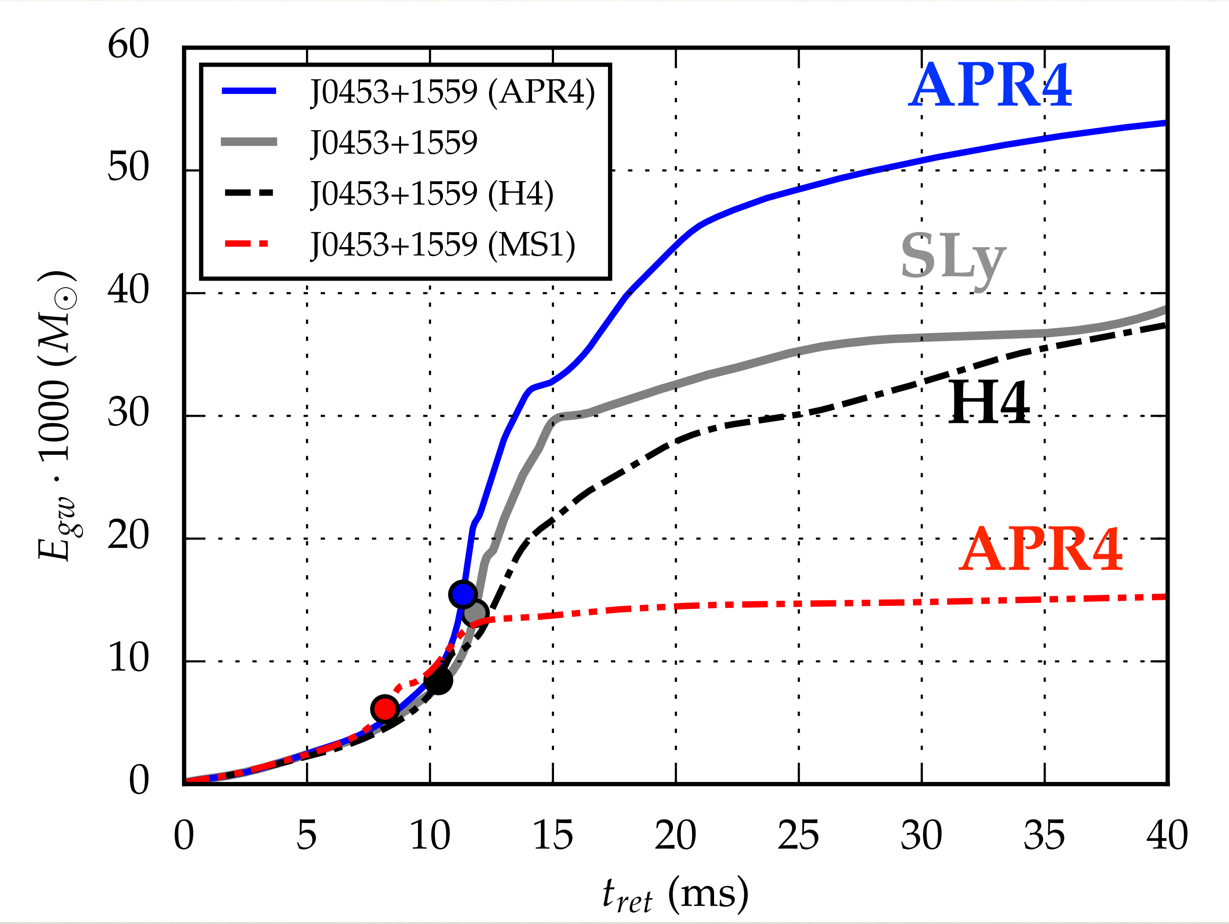




and the SOUND!

Different EOS – same stellar model

GW - ENERGY



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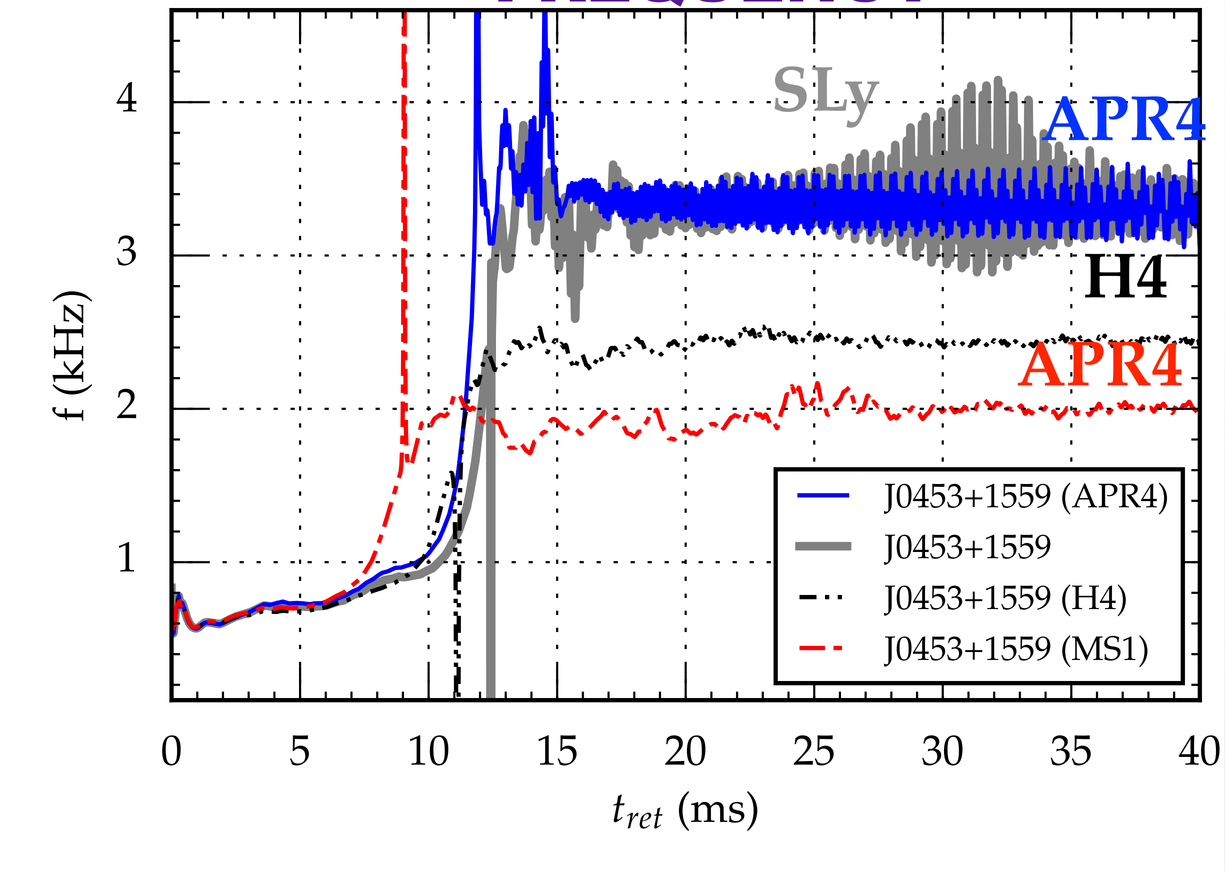
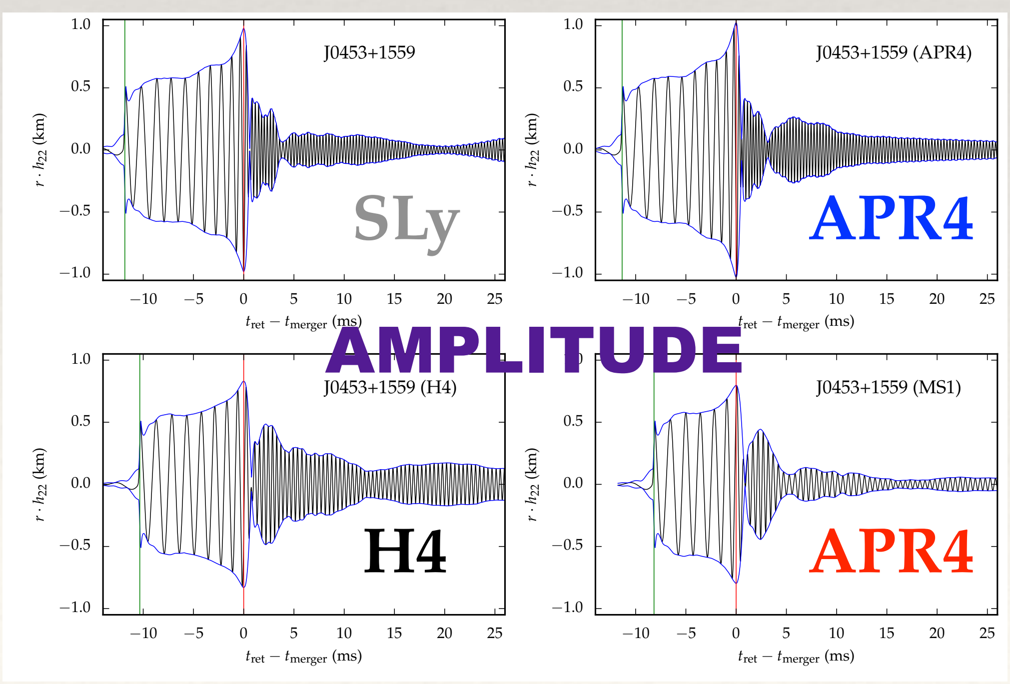
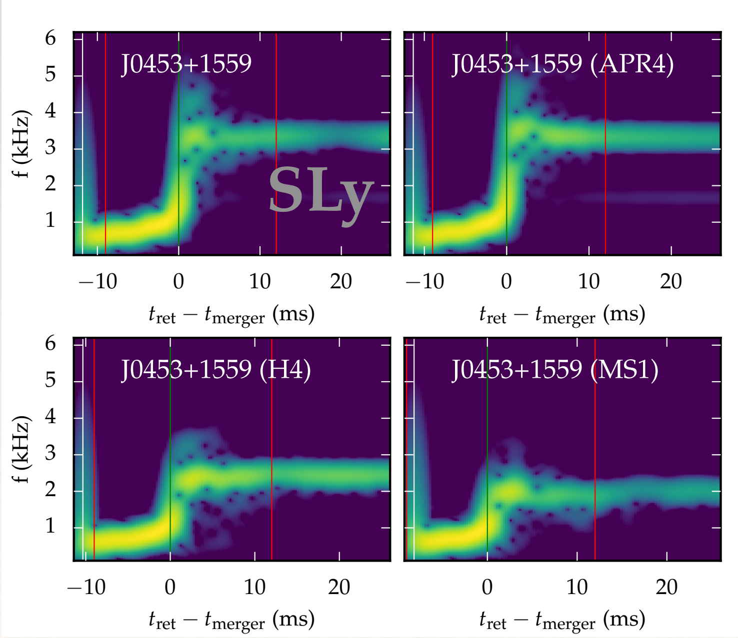


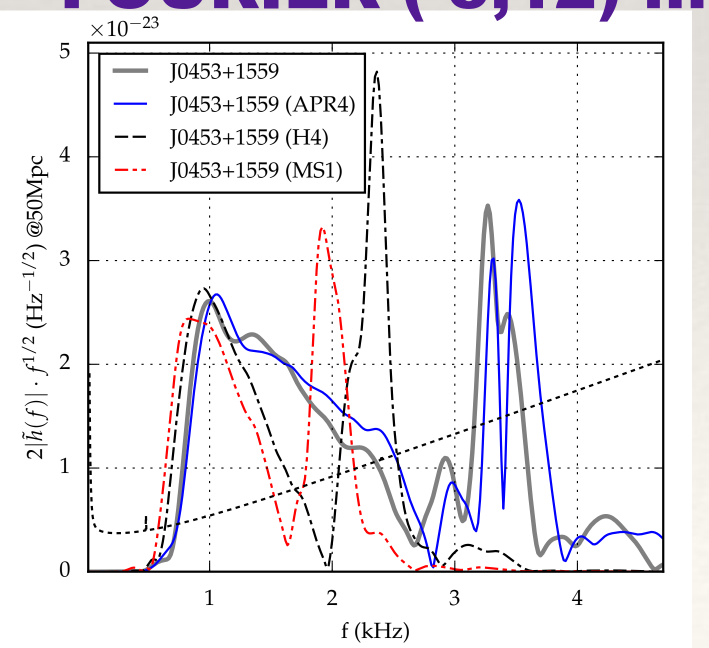
AMPLITUDE



SPECTROGRAM



FOURIER (-5,12) ms



Conclusions

- With the first detection of GWs the era of Gravitational waves astronomy just started.
- Long term simulation of BNS mergers using only public codes: **You can re-run all the models on your own**.
- It is possible to check the code on a laptop … (Using our setting).
- All the simulation presented here were performed on Tier-1 system.
- More insight improving the resolution of the simulation.
- Confirmation of previous results published in literature.
- New results for un-equal mass BNS systems, and the evolution of six galactic systems with different EOS.
- Just a starting point for new research ….

What's next ?

- Investigate dependence of collapse time on resolution and EoS.
- Matter expelled not-axisymmetrically during merger => study accretion disk formation, mass, composition and development to an equilibrium configuration.
- Can (magneto)hydrodynamical instabilities develop in the disk?
- (Black hole like) kicks from linear momentum emitted in gravitational waves and unbound matter expelled not-axisymmetrically.
- Realistic treatment of EOS thermal component (ex. Using finite temperature EOS from relativistic mean field theory like Shen EOS).
- Simulations with magnetic fields to study the development of magnetic instabilities during the merger (Kelvin-Helmotz), in the hypermassive NS and the accretion disk (MRI).
- Studying possible electromagnetic and jet emissions after collapse.
- Use of OpenMP4 to test at least part of the code on GPUs and Intel MICs.

The end …