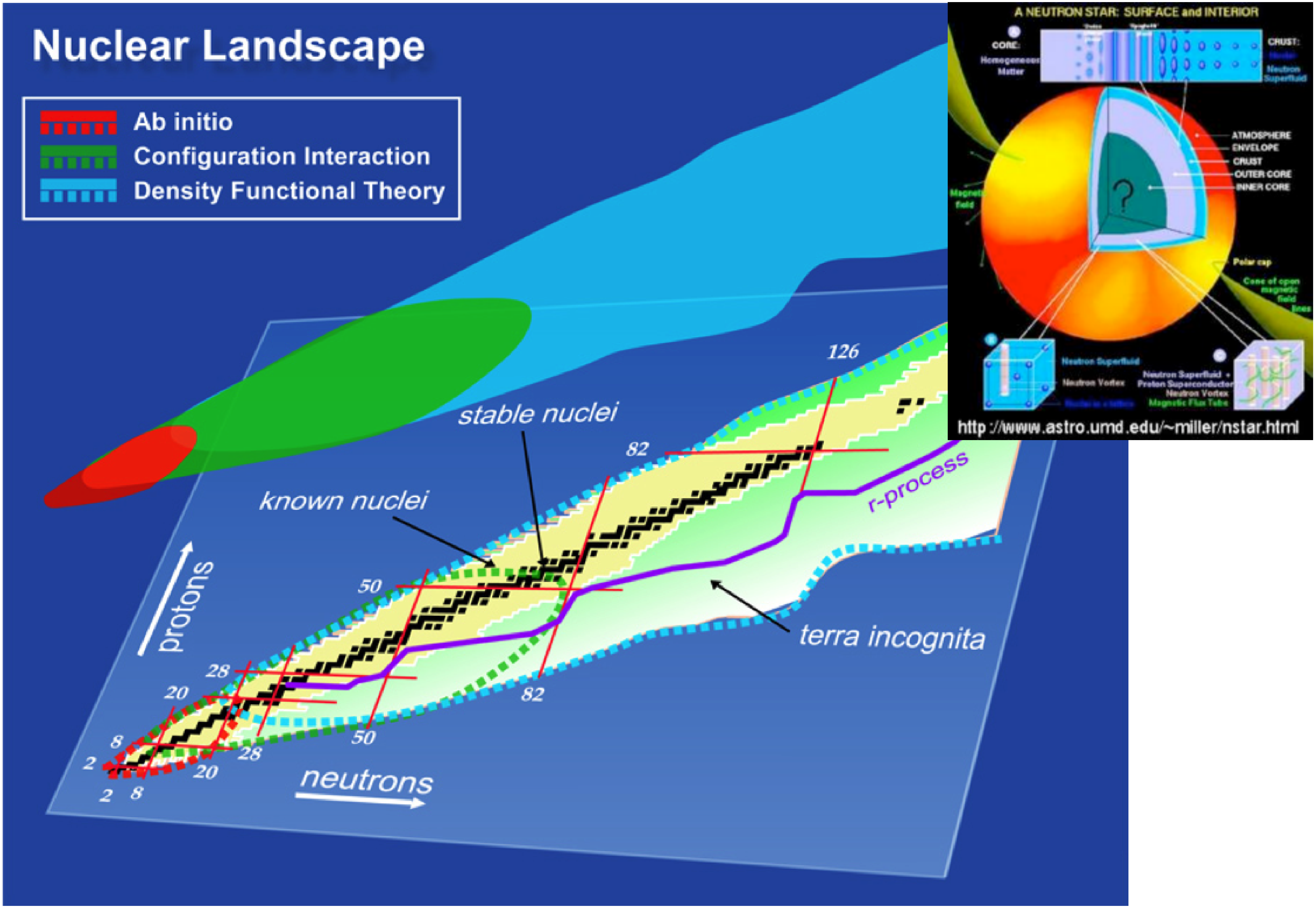
Recent developments of nuclear interactions within chiral EFT and applications to nuclear matter and nuclei

Kai Hebeler

Geneva, December 6, 2016

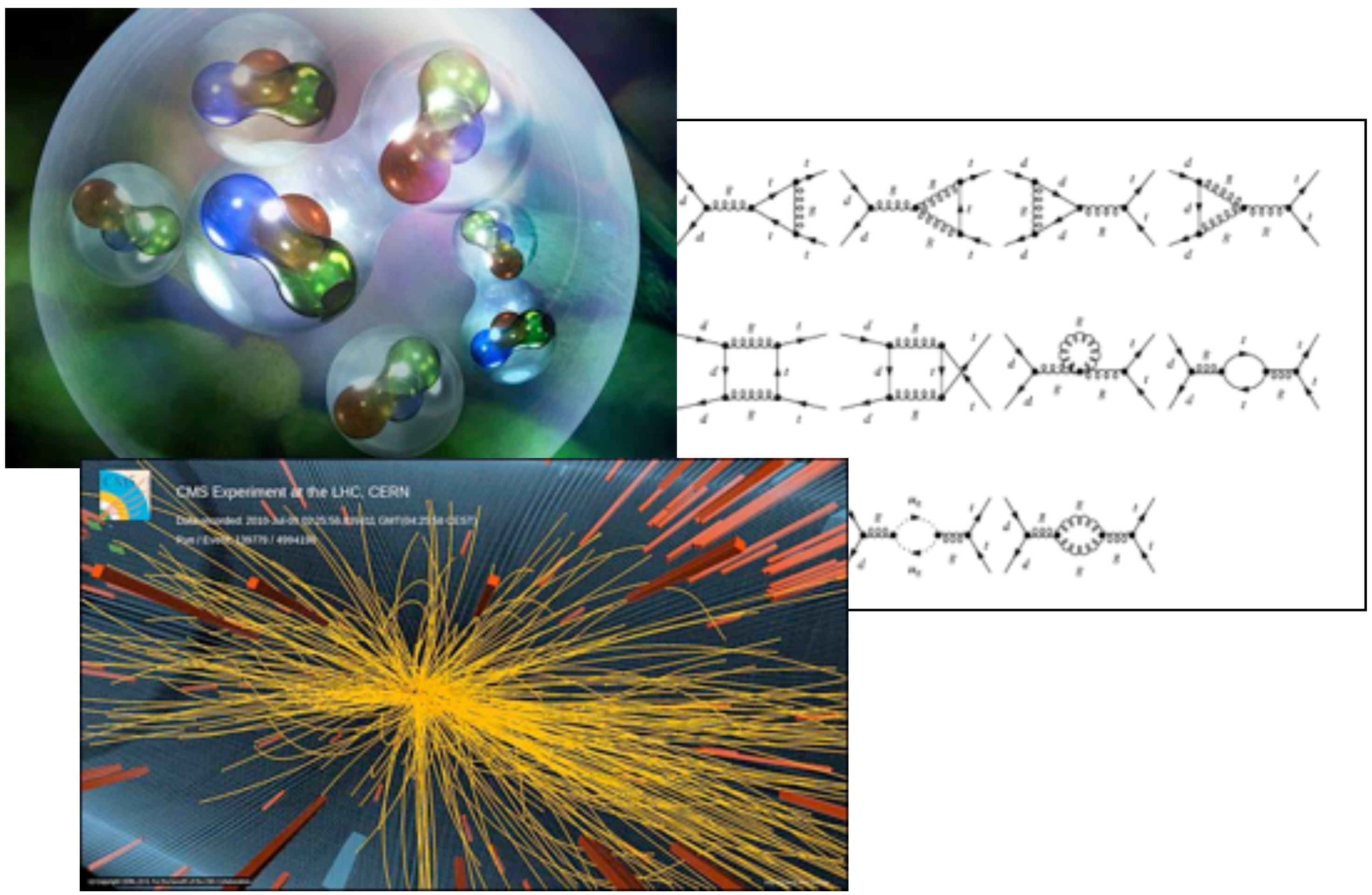
From quarks to gravitational waves: Neutron stars as a laboratory for fundamental physics

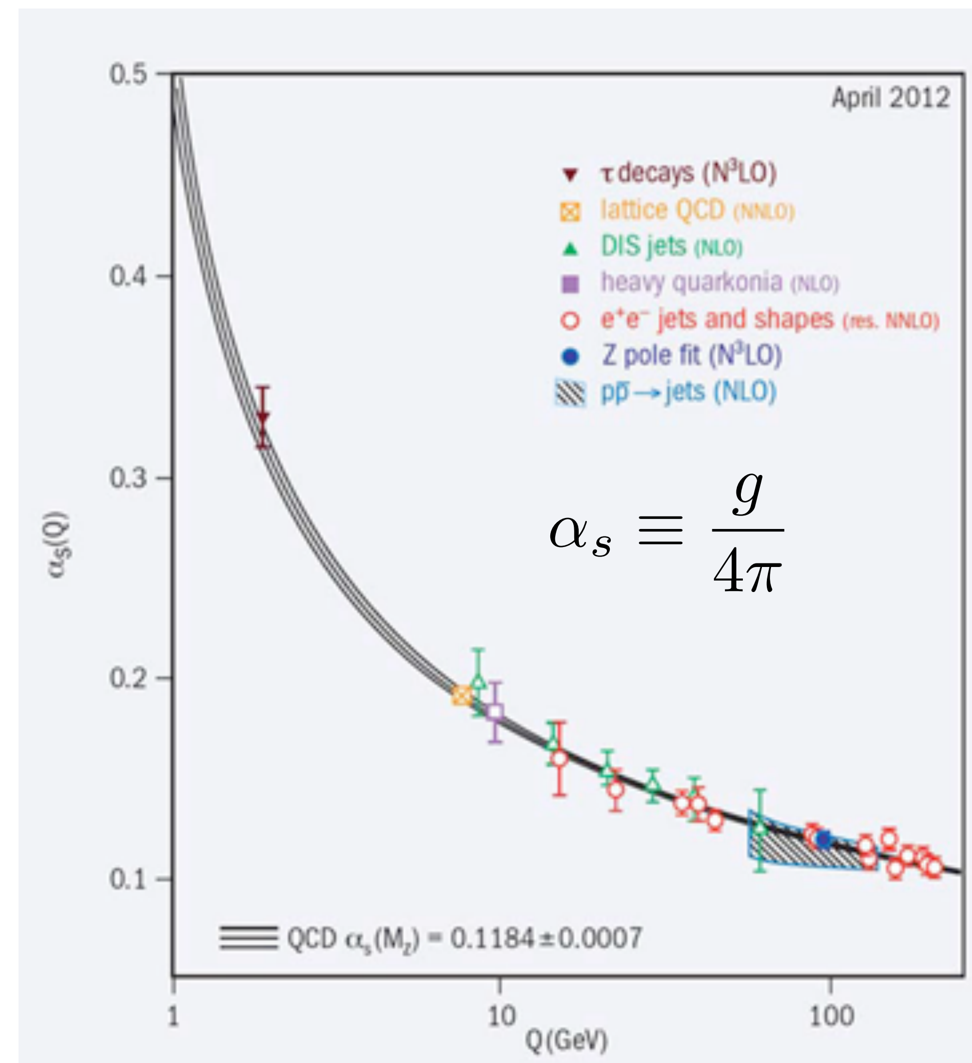
The theoretical nuclear landscape several years ago...



Theory of the strong interaction: Quantum chromodynamics

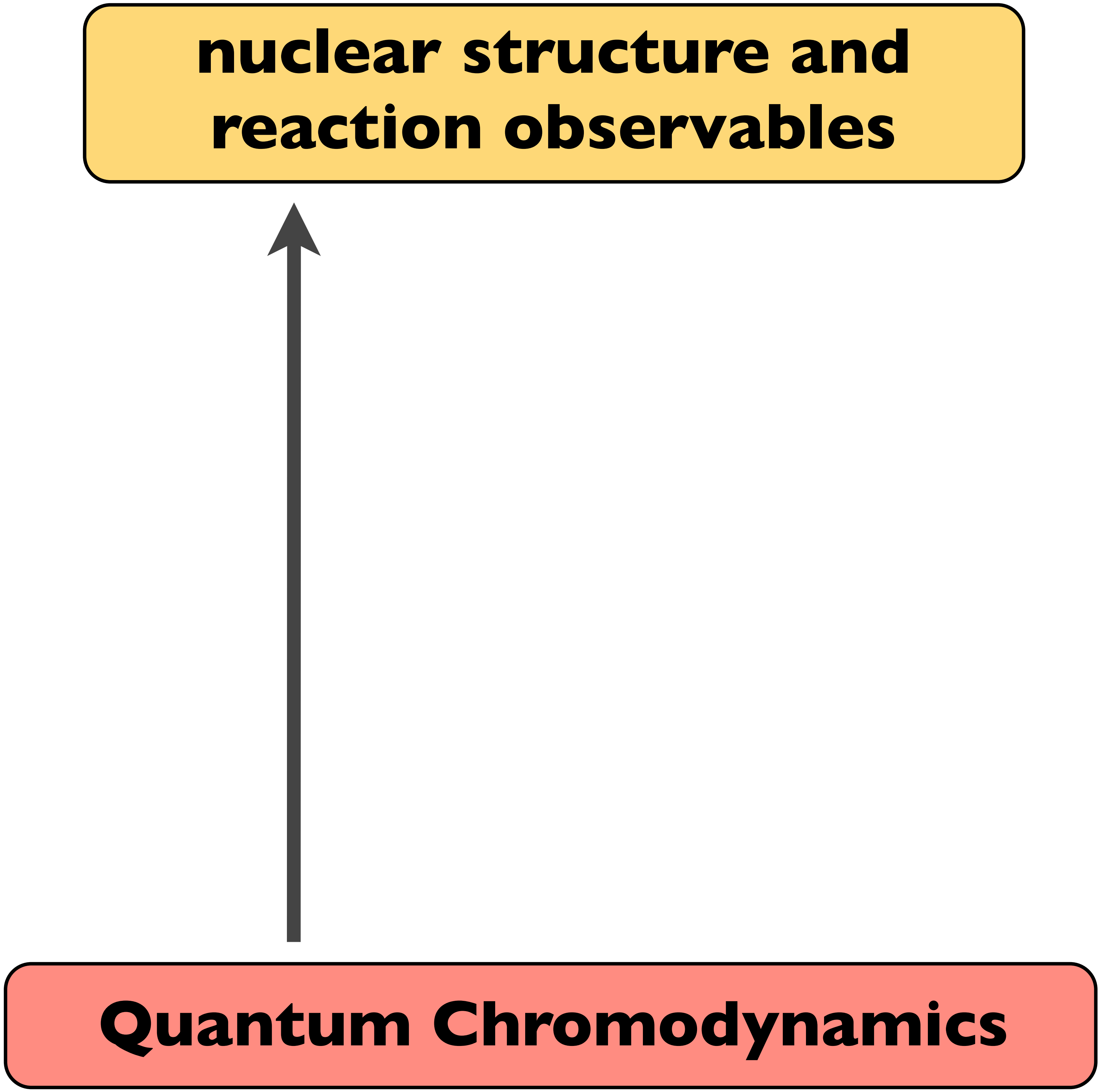
$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{q}(i\gamma^\mu \partial_\mu - m)q + g\bar{q}\gamma^\mu T_a q A^a_\mu$$





- theory perturbative at high energies
- highly non-perturbative at low energies

Ab initio nuclear structure and reaction theory



Ab initio nuclear structure and reaction theory

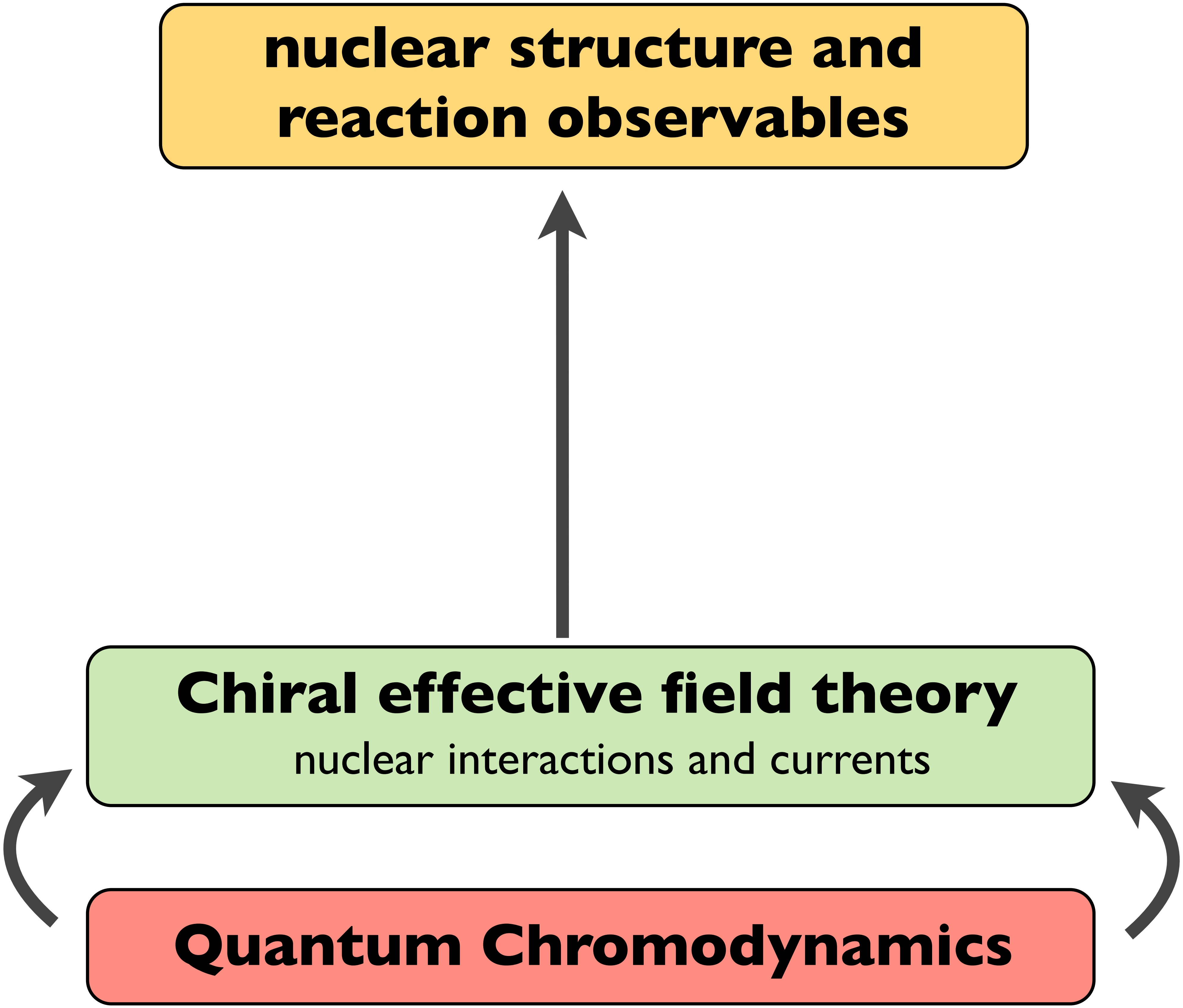
nuclear structure and reaction observables

Lattice QCD

- requires extreme amounts of computational resources
- currently limited to 1- or 2-nucleon systems
- current accuracy insufficient for precision nuclear structure

Quantum Chromodynamics

Ab initio nuclear structure and reaction theory



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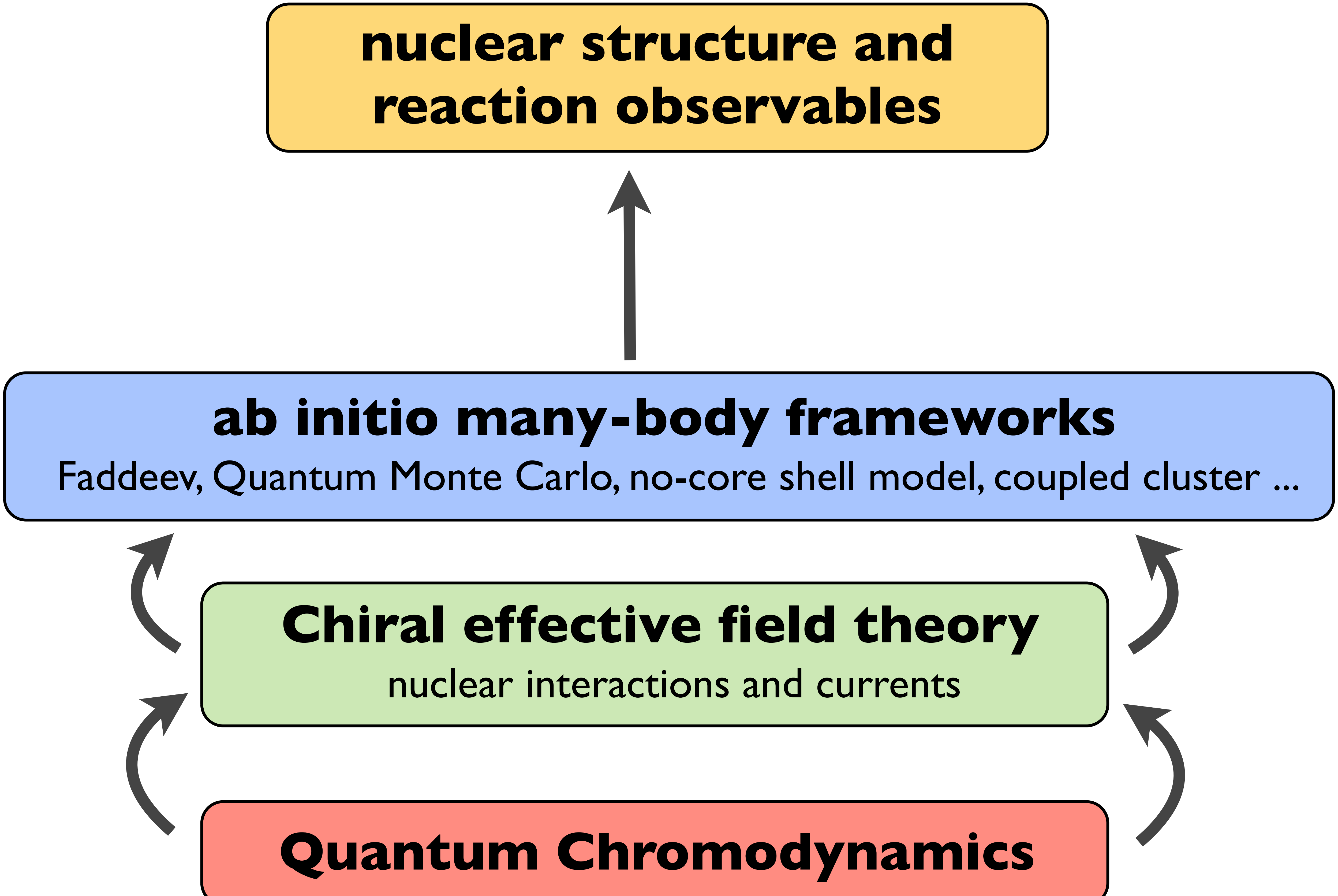

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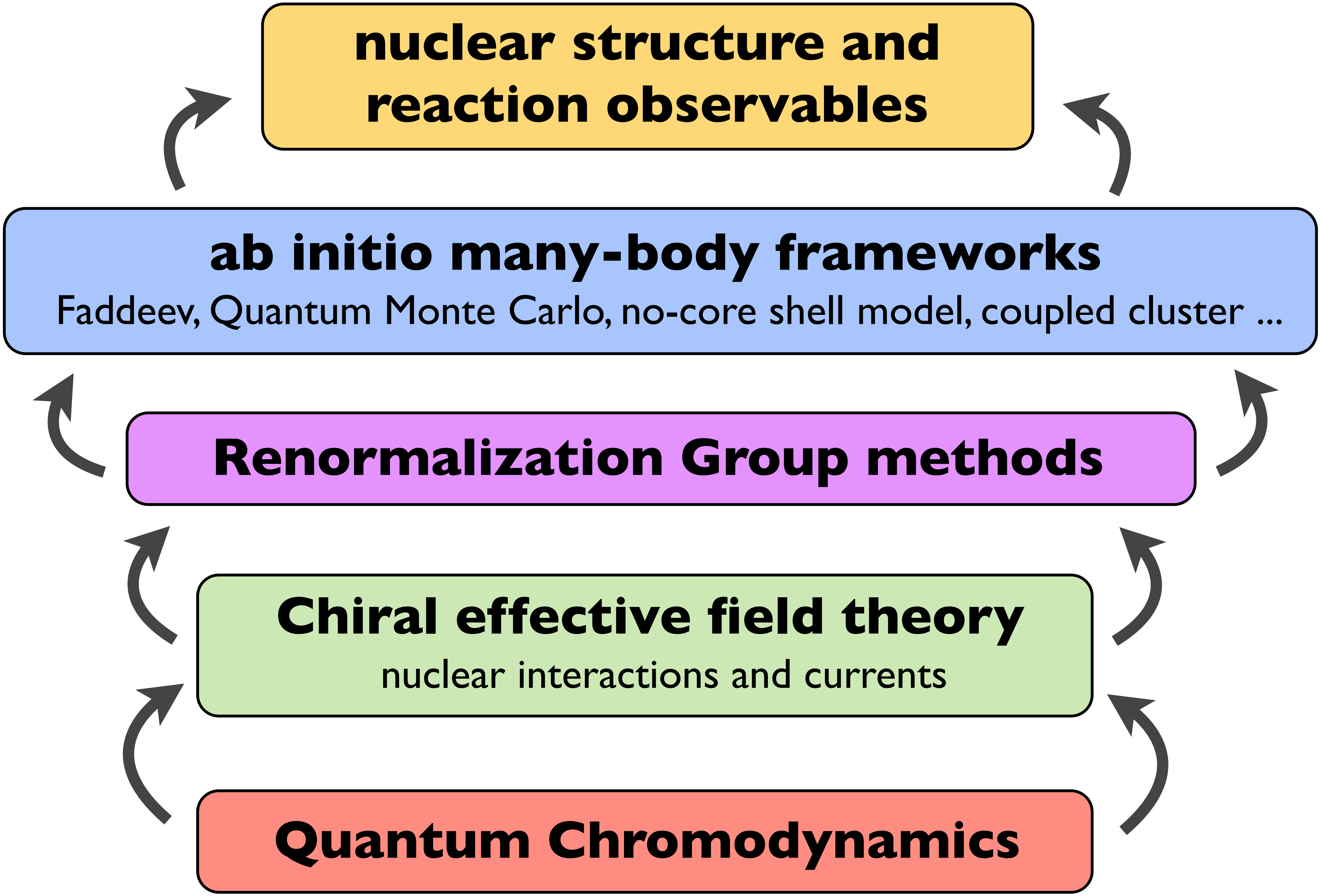
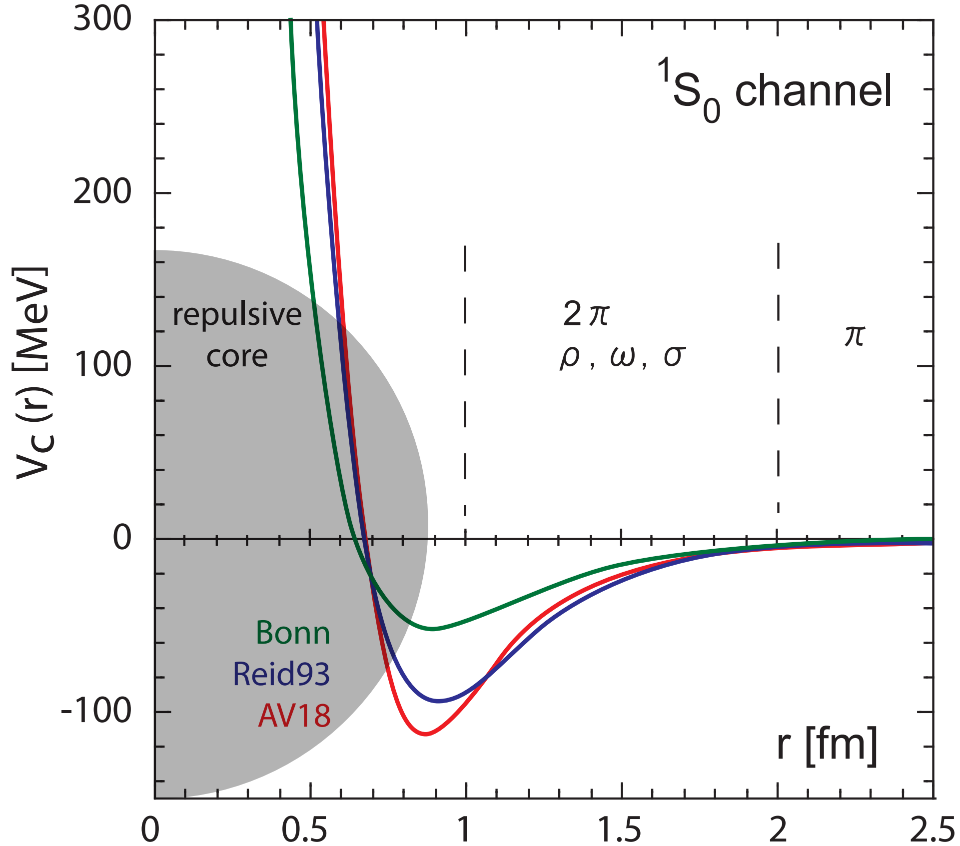
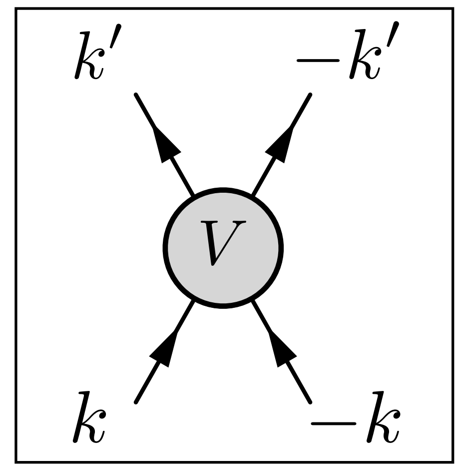
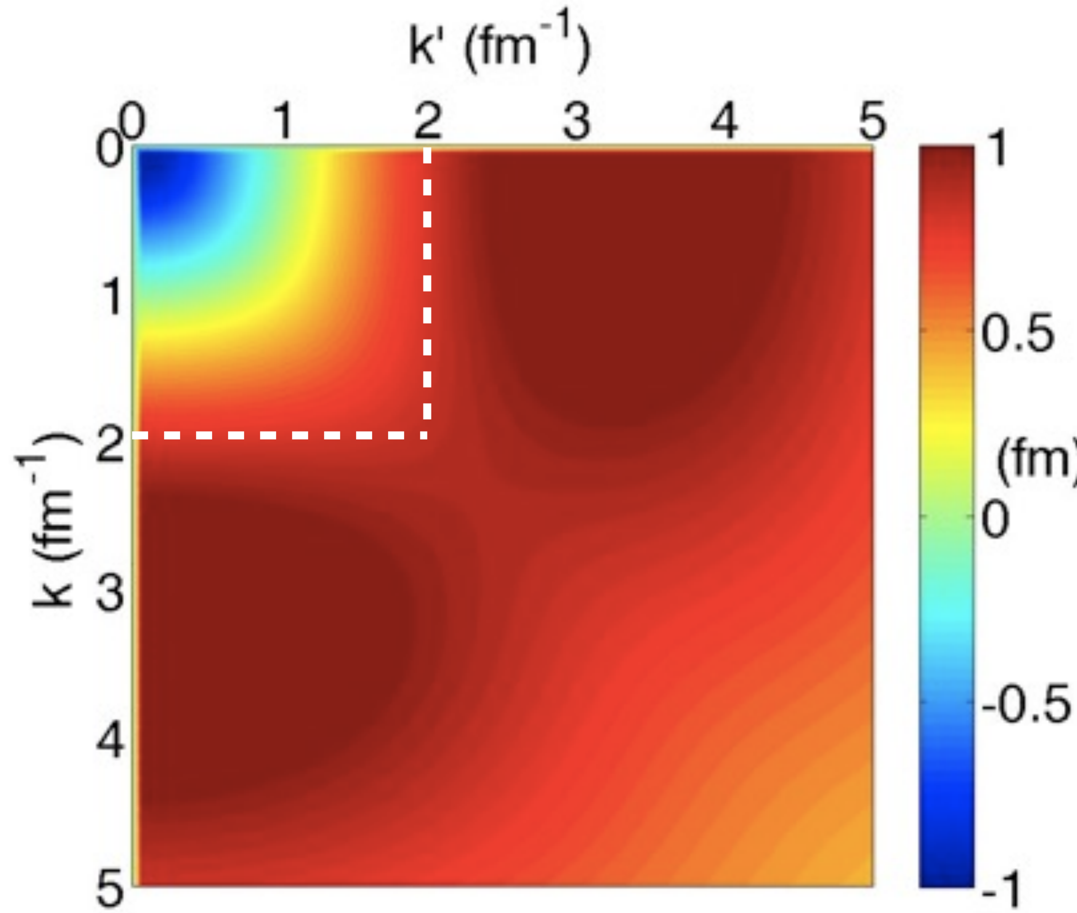


"Traditional" empirical NN interactions



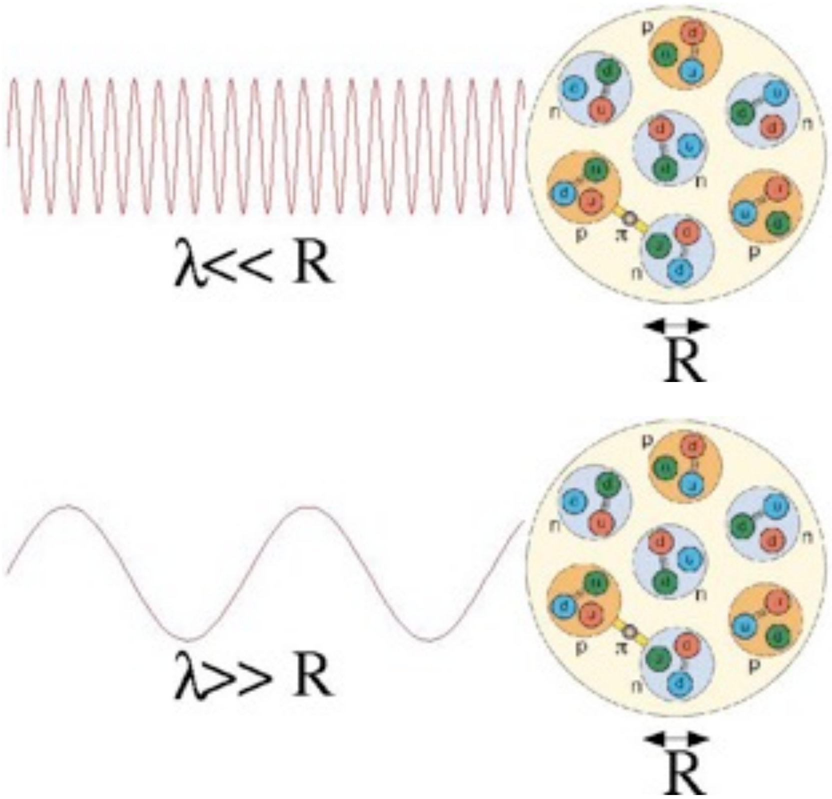


$\langle k' | V | k \rangle$



- constructed to fit scattering data (long-wavelength information!)
- long-range pion exchange part agrees in all potentials
- short range part strongly scheme dependent!
- "hard" NN interactions contain repulsive core at small relative distance
- strong coupling between low and high-momentum components, hard to solve!

Nuclear effective degrees of freedom



- if a nucleus is probed at high energies, nucleon substructure is resolved
- at low energies, details are not resolved

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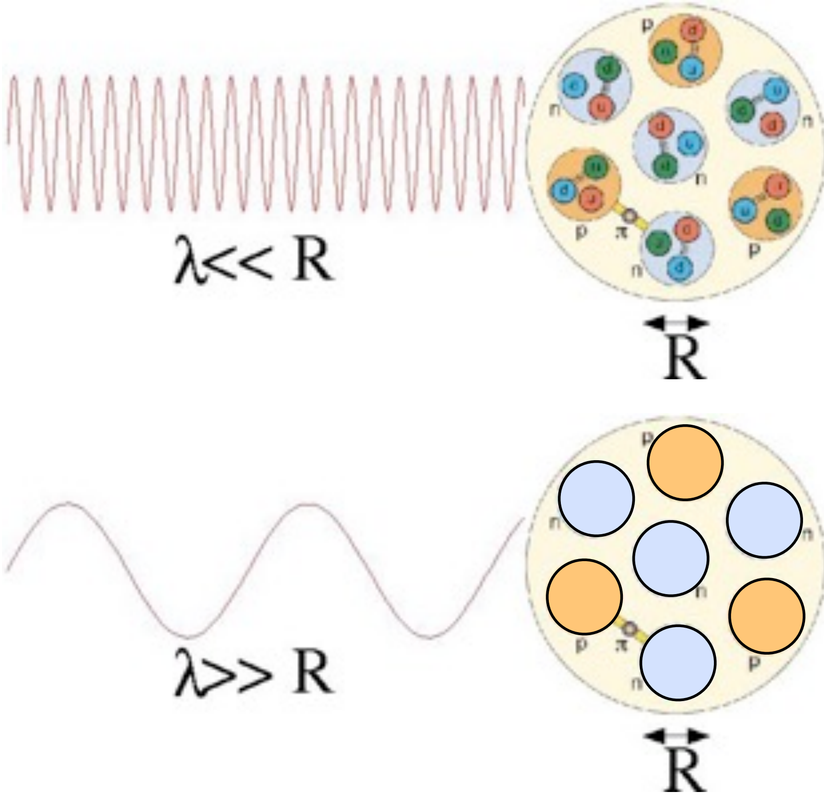
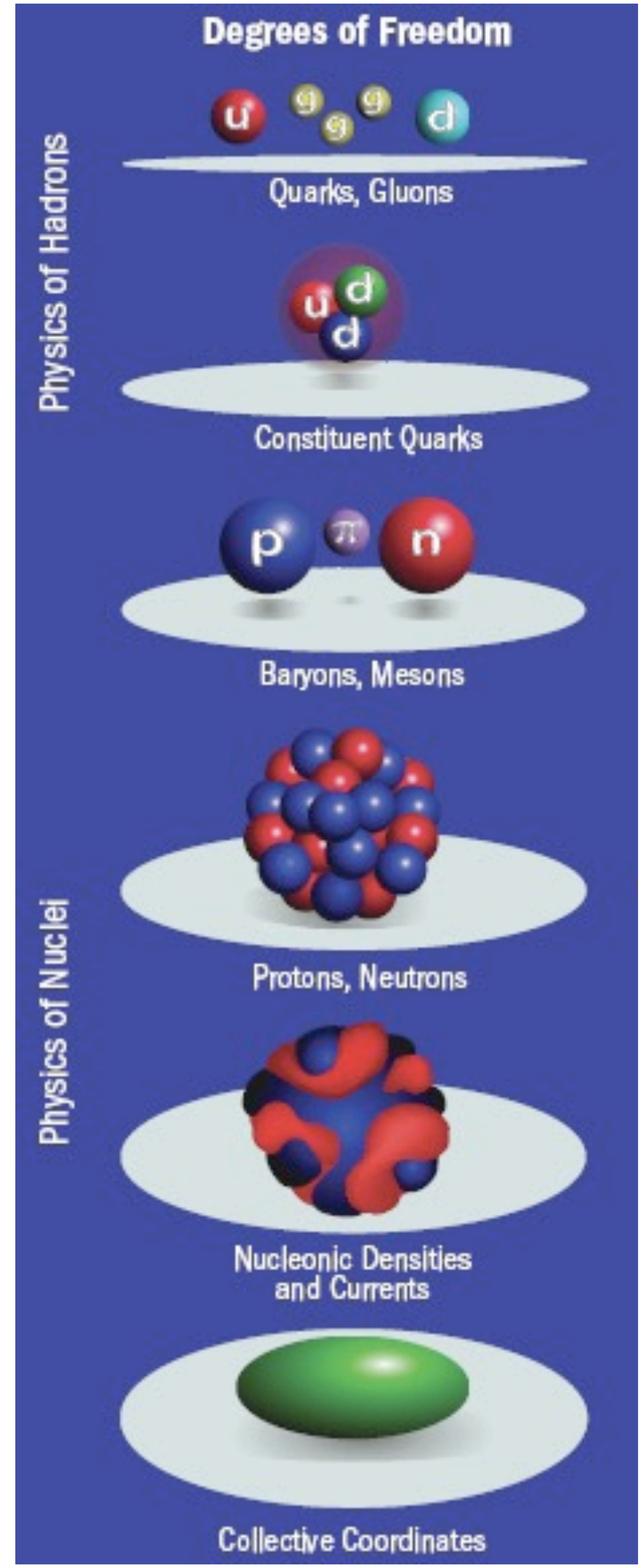




Resolution

- if a nucleus is probed at high energies, nucleon substructure is resolved
- at low energies, details are not resolved
- replace fine structure by something simpler (like multipole expansion), low-energy observables unchanged

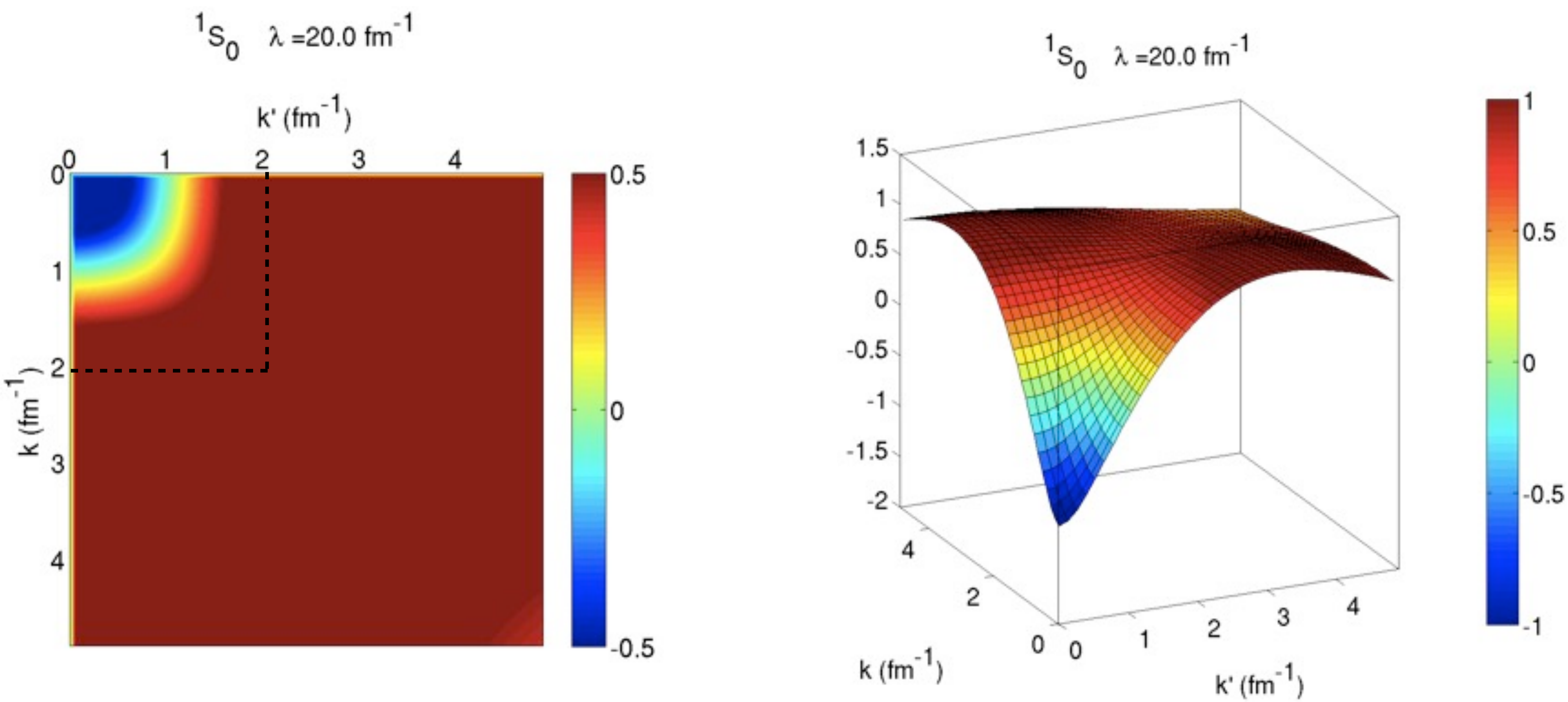
⟶ effective field theory

Changing the resolution scale: the Similarity Renormalization Group

- generate unitary transformation which decouples low- and high momenta

$$H_\lambda = U_\lambda H U_\lambda^\dagger \quad \text{with the resolution parameter } \lambda$$

- basic idea: change resolution successively in small steps: $\frac{dH_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda]$
- generator η_λ can be chosen and tailored to different applications
- observables are preserved due to unitarity of transformation



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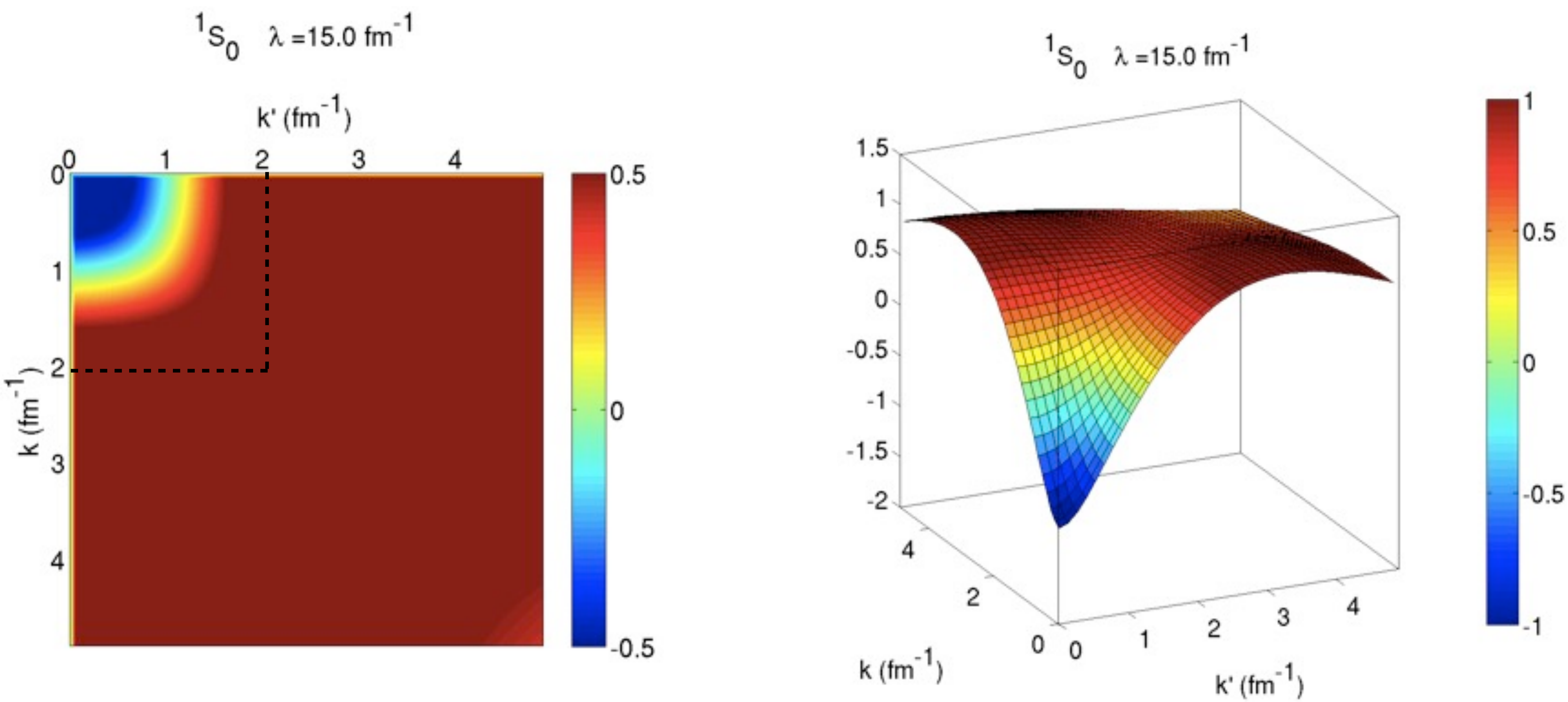

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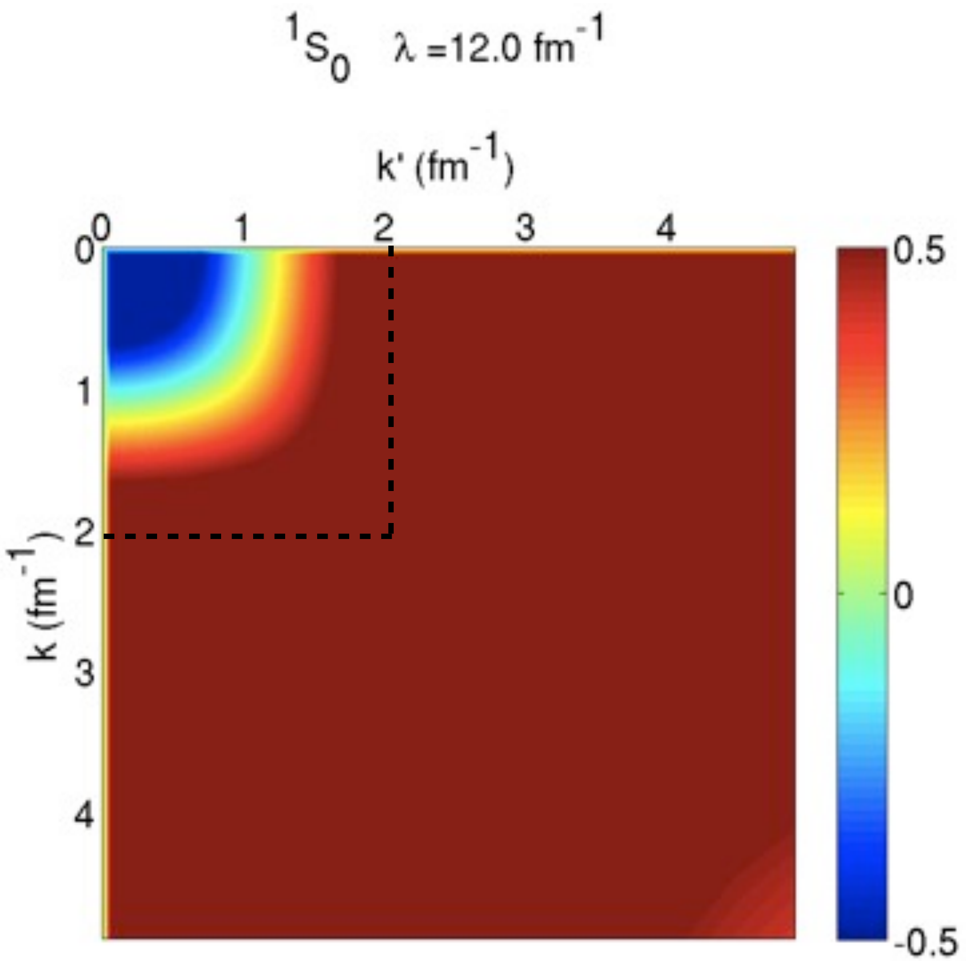

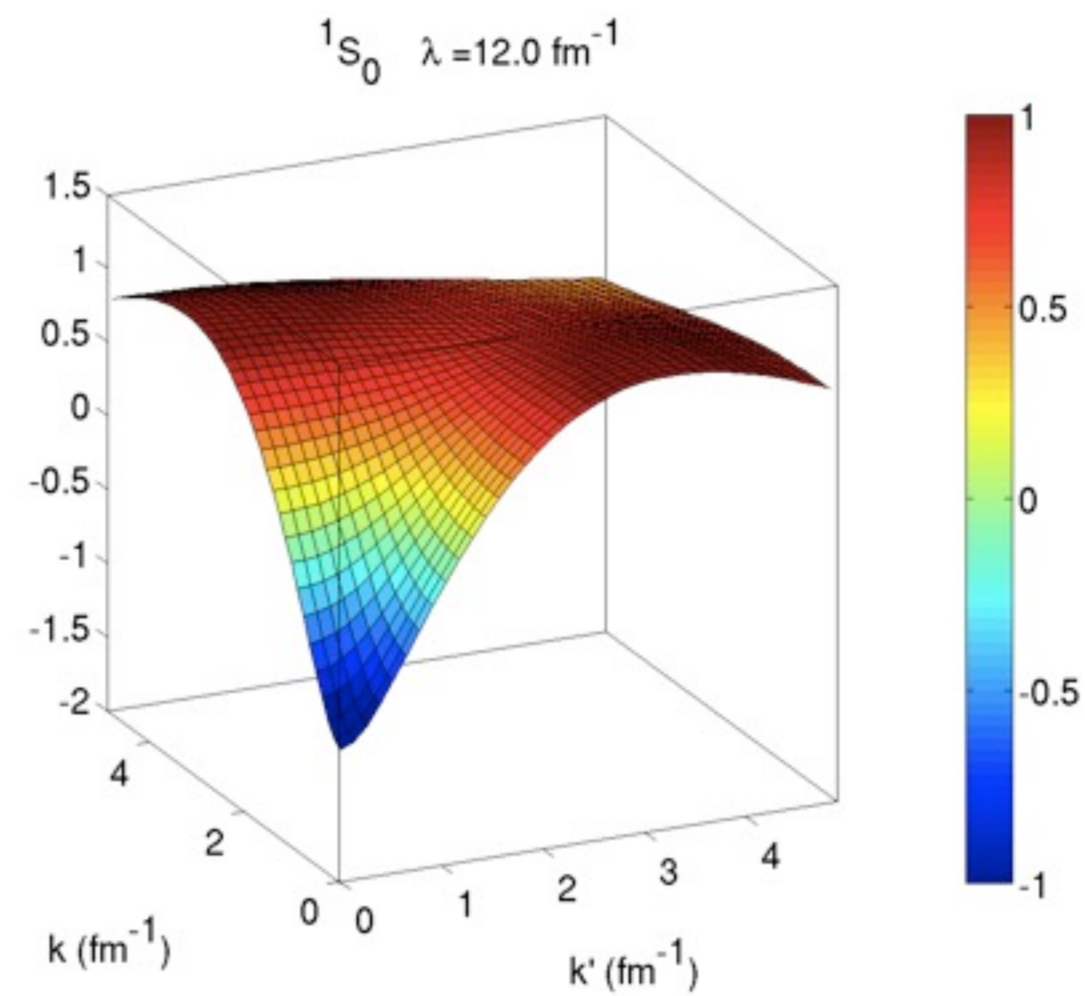

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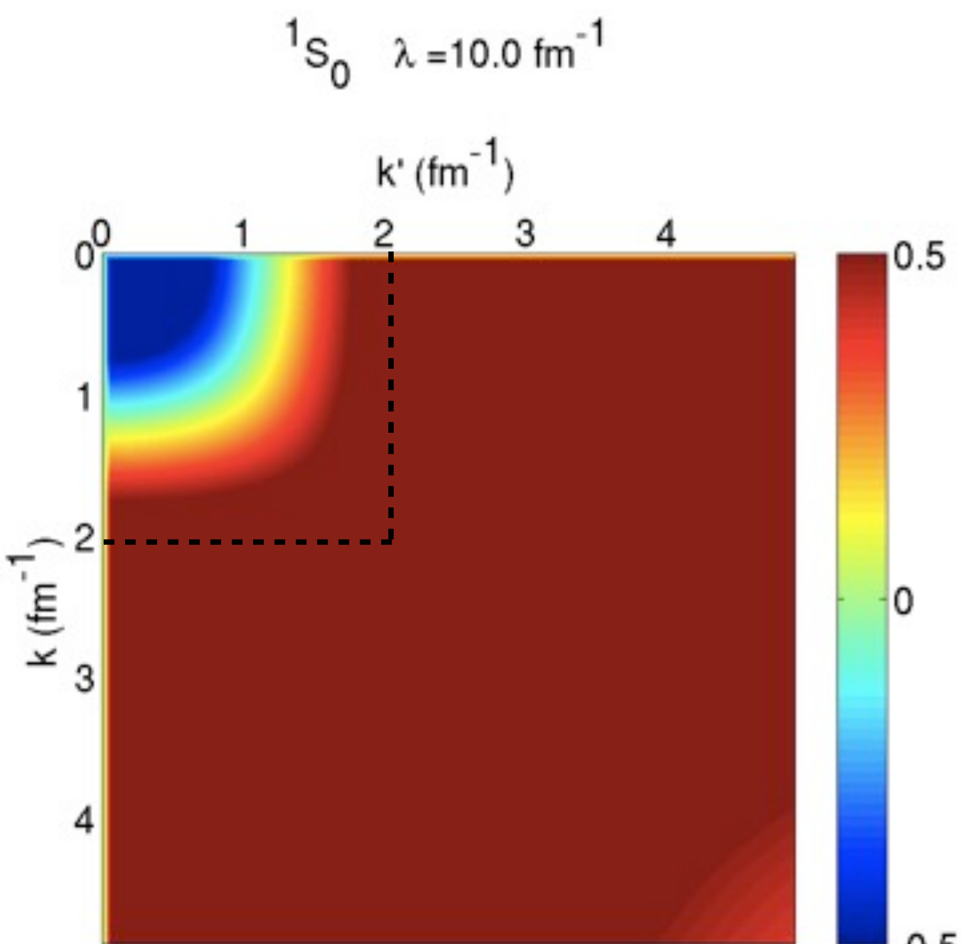

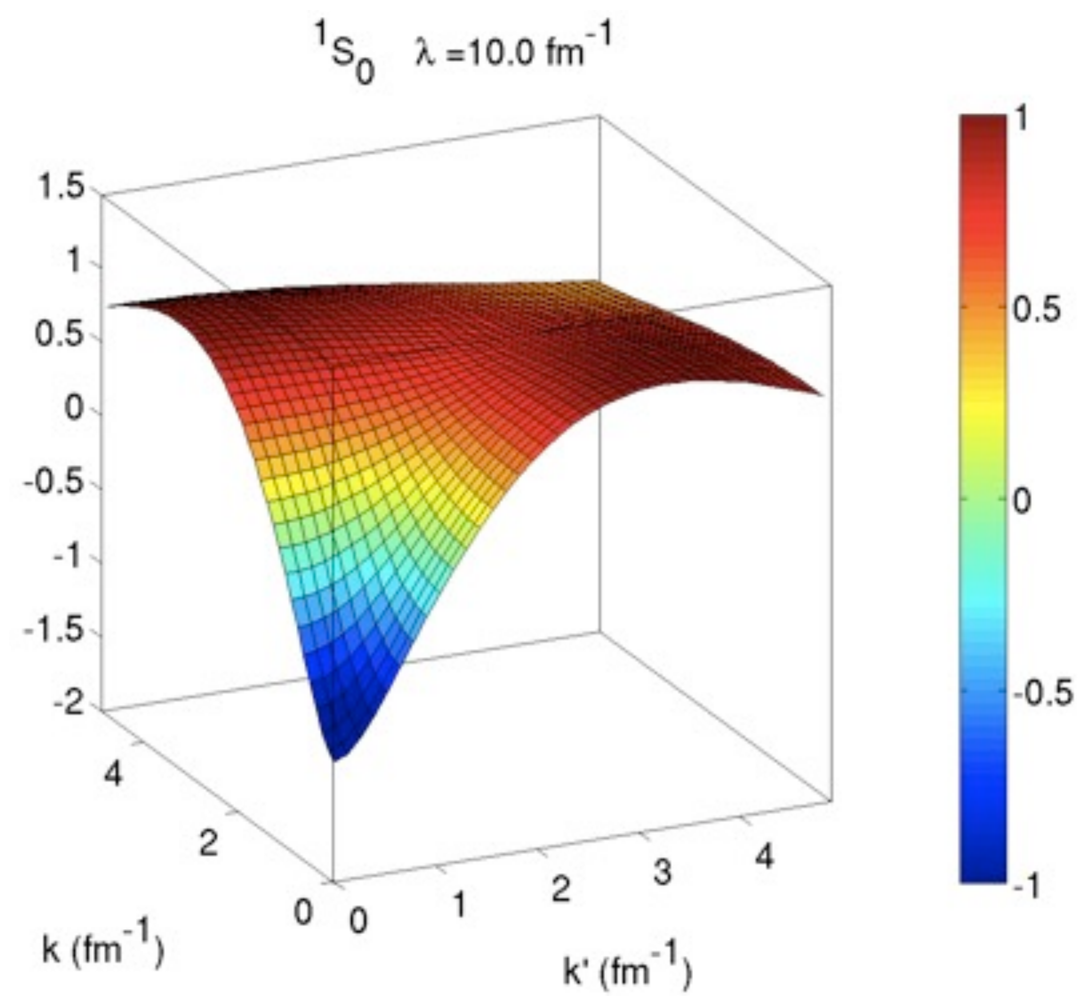

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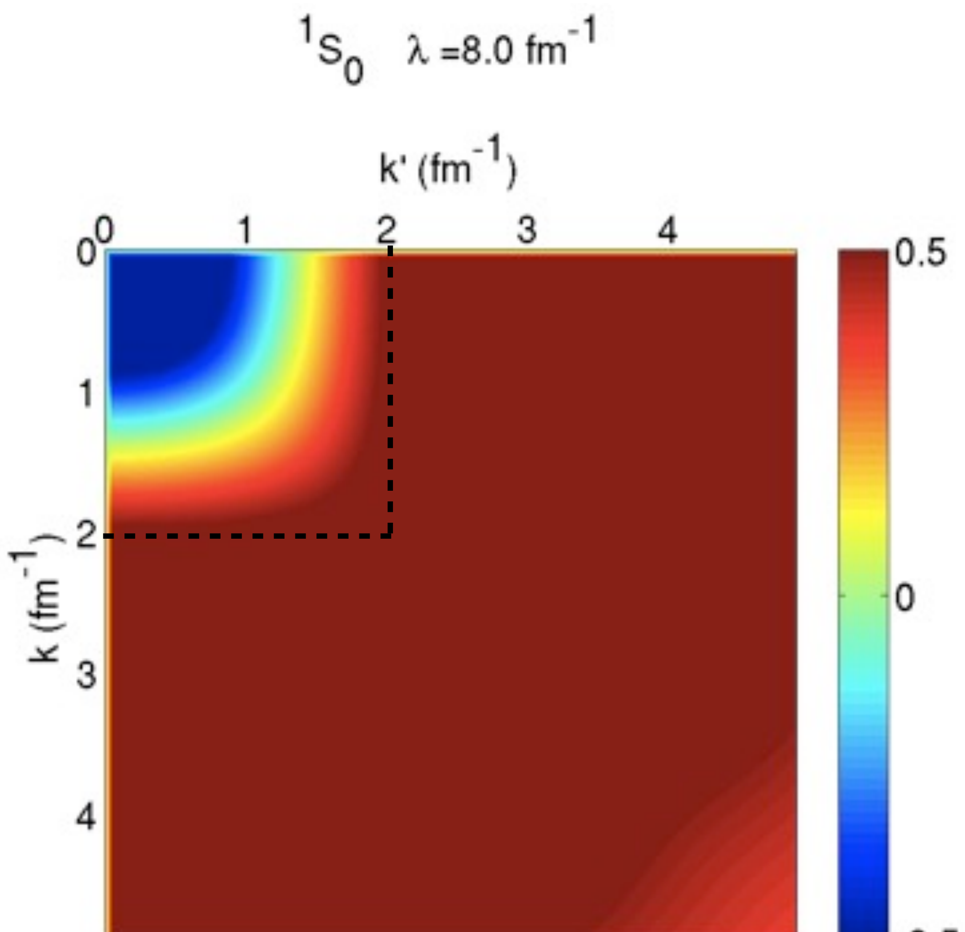

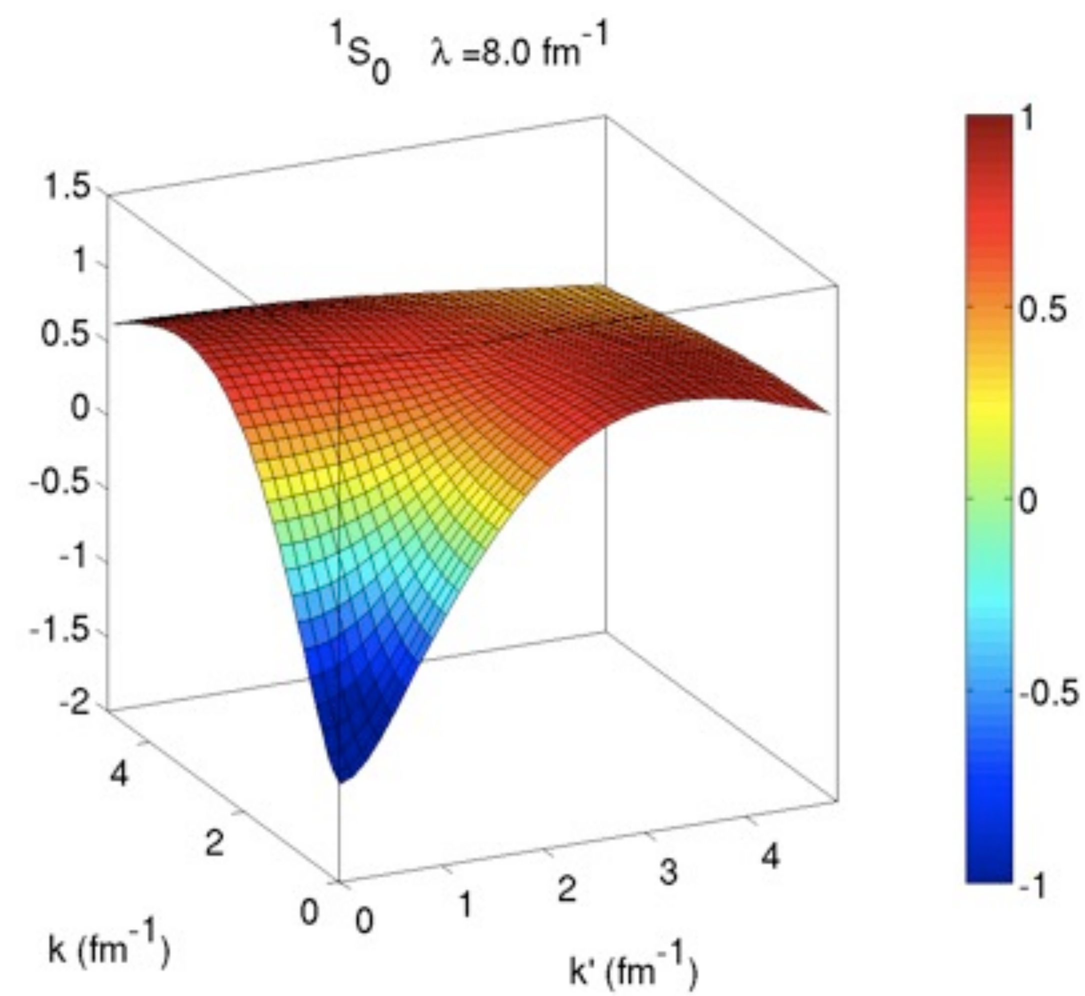

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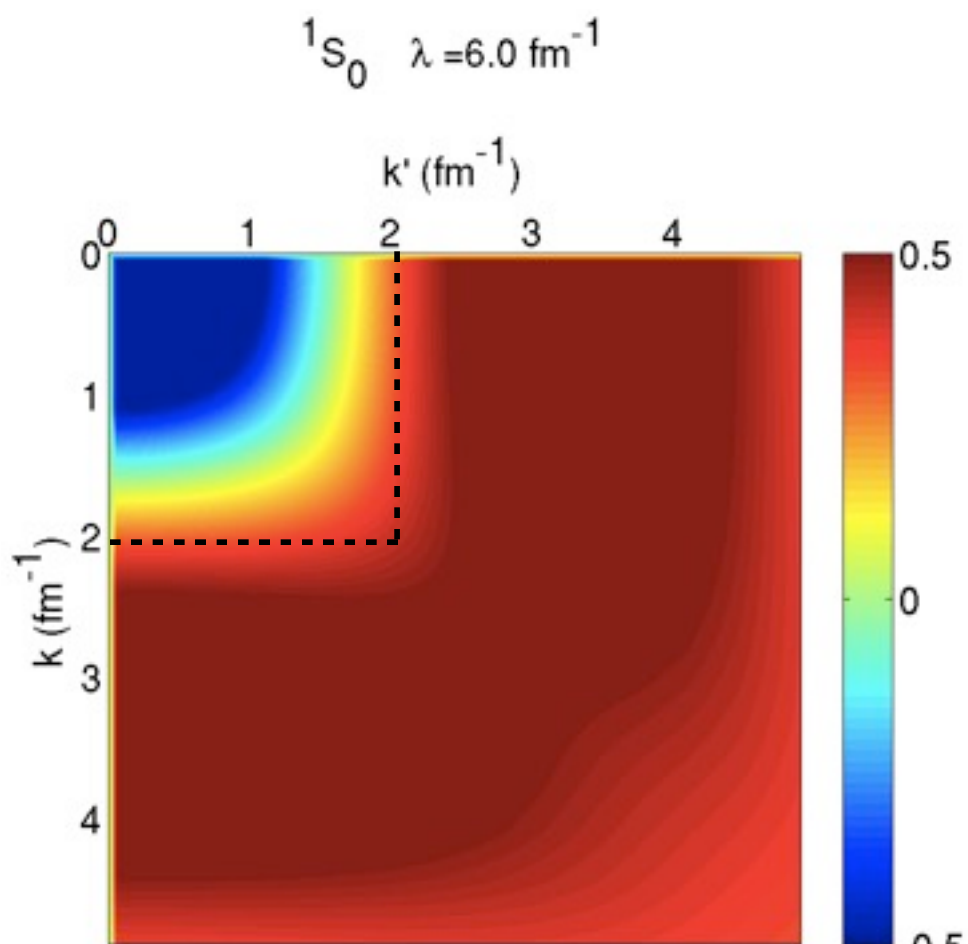

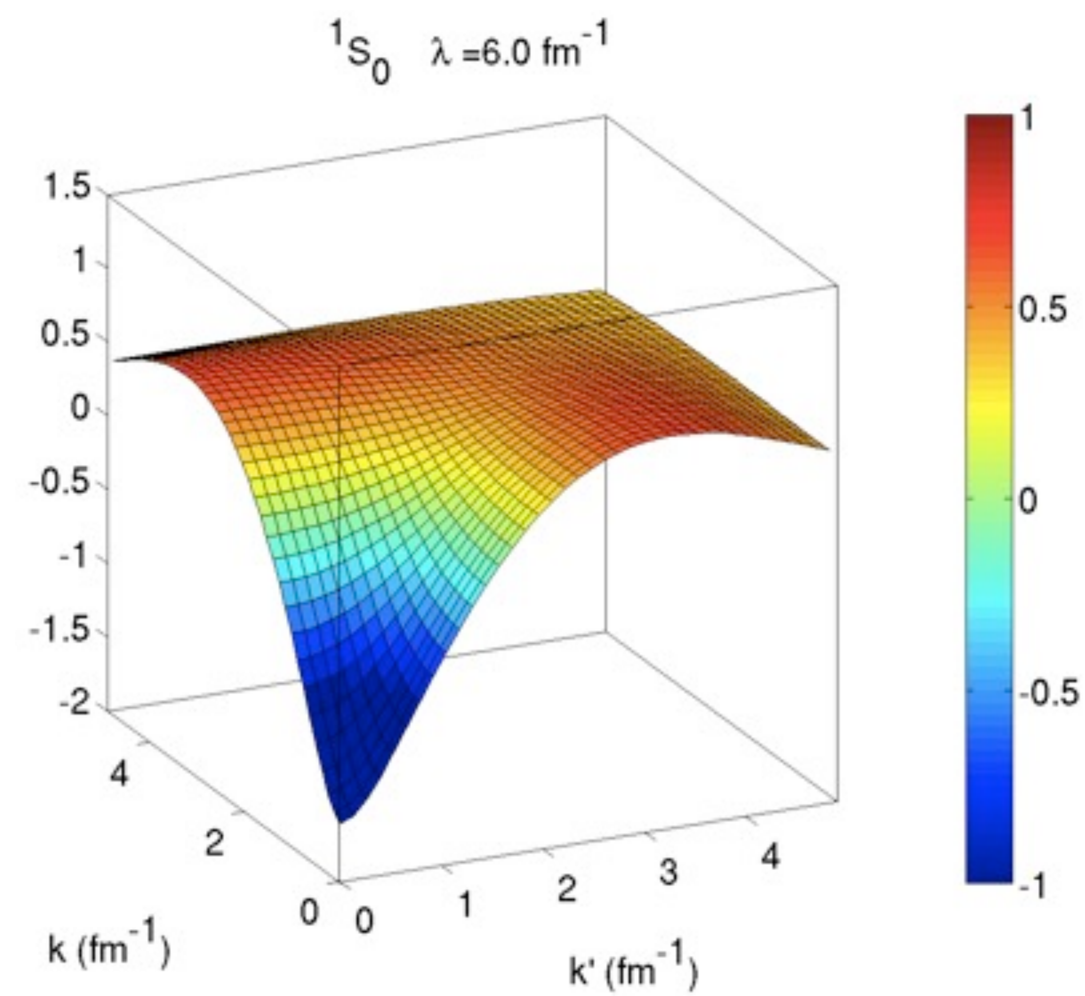

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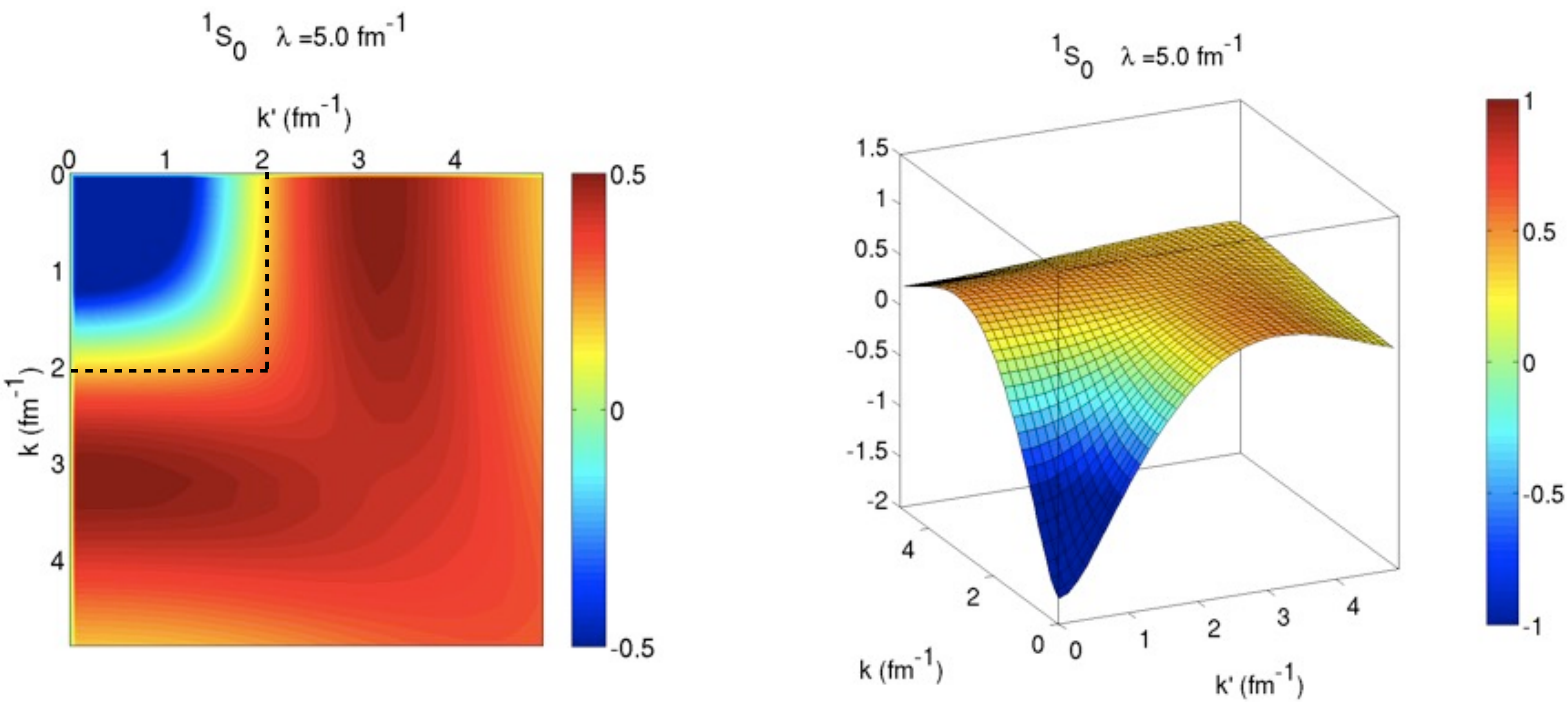

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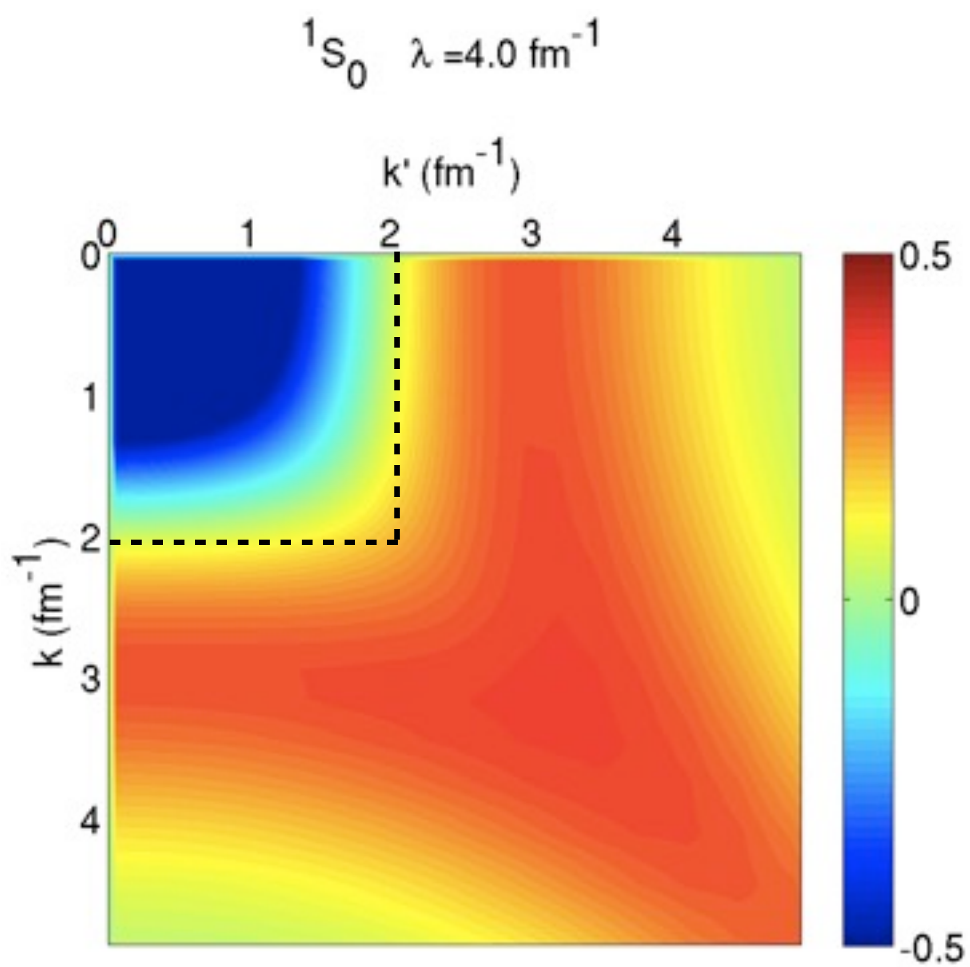

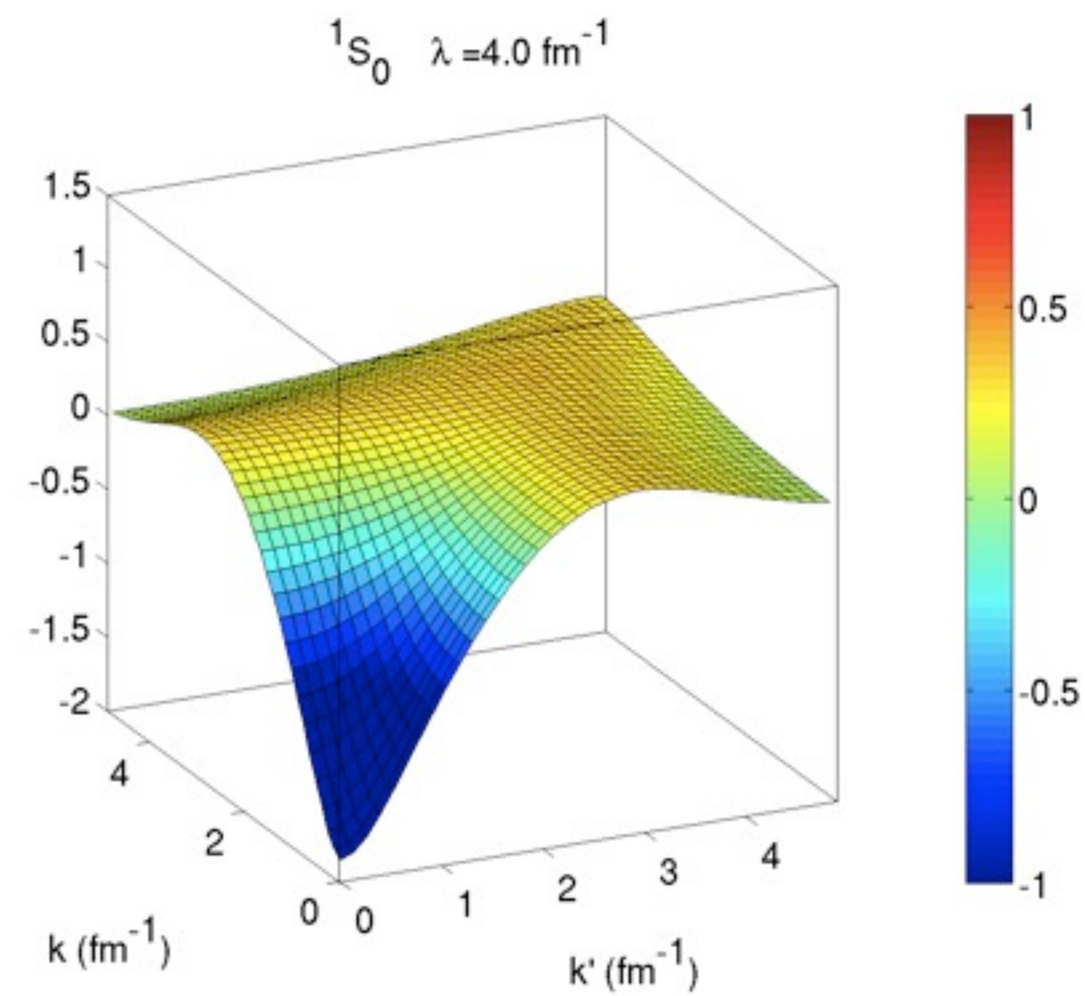

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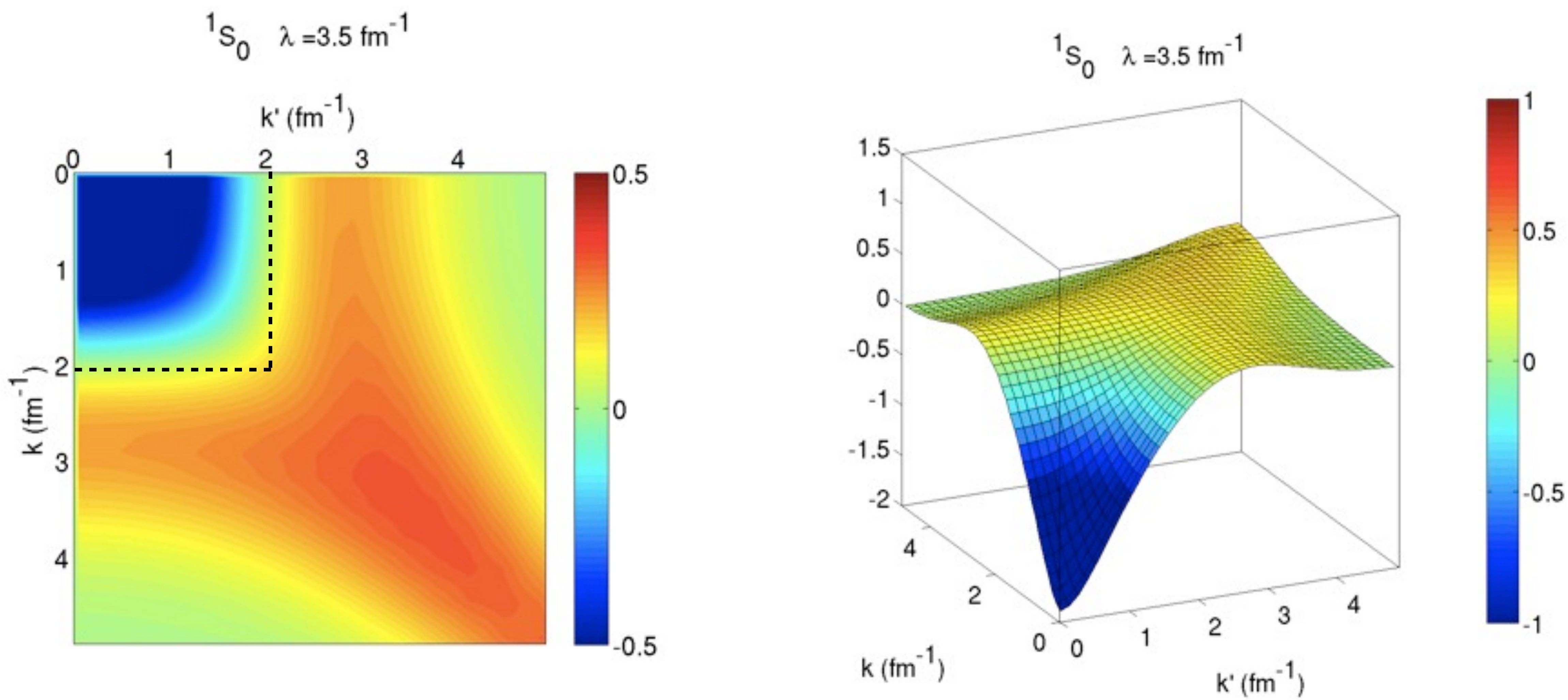

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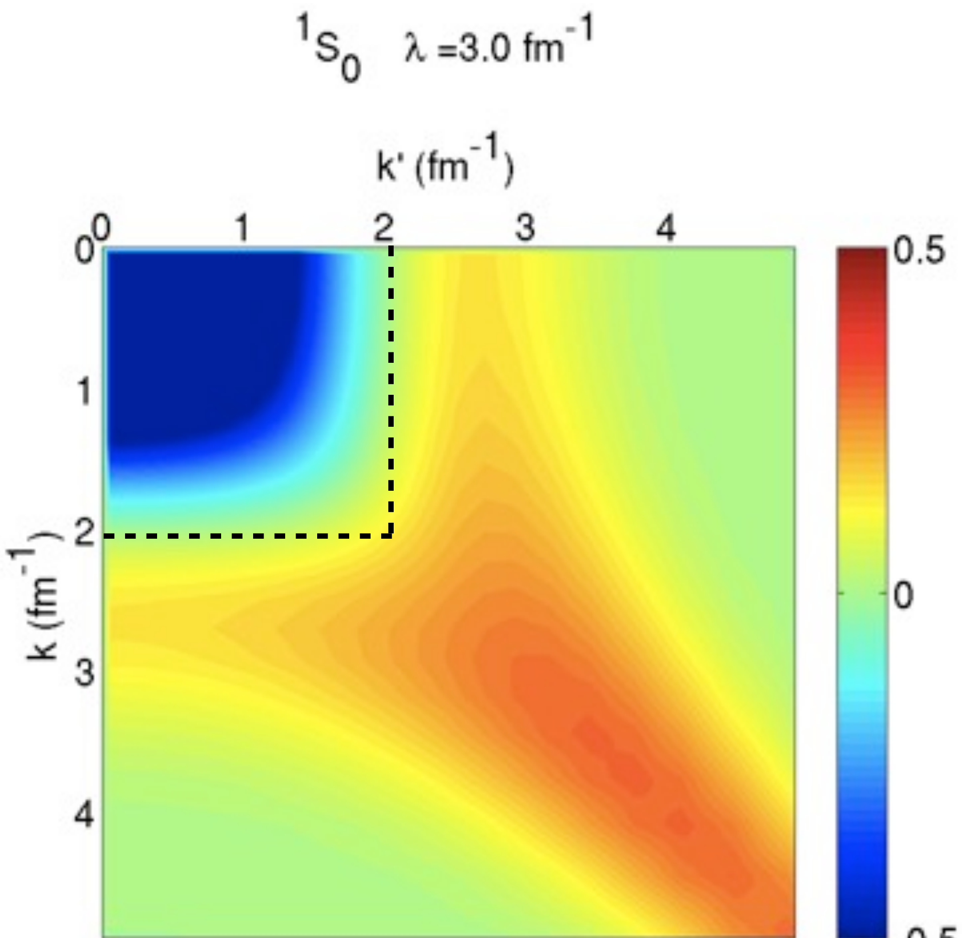

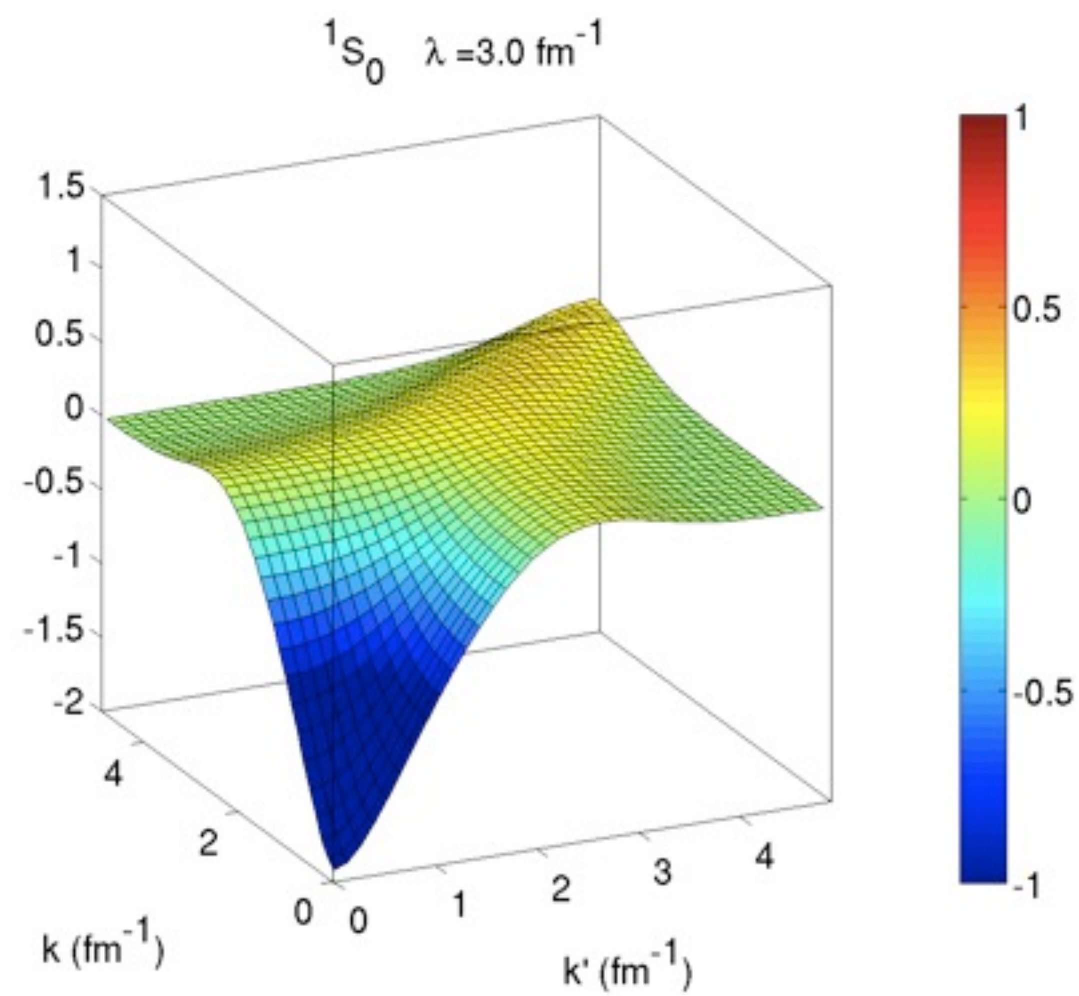

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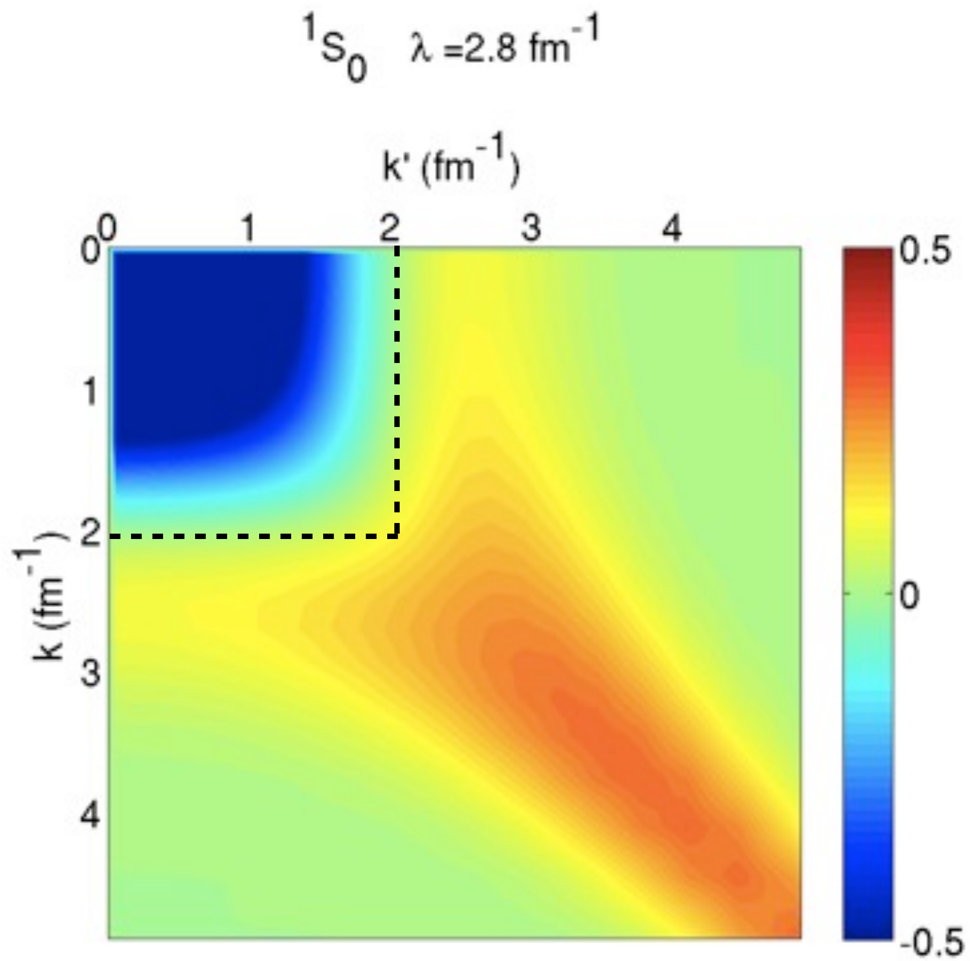

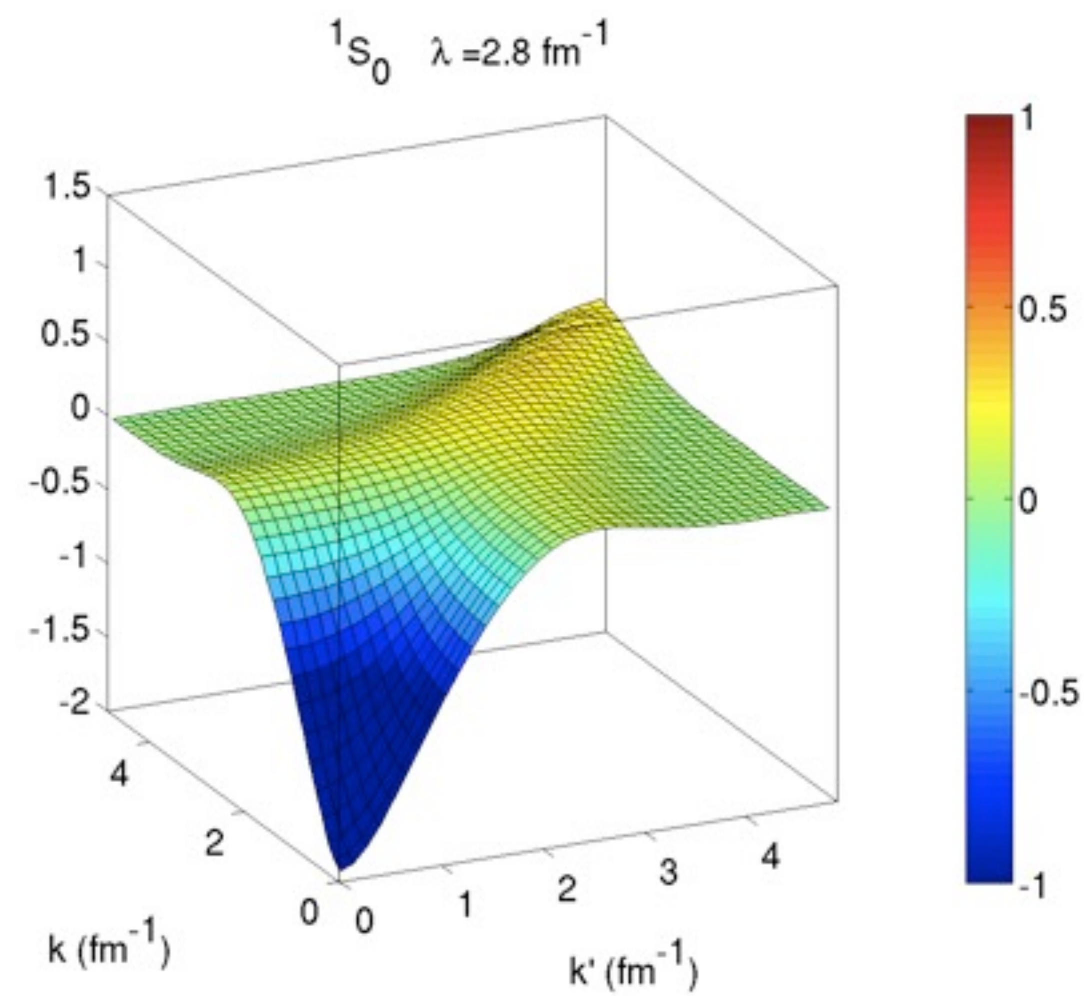

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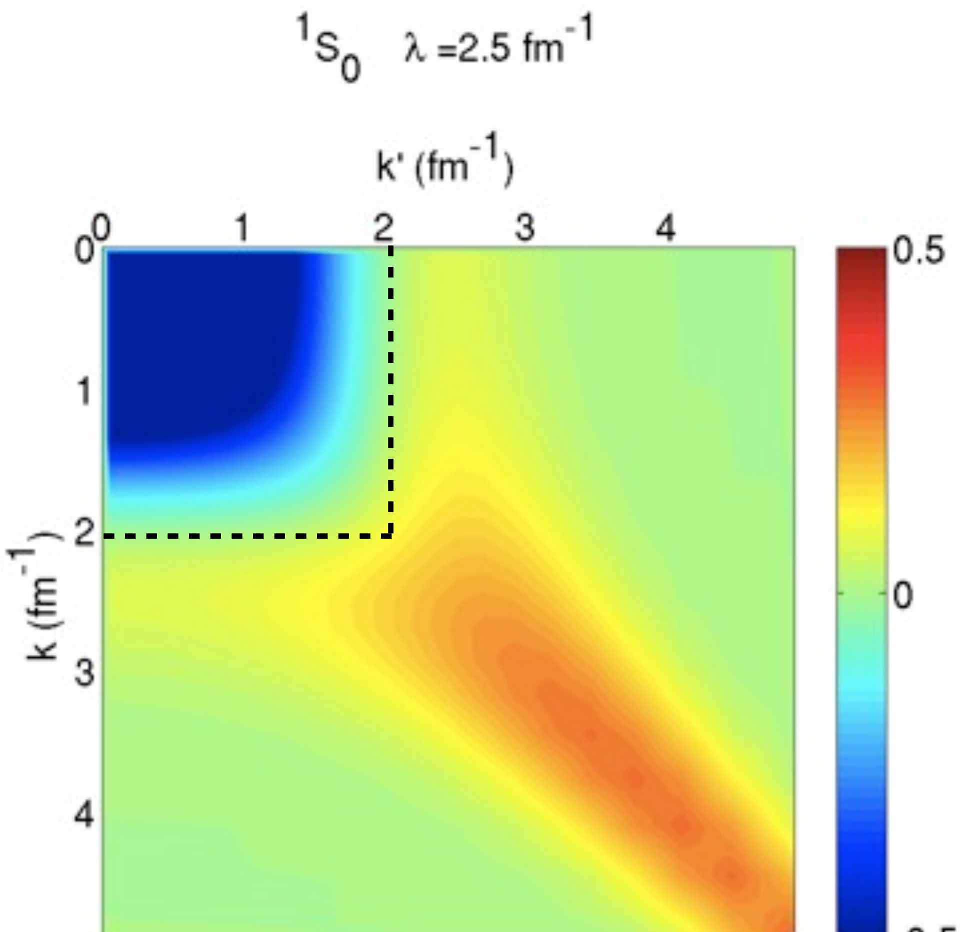

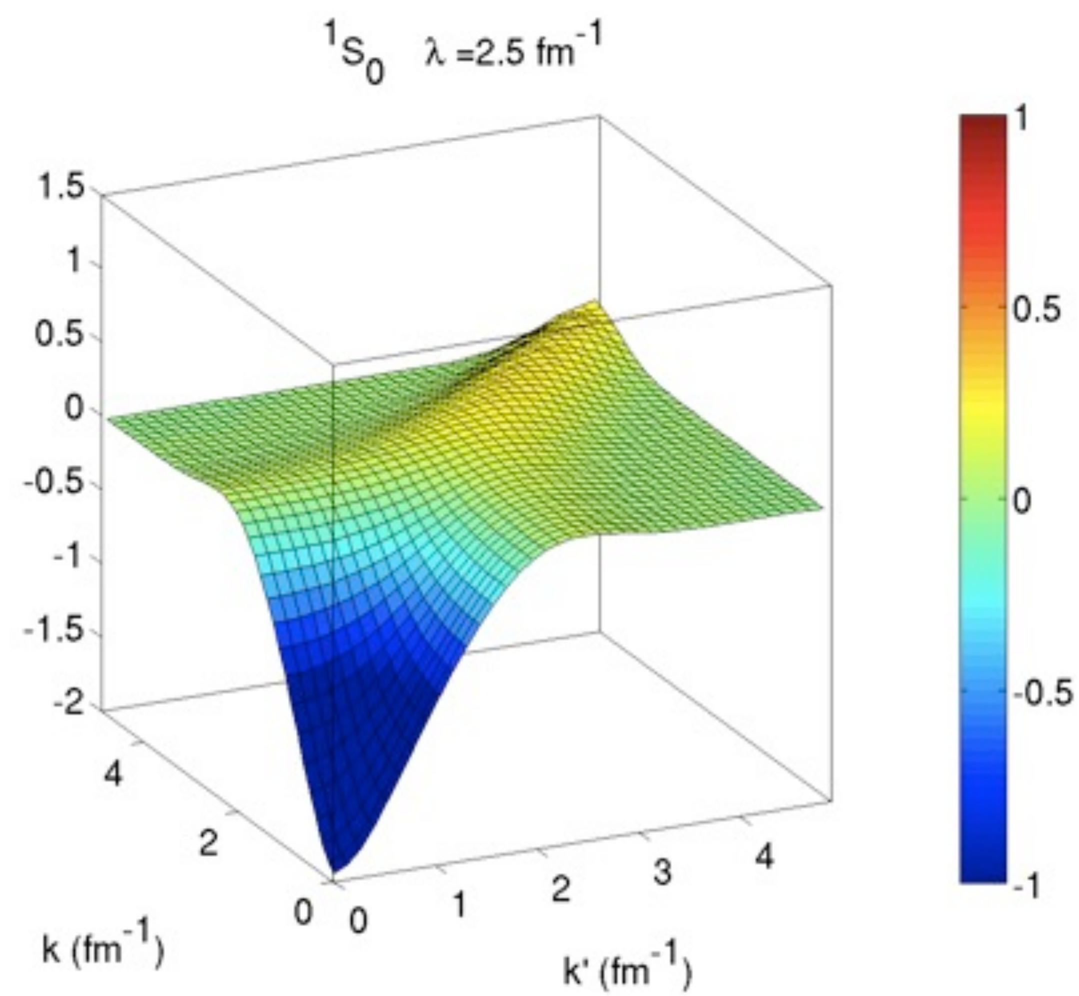

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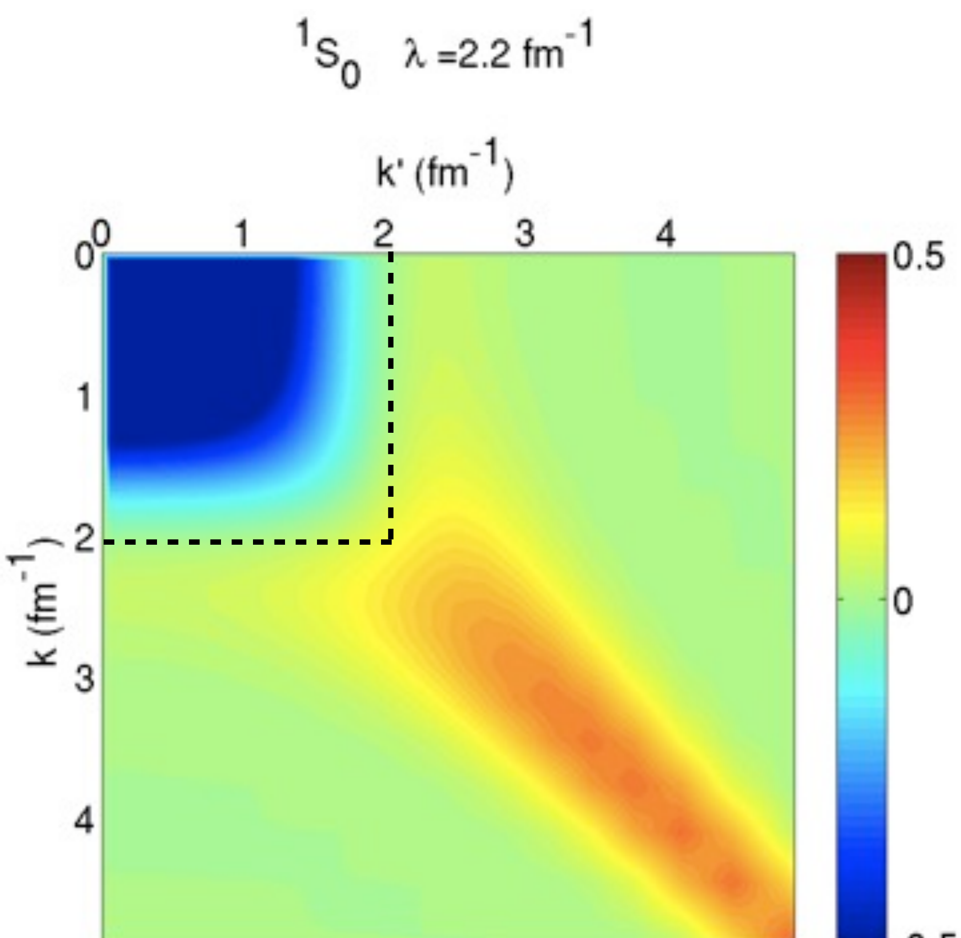

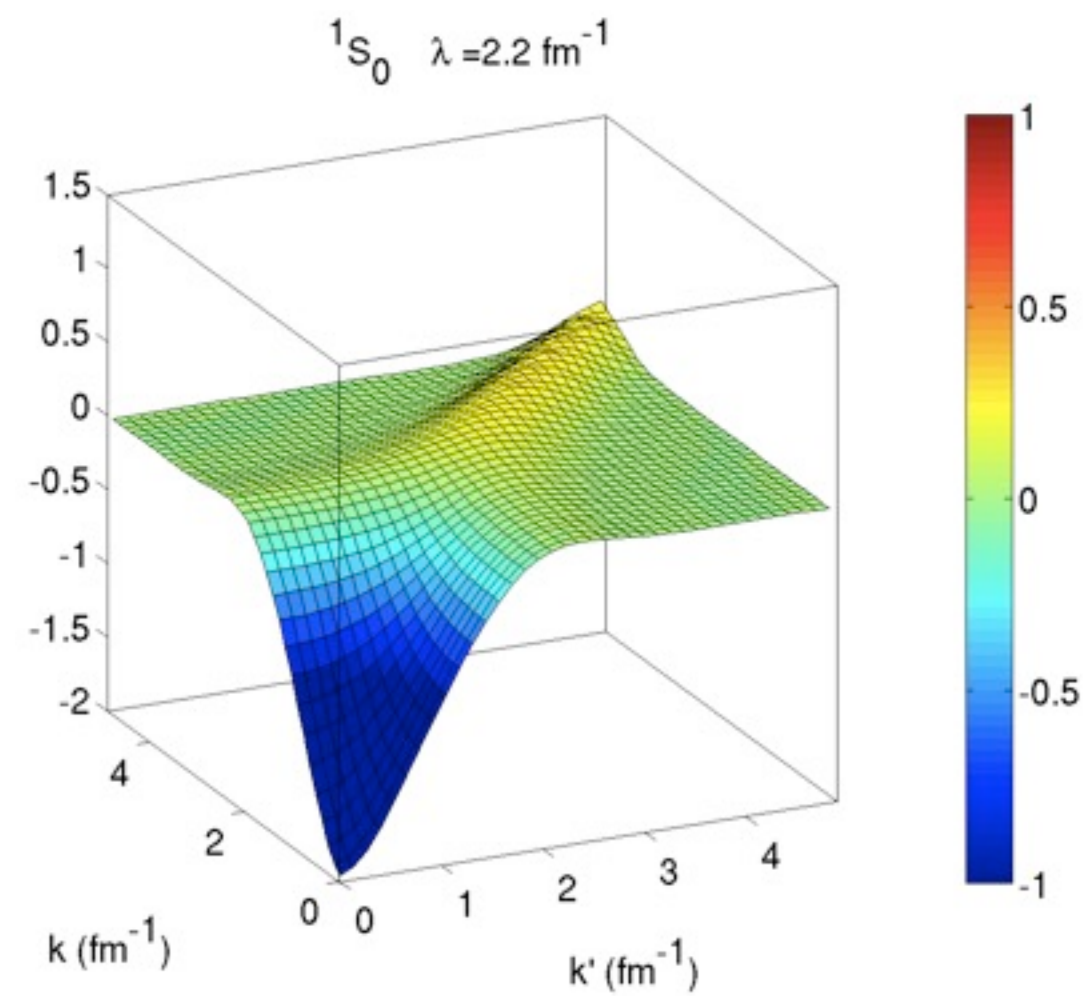

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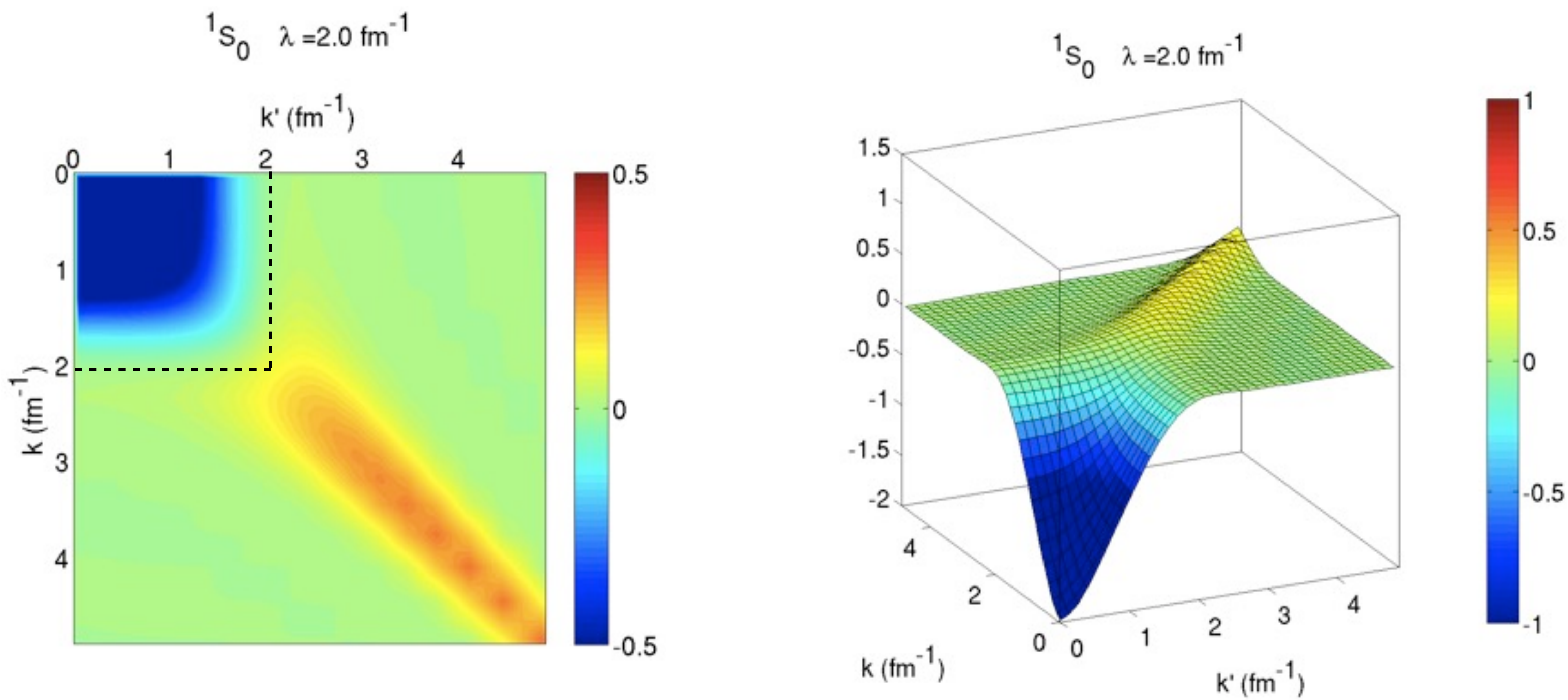
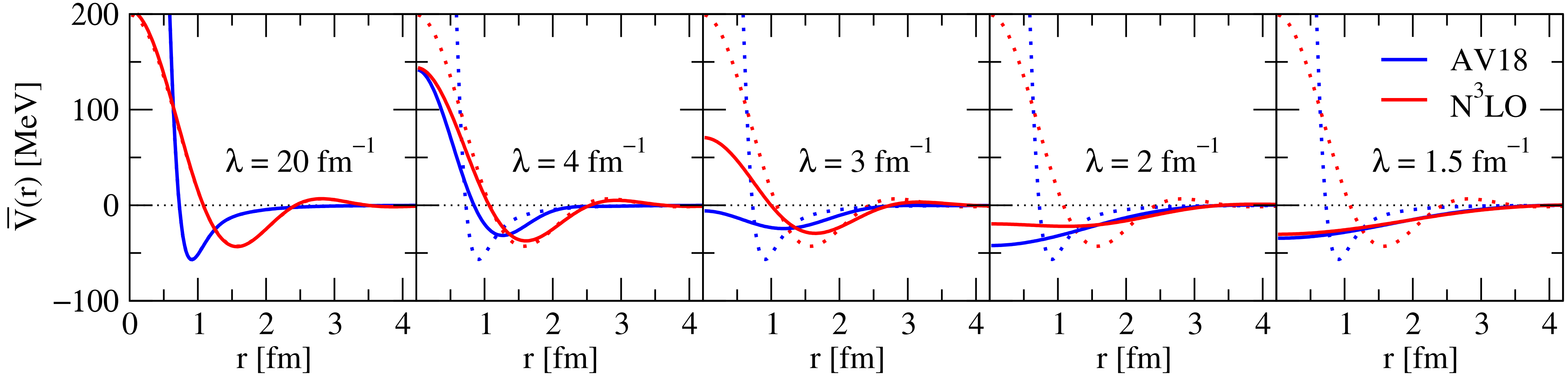
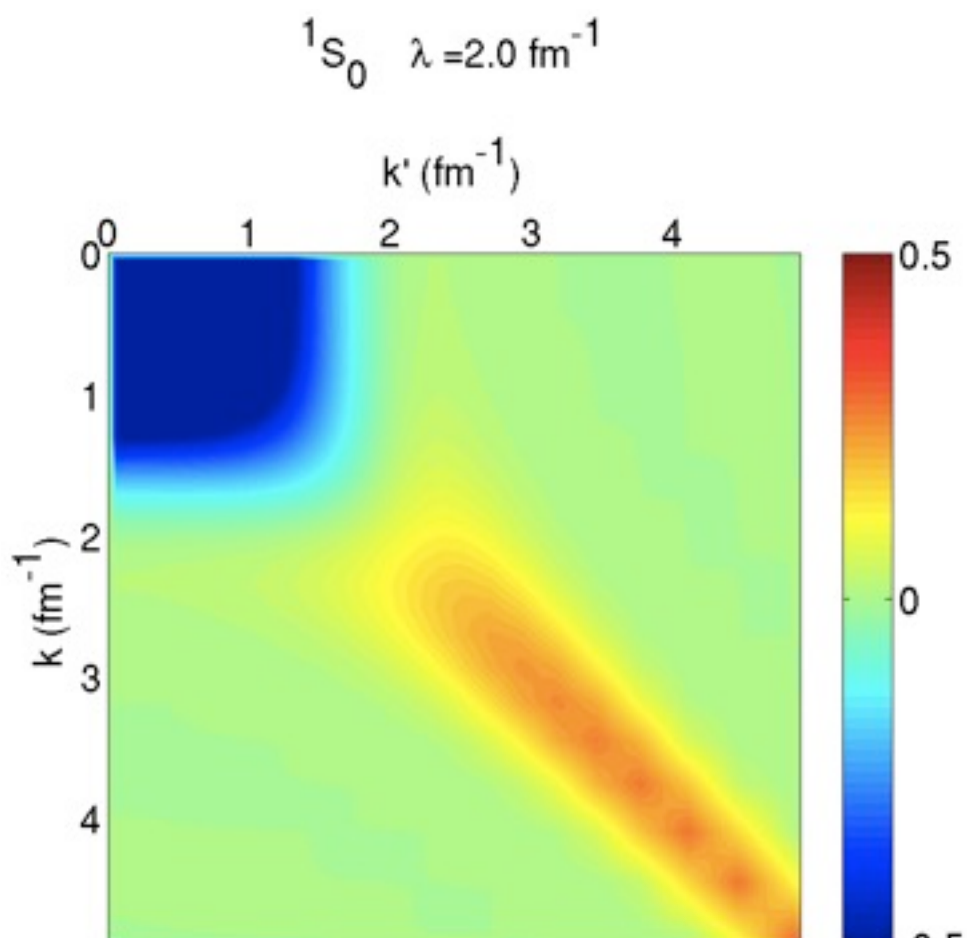
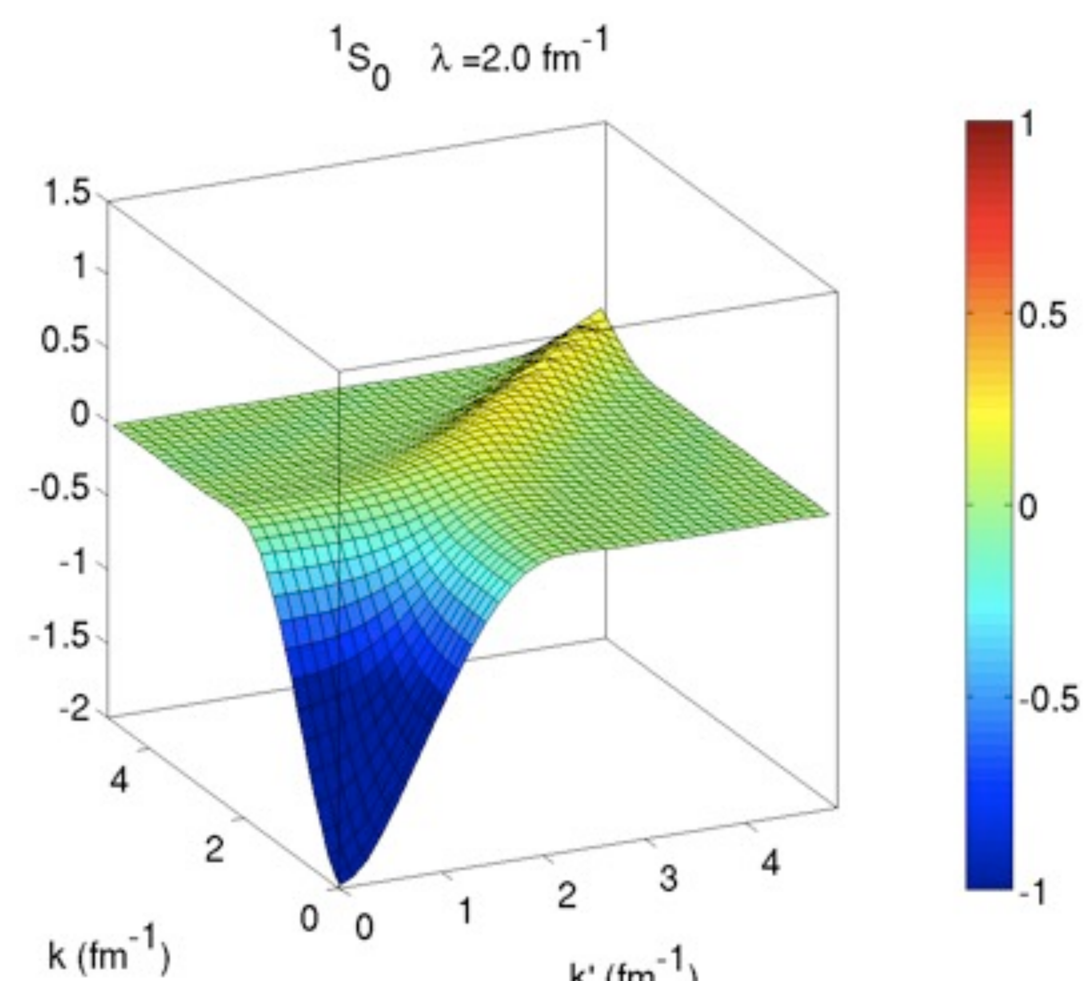


Systematic decoupling of high-momentum physics: The Similarity Renormalization Group

$$\overline{V}_\lambda(r) = \int dr' r'^2 V_\lambda(r, r')$$







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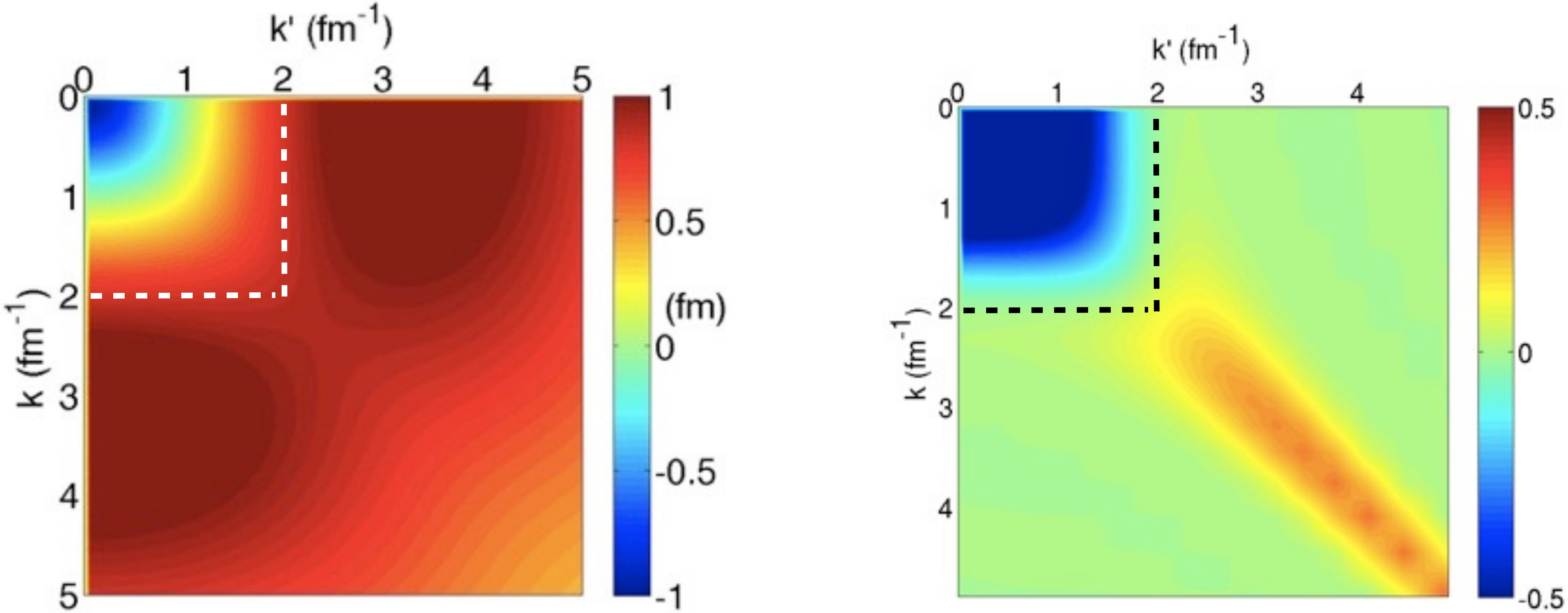
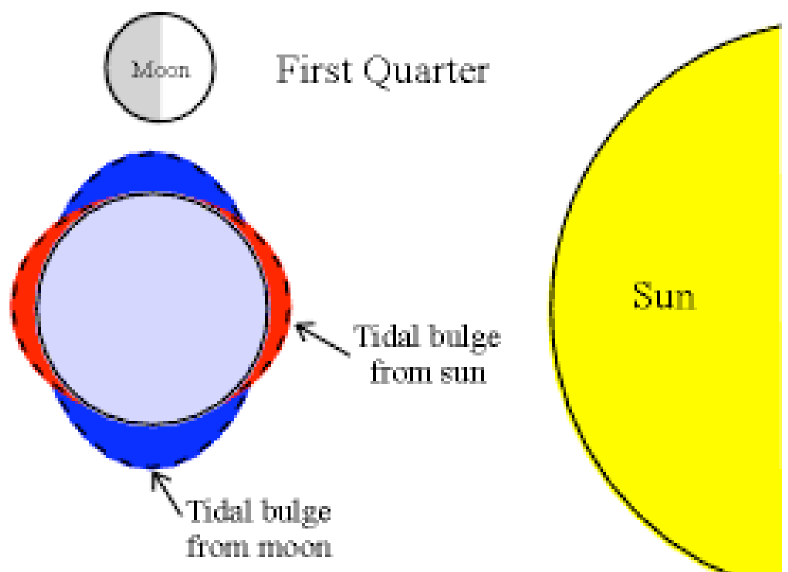


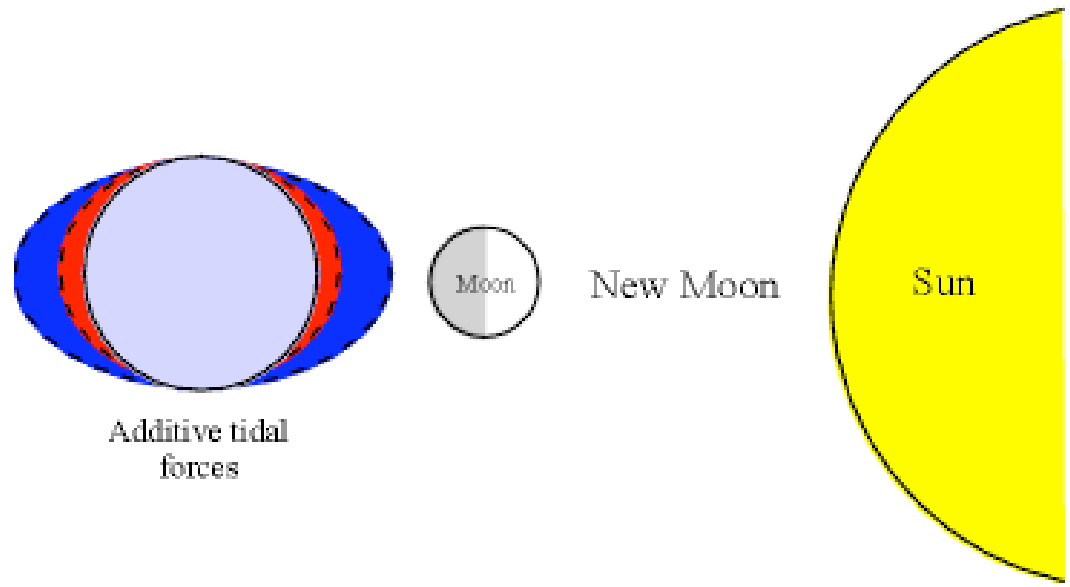
- elimination of coupling between low- and high momentum components, $\longrightarrow$ simplified many-body calculations, smaller required model spaces
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

Not the full story:
RG transformation also changes three-body (and higher-body) interactions.

Aren't 3N forces unnatural? Do we really need them?

Consider classical analog: tidal effects in earth-sun-moon system



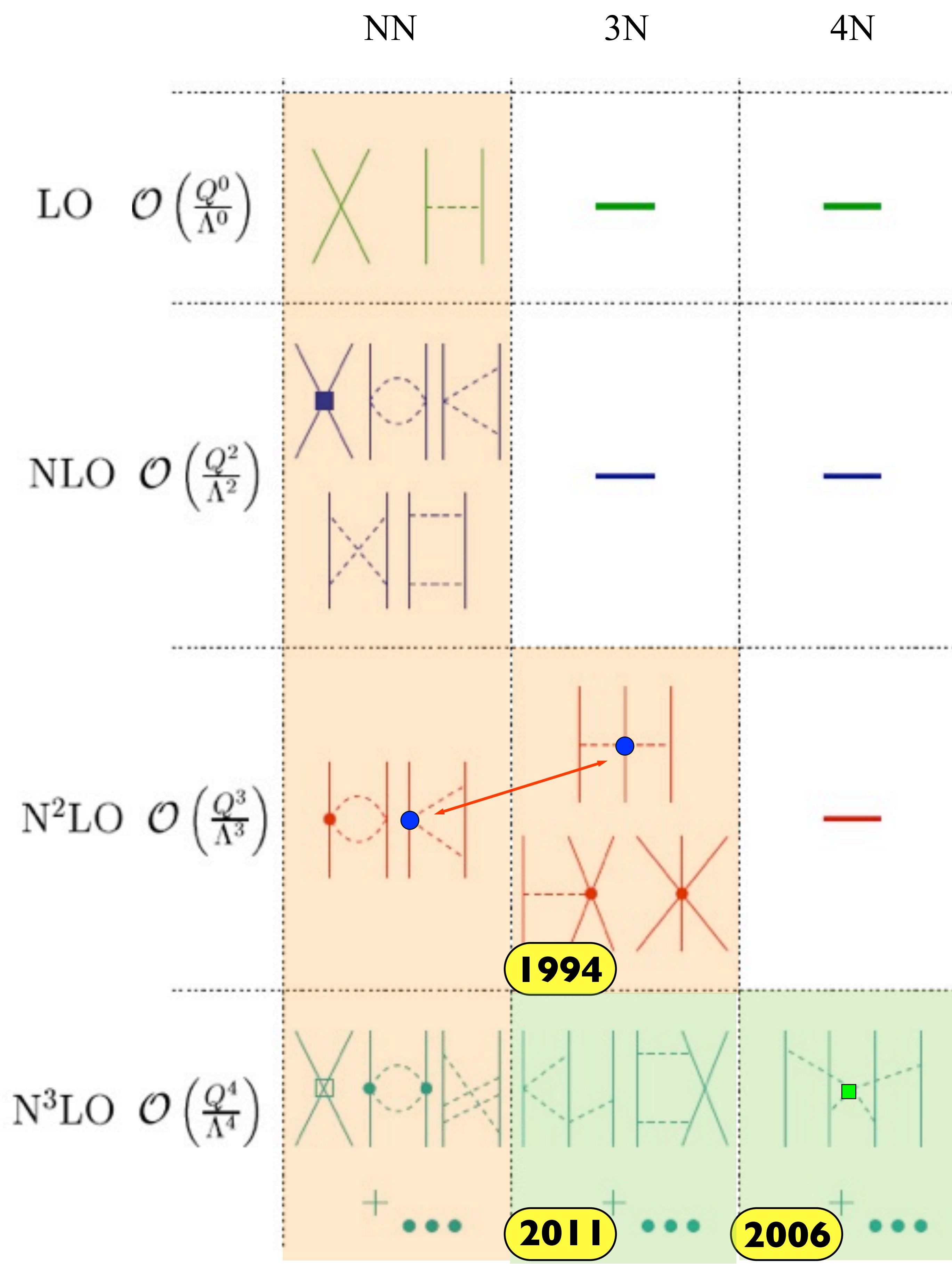


- force between earth and moon depends on the position of sun
- tidal deformations represent internal excitations
- describe system using point particles $\longrightarrow$ 3N forces inevitable!

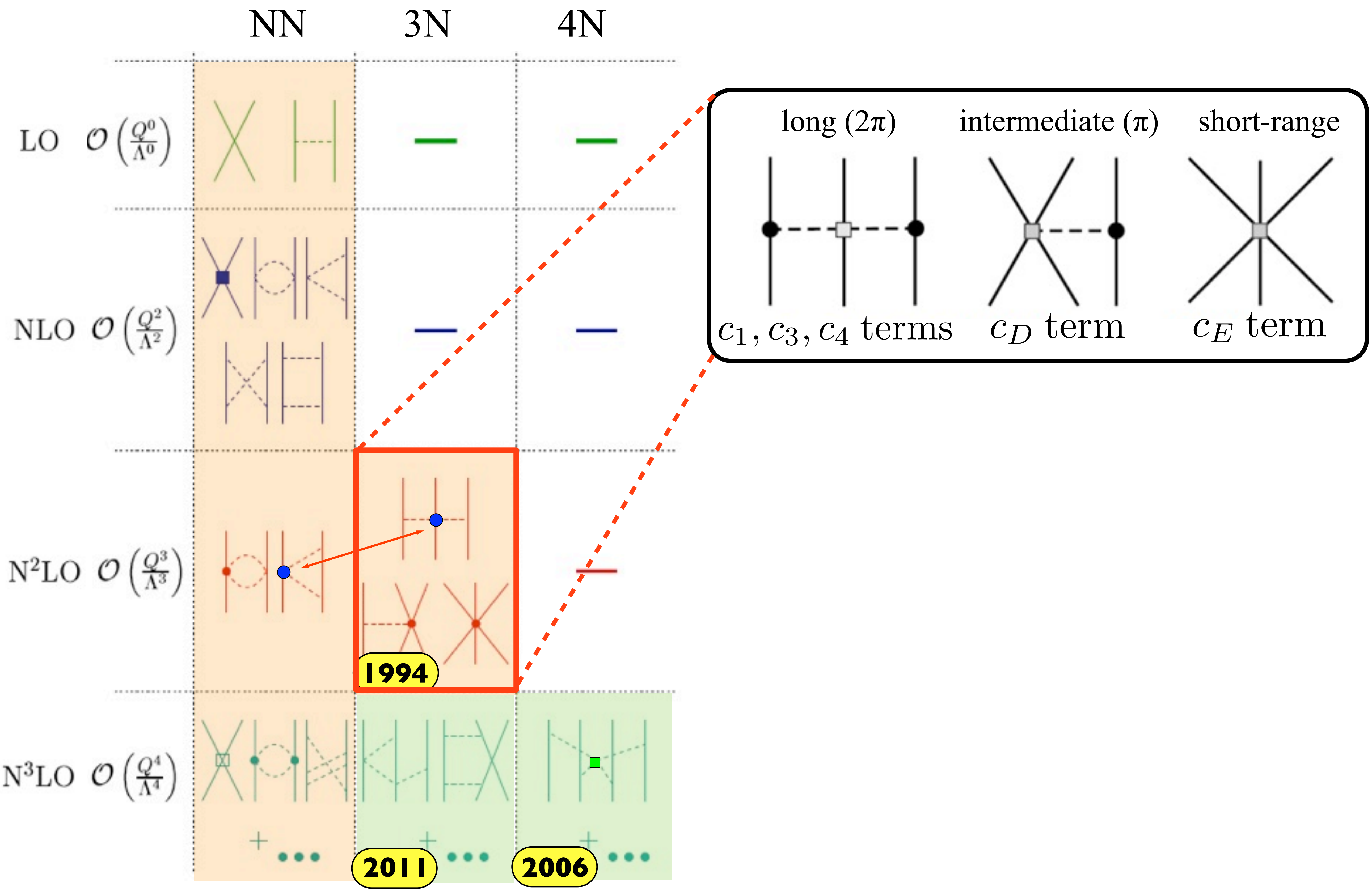
- nucleons are composite particles, can also be excited
- change of resolution change excitations that can be described explicitly
- existence of three-nucleon forces natural
- crucial question: How important are their contributions?

Chiral effective field theory for nuclear forces

- choose relevant degrees of freedom: here nucleons and pions
- operators constrained by symmetries of QCD
- short-range physics captured in few short-range couplings
- separation of scales: Q << Λ_b, breakdown scale Λ_b~500 MeV
- power-counting: expand in powers Q/Λ_b
- systematic: work to desired accuracy, obtain error estimates



Many-body forces in chiral EFT



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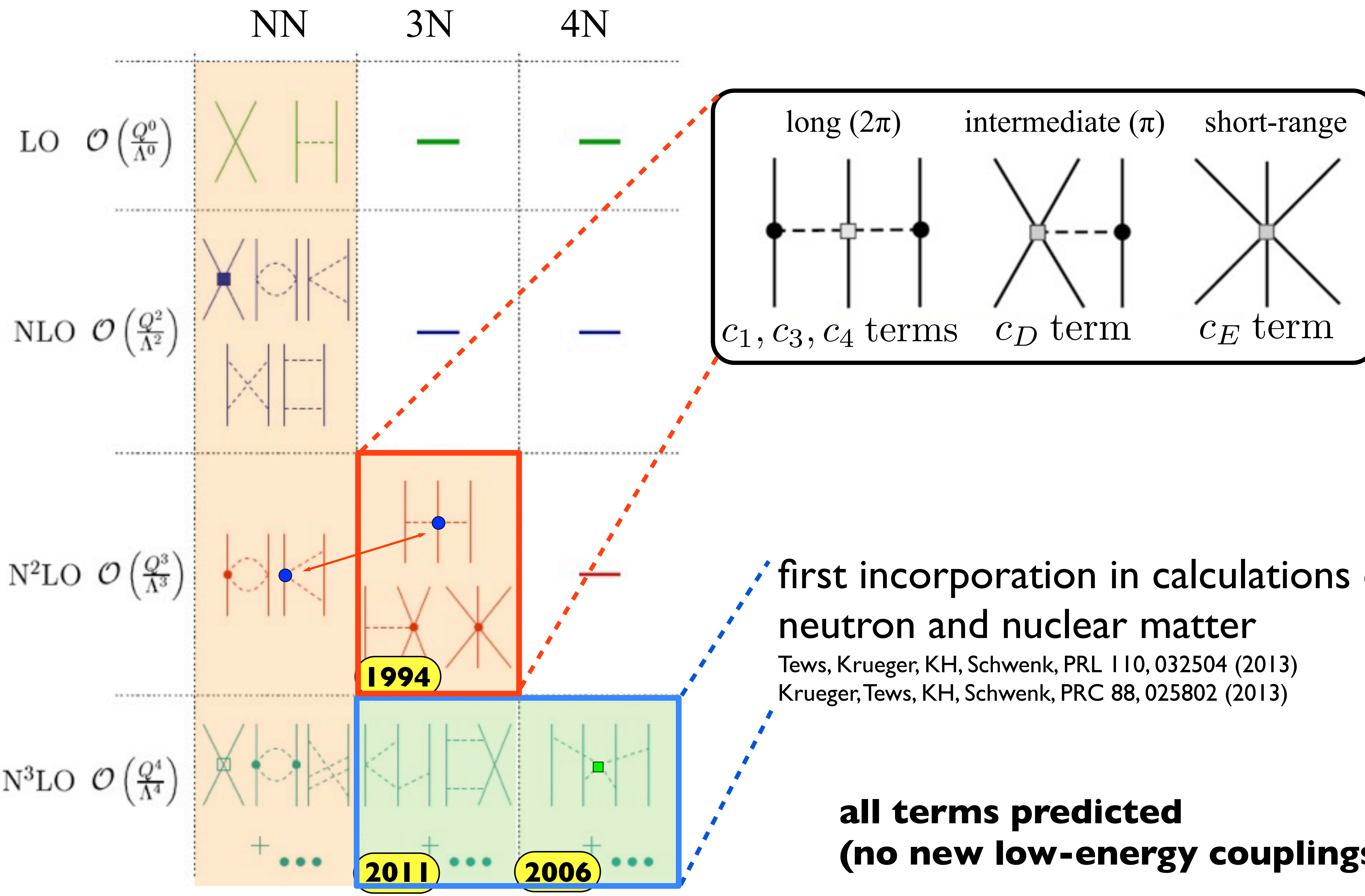
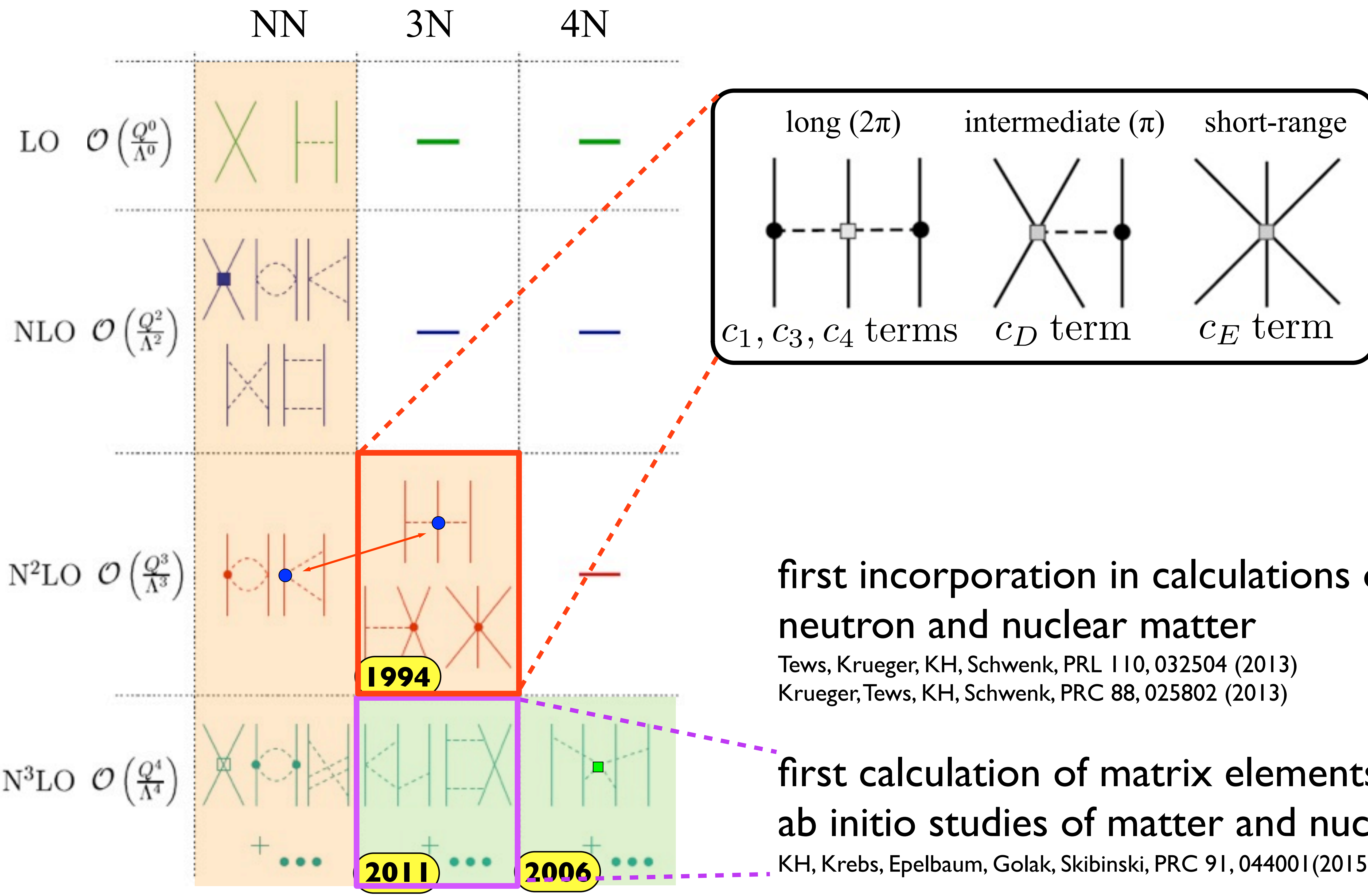


first incorporation in calculations of neutron and nuclear matter

Tews, Krueger, KH, Schwenk, PRL 110, 032504 (2013)
Krueger, Tews, KH, Schwenk, PRC 88, 025802 (2013)

all terms predicted
(no new low-energy couplings)

Many-body forces in chiral EFT



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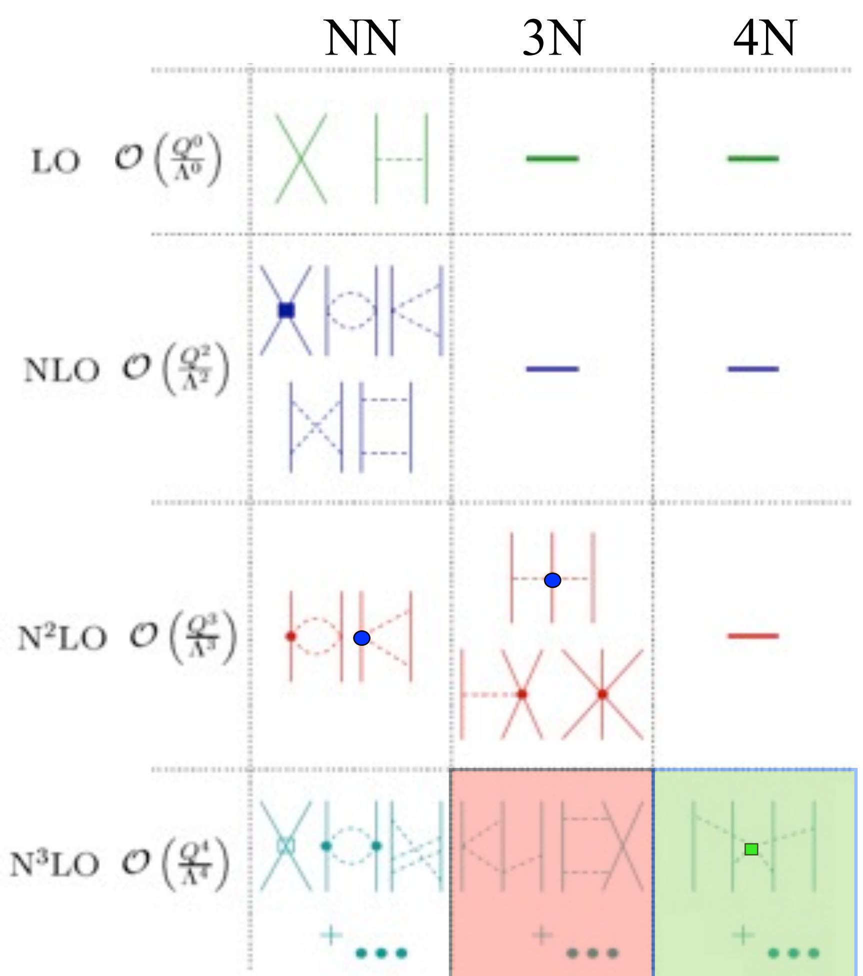
first calculation of matrix elements for ab initio studies of matter and nuclei

KH, Krebs, Epelbaum, Golak, Skibinski, PRC 91, 044001(2015)

Contributions of many-body forces at N^{3}LO in neutron matter



- first calculations of N^{3}LO 3NF and 4NF contributions to EOS of neutron matter
- found large contributions in Hartree Fock appr., comparable to size of N^{2}LO contributions, (power counting?)

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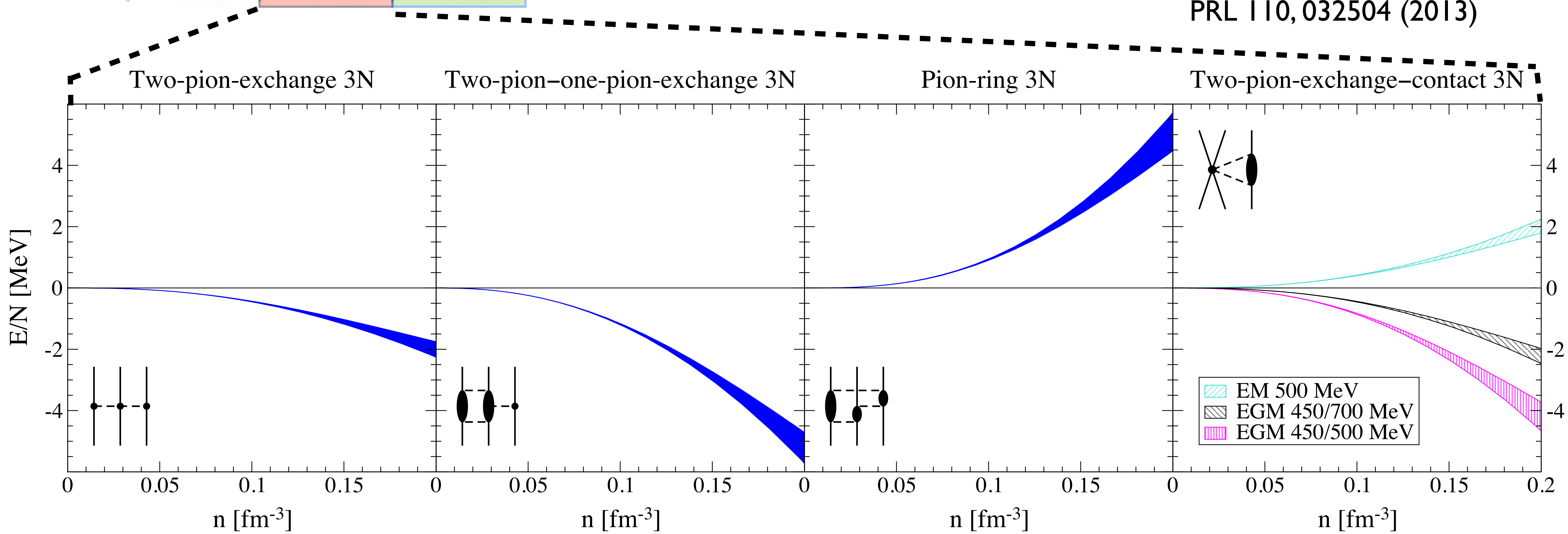

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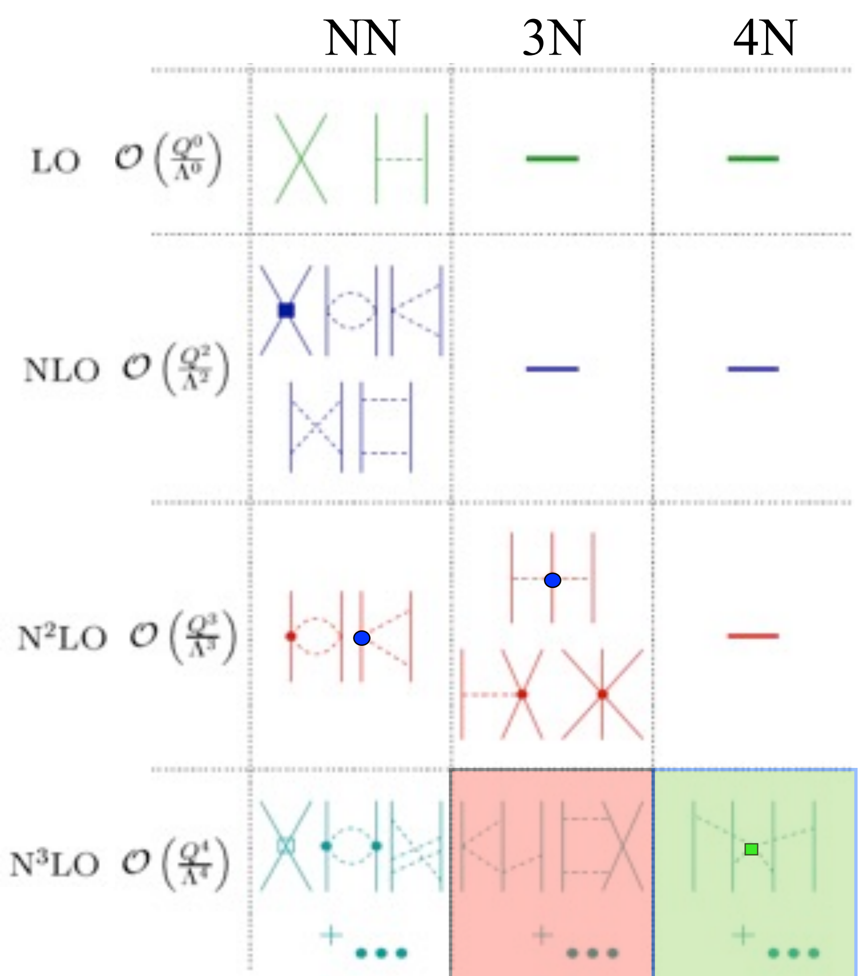

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- 4NF contributions small

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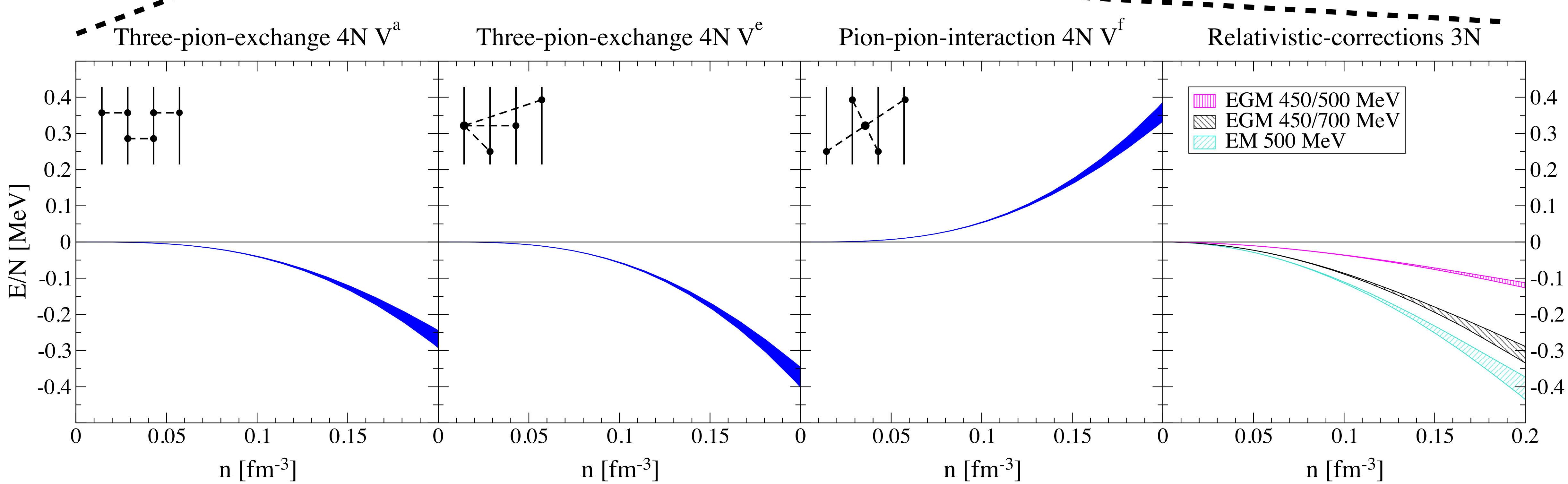
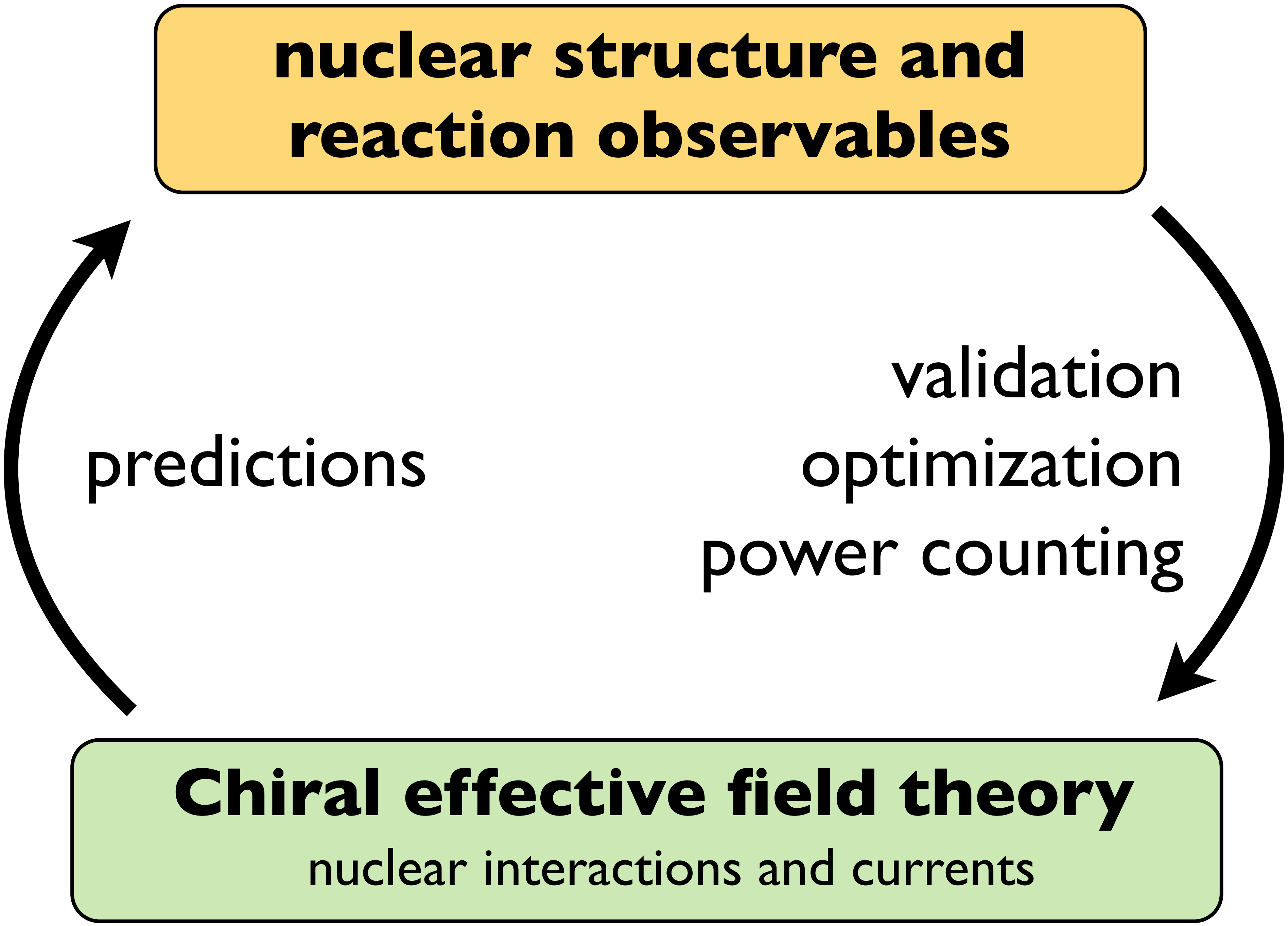
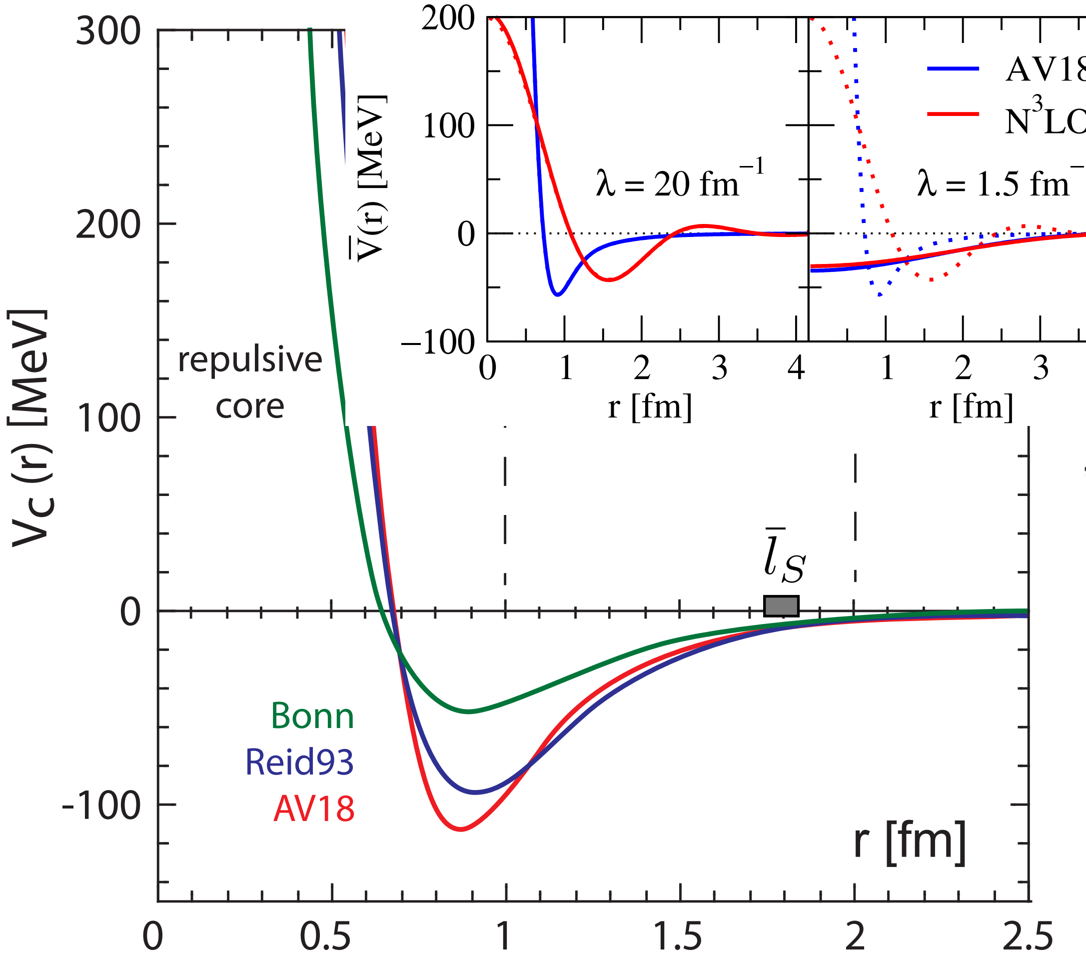
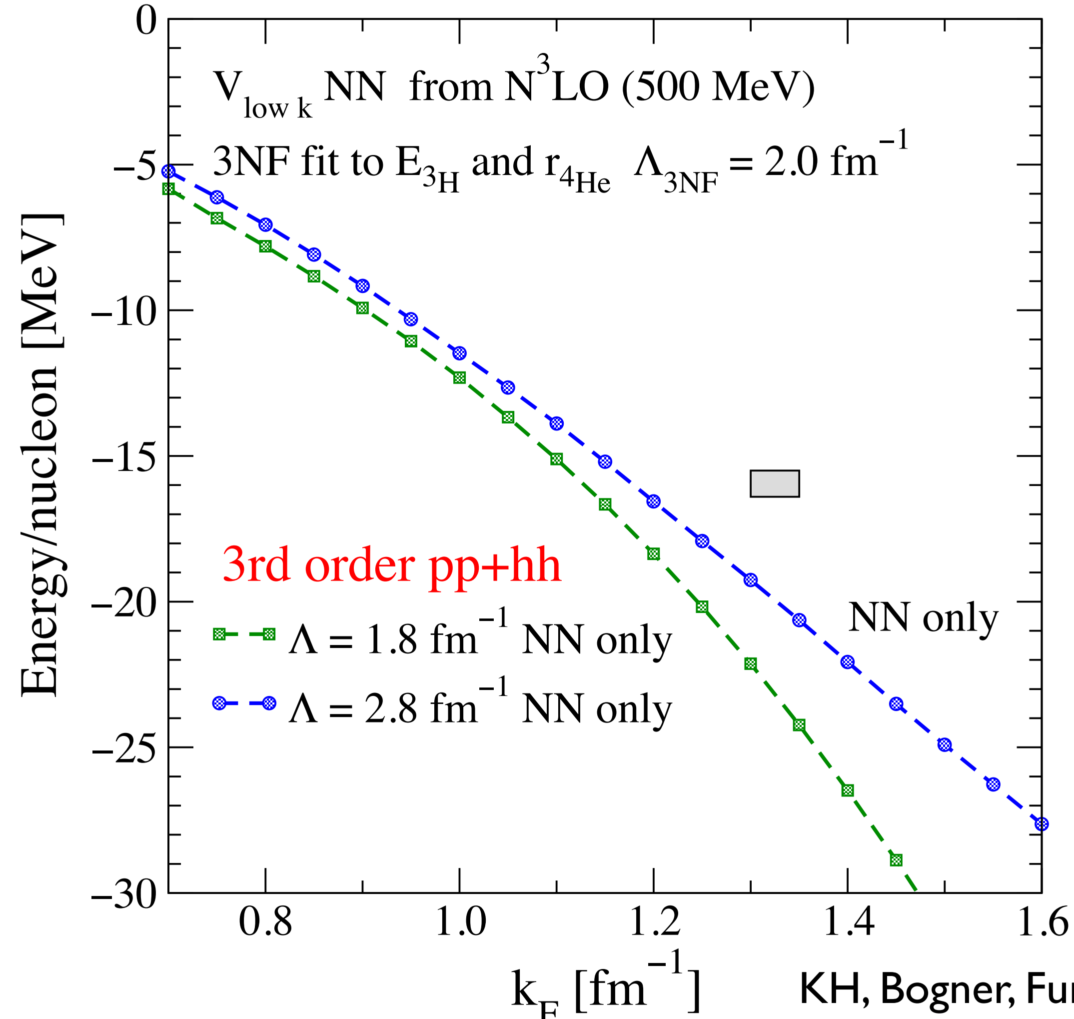


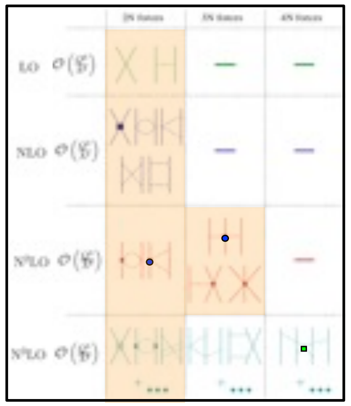
Development of nuclear interactions



Equation of state of symmetric nuclear matter

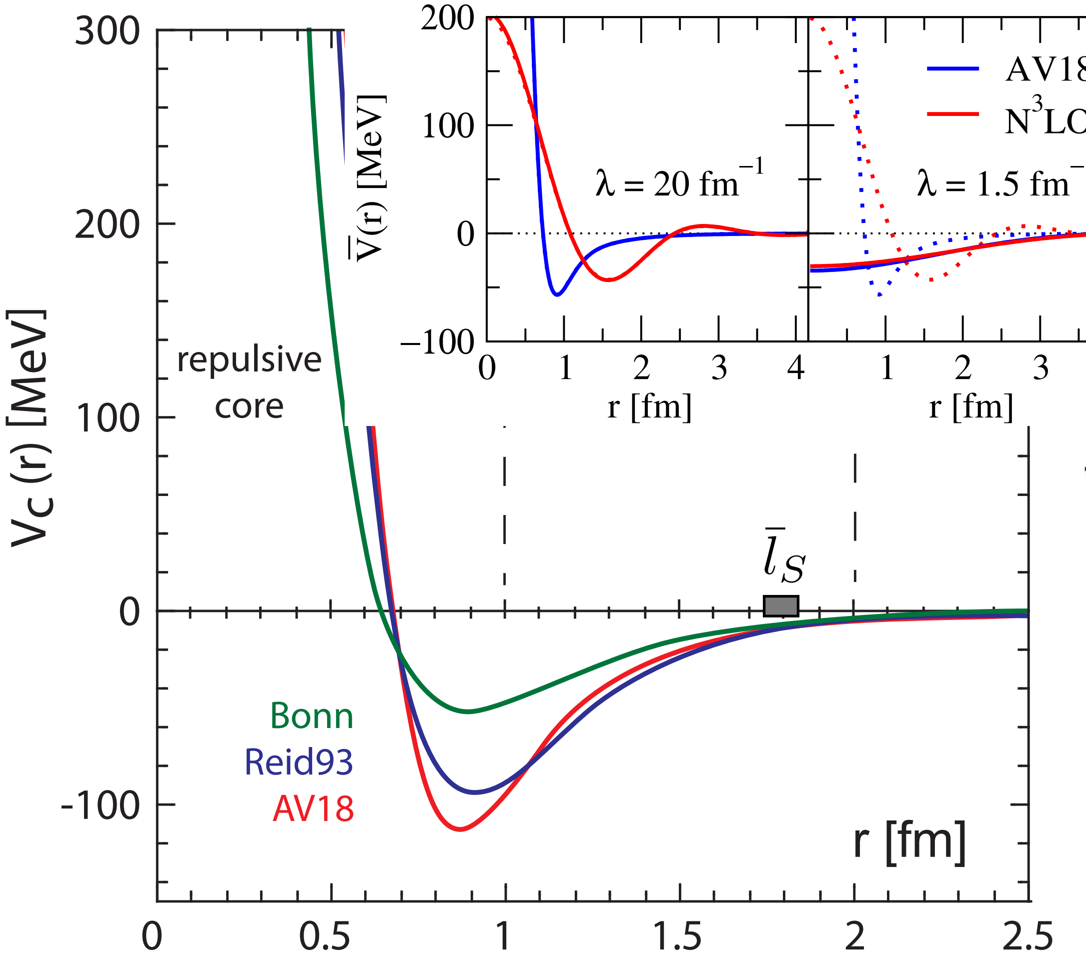


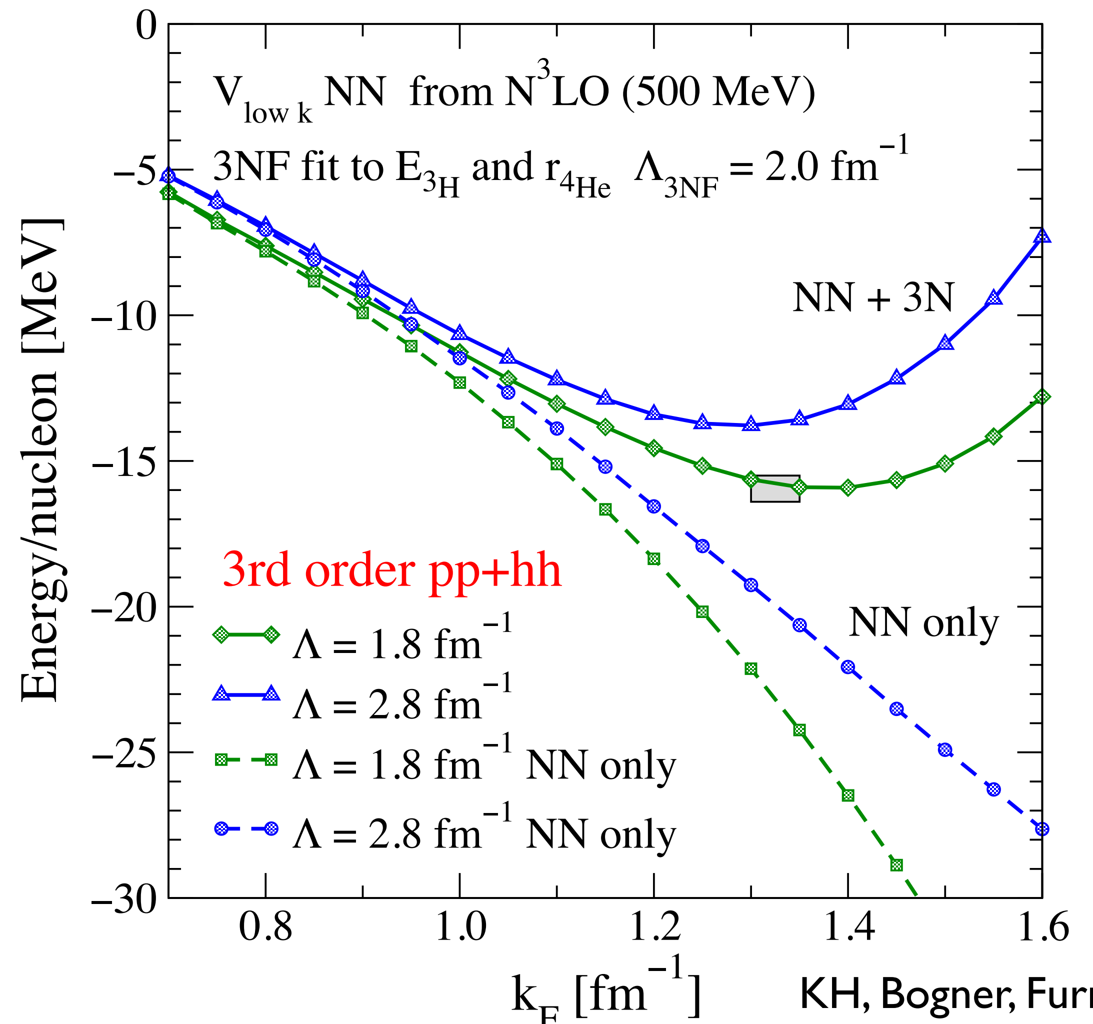




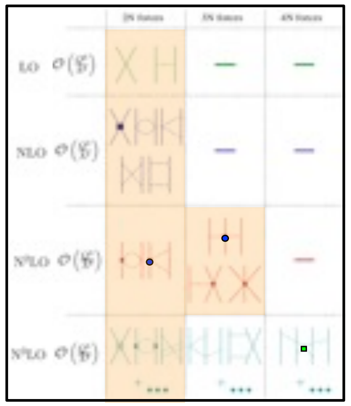
KH, Bogner, Furnstahl, Nogga,
PRC(R) 83, 031301 (2011)

Equation of state of symmetric nuclear matter



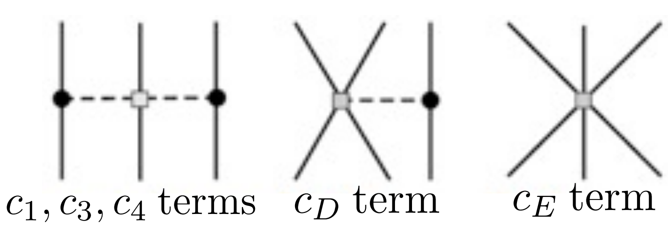


KH, Bogner, Furnstahl, Nogga,
PRC(R) 83, 031301 (2011)

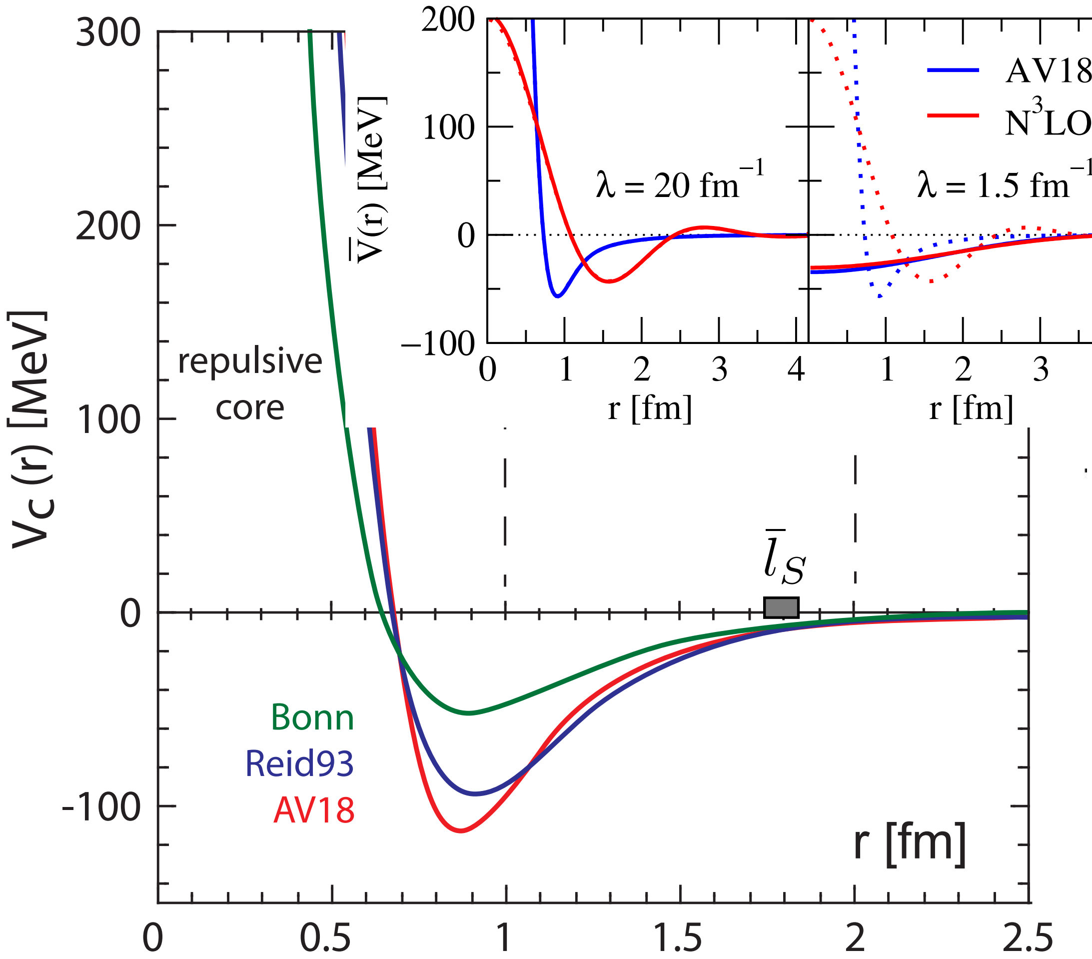


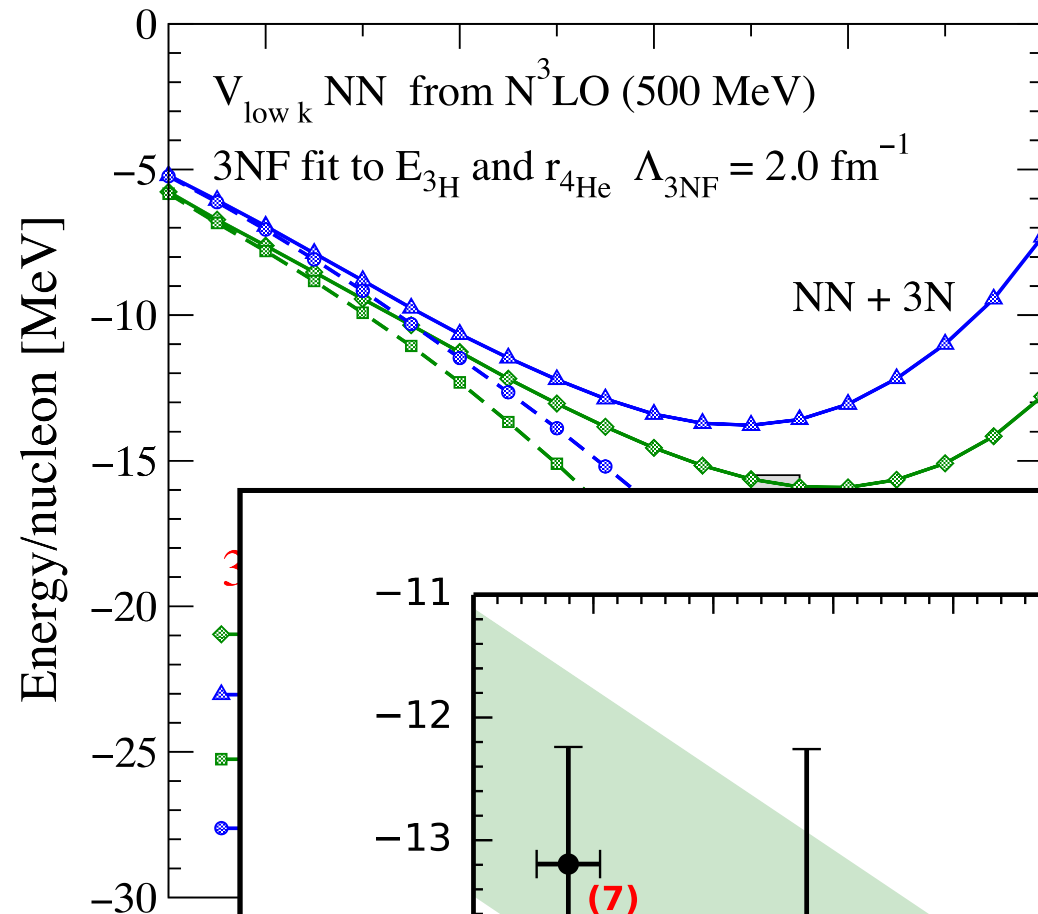
intermediate (c_D) and short-range (c_E) 3NF couplings fitted to few-body systems at different resolution scales:

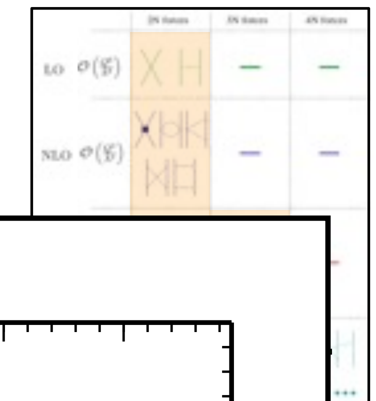
$$E_{^3\text{H}} = -8.482\,\text{MeV} \qquad r_{^4\text{He}} = 1.464\,\text{fm}$$

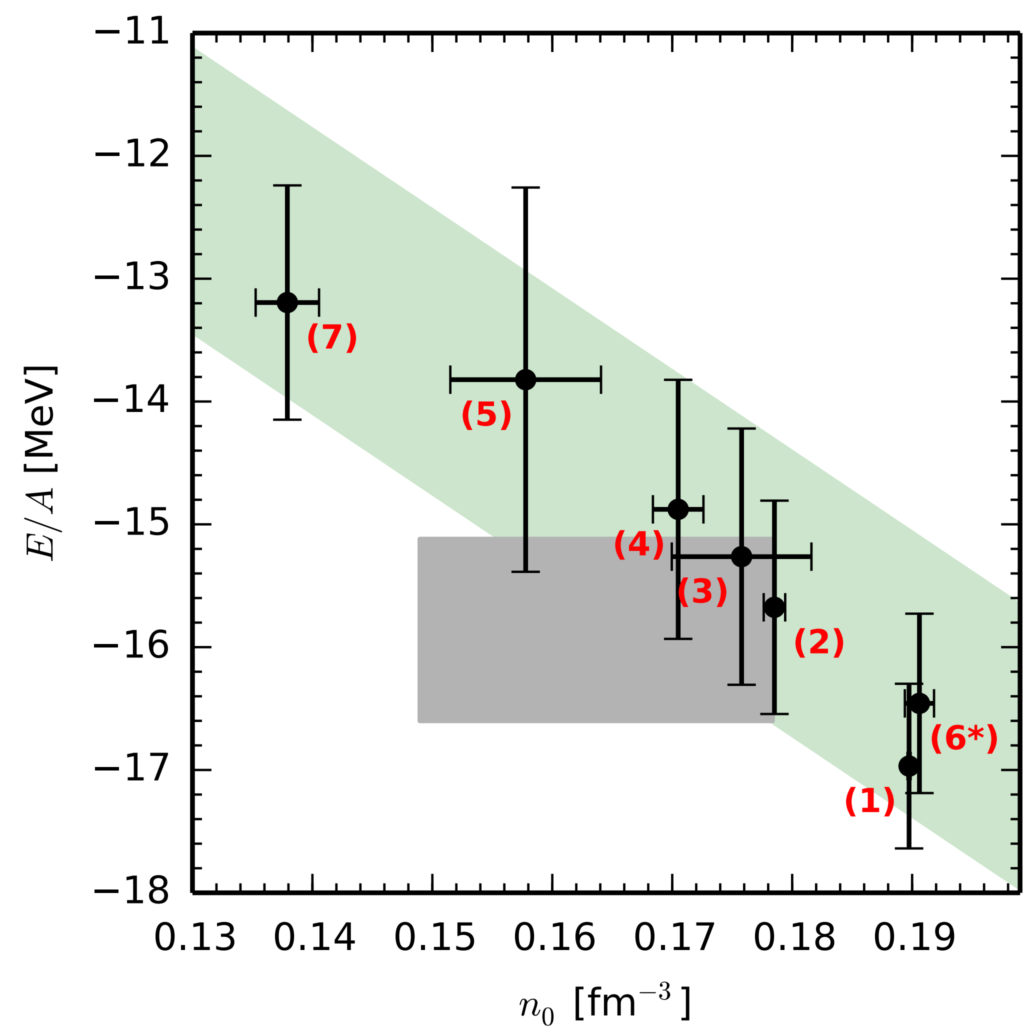


Equation of state of symmetric nuclear matter









Drischler, KH, Schwenk, PRC93, 054314 (2016)

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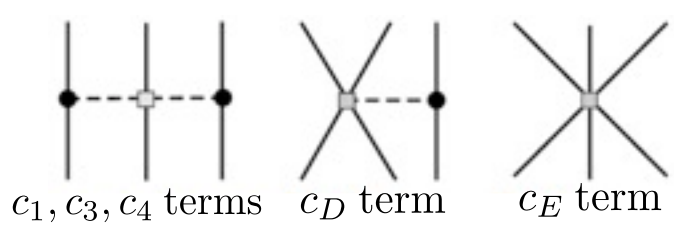
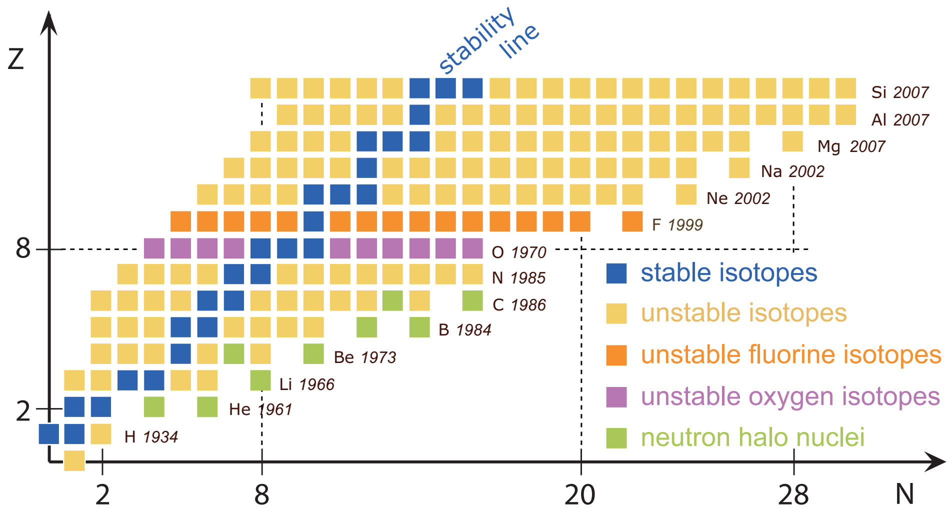
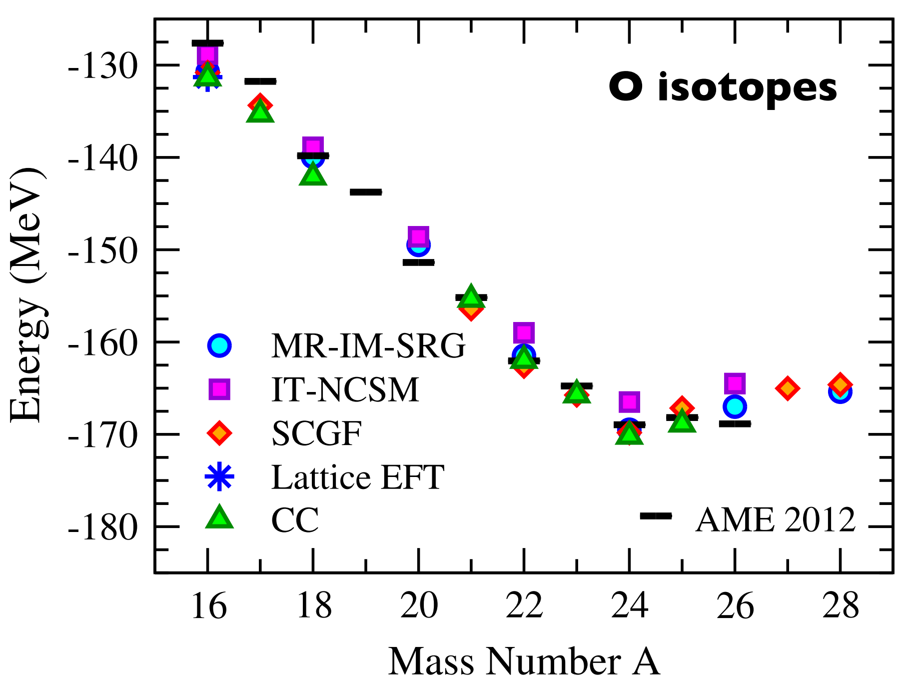
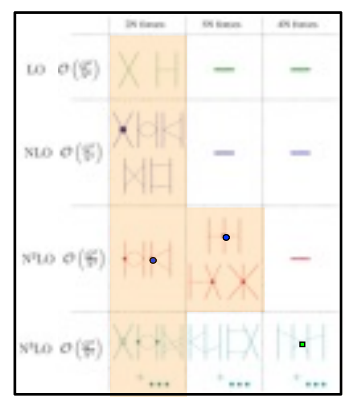


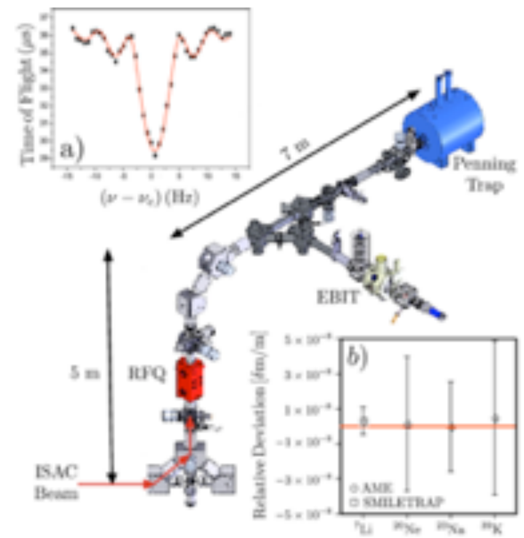
Studies of neutron-rich nuclei





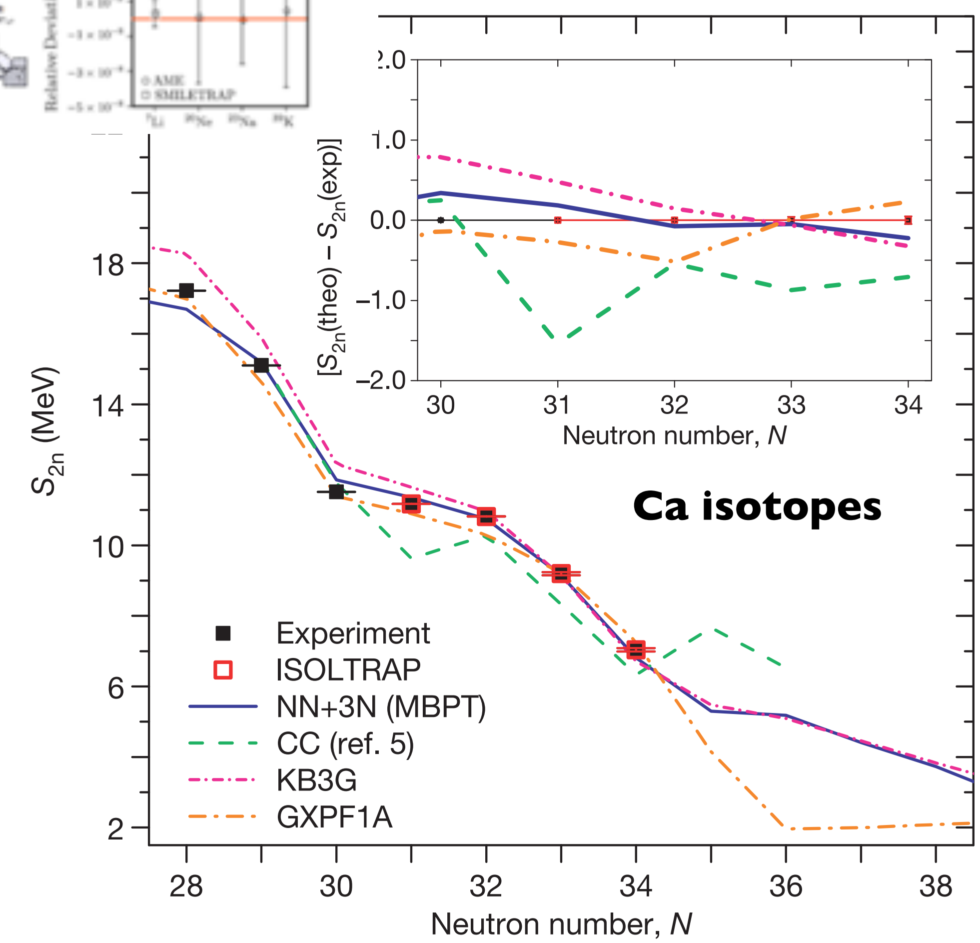


KH et al. , Ann. Rev. Nucl. Part. Sci.165, 457 (2015)



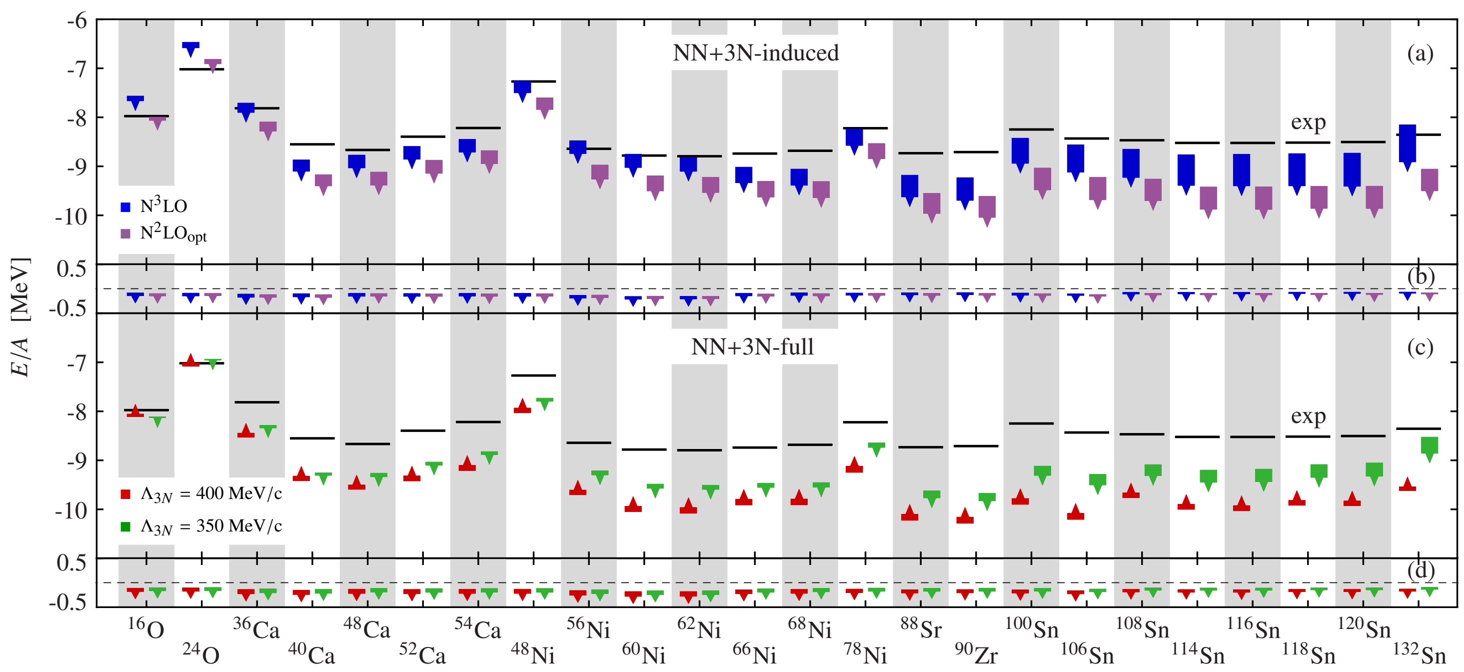
Gallant et al.
PRL 109, 032506 (2012)

Wienholtz et al.
Nature 498, 346 (2013)

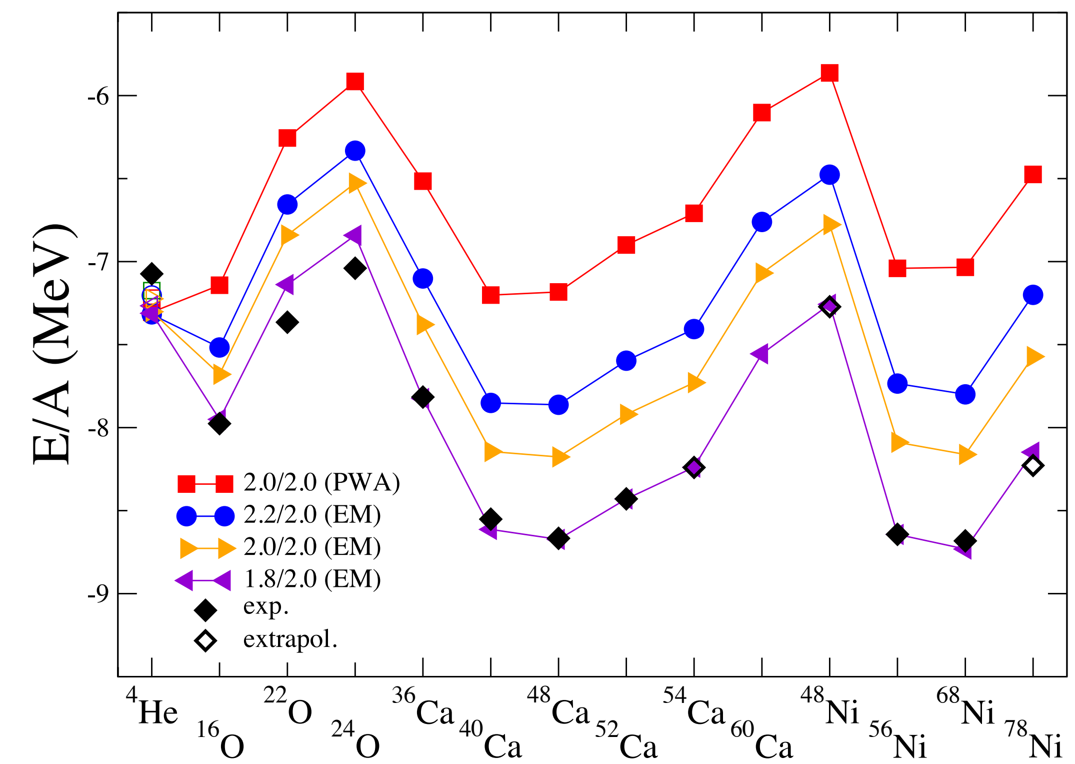


- remarkable agreement between different many-body frameworks
- excellent agreement between theory and experiment for masses of oxygen and calcium isotopes based on specific chiral interactions
- need to quantify theoretical uncertainties

Ab-initio calculations of heavy nuclei

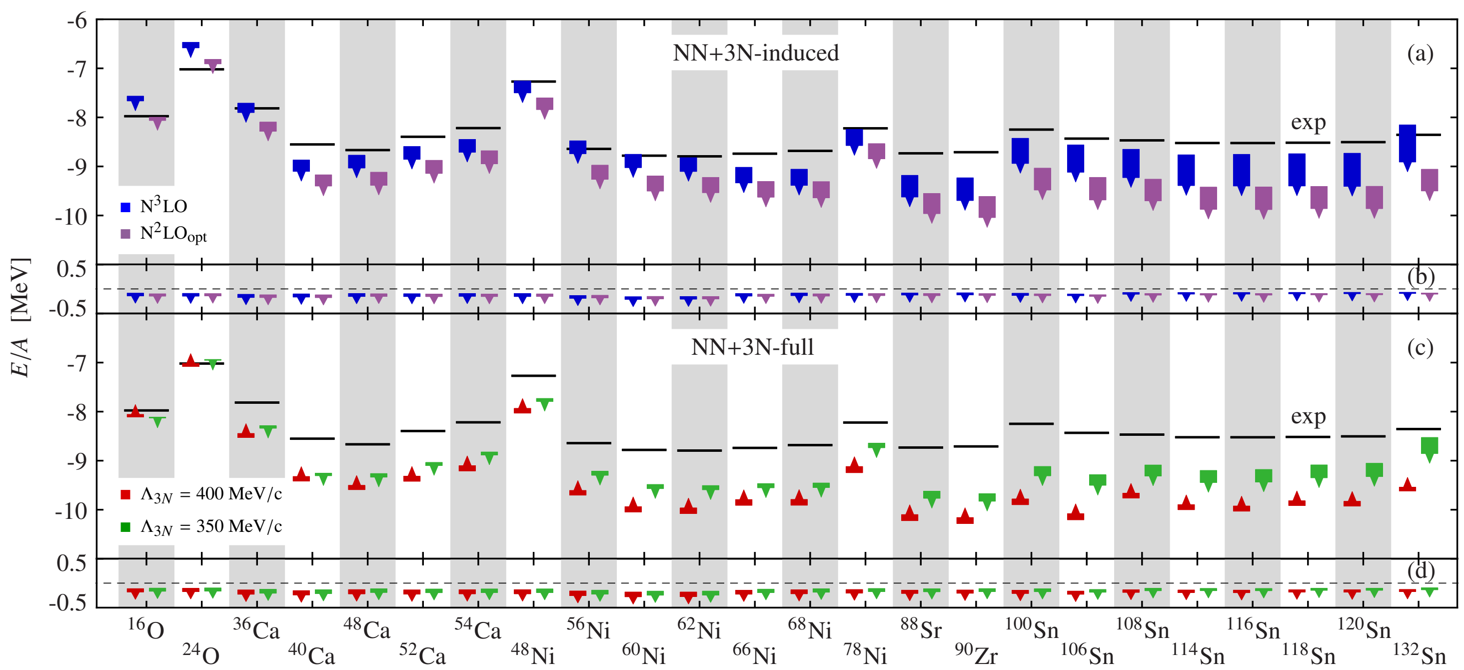


Binder et al., Phys. Lett B 736, 119 (2014)

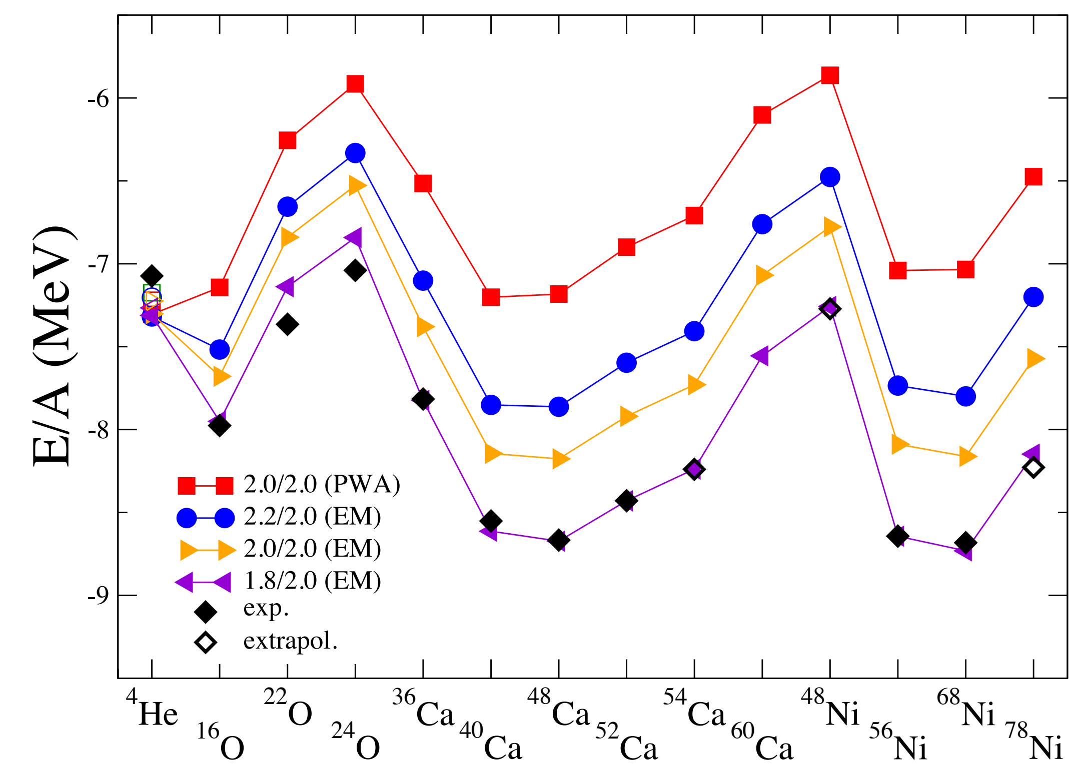


Simonis, Stroberg et al., in preparation

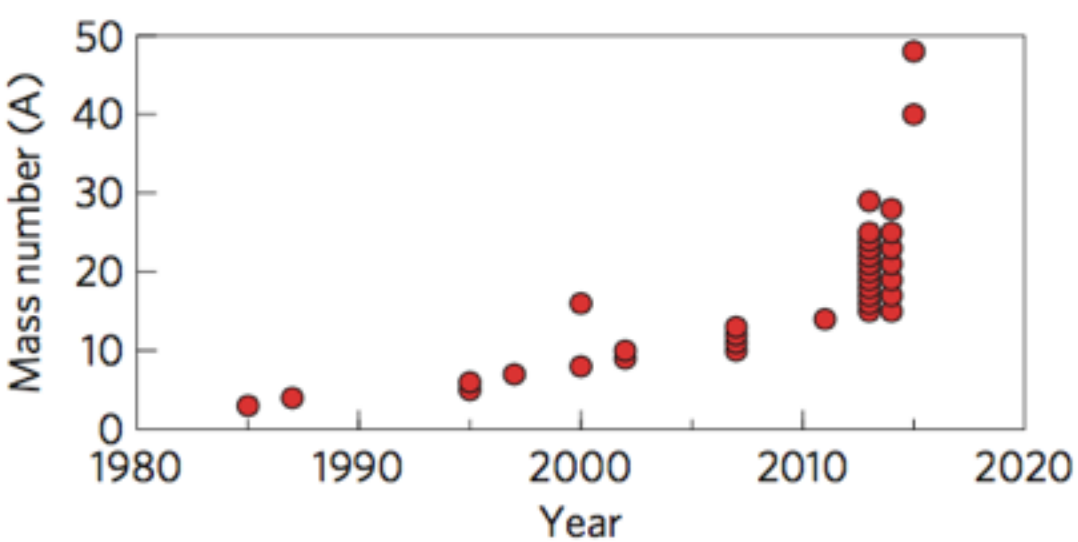
Ab-initio calculations of heavy nuclei



Binder et al., Phys. Lett B 736, 119 (2014)



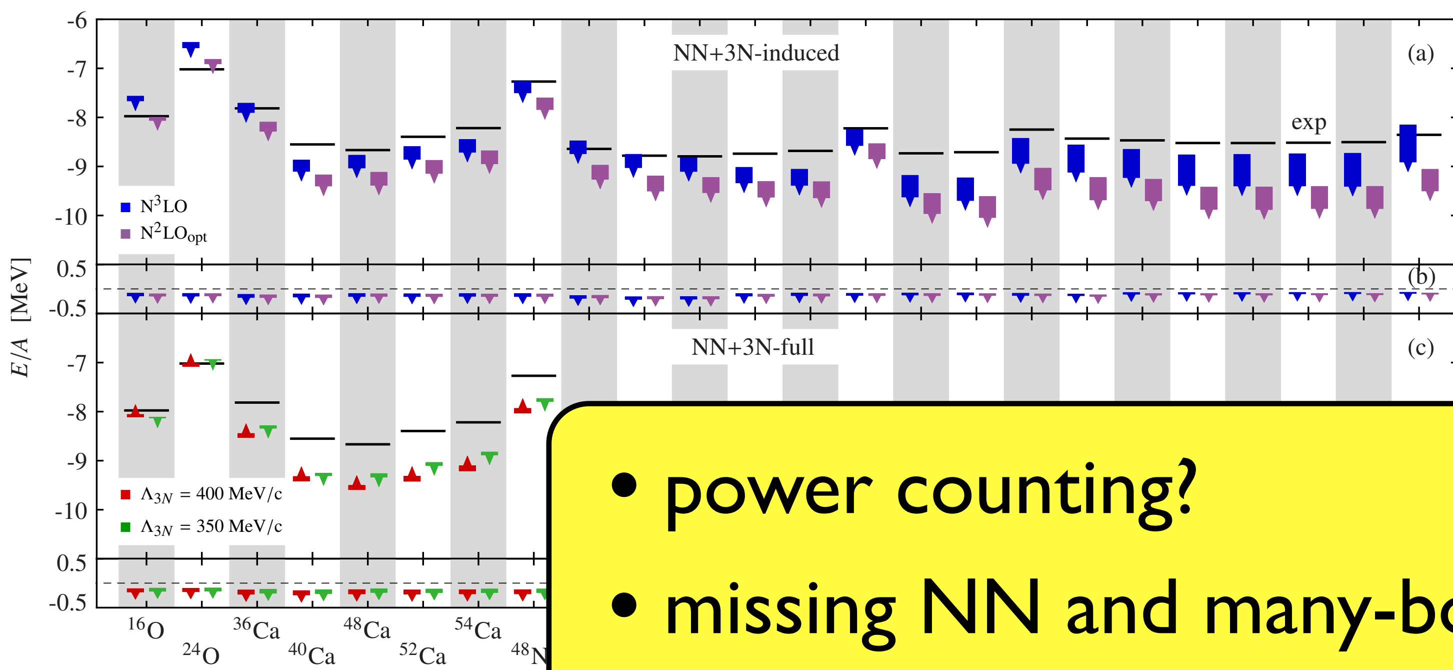
Simonis, Stroberg et al., in preparation



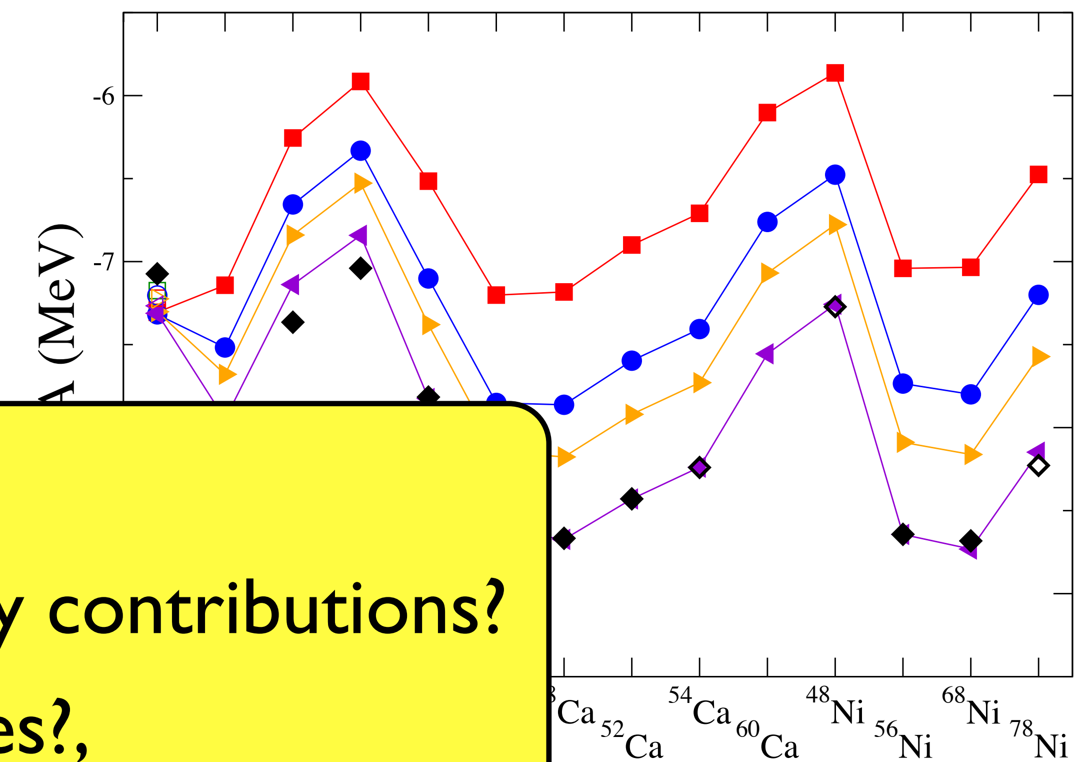
Hagen et al., Nature Physics 12, 186 (2016)

- spectacular increase in range of applicability of *ab initio* many body frameworks
- significant discrepancies to experimental data for heavy nuclei for (most of) presently used nuclear interactions

Ab-initio calculations of heavy nuclei

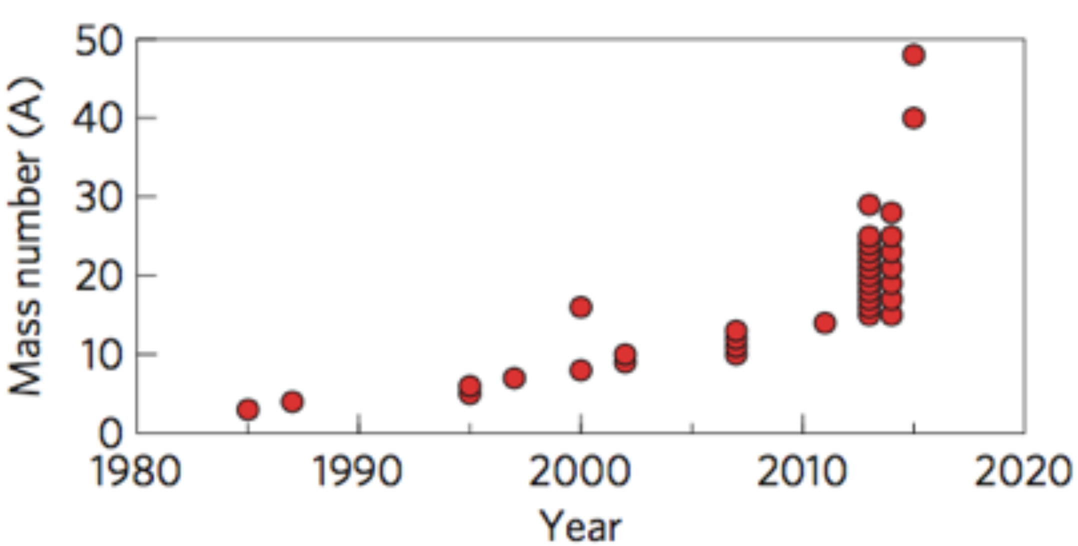


Binder et al., Phys. Lett B



., in preparation

- power counting?
- missing NN and many-body contributions?
- optimized fitting procedures?,
- selection of fitting observables?



Hagen et al., Nature Physics 12, 186 (2016)

- spectacular increase in range of applicability of *ab initio* many body frameworks
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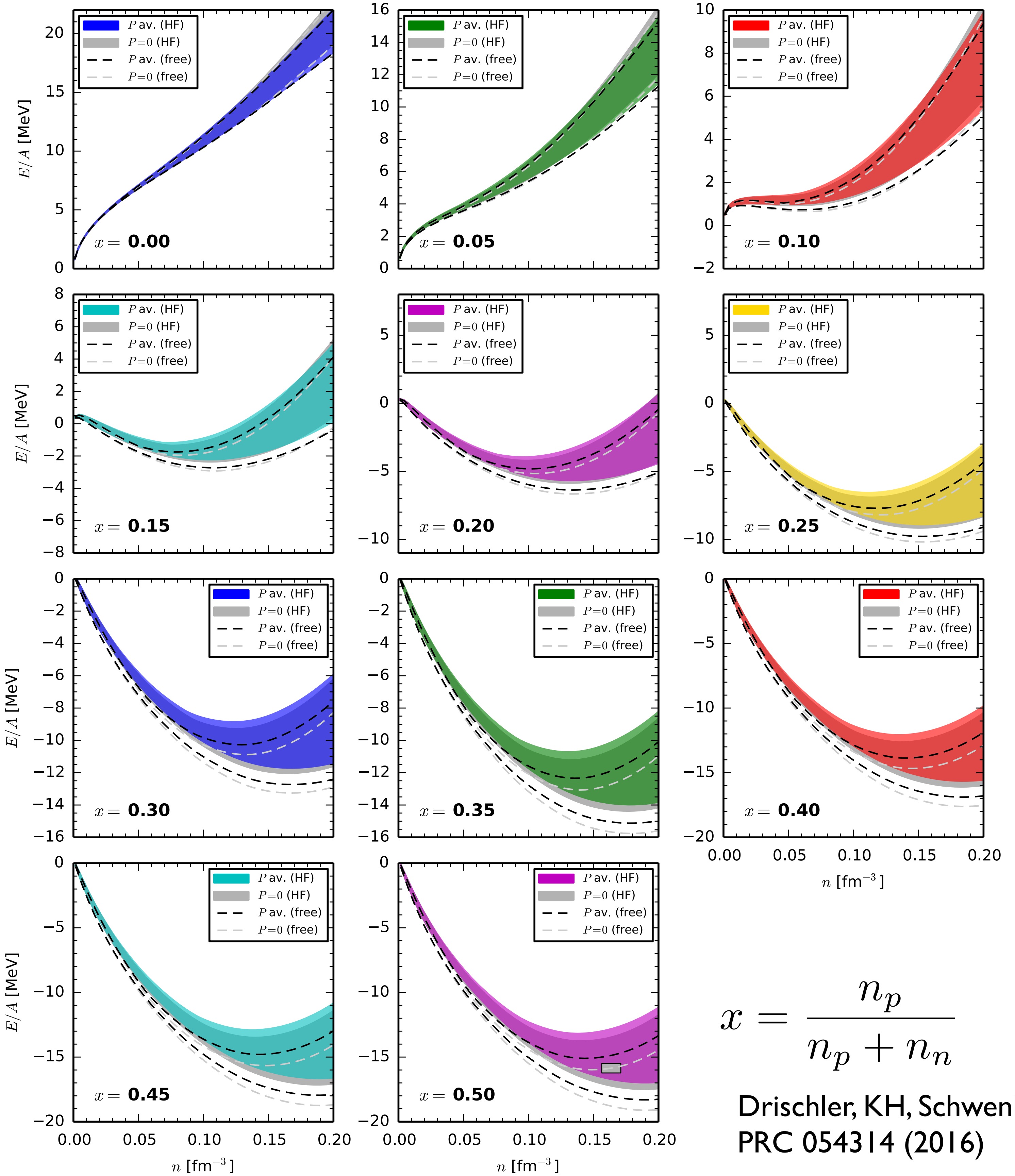
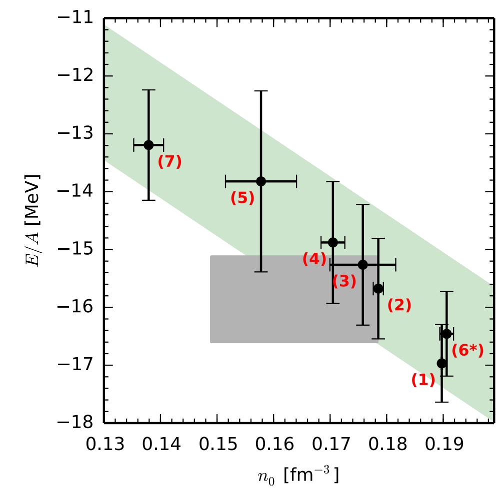
First application to isospin asymmetric nuclear matter



$$x = \frac{n_p}{n_p + n_n}$$

Drischler, KH, Schwenk, PRC 054314 (2016)

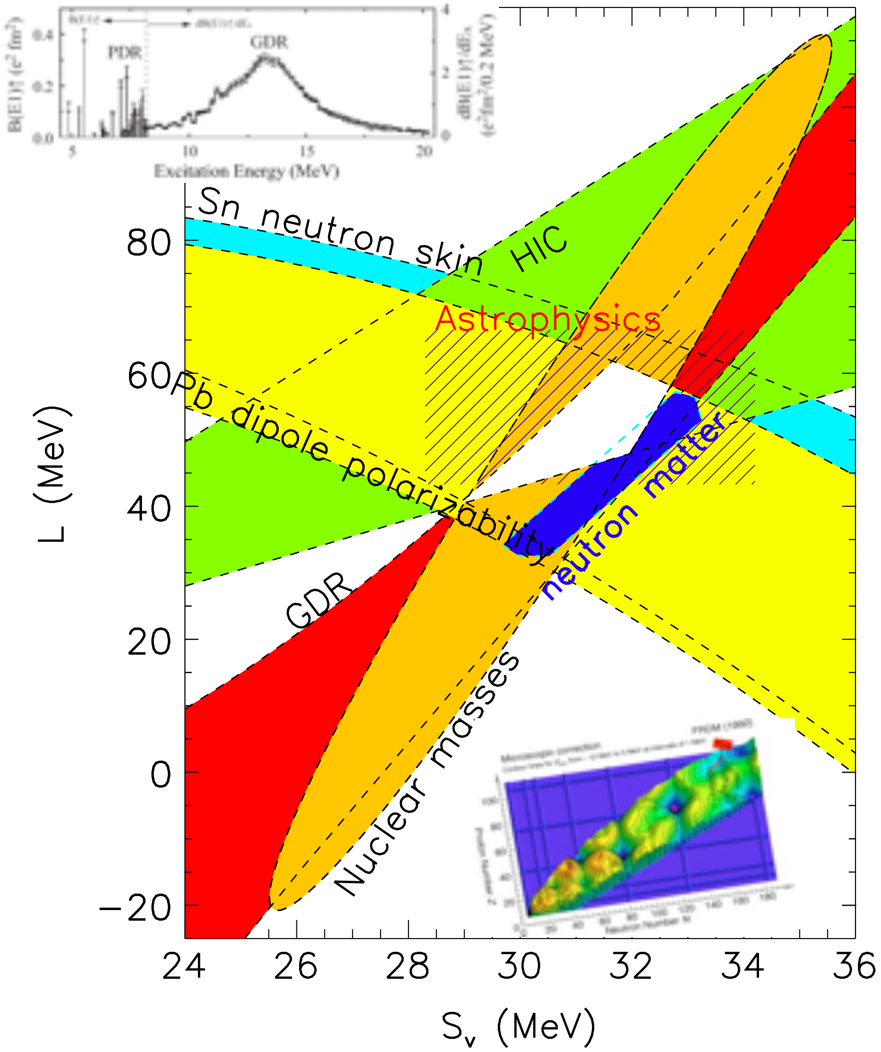
- uncertainty bands determined by set of 7 Hamiltonians



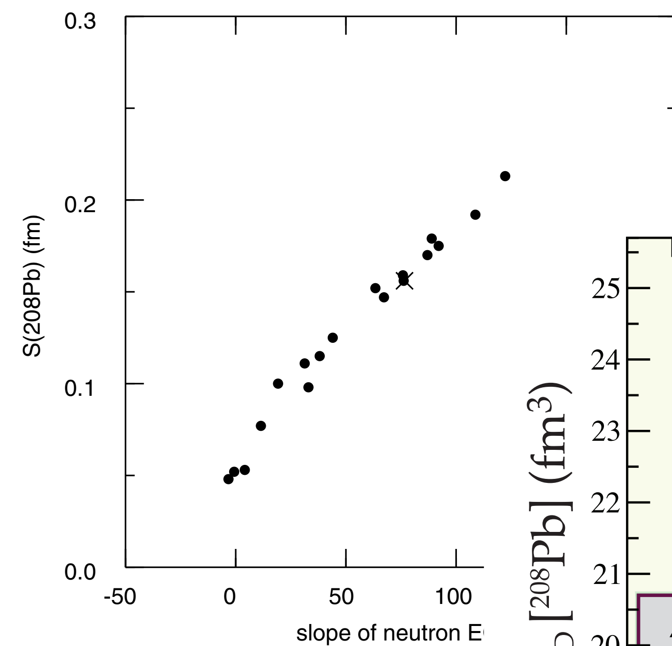
- many-body framework allows treatment of any decomposed 3N interaction

- generalization to finite temperature work in progress

Symmetry energy and neutron skin constraints

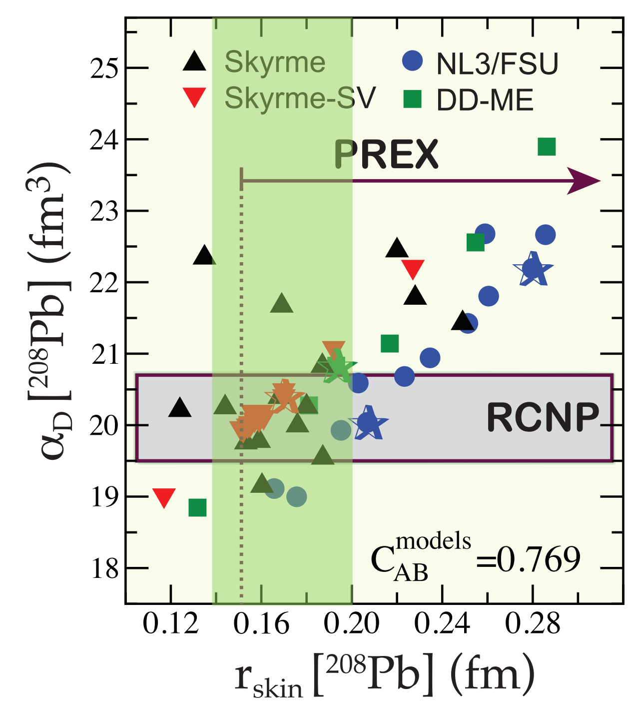


KH, Lattimer, Pethick, Schwenk, ApJ 773,11 (2013)



Brown,
PRL 85, 5296 (2000)

Piekarewicz,
PRC 85, 041302 (2012)



$$S_v = \left.\frac{\partial^2 E/N}{\partial^2 x}\right|_{\rho=\rho_0, x=1/2}$$

$$L = \frac{3}{8}\left.\frac{\partial^3 E/N}{\partial\rho\partial^2 x}\right|_{\rho=\rho_0, x=1/2}$$

constraint from neutron matter results:

$$r_{\rm skin}[^{208}{\rm Pb}] = 0.14 - 0.2 \text{ fm}$$

KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

- neutron matter give tightest constraints
- in agreement with all other constraints

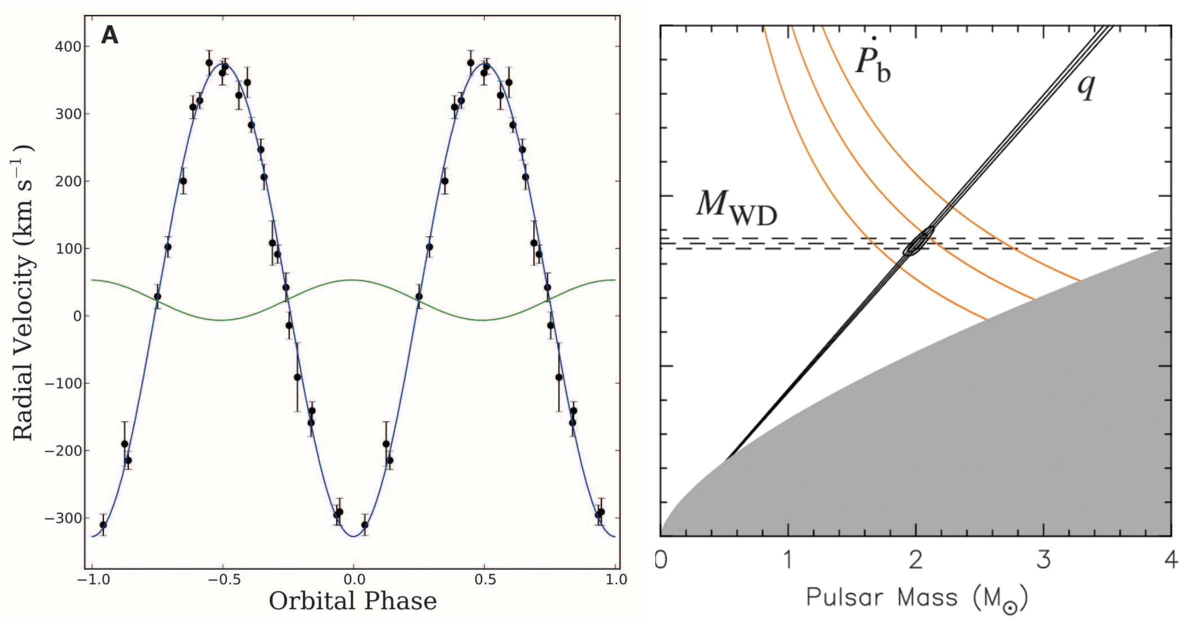
current constraints from PREX:

$$r_{\rm skin}[^{208}{\rm Pb}] = 0.15 - 0.49 \text{ fm}$$

Abrahamyan et al., PRL 108, 112502 (2012)

Constraints on the nuclear equation of state (EOS)

Science A Massive Pulsar in a Compact Relativistic Binary

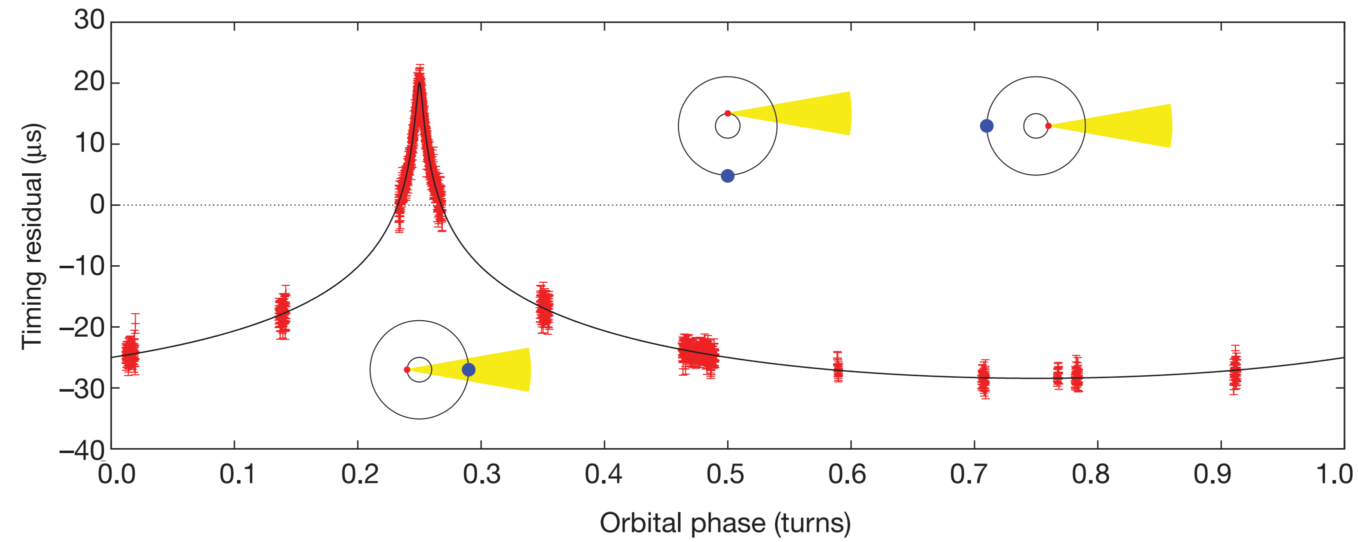


Antoniadis et al., Science 340, 448 (2013)

nature

A two-solar-mass neutron star measured using Shapiro delay

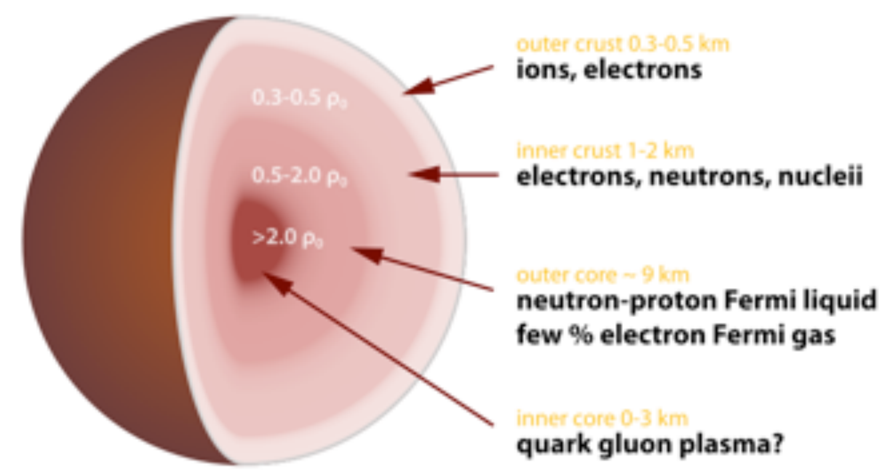
P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]



Demorest et al., Nature 467, 1081 (2010)

High-density constraints from observations:

$$M_{\max} = 1.65 M_\odot \rightarrow 1.97 \pm 0.04\, M_\odot$$
$$\rightarrow 2.01 \pm 0.04\, M_\odot$$



Calculation of neutron star properties require EOS up to high densities.

Strategy:

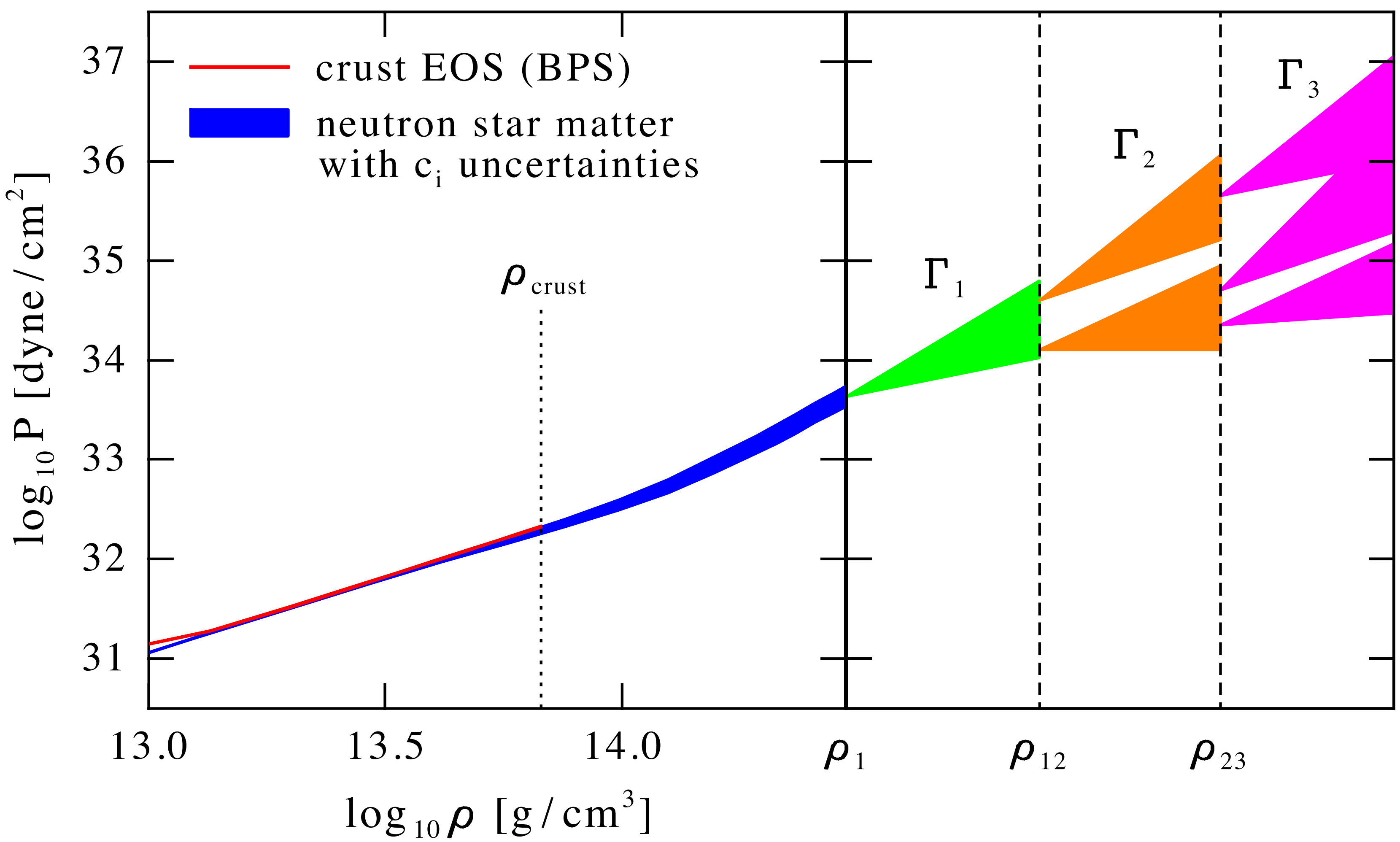
Use observations to constrain the high-density part of the nuclear EOS.

Neutron star radius constraints

incorporation of beta-equilibrium: neutron matter $\longrightarrow$ neutron star matter

parametrize piecewise high-density extensions of EOS:

- use polytropic ansatz $p \sim \rho^{\Gamma}$
- range of parameters $\Gamma_1, \rho_{12}, \Gamma_2, \rho_{23}, \Gamma_3$ limited by physics



KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)
KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

Constraints on the nuclear equation of state

use the constraints:

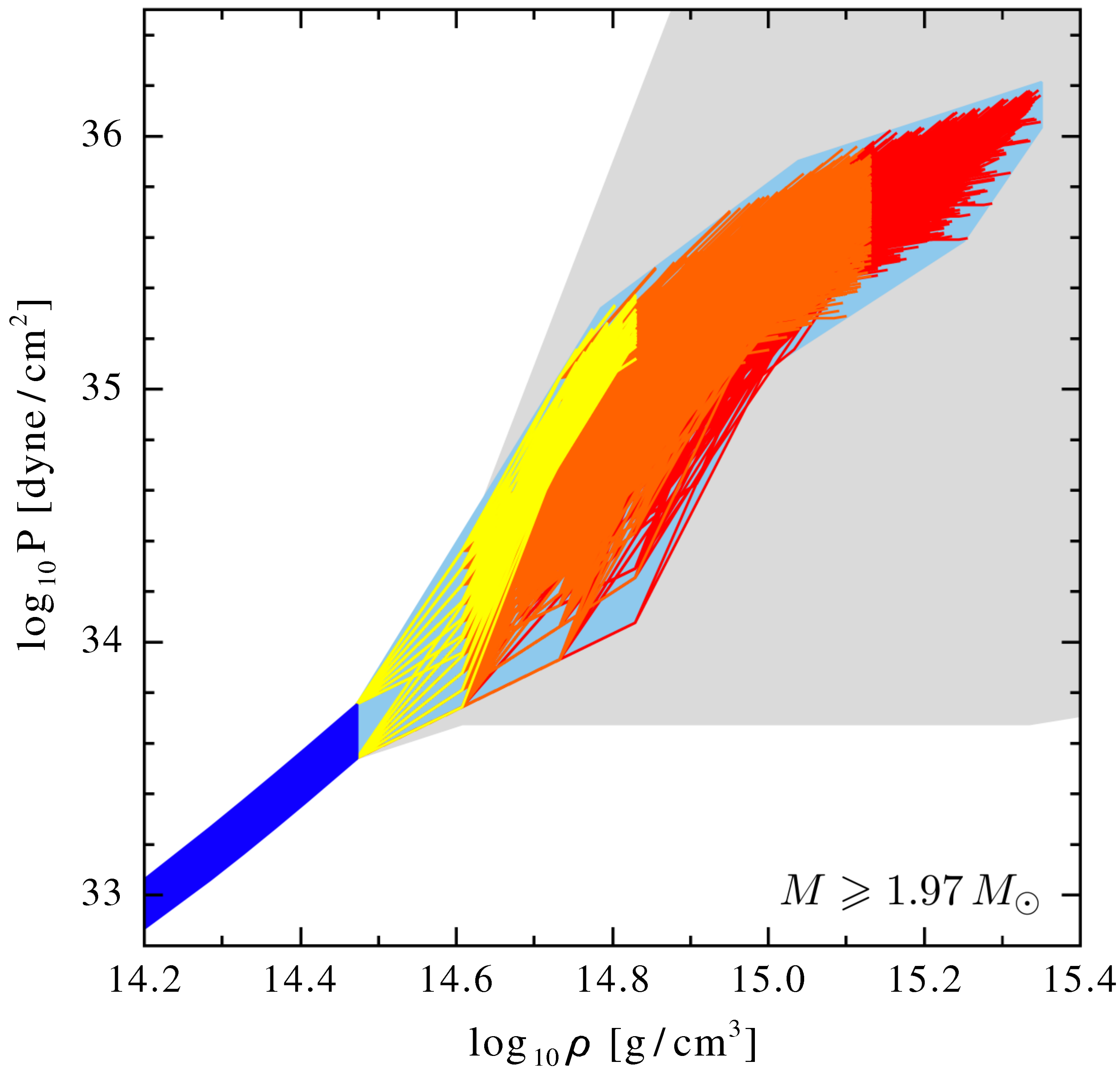
recent NS observations

$$M_{\max} > 1.97\, M_\odot$$

causality

$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)



constraints lead to significant reduction of EOS uncertainty band

Constraints on the nuclear equation of state

use the constraints:

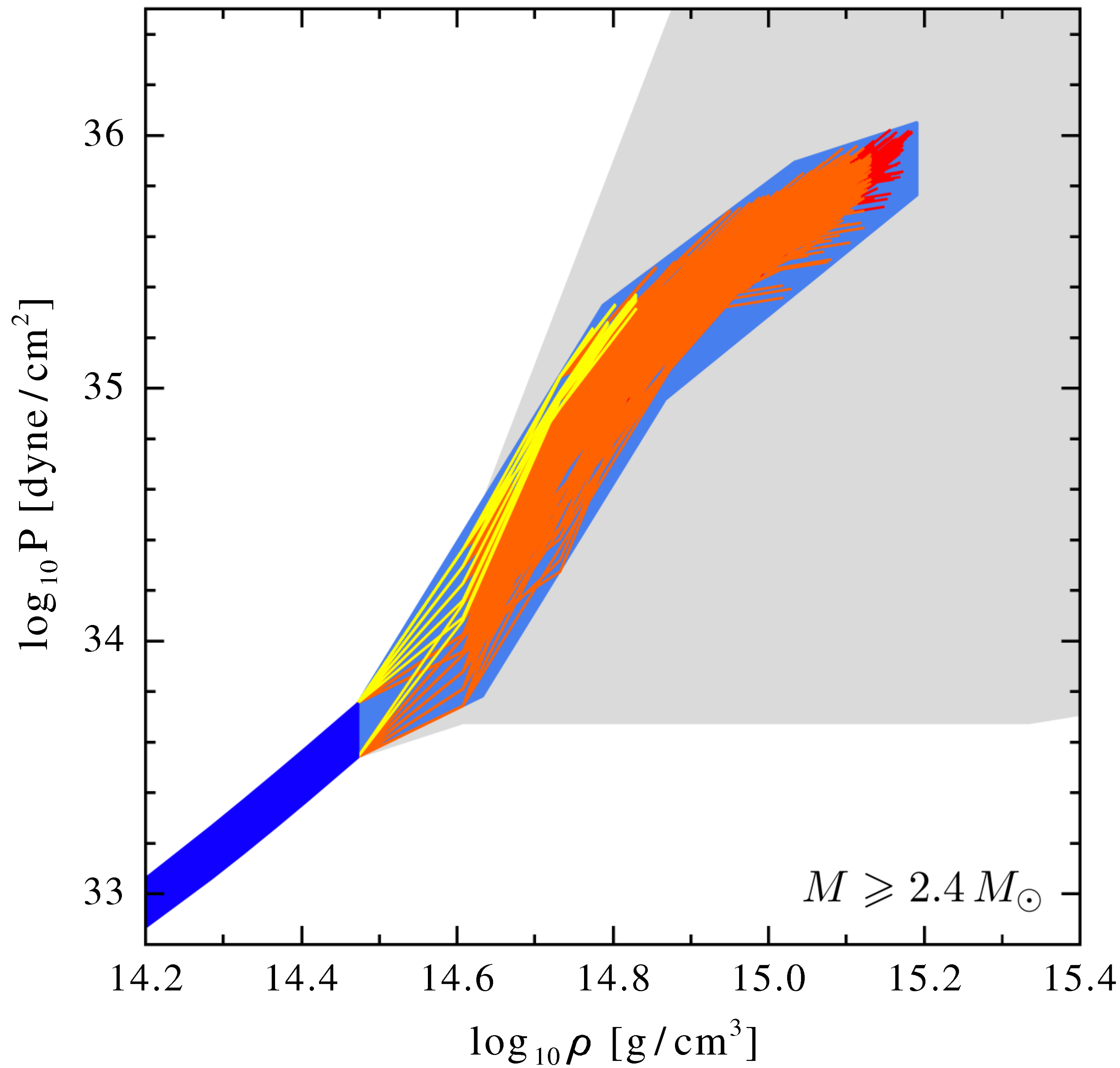
fictitious NS mass

$$M_{\max} > 2.4\, M_{\odot}$$

causality

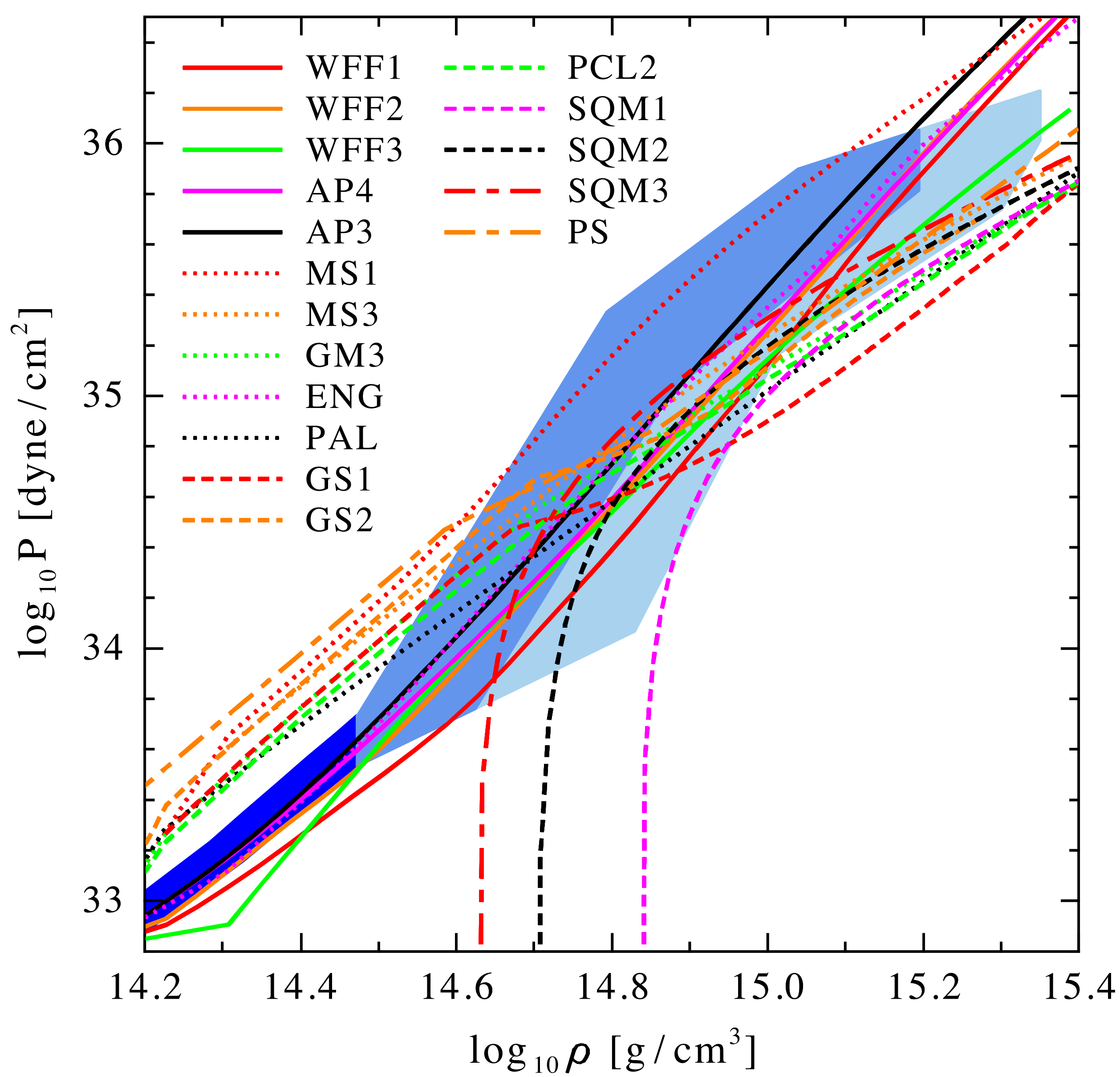
$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

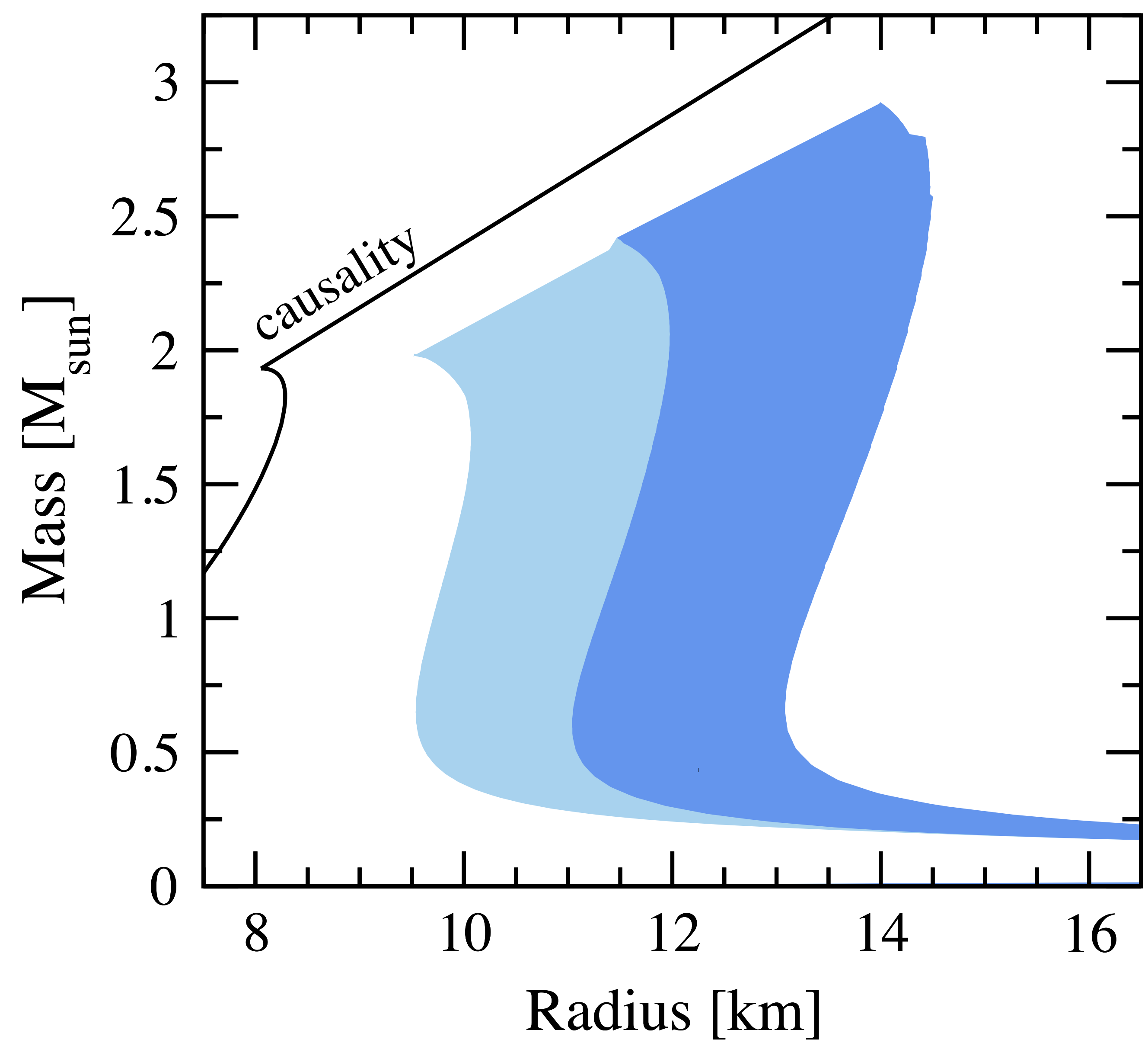
KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)



increased $M_{\max}$ systematically reduces width of band

Constraints on neutron star radii

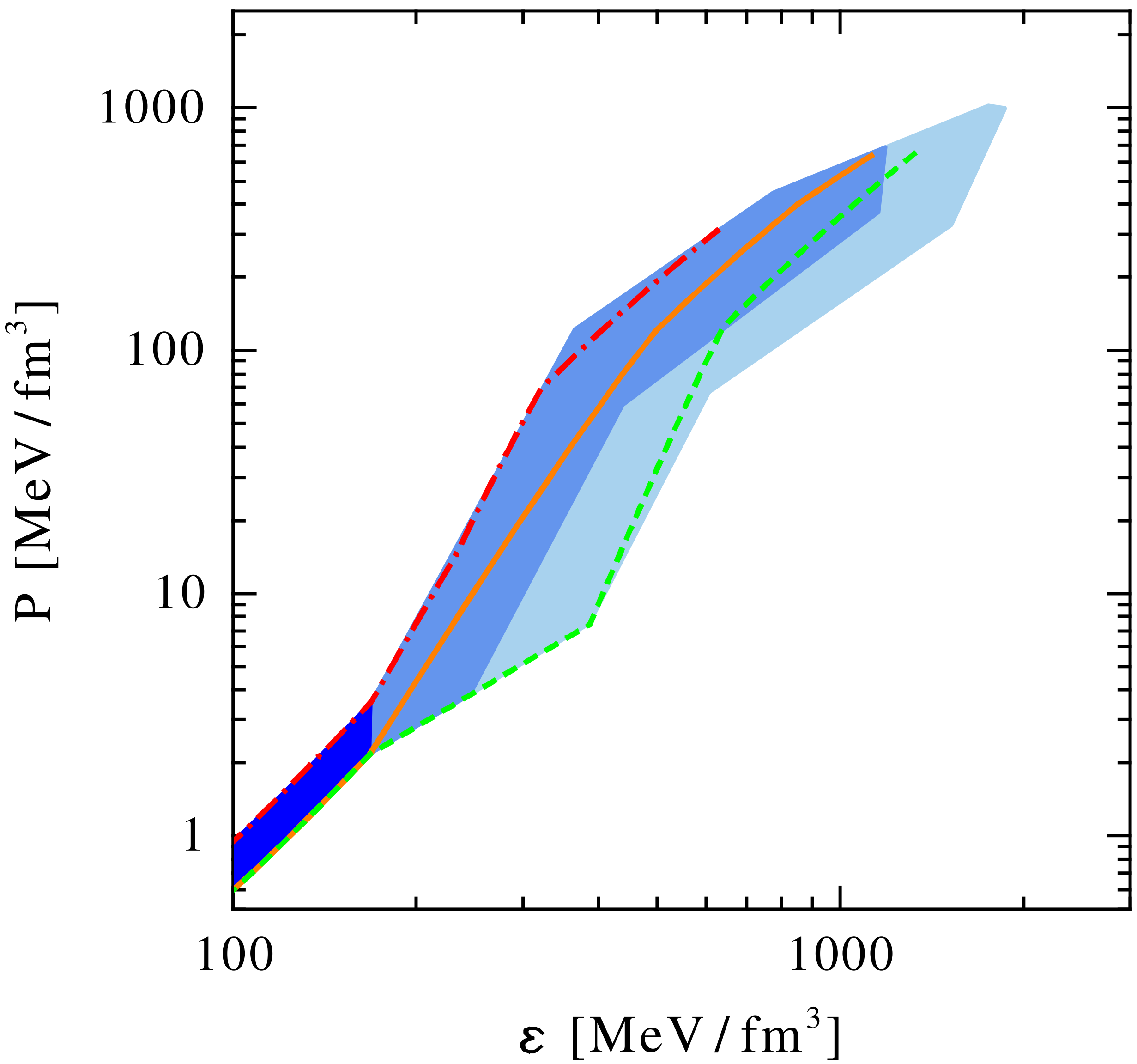


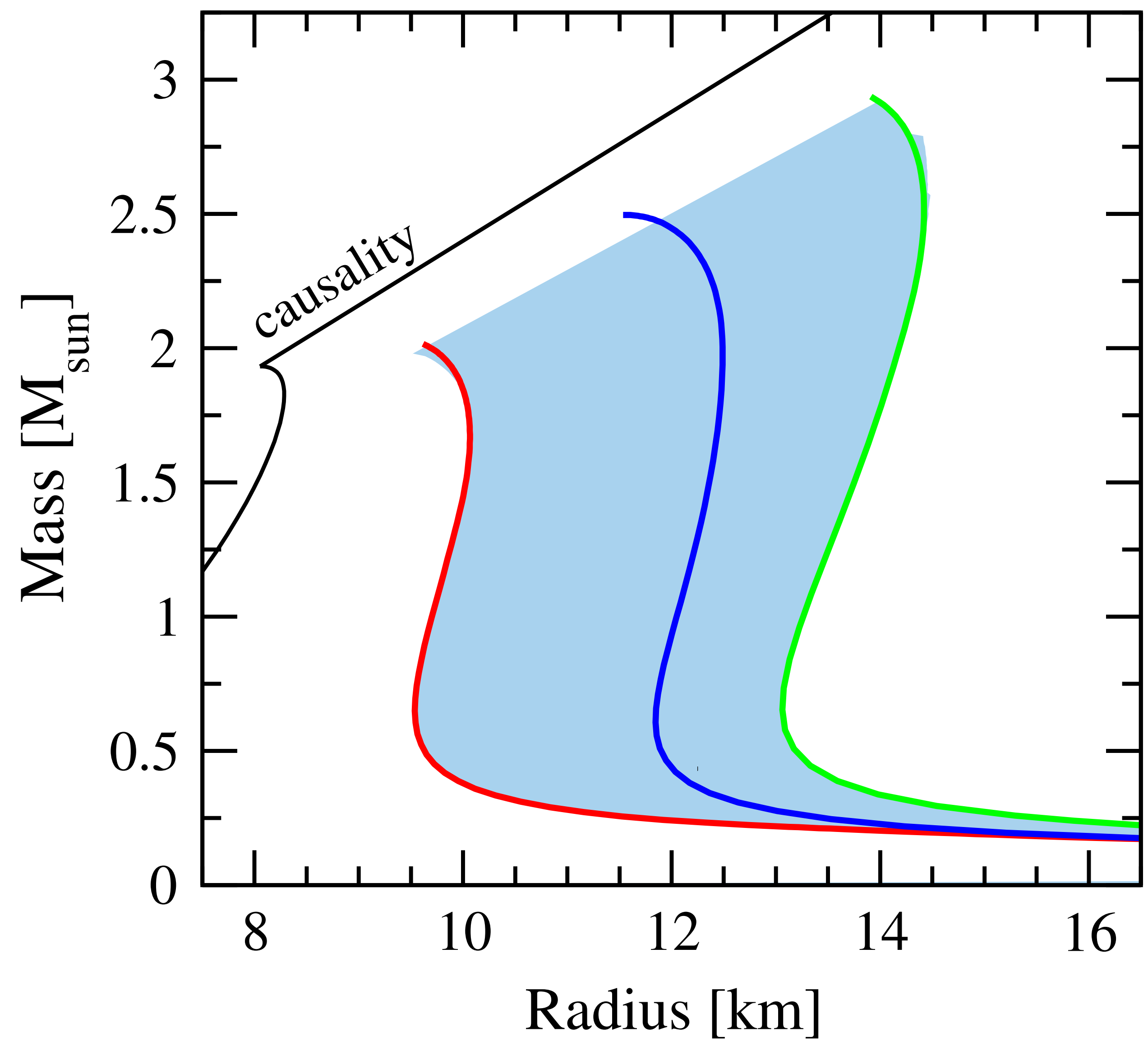


KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)
KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

- low-density part of EOS sets scale for allowed high-density extensions
- current radius prediction for typical $1.4\, M_\odot$ neutron star: $9.7 - 13.9\,\mathrm{km}$

Representative set of EOS

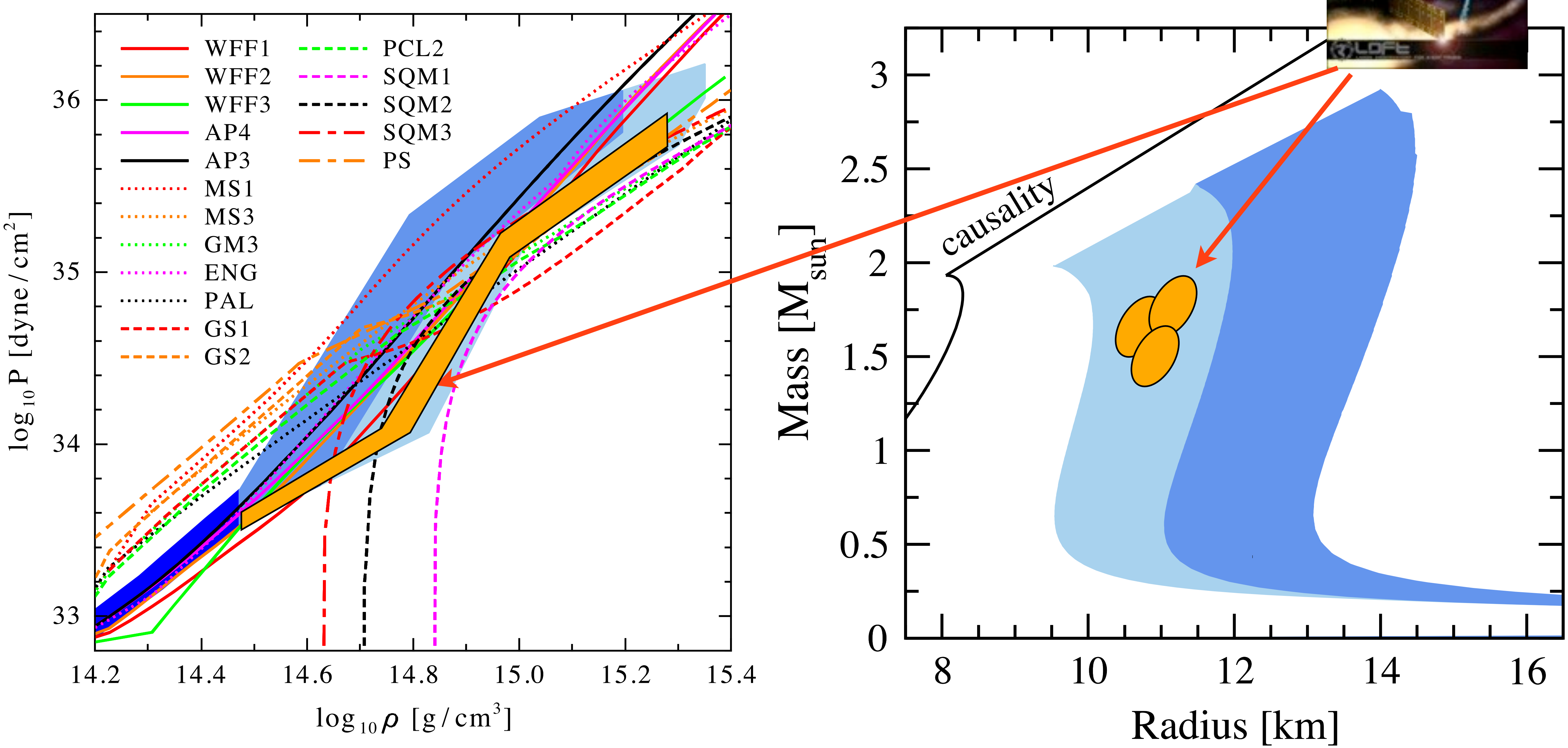




KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)

- constructed 3 representative EOS compatible with uncertainty bands for astrophysical applications: soft, intermediate and stiff
- allows to probe impact of current theoretical EOS uncertainties on astrophysical observables

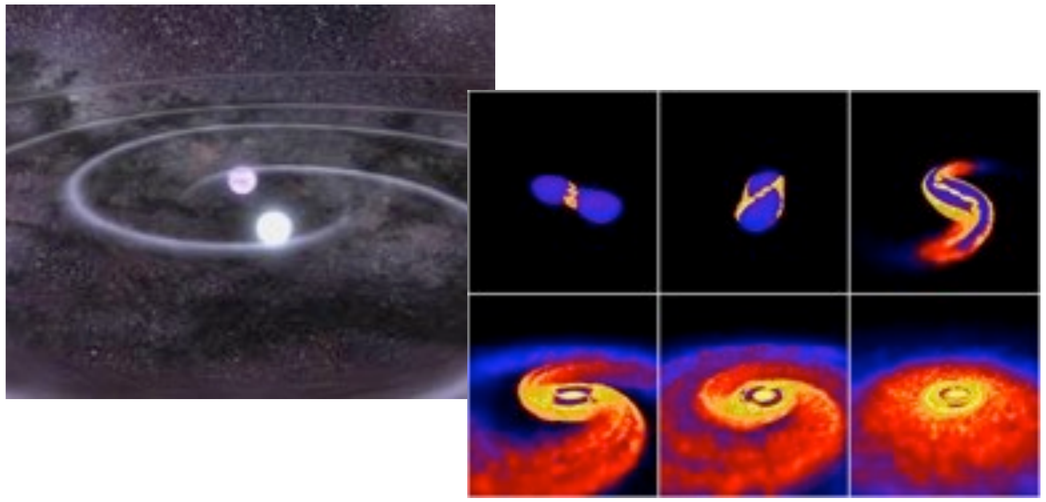
Constraints on EOS from M-R measurements

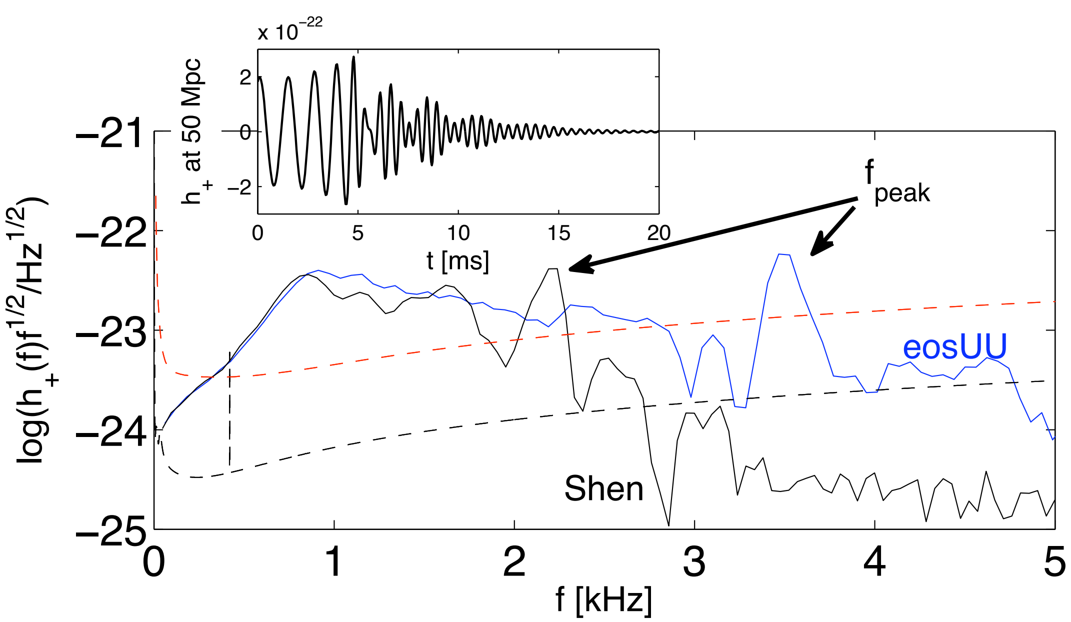


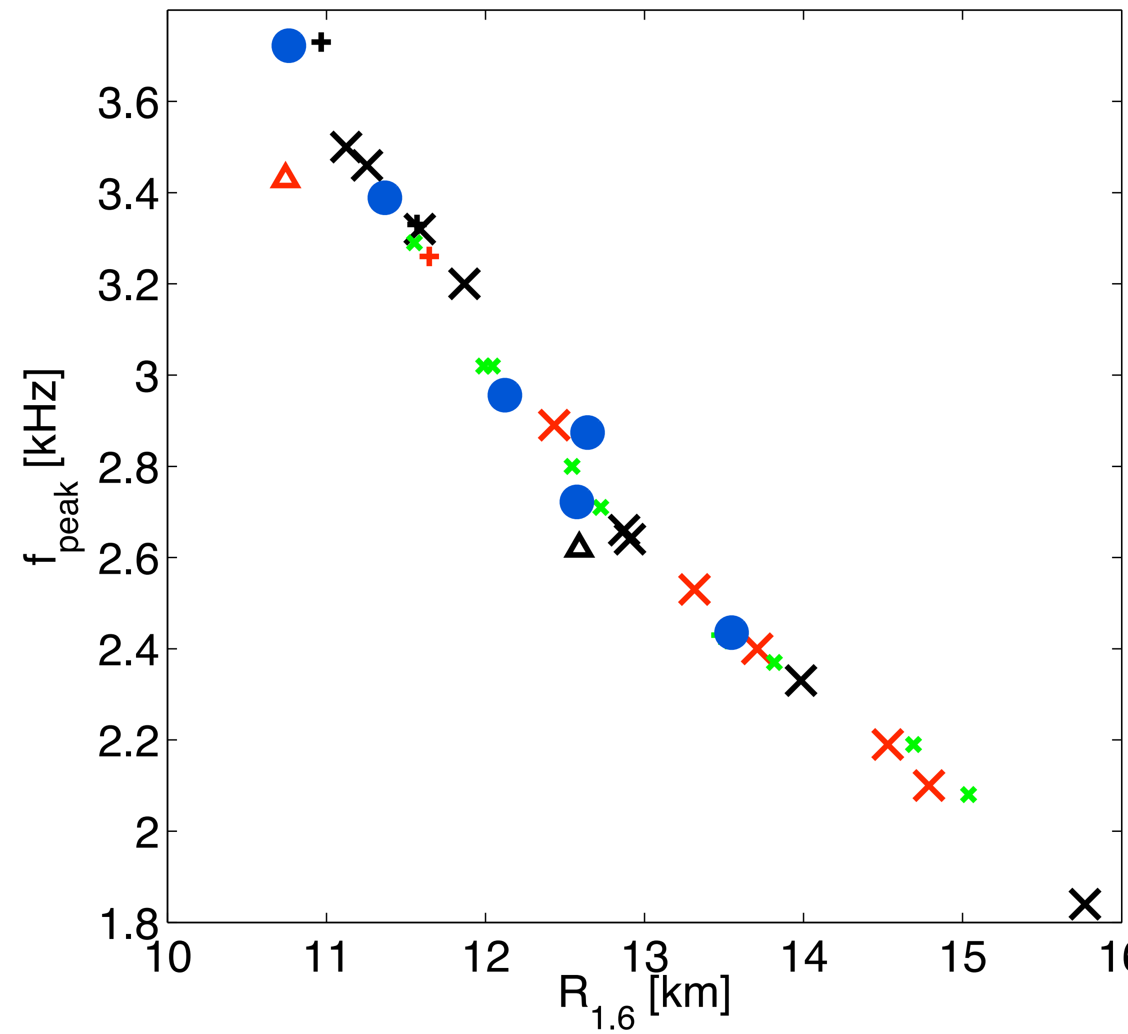
KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)
KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

- low-density part of EOS sets scale for allowed high-density extensions
- current radius prediction for typical $1.4\,M_\odot$ neutron star: $9.7 - 13.9\,\mathrm{km}$
- proposed LOFT mission could significantly improve constraints

Gravitational wave signals from neutron star binary mergers







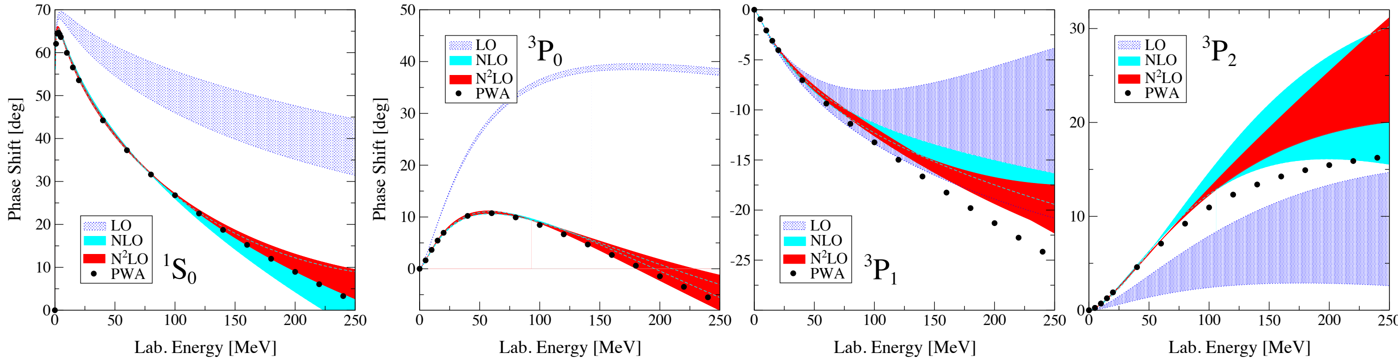
Bauswein and Janka, PRL 108, 011101 (2012),
Bauswein, Janka, KH, Schwenk, PRD 86, 063001

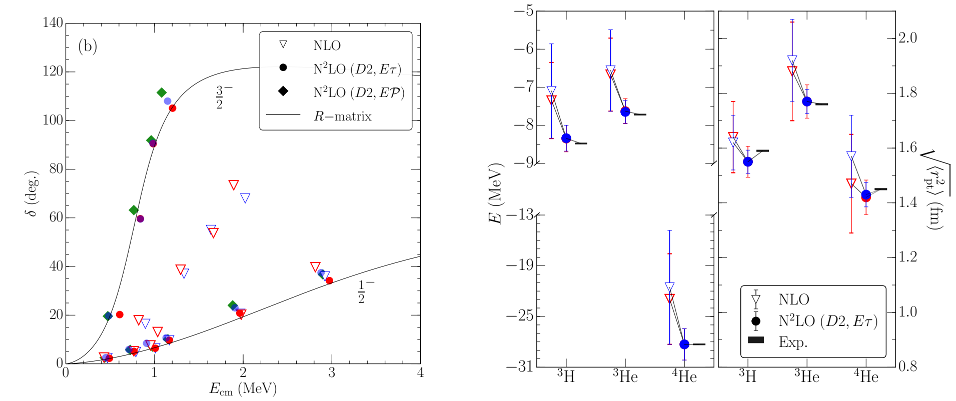
- simulations of NS binary mergers show strong correlation between between $f_{\rm peak}$ of the GW spectrum and the radius of a NS
- measuring $f_{\rm peak}$ is key step for constraining EOS systematically at large ρ

Recent and current developments of novel nuclear interactions

1. local EFT interactions, suitable for Quantum Monte Carlo calculations

status: NN plus 3N up to N2LO





Gezerlis et al.,
PRL 111, 032501 (2013)

Gezerlis et al.,
PRC 90, 054323 (2014)

Lynn et al.,
PRL 116, 062501 (2016)

Recent and current developments of novel nuclear interactions

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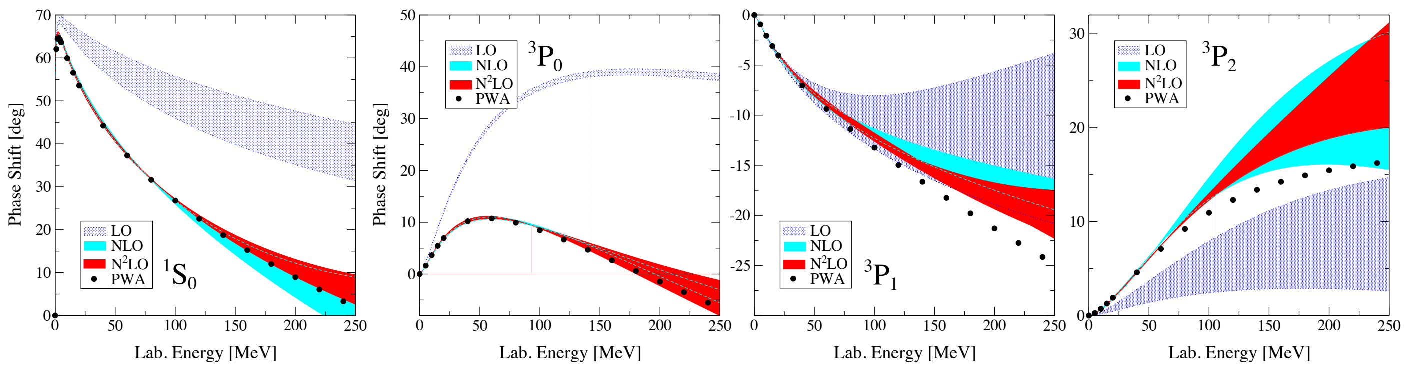

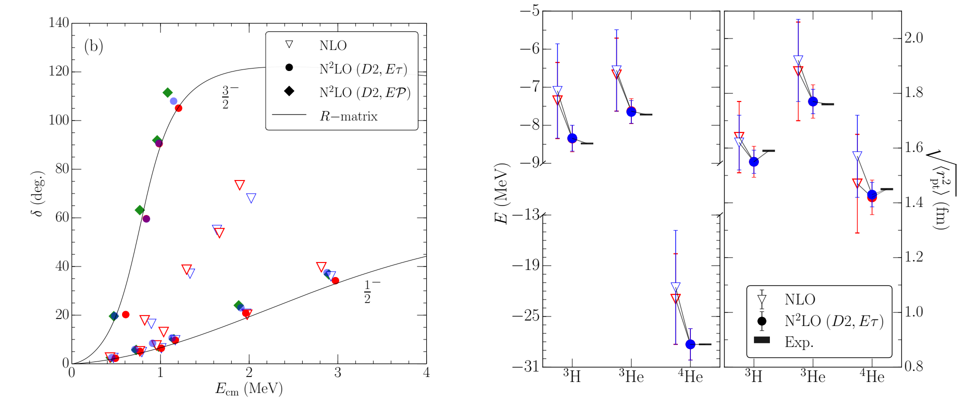

Gezerlis et al.,
PRL 111, 032501 (2013)

Gezerlis et al.,
PRC 90, 054323 (2014)

Lynn et al.,
PRL 116, 062501 (2016)

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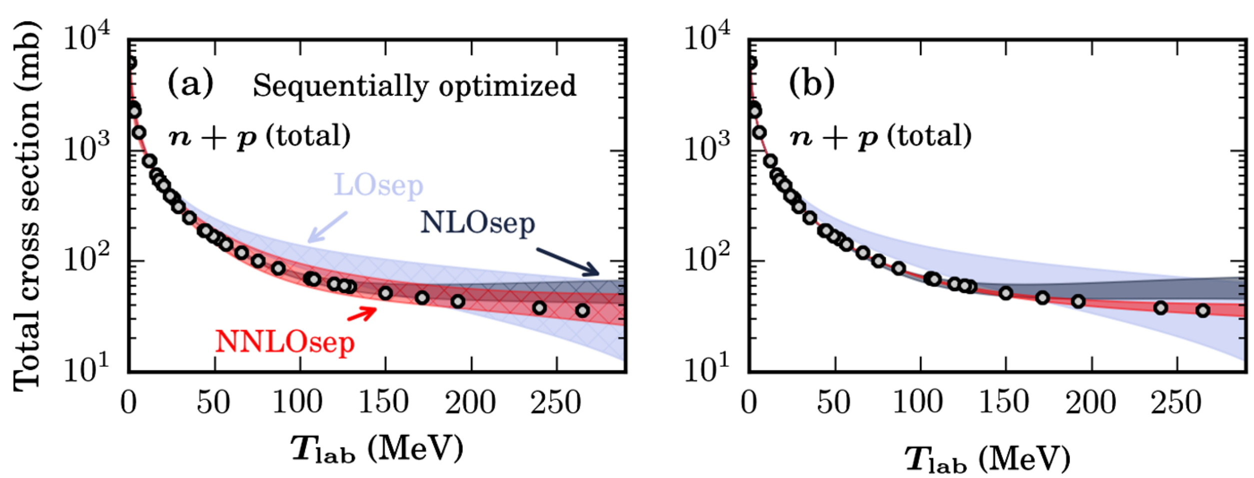
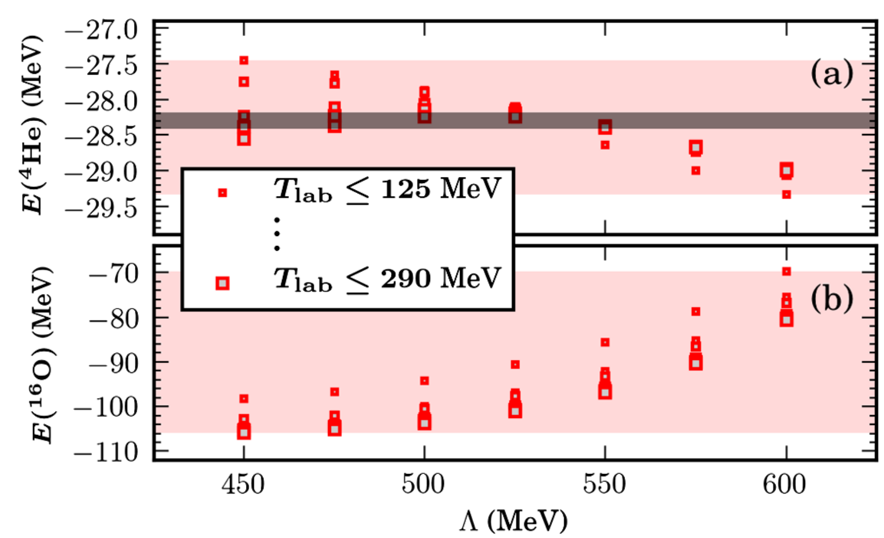
2. simultaneous fit of NN and 3N forces to two- and few-body observables

status: NN plus 3N up to N2LO

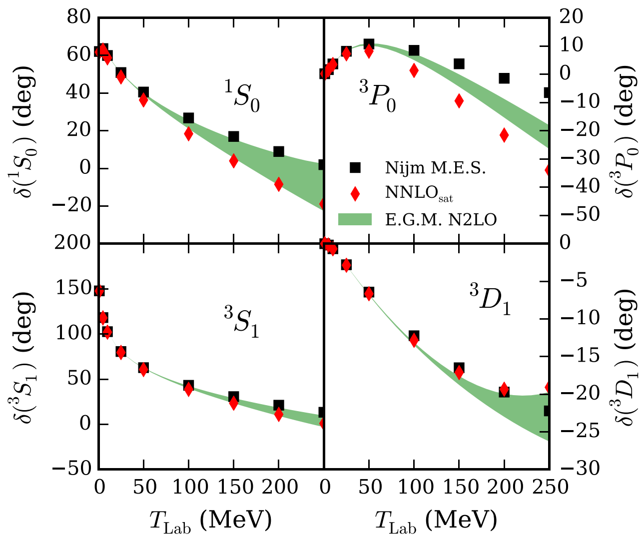


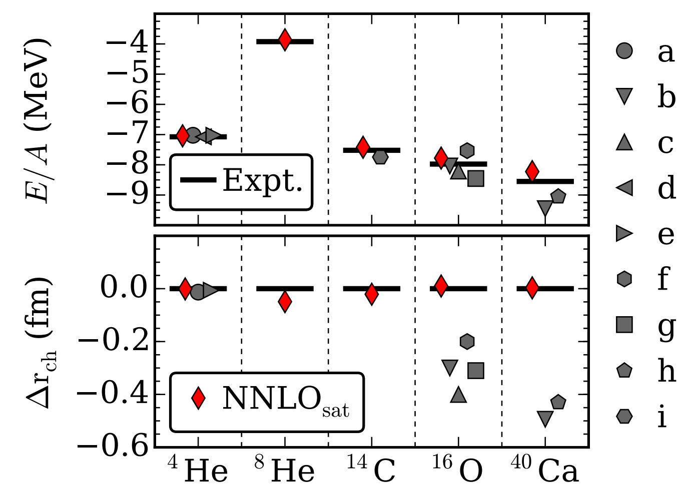


Carlsson et al.,
PRX 6, 011019 (2016)

Recent and current developments of novel nuclear interactions

3. fits of NN plus 3N forces to two-, few- and many-body observables
   status: NN plus 3N up to N2LO





Ekström et al.,
PRC91, 051301 (2015)

Recent and current developments of novel nuclear interactions

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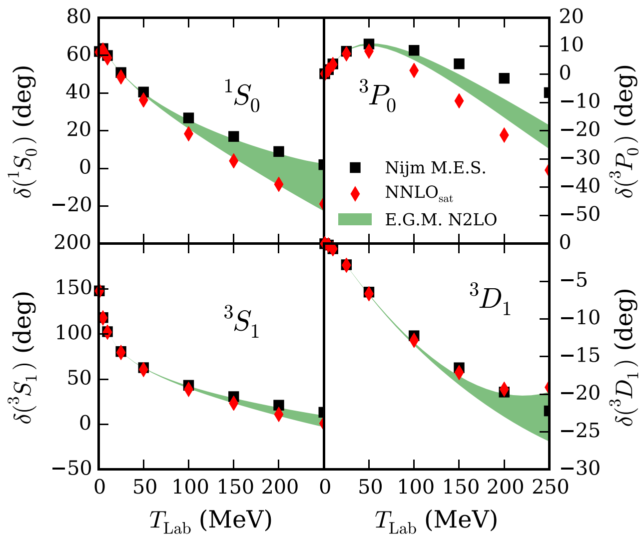
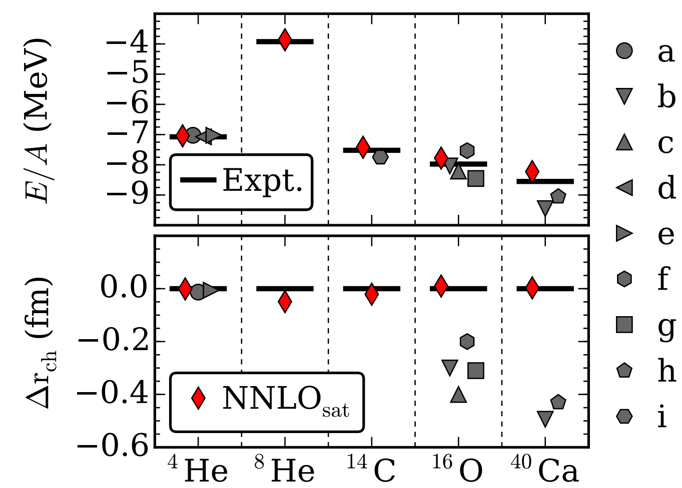
status: NN plus 3N up to N2LO

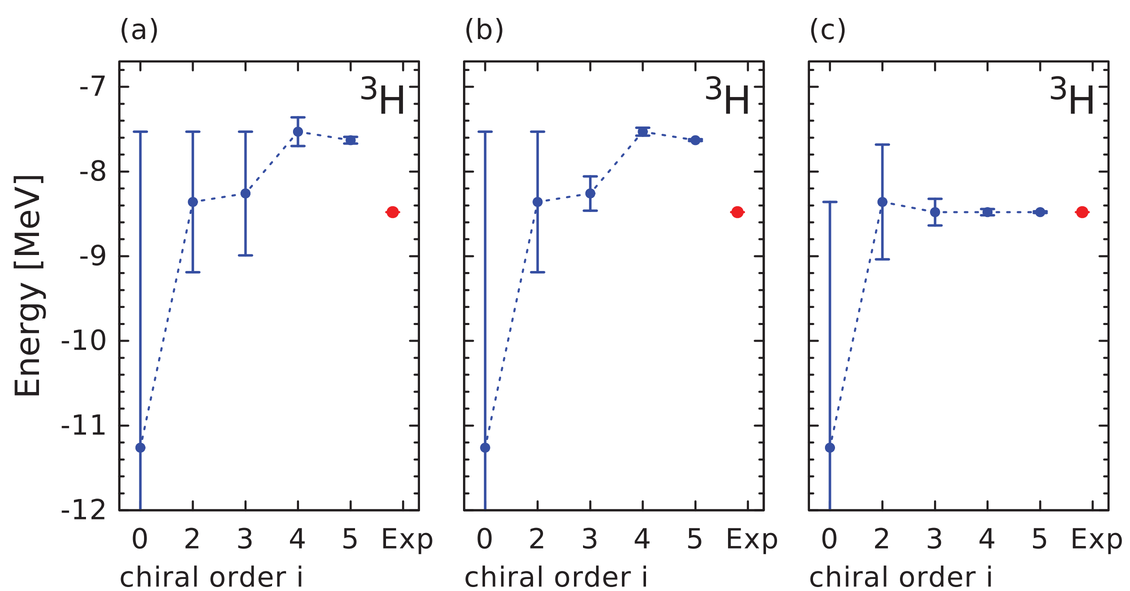


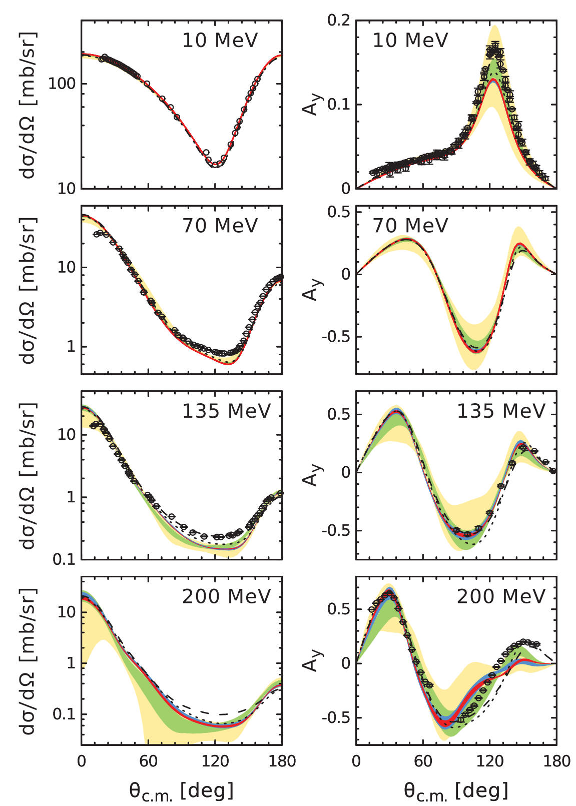


Ekström et al.,
PRC91, 051301 (2015)

4. fit of semilocal NN forces, development of novel way of estimating uncertainties

status: NN up to N4LO, no 3NF yet

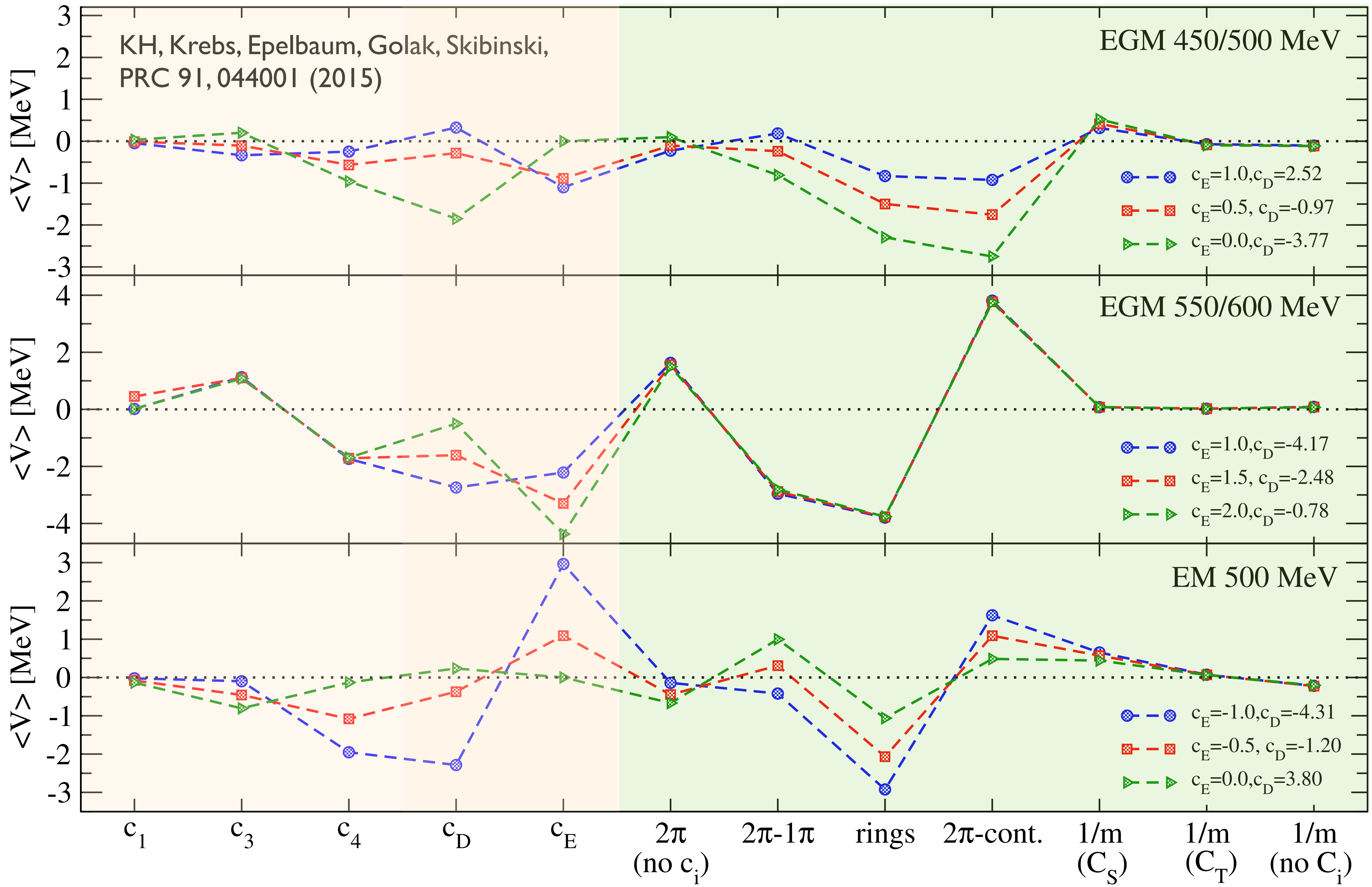


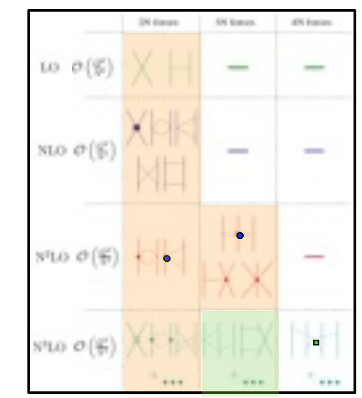


Epelbaum, Krebs, Meißner,
PRL 115, 122301 (2015)

Binder et al.,
PRC 93, 044002 (2016)

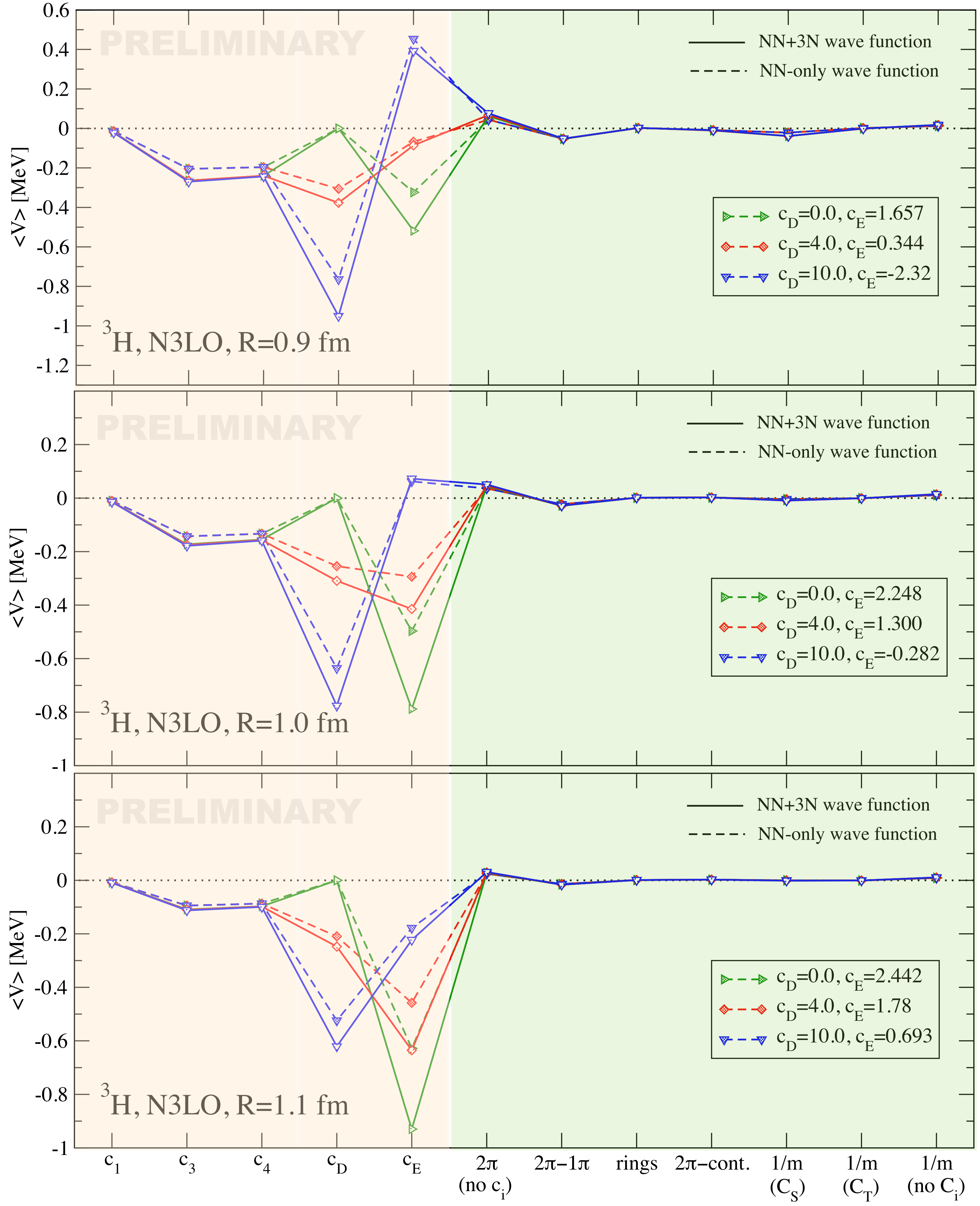
Contributions of individual topologies in ^{3}H (nonlocal)

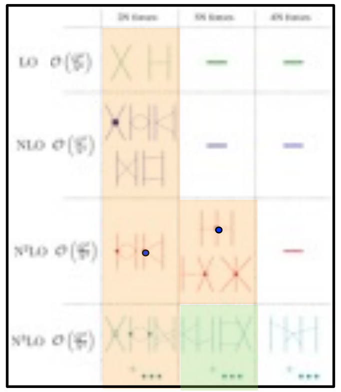




- contributions of individual contributions depend sensitively on details
- N3LO contributions not suppressed compared to N2LO
- perturbativeness of 3NF strongly depends on NN interaction

Contributions of individual topologies in ^3H (semi-local)





- contributions of individual topologies very similar for all cutoffs R at N3LO
- N3LO contributions significantly suppressed compared to N2LO
- 3NF behave perturbatively

Summary

- presently-used nuclear interactions show deficiencies for heavier nuclei
- remarkable agreement between different many-body methods for a given low-resolutions Hamiltonian
- currently active efforts to develop improved NN interactions
- results for matter and nuclei depend sensitively on regularization scheme of NN and 3N interactions, upper density limit for nuclear matter calculations?
- power counting: contributions of N3LO 3NF topologies in 3H:
  - ✦ not suppressed for non-local NN+3N interactions
  - ✦ suppressed for semi-local NN+3N interactions

Outlook and open questions

- understanding of chiral power counting for different regularization schemes
- fitting of LECs in chiral EFT interactions
- explore novel NN+3N interactions for structure of medium-mass nuclei, heavy nuclei and nuclear matter and few-body reactions