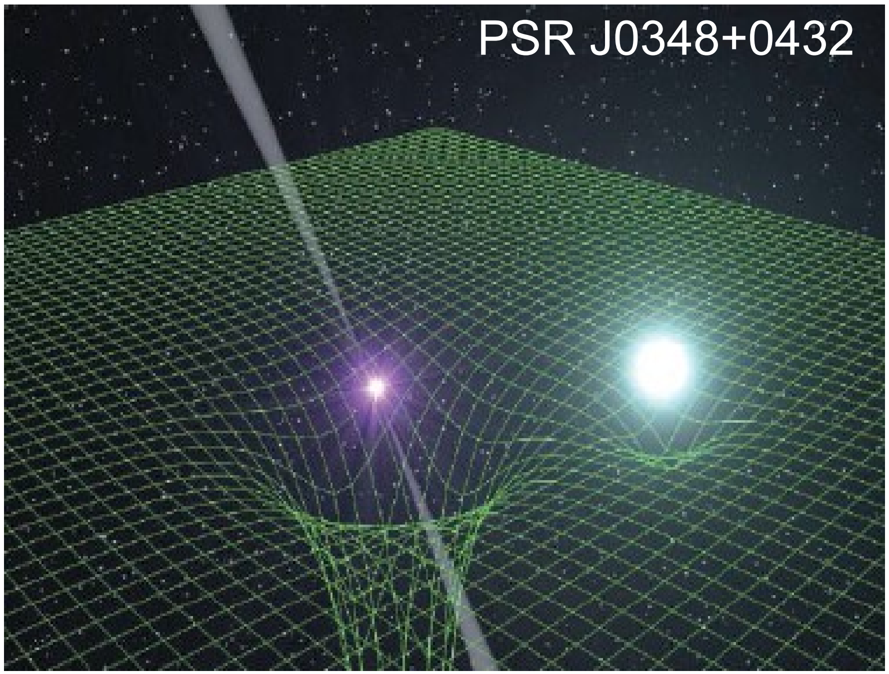
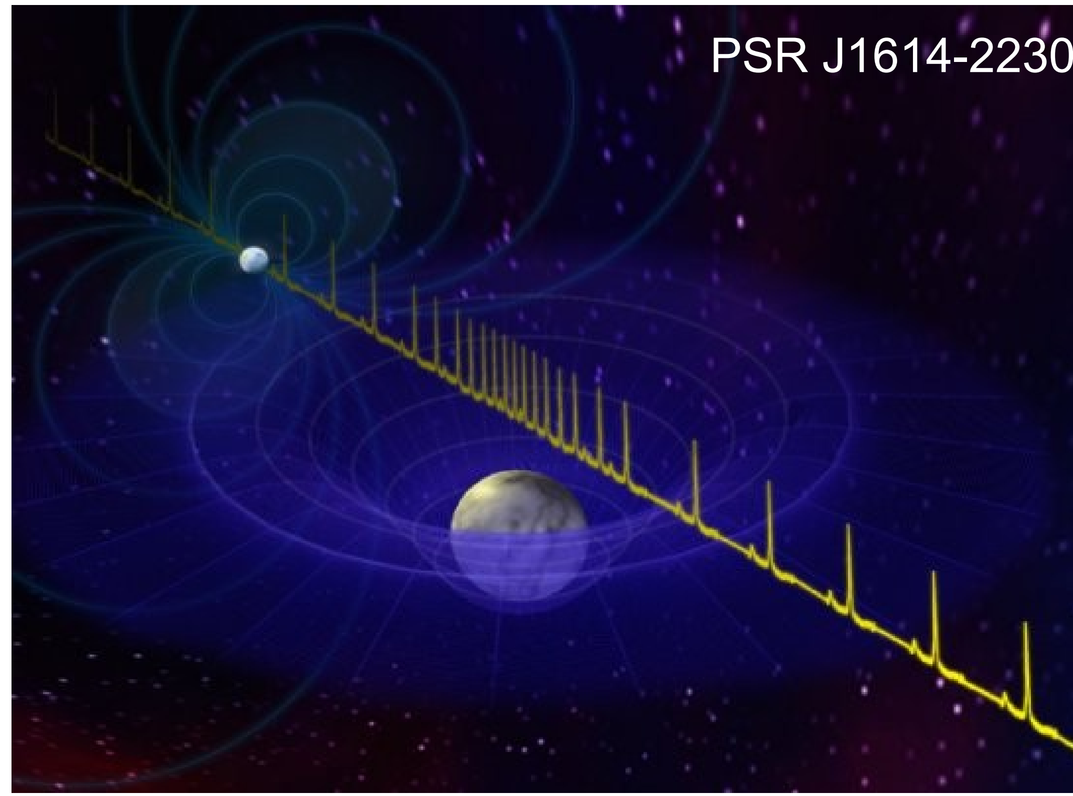
# High-mass neutron stars with quark matter cores-constraints for the equation of state

**David Blaschke (University Wroclaw, JINR Dubna & MEPhI Moscow)**




Antoniadis et al., Science 340 (2013) 448




Demorest et al., Nature 467 (2010) 1081

“From quarks to gravitational waves”, CERN TH Institute, 07.12.2016

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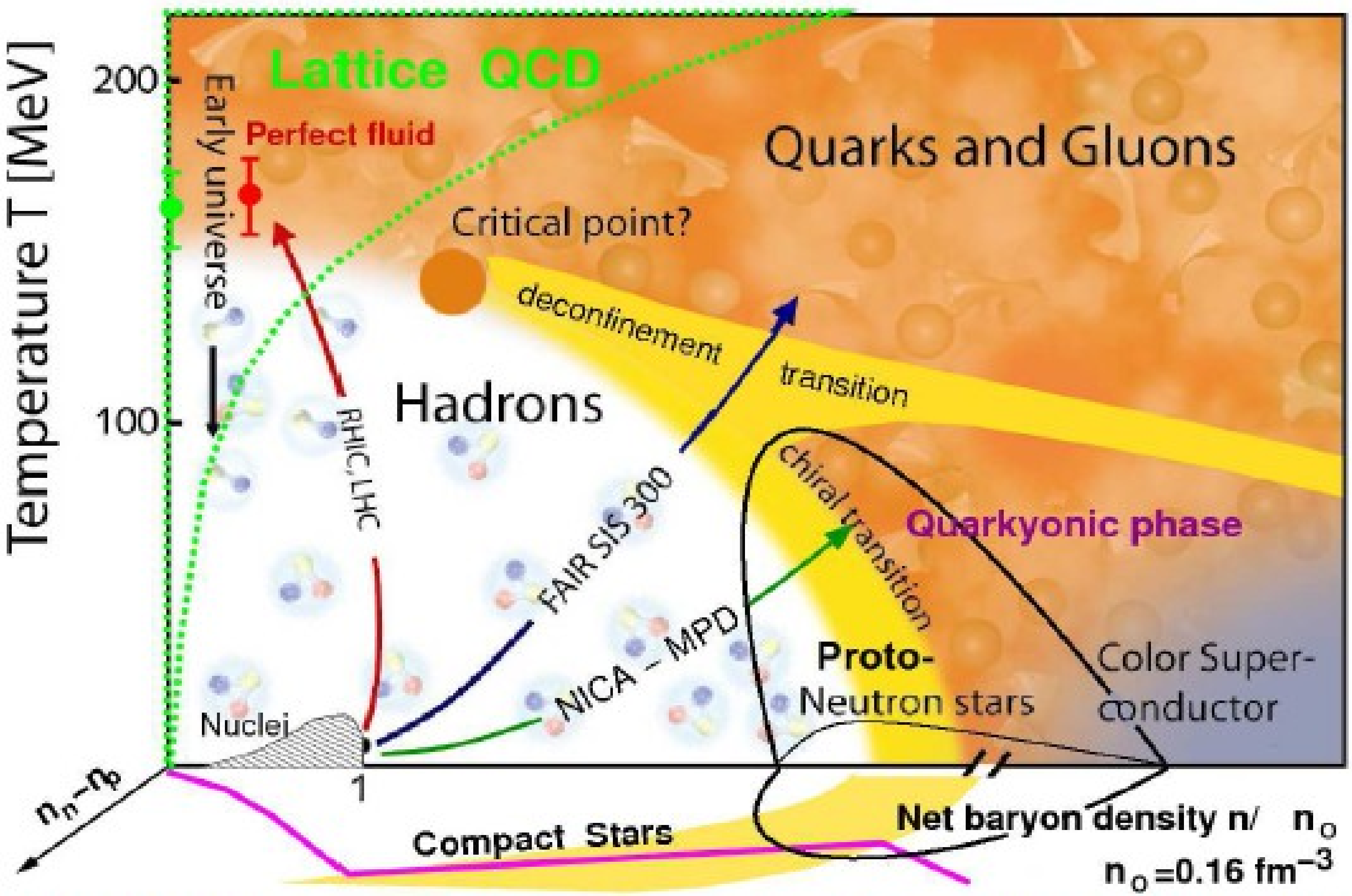
# Support a CEP in QCD phase diagram with Astrophysics?

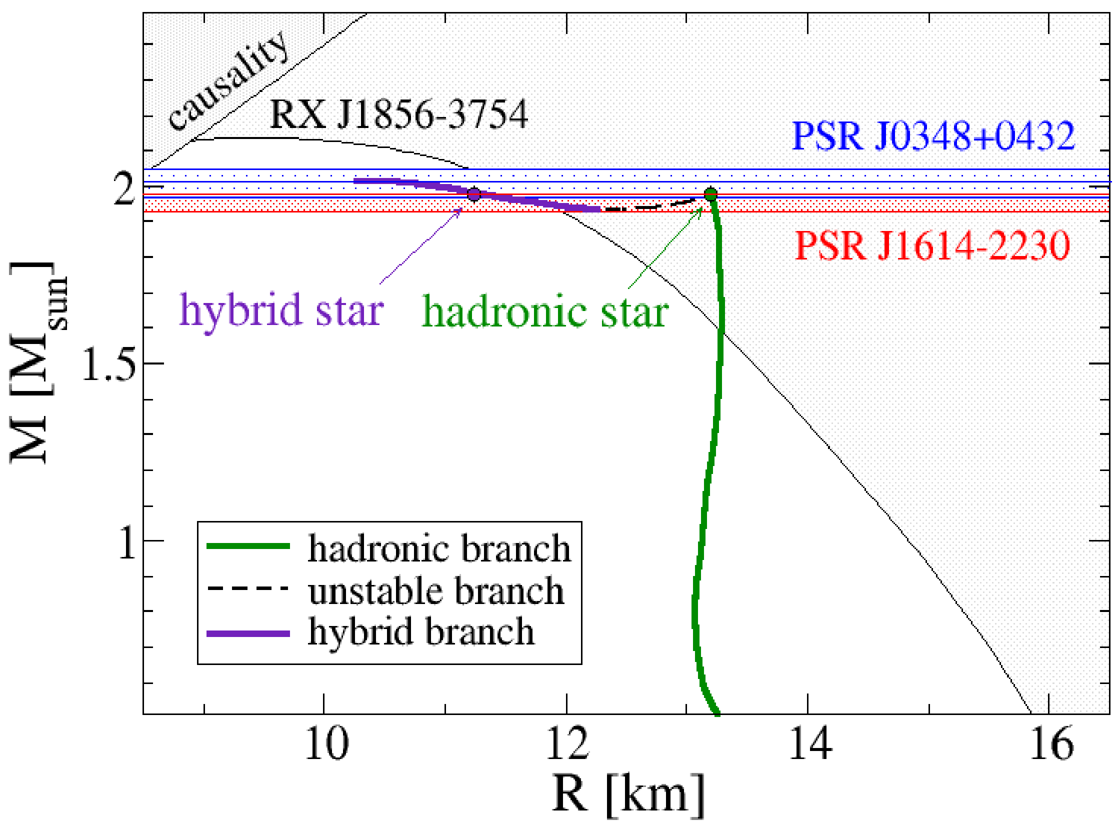


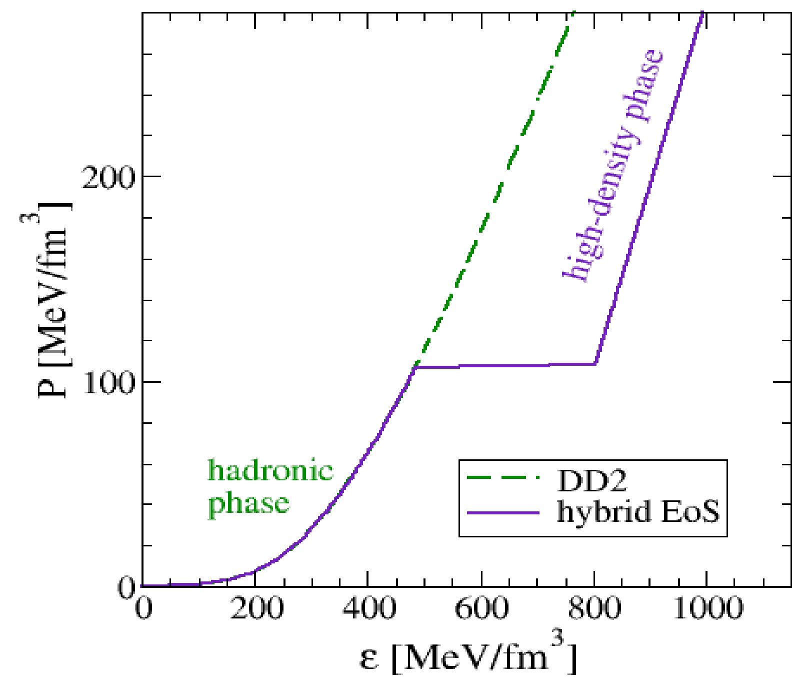

NICA White Paper, http://theor.jinr.ru/twiki-cgi/view/NICA/WebHome

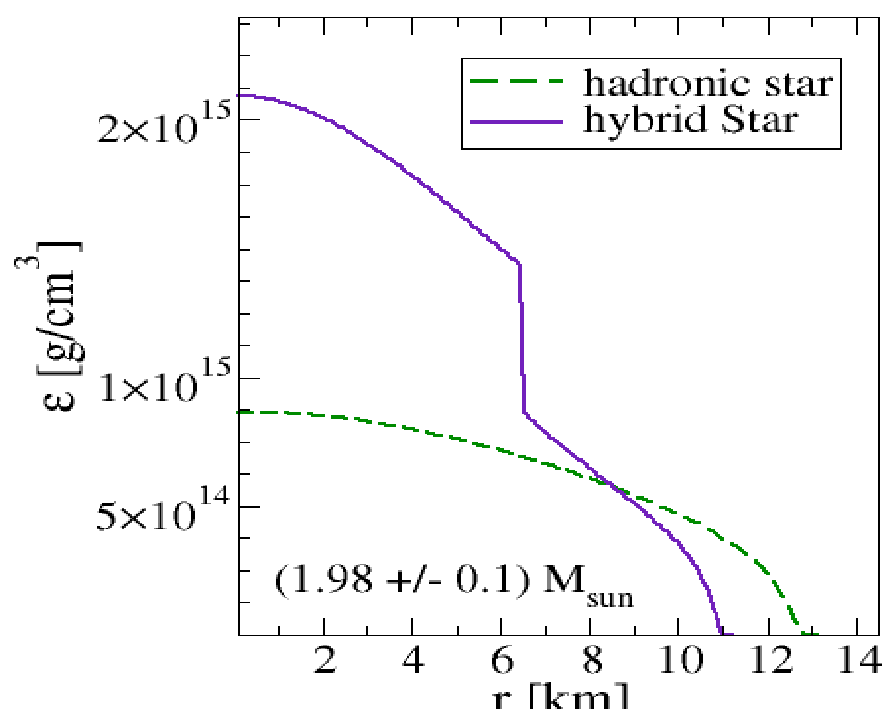
**Crossover at finite T (Lattice QCD) + First order at zero T (Astrophysics) = Critical endpoint exists!**

# “Holy Grail” - High-Mass Twin Stars

**Twins** prove existence of **disconnected populations** (third family) in the M-R diagram
Consequence of a **first order phase transition**
**Question:** Do twins prove the 1st order phase trans.?




Alvarez & Blaschke, arxiv:1304.7758

# High-mass twins & quark matter in compact stars

**David Blaschke (University Wroclaw, JINR Dubna & MEPhI Moscow)**

1. css model for quark matter
2. higher order NJL model
3. piecewise polytrope approach
4. density functional model

*The New is often the well-forgotten Old*

cost
EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

"From quarks to gravitational waves", CERN TH Institute, 07.12.2016

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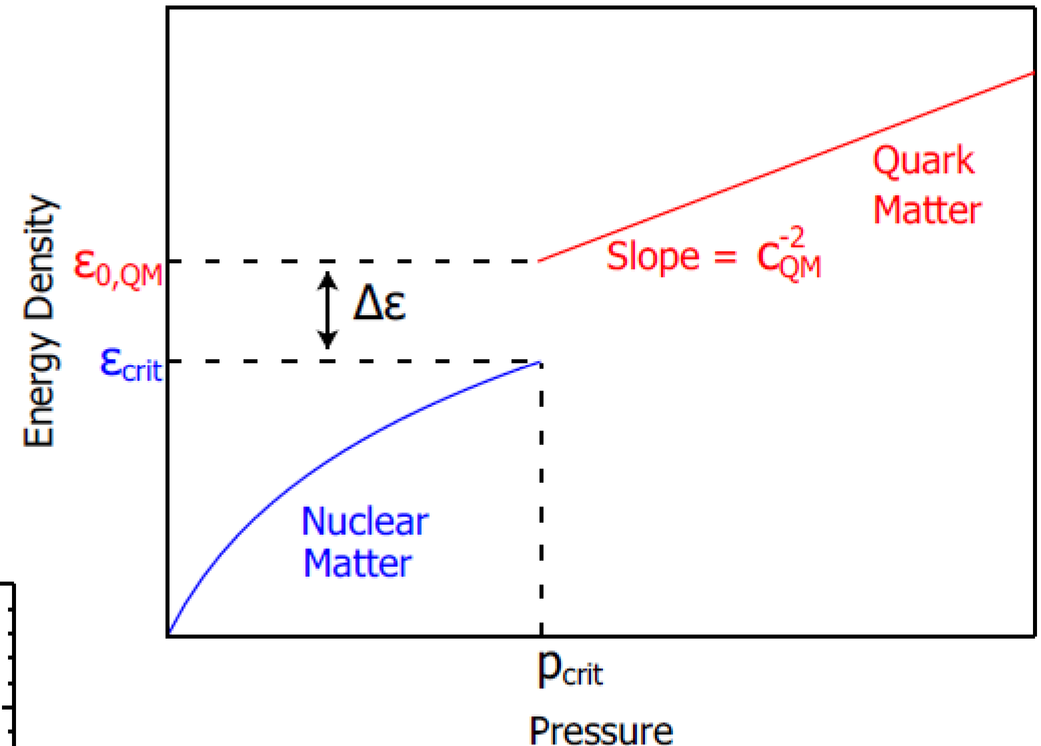
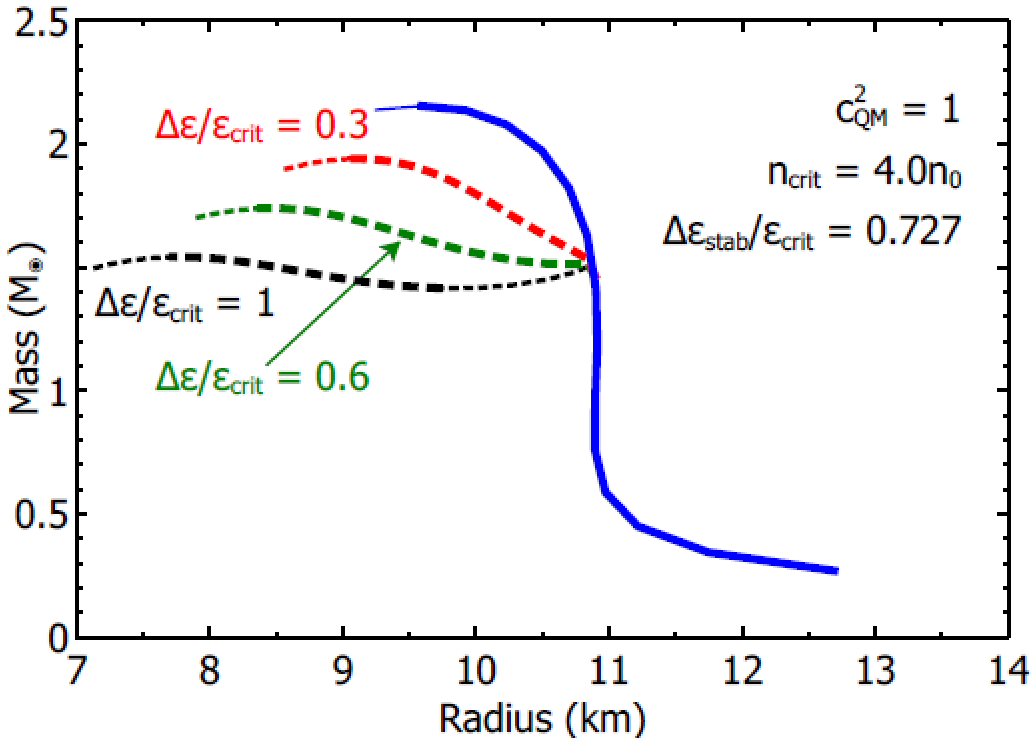
# 1. Constant speed of sound (css) model

**Alford, Han, Prakash, arxiv:1302.4732**

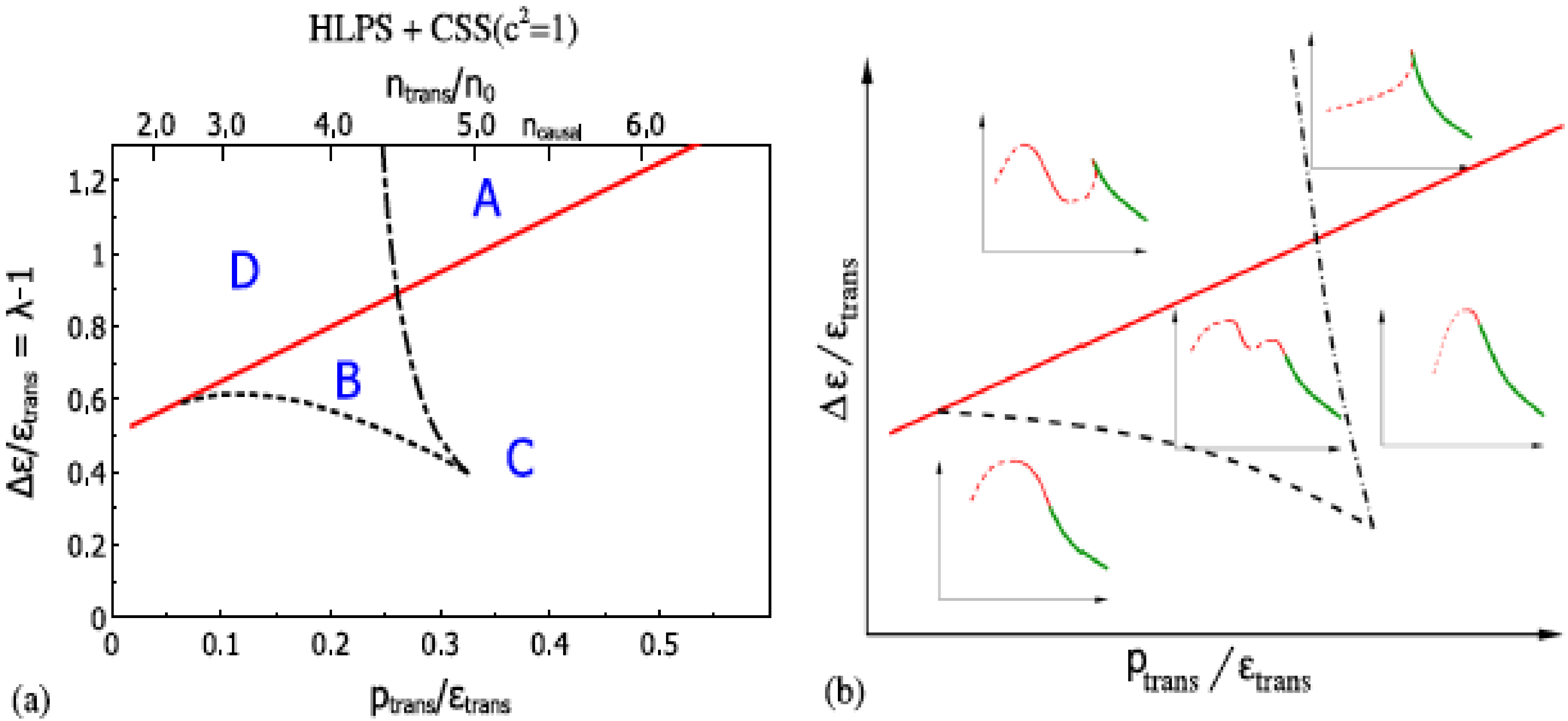
First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the “latent heat” (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → **“third family of CS”**.




Measuring two **disconnected populations** of compact stars in the M-R diagram would be the **detection of a first order phase transition** in compact star matter and thus the indirect proof for the existence of a **critical endpoint (CEP) in the QCD phase diagram**!

# Key fact: Mass “twins” ↔ 1st order PT



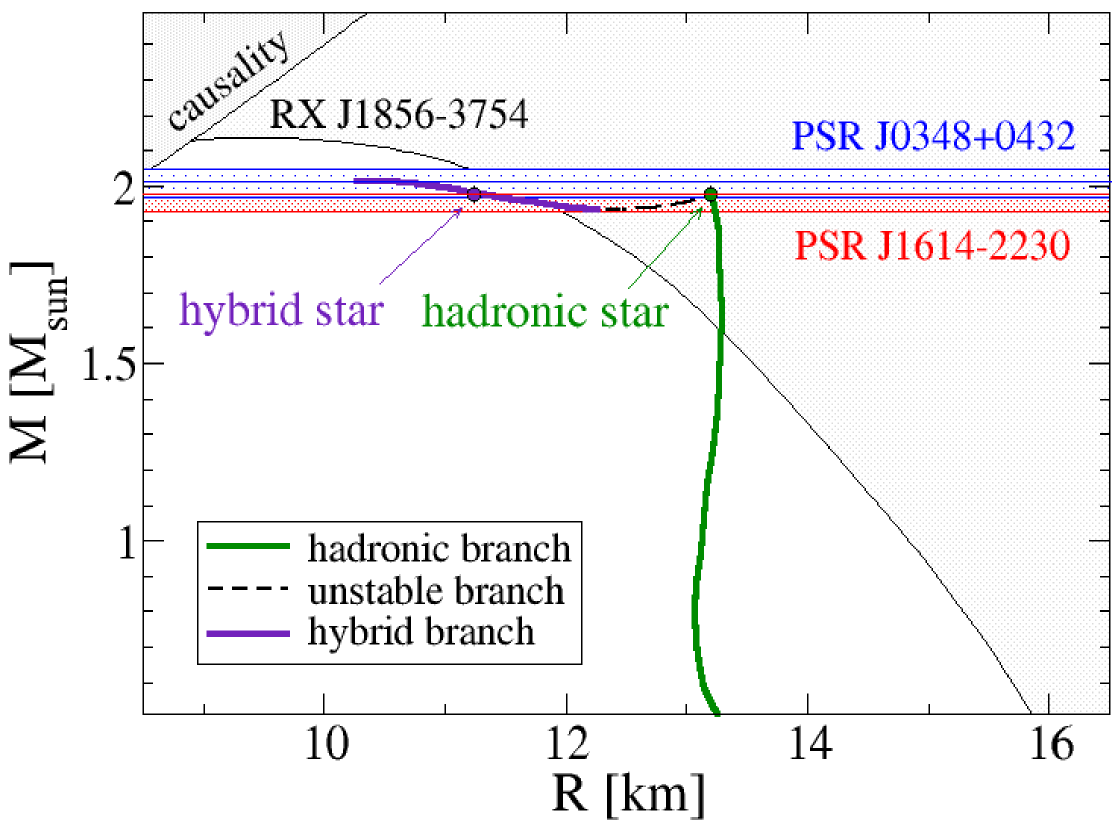

**Systematic Classification [Alford, Han, Prakash: PRD88, 083013 (2013)]**

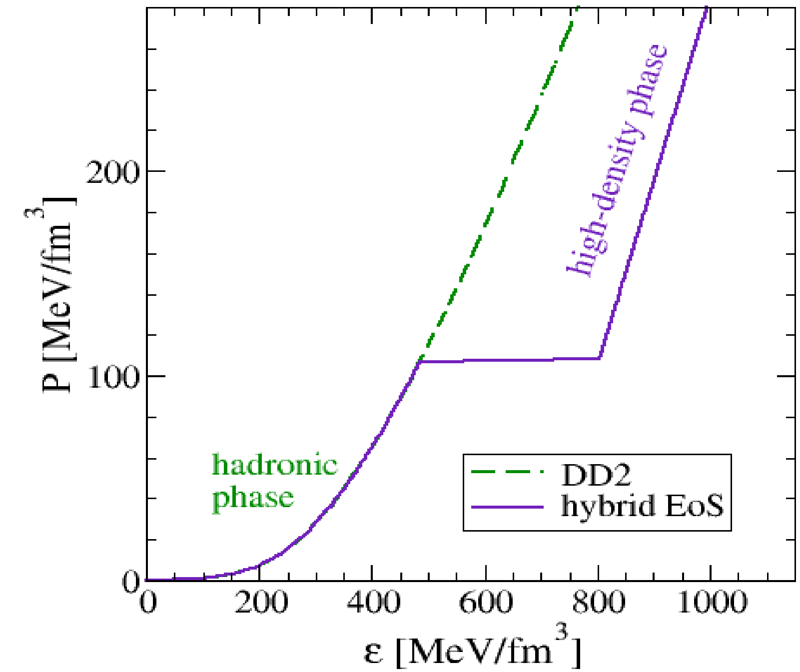
## EoS P(ε) <--> Compact star phenomenology M(R)

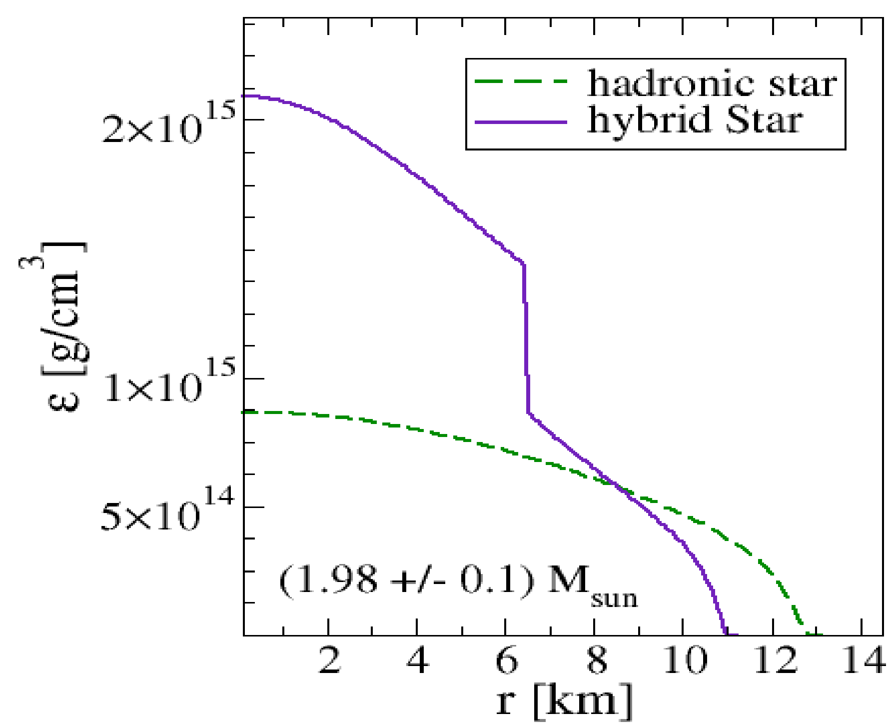
**Most interesting and clear-cut cases: (D)isconnected and (B)oth – high-mass twins!**

# “Holy Grail” - High-Mass Twin Stars

**Twins** prove exitence of **disconnected populations** (third family) in the M-R diagram
Consequence of a **first order phase transition**
**Question:** Do twins prove the 1st order phase trans.?

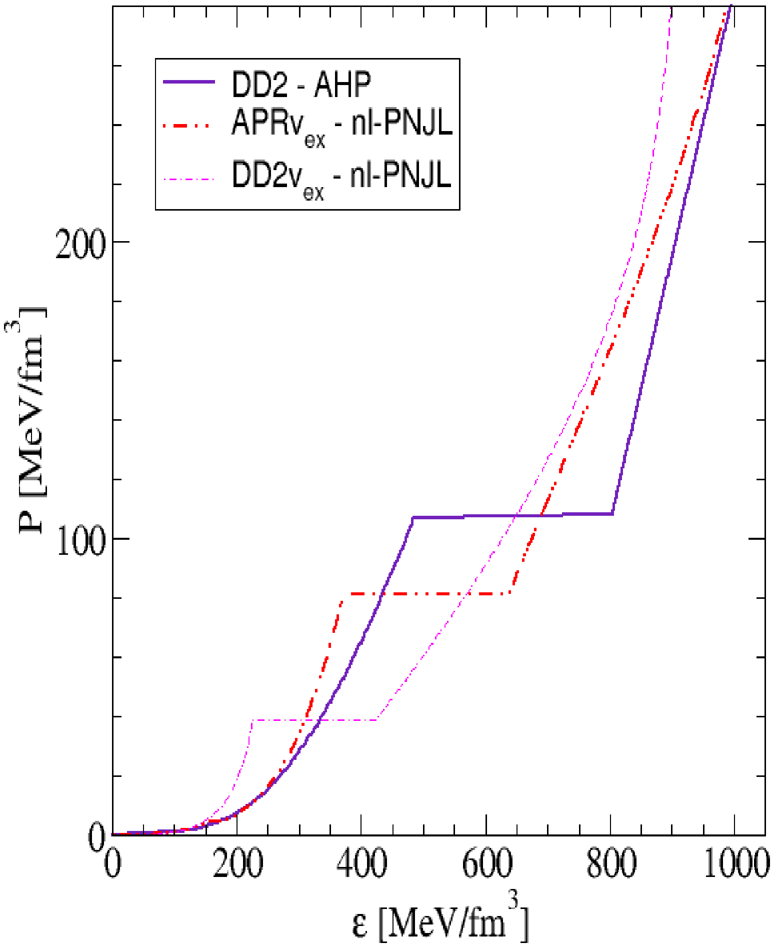


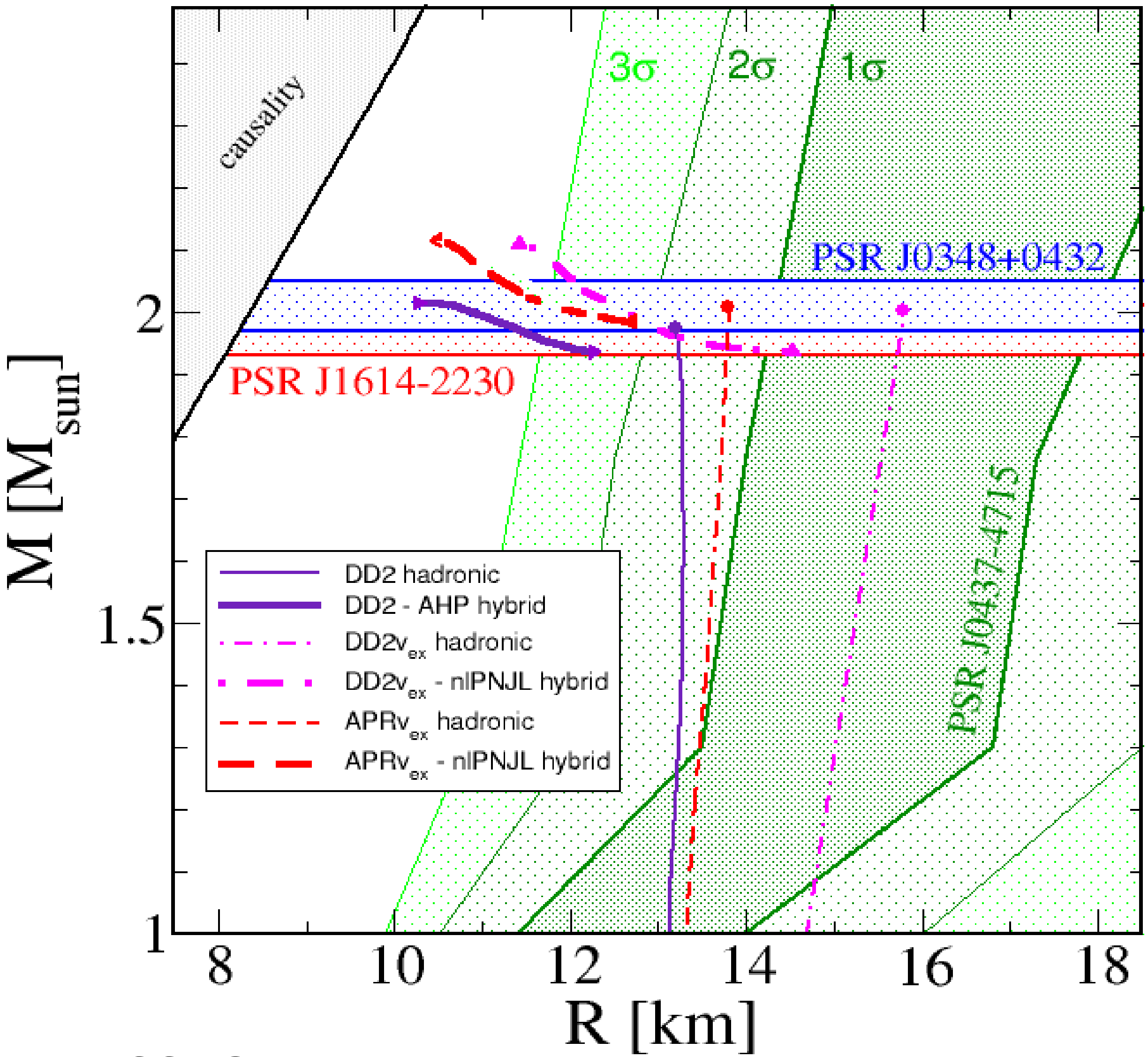




**Alvarez & Blaschke, arxiv:1304.7758**

# High mass twins: more examples !




**DB, Alvarez, Benic, arxiv:1310.3803 Proceedings of CPOD 2013**

**MESSAGE:**

- excluded volume (quark Pauli blocking) important
- high-density quark matter slightly stiffer eta_v=0.25
- the scaled energy density jump (0.65) fulfills the twin condition of the schematic model by Alford et al. (2013)

→ **Astronomers: Find disconnected star branches !!**

# 2. Higher order NJL model (hNJL)

S. Benic, Eur. Phys. J. A 50, 111 (2014)

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - m)q + \mu_q \bar{q}\gamma^0 q + \mathcal{L}_4 + \mathcal{L}_8 \ , \ \mathcal{L}_4 = \frac{g_{20}}{\Lambda^2}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2] - \frac{g_{02}}{\Lambda^2}(\bar{q}\gamma_\mu q)^2 \ ,$$

$$\mathcal{L}_8 = \frac{g_{40}}{\Lambda^8}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]^2 - \frac{g_{04}}{\Lambda^8}(\bar{q}\gamma_\mu q)^4 - \frac{g_{22}}{\Lambda^8}(\bar{q}\gamma_\mu q)^2[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$

**Meanfield approximation:** $\mathcal{L}_{\mathrm{MF}} = \bar{q}(i\partial\!\!\!/ - M)q + \tilde{\mu}_q \bar{q}\gamma^0 q - U \ ,$

$$M = m + 2\frac{g_{20}}{\Lambda^2}\langle\bar{q}q\rangle + 4\frac{g_{40}}{\Lambda^8}\langle\bar{q}q\rangle^3 - 2\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle\langle q^\dagger q\rangle^2 \ ,$$

$$\tilde{\mu}_q = \mu_q - 2\frac{g_{02}}{\Lambda^2}\langle q^\dagger q\rangle - 4\frac{g_{04}}{\Lambda^8}\langle q^\dagger q\rangle^3 - 2\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle^2\langle q^\dagger q\rangle \ ,$$

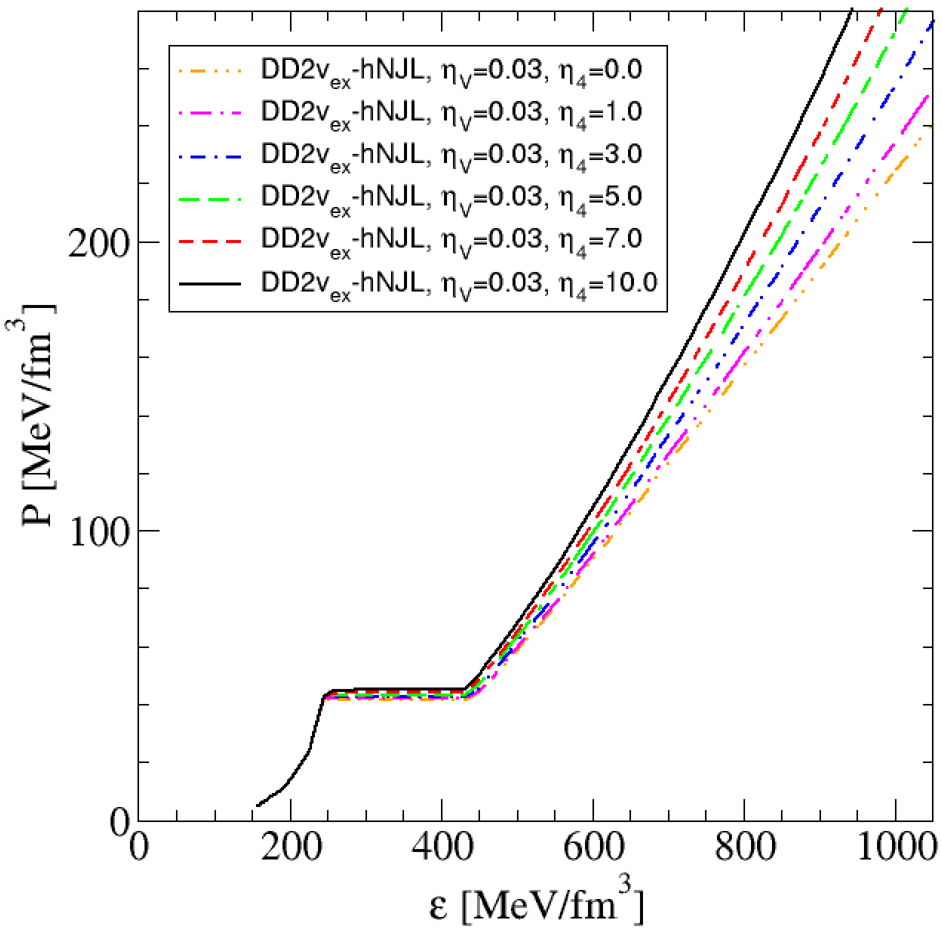
$$U = \frac{g_{20}}{\Lambda^2}\langle\bar{q}q\rangle^2 + 3\frac{g_{40}}{\Lambda^8}\langle\bar{q}q\rangle^4 - 3\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle^2\langle q^\dagger q\rangle^2 - \frac{g_{02}}{\Lambda^2}\langle q^\dagger q\rangle^2 - 3\frac{g_{04}}{\Lambda^8}\langle q^\dagger q\rangle^4 \ .$$

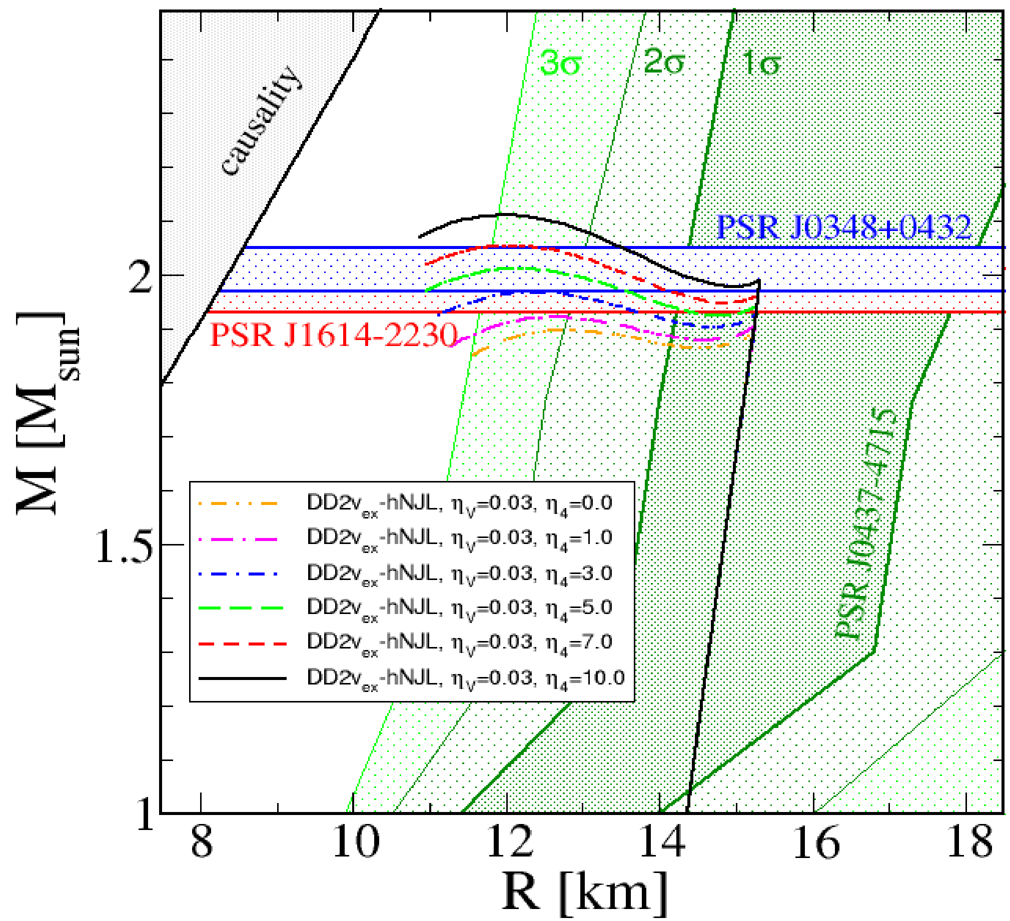
**Thermodynamic Potential:**

$$\Omega = U - 2N_f N_c \int \frac{d^3p}{(2\pi)^3}\left\{E + T\log[1 + e^{-\beta(E-\tilde{\mu}_q)}] + T\log[1 + e^{-\beta(E+\tilde{\mu}_q)}]\right\} + \Omega_0$$

# Result: high-mass twins ↔ 1st order PT

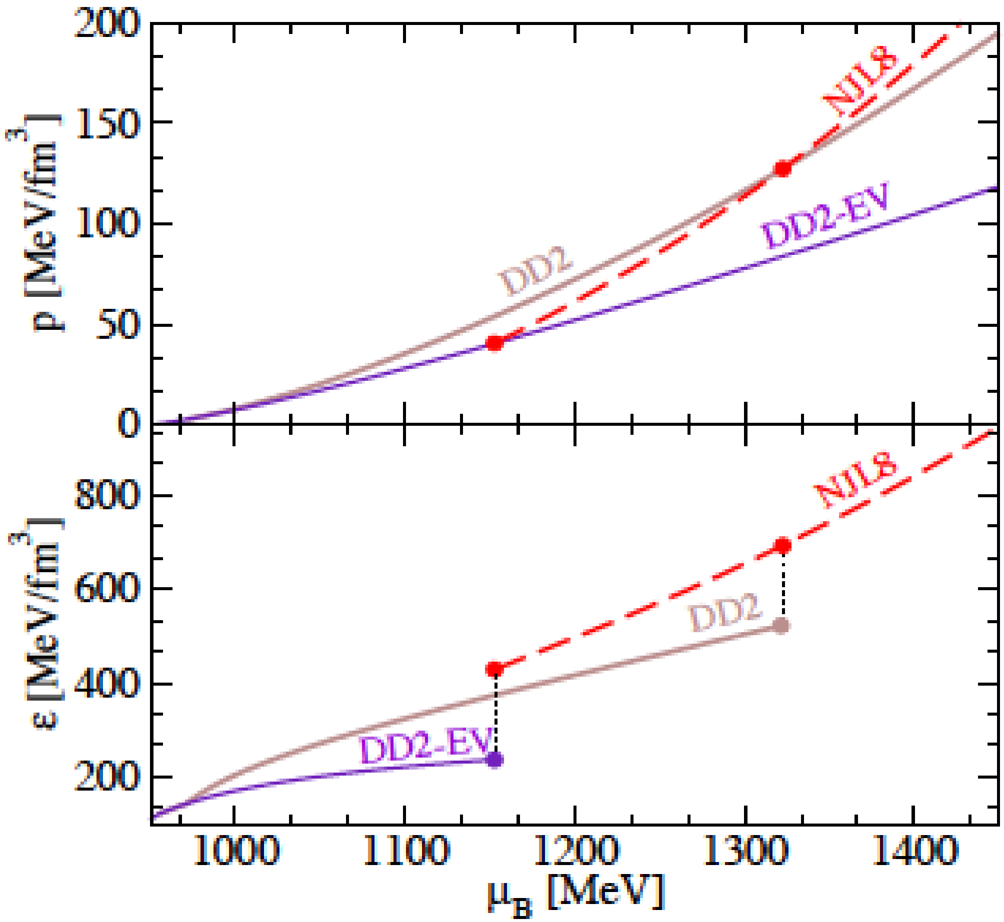
Benic, D.B., Alvarez-Castillo, Fischer, Typel, A&A 577, A40 (2015) [arxiv:1411.2856]

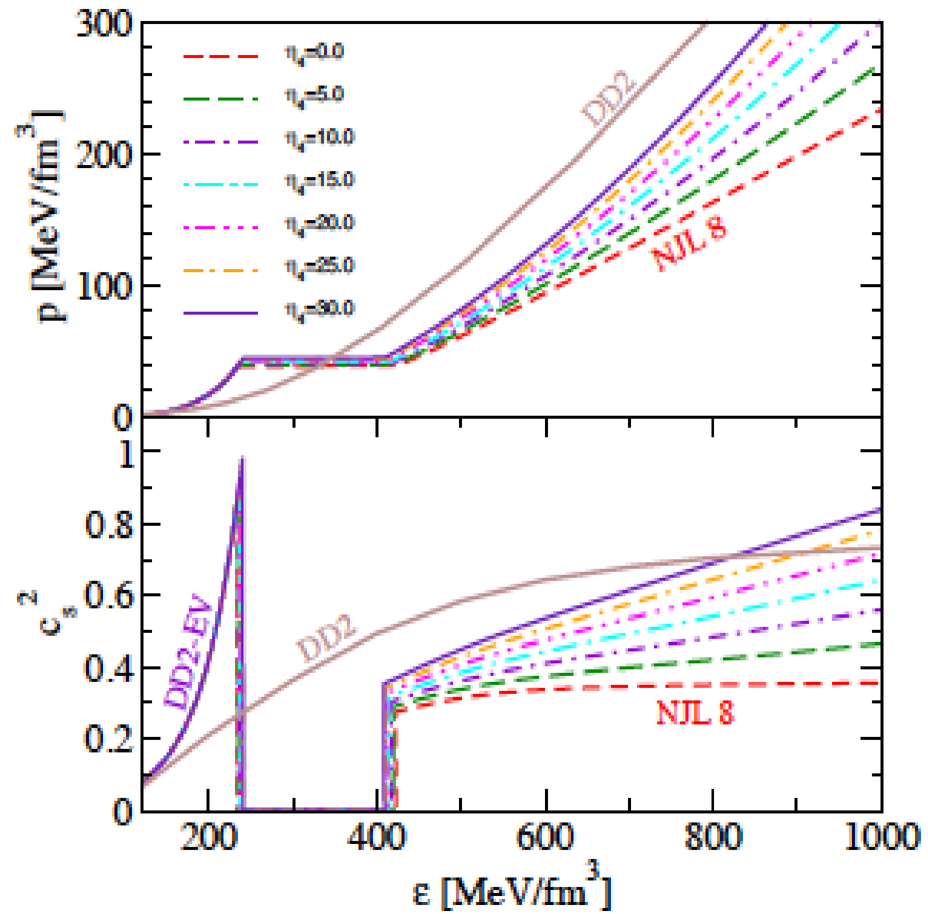



**Hybrid EoS supports M-R sequences with high-mass twin compact stars**

# 2. hNJL model: stiff quark matter @ high densities



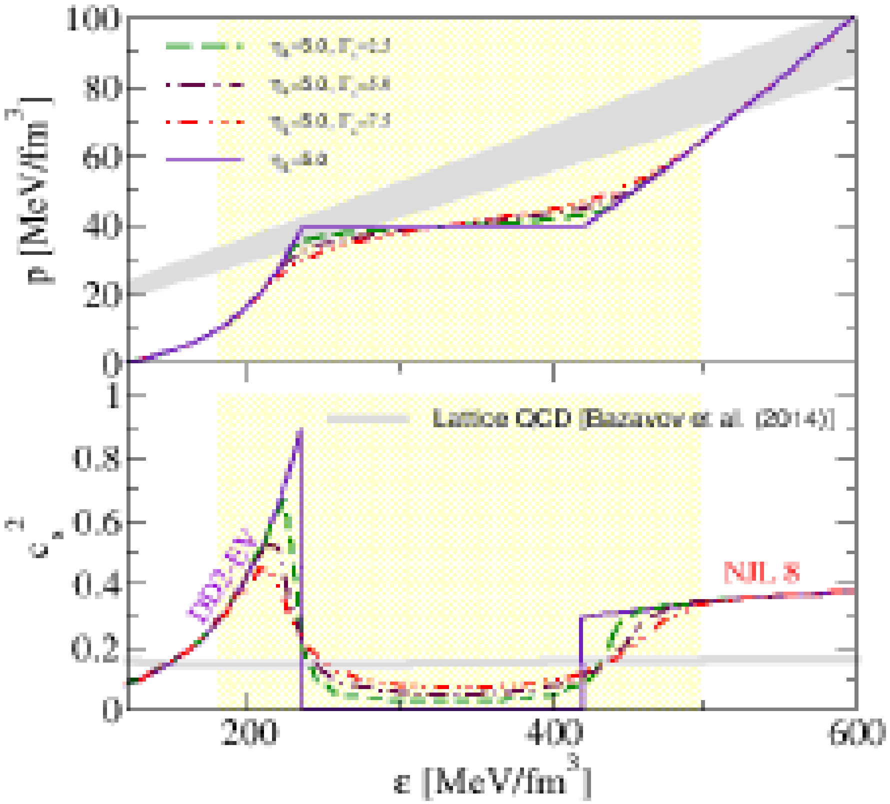



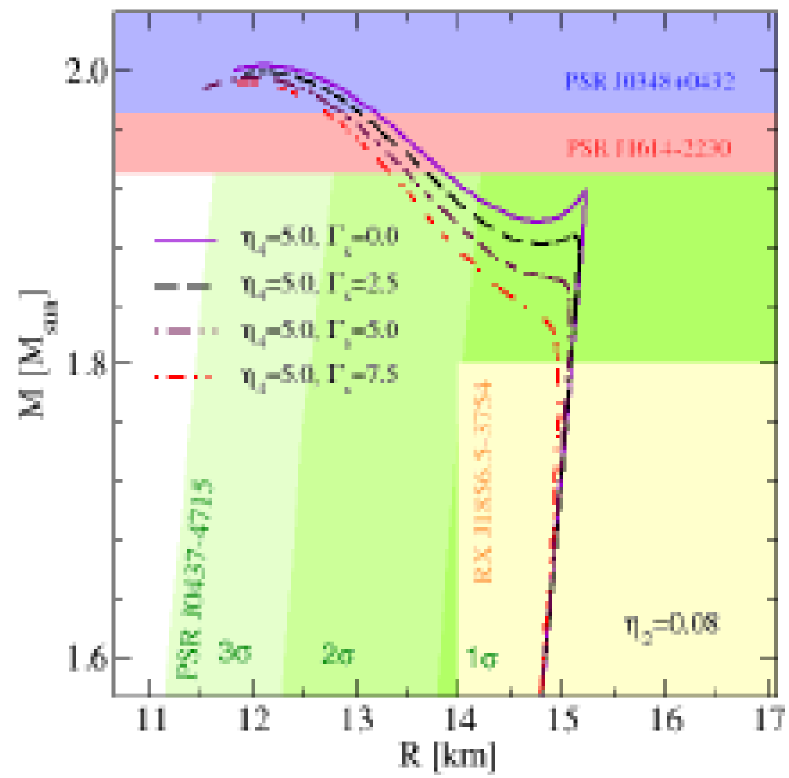

Here: Stiffening of dense hadronic matter by excluded volume in density-dependent RMF

S. Benic, D.B., D. Alvarez-Castillo, T. Fischer, S. Typel, A&A 577, A40 (2015)

# 2. hNJL model: stiff quark matter @ high densities

Estimate effects of structures in the phase transition region (“pasta”)



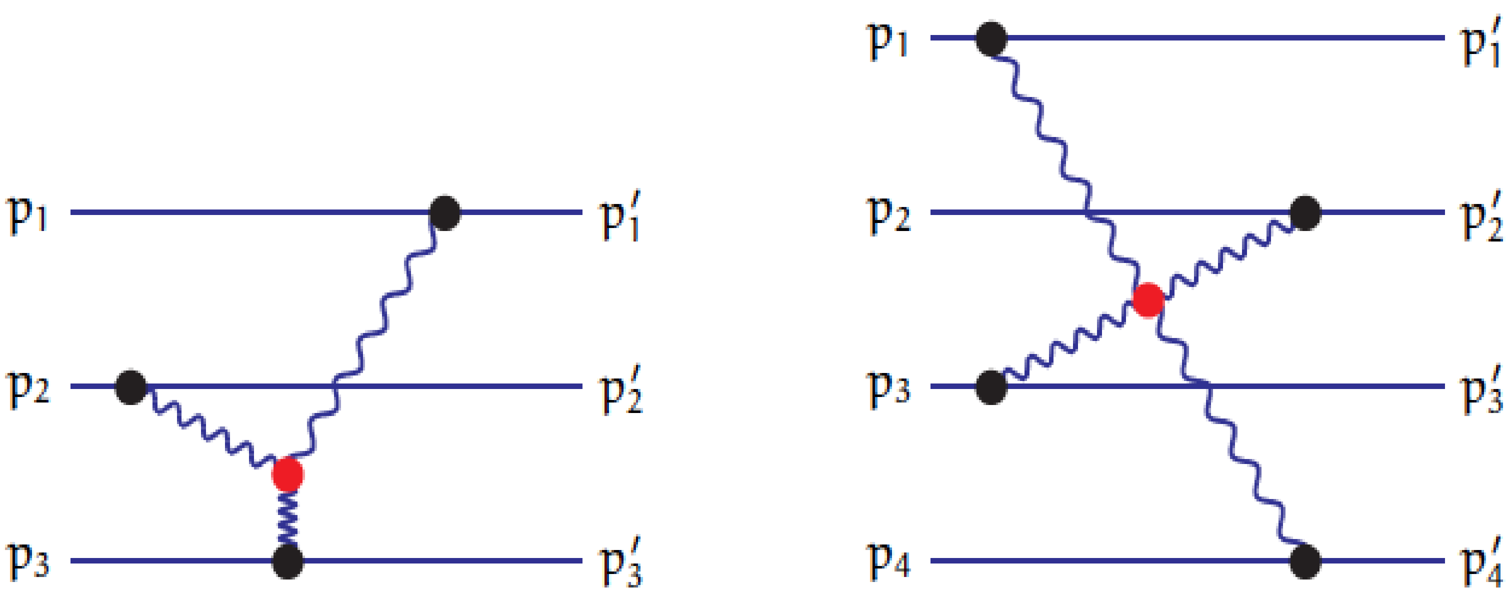

High-mass Twins relatively robust against “smoothing” the Maxwell transition construction

D. Alvarez-Castillo, D.B., arxiv:1412.8463; Phys. Part. Nucl. 46 (2015) 846

# 2. Support for universal stiffening of hadron and quark matter at high densities

Universal repulsion from multi-pomeron exchanges in baryon-baryon scattering.
(Th. A. Rijken, private communication)

Y. Yamamoto, T. Furumoto, N. Yasutake, Th.A. Rijken; EPJA 52 (3) (2016)




(a) Triple-pomeron vertex (b) Quartic-pomeron vertex

The pomerons (wavy lines) couple to different quarks (solid lines) in quark matter (as in the hNJL Lagrangian) or to quarks in different baryons in nuclear matter (giving rise to repulsive 3- and 4- body interactions).

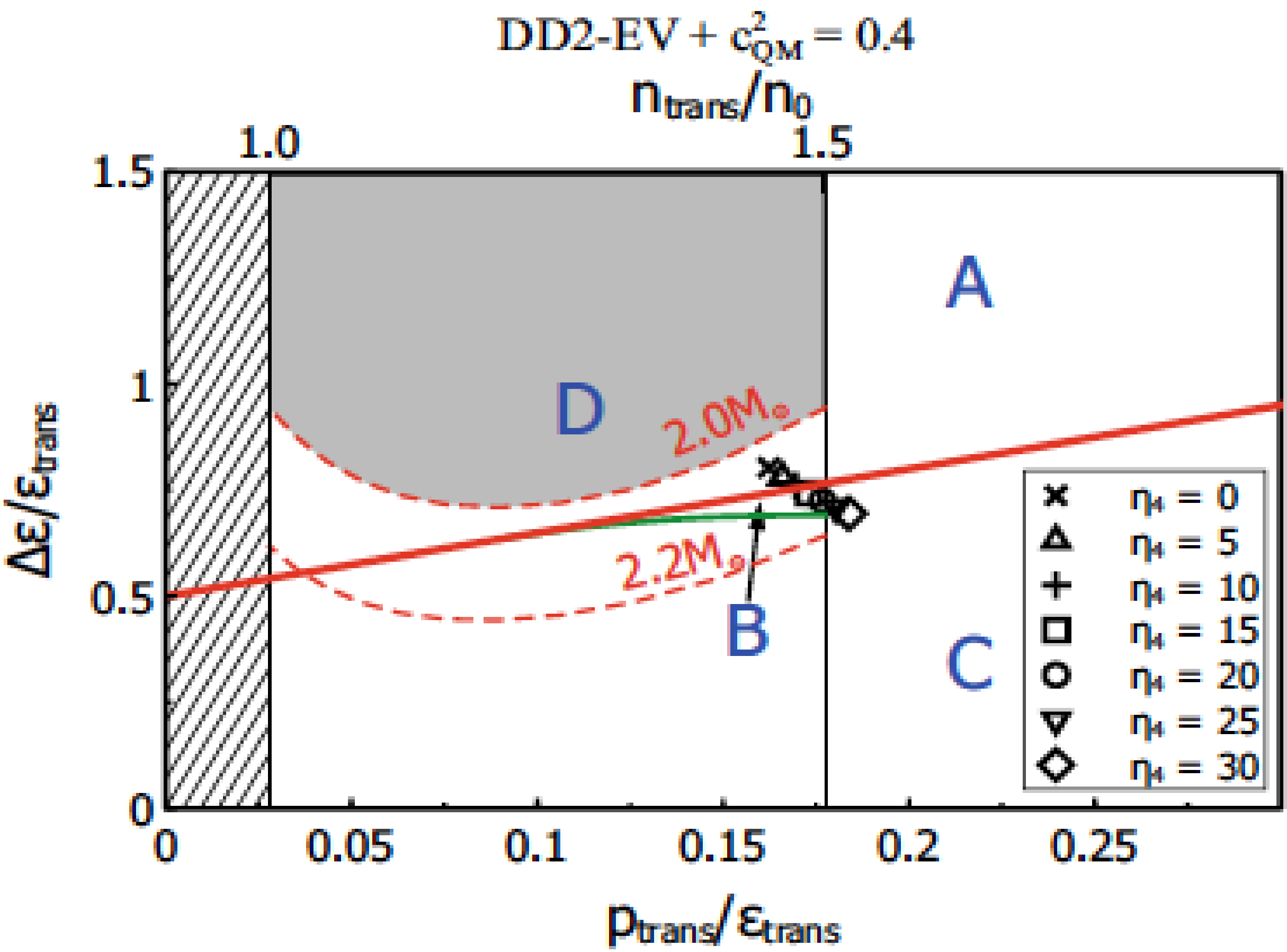
# NICA White Paper – selected topics ...

Many cross-relations with astrophysics of compact stars! High-mass twin stars prove CEP !

**#22 Neutron star mass limit at $2M_\odot$ supports the existence of a CEP**

D. Alvarez-Castillo[1,a], S. Benic[2,b], D. Blaschke[1,3,4], Sophia Han[5,6], and S. Typel[7]

EPJA 52 (2016) 22




Seidov criterion for star Instability at 1st oder PT:

$$\frac{\Delta\varepsilon}{\varepsilon_{\rm trans}} = \frac{1}{2} + \frac{3}{2}\frac{p_{\rm trans}}{\varepsilon_{\rm trans}}$$

(the red line)

Based on Hybrid EoS

S. Benic et al. (2015), A&A **577**, A40

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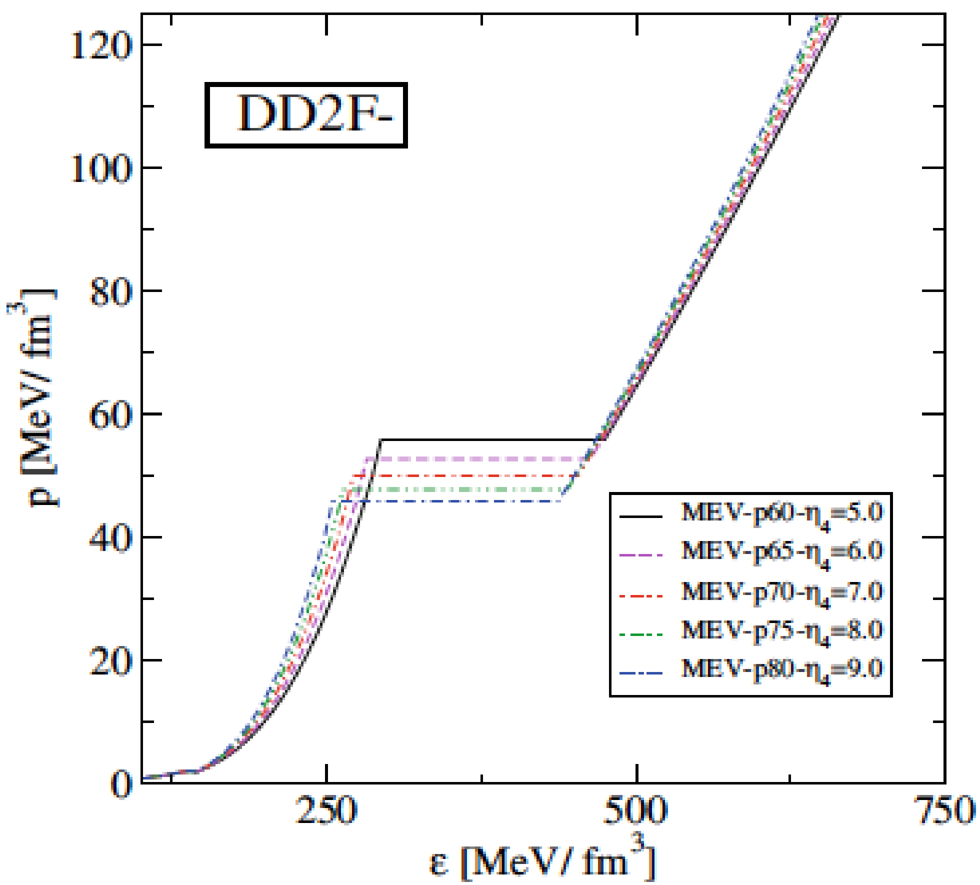
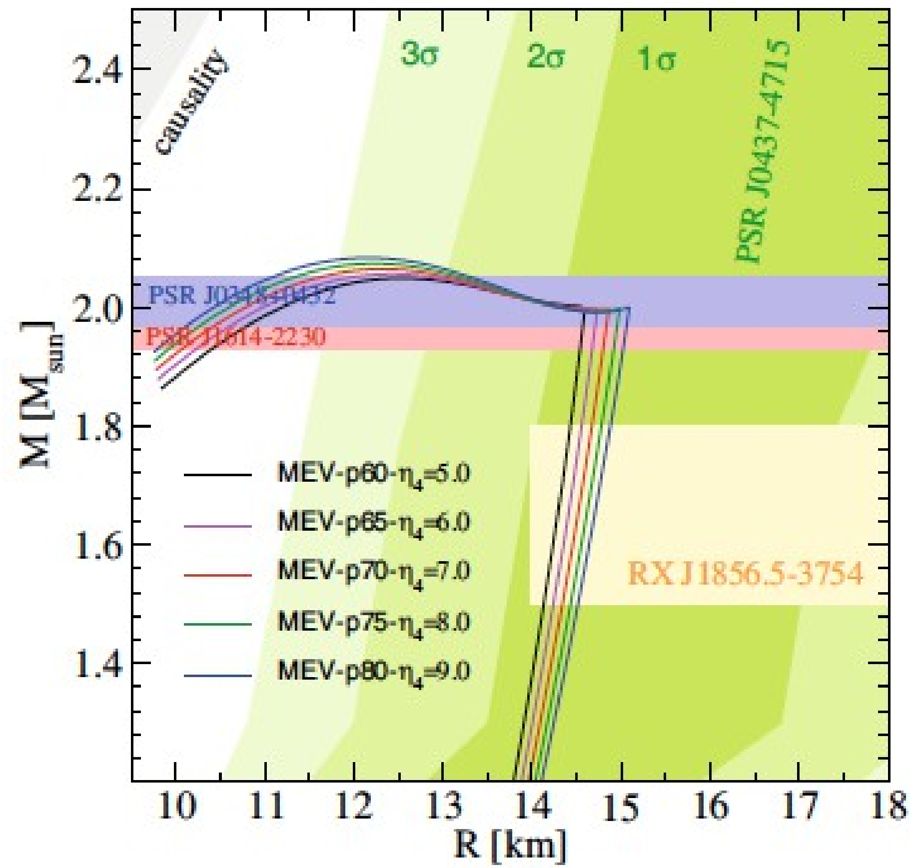
EPJA 52 (2016) 22




Vary stiffness of hadronic and quark EoS so that onset of instability at 2 M_sun.

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Many cross-relations with astrophysics of compact stars! High-mass twin stars prove CEP !

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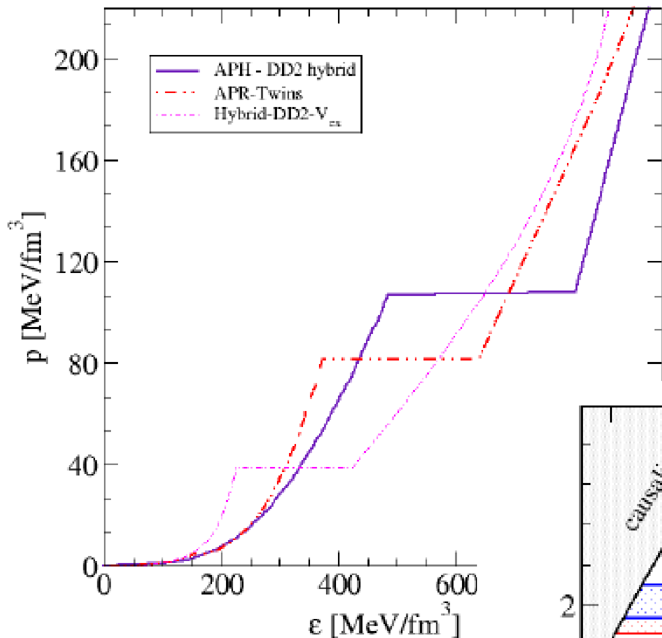
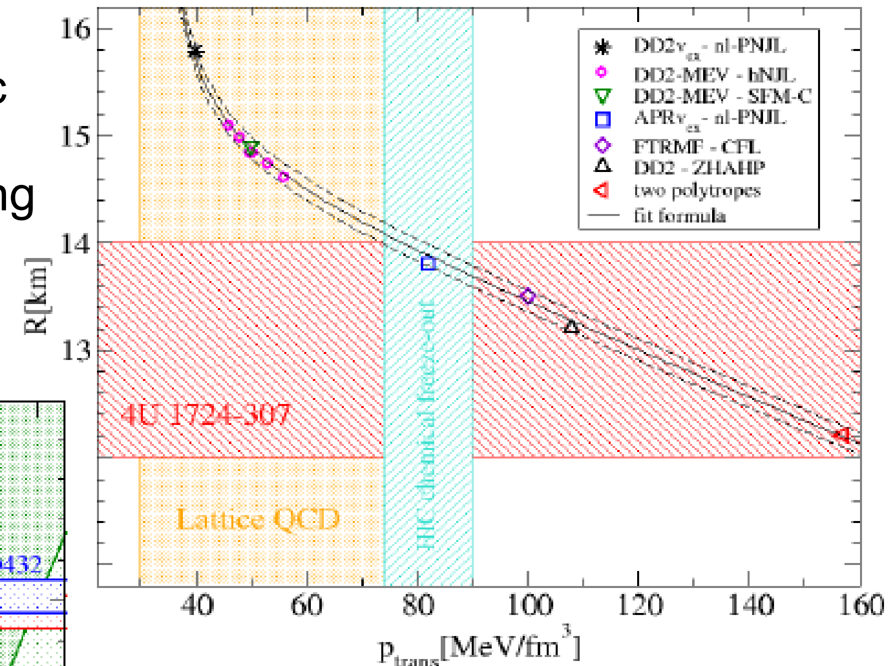
EPJA 52 (2016) 22

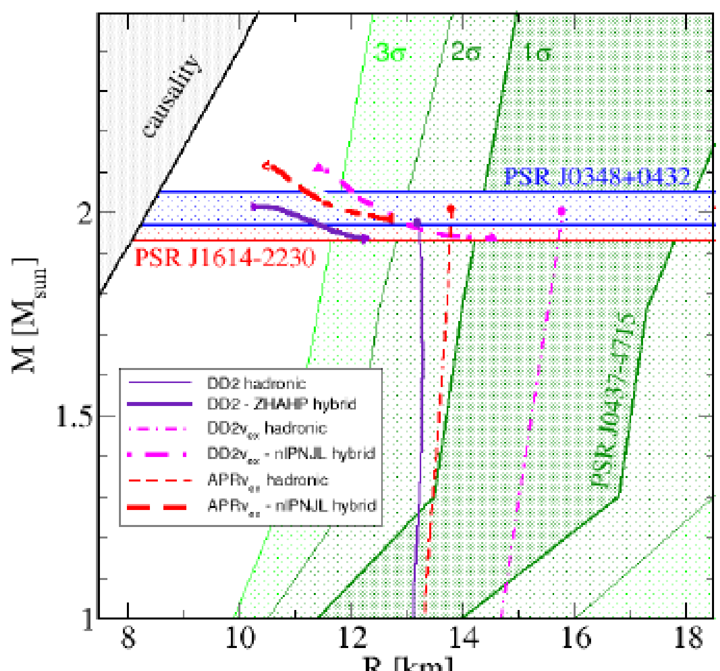



Endpoint of hadronic
Neutron star config.
At 2Msun, then strong
Phase transition




Strong phase trans.

High-mass twin stars




**Universal transition pressure ?**

Petran & Rafelski, PRC 88, 021901

$P_{trans}$ = 82 +/- 8 MeV/fm$^3$

# 2. hNJL model+ Rotatio

- existence of 2 M_sun pulsars and possibility of high-mass twins raises question for their inner structure: (Q)uark or (N)ucleon core ??
  --> degenerate solutions
  --> transition from N to Q branch

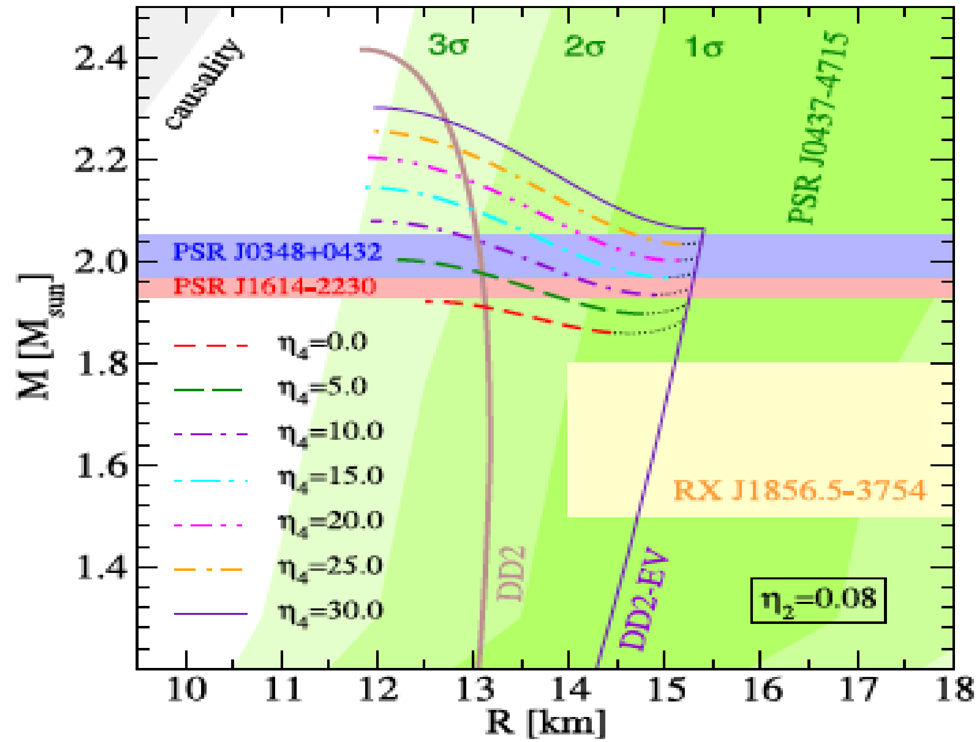
- PSR J1614-2230 is millisecond pulsar, period P = 3.41 ms, consider rotation !

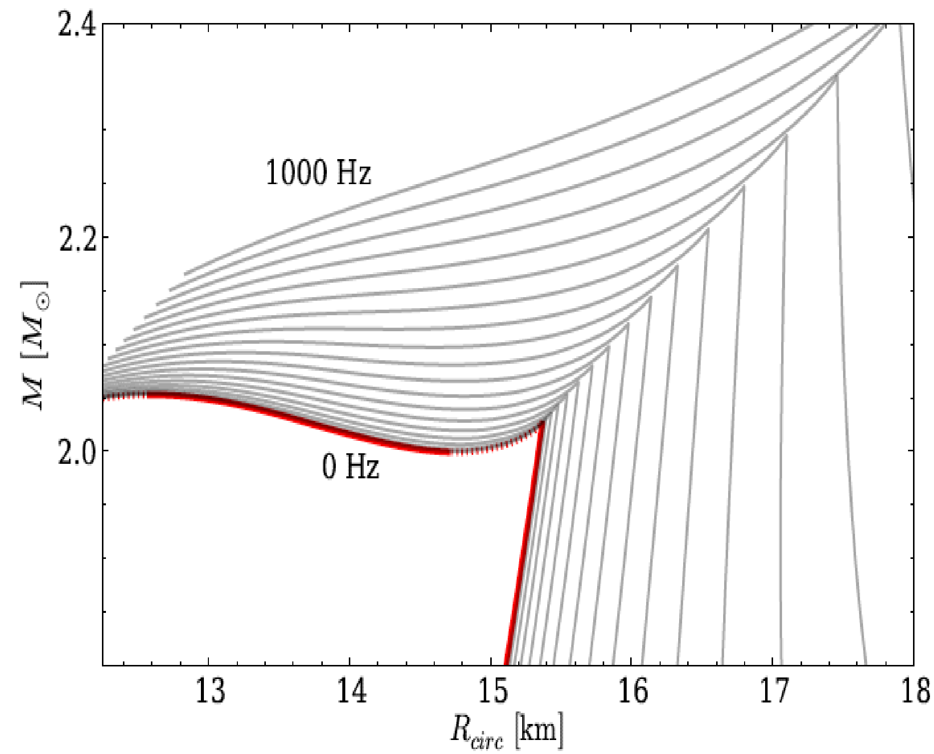
- transitions N --> Q must be considered for rotating configurations:
  --> how fast can they be?

(angular momentum J and baryon mass should be conserved simultaneously)

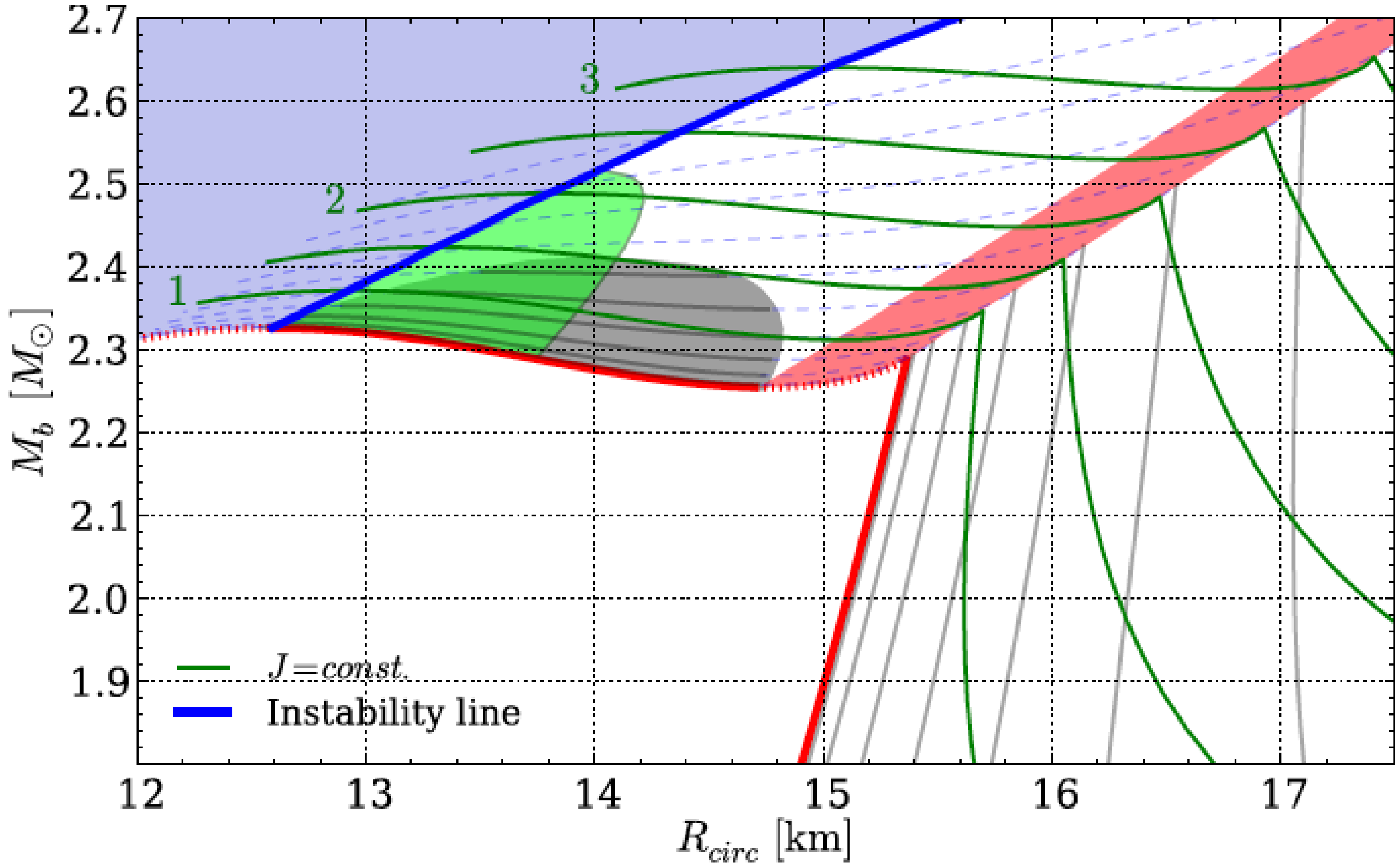
- similar scenario as fast radio bursts (Falcke-Rezzolla, 2013) or braking index (Glendenning-Pei-Weber, 1997)

**M. Bejger, D.B. et al., arxiv:1608.07049**



# 2. Rotation and stability




Red region - strong phase-transition instability,
Blue region - unstable w.r.t axisymmetric oscillations,
Grey region - no back-bending,
Green region - stable twin branch reached after the mini-collapse from the tip of $J = const$. curve, along $M_b = const$.

# 2. hNJL model + Rotation - summary

This type of instability EOS provides a ”natural” explanation for:

- ⋆ Lack of back-bending in radiopulsar timing,
- ⋆ Spin frequency cut-off at some moderate (but >716 Hz) frequency,
- ⋆ Falcke & Rezzolla Fast Radio Burst (FRB) engine
  - ⋆ catastrophic mini-collapse to the second branch (or to a black hole),
  - ⋆ massive rearrangement of the magnetic field $\rightarrow$ energy emission.

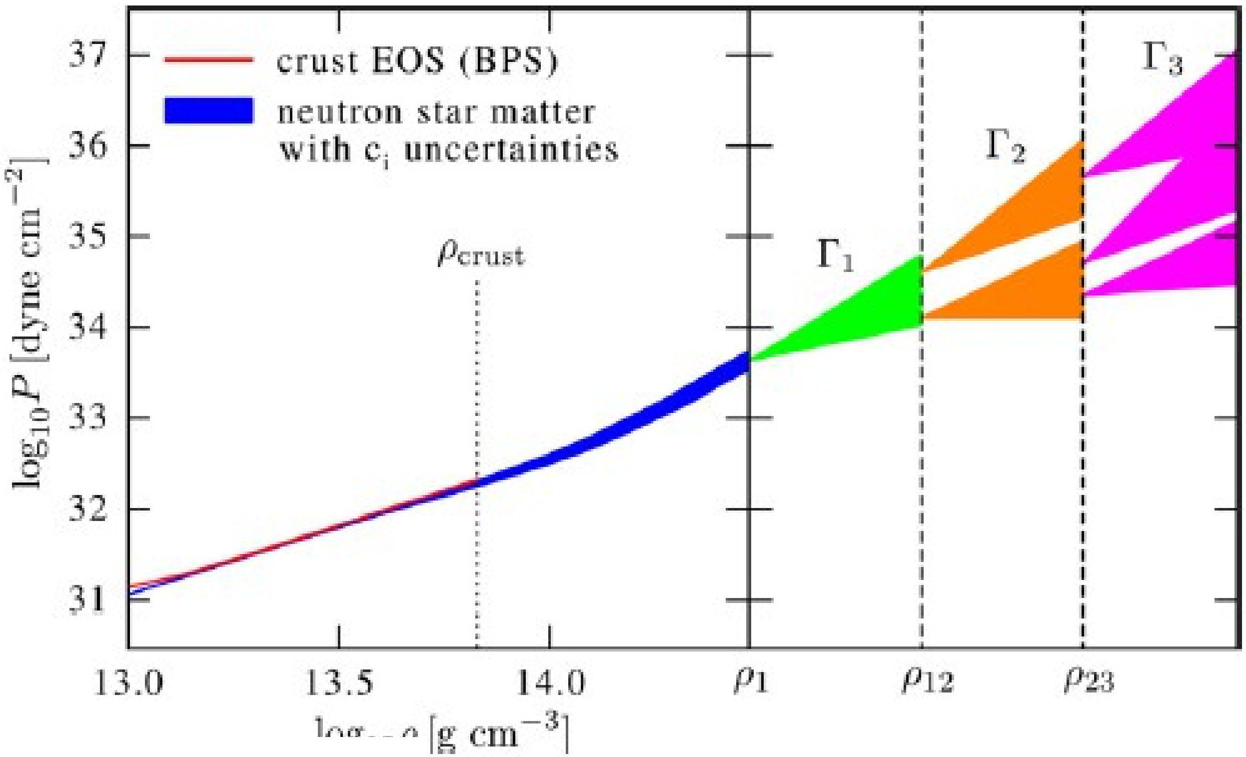
Astrophysical predictions:

- ⋆ Way to constraint on $M_b$, $J$, $I$, core EOS etc.,
- ⋆ Specific shape of NS-BH mass function (no mass gap?)
- $\rightarrow$ population of massive, low B-field NSs (radio-dead?),
- $\rightarrow$ population of massive, high B-field NSs (collapse enhances the field?),
- ⋆ Characteristic burst-like signature in GW emission during the mini-collapse.

**M. Bejger, D.B. et al., arxiv:1608.07049**

# 3. Piecewise polytrope EoS – high mass twins?

Hebeler et al., ApJ 773, 11 (2013)




$$P_i(n) = \kappa_i n^{\Gamma_i}$$

$$\begin{aligned}
i = 1 &: n_1 \leq n \leq n_{12} \\
i = 2 &: n_{12} \leq n \leq n_{23} \\
i = 3 &: n \geq n_{23} \ ,
\end{aligned}$$

Here, 1st order PT in region 2:

$$\Gamma_2 = 0 \text{ and } P_2 = \kappa_2 = P_{\text{crit}}$$

$$\begin{aligned}
P(n) &= n^2 \frac{d(\varepsilon(n)/n)}{dn}, \\
\varepsilon(n)/n &= \int dn \, \frac{P(n)}{n^2} = \int dn \, \kappa n^{\Gamma-2} = \frac{\kappa \, n^{\Gamma-1}}{\Gamma-1} + C, \\
\mu(n) &= \frac{P(n) + \varepsilon(n)}{n} = \frac{\kappa\Gamma}{\Gamma-1} n^{\Gamma-1} + m_0,
\end{aligned}$$

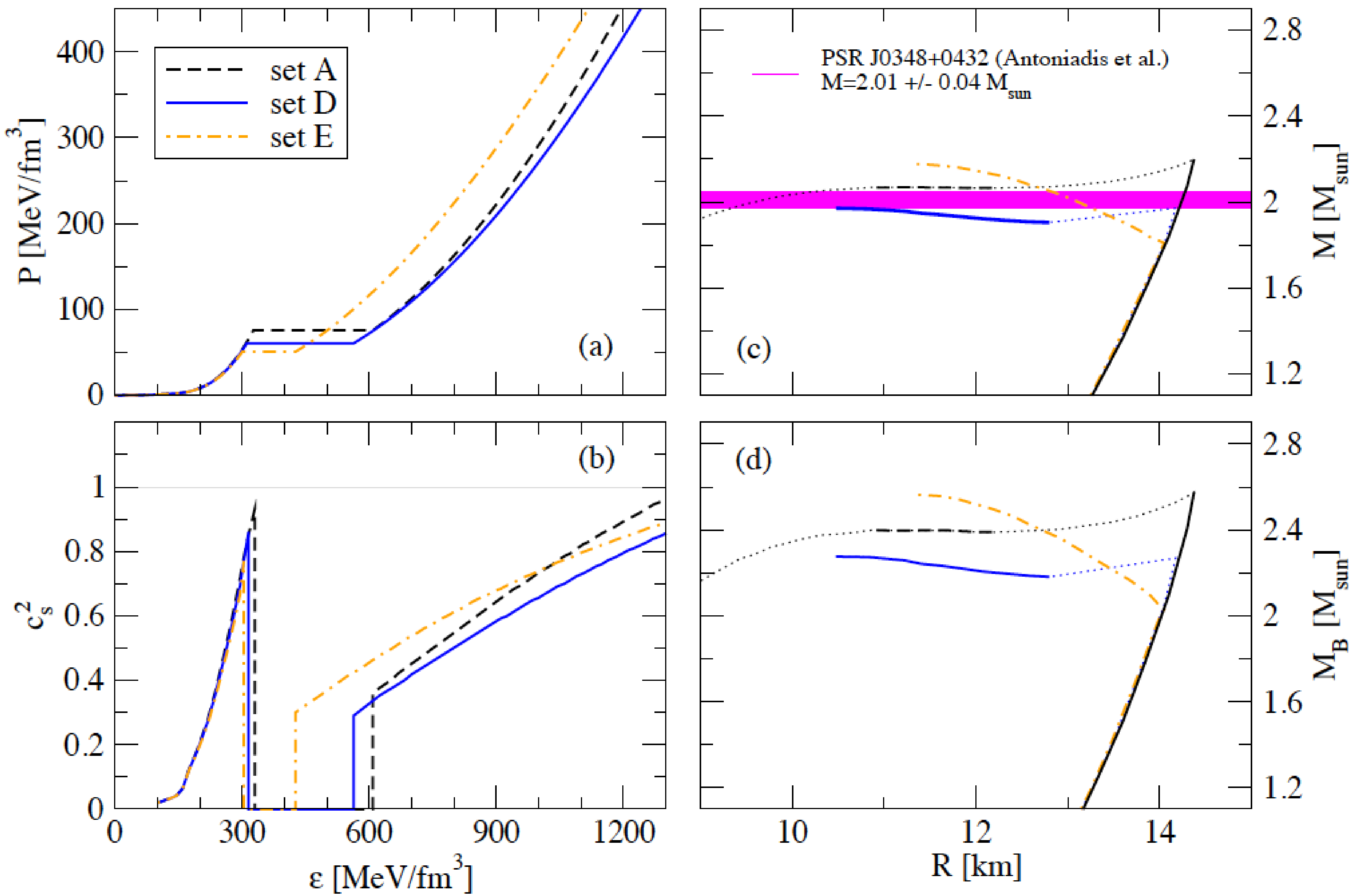
$$\begin{aligned}
n(\mu) &= \left[(\mu - m_0)\frac{\Gamma-1}{\kappa\Gamma}\right]^{1/(\Gamma-1)} \\
P(\mu) &= \kappa \left[(\mu - m_0)\frac{\Gamma-1}{\kappa\Gamma}\right]^{\Gamma/(\Gamma-1)}
\end{aligned}$$

Maxwell construction:

$$\begin{aligned}
P_1(\mu_{\text{crit}}) &= P_3(\mu_{\text{crit}}) = P_{\text{crit}} \\
\mu_{\text{crit}} &= \mu_1(n_{12}) = \mu_3(n_{23})
\end{aligned}$$

Seidov criterion for instability: $$\frac{\Delta\varepsilon}{\varepsilon_{\text{crit}}} \geq \frac{1}{2} + \frac{3}{3}\frac{P_{\text{crit}}}{\varepsilon_{\text{crit}}}$$

# 3. Piecewise polytrope EoS – high mass twins?




Third family solutions in the 2M_sun mass range (high-mass twins) exist !! **[Alvarez&D.B. (2016)]**

# 4. Density functional approach to quark matter

[M.A. Kaltenborn & D.B., in preparation (2016)]

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm free} + \mathcal{L}_{\rm int} = \bar{q}\left(i\partial\!\!\!/ - m_0\right) q \; + \mu_0\, \bar{q}\gamma^0 q + U\left(\bar{q}q, \bar{q}\gamma_0 q\right)$$

$$U\left(\bar{q}q, \bar{q}\gamma_0 q\right) = U\left(\langle\bar{q}q\rangle, \langle\bar{q}\gamma_0 q\rangle\right) + \left.\frac{\partial U\left(\bar{q}q, \bar{q}\gamma_0 q\right)}{\partial \bar{q}q}\right|_{\substack{\bar{q}q=\langle\bar{q}q\rangle \\ \bar{q}\gamma_0 q=\langle\bar{q}\gamma_0 q\rangle}} \left(\bar{q}q - \langle\bar{q}q\rangle\right)$$

$$+ \left.\frac{\partial U\left(\bar{q}q, \bar{q}\gamma_0 q\right)}{\partial \bar{q}\gamma_0 q}\right|_{\substack{\bar{q}q=\langle\bar{q}q\rangle \\ \bar{q}\gamma_0 q=\langle\bar{q}\gamma_0 q\rangle}} \left(\bar{q}\gamma_0 q - \langle\bar{q}\gamma_0 q\rangle\right) + \dots$$

$$\mathcal{L}_{\rm eff} \approx \mathcal{L}_{\rm free} + U + U_s \bar{q}q - U_s\langle\bar{q}q\rangle + U_v \bar{q}\gamma_0 q - U_v\langle\bar{q}\gamma_0 q\rangle$$

$$\bar{n}_s = \langle\bar{q}q\rangle \quad , \quad \bar{n}_v = \langle\bar{q}\gamma^0 q\rangle \; , \qquad \left.\frac{\partial U\left(\bar{q}q, \bar{q}\gamma_0 q\right)}{\partial \bar{q}q, \bar{q}\gamma_0 q}\right|_{\substack{\bar{q}q=\langle\bar{q}q\rangle \\ \bar{q}\gamma_0 q=\langle\bar{q}\gamma_0 q\rangle}} = U_{s,v}$$

$$P = -\left.\frac{\partial\Omega}{\partial V}\right|_{\mu,T} = g\int \frac{d^3p}{(2\pi)^3}\left[T\ln(1+e^{-\beta(E-\mu^*)}) + T\ln(1+e^{-\beta(E+\mu^*)})\right] + \Theta \qquad E = \sqrt{\vec{p}^2 + m^{*2}}.$$

$$m_i^* = m_{0,i} - \Sigma_{s,i}(\bar{n}_s) = m_{0,i} + \underbrace{D(\bar{n}_s)\bar{n}_s^{-\frac{1}{3}}}_{\text{confinement}} \qquad D(\bar{n}_s,\bar{n}_v) \;=\; D_0\Phi(\bar{n}_s,\bar{n}_v)$$

Available volume fraction:

$$\Phi(\bar{n}_s,\bar{n}_v) \;=\; e^{-\alpha(\bar{n}_v - n_0)^2}$$

$$\mu_i^*(\bar{n}_v) \;=\; \mu_{0,i} - \Sigma_{v,i}(\bar{n}_v) \qquad \Sigma_{v,i}(\bar{n}_v) \;=\; a\bar{n}_v + \frac{b\bar{n}_v^3}{1 + c\bar{n}_v^2}$$

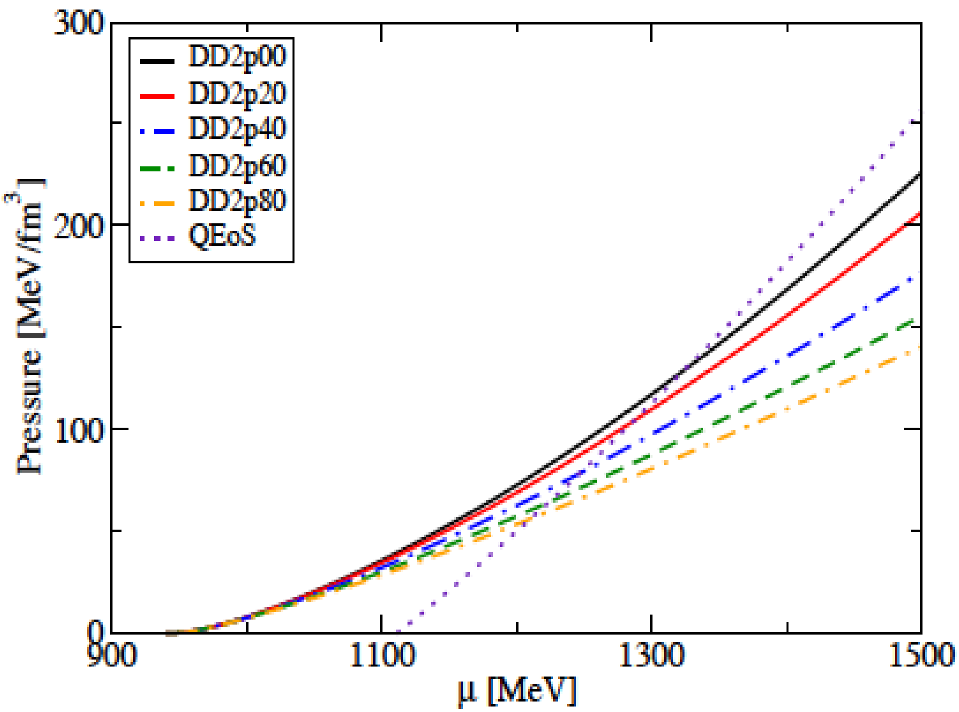
Thermodynamic consistency -->
Rearrangement selfenergies

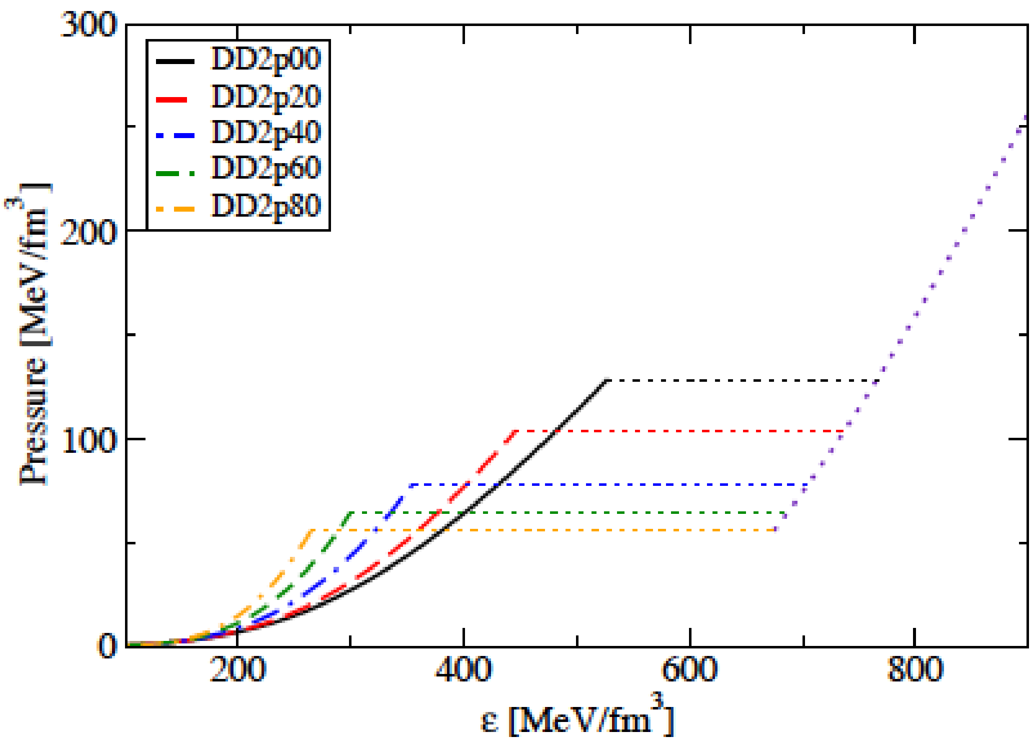
# 4. Density functional approach to quark matter

Hadronic matter: DD2 with excluded volume **[S. Typel, EPJA 52 (3) (2016)]**

$$\Phi_n = \Phi_p = \begin{cases} 1, & \text{if } n_{\mathrm{B}} < n_0 \\ e^{-\frac{v|v|}{2}(n_{\mathrm{B}}-n_0)^2}, & \text{if } n_{\mathrm{B}} > n_0 \end{cases}$$

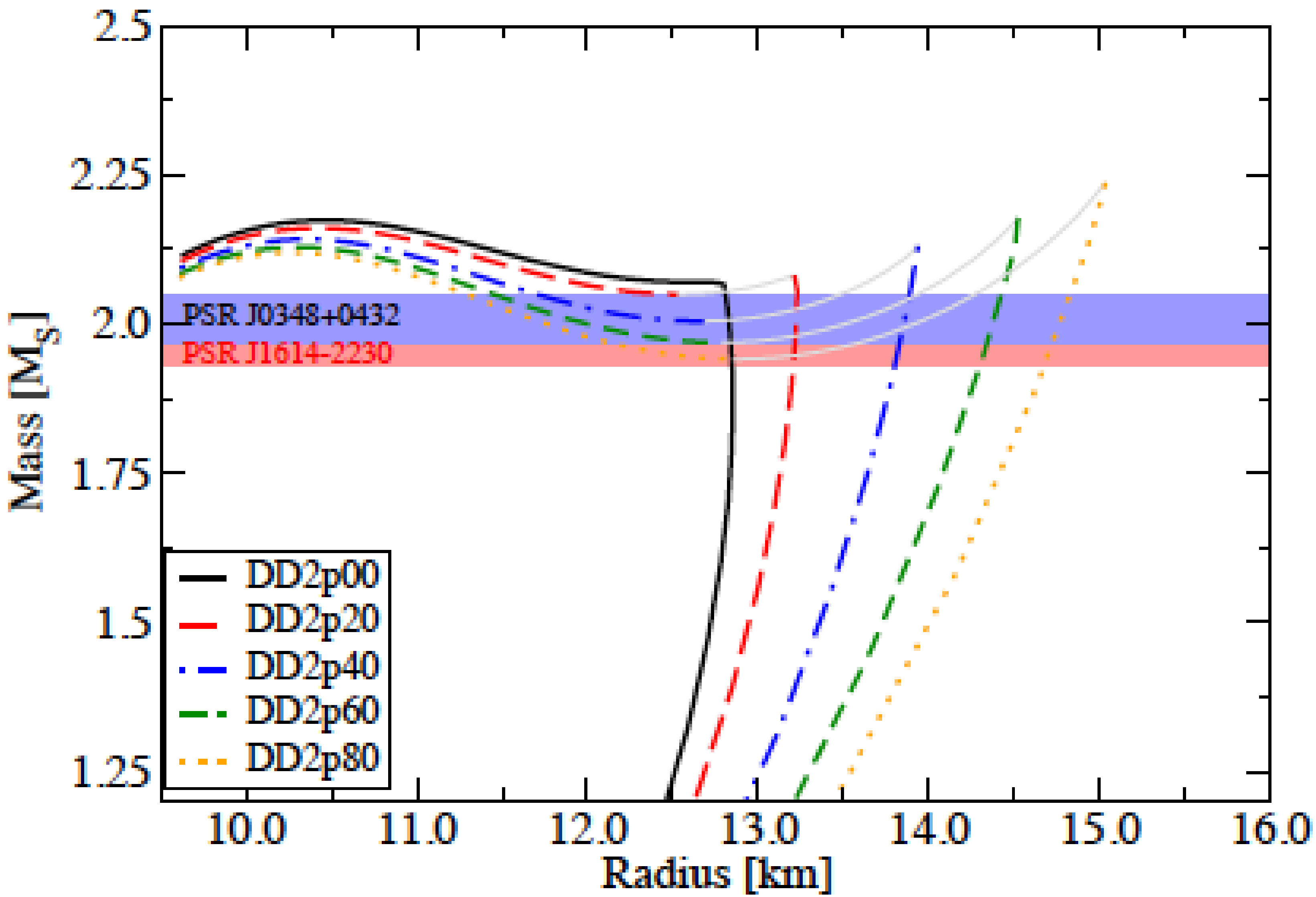
Varying the hadronic excluded volume parameter, p00 → v=0, … , p80 → v=8 fm^3



# 4. Density functional approach to quark matter

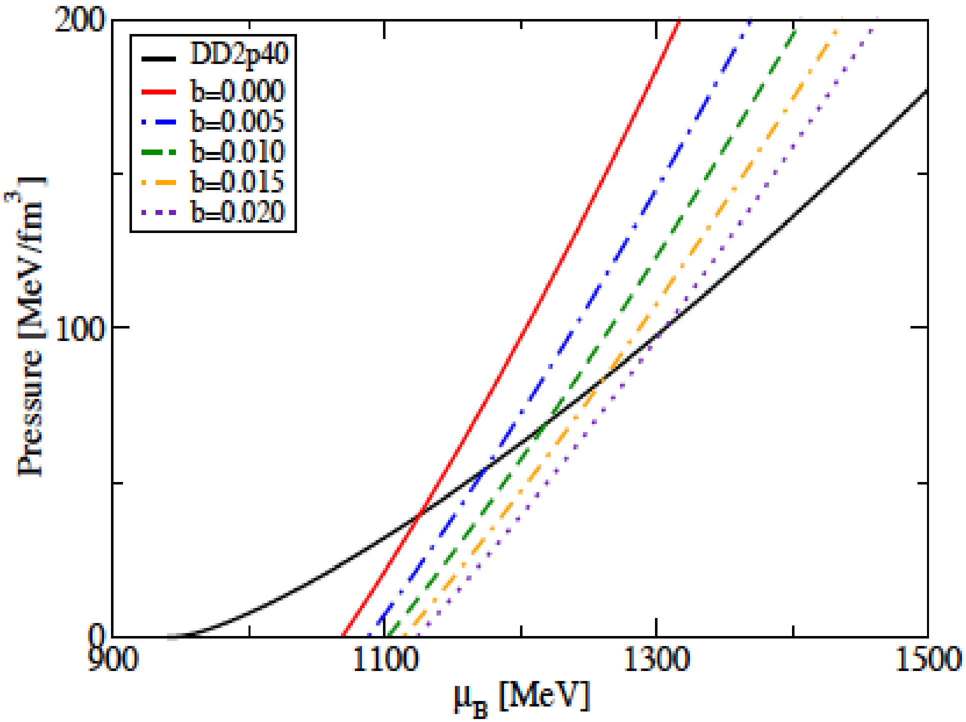
Varying the hadronic excluded volume parameter, p00 → v=0, … , p80 → v=8 fm^3

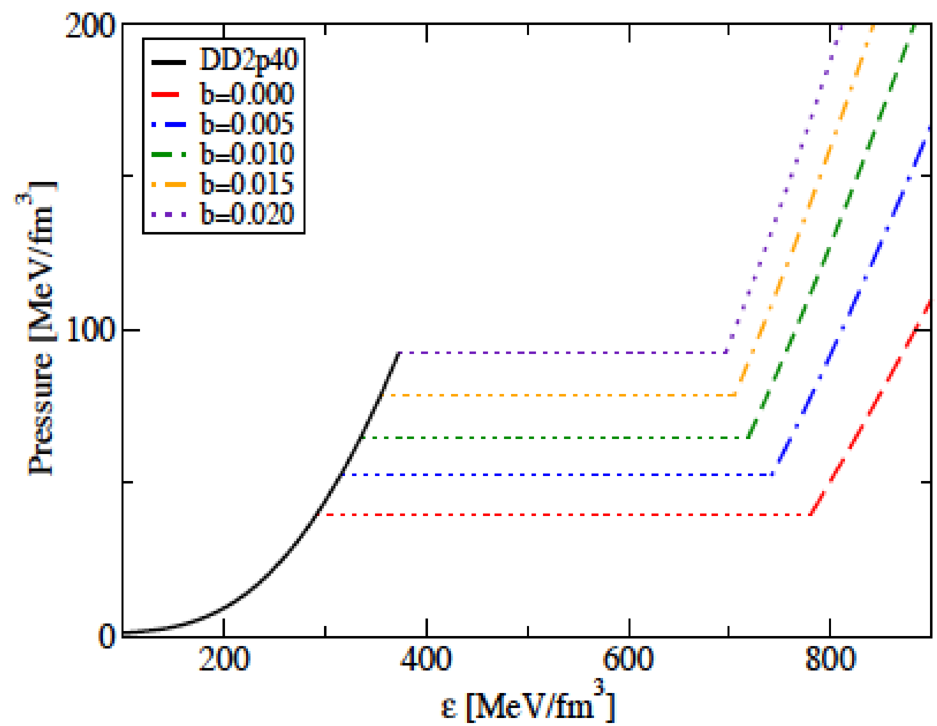



[M.A. Kaltenborn & D.B., in preparation (2016)]

# 4. Density functional approach to quark matter

Varying the higher order density coupling parameter b

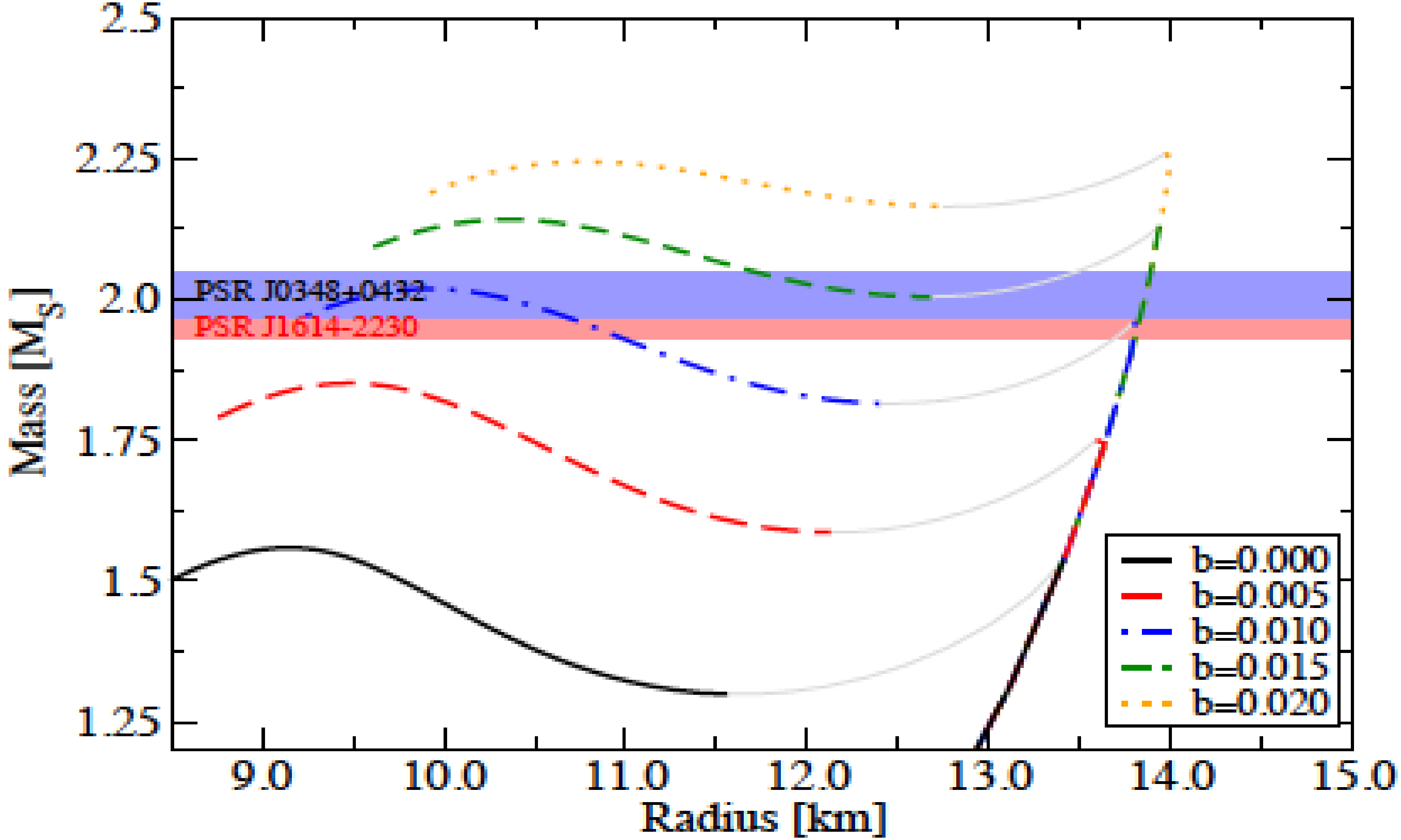



[M.A. Kaltenborn & D.B., in preparation (2016)]

# 4. Density functional approach to quark matter

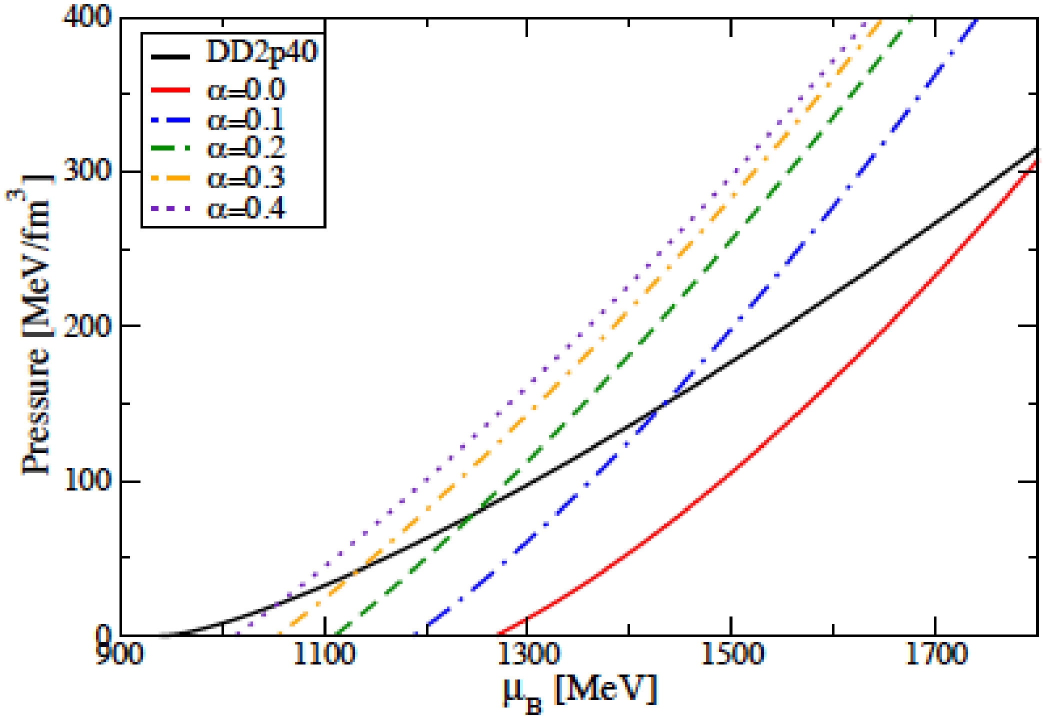
Varying the higher order density coupling parameter b

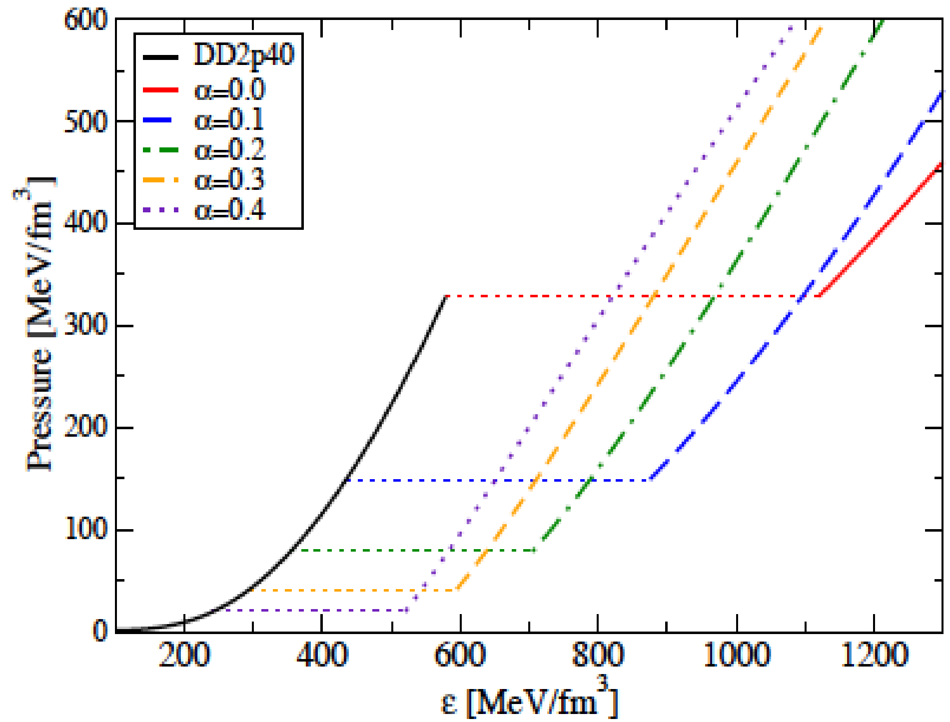



[M.A. Kaltenborn & D.B., in preparation (2016)]

# 4. Density functional approach to quark matter

Varying the available volume parameter alpha

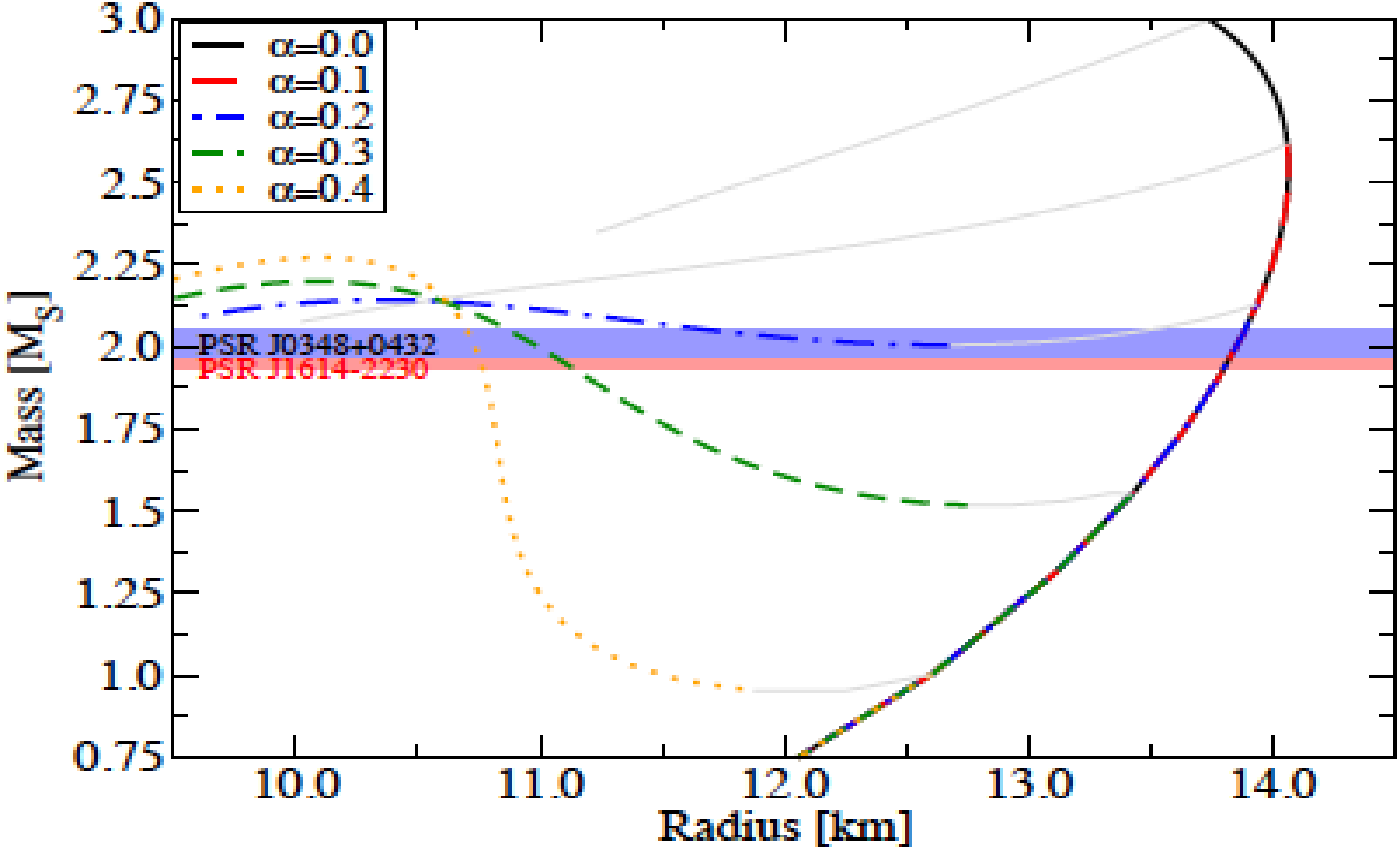



[M.A. Kaltenborn & D.B., in preparation (2016)]

# 4. Density functional approach to quark matter

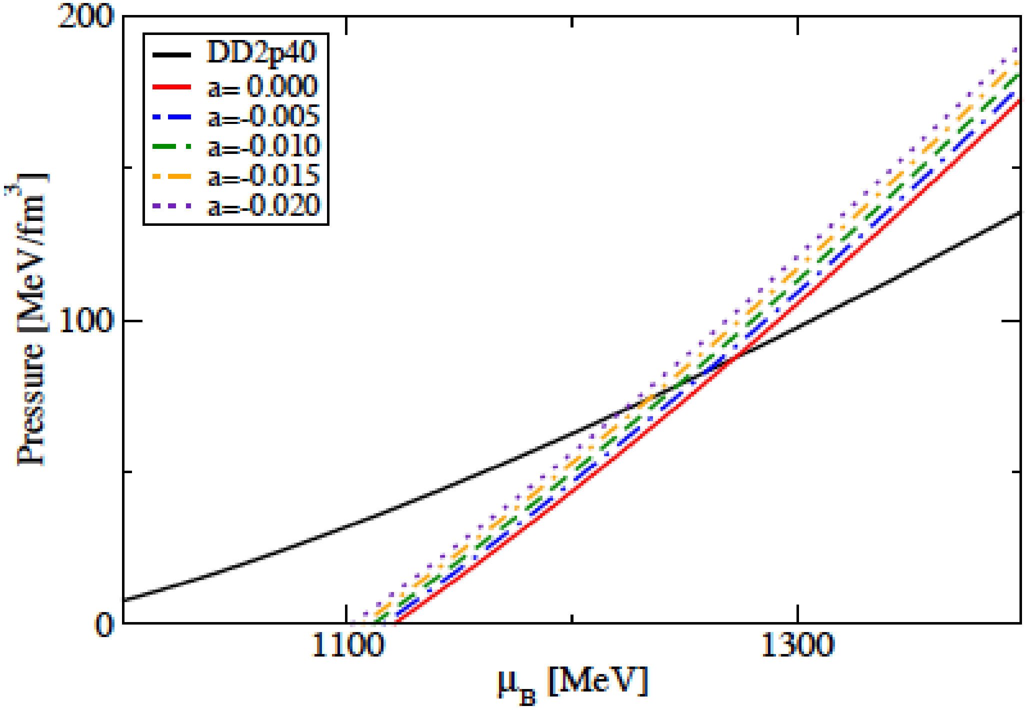
Varying the available volume parameter alpha

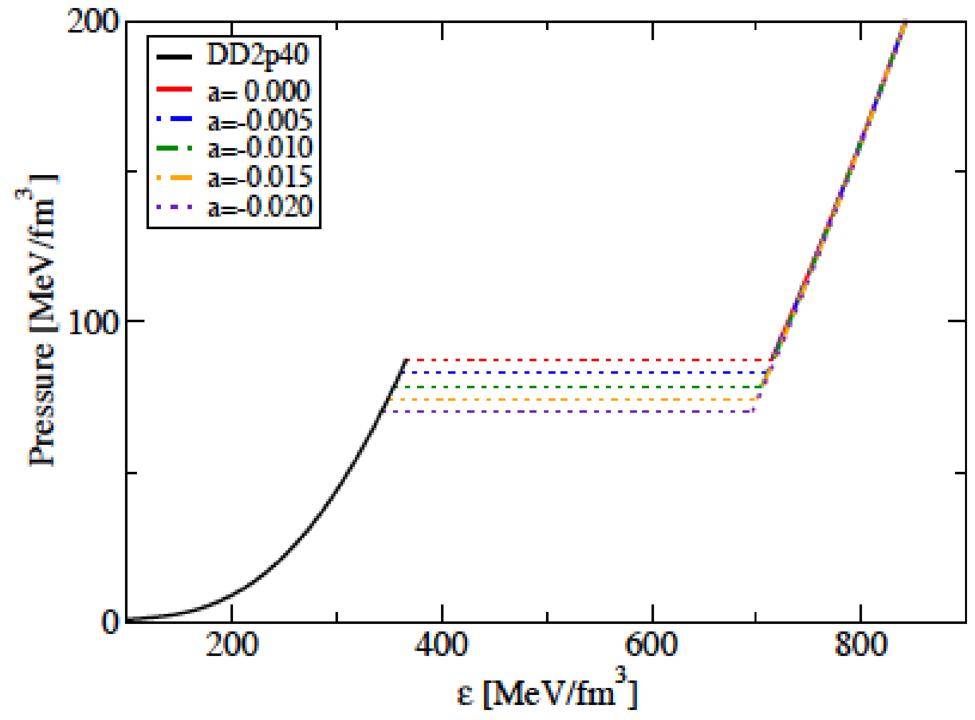



[M.A. Kaltenborn & D.B., in preparation (2016)]

# 4. Density functional approach to quark matter

Varying the 4-quark coupling parameter a

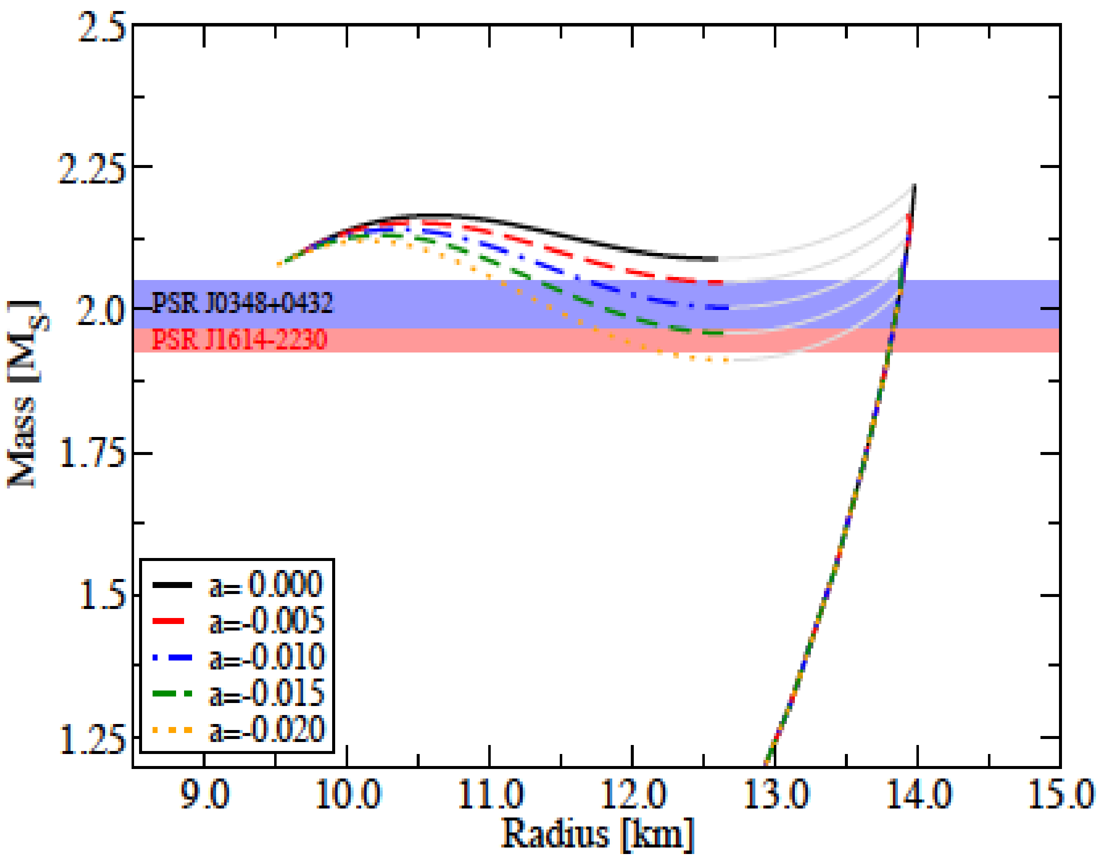


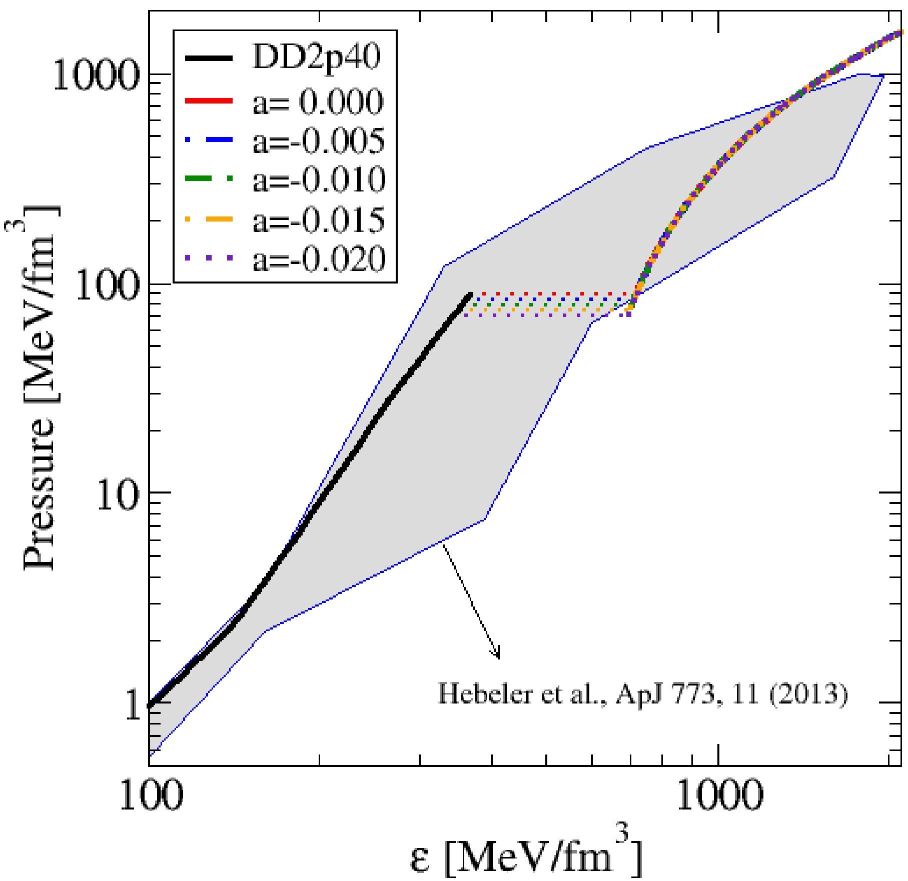




[M.A. Kaltenborn & D.B., in preparation (2016)]

# 4. Density functional approach to quark matter

Varying the 4-quark coupling parameter a

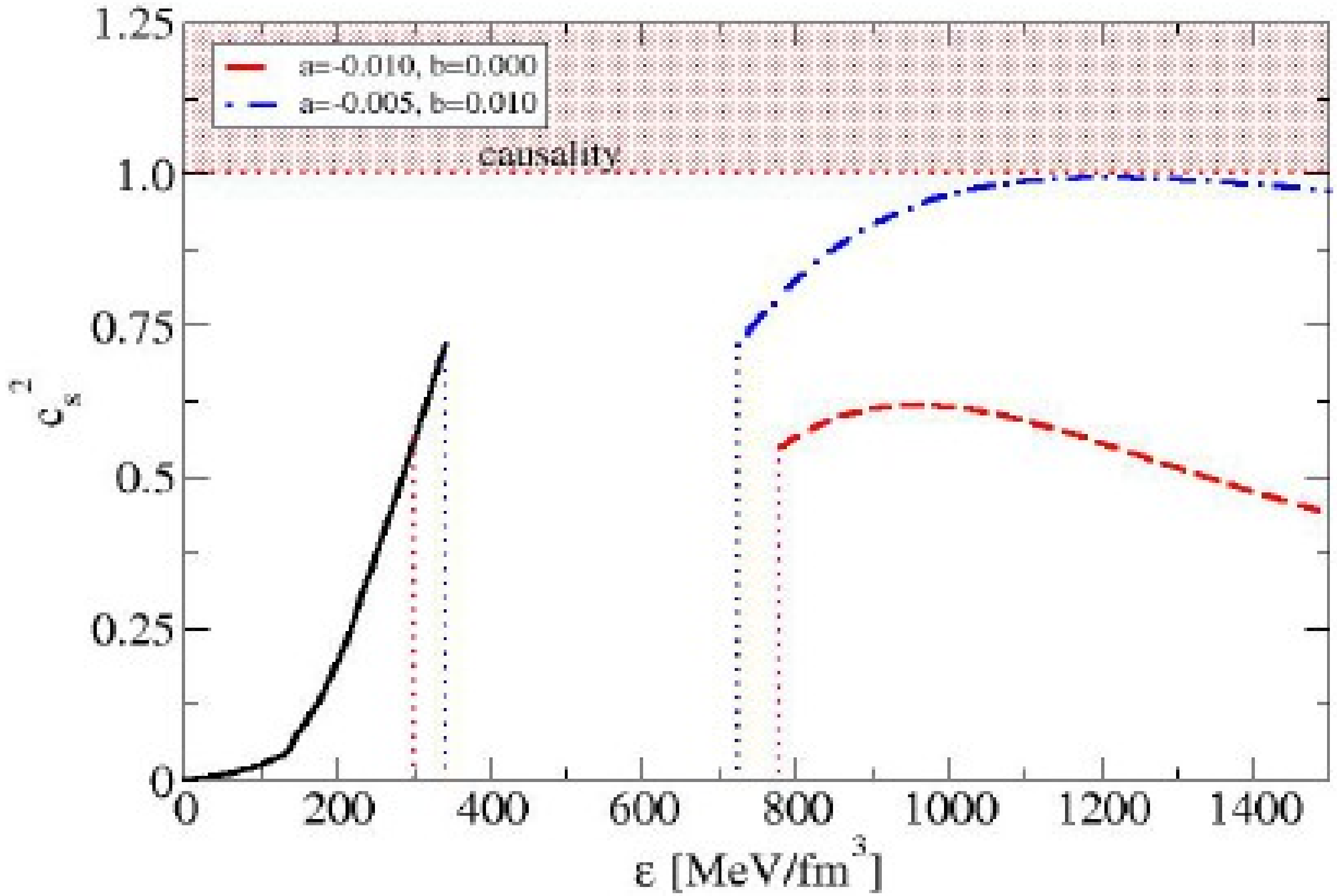



[M.A. Kaltenborn & D.B., in preparation (2016)]

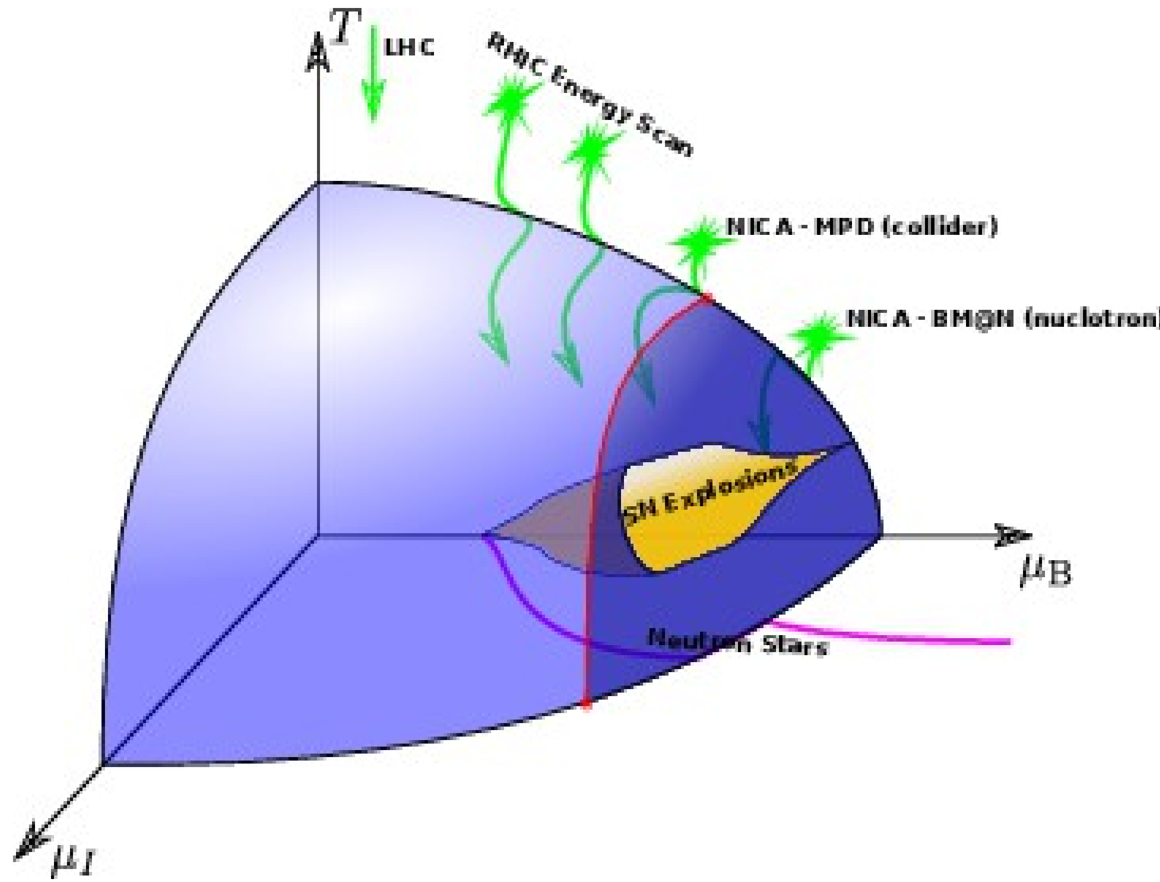
# 4. Density functional approach to quark matter

Check the speed of sound (causality)




[M.A. Kaltenborn & D.B., in preparation (2016)]

# Conclusion:



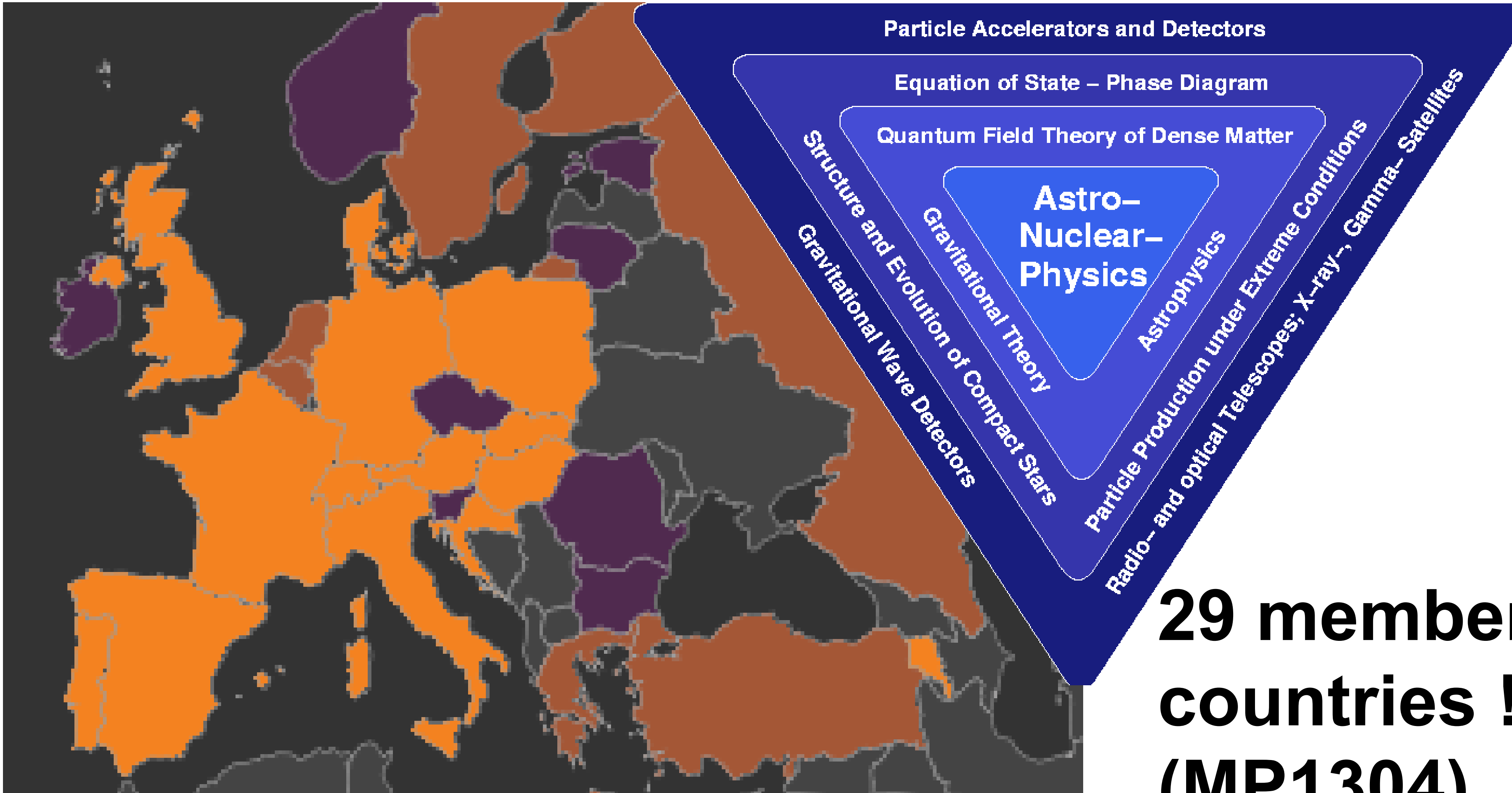

High-mass twins (HMTs) with quark matter cores can be obtained within different hybrid star EoS models, e.g.,
- constant speed of sound
- higher order NJL
- piecewise polytrope
- density functional

HMTs require stiff hadronic and quark matter EoS with a strong phase transition (PT)

Existence of HMTs can be verified, e.g., by precise compact star mass and radius observations (and a bit of good luck) → Indicator for strong PT !!

Extremely interesting scenarios possible for dynamical evolution of isolated (spin-down and accretion) and binary (NS-NS merger) compact stars

**Critical endpoint search in the QCD phase diagram with Heavy-Ion Collisions goes well together with Compact Star Astrophysics**



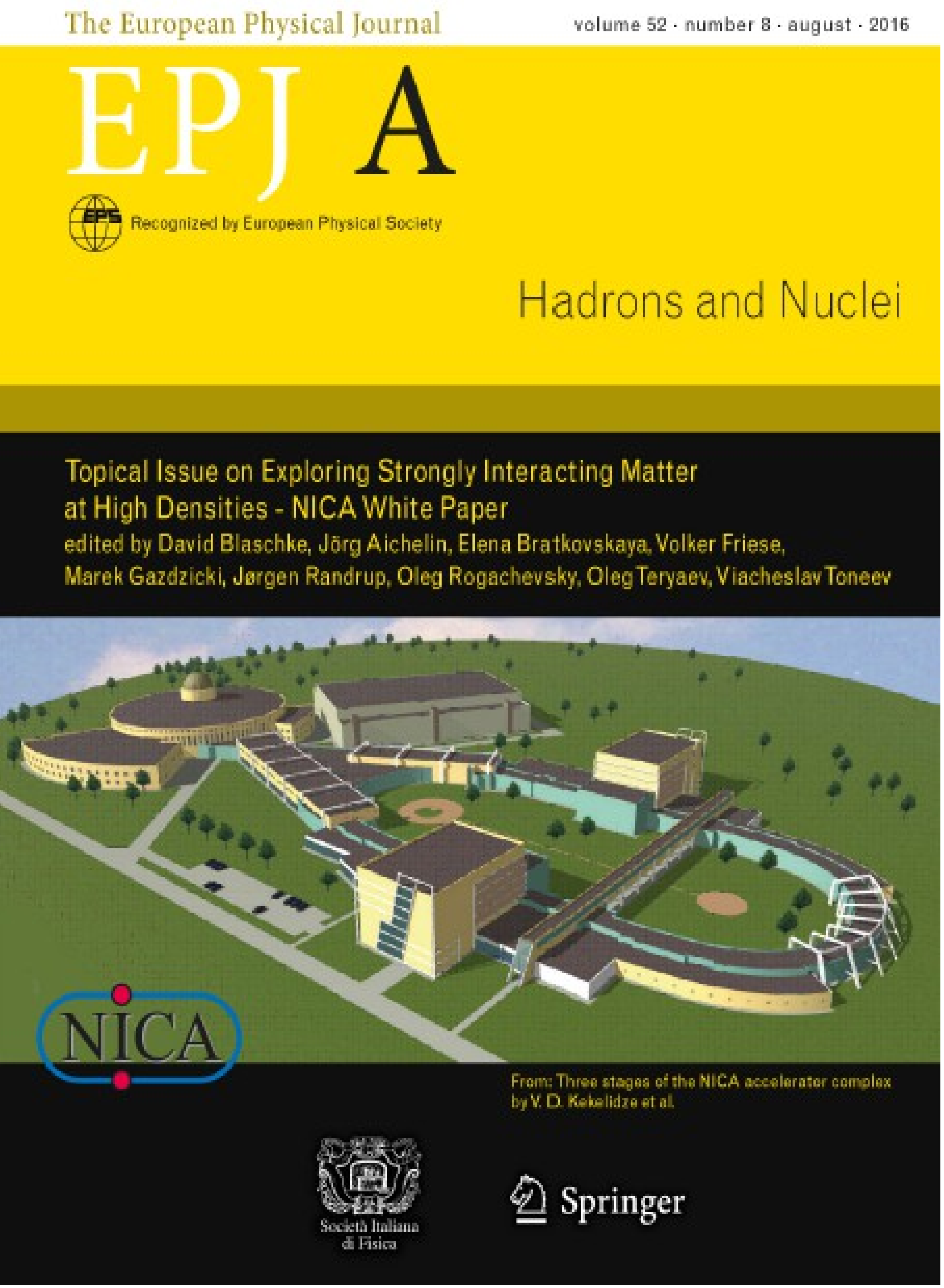

**29 member countries !! (MP1304)**

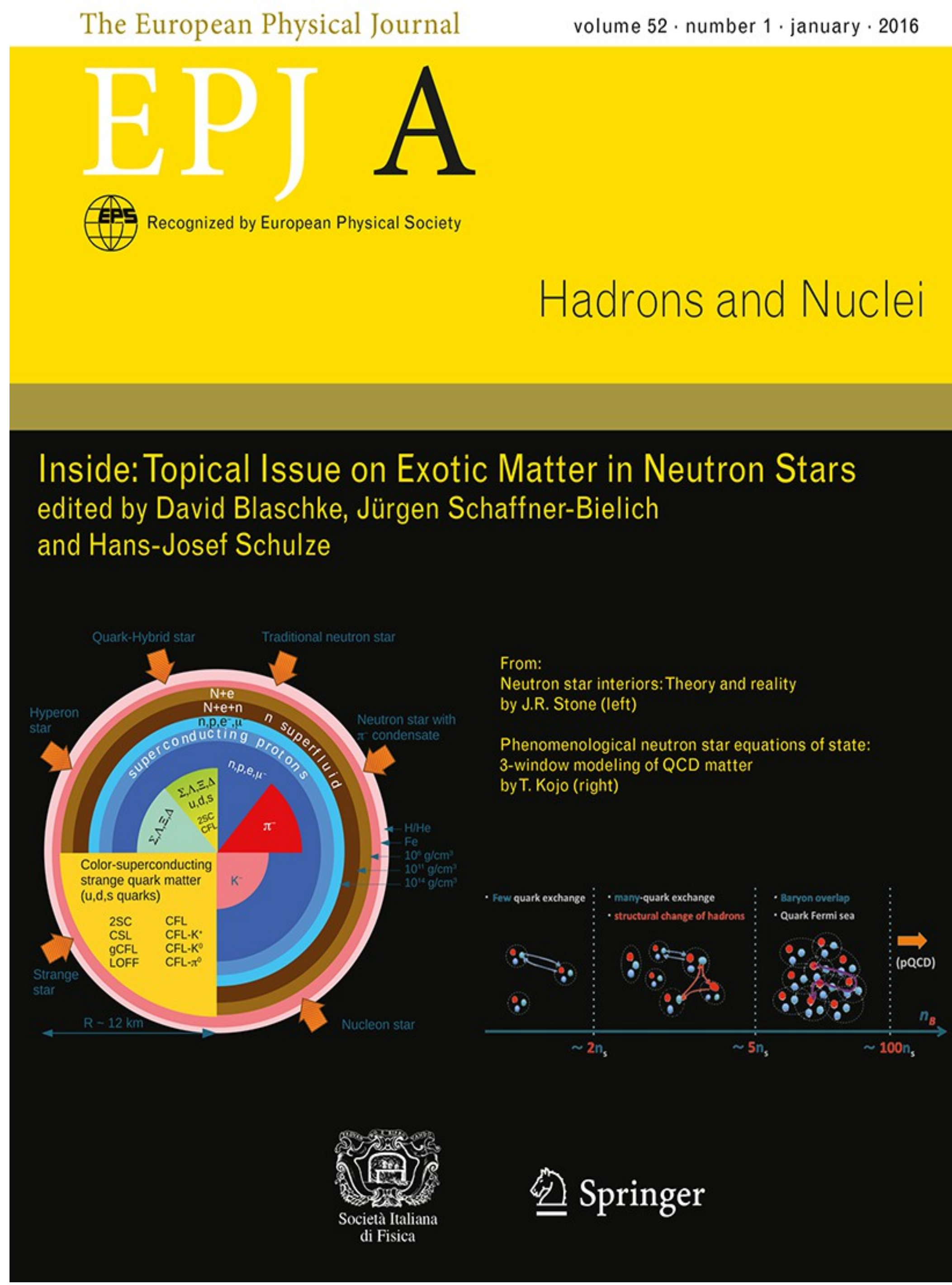
**New** compstar **!**

cost
EUROPEAN COOPERATION
IN SCIENCE AND TECHNOLOGY

**Kick-off: Brussels, November 25, 2013**

International Conference “Critical Point and Onset of Deconfinement”
University of Wroclaw, May 29 – June 4, 2016

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