EBERHARD KARLS UNIVERSITÄT TÜBINGEN

‘Beyond’ General Relativity

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Theoretical Astrophysics

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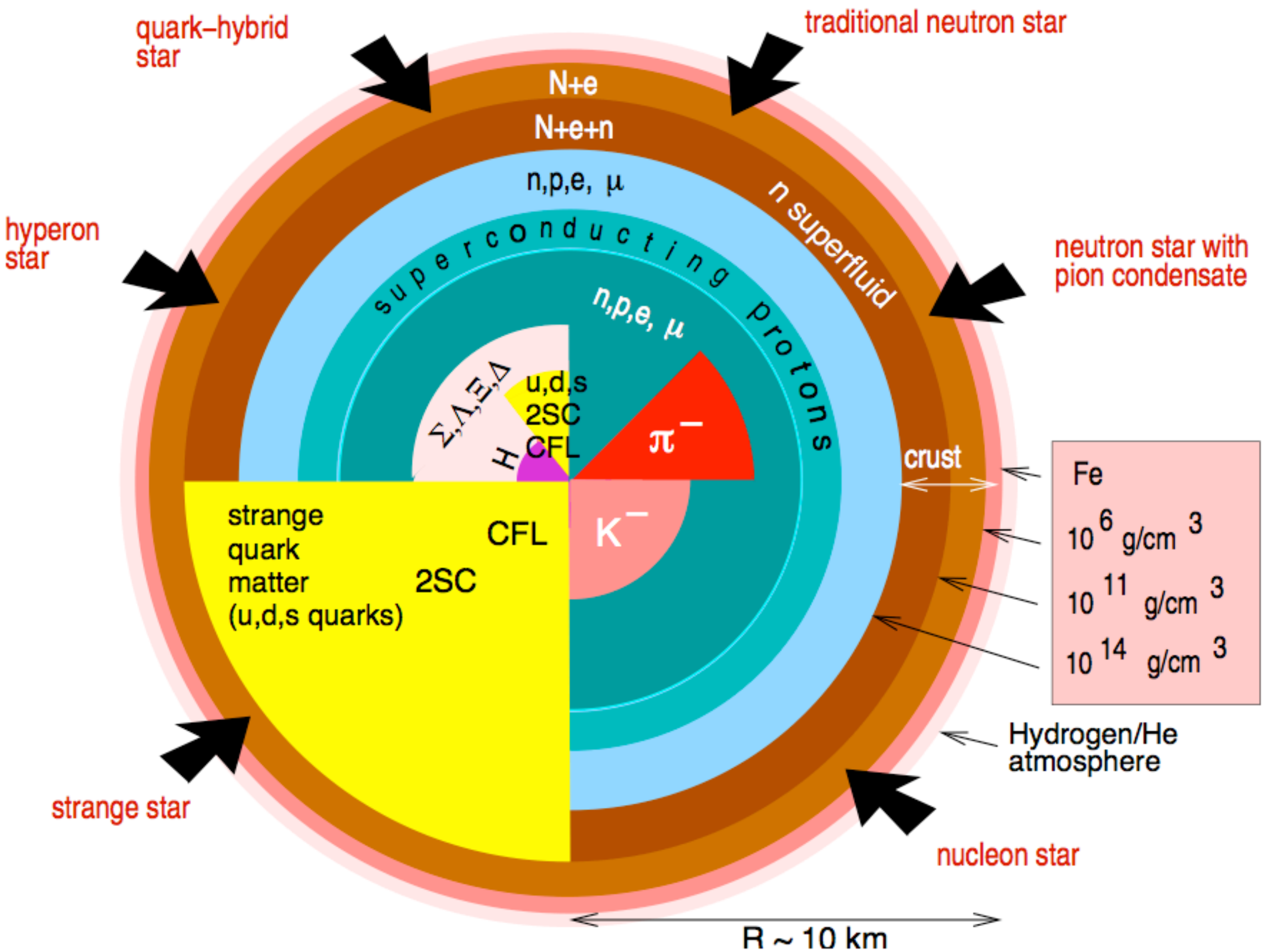
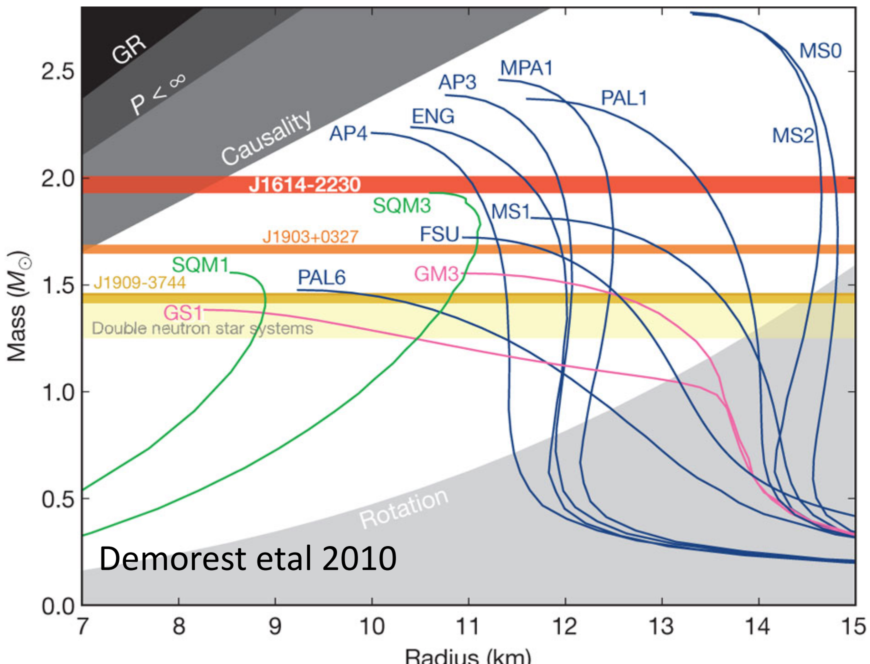
Zooming into a Neutron Star

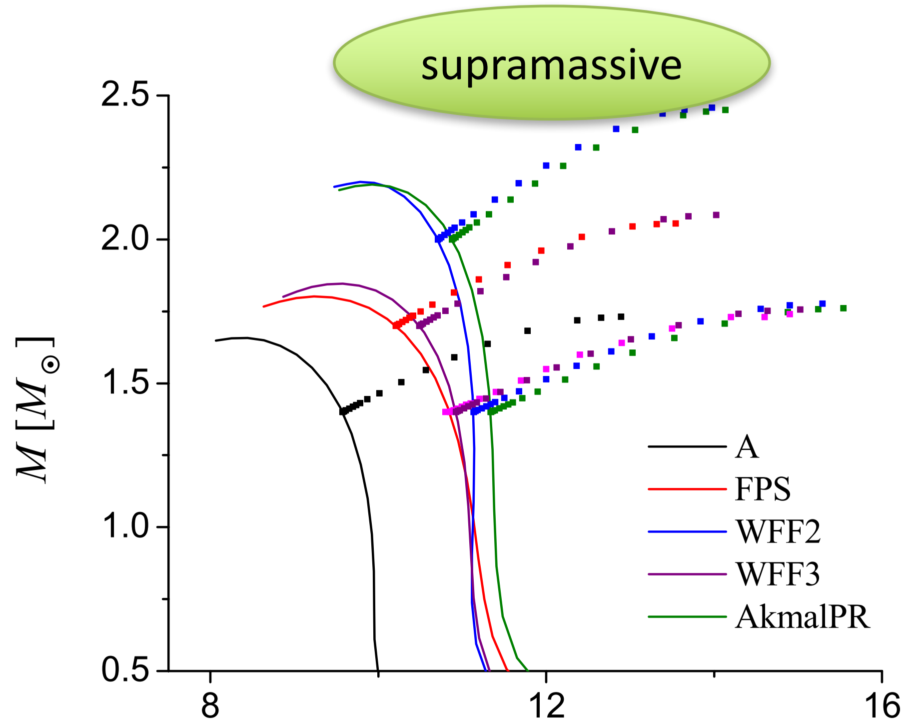


Neutron Stars: Mass vs Radius

Static Models



Rotating Models



The holy grail of NS astrophysics… is the determination of the equation of state (EOS) of matter at supra-nuclear densities.
The most direct way of constraining the EOS is to measure simultaneously the neutron star mass and radius.

Constraints on NS Radius

Main methods in EM spectrum:

- Thermonuclear X-ray bursts (photospheric radius expansion)
- Burst oscillations (rotationally modulated waveform)
- Fits of thermal spectra to cooling neutron stars
- khZ QPOs in accretion disks around neutron stars
- Pericenter precession in relativistic binaries (double pulsar J0737)

Soon also via observations in the GW spectrum

Main methods in GW spectrum:

- Tidal effects on waveform during inspiral phase of NS-NS mergers
- Tidal disruption in BH-NS mergers
- Post-merger phase of NS-NS mergers and Oscillations

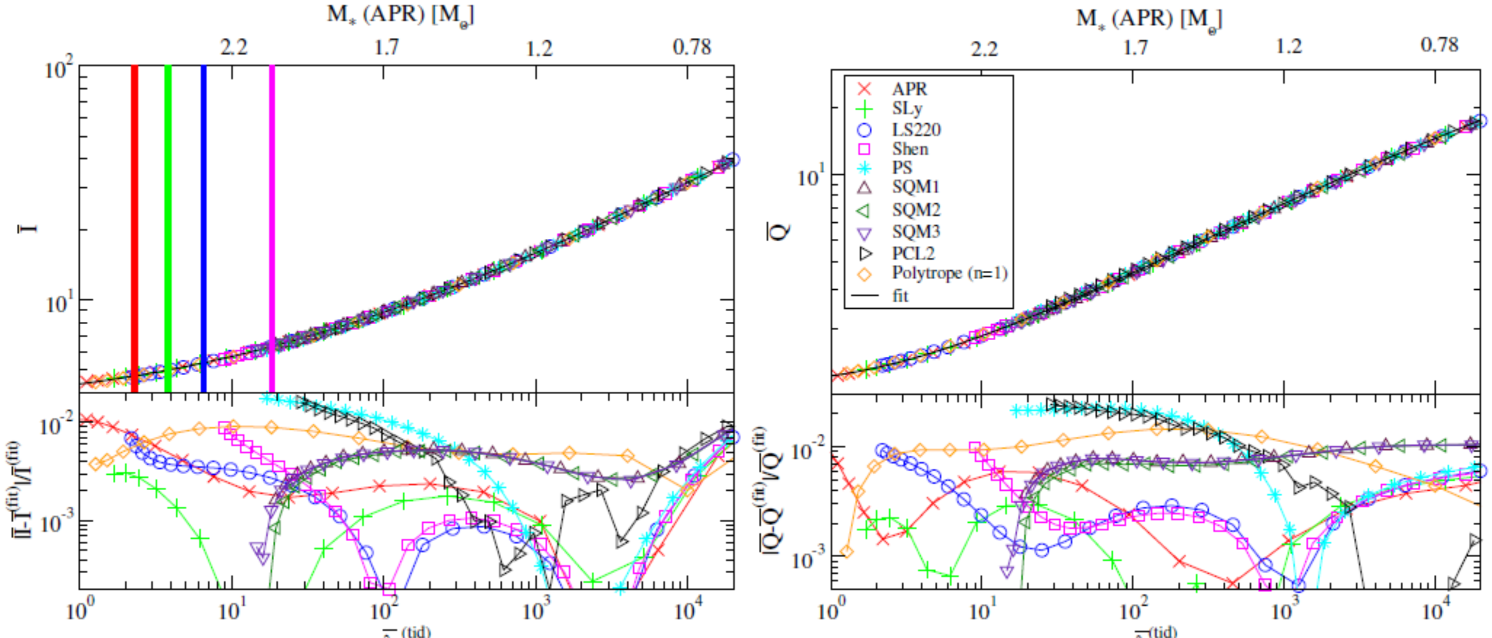
Neutron Stars & “universal relations”

Need for relations between the “observables” and the “fundamentals” of NS physics

Average Density	$\bar{\rho} \sim M / R^3$	
Compactness	$z \sim M / R$	$\eta = \sqrt{M^3 / I}$
Moment of Inertia	$I \sim MR^2$	$I \sim J / \Omega$
Quadrupole Moment	$Q \sim R^5 \Omega^2$	
Tidal Love Numbers	$\lambda \sim I^2 Q$	

I-Love-Q relations

EOS independent relations were derived by **Yagi & Yunes(2013**) for non-magnetized stars in the slow-rotation and small tidal deformation approximations.



… the relations proved to be valid (*with appropriate normalizations*) even for *fast rotating* and *magnetized* stars

✓ Yagi-Yunes Phys. Reports (arXiv:1608.02582)

Oscillations & Instabilities

p-modes: main restoring force is the **pressure** (**f-mode**) ***(>1.5 kHz)***

$$\sigma \approx \sqrt{\frac{GM}{R^3}}$$

Inertial modes: **(r-modes)** main restoring force is the **Coriolis force**

$$\sigma \approx \Omega$$

w-modes: pure **space-time modes** (only in GR) ***(>5kHz)***

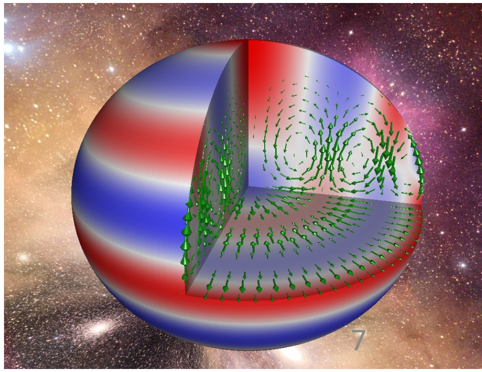
$$\sigma \approx \frac{1}{R}\left(\frac{GM}{Rc^2}\right)$$

Torsional modes (t-modes) ***(>20 Hz)*** shear deformations. **Restoring force, the weak Coulomb force of the crystal ions.**

$$\sigma \approx \frac{v_S}{R} \sim 16\ \ell\ \mathrm{Hz}$$

… and many more

shear, g-, Alfven, interface, … modes

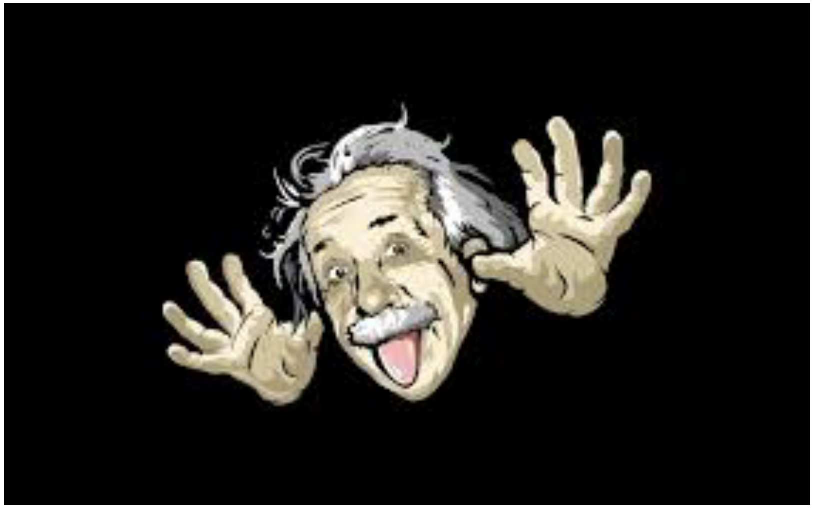


Famous Men Words

- EINSTEIN:
  - I would feel sorry for the good Lord. The theory is correct
- CHANDRASEKHAR (to C.M. Will)
  - Why do you spend so much time and energy testing GR? We **know** that the theory is right.

However, there is growing theoretical and experimental evidence that modifications of GR at **small** and **large** energies are somehow inevitable.

NEUTRON STARS &
ALTERNATIVE THEORIES OF GRAVITY



Expect (and Prepare for) The Unexpected

ATG and Neutron Stars

The **enormous gravitational field** of NSs, the **high density of matter** at their cores and the existence of pulsars with **fast spin** and **large magnetic fields** make them ideal laboratories to study all fundamental interactions

- The structure of compact stars depends on the **coupling of gravity with matter** in strong-field regions.
- NSs are a valuable alternative to BHs in tests of strong-field gravity, because **they can probe** (and possibly rule out) those theories that are close to GR in vacuum, but differ in the description of the coupling between matter and gravity.

Alternative theories of gravity: Motivation

Motivation for modifying General Relativity

Theory

Theories trying to unify all interactions: Kaluza-Klein theories, higher dimensional gravity, etc.

Quantum corrections in the strong field regime

Observations

Dark energy and dark matter does not fit well in the standard GR framework

The strong field regime of gravity is essentially unconstrained

Studying alternative theories of gravity can give us a deeper insight in GR itself

Lovelock's theorem

- **GR emerges as the unique theory of gravity** under specific assumptions
- In 4D spacetimes the only **divergence-free symmetric rank-2 tensor constructed solely** from the metric $\mathbf{g}_{\mu\nu}$ and its derivatives up to **second** differential order, and preserving diffeomorphism invariance, is the **Einstein tensor** plus **a cosmological term**.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \qquad \nabla_\mu T^{\mu\nu} = 0$$

Einstein-Hilbert action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\,R + S_M\left[\Psi,\, g_{\mu\nu}\right]$$

The main alternative theories (ATG)

1. **Scalar-tensor theories and their generalizations.** Including multiscalar and Horndeski theories
2. **F(R) theories**
3. **Theories whose action contains terms quadratic in curvature.** Including Einstein-dilaton-Gauss-Bonnet (EdGB) and dynamical Chern-Simons (dCS) theories
4. **Lorentz-violating theories.** Including Einstein-Aether, Hořava and n-Dirac-Born-Infeld (n-DBI) gravity.
5. **Massive gravity theories**
6. **Theories involving non-dynamical fields.** Including the Palatni formulation of F(R) gravity and Eddington-inspired Born-Infeld (EiBI) gravity.

Berti etal (2015)

Catalog of ATG vs Lovelock

Table 1. Catalog of several theories of gravity and their relation with the assumptions of Lovelock's theorem. Each theory violates at least one assumption (see also figure 1), and can be seen as a proxy for testing a specific principle underlying GR.

Theory	Field content	Strong EP	Massless graviton	Lorentz symmetry	Linear $T_{\mu\nu}$	Weak EP	Well- posed?	Weak-field constraints
Extra scalar field								
Scalar–tensor	S	×	✓	✓	✓	✓	✓[34]	[35–37]
Multiscalar	S	×	✓	✓	✓	✓	✓[38]	[39]
Metric $f(R)$	S	×	✓	✓	✓	✓	✓[40, 41]	[42]
Quadratic gravity								
Gauss–Bonnet	S	×	✓	✓	✓	✓	✓?	[43]
Chern–Simons	P	×	✓	✓	✓	✓	×✓? [44]	[45]
Generic	S/P	×	✓	✓	✓	✓	?	
Horndeski	S	×	✓	✓	✓	✓	✓?	
Lorentz-violating								
Æ-gravity	SV	×	✓	×	✓	✓	✓?	[46–49]
Khronometric/ Hořava–Lifshitz	S	×	✓	×	✓	✓	✓?	[48–51]
n-DBI	S	×	✓	×	✓	✓	?	none ([52])
Massive gravity								
dRGT/Bimetric	SVT	×	×	✓	✓	✓	?	[17]
Galileon	S	×	✓	✓	✓	✓	✓?	[17, 53]
Nondynamical fields								
Palatini $f(R)$	—	✓	✓	✓	×	✓	✓	none
Eddington–Born–Infeld	—	✓	✓	✓	×	✓	?	none
Others, not covered here								
TeVeS	SVT	×	✓	✓	✓	✓	?	[37]
$f(R)\mathcal{L}_m$	?	×	✓	✓	✓	×	?	
$f(T)$	?	×	✓	×	✓	✓	?	[54]

Note. See text for details of the entries. Key to abbreviations: S: scalar; P: pseudoscalar; V: vector; T: tensor; ?: unknown; ✓?: not explored in detail or not rigorously proven, but there exist arguments to expect ✓. The occurrence of ×✓? means that there exist arguments in favor of well-posedness within the EFT formulation, and against well-posedness for the full theory. Weak-field constraints (as opposed to strong-field constraints, which are the main topic of this review) refer to Solar System and binary pulsar tests. Entries below "Others, not covered here" are not covered in this review.

Berti etal (2015)

Catalog of NS properties in ATG

Table 3. Catalog of NS properties in several theories of gravity. Symbols and abbreviations are the same as in table 2.

Theory	Structure			Collapse	Sensitivities	Stability	Geodesics
	NR	SR	FR				
Extra scalar field							
Scalar–tensor	[26, 114–118]	[116, 119, 120]	[121–123]	[124–131]	[132]	[133–143]	[122, 144]
Multiscalar	?	?	?	?	?	?	?
Metric $f(R)$	[145–157]	[158]	[159]	[160, 161]	?	[162, 163]	?
Quadratic gravity							
Gauss–Bonnet	[164]	[164]	[82]	?	?	?	?
Chern–Simons	≡GR	[27, 45, 165–167]	?	?	[166]	?	?
Horndeski	?	?	?	?	?	?	?
Lorentz-violating							
Æ-gravity	[168, 169]	?	?	[170]	[48, 49]	[162]	?
Khronometric/ Hořava–Lifshitz	[171]	?	?	?	[48, 49]	?	?
n-DBI	?	?	?	?	?	?	?
Massive gravity							
dRGT/Bimetric	[172, 173]	?	?	?	?	?	?
Galileon	[174]	[174]	?	[175, 176]	?	?	?
Nondynamical fields							
Palatini $f(R)$	[177–181]	?	?	?	—	?	?
Eddington–Born–Infeld	[182–188]	[182, 183]	?	[183]	—	[189, 190]	?

Berti etal (2015)

Alternative theories of gravity: Motivation

- ✓ There is a very wide range of alternative theories of gravity constructed from different generalizations/modifications of Einstein's theory.

- ✓ We will concentrate **on the most natural and widely used generalizations**:

  - ➢ **Scalar-tensor theories of gravity**
  - ➢ $\boldsymbol{f(R)}$ **theories of gravity**

- ✓ They are in agreement with all the observations and do not posses any intrinsic problems.

- ✓ Widely used as an alternative explanation of the dark energy phenomena.

- ✓ Scalar-tensor theories can be consider as an Einstein theory of gravity but with variable gravitational constant.

Alternative theories of gravity: Overview

Scalar-tensor theories

- **Essence:** one or several scalar field that can be viewed as mediators of the gravitational interaction in addition to the spacetime metric

- **Action:**

Jordan frame
Physical one

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-\tilde{g}} \left[F(\Phi)\tilde{R} - Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi) \right] + S_m\left[\Psi_m; \tilde{g}_{\mu\nu}\right]$$

✓ Conformal transformation of the metric
✓ Redefinition of the scalar field

Einstein frame
Much simpler!

$$S = \frac{1}{16\pi G_*} \int d^4x \sqrt{-g} \left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi) \right) + S_m\left[\Psi_m; \boxed{\mathcal{A}^2(\varphi) g_{\mu\nu}}\right]$$

The price we pay for simplicity: Explicit coupling between the matter and the scalar field

Alternative theories of gravity: Overview

Field equations in STT (Einstein frame)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_* T_{\mu\nu} + 2\partial_\mu \varphi \partial_\nu \varphi - g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi - 2V(\varphi) g_{\mu\nu}$$

$$\nabla^\mu \nabla_\mu \varphi = -4\pi G_* k(\varphi) T + \frac{dV(\varphi)}{d\varphi}.$$

$$k(\varphi) = \frac{d \ln(\mathcal{A}(\varphi))}{d\varphi}$$

These equations have to be supplemented with:

- Equation for **hydrostatic equilibrium**
- **Equation of state** of the nuclear matter

We set the potential to zero $V(\varphi) = 0$

Equilibrium neutron star solutions: Scalar-Tensor Theory

Scalar-tensor theories with massless scalar field

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \cancel{4V(\varphi)}\right) + S_m\left[\Psi_m; \boxed{A^2(\varphi)}g_{\mu\nu}\right]$$

Coupling function $\alpha(\varphi) = \frac{d\, lnA(\varphi)}{d\, \varphi}$

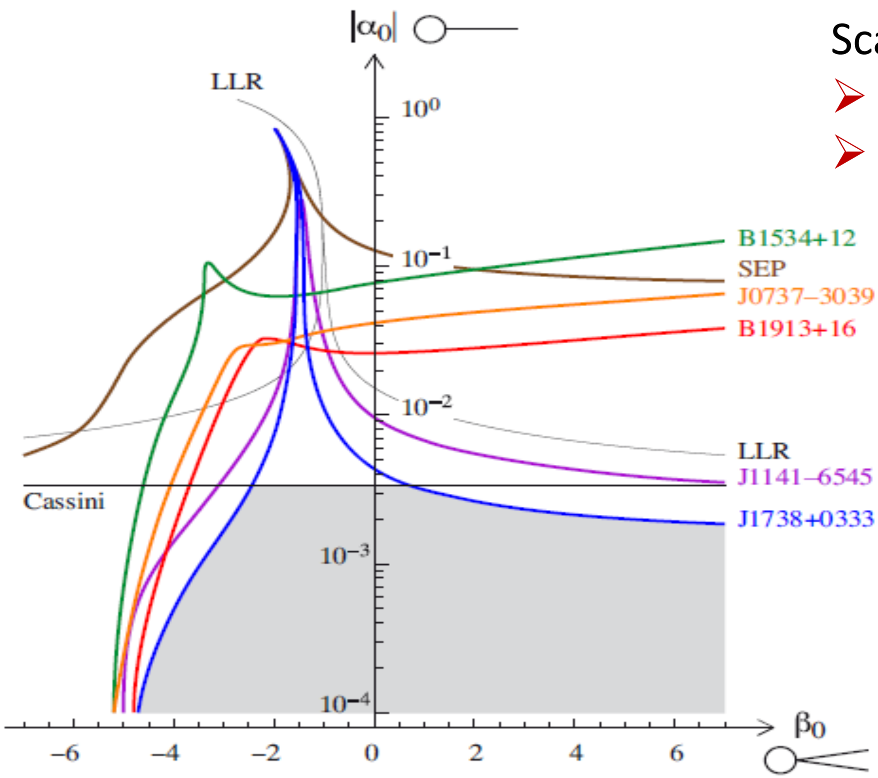
- The coupling function can be expanded as $\boldsymbol{\alpha(\varphi) = \alpha_0 + \beta\varphi +}$ **higher order terms**
  1. $\boldsymbol{\alpha(\varphi) = \alpha_0}$
     - Equivalent to the Brans-Dicke theory.
     - Differs from GR in the weak field regime.
     - Neutron stars have nontrivial scalar field for every $\alpha_0 \neq 0$
  2. $\boldsymbol{\alpha(\varphi) = \beta\varphi}$ $(\alpha_0 = 0)$
     - Equivalent to GR in the weak field regime.
     - Can differ significantly when strong fields are considered.
     - Nonuniqueness of the neutron star solutions can exist – one solution with trivial scalar field and one or several others with nontrivial scalar field.
- **Higher order terms** in $\alpha(\varphi)$ lead to qualitatively similar results

Equilibrium neutron star solutions: Scalar-Tensor Theory

Observational constraints

$\boldsymbol{\alpha_0 < 0.0035}$ $(Cassini)$ and $\boldsymbol{\beta > -4.5}$

(Damour & Esposito-Farese (1996,1998), Will (2006), Freire et al (2012), Antoniadis et al (2013))



Freire et al (2012)

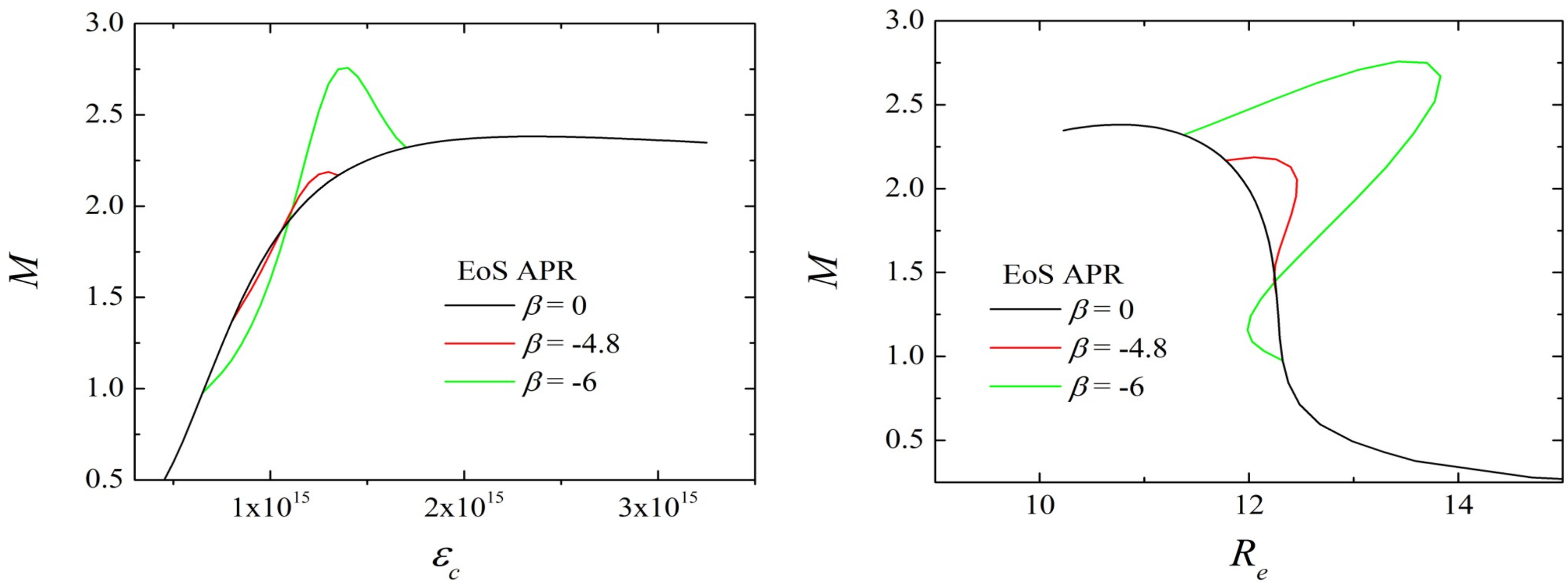
Scalarized solutions exist only for

- $\beta < -4.35$ in the static case and
- $\beta < -3.9$ in the rapidly rotating case.

Equilibrium neutron star solutions: Scalar-Tensor Theory

Spontaneous Scalarizarion is possible for β<-4.35 **(Damour+Esposito-Farese 1993)** introducing macroscopically (and observationally) significant modifications to the structure of the star[52].

The solutions become nonunique: for certain ranges of the parameter space: NS solutions in GR coexist with scalarized NSs.

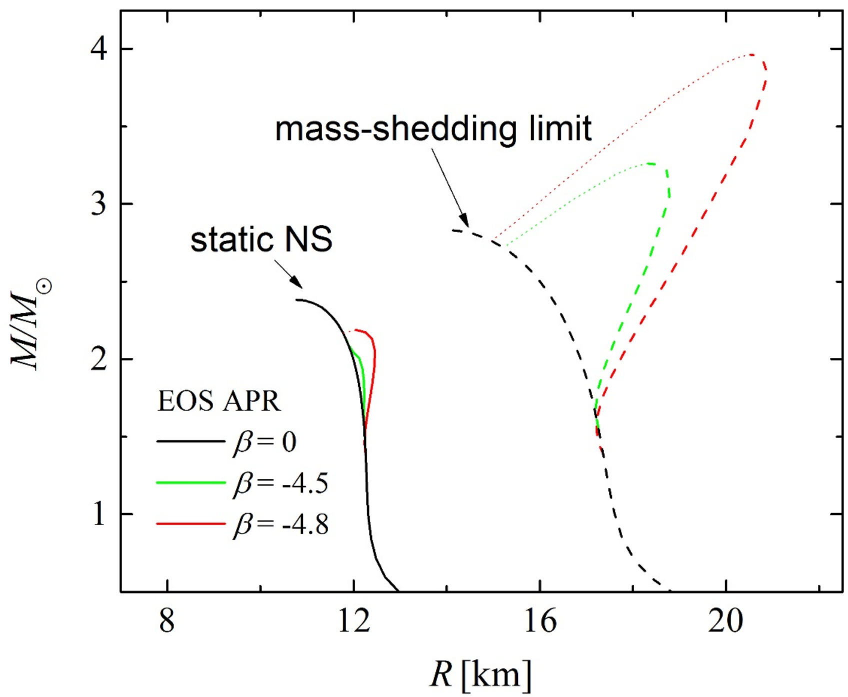


The solutions with nontrivial scalar field are *energetically more favorable* than their GR counterpart (Harada 1997, Harada 1998, Sotani+Kokkotas 2004)**.**

Equilibrium neutron star solutions:
Scalar-Tensor Theory

- **Slow rotation approximation** was also considered (Damour&Esposito-Farese (1996), Sotani (2012), Pani & Berti(2014)).
- **Rapid rotation** – changes the picture significantly (Doneva , Yazadjiev, Stergioulas, KK (2013)

Coupling function $\boldsymbol{\alpha(\varphi) = \beta\varphi}$



- ➢ Scalarization possible also for **positive $\boldsymbol{\beta}$** and negative trace of the energy momentum tensor. Possible for stiff EOS and very massive stars, not fully studied yet (Mendes (2015), Mendes & Ortiz (2016), Palenzuela & Liebling (2015)).
- ➢ **Tensor-multi-scalar theories** (Horbatsch et al (2015)) – new interesting phenomena, still in development.

Equilibrium neutron star solutions:
Scalar-Tensor Theory

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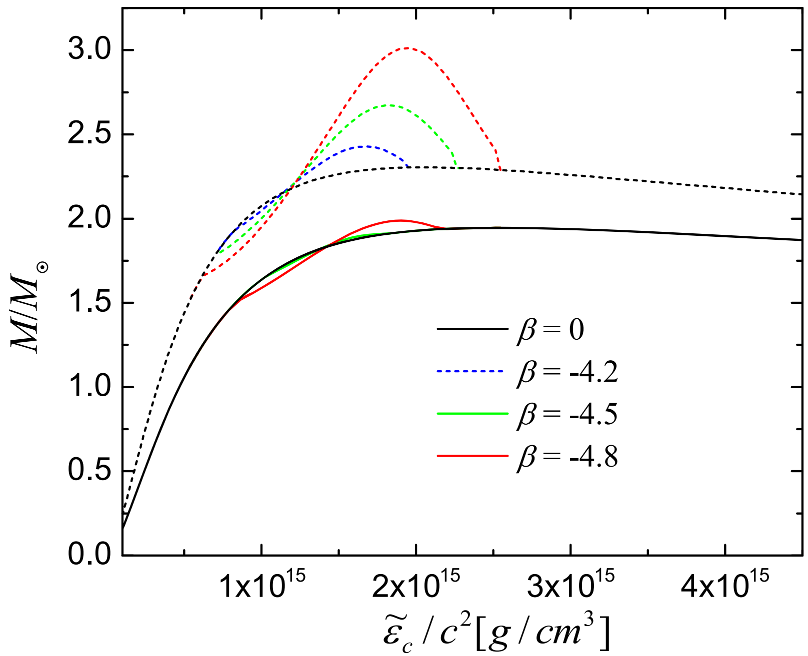

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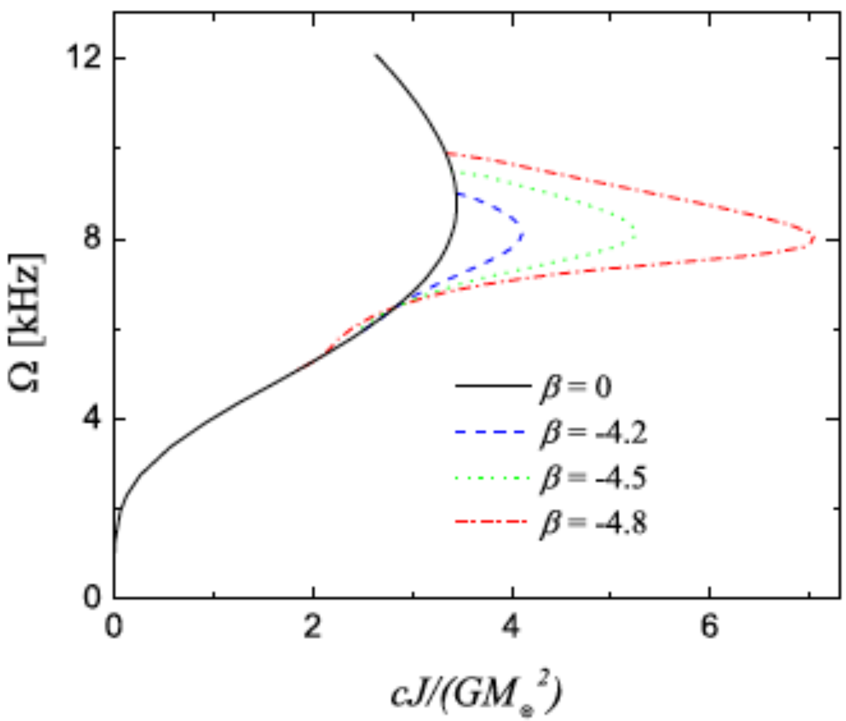
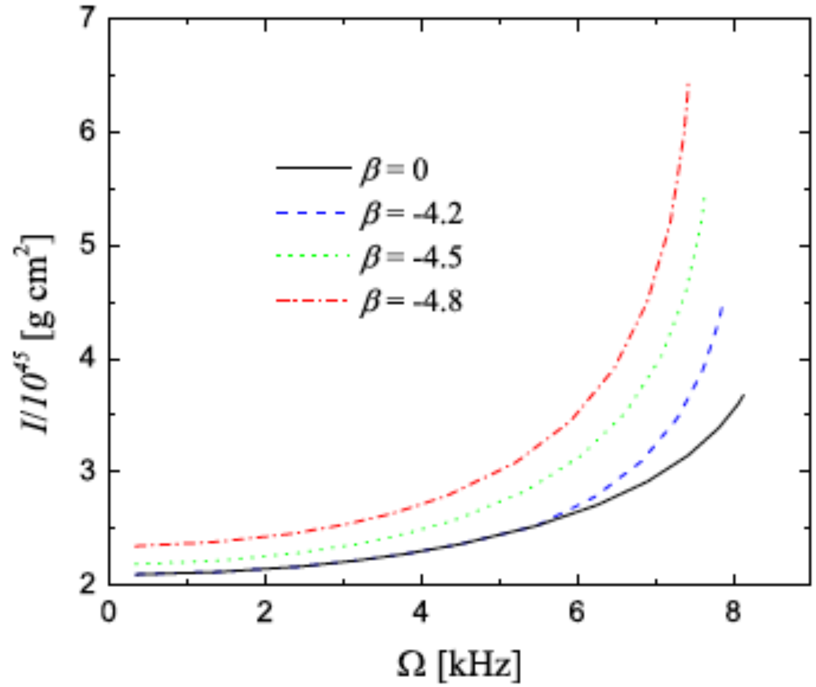
Equilibrium neutron star solutions:
Scalar-Tensor Theory



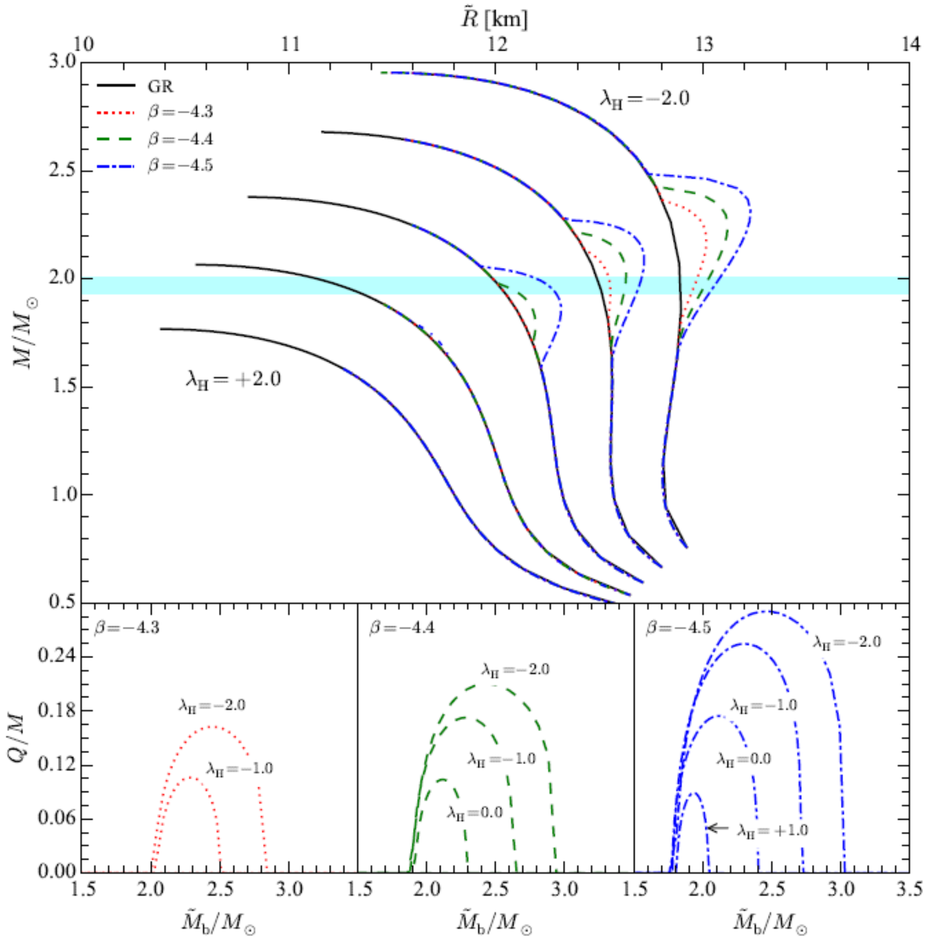
The **angular velocity** as a function of the **angular momentum** for sequences of stars rotating at the mass-shedding limit.



Moment of inertia as function of **angular velocity**

Equilibrium neutron star solutions:
Scalar-Tensor Theory

Anisotropic scalar-tensor neutron stars (Silva et al (2015)) – the deviations from GR are magnified significantly for strong degree of anisotropy



Scalar Tensor Theories with a massive scalar field

Neutron stars, with a massive scalar field could, in principle, have rather different structure and properties compared to their counterparts in the massless case.

Ramazanoglu, Pretorious *Spontaneous scalarization with massive field* (2016)

Yazadjiev, Doneva & Popchev *Slowly rotating neutron stars in scalar-tensor theories with a massive scalar field* (2016)

Doneva & Yazadjiev *Rapidly rotating neutron stars with a massive scalar field - structure and universal relations* (2016)

Scalar-Tensor Theory
with massive scalar field

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi)\right) + S_m[\Psi_m; \boxed{\mathcal{A}^2(\varphi)} g_{\mu\nu}]$$

Coupling function ($k(\varphi) = \frac{d\, lnA(\varphi)}{d\,\varphi}$

Two types of coupling functions :

1) Brans-Dicke coupling $k(\varphi) = \alpha_0 \Leftrightarrow A(\varphi) = \exp(\alpha_0\varphi)$

2) Theory with spontaneous scalarization $k(\varphi) = \beta\varphi \Leftrightarrow A(\varphi) = \exp\left(\frac{\beta}{2}\varphi^2\right)$, where $\beta < 0$

✓ **Massive scalar field with a potential** $V(\varphi) = \frac{1}{2}m_\varphi^2\varphi^2$

Theoretical & observational bounds on the parameters

L. Perivolaropoulos, PRD ***81****, 047501 (2010); J. Alsing, E. Berti, C. M. Will, H. Zaglauer, PRD* ***85****, 064041 (2012); M. Hohmann, L. J¨arv, P. Kuusk, E. Randla, PRD* ***88****, 084054 (2013); A. Scharer, R. Ang´elil, R. Bondarescu, P. Jetzer, and A. Lundgren, PRD* ***90****, 123005 (2014); L. Jarv, P. Kuusk, M. Saal, and O. Vilson, PRD* ***91****, 024041 (2015)*

The recent astrophysical and cosmological observations have severely constrained the basic parameters of the scalar-tensor theories with a massless scalar field leaving a narrow window for new physics beyond general relativity.

The situation changes drastically if we consider a massive scalar field.

The scalar field mass m_φ leads to a finite range of the scalar field of the order of its Compton wavelength $\lambda_\varphi = 2\pi/m_\varphi$.

- The presence of the scalar field will be suppressed outside the compact objects at distances $D > \lambda_\varphi$.
- This means in turn that all observations of compact objects involving distances greater than λ_φ cannot put constraints, or at least stringent constraints, on the scalar tensor theories.

Theoretical & observational bounds on the parameters

Massive Brans-Dicke theory

- For massive Brans-Dicke theory with $m_\varphi \geq 2\times10^{-25}GeV$ $(or\ \lambda_\varphi \leq 10^{11}m)$ the Solar System observations cannot put constraints on the Brans-Dicke parameter α_0 and all values of α_0 $(\omega_{BD} > -3/2)$ are observationally allowed.
- **The massive gravitational scalar suppresses also the dipole radiation and the compact binaries** cannot constrain severely the Brans-Dicke parameter if their orbit radius is significantly greater than λ_φ.

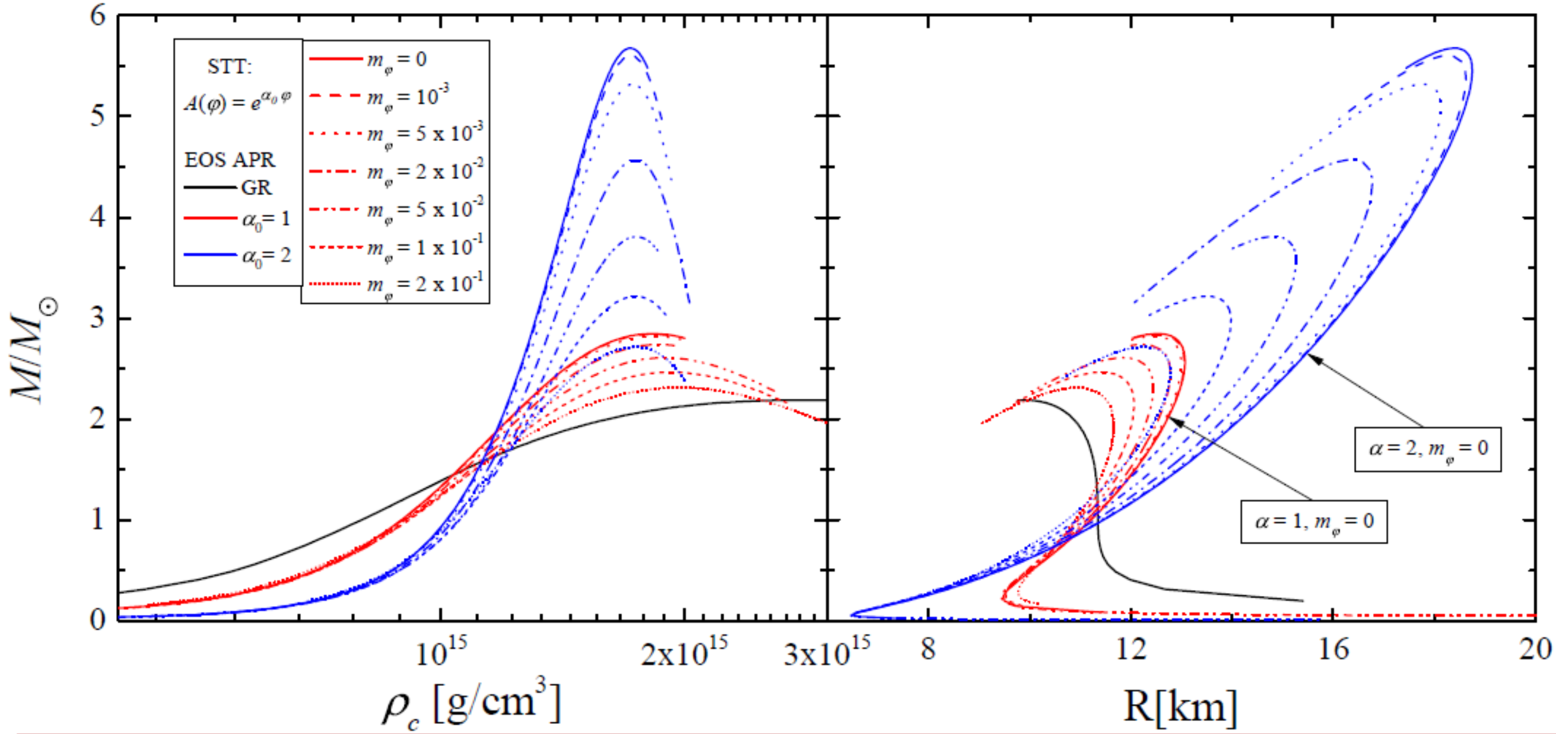
Scalar-tensor theory with $k(\varphi) = \beta\varphi$

- **The mass of the scalar field can effectively suppress the scalar gravitational waves** and reconcile the scalar-tensor theories with the binary neutron star observations for a much larger range of $\boldsymbol{\beta}$.
- **If the Compton wavelength of the scalar field λ_φ is much smaller than the separation of the two stars** in the binary system **the emitted scalar gravitational radiation will be negligible.**

Neutron stars in massive Brans-Dicke theory

$$m_\varphi \to m_\varphi R_0 = 2\pi\, R_0/\lambda_\varphi \qquad R_0 = 1.47664\, km$$

Brans-Dicke coupling $k(\varphi) = \alpha_0 \Leftrightarrow A(\varphi) = \exp(\alpha_0 \varphi)$

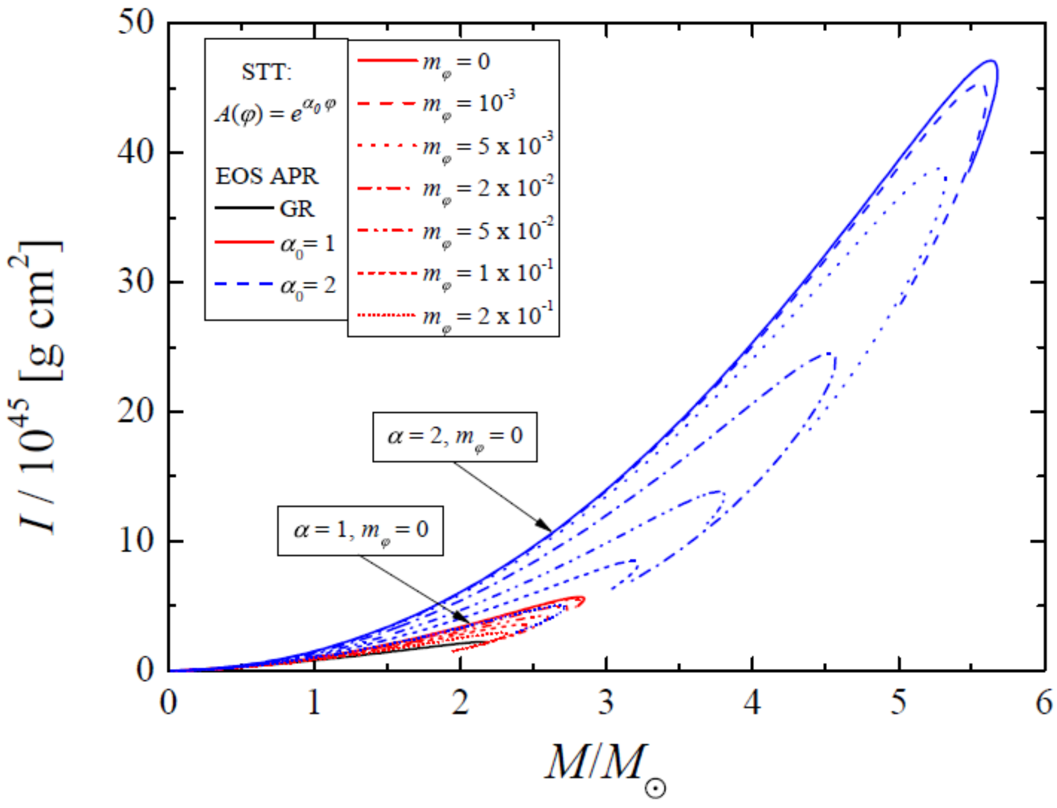


The allowed range for m_φ is $\mathbf{10^{-16}\, eV \le m_\varphi \le 10^{-9}\, eV}$

… normalized $\mathbf{7x10^{-7} \le m_\varphi \le 7}$

STT of gravity - observations

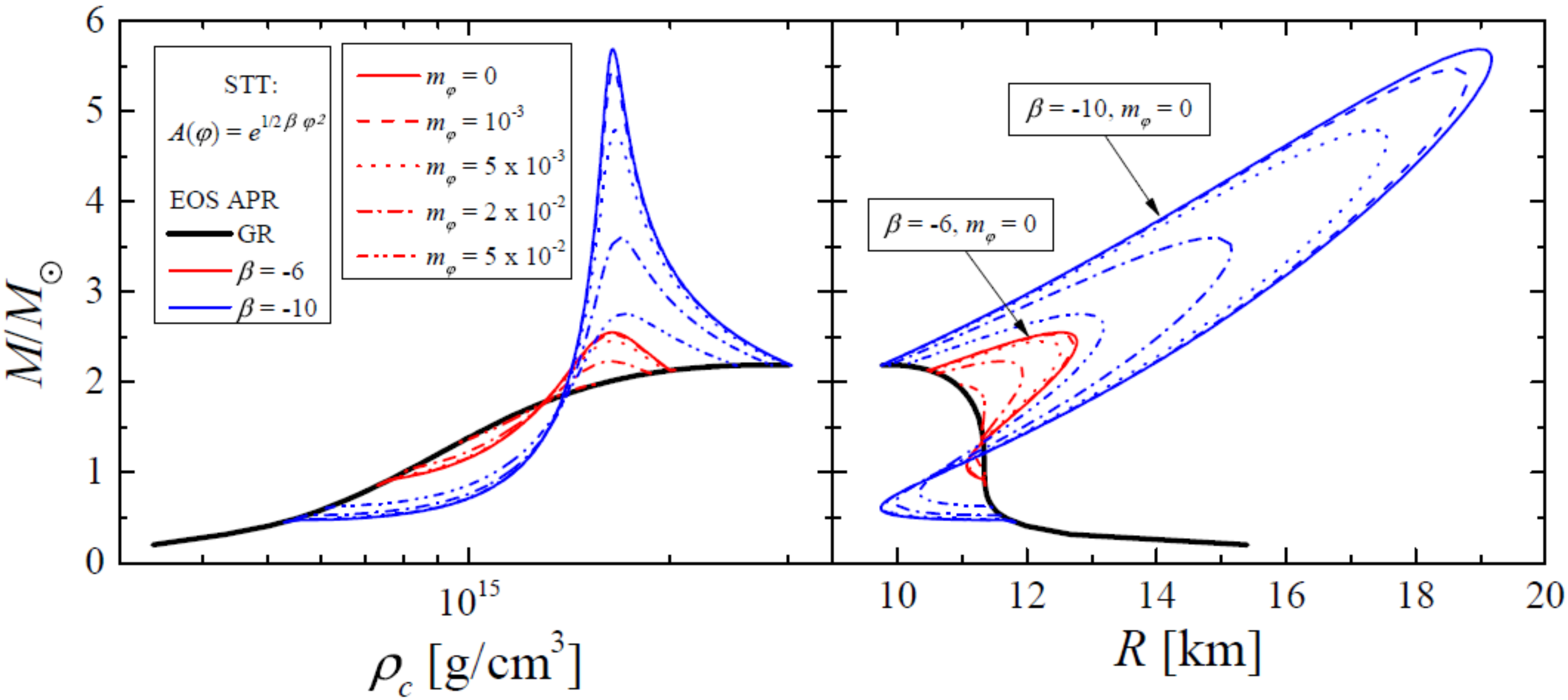
Brans-Dicke coupling $k(\varphi) = \alpha_0 \Leftrightarrow A(\varphi) = \exp(\alpha_0\varphi)$



STT of gravity - observations

Theory with **spontaneous scalarization**

$$k(\varphi) = \beta\varphi \Leftrightarrow A(\varphi) = \exp\left(\frac{\beta}{2}\varphi^2\right), \beta < 0$$



STT of gravity - observations

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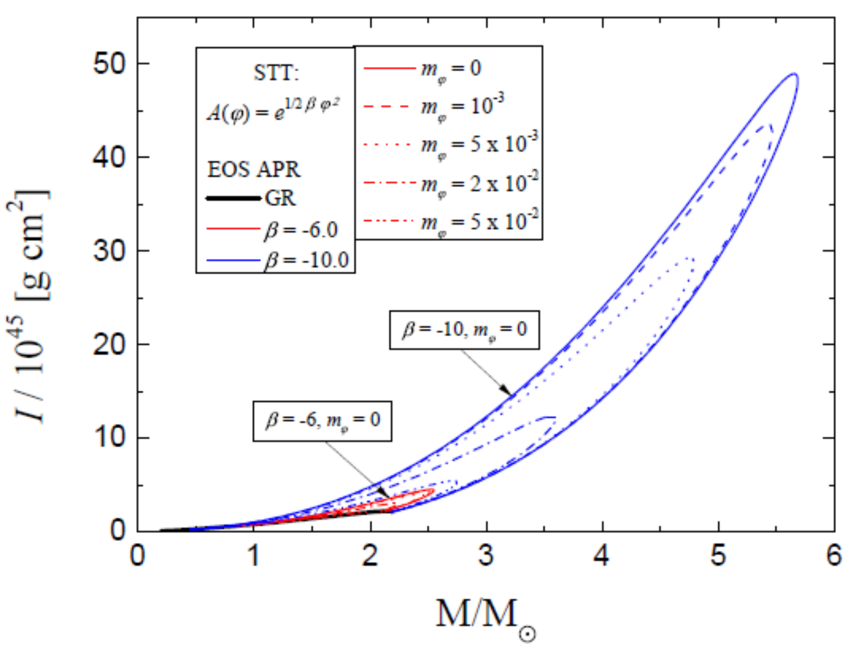
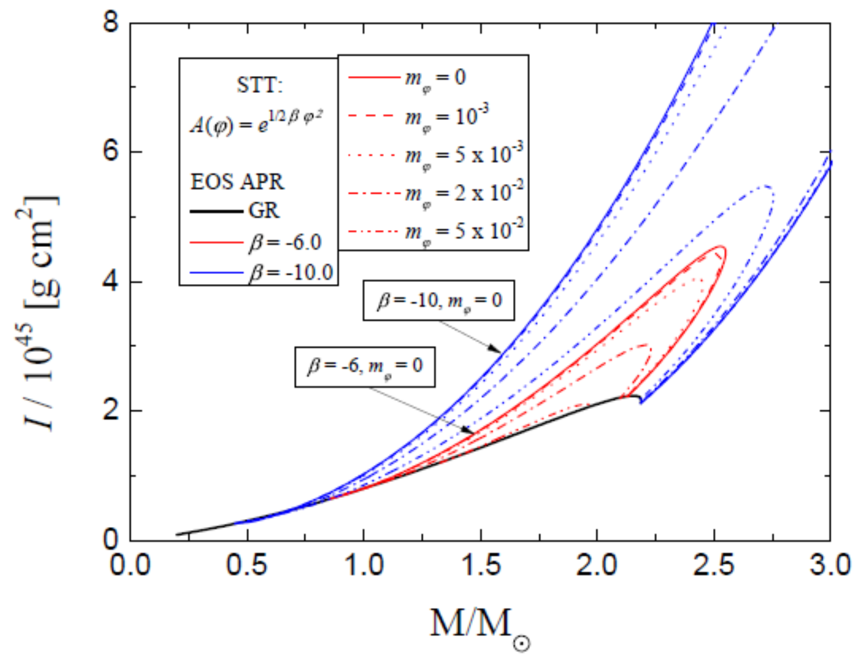




Conclusions (massive field)

- ✓ In scalar-tensor theories with a **massless scalar field neutron stars differ almost marginally from GR** if one considers coupling parameters that are in agreement with the present observations.

- ✓ The inclusion of **scalar field mass changes the picture dramatically**. It suppresses the scalar field at length scale of the order of the Compton wavelength which helps us reconcile the theory with the observations for a much broader range of the coupling parameters.

- ✓ **The structure and the properties of the neutron stars in massive STT can differ drastically from the pure GR solutions if sufficiently large masses of the scalar field are considered.**

$f(R)$ theories of gravity

Alternative theories of gravity: $f(R)$ theories

- **Motivation:** widely used as an alternative explanation of the *accelerated expansion of the universe*

- **Studied mainly at cosmological scales**, but every theory of gravity should pass via the observations at astrophysical scale too

- **Action:** $$\boxed{S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\, f(R) + S_{\text{matter}}(g_{\mu\nu}, \chi)}$$

- Free of tachyonic instabilities and the appearance of ghosts when:

$$\boxed{\frac{d^2 f}{dR^2} \geq 0, \qquad \frac{df}{dR} > 0}$$

- **Mathematical treatment** of the problem: $f(R)$ *theories are mathematically equivalent to a particular class of massive scalar-tensor theories.*

Alternative theories of gravity: Overview

$f(R)$ theories

- **Example:** R^2 gravity ($f(R) = R + aR^2$)

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-\tilde{g}}\left[\boxed{F(\Phi)}\tilde{R} - \cancel{Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi} - \boxed{2U(\Phi)}\right] + S_m\left[\Psi_m;\tilde{g}_{\mu\nu}\right]$$

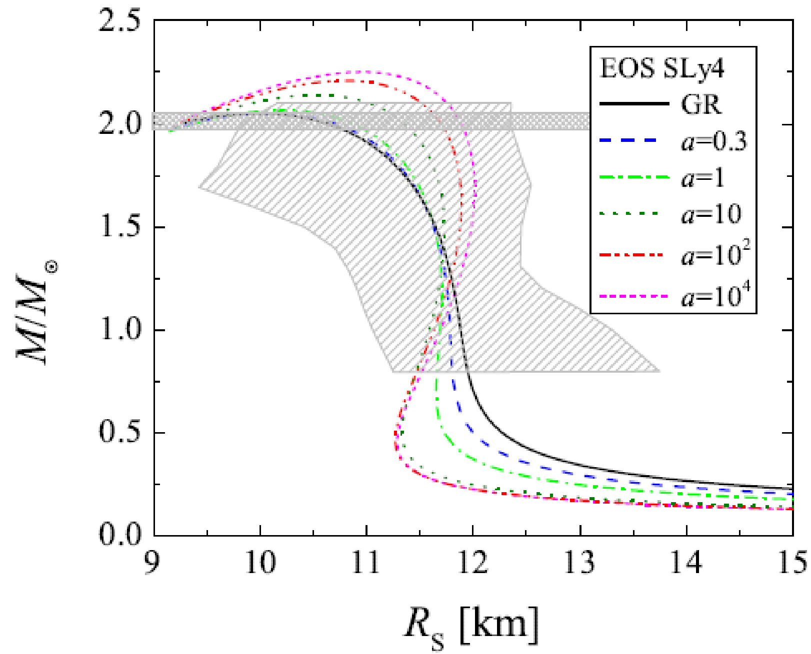
$$\boxed{= \Phi = f'(R)}$$

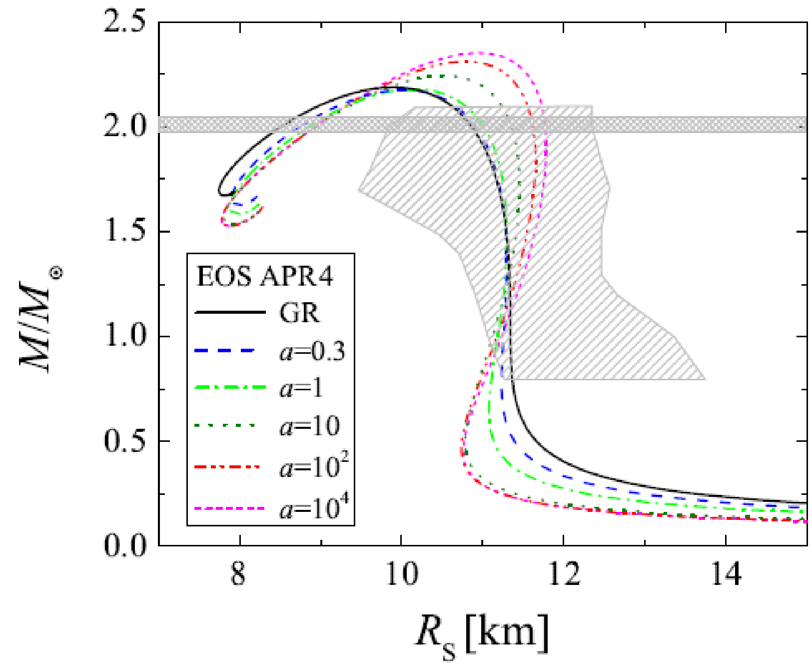
$$\boxed{= \frac{1}{8a}(\Phi - 1)^2 \;\Rightarrow\; m_\Phi = \frac{1}{\sqrt{6a}}}$$

Equilibrium neutron star solutions:
$f(R)$ theories of gravity

- We will concentrate on the $\boldsymbol{R^2}$ **gravity** ($f(R) = R + a\, R^2$) case, that is expected to give the dominant contribution at astrophysical scales.
- **Pertubative approach**, assuming that $\boldsymbol{a}$ is a small number, (Cooney, DeDeo, Psaltis (2010)) widely used in the past, but recently it was shown to be "**misleading**" (Yazadjiev, Doneva, KK, Staykov (2014))
- **Observational constraints** – the most severe coming from the Gravity Probe B experiments $a < 2.5{\times}10^5$ (or $a < 5{\times}10^{11} m^2$ in physical units).
- The **scalar-tensor representation** of $f(R)$ theories is commonly employed.
- *The field equation for the Ricci scalar curvature (or equivalently the scalar field) is stiff which poses a computational difficulty.*

NSs in f(R)-gravity: Static Models





$$f(R) = R + aR^2$$

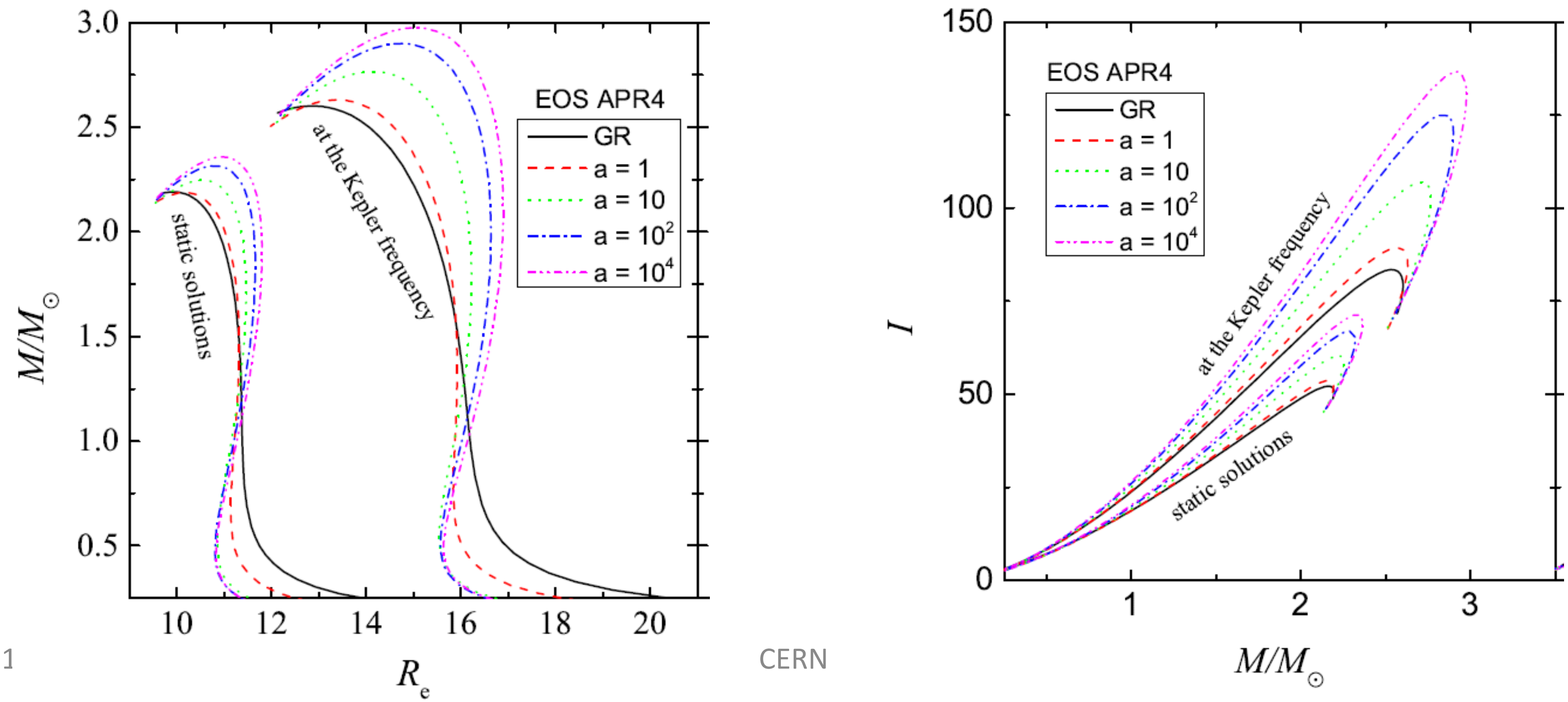
Yazadjiev, Doneva, Kokkotas, Staykov (2014)

- The differences between the R^2 and GR are comparable with the uncertainties in the nuclear matter equations of state.
- The current observations of the NS masses and radii alone can not put constraints on the value of the parameters a, **unless the EoS is better constrained in the future.**

See also: Capozziello, De Laurentis, Farinelli, Odintsov (2015)

Equilibrium neutron star solutions

- **Non-perturbative approach**: reported in Babichev&Langlois(2010), Jaime et al (2011), and the first detailed study of realistic NS models was done in Yazadjiev, Doneva, Kokkotas, Staykov (2014)
- **Rotating models** are also studied (Staykov, Doneva, Yazadjiev, Kokkotas (2014), Yazadjiev, Doneva, Kokkotas (2015))
- **Non-negligible deviation** for the allowed values of a. The **moment of inertia** is very sensitive and can be used to set constraints on the parameters.

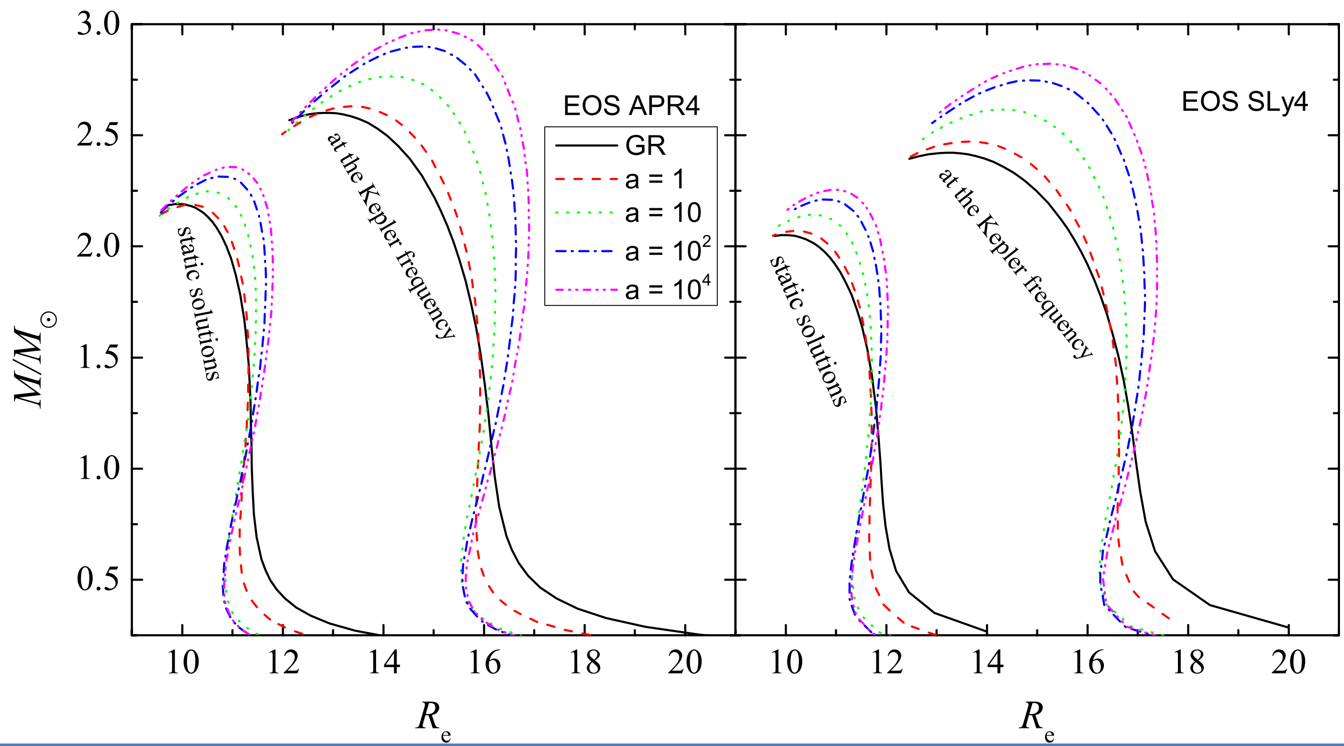


NSs in f(R)-gravity: Fast Rotation

$$f(R) = R + aR^2$$

Mass vs Radius diagrams for two realistic EOS

Yazadjiev, Doneva, Kokkotas, (2015)

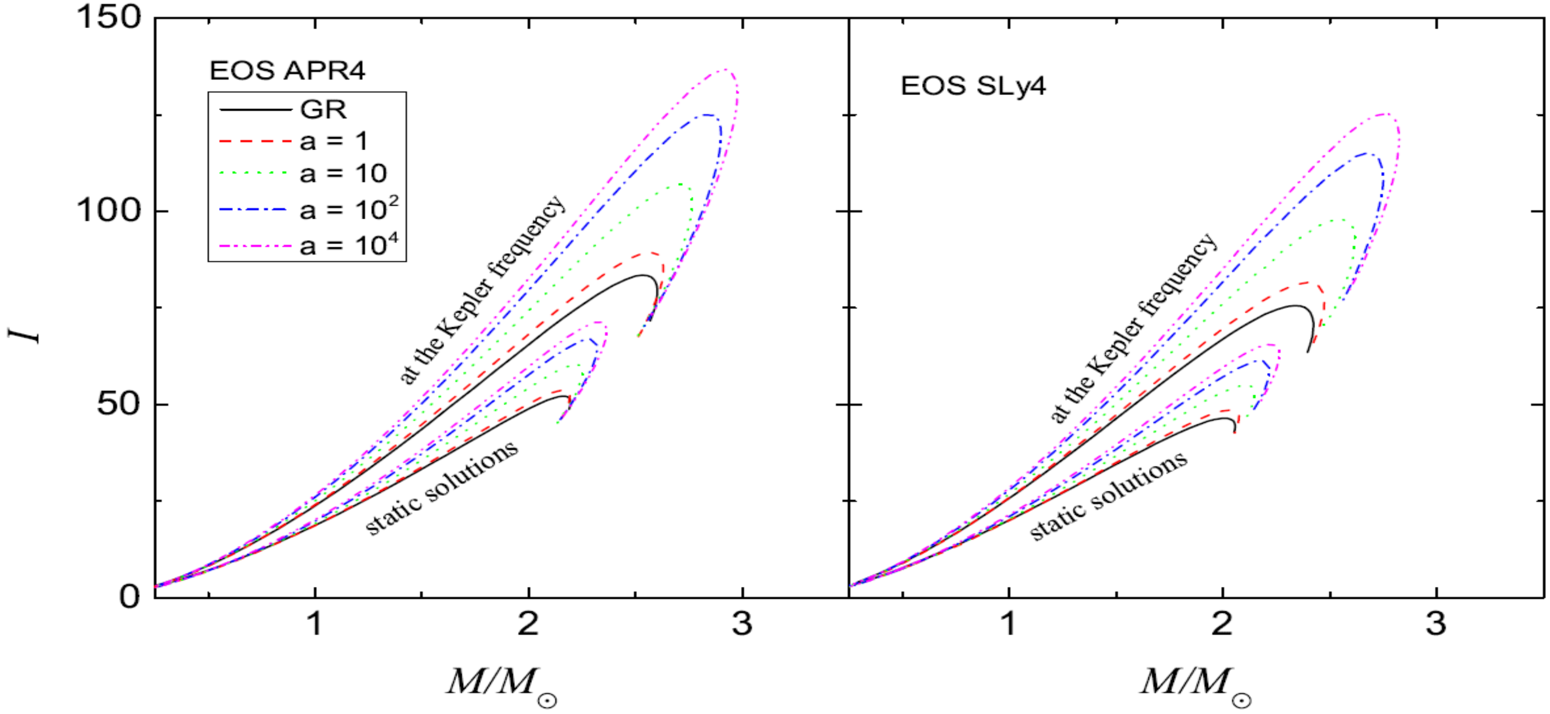


Difficult to set constraints on the *f* (*R*) theories using measurement of the neutron star **M** and **R** alone, until the EOS can be determined with smaller uncertainty.

NSs in f(R)-gravity: Fast Rotation

$$f(R) = R + aR^2$$

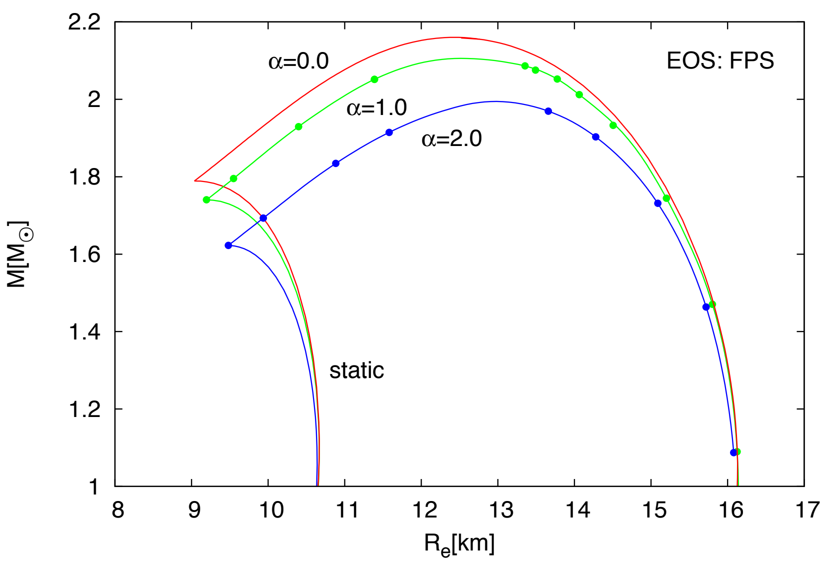
Yazadjiev, Doneva, Kokkotas (2015)

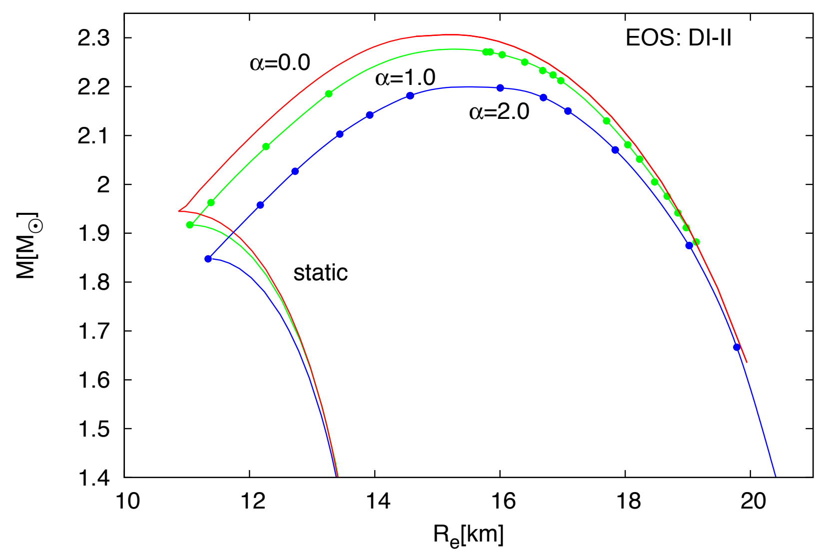


- ✓ The differences in the neutron star moment of inertia on the other hand can be much more dramatic. **BEYOND THE UNCERTAINTY DUE TO THE EOS**
- ✓ Large deviations can be potentially measured by the forthcoming observations of the NS moment of inertia [Lattimer-Schutz 2005, Kramer-Wex 2009] that can lead to a direct test of the R^2 gravity.

Dilatonic Einstein-Gauss-Bonnet Theory

- The theory is **motivated from string theory**.
- String theory predicts the presence of **higher curvature terms** in the action as well as further fields.
- In the low energy effective action obtained from heterotic string theory contains as **basic ingredients a Gauss-Bonnet** (GB) term and a dilaton field

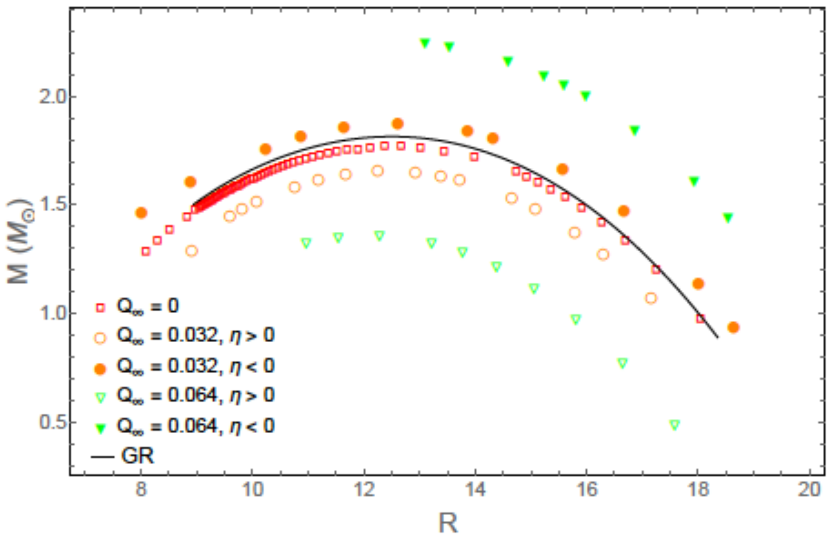


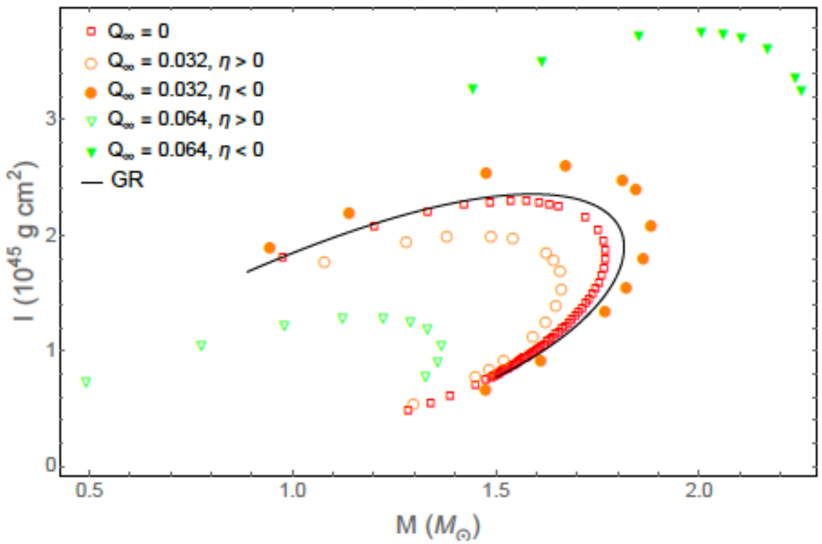


Kleihaus, Kunz, Mojica, Zagermann 2016

Hondersky gravity

The most **general extension of Einstein's theory** of general relativity with a **single scalar degree of freedom** and **second-order field equations**.



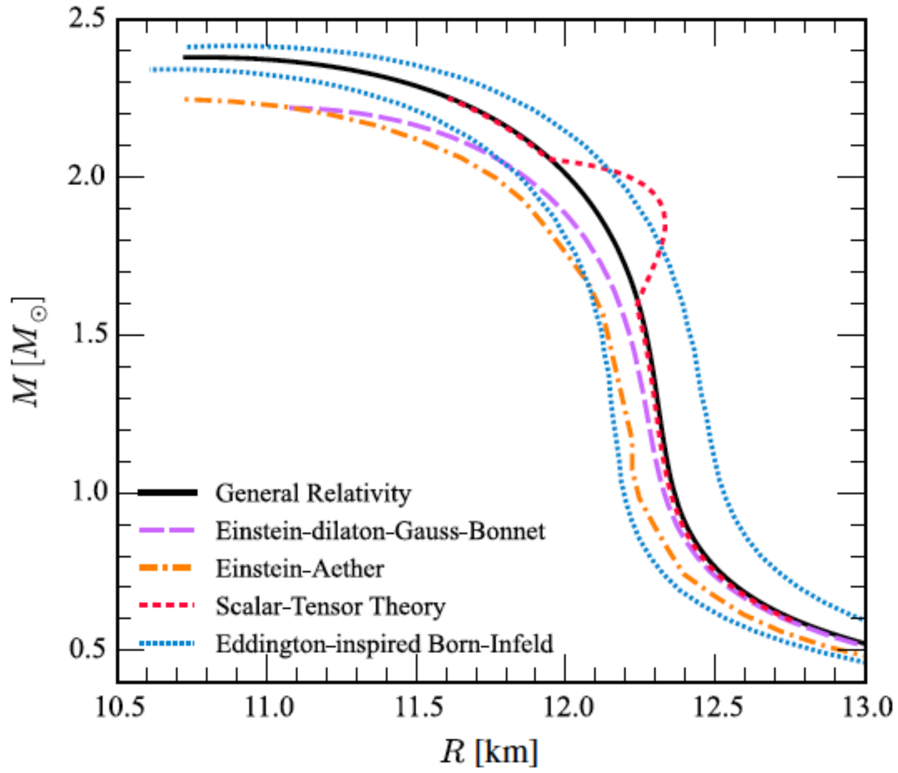


Babichev and Charmousis (2014)
Babichev, Charmousis, Lehebel (2016)
Babichev etal (2016)
Barausse and Yagi (2015)
Cisterna et al. (2015)
Maselli et al. (2016a, 2016b)
…

Post-TOV approximation

The gravity theory degeneracy problem:

Exists even if we do know the correct equation of state



The logic underpinning the formalism is that by parametrizing the deviation of the stellar structure equations from their GR counterparts, thus producing a set of post-TOV equations.

Glampedakis, Pappas, Silva, Berti (2015, 2016)

Post-TOV approximation

Post-TOV equations: *describe smooth modifications of he TOV equations, parametrized by the post-TOV parameters*

$$\frac{dp}{dr} = \left(\frac{dp}{dr}\right)_{\mathrm{GR}} - \frac{\rho m}{r^2}\mathcal{P} \qquad \frac{dm}{dr} = \left(\frac{dm}{dr}\right)_{\mathrm{GR}} + 4\pi r^2 \rho \mathcal{M}$$

where

$$\mathcal{P} \equiv \pi_1 \frac{m^3}{r^5\rho} + \pi_2 \frac{m^2}{r^2} + \pi_3 r^2 p + \pi_4 \frac{\Pi p}{\rho}$$

$$\mathcal{M} \equiv \mu_1 \frac{m^3}{r^5\rho} + \mu_2 \frac{m^2}{r^2} + \mu_3 r^2 p + \mu_4 \frac{\Pi p}{\rho} + \mu_5 \frac{\Pi^3 r}{m}$$

Glampedakis, Pappas, Silva, Berti (2015, 2016)

Astrophysical Implications

Astrophysical Implications

- **Final goal** – test the strong field regime of gravity via neutron star observations and impose constraints on the alternative theories

- **Obstacles**:
  - ➢ Accuracy of observations
  - ➢ Accurate models of the observed phenomena
  - ➢ EOS uncertainty

- **Ways out:**
  - ➢ Deviation from GR stronger than the EOS uncertainty for the allowed range of parameters
  - ➢ EOS independent relations

Astrophysical Implications

Possible approaches for testing alternative theories of gravity

- **Direct observation of the mass and radius.**
- **Observations of the moment of inertia**: applicable for example for $f(R)$ theories Staykov at al (2014) and Eddington inspired gravity Pani, Cardoso, Delsate (2011)
- **Quasiperiodic oscillations** DeDeo&Psaltis (2004), Doneva etal (2014), Staykov, Doneva, Yazadjiev (2015)
- **The redshift of surface spectral lines** in X-rays and γ-rays DeDeo&Psaltis(2003)
- **Gravitational wave emission of oscillating neutron stars**
- **Neutron star mergers**
- **Universal relations**

Astrophysical Implications

Universal relations

- **EOS independent relations** between the properties, including the oscillation spectrum, of neutron stars. Normally a proper normalization of the quantities is required.

- Very convenient way to **circumvent the EOS uncertainty**.

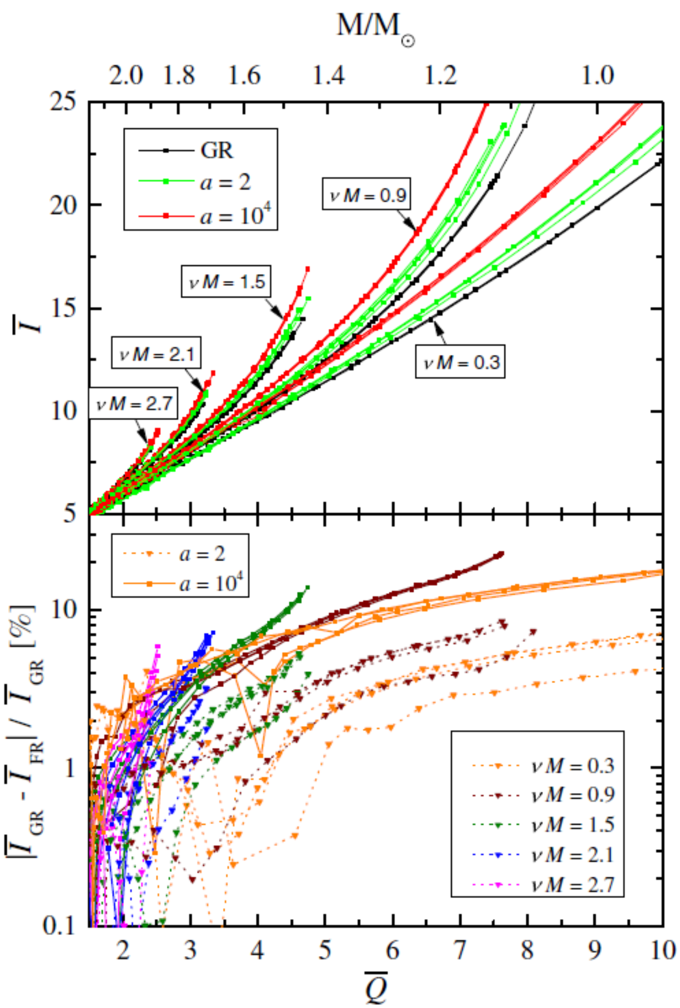
- Attracted particular attraction with the paper of *Yagi&Yunes (2013)*

- The focus is on the **I-Love-Q relations** but many other universal relations exist (Lattimer&Schutz(2005),Yagi et al (2014), AlGendy&Morsink(2014), Breu&Rezzolla(2016))

- **General idea** for **testing the strong field** regime of gravity: if the two parameters that enter in a universal relation are measured independently, then a possible deviation from the GR EOS independent relations can be measured.

Astrophysical Implications

Example R^2 theories:



- **I-Love-Q relations**: appreciable deviations from GR only for some alternative theories of gravity (**dynamical Chern-Simons gravity** Yagi&Yunes(2013), $f(R)$ gravity Doneva, Yazadjiev, Kokkotas (2015), **massive STT** Yazadjie,Doneva arXiv:1607.03299)

- **Most of the studied alternative theories of gravity give only marginal deviations from GR** (eg. Sham, Lin, Leung(2014); Kleinhaus, Kunz, Mojica (2014), Pani, Berti (2014), Pappas, Sotiriou (2015)).

- **Unnormalized relations** STILL differ significantly from GR. Solution:
  - Different normalization
  - Different universal relation

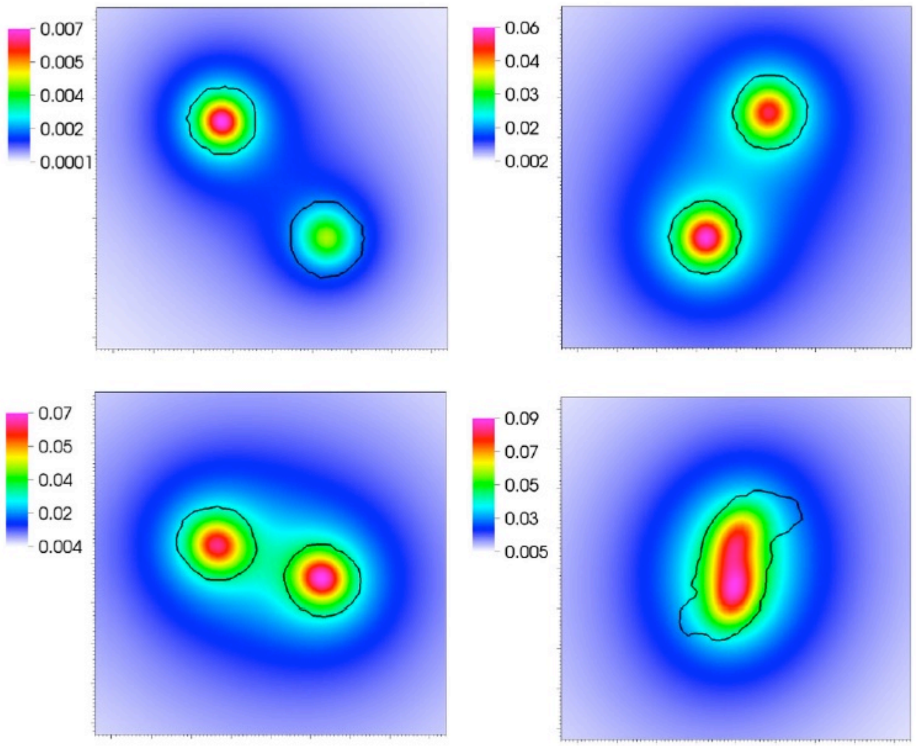
- **Strong point:** these relations are also theory independent up to a good extend that might have different application.

Astrophysical Implications

Dynamical scalarization – NS mergers

Even if the two NS are not scalarized when separated, in close binary system they **develop strong scalar field**.

Coupling function $\boldsymbol{\alpha(\varphi) = \beta\varphi}$



The observational signature of the scalarized merging neutron stars has been studied in Barause et al (2013), Palenzuela et al (2014), Shibata et al (2014), Sampson (2014), Taniguchi et al (2015).

Astrophysical Implications

Neutron star oscillations

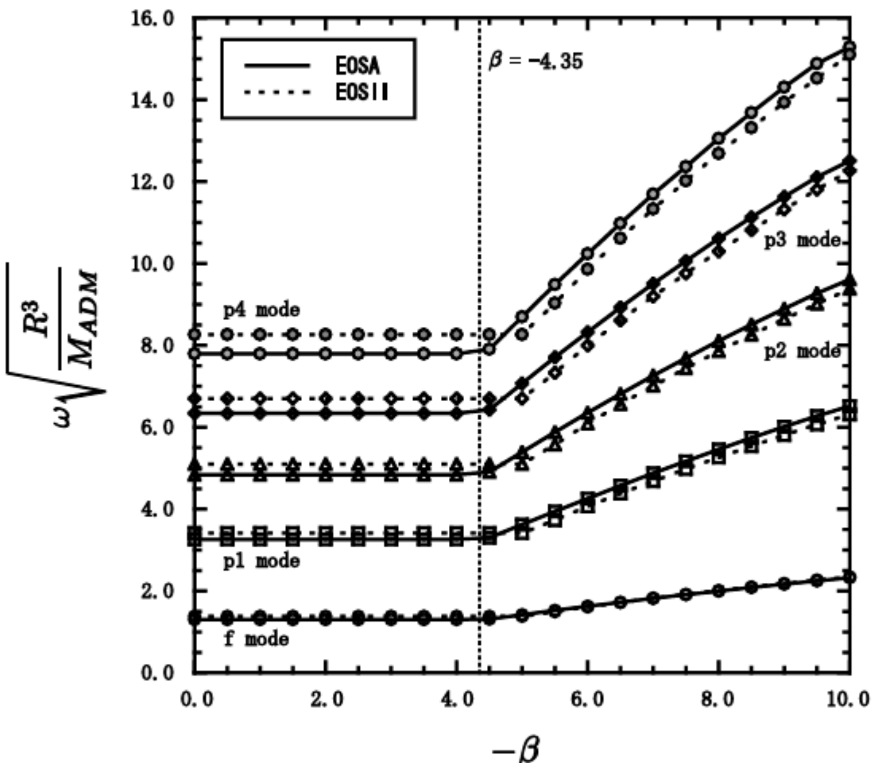
- The study was **initiated** with the work of Sotani & Kokkotas (2004,2005) for *f*-, *p*- and *w*-modes in STT.

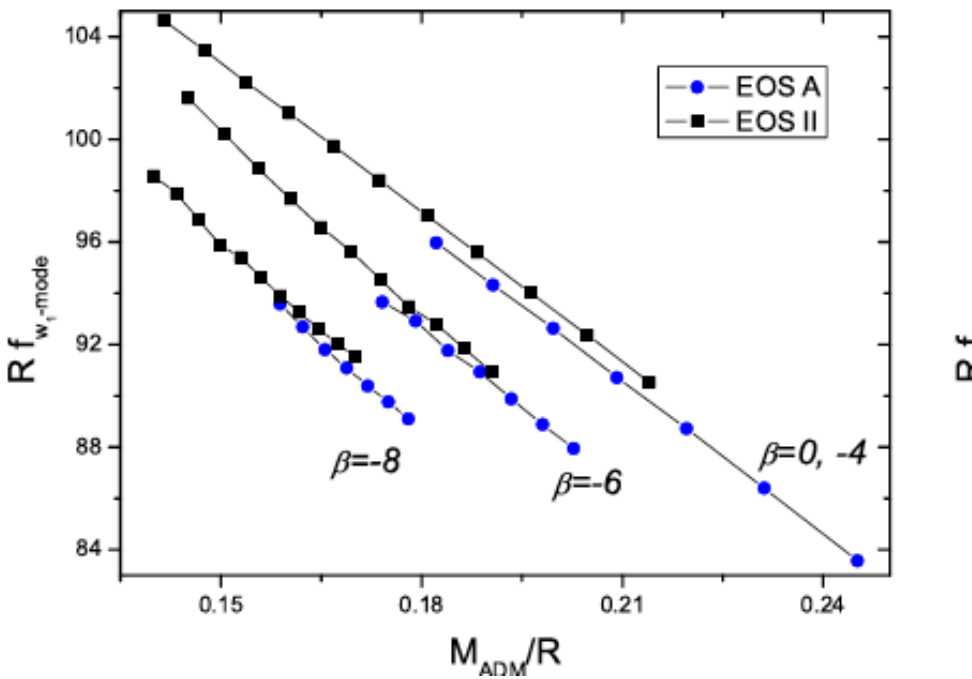
- The main idea is to constrain the deviations from GR using the emitted gravitational wave signal or in some cases electromagnetic signal, related to neutron star oscillations

- Several alternative theories studied until now – **STT** Sotani&Kokkotas (2004, 2005), Silva et al (2014), **TeVeS** Sotani (2010, 2011, 2009), $\boldsymbol{f(R)}$ Staykov et al (2015), **Einstein-Gauss-Bonnet-dilaton gravity** Blázquez-Salcedo et al (2016)

- Fundamental ***f*-modes**, **torsional** modes, ***w*-modes** and others are studied. In many cases the Cowling approximation is employed.

Stellar Oscillations in STT





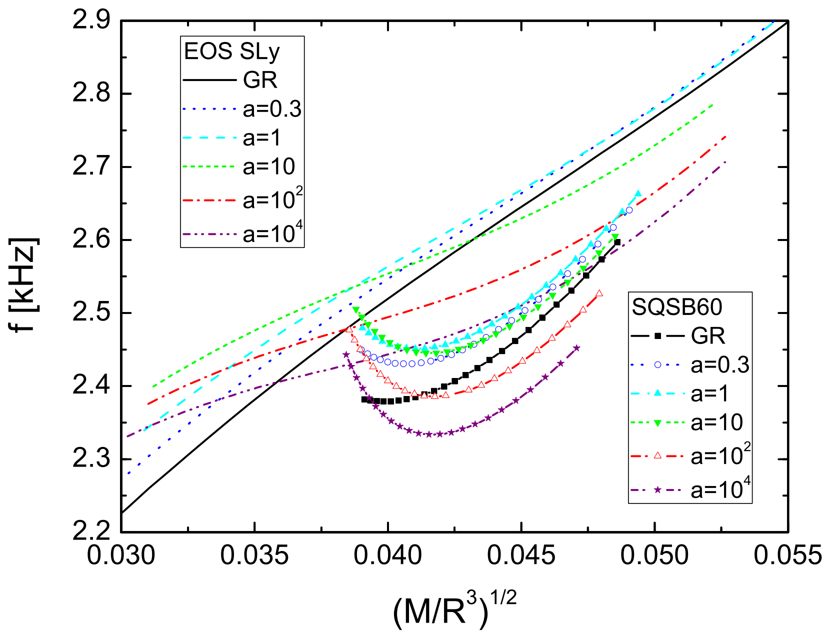
$$\frac{\partial \omega_n}{\partial(-\beta)} \approx \frac{n}{4}$$

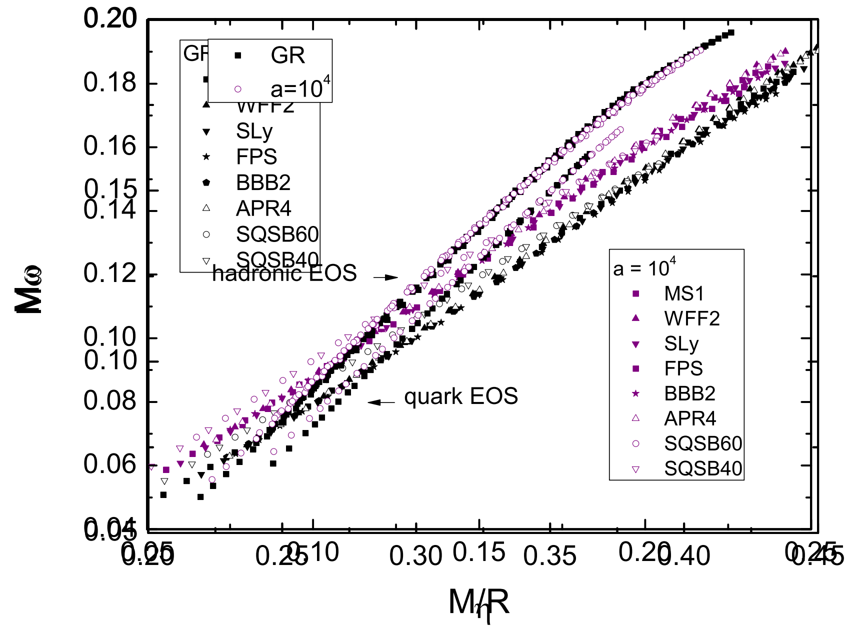
Sotani-Kokkotas (2004,2005)

Asteroseismology in ATG

Asteroseismology relations in R^2 theories

- ***f*-mode** oscillation frequencies, nonrotating case
- Quite **EOS independent** with suitable choice of normalization



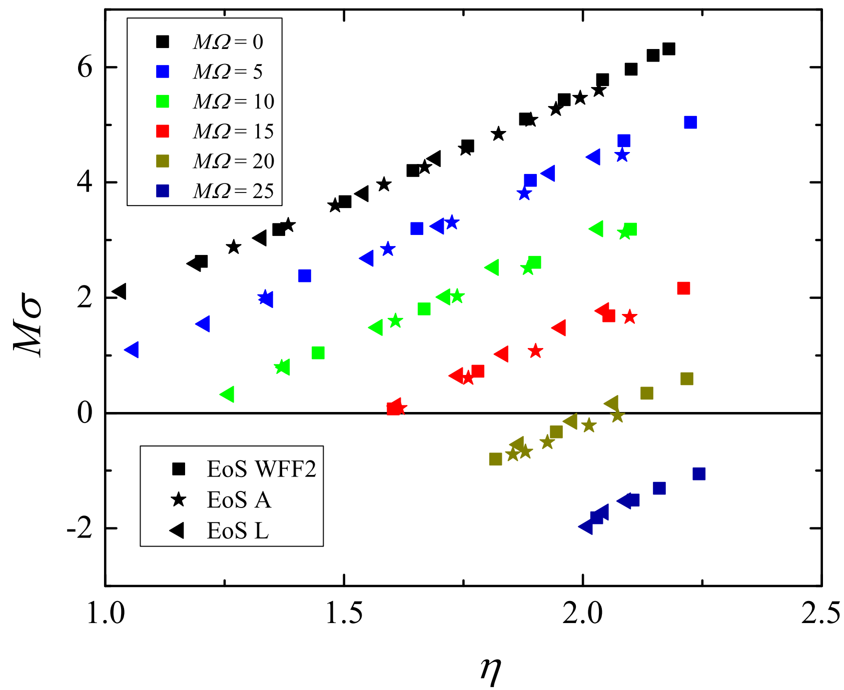


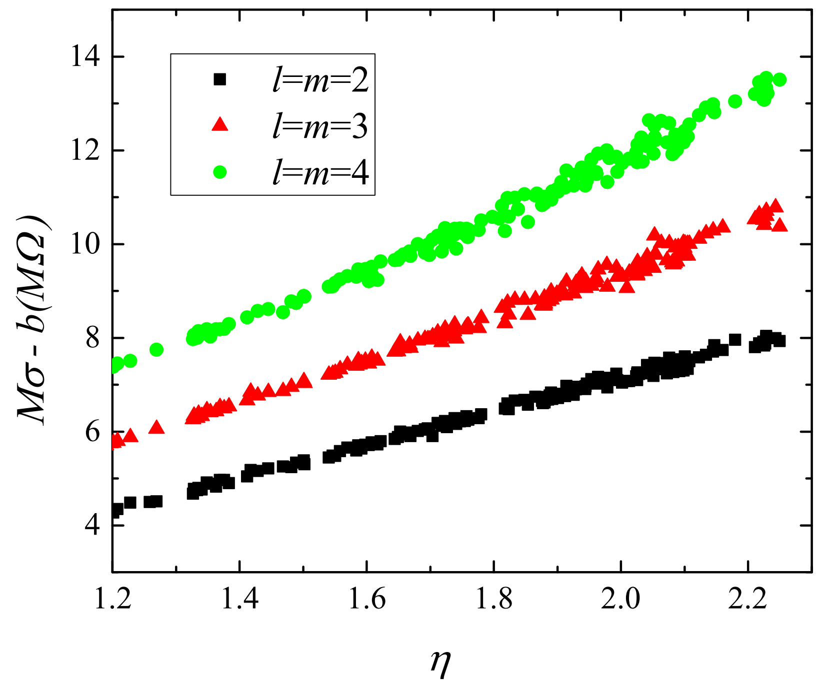
$$\eta = \sqrt{M^3 / I}$$

- The maximum deviation between the f-mode frequencies in GR and R^2 gravity is up to 10% and depends on the value of the R^2 gravity parameter *a*.
- Alternative normalizations show nicer relations

Asteroseismology: but in GR

$$M\sigma_i^{unst} = [(0.56 - 0.94\ell) + (0.08 - 0.19\ell)M\Omega + 1.2(\ell + 1)\eta]$$



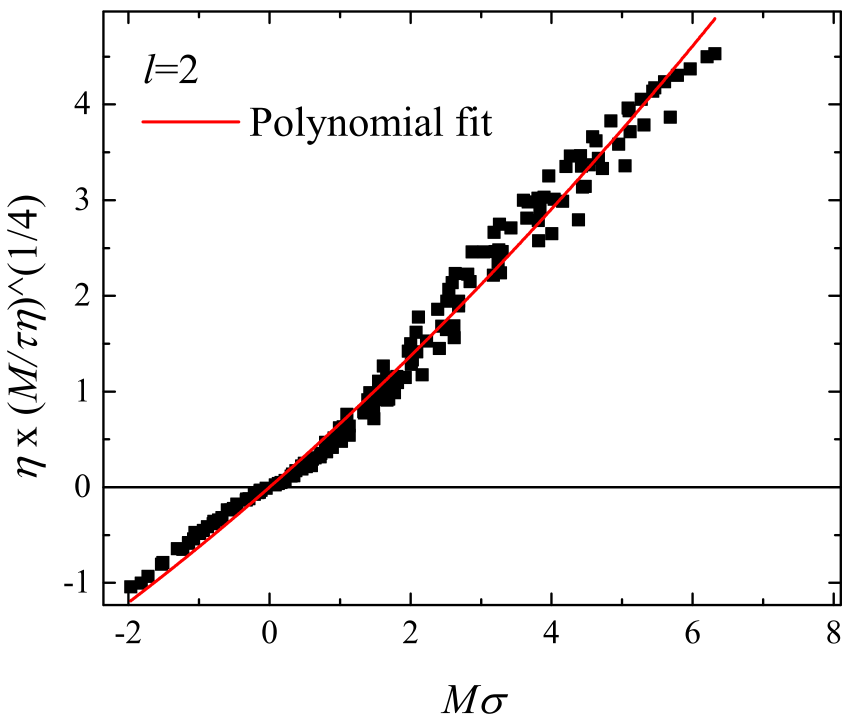


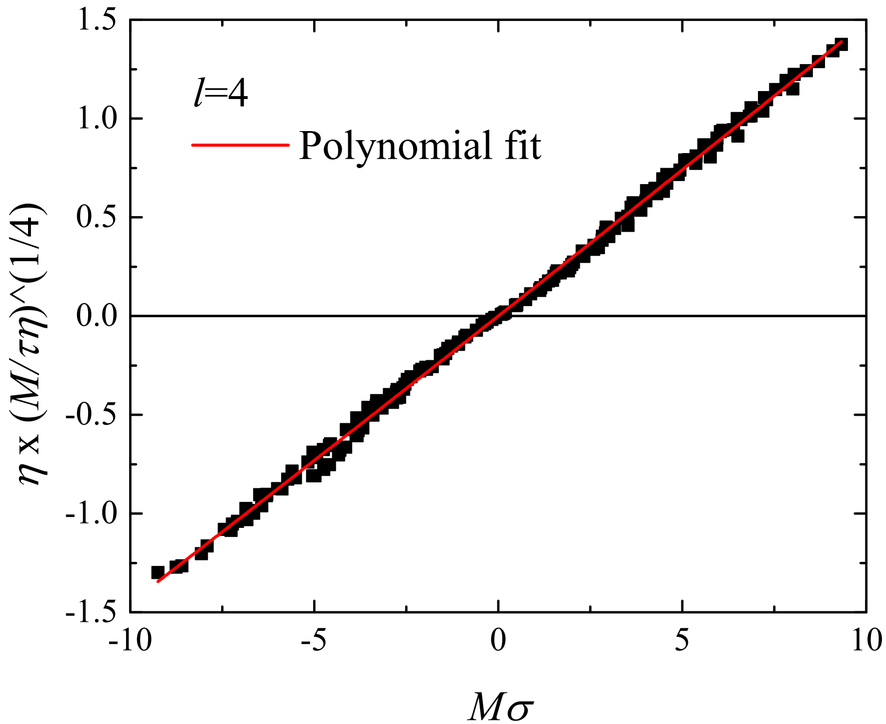
The l = 2 f-mode oscillation frequencies as functions of the parameter η

$$\eta = \sqrt{M^3 / I}$$

Doneva-Kokkotas 2015

Asteroseismology: but in GR



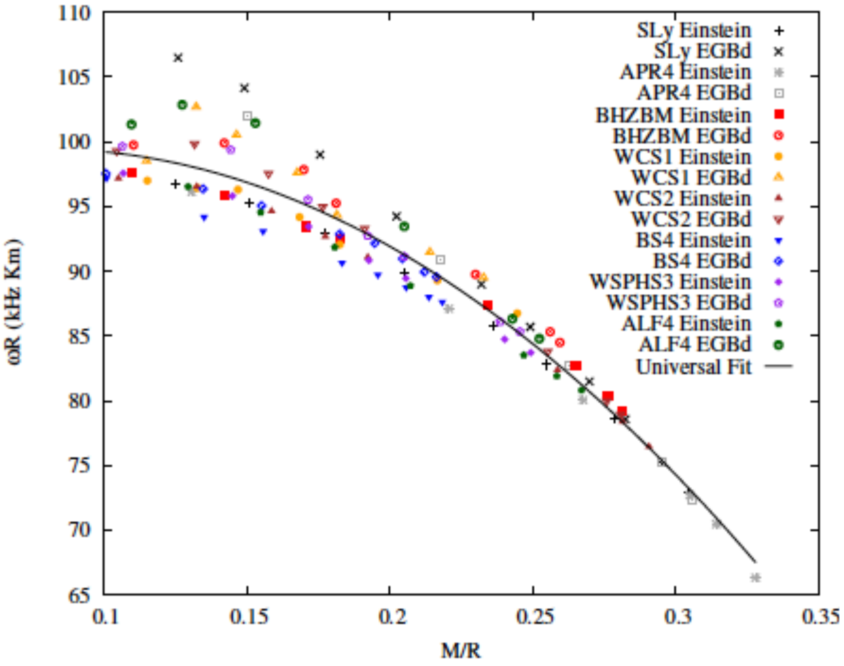


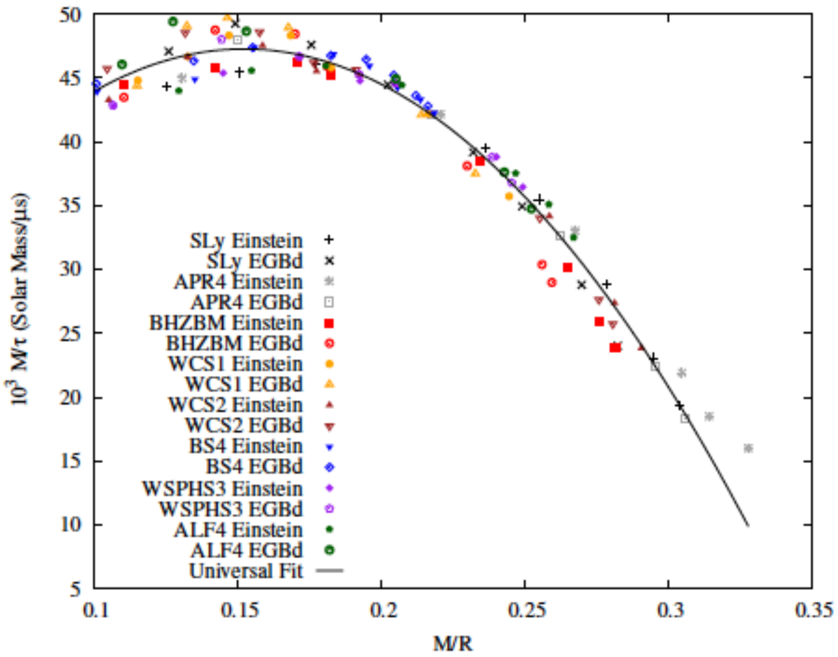
The **normalized damping time** $\eta\left(\frac{M}{\tau\eta^2}\right)^{(1/2\ell)}$ where $\eta=\sqrt{M^3/I}$

as a function of the normalized oscillation frequency ***Mσ*** for l = m = 2 & l = m = 4 f-modes.

Doneva-KK 2015

Dilatonic Einstein-Gauss-Bonnet Theory
Axial w-modes





Blazquez-Salcedo, Gonzalez-Romero, Kunz, Mojica, Navarro-Lerida (2016)

Conclusions

- ✓ **Neutron stars in alternative theories of gravity can have significantly different properties compared to their general relativistic counterparts.**
- ✓ **Rotation can magnify the deviations and lead to new observational consequences.**
- ✓ **A further study of the astrophysical implications is required in order to check what are the most promising astrophysical implications.**
- ✓ **Further info: Berti et al (2015), Yagi & Yunes (2016)**

Thank you