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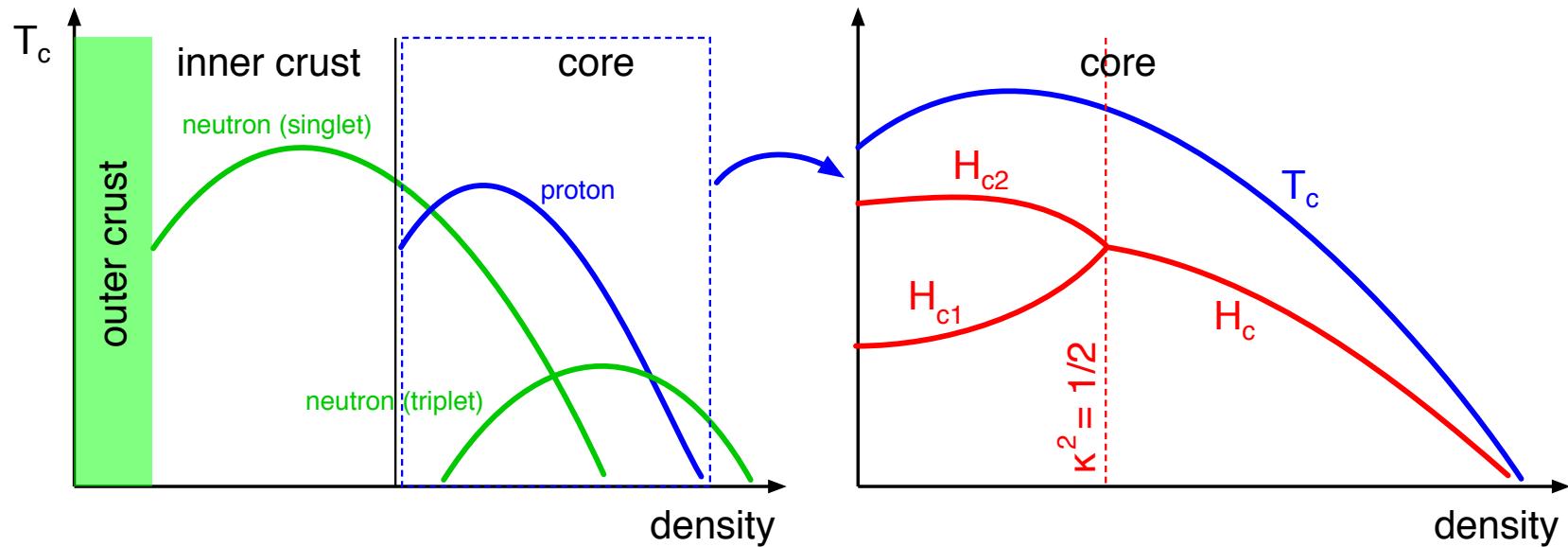


Critical magnetic fields in a superconductor coupled to a superfluid

A. Haber, A. Schmitt, arXiv:1612.01865

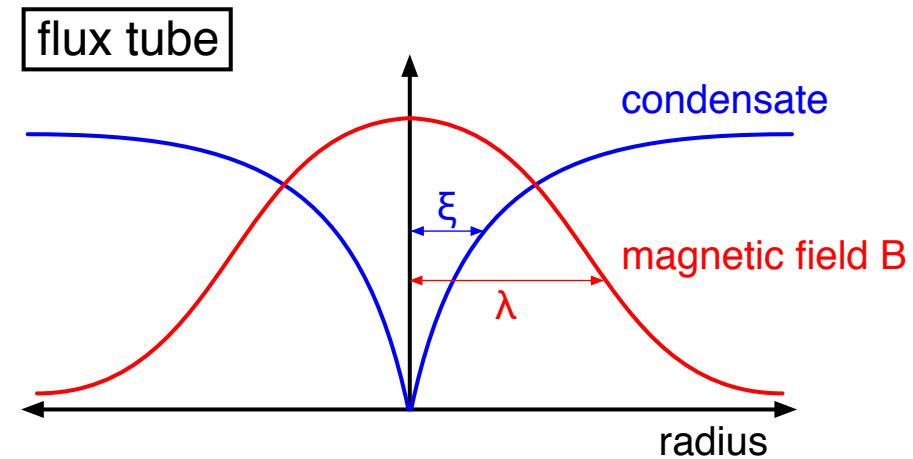
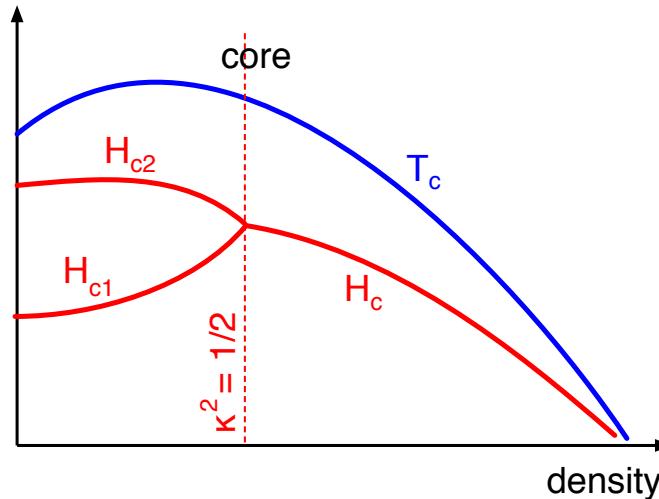
- astrophysical motivation:
coexisting superconductor and superfluid in neutron stars
- theoretical setup:
charged and neutral complex scalar fields with cross-coupling
- main results:
effect of superfluid on H_{c1} , H_{c2} , H_c and type-I/type-II transition

- Coexistence of neutron and proton Cooper pairing



- open questions:
transition to type-I superconductivity? transition to quark matter?
magnetic field evolution? interfaces between phases?
- this work:
two-fluid model, determine ground state for all H, T (no time evolution), include flux tubes (but no rotation and vortices)

- Remember type-I/type-II superconductivity



- Ginzburg-Landau parameter

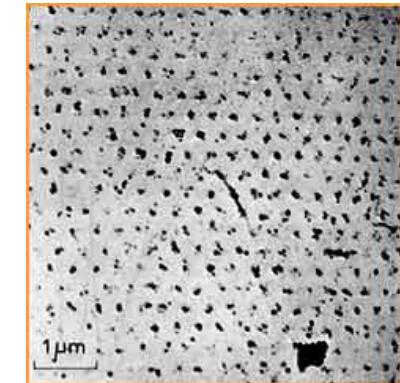
$$\kappa = \frac{\lambda}{\xi}$$

- type-II superconductivity for $\kappa > 1/\sqrt{2}$:

flux tube lattice for $H_{c1} < H < H_{c2}$

A.A. Abrikosov, Soviet Physics JETP 5, 1174 (1957)

- neutron star: spatially varying κ



first image of flux tube lattice:
U. Essmann and H. Träuble
Phys. Lett. A 24, 526 (1967)

● Model

see also: M. A. Alpar, S. A. Langer and J. A. Sauls, *Astrophys. J.* 282, 533 (1984)

M. G. Alford and G. Good, *PRB* 78, 024510 (2008)

A. Haber, A. Schmitt and S. Stetina, *PRD* 93, 025011 (2016)

$$\text{Lagrangian} \quad \mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_{\text{int}} - \frac{F_{\mu\nu}F^{\mu\nu}}{16\pi}$$

with

$$\mathcal{L}_i = (\partial_\mu + iq_i A_\mu)\varphi_i(\partial^\mu - iq_i A^\mu)\varphi_i^* - m_i^2|\varphi_i|^2 - \lambda_i|\varphi_i|^4 \quad i = 1, 2$$

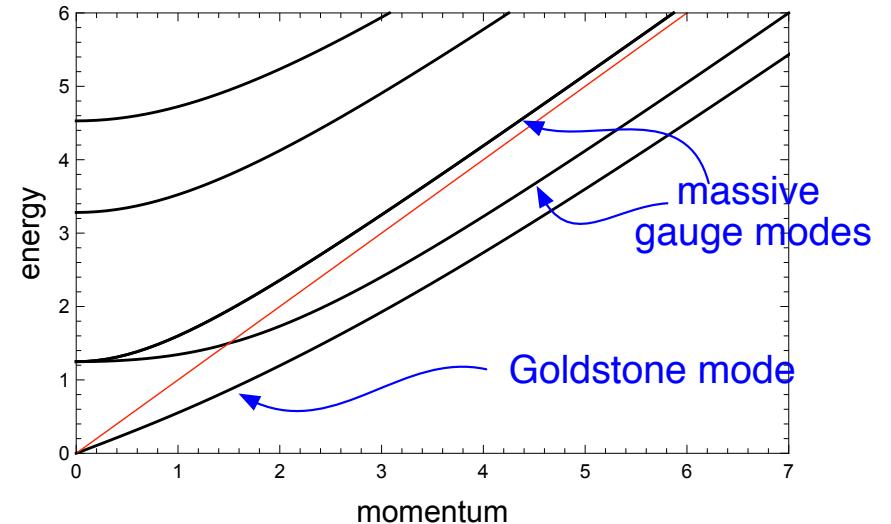
$$\mathcal{L}_{\text{int}} = 2h|\varphi_1|^2|\varphi_2|^2 - \frac{g_1}{2} \left[\varphi_1\varphi_2(\partial_\mu - iq_1 A_\mu)\varphi_1^*(\partial^\mu - iq_2 A^\mu)\varphi_2^* + \text{c.c.} \right]$$

$$- \frac{g_2}{2} \left[\varphi_1\varphi_2^*(\partial_\mu - iq_1 A_\mu)\varphi_1^*(\partial^\mu + iq_2 A^\mu)\varphi_2 + \text{c.c.} \right]$$

- coupling to gauge field $q_1 = 2e$ and $q_2 = 0$
(proton and neutron Cooper pairs)
- cross-couplings: density coupling h and gradient couplings g_1, g_2

● Nonzero-temperature effects

- compute excitation energies
(here shown for coexistence of charged and neutral condensates)
- apply large- T approximation



⇒ Ginzburg-Landau free energy ($g_1 = g_2$, $G \equiv (g_1 + g_2)/2$ small)

$$\begin{aligned} U(\vec{x}) = & \frac{(\nabla\rho_1)^2}{2} + \frac{(\nabla\rho_2)^2}{2} - \frac{\mu_1^2 - (\nabla\psi_1 - q\vec{A})^2 - m_{1,T}^2}{2}\rho_1^2 - \frac{\mu_2^2 - m_{2,T}^2}{2}\rho_2^2 + \frac{\lambda_1}{4}\rho_1^4 + \frac{\lambda_2}{4}\rho_2^4 \\ & - \frac{h_T}{2}\rho_1^2\rho_2^2 - \frac{G}{2}\rho_1\rho_2\nabla\rho_1 \cdot \nabla\rho_2 + \frac{B^2}{8\pi} \end{aligned}$$

with

$$m_{1,T}^2 = m_1^2 + \frac{2\lambda_1 - h + 6\pi q^2}{6}T^2, \quad m_{2,T}^2 = m_2^2 + \frac{2\lambda_2 - h}{6}T^2, \quad h_T = h \left(1 + \frac{GT^2}{6} \right)$$

[generalization of J. I. Kapusta, PRD 24, 426 (1981)]

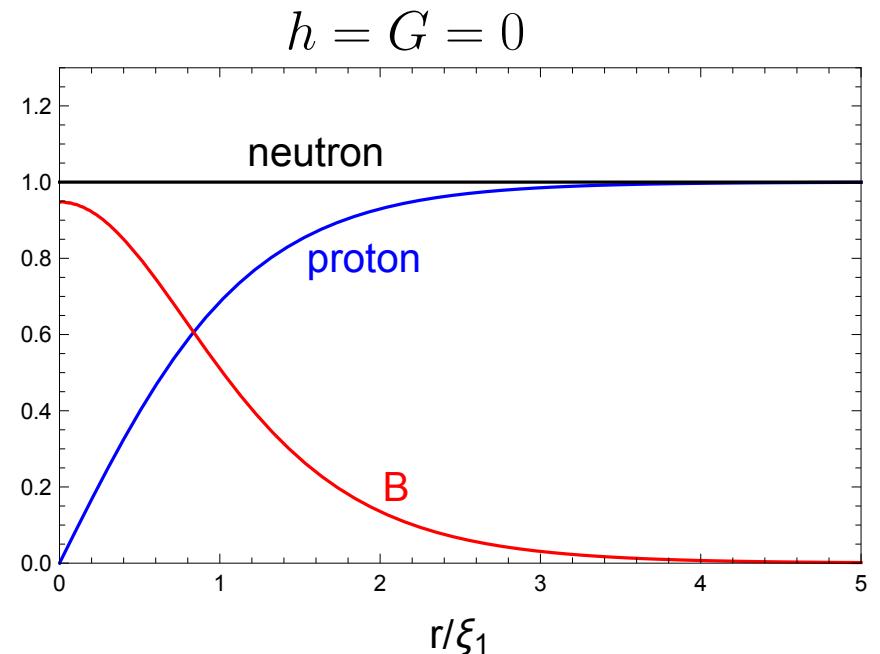
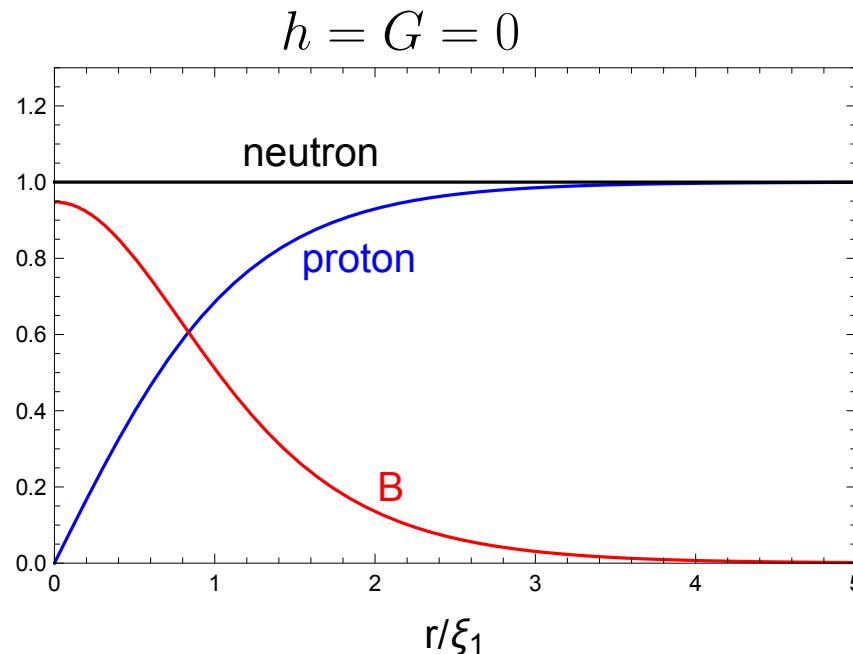
- **Flux tube profiles**

- with the condensates

$$\varphi_i(r, \theta) = \frac{\rho_i(r)}{\sqrt{2}} e^{i\psi_i(\theta)}, \quad \psi_1 = n\theta, \quad \psi_2 = 0$$

solve equations of motion for ρ_1, ρ_2, \vec{A}

- flux tube profiles $\rho_i(r)$:



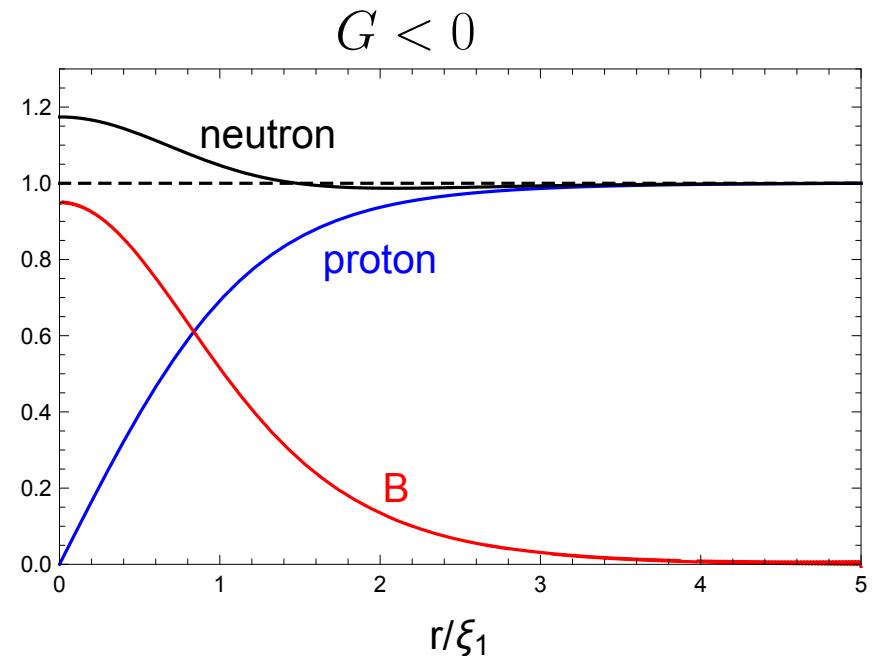
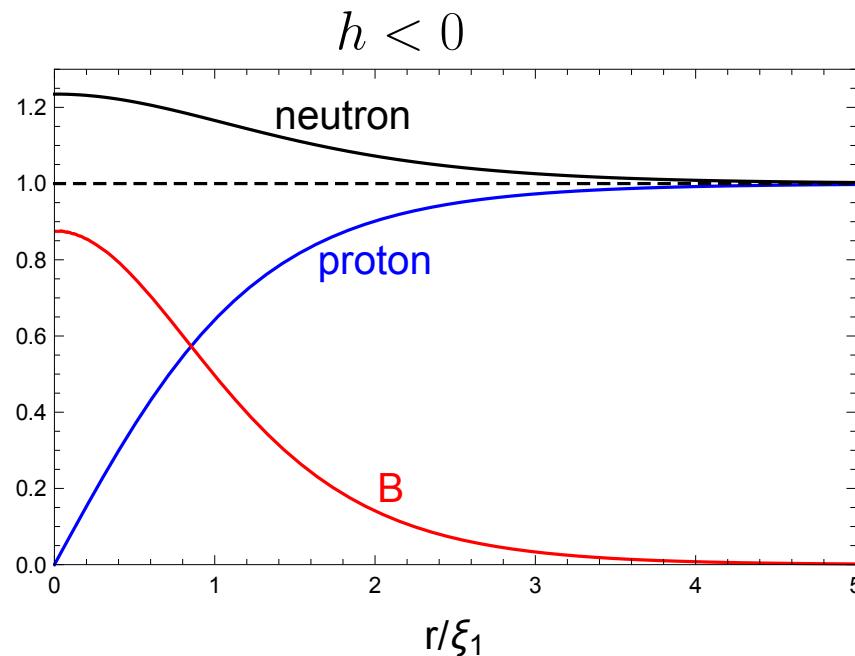
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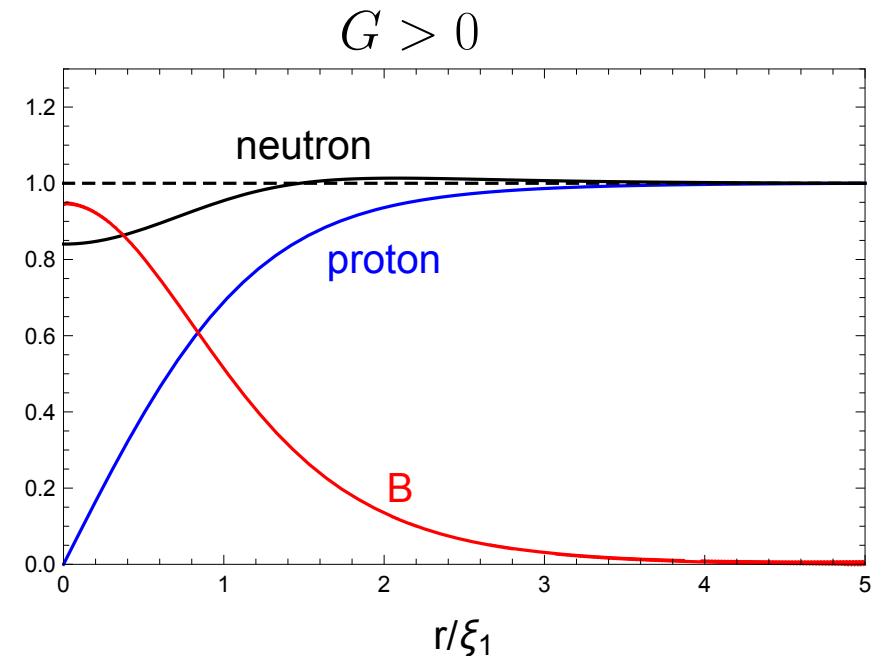
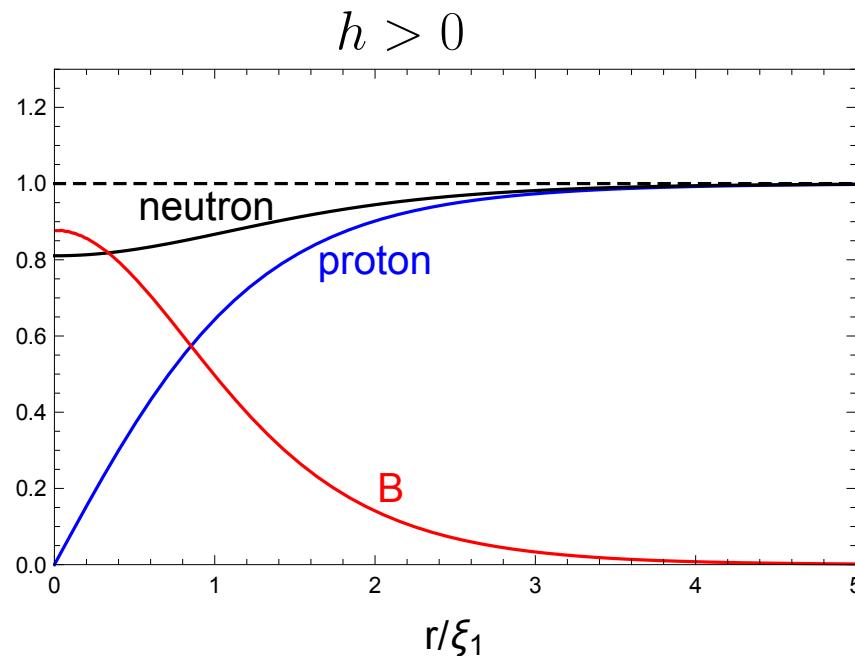
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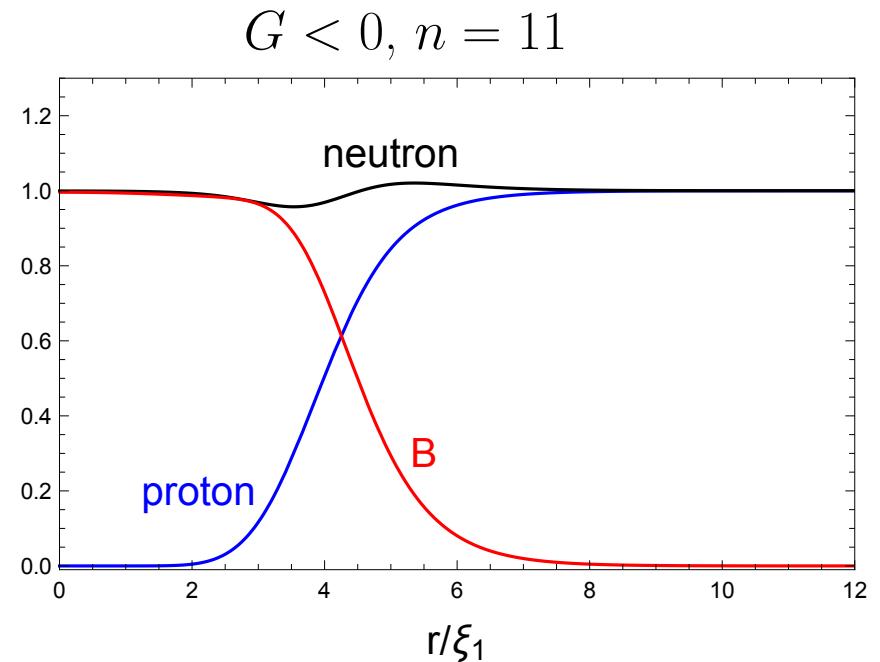
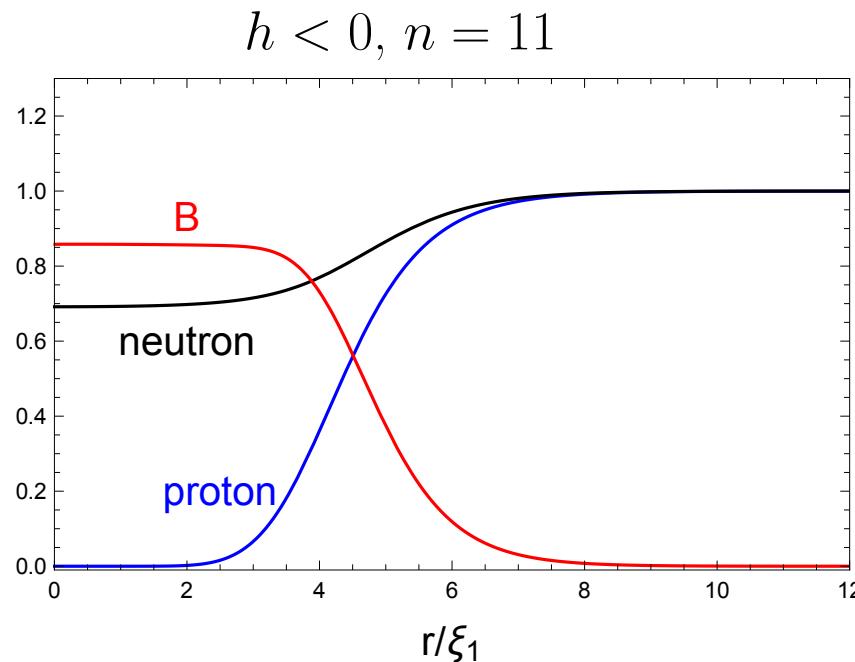
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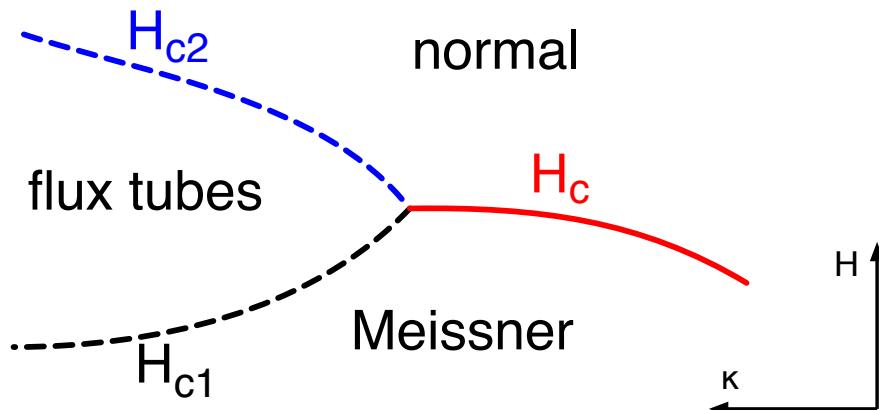
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solve equations of motion for ρ_1, ρ_2, \vec{A}

- flux tube profiles $\rho_i(r)$:



- Critical magnetic fields



- Gibbs free energies of Meissner and normal phase identical:

$$H_c = \sqrt{2\pi\lambda_1} \rho_1^2 \sqrt{1 - \frac{h_T^2}{\lambda_1\lambda_2}}$$

- just below H_{c2} condensation becomes favorable

$$H_{c2} = H_c \sqrt{2\kappa} \sqrt{1 - \frac{h_T^2}{\lambda_1\lambda_2}}$$

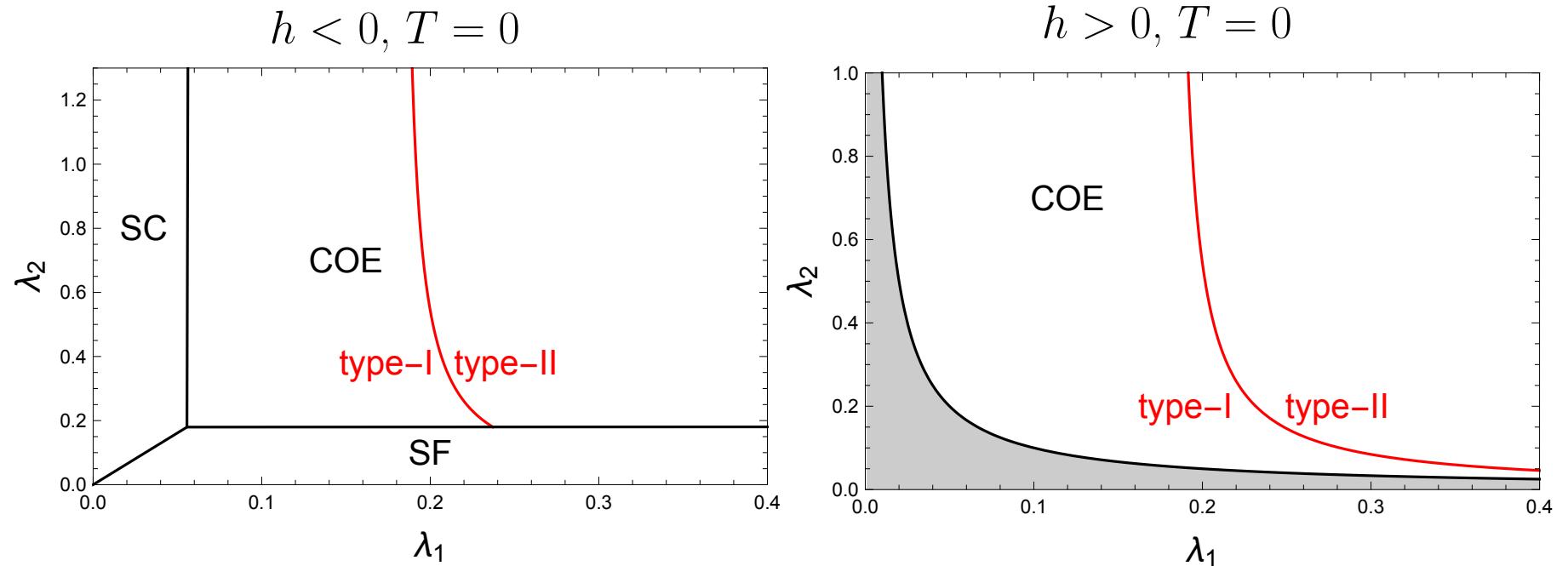
→ coupling to superfluid appears to simply shift critical κ

M. Sinha, A. Sedrakian, PRC 91, 035805 (2015)

- just above H_{c1} a single flux tube becomes favorable
(needs to be computed numerically)

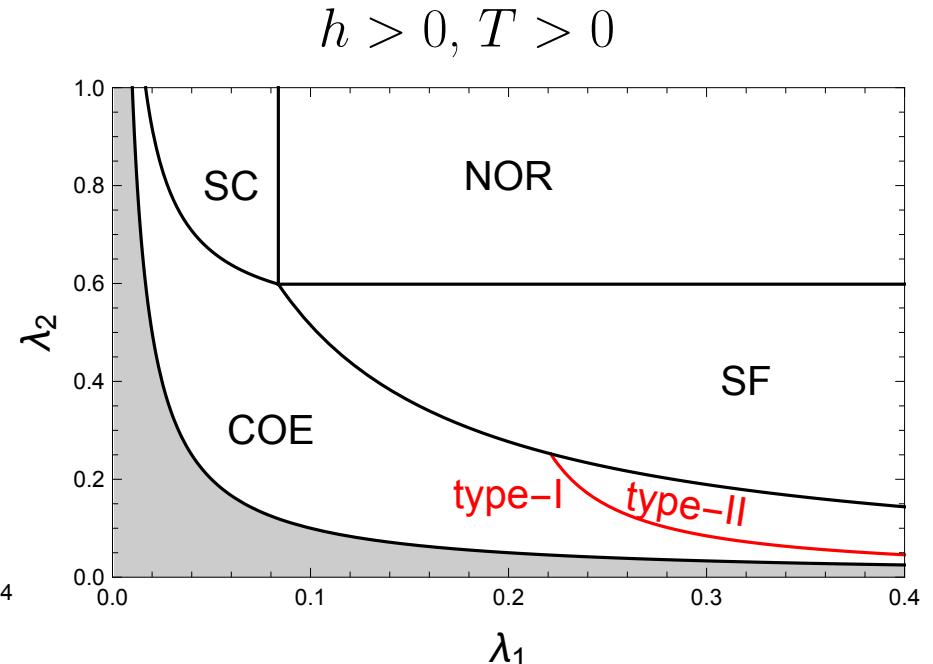
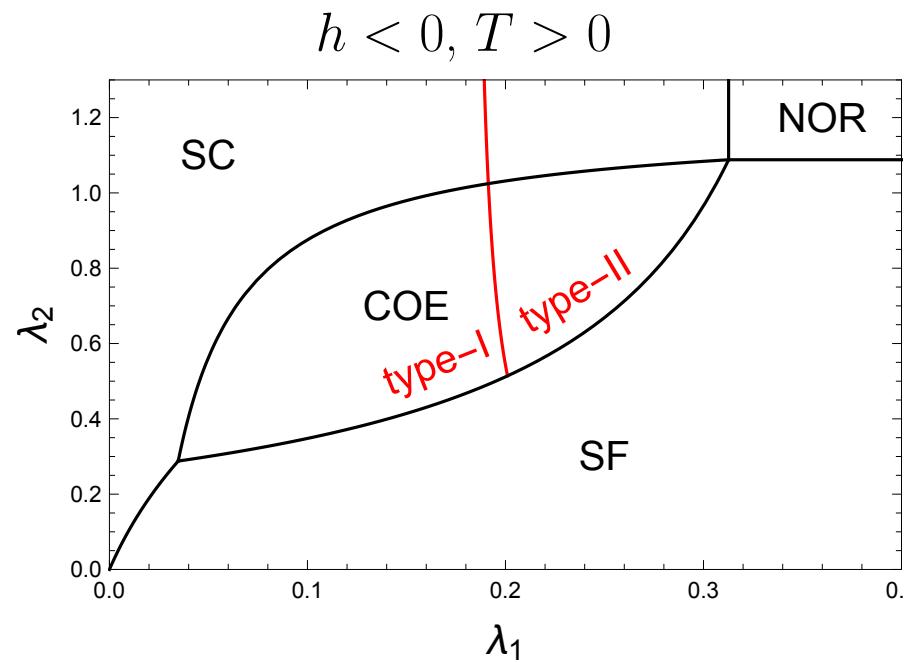
- Towards computing H - T phase structure

- in general: multi-dimensional parameter space $(m_1, m_2, \lambda_1, \lambda_2, q, h, G)$ and (μ_1, μ_2, T, H)
- here: $m \equiv m_1 = m_2$, $\mu_1 > m_1$, $\mu_2 > m_2$, $q = 2e$, $G = 0$
- focus on transition between type-I and type-II superconductivity



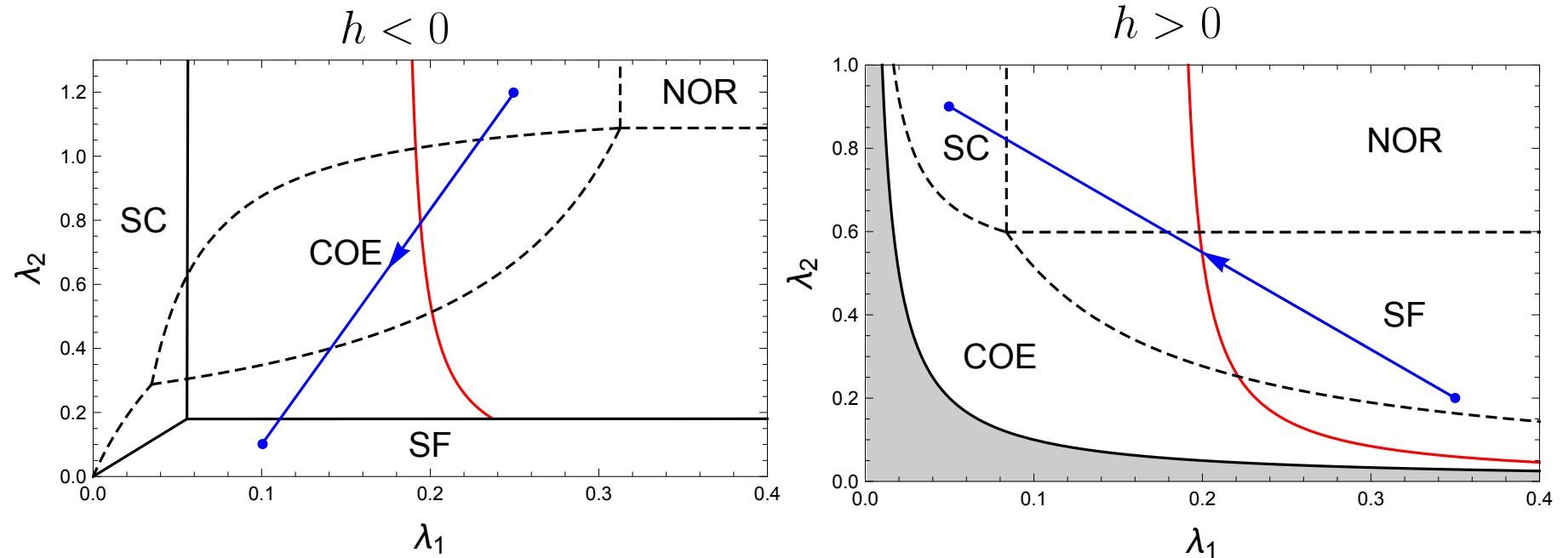
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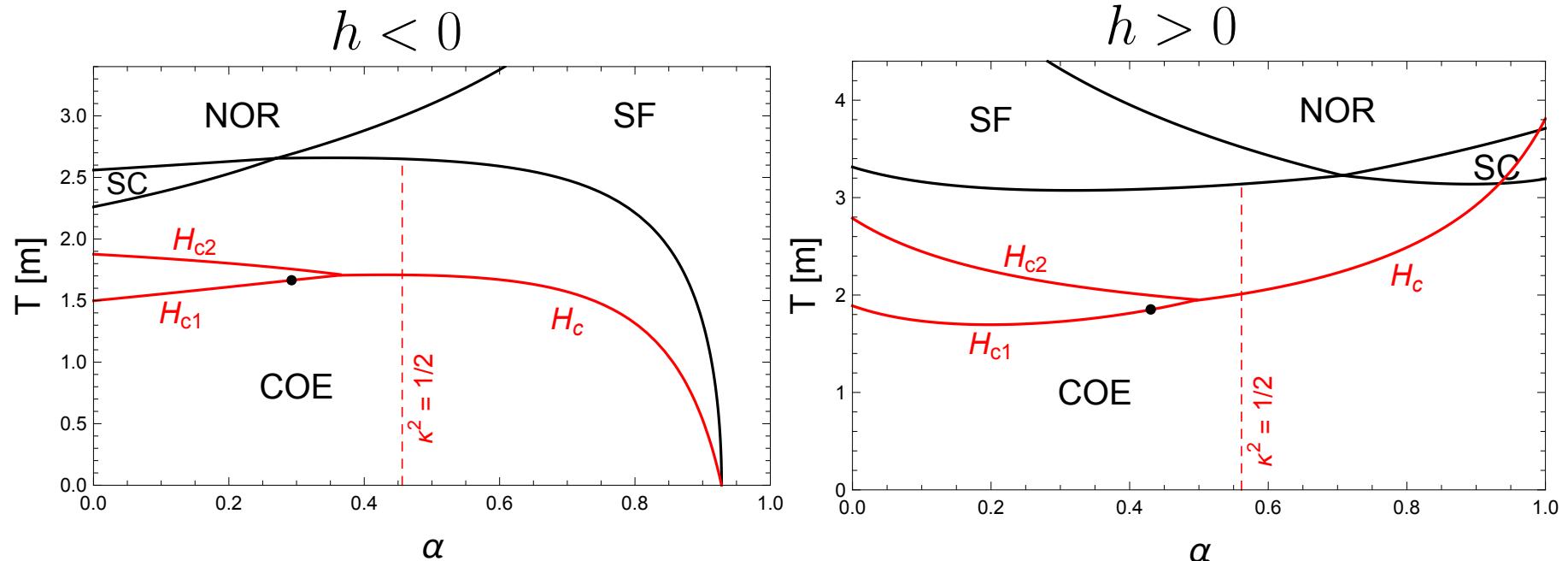
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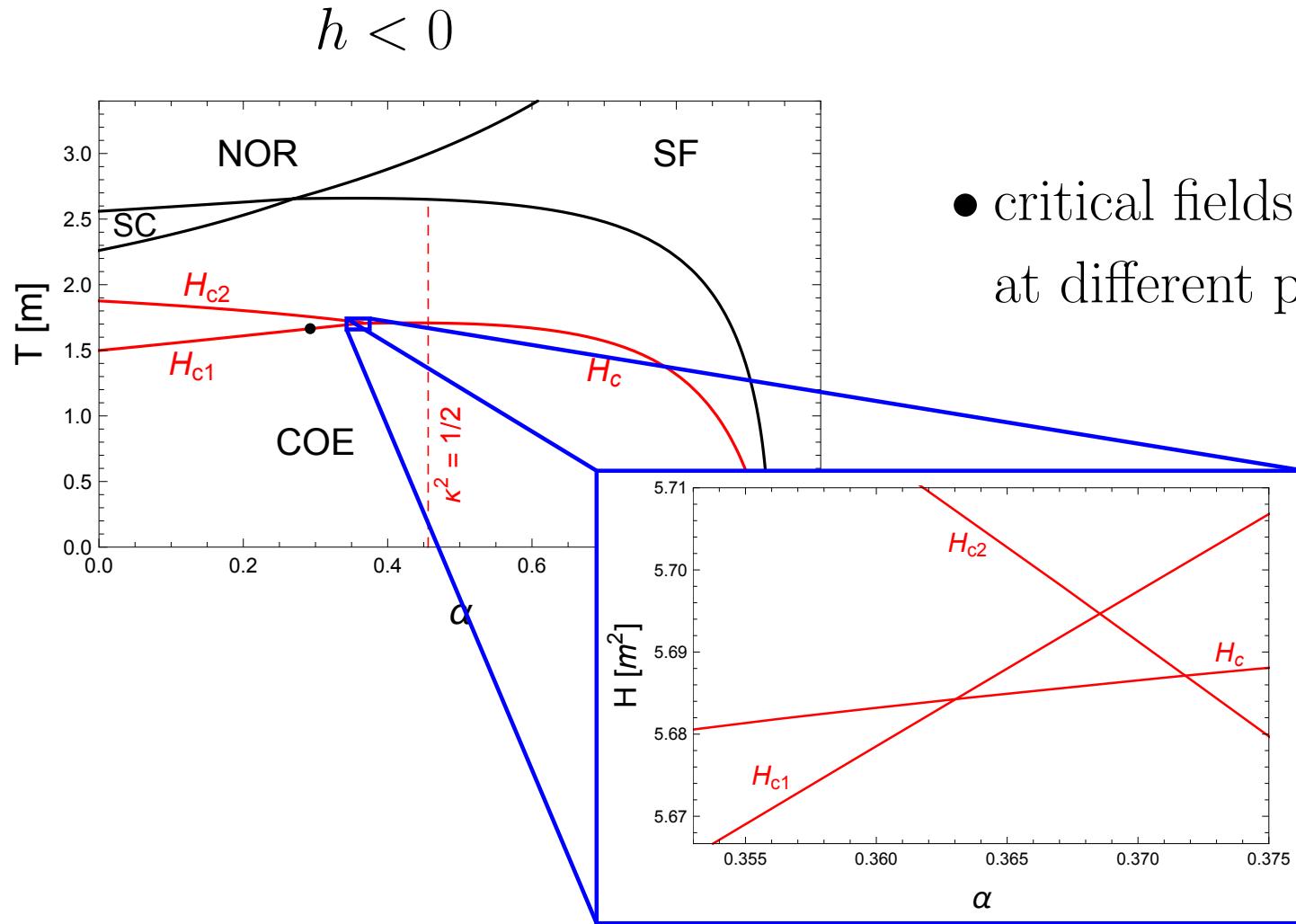
- choose path through λ_1 , λ_2 plane parameterized by "density" α

- Critical temperatures and magnetic fields (page 1/2)



- possible to construct full α - T - H phase diagram
 - increase T : eventually all condensates vanish \rightarrow NOR
 - increase H : eventually all *charged* condensates vanish \rightarrow SF or NOR
- type-I/type-II transition ...
 - ... does not occur at $\kappa^2 = 1/2$
 - ... is qualitatively different due to coupling to superfluid \rightarrow zoom in

- Critical temperatures and magnetic fields (page 2/2)



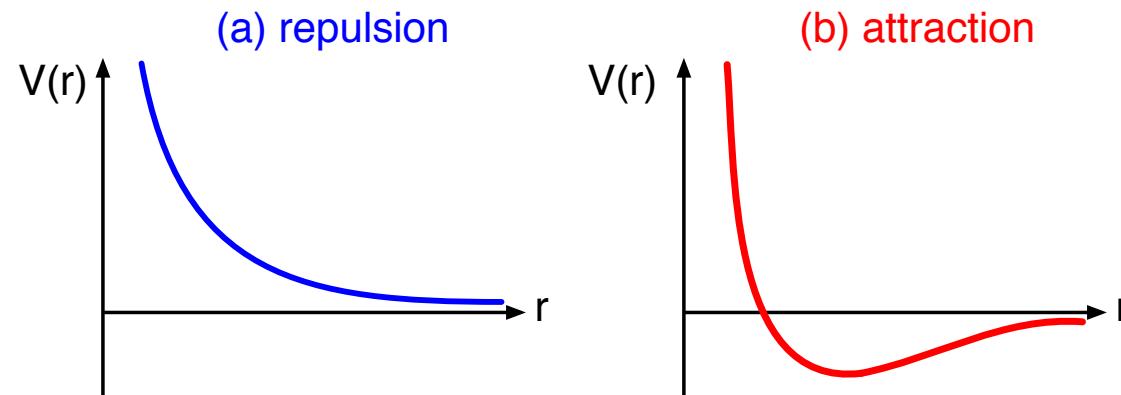
- critical fields intersect at different points

- phase structure? compute flux tube interactions ...

- **Flux tube interaction (page 1/2)**
- interaction between two flux tubes with large separation:
approximate profiles and free energy around fluxtube 1 by mainly coming from flux tube 1 plus small corrections from flux tube 2
L. Kramer, PRB 3, 3821 (1971)
K. B. W. Buckley, M. A. Metlitski and A. R. Zhitnitsky, PRC 69, 055803 (2004)
- generalization of previous work to arbitrary coupling λ_1, λ_2, h , plus entrainment G A. Haber, A. Schmitt, arXiv:1612.01865
- analytical expression in the absence of entrainment, $G = 0$
 \Rightarrow long-range interaction changes from repulsion to attraction at
$$1 = \frac{H_{c2}}{H_c} \left[1 - \frac{h^2}{2\lambda_2^2 x^2} + \mathcal{O}\left(\frac{1}{x^4}\right) \right] \quad \text{where} \quad x \equiv \frac{\rho_{\text{neutron}}}{\rho_{\text{proton}}}$$

→ different from effective critical $\kappa!$
(neutron star: x large \Rightarrow effect small?)

- Flux tube interaction (page 2/2)

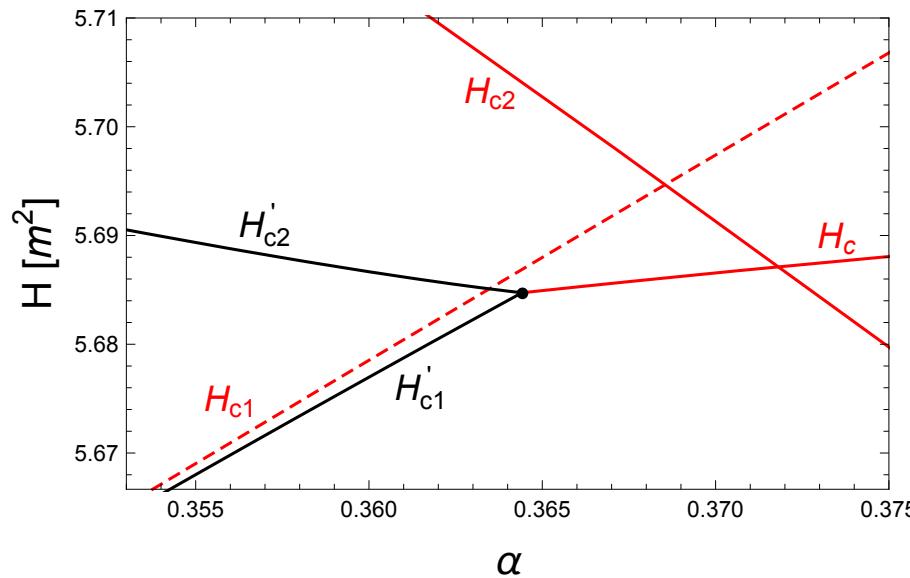


- textbook scenario: (a) \rightarrow (b) at $\kappa^2 = 1/2$ where $H_c = H_{c1} = H_{c2}$
 \rightarrow type-I/type-II transition
- coupling to superfluid: (a) \rightarrow (b) in type-II region

\Rightarrow first-order flux tube onset

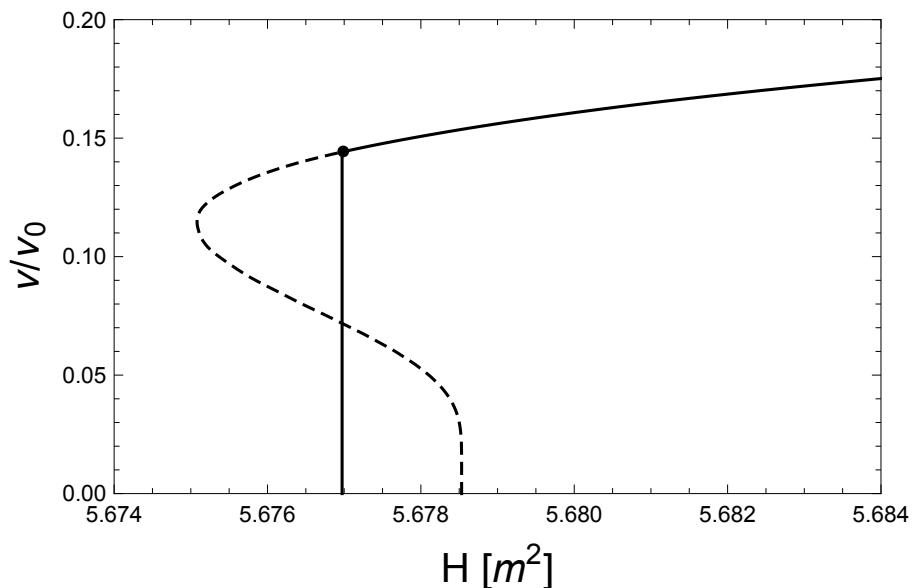
\rightarrow compute first-order transition: use above approximation for interaction and nearest-neighbor approximation (and hexagonal flux tube lattice)

- First-order flux tube onset

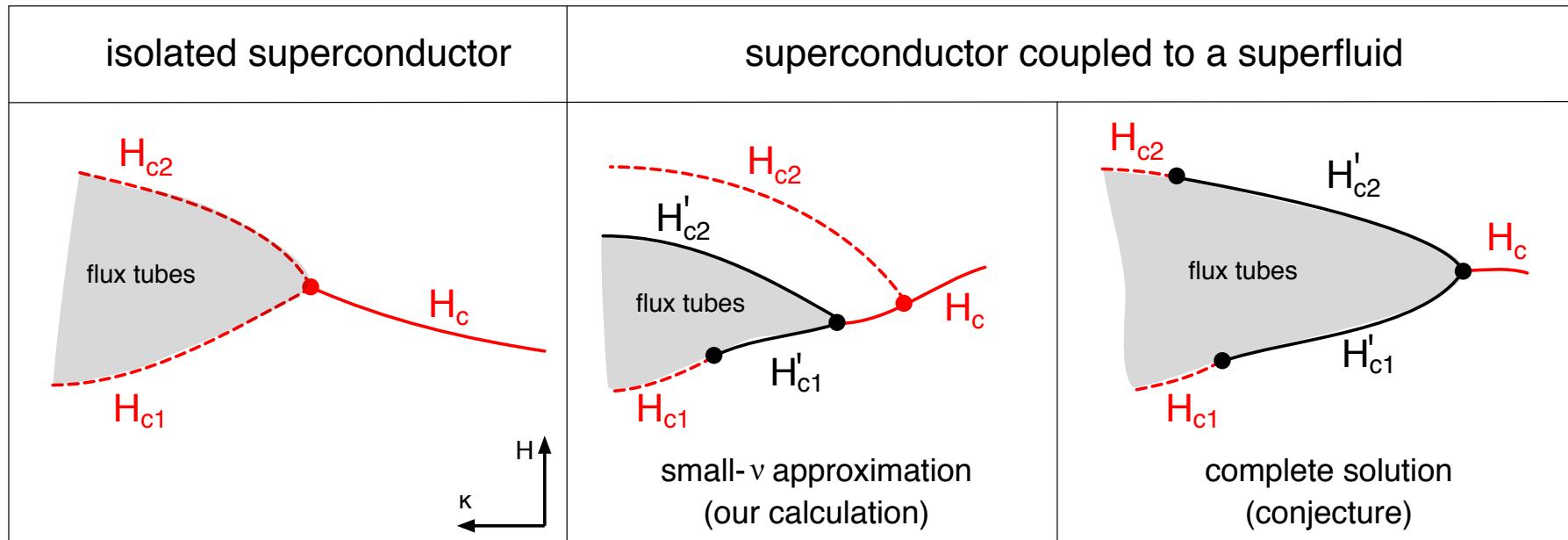


- flux tube area density ν discontinuous at onset
- analogy:
 $H \leftrightarrow$ chemical potential
 $B \leftrightarrow$ density

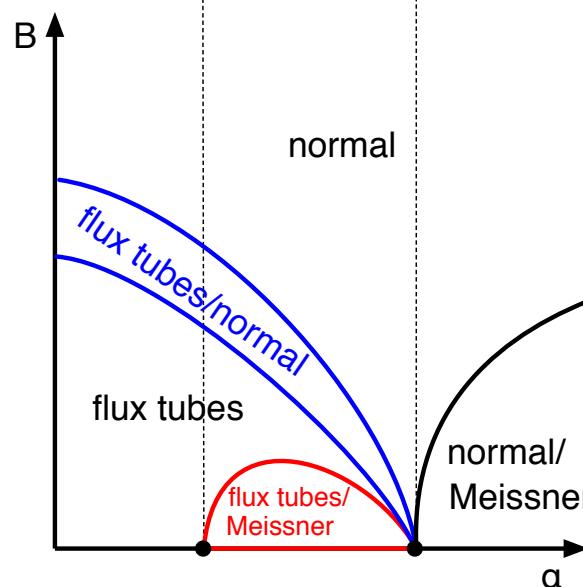
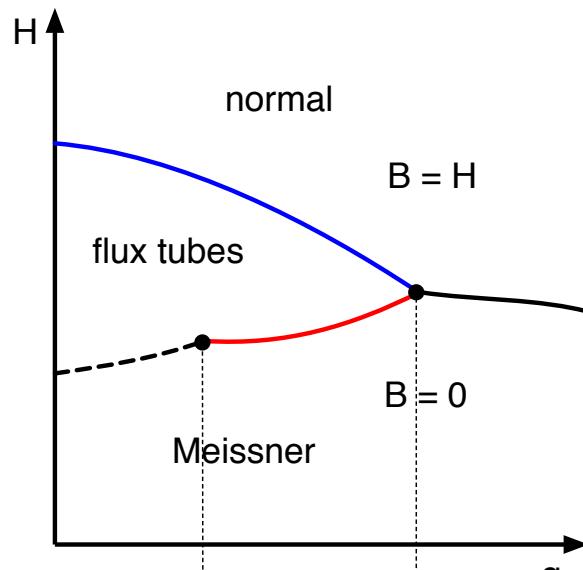
- H'_{c1}, H'_{c2} : first-order boundaries of flux tube phase
- $H'_{c2} < H_{c2}$: artifact of approximation



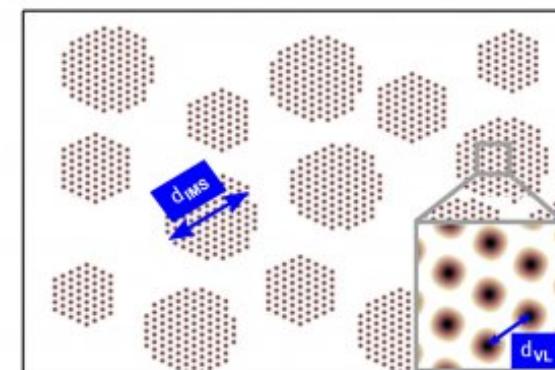
- Conjectured phase structure (page 1/2)



● Conjectured phase structure (page 2/2)



- discontinuity in $B \rightarrow$ mixed phases with flux tube lattice
- see also multi-component superconductors
S.Z. Lin, X. Hu, PRB 84, 214505 (2011)
Y. Takahashi, Z. Huang, X. Hu, 1309.1570
 - observation of "flux tube clusters"
T. Reimann *et al.*, Nature Comm. 6, 8813 (2015)



- **Summary**

- **Model:** two bosonic fields interacting
 - with themselves and
 - with each other (density and gradient coupling) and
 - with a gauge field (one of them)
- **Results:**
 - phase structure at all T and H (in particular H_{c1} , H_{c2} , H_c), consistently taking into account all possible phases
 - type-I/type-II transition qualitatively different through coupling to superfluid \rightarrow first order flux tube onset

● Discussion/Outlook

● check conjecture about phase diagram

→ need free energy of flux tube lattice

M. Alford, A. Haber, A. Schmitt, A. Windisch, work in progress

● apply to neutron star core

– fix parameters, e.g., by starting from fermionic theory

M. Alford, G. Good and S. Reddy, PRC 72, 055801 (2005)

– time scale of magnetic field expulsion? flux tube phase ever realized?

– mixed phase flux tubes/Meissner relevant? very small layer in the star?

– relevant for color flux tubes in CFL- K^0 quark matter?

M. G. Alford and A. Sedrakian, JPG 37, 075202 (2010)

K. Glampedakis, D. I. Jones and L. Samuelsson, PRL 109, 081103 (2012)

● application to ultracold atoms

– two-superfluid systems & single-component "charged" system

I. Ferrier-Barbut, *et al.*, Science 345, 1035 (2014)

Y.J. Lin, *et al.*, Nature 462, 628 (2009)

– neutral/"charged" mixture possible?