The Hearty Shapes of Quark Stars

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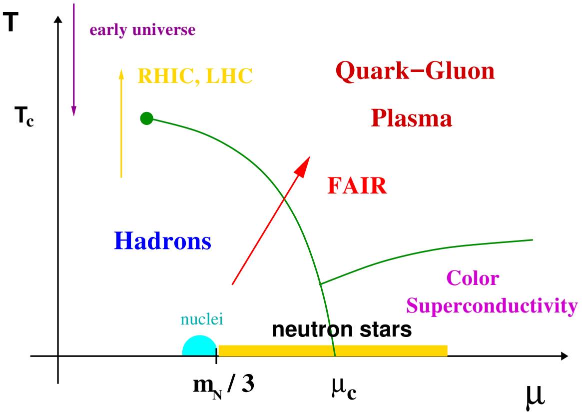
TH Institute 'From quarks to gravitational waves: Neutron stars as a laboratory for fundamental physics'
CERN, Geneva, December 5-9, 2016

compstar HIC for FAIR Helmholtz International Center HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research

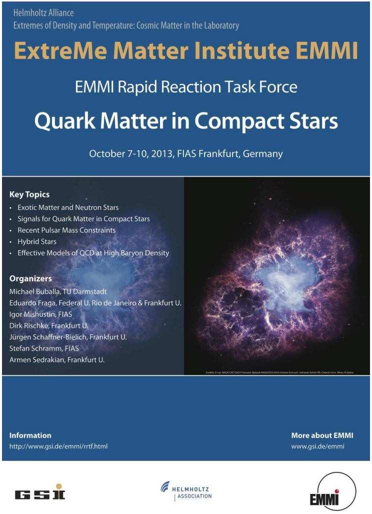
Content

Phase Diagram of Quantum Chromodynamics QCD



- early universe at zero density and high temperature
- neutron star matter at small temperature and high density
- rapid crossover at zero density (lattice simulations)

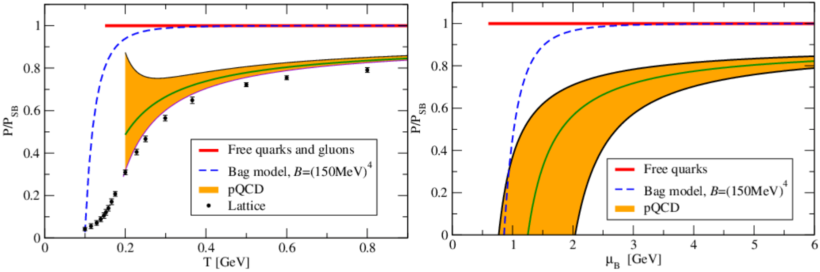
Task Force Meeting on Quark Matter in Compact Stars



EMMI Rapid Reaction Task Force Meeting on 'Quark Matter in Compact Stars', FIAS, October 6-10, 2013

(Buballa, Dexheimer, Drago, Fraga, Haensel, Mishustin, Pagliara, JSB, Schramm, Sedrakian, Weber, arXiv:1402.6911, JPG 41 (2014) 12)

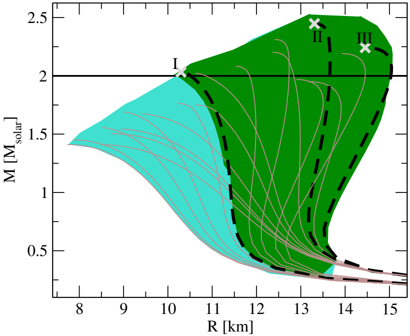
High-Density Side: perturbative QCD



(Fraga, Kurkela, Vuorinen, 2013)

- asymptotic freedom of strong interactions (QCD): weakly interacting at large scales (temperature and/or chemical potential)
- perturbative calculations up to $\mathcal{O}(\alpha_s^2)$:
  follows lattice simulation at nonzero temperature
- band of uncertainty for the pQCD equation of state at nonzero μ (from choice of renormalization scale)

Interpolated Mass-Radius Relation



(Kurkela, Fraga, JSB, Vuorinen (2014))

- corresponding band of the interpolated EoS in mass-radius plot
- cyan band: excluded by $2M_\odot$ mass constraint
- green band: masses of up to $2.5M_\odot$ possible
- typical radii: 11-14.5km for $1.4M_\odot$ and 10-15 km for $2M_\odot$

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Chiral Effective Lagrangian

Investigation of the chiral phase transition of QCD within a chiral effective Lagrangian (Pisarski and Wilczek 1984):

$$\begin{aligned}\mathcal{L}_{\mathrm{SU(3)}} &= \frac{1}{2}\mathrm{Tr}\left[\partial_\mu \mathrm{M}\partial^\mu \mathrm{M}^\dagger\right] + \frac{1}{2}\mu^2 \mathrm{Tr}\left[\mathrm{M}\mathrm{M}^\dagger\right] - \lambda \mathrm{Tr}\left[\mathrm{M}\mathrm{M}^\dagger \mathrm{M}\mathrm{M}^\dagger\right] \\ &\quad -\lambda'\left(\mathrm{Tr}\left[\mathrm{M}\mathrm{M}^\dagger\right]\right)^2 + c\left[\det \mathrm{M} + \det \mathrm{M}^\dagger\right]\end{aligned}$$

equivalent to the linear σ model (for SU(3): Levy, 1967)

- $\mathrm{Tr} M^\dagger M \longrightarrow O(18)$ (norm of a vector)
- $\mathrm{Tr} M^\dagger M M^\dagger M \longrightarrow U(3) \times U(3)$ ($M \to U M U^{-1}$)
- $\det M + \det M^\dagger \longrightarrow SU(3) \times SU(3)$
  ($\det \exp i\lambda_0 = \exp \mathrm{Tr}\lambda \neq 1$) breaks $U_A(1)$ symmetry

Breaking of Chiral Symmetry and Masses

- spontaneous breaking of chiral symmetry by two order parameters

$$< \bar{u}u + \bar{d}d > \to \sigma_u + \sigma_d \qquad \text{and} \qquad < \bar{s}s > \to \sigma_s$$

- explicit breaking of chiral symmetry:

$$\mathcal{L}_{\text{esb}} = \epsilon \cdot (\sigma_u + \sigma_d) + \epsilon' \cdot \sigma_s$$

- vacuum values fixed by PCAC relations:
  $\sigma_u^0 = \sigma_d^0 = f_\pi = 92.4$ MeV
  $\sigma_s^0 = \sqrt{2} f_K - f_\pi / \sqrt{2} = 94.5$ MeV
- coupling constants μ^2, λ, λ', c fitted to meson masses: m_π, m_K, $m_\eta^2 + m_{\eta'}^2$, m_σ.

Quark contribution

Add quarks to the thermodynamic potential including the coupling to the Polyakov-loop and meson fields:

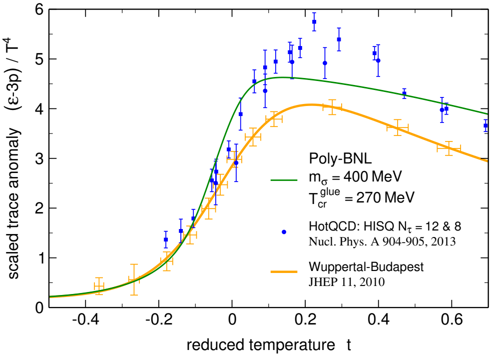
$$\Omega_q^{th} = -2T \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \times$$
$$\left\{ \ln\left[1 + 3\left(\Phi + \bar{\Phi} e^{-(E_f-\mu_f)/T}\right) \times e^{-(E_f-\mu_f)/T} + e^{-3(E_f-\mu_f)/T}\right]\right.$$
$$\left. + \ln\left[1 + 3\left(\bar{\Phi} + \Phi e^{-(E_f+\mu_f)/T}\right) \times e^{-(E_f+\mu_f)/T} + e^{-3(E_f+\mu_f)/T}\right]\right\}$$

Quark dispersion relation: $E_f = \sqrt{k^2 + m_f^2}$ with

$$m_u = \frac{g}{2}\sigma_u \qquad m_d = \frac{g}{2}\sigma_d \qquad m_s = \frac{g}{\sqrt{2}}\sigma_s$$

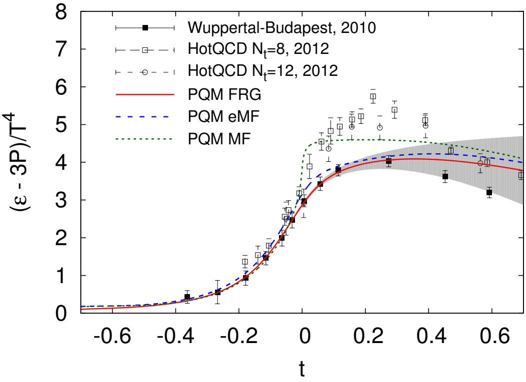
Fix Yukawa coupling g by setting $m_q = 300$ MeV

Comparison to lattice simulation



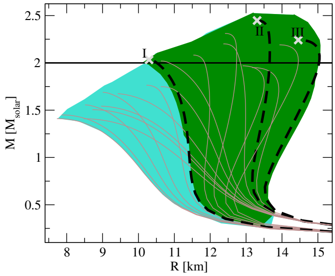
- check lattice simulation at zero chemical potentials
- reduced temperature $t = (T - T_c)/T_c$
- use FRG-improved description of Polyakov loop potential (Haas, Stiele, Braun, Pawlowski, JSB 2013)
- reasonable description of scaled trace anomaly

Comparison to lattice simulation II

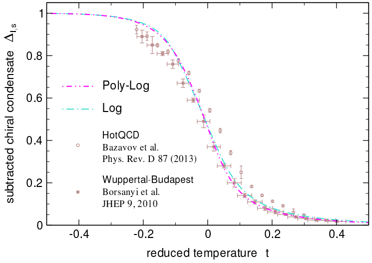


- comparison to functional renormalisation group (FRG) (Herbst, Mitter, Pawlowski, Schaefer, Stiele 2013)
- mean-field results are close to FRG results
- some difference without (MF) and with zero-point energy (eMF)

Central Number and Energy Densities



(Stiele and JSB 2016)



EoS	n/n_0	ϵ [GeV/fm^{-}3]
I	7.5	1.54
II	4.5	0.87
III	3.9	0.72

energy density in the crossover region from lattice simulation ($T = 145 - 163$ MeV):

$$\epsilon_c = (0.18 - 0.5)\ \mathrm{GeV/fm^3}$$

(Bazavov et al. 2014)

Extrapolation to High Densities

- lattice simulation: dominated by Polyakov-loop potential
  $\Longrightarrow$ vanishes for $T = 0$
- lattice simulation: small vector interaction term from susceptibilities
  $\Longrightarrow$ need vector interactions (repulsion) to get massive compact stars
- lattice simulation: zero total energy in the vacuum
  $\Longrightarrow$ gravity couples to vacuum energy

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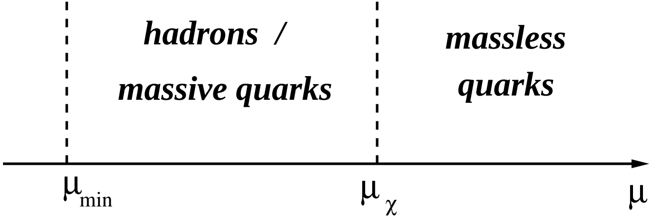
Matching to low density EoS

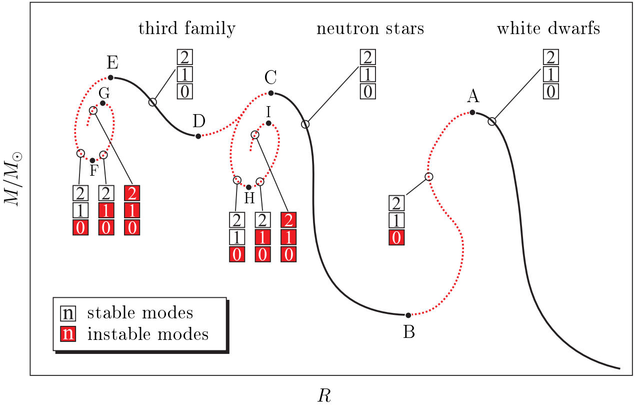


Two possibilities for a first-order chiral phase transition:

- A weakly first-order chiral transition (or no true phase transition),
  $\Longrightarrow$ one type of compact star:
  hybrid stars masquerade as neutron stars
- A strongly first-order chiral transition
  $\Longrightarrow$ two types of compact stars:
  a new stable solution with smaller masses and radii

Third Family of Compact Stars (Gerlach 1968)

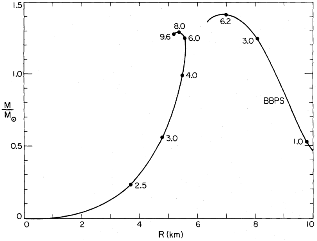
(Glendenning, Kettner 2000; Schertler, Greiner, JSB, Thoma 2000)



- third solution to the TOV equations besides white dwarfs and neutron stars, solution is stable!
- generates stars more compact than neutron stars
- possible for any first order phase transition!

Selfbound Star versus Ordinary Neutron Star

(Hartle, Sawyer, Scalapino (1975!))



selfbound stars:

- vanishing pressure at a finite energy density
- mass-radius relation starts at the origin (ignoring a possible crust)
- arbitrarily small masses and radii possible

neutron stars:

- bound by gravity, finite pressure for all energy density
- mass-radius relation starts at large radii
- minimum neutron star mass: $M \sim 0.1 M_\odot$ with $R \sim 200$ km

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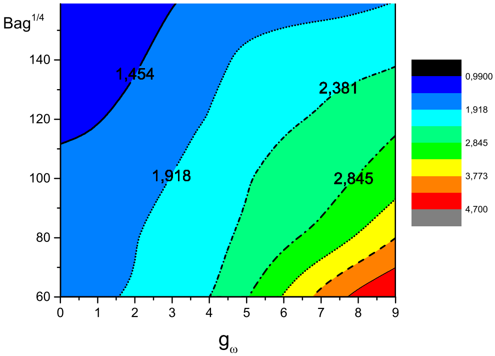
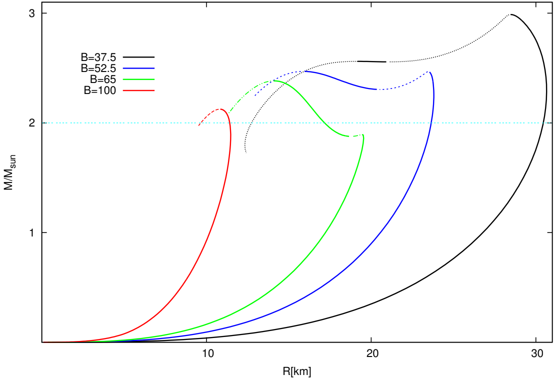
Quark Star Masses in the Quark-Meson Model

(Zacchi, Stiele, JSB, 2015)



- chiral quark-meson model as an effective model
- maximum masses for different vacuum energy and vector coupling
- repulsive vector interaction needed to get massive compact stars

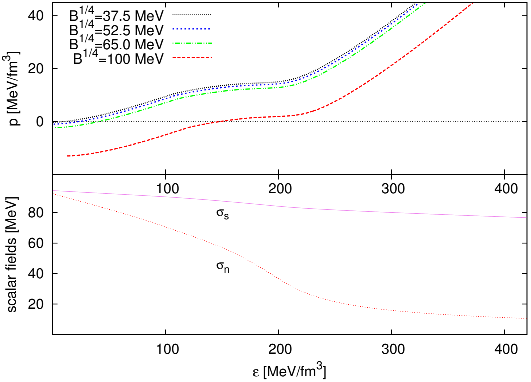
Mass-Radius Relation for Quark Stars



(Zacchi, Tolos, JSB 2016)

- pure quark stars: curves start at origin
- nontrivial scaling with the vacuum energy B due to interactions
- twin star solution from chiral crossover transition

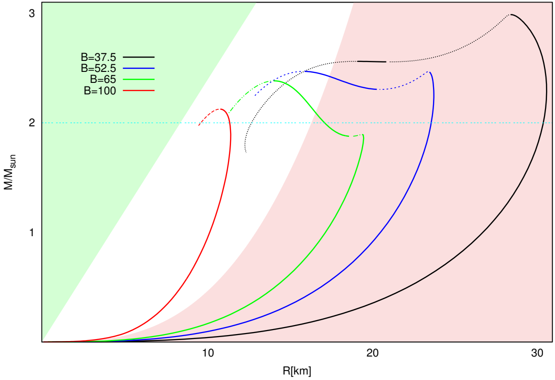
Chiral Phase Transition for Compact Stars



(Zacchi, Tolos, JSB 2016)

- two chiral condensates: σ_n (nonstrange) and σ_s (strange)
- smooth dropping of σ_n and a corresponding plateau in the pressure
- appearance of chiral transition plateau for compact star matter hinges on vacuum pressure B

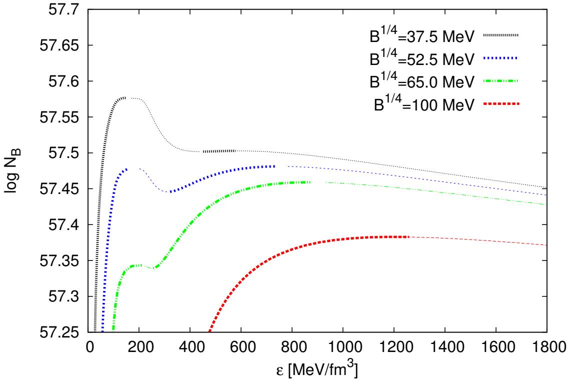
Constraints on Mass-Radius Relation



(Zacchi, Tolos, JSB 2016)

- constraints on mass-radius curves
- left side (green): causality limit on EoS (3GM)
- right side (red): mass-shedding rotation limit (for 716 Hz)

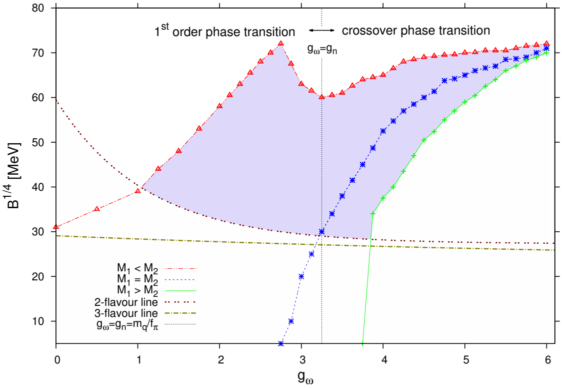
Comparison of Twin Quark Star Solution



(Zacchi, Tolos, JSB 2016)

- baryon number N_B of the compact star versus central energy density
- look at constant N_B for a transition to the twin star (conserving baryon number)
- transition possible for an intermediate range of vacuum energy B

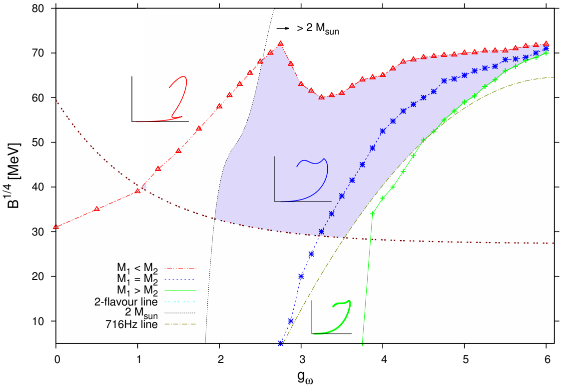
Parameter Scan for Twin Quark Stars



(Zacchi, Tolos, JSB 2016)

- vacuum pressure B versus vector coupling constant g_ω (repulsion)
- first order phase transition for $g_\omega \lesssim g_n = m_q/f_\pi$ (scalar coupling constant)
- shaded region: parameter space for twin star solutions

Parameter Scan for Twin Stars II



(Zacchi, Tolos, JSB 2016)

- more than $2M_{\odot}$ maximum mass for $g_{\omega} \gtrsim 2$
- below dotted curve: two-flavour quark matter more stable than ^{56}Fe
- shaded region: heart shaped mass radius curves

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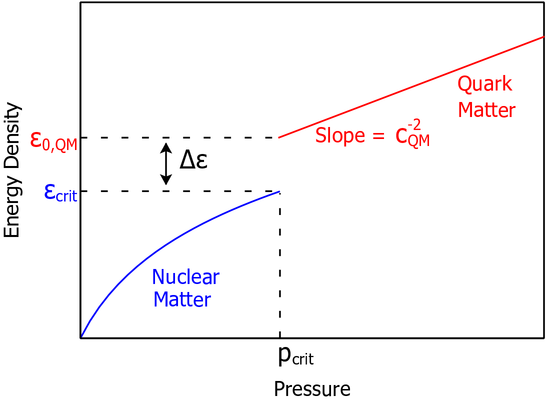
Constant Speed of Sound Parameterization

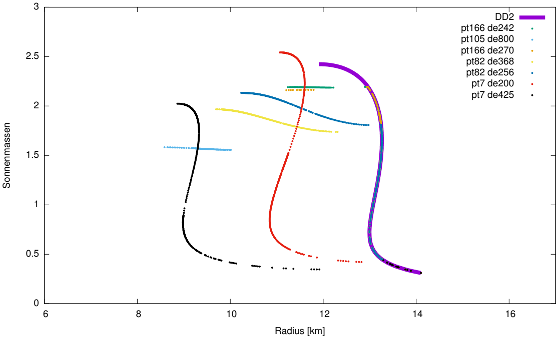


(Alford, Han, Prakash 2013)

- use low density nuclear EoS (in the following DD2)
- explore possible twin star solutions for given jump in energy density at a given pressure (choose $c_s^2 = 1$ for maximal effect)

Possible Classes of Twin Star Configurations

(Christian, JSB 2016)



- class I (low transition pressure): small radii down to around 9 km
- class II (higher transition pressure and moderate jump in energy density): flat mass-radius curves around $1.8 - 2M_{\odot}$
- class III (phase transition at $2M_{\odot}$ and large jump in energy density): true twin stars around $1.6M_{\odot}$ with $\Delta R \approx 4$ km
- class IV (phase transition above $2M_{\odot}$): true twin stars around $2M_{\odot}$

Summary

- conventional range of possible masses and radii (from interpolation):
  $R = 11 - 14.5$ km for $1.4 M_\odot$,
  $R = 10 - 15$ km for $2 M_\odot$
- softening of EoS due to crossover transition:
  heart shaped mass-radius curves,
  twin stars around $2 M_\odot$ and above
- possible classes of twin stars from first order phase transition:
  - radii down to 9km
  - flat mass radius curves around $2 M_\odot$
  - twin stars around $1.6 M_\odot$ with large radius difference
  - twin stars around $2.0 M_\odot$ with moderate radius difference

  warning: these are extreme cases!