Introduction to Accelerator Physics

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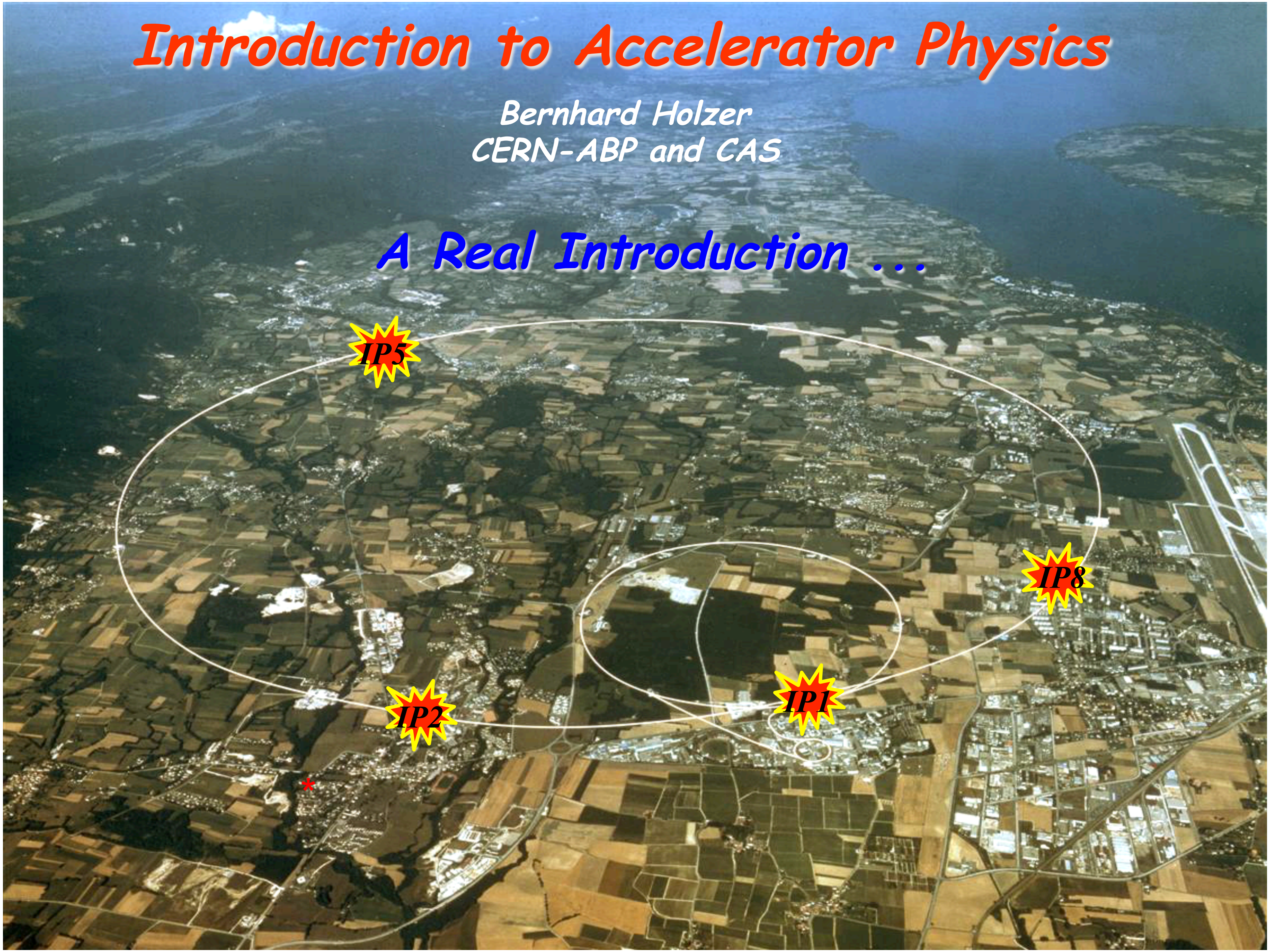
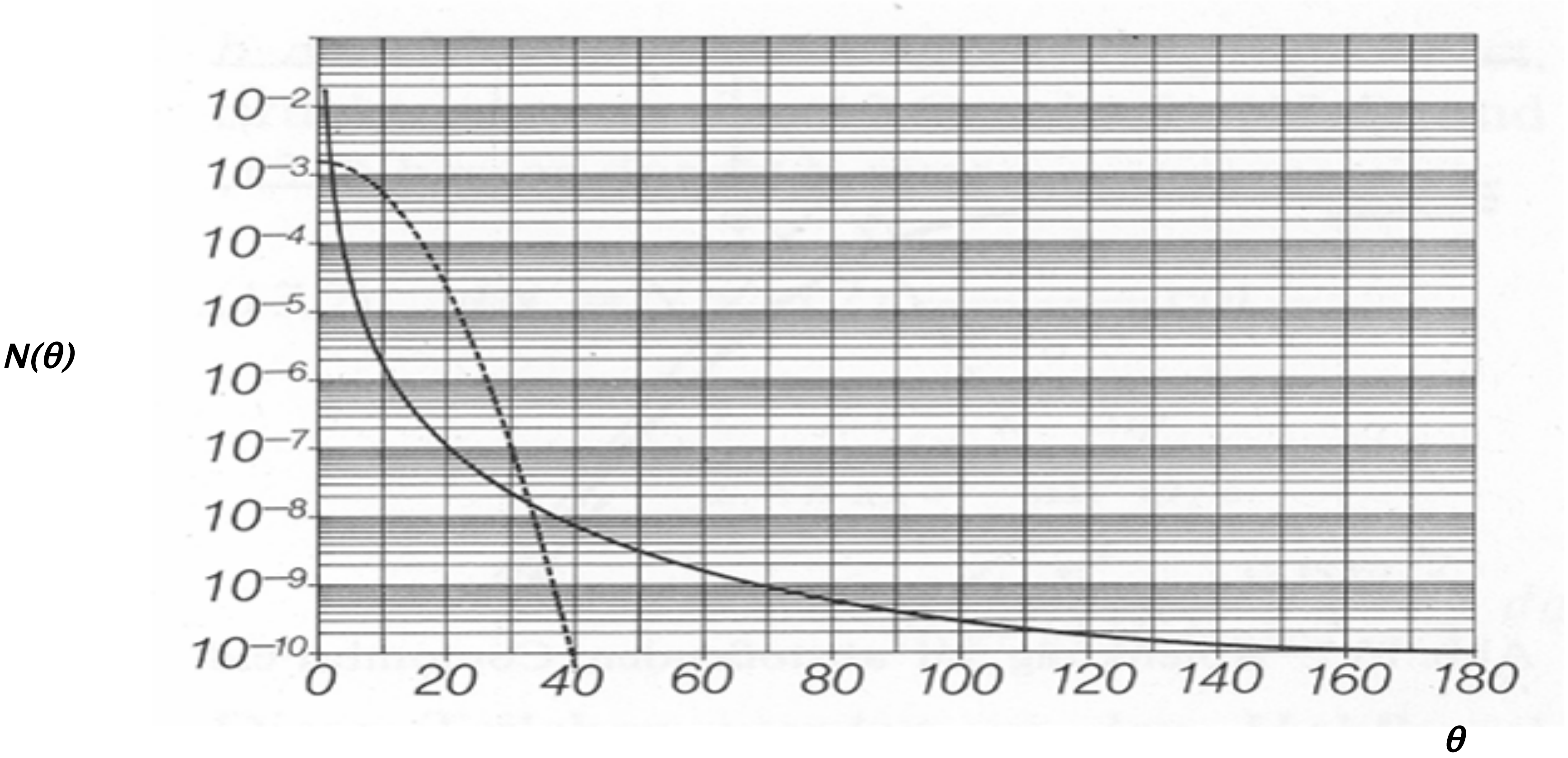
CERN-ABP and CAS

A Real Introduction ...

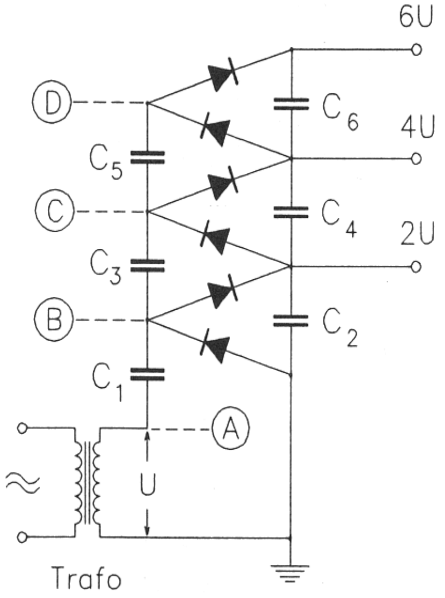


I.) A Bit of History

$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta/2)}$$

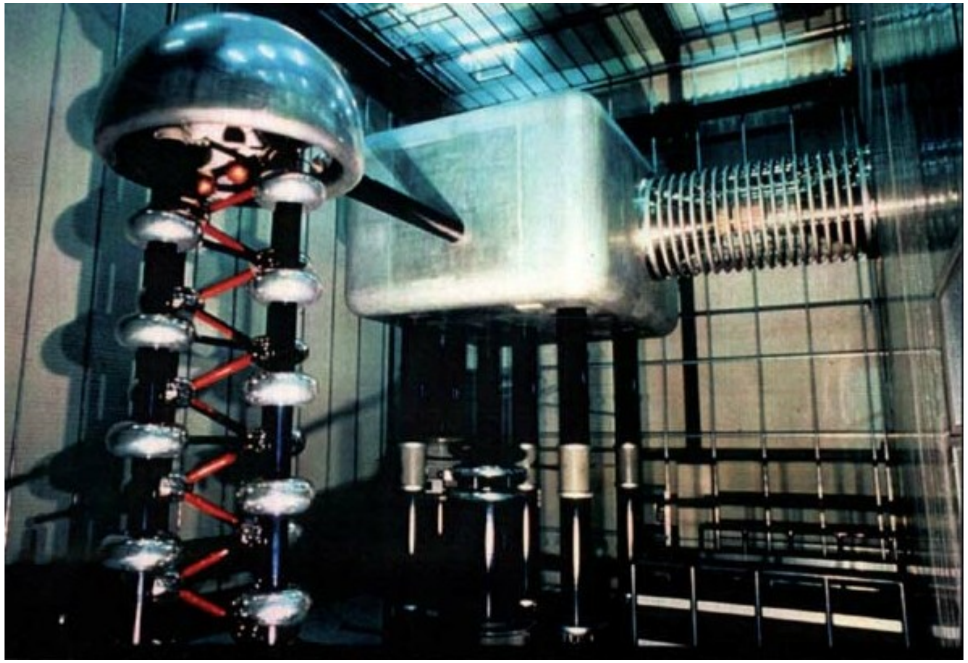


1.) Electrostatic Machines:
The Cockcroft-Walton Generator



1928: *Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam*

1932: *First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV*



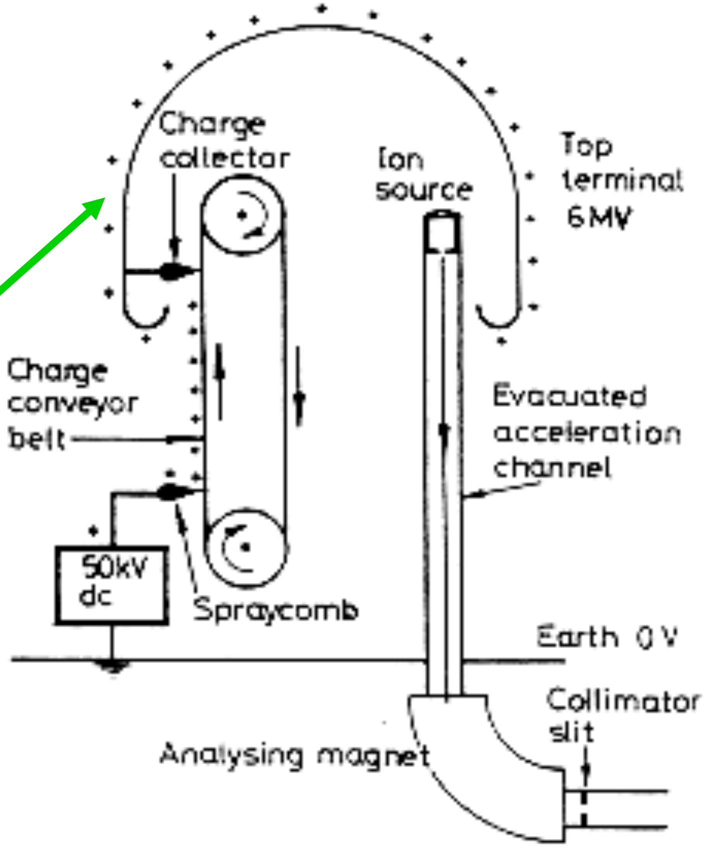
Particle source: *Hydrogen discharge tube on 400 kV level*
Accelerator: *evacuated glas tube*
Target: *Li-Foil on earth potential*

Technically: rectifier circuit, built of capacitors and diodes (Greinacher)

Problem:
DC Voltage can only be used once

2.) Electrostatic Machines:
(Tandem -) van de Graaff Accelerator (1930 ...)

creating high voltages by mechanical transport of charges



** Terminal Potential: U ≈ 12 ...28 MV*
using high pressure gas to suppress discharge (SF_6)

*Problems: * Particle energy limited by high voltage discharges*
** high voltage can only be applied once per particle ...*
... or twice ?

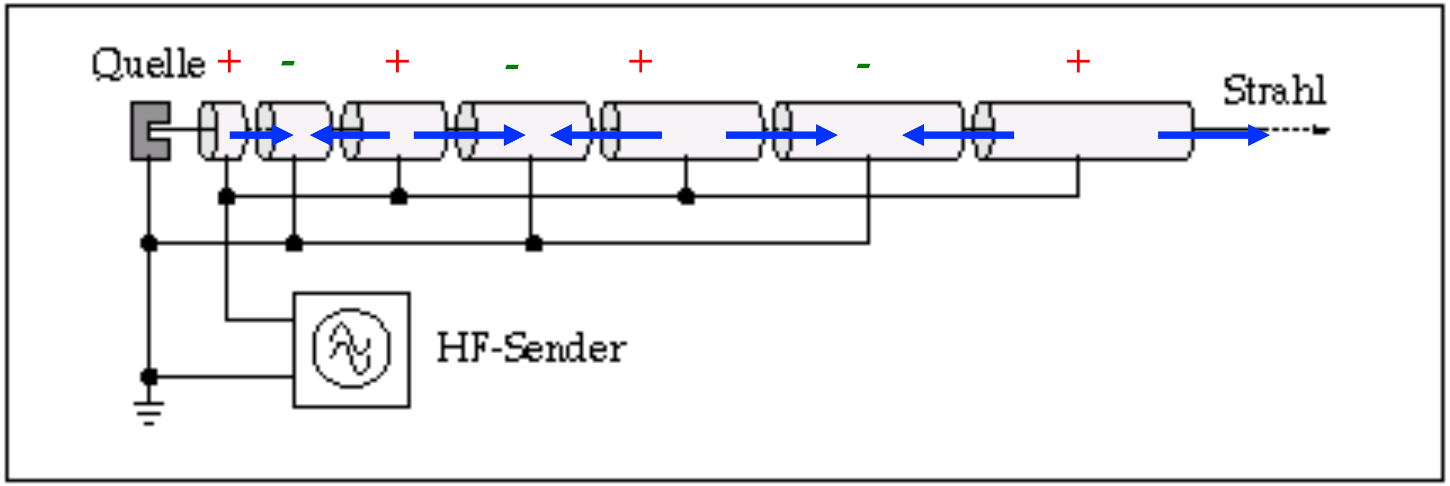
The „Tandem principle": Apply the accelerating voltage twice ...
... by working with negative ions (e.g. H^-) and stripping the electrons in the centre of the structure

Example for such a „steam engine": 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

3.) The first RF-Accelerator: „Linac"

1928, Wideroe: ***how can the acceleration voltage be applied several times to the particle beam***

schematic Layout:



Energy gained after n acceleration gaps

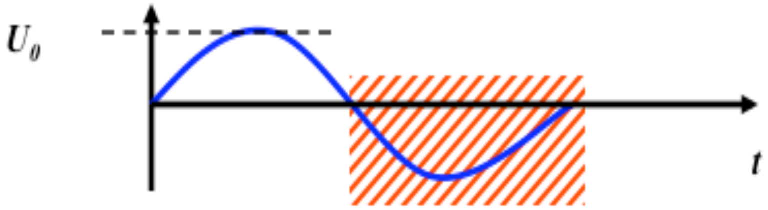
$$E_n = n * q * U_0 * \sin\psi_s$$

n *number of gaps between the drift tubes*
q *charge of the particle*
$\boldsymbol{U_0}$ *Peak voltage of the RF System*
$\boldsymbol{\Psi_S}$ *synchronous phase of the particle*

**** acceleration of the proton in the first gap***
**** voltage has to be „flipped" to get the right sign in the second gap → RF voltage***
→ shield the particle in drift tubes during the negative half wave of the RF voltage

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



Time span of the negative half wave: $\tau_{RF}/2$

Length of the Drift Tube: $l_i = v_i * \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_i = \frac{1}{2} m v^2$

$$\rightarrow v_i = \sqrt{2E_i / m}$$

$$l_i = \frac{1}{\nu_{rf}} * \sqrt{\frac{i * q * U_0 * \sin \psi_s}{2m}}$$

valid for non relativistic particles ...

Alvarez-Structure: 1946, surround the whole structure by a rf vessel

Energy: ≈ 20 MeV per Nucleon $\beta \approx 0.04 \ldots 0.6$, Particles: Protons/Ions

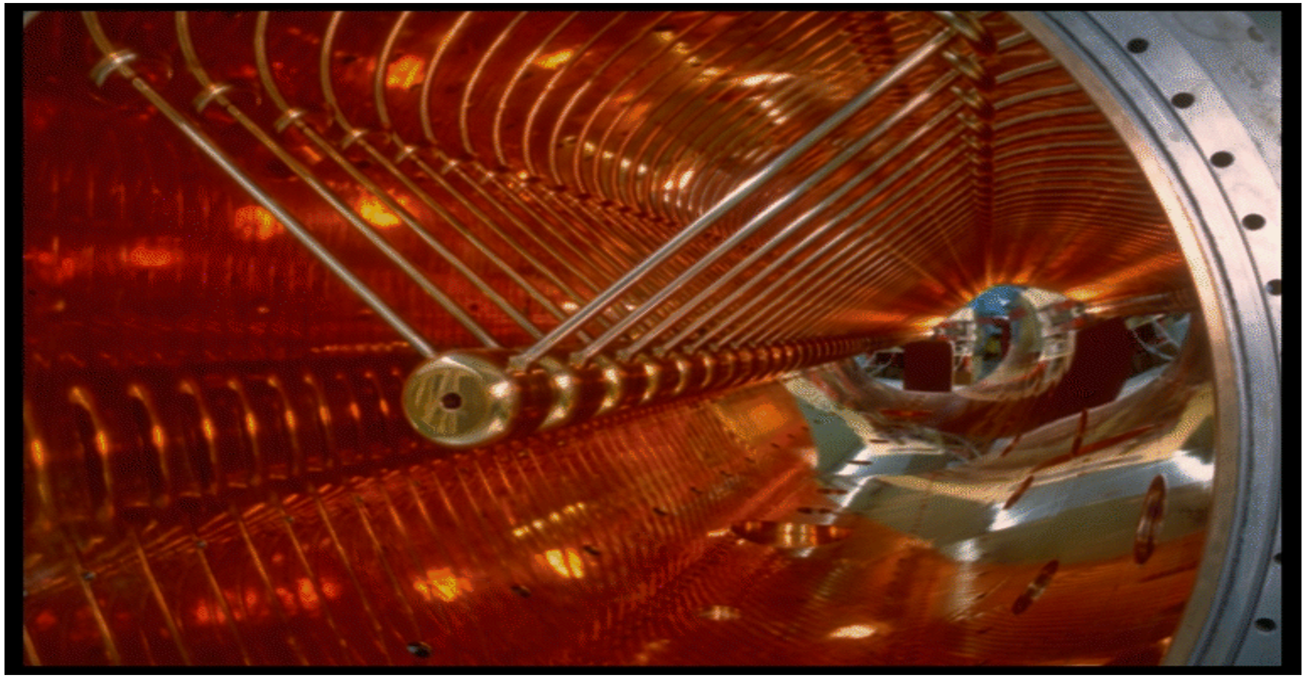
Accelerating structure of a Proton Linac (DESY Linac III)

$E_{total} = 988\,MeV$

$m_0c^2 = 938\,MeV$

$p = 310\,MeV/c$

$E_{kin} = 50\,MeV$



Beam energies

Energy Gain per „Gap“: $\boxed{W = q\,U_0 \sin \omega_{RF} t}$

1.) reminder of some relativistic formula

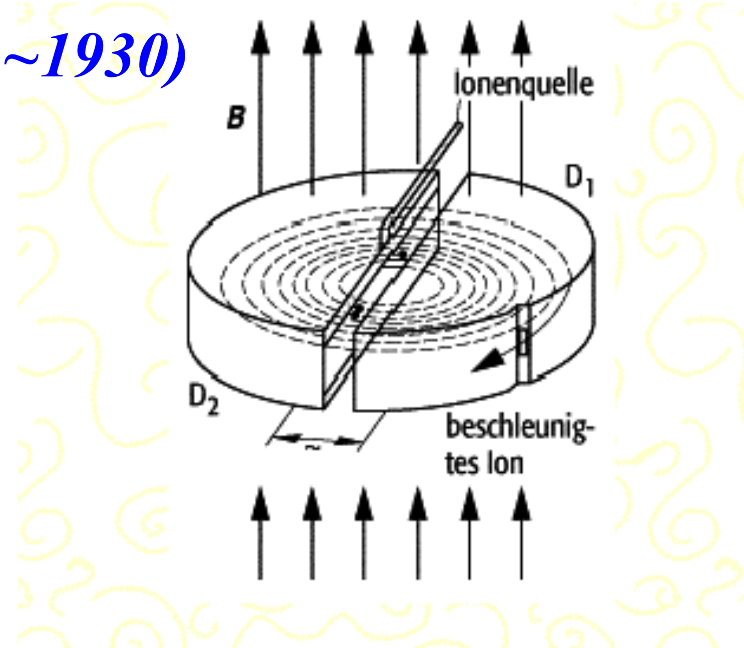
rest energy $E_0 = m_0c^2$

total energy $E = \gamma * E_0 = \gamma * m_0c^2$

kinetic energy $E_{kin} = E_{total} - m_0c^2$

momentum $E^2 = c^2p^2 + m_0^2c^4$

3.) The Cyclotron: (Livingston / Lawrence ~1930)



Idea: B = const, RF = const
Synchronisation particle / RF via orbit

Lorentzforce

$$\vec{F} = q * (\vec{v} \times \vec{B}) = q * v * B$$

circular orbit

$$q * v * B = \frac{m * v^2}{R} \quad \longrightarrow \quad B * R = p/q$$

increasing radius for increasing momentum
➔ ***Spiral Trajectory***

revolution frequency

$$\omega_z = \frac{v}{R} = \frac{q}{m} * B_z$$

the cyclotron (rf-) frequency is independent of the momentum

rf-frequency = h* revolution frequency,   h = "harmonic number"

Cyclotron:

exact equation for revolution frequency:

$$\omega_z = \frac{v}{R} = \frac{q}{\boxed{\gamma *} m} * B_z$$

Cyclotron SPIRAL at GANIL

1.) if $v << c \Rightarrow \gamma \cong 1$

2.) γ increases with the energy
$\Rightarrow$ no exact synchronism

"synchronisation" with the acceleration potential is established via the spiraling orbit length

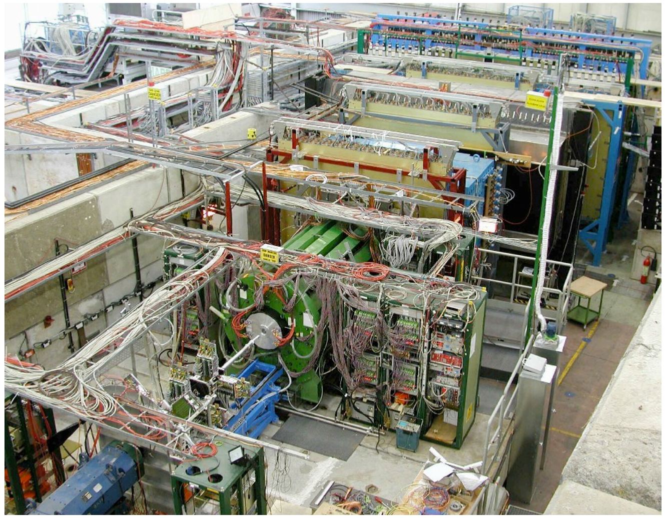
Syn on

B = constant
$\gamma\, \omega_{RF}$ = constant
ω_{RF} decreases with time

$$\omega_s(t) = \omega_{rf}(t) = \frac{q}{\gamma(t) * m_0} * B$$

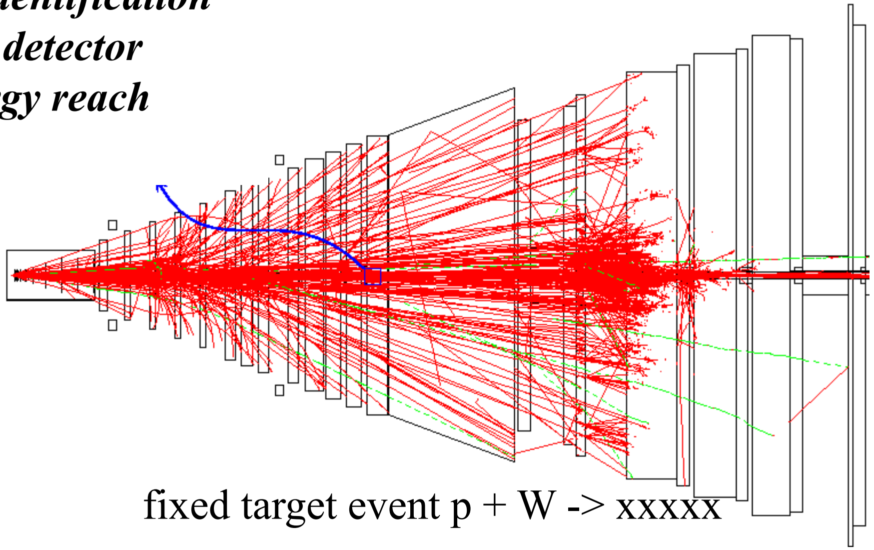
keep the synchronisation condition by varying the rf frequency

Fixed target experiments:



HARP Detector, CERN

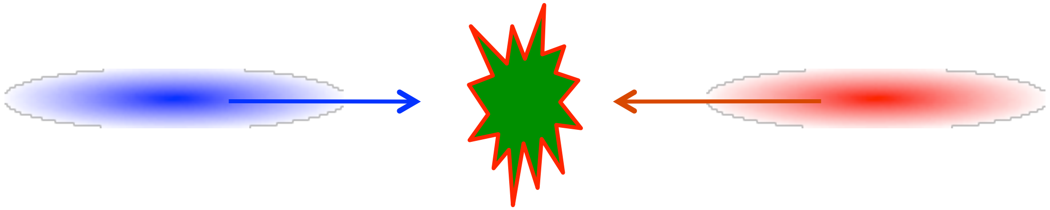
high event rate
easy track identification
asymmetric detector
limited energy reach



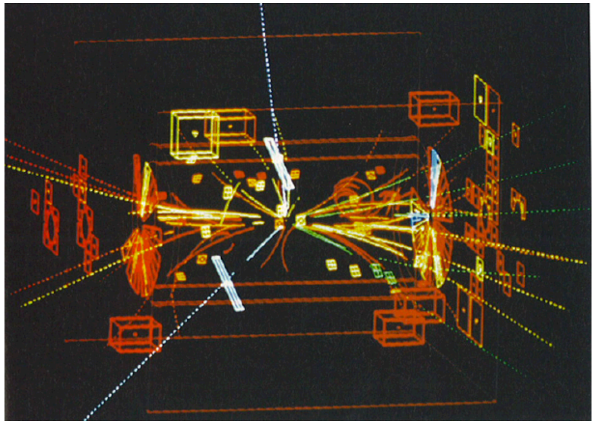
fixed target event p + W -> xxxxx

Collider experiments:

$E=mc^2$



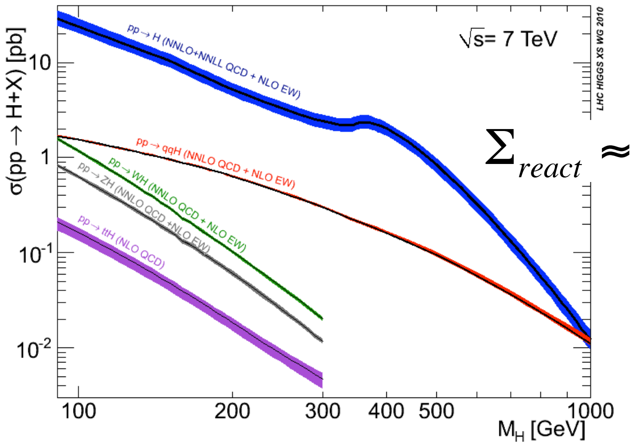
low event rate (luminosity)
challenging track identification
symmetric detector
$E_{lab} = E_{cm}$



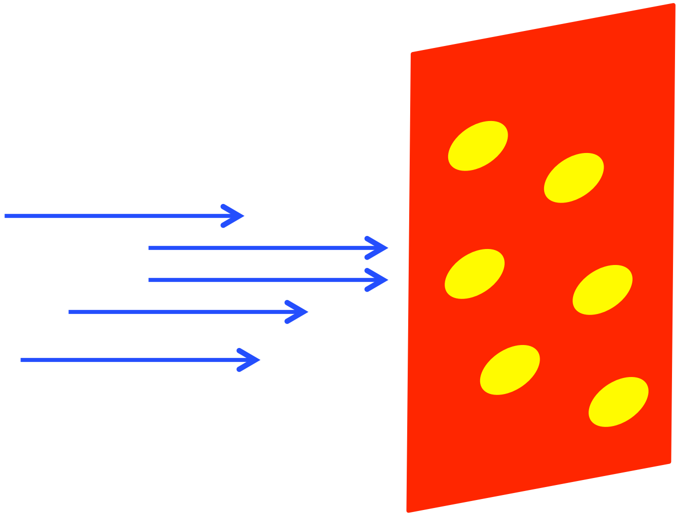
Z_0 boson discovery at the UA2 experiment (CERN).
The Z_0 boson decays
into a e+e− pair, shown as white dashed lines.

Problem: Our particles are VERY small !!

Overall cross section of the Higgs:



$$\Sigma_{react} \approx 1\, pb$$

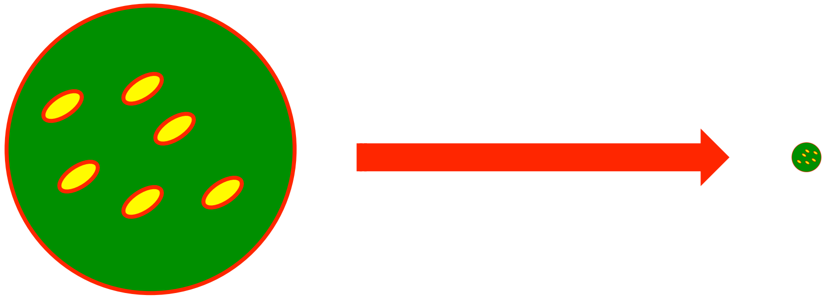


$$1b \ = 10^{-24}\, cm^2$$

$$1pb = \ 10^{-12} * 10^{-24}\, cm^2 = 1/mio * 1/mio * 1/mio * 1/mio * 1/mio * 1/10000\ mm^2$$

The particles are "very small"

The only chance we have:
compress the transverse beam size … at the IP



LHC typical:

σ = 0.1 mm → 16 µm

1.) Introduction and Basic Ideas

„ … in the end and after all it should be a kind of circular machine"

→ need transverse deflecting force

Lorentz force

$$\vec{F} = q * (\cancel{\vec{E}} + \vec{v} \times \vec{B})$$

typical velocity in high energy machines:

$$v \approx c \approx 3*10^{8}\, {}^{m}\!/\!{}_{s}$$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q*3*10^{8}\frac{m}{s}*1\frac{Vs}{m^{2}}$$

$$F = q*\underbrace{300\frac{MV}{m}}$$

equivalent electrical field: E

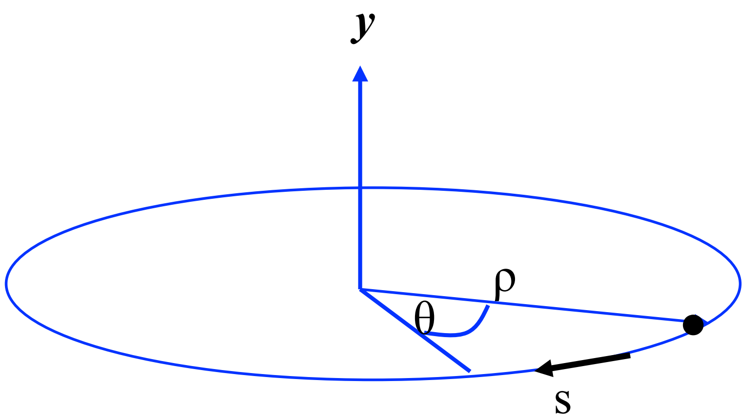
Technical limit for electrical fields:

$$E \leq 1\frac{MV}{m}$$

Pearl of Wisdom:

if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit



circular coordinate system

condition for circular orbit:

Lorentz force $$F_L = e\,v\,B$$

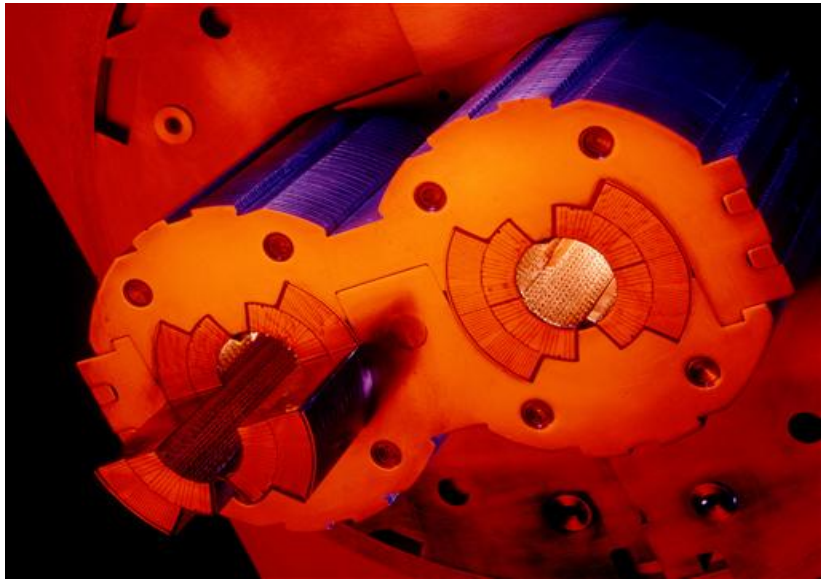
centrifugal force $$F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$$

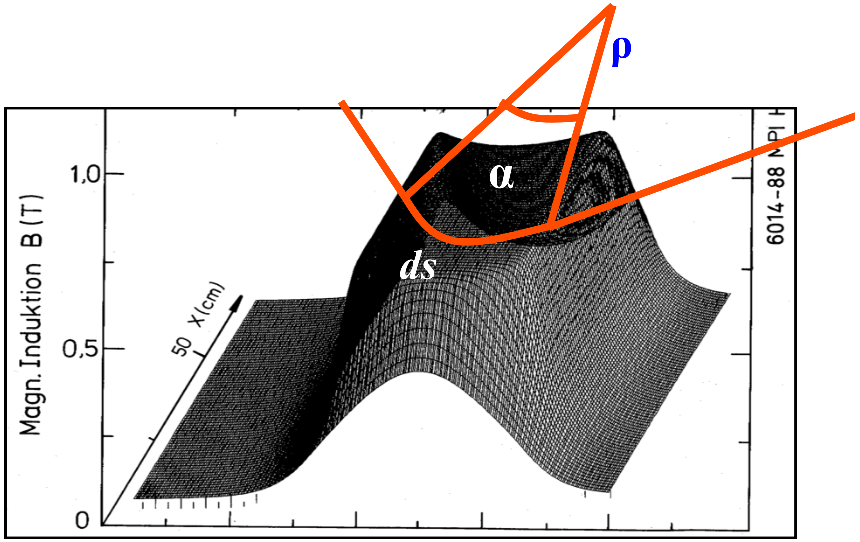
$$\left.\frac{\gamma\, m_0\, \cancel{v^2}}{\rho} = e\,\cancel{v}\,B \right\}$$

$$\boxed{\frac{p}{e} = B\,\rho}$$

$B\,\rho =$ ***"beam rigidity"***

The Magnetic Guide Field





field map of a storage ring dipole magnet

$\rho = 2.8\ km$ → $2\pi\rho = 17.6\ km$ $\approx 66\%$

$B \approx 1 \ldots 8\ T$

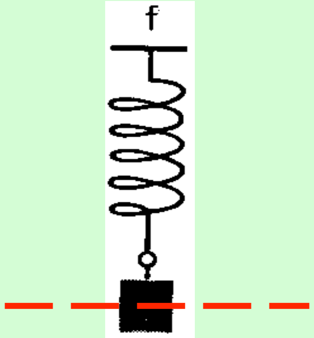
rule of thumb:

$$\boxed{\frac{1}{\rho} \approx 0.3\,\frac{B[T]}{p[GeV/c]}}$$

„normalised bending strength“

Focusing Properties and Quadrupole Magnets

classical mechanics:
pendulum



there is a restoring force, proportional to the elongation x:

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general solution: free harmonic oszillation

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this is how grandma‘s Kuckuck‘s clock is working!!!

Storage Rings: *linear increasing Lorentz force to keep trajectories in vicinity of the ideal orbit*

linear increasing magnetic field $B_y = g\,x$ $B_x = g\,y$

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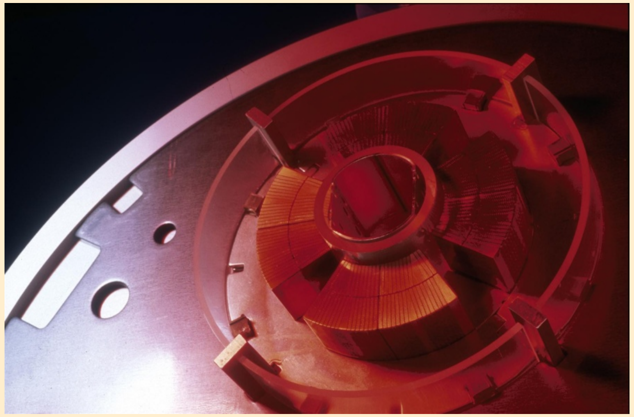


LHC main quadrupole magnet $g \approx 25 \ldots 220 \;\; T/m$

Focusing forces and particle trajectories:

normalise magnet fields to momentum
(remember: $\mathbf{B*\rho = p / q}$ *)*

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

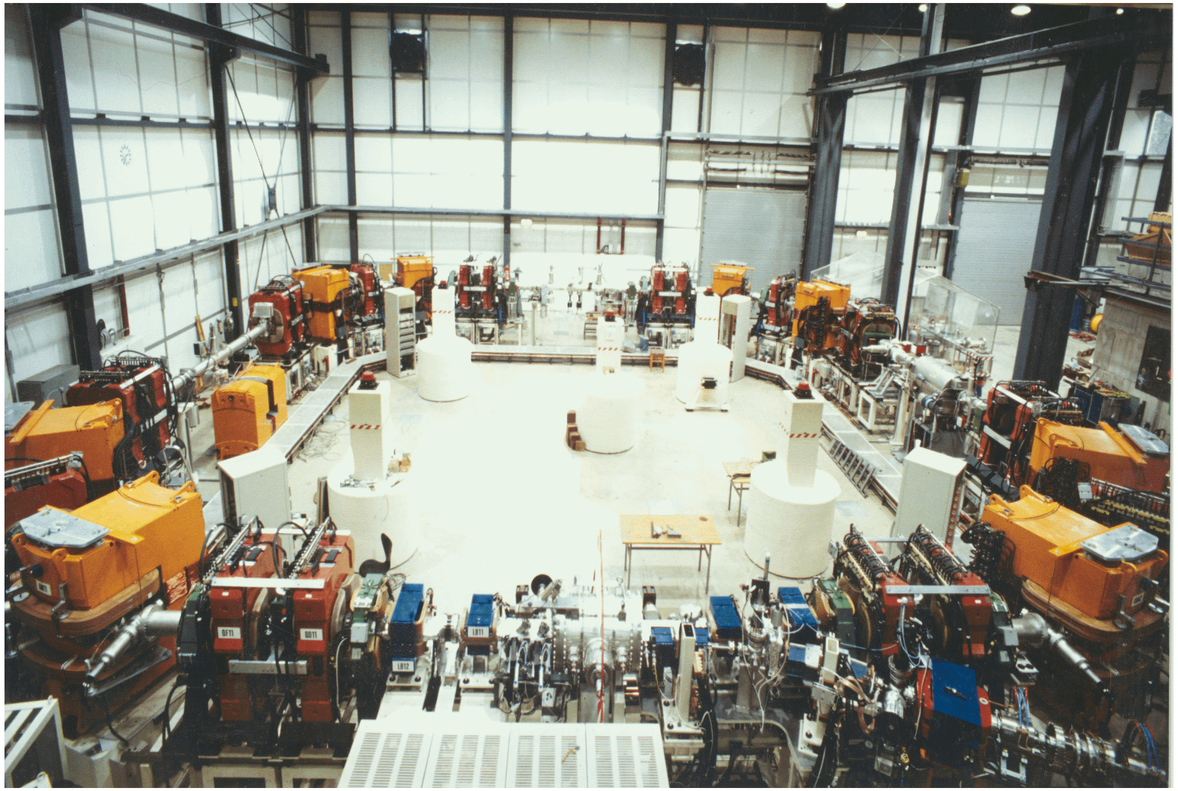
Quadrupole Magnet

$$k := \frac{g}{p/q}$$

3.) The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \cancel{\frac{1}{2!}\,m\,x^2} + \cancel{\frac{1}{3!}\,n\,x^3} + \dots$$

only terms linear in x, y taken into account *dipole fields*
quadrupole fields



Separate Function Machines:

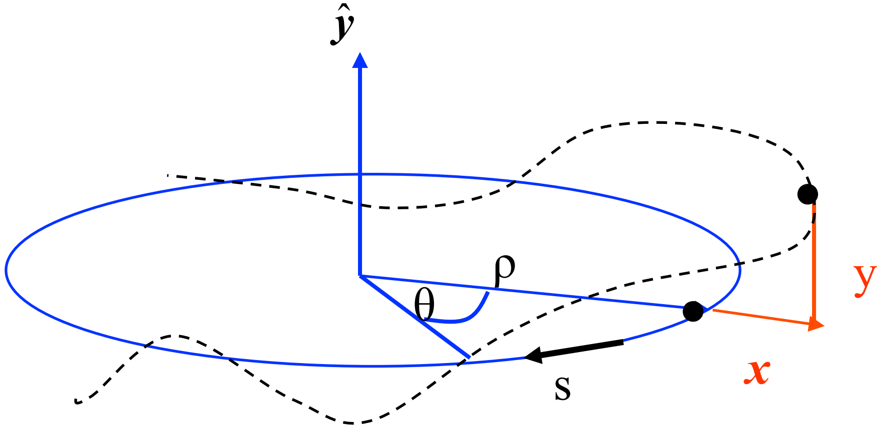
Split the magnets and optimise them according to their job:

bending, focusing etc

Example:
heavy ion storage ring TSR

* **man sieht nur dipole und quads → linear**

The Equation of Motion:



- ***Equation for the horizontal motion:***

$$x'' + x\left(\frac{1}{\rho^2} + k\right) = 0$$

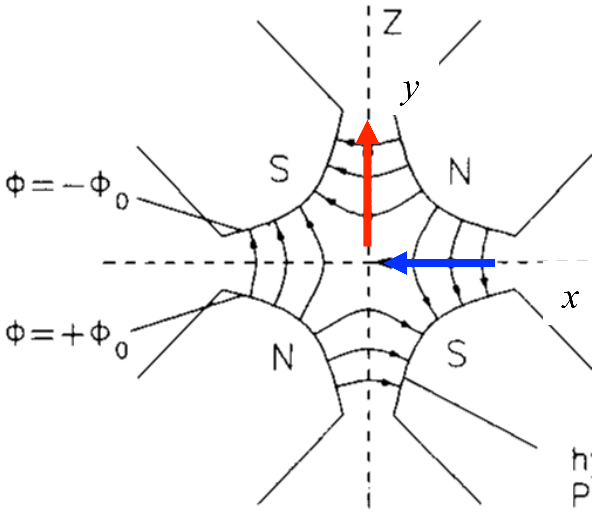
x = particle amplitude
x' = angle of particle trajectory (wrt ideal path line)

- ***Equation for the vertical motion:***

$$\frac{1}{\rho^2} = 0$$ ***no dipoles … in general …***

$$k \leftrightarrow -k$$ ***quadrupole field changes sign***

$$y'' - k\,y = 0$$



4.) Solution of Trajectory Equations

$$\left.\begin{array}{l} \textit{Define … hor. plane: } K = 1/\rho^2 + k \\ \textit{… vert. Plane: } K = -k \end{array}\right\}$$

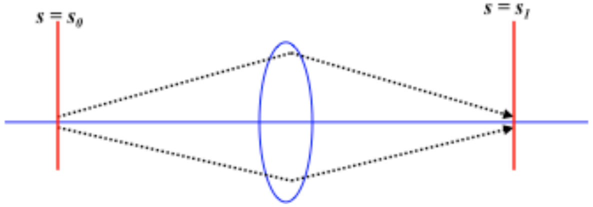
$$\boxed{x'' + K\,x = 0}$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: *Hor. Focusing Quadrupole K > 0:*

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x_0' \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x_0' \cdot \cos(\sqrt{|K|}s)$$



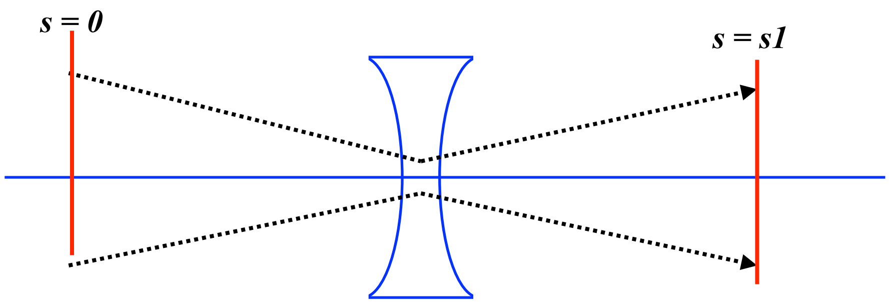
For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$\boxed{M_{foc} = \begin{pmatrix} \cos\left(\sqrt{|K|}l\right) & \frac{1}{\sqrt{|K|}} \sin\left(\sqrt{|K|}l\right) \\ -\sqrt{|K|}\sin\left(\sqrt{|K|}l\right) & \cos\left(\sqrt{|K|}l\right) \end{pmatrix}}$$

hor. defocusing quadrupole:

$$x'' - K\,x = 0$$



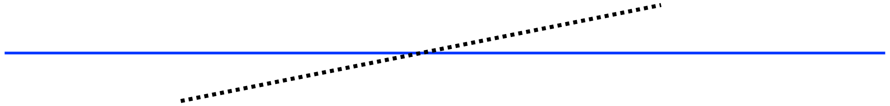
Ansatz: ***Remember from school***

$$x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}\,l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}\,l \\ \sqrt{|K|}\sinh\sqrt{|K|}\,l & \cosh\sqrt{|K|}\,l \end{pmatrix}$$

drift space:

$$K = 0$$



$$x(s) = x'_0 * s$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

!   with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“

Ok … ok … it's a bit complicated and cosh and sinh and all that is a pain.
BUT … compare …

Weak Focusing / Strong Focusing

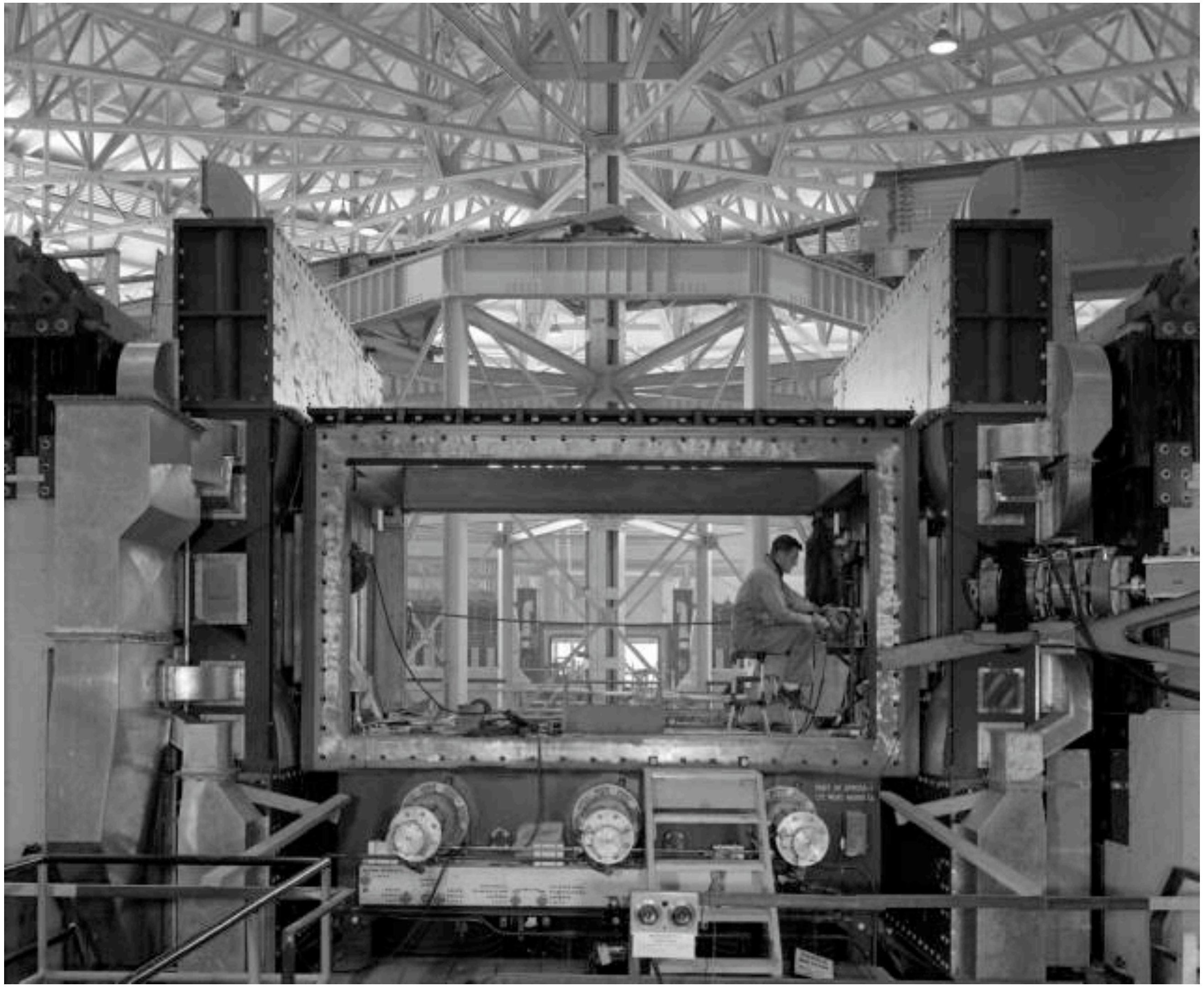
weak focusing term = 1/ρ²

$$x'' + x\left(\frac{1}{\rho^2} + \cancel{k}\right) = 0$$

Problem: the higher the energy, the larger the machine

The last weak focusing
high energy machine …
BEVATRON

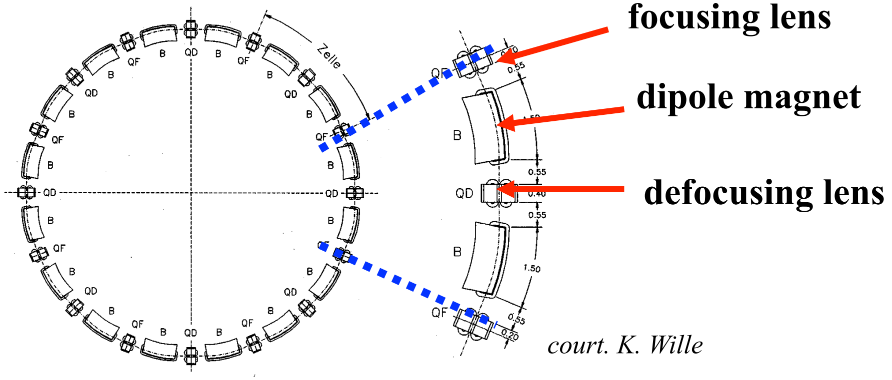
- → *large apertures needed*
- → *very expensive magnets*



Transformation through a system of lattice elements

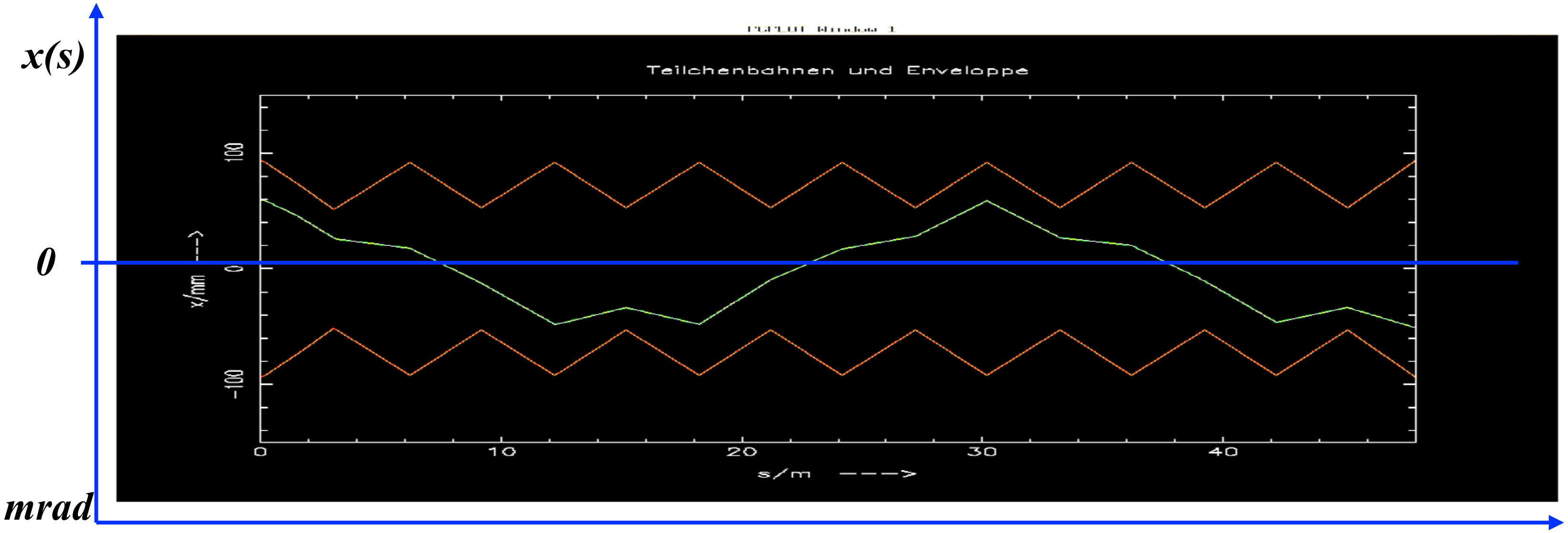
combine the single element solutions by multiplication of the matrices

$$M_{total} = M_{QF} * M_{D} * M_{QD} * M_{Bend} * M_{D^*}.....$$



$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$

in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „

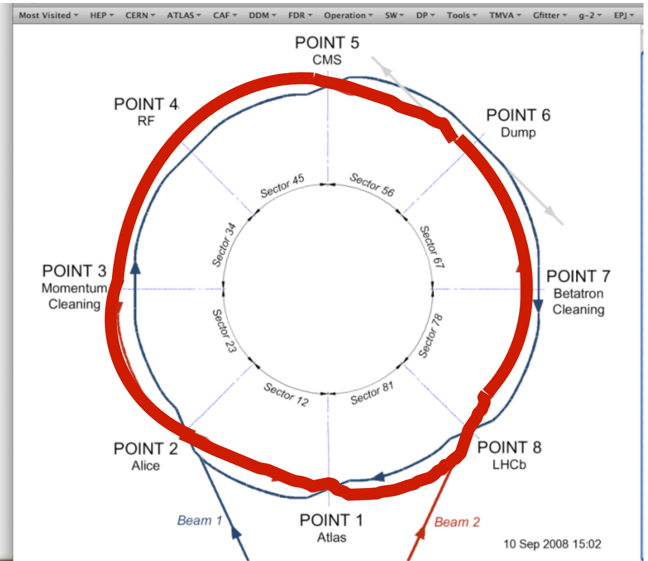


typical values in a strong foc. machine:

x ≈ mm, x´ ≤ mrad

LHC Operation: Beam Commissioning

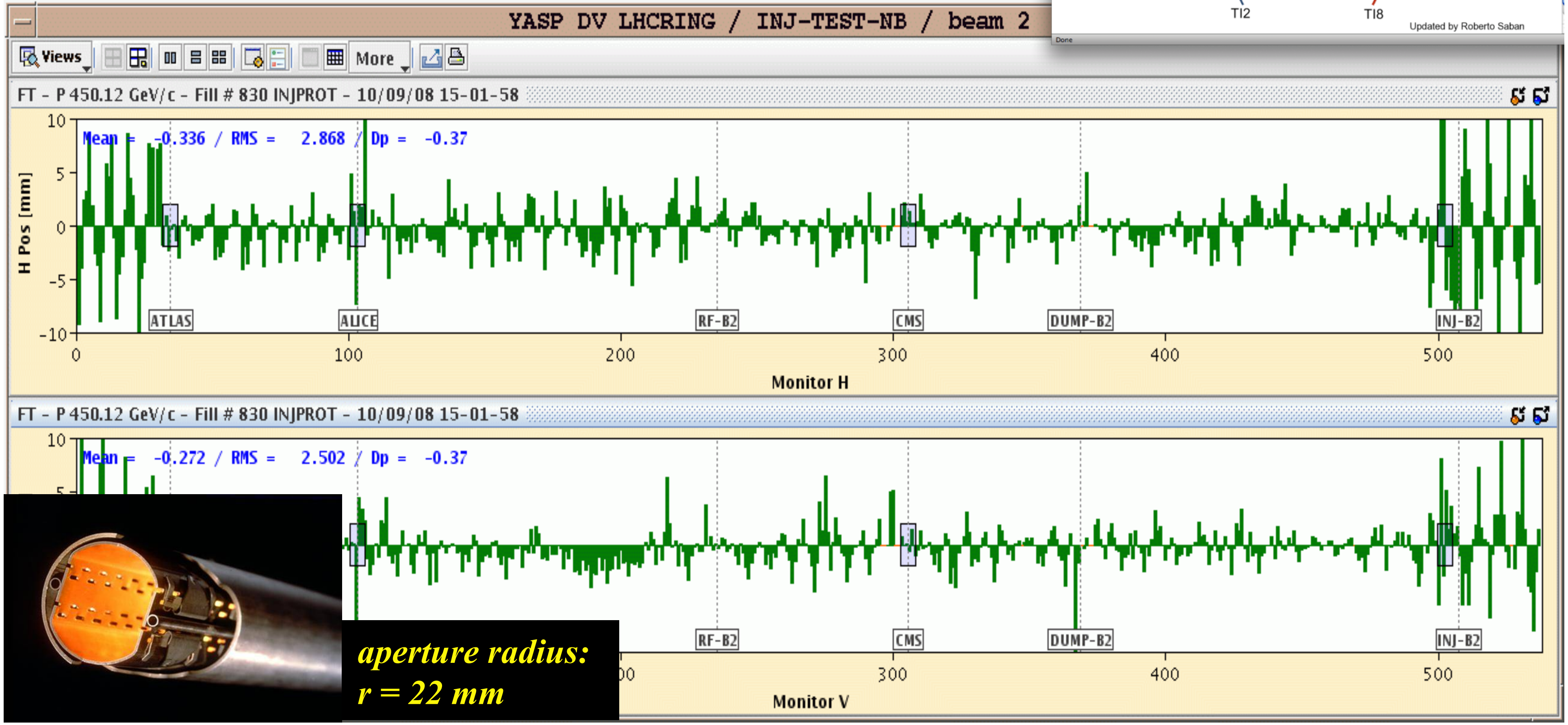
The transverse focusing fields create a harmonic oscillation of the particles with a well defined "Eigenfrequency" which is called tune



10 Sep 2008 15:02

Updated by Roberto Saban

First turn steering "by sector:"



FT - P 450.12 GeV/c - Fill # 830 INJPROT - 10/09/08 15-01-58

Mean = -0.336 / RMS = 2.868 / Dp = -0.37

FT - P 450.12 GeV/c - Fill # 830 INJPROT - 10/09/08 15-01-58

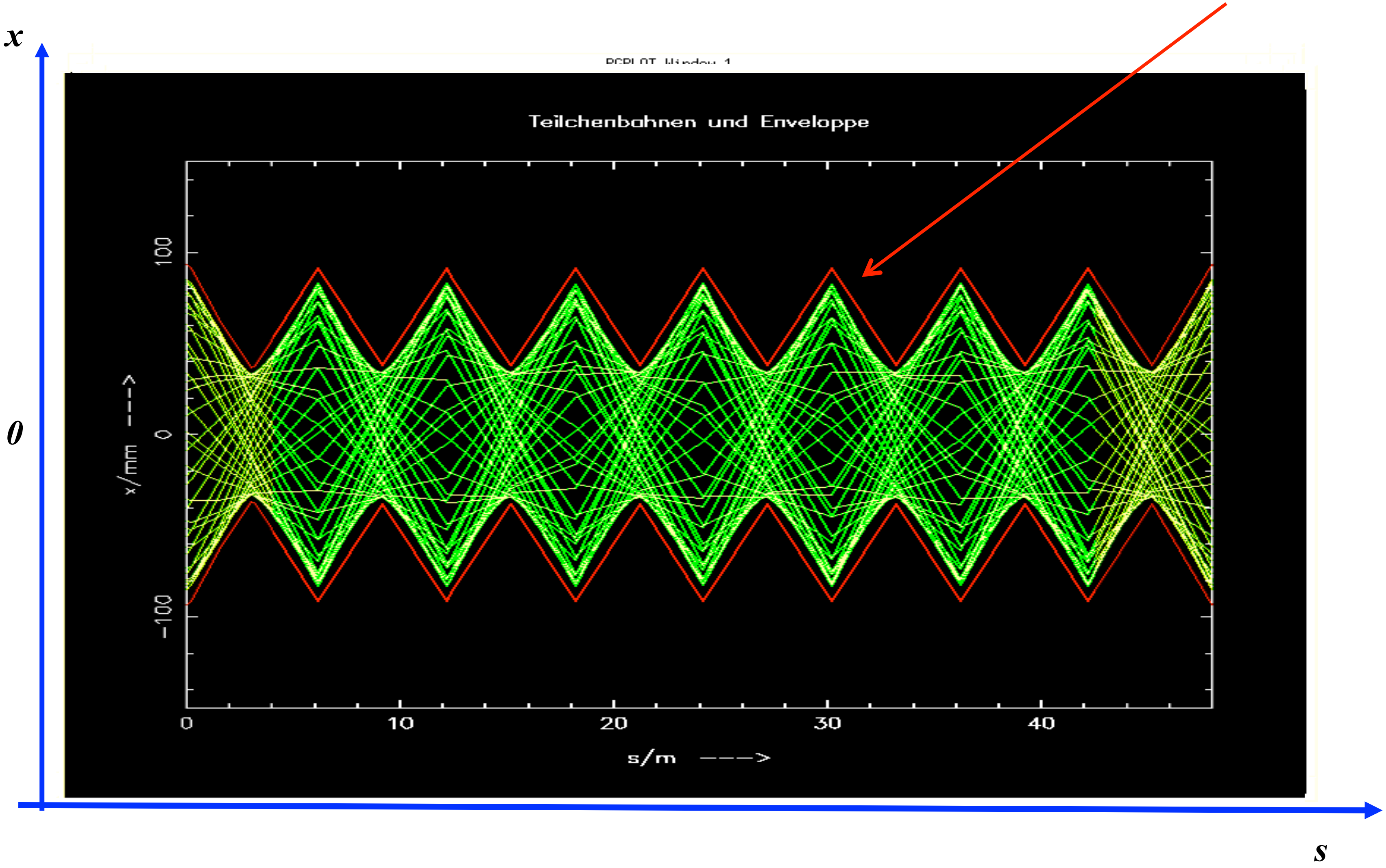
Mean = -0.272 / RMS = 2.502 / Dp = -0.37

aperture radius:
r = 22 mm

Question: what will happen, if the particle performs a second turn ?

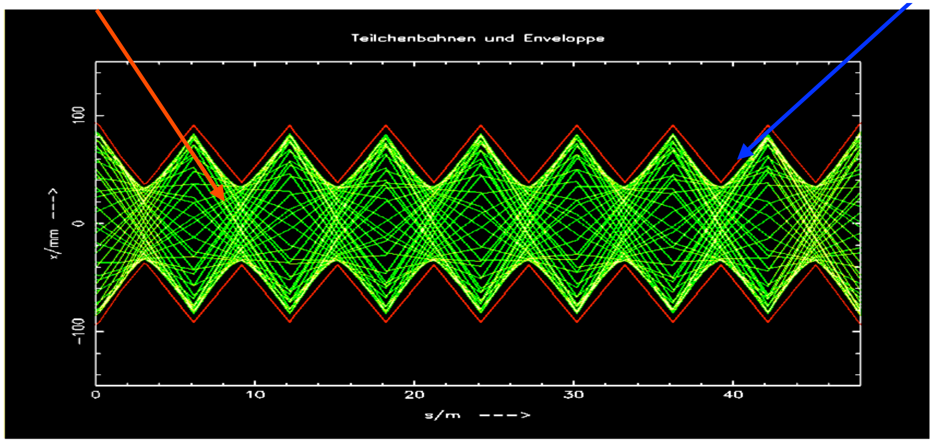
... or a third one or ... 10^{10} turns

$$\sigma = \sqrt{\varepsilon\beta}$$



Emittance of the Particle Ensemble:

$$x(s) = \sqrt{\varepsilon}\sqrt{\beta(s)} \cdot \cos(\Psi(s) + \phi) \qquad \hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$$



single particle trajectories, N ≈ 10 ^{11} per bunch

Gauß Particle Distribution:

$$\rho(x) = \frac{N \cdot e}{\sqrt{2\pi}\sigma_x} \cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x{}^2}}$$

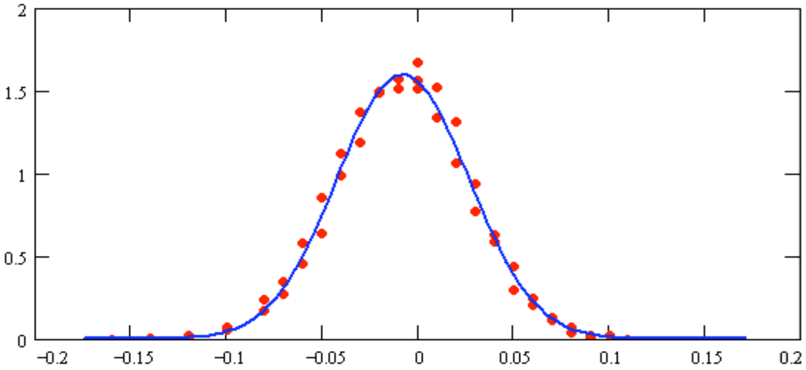
particle at distance 1 σ from centre
↔ 68.3 % of all beam particles

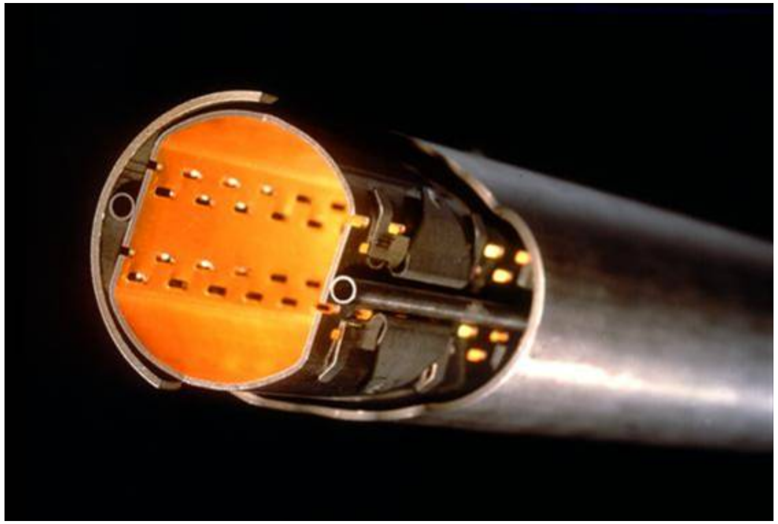
LHC:

$$\beta = 180\,m$$

$$\varepsilon = 5 * 10^{-10}\,m\,rad$$

$$\sigma = \sqrt{\varepsilon * \beta} = \sqrt{5 * 10^{-10}\,m * 180\,m} = 0.3\,mm$$



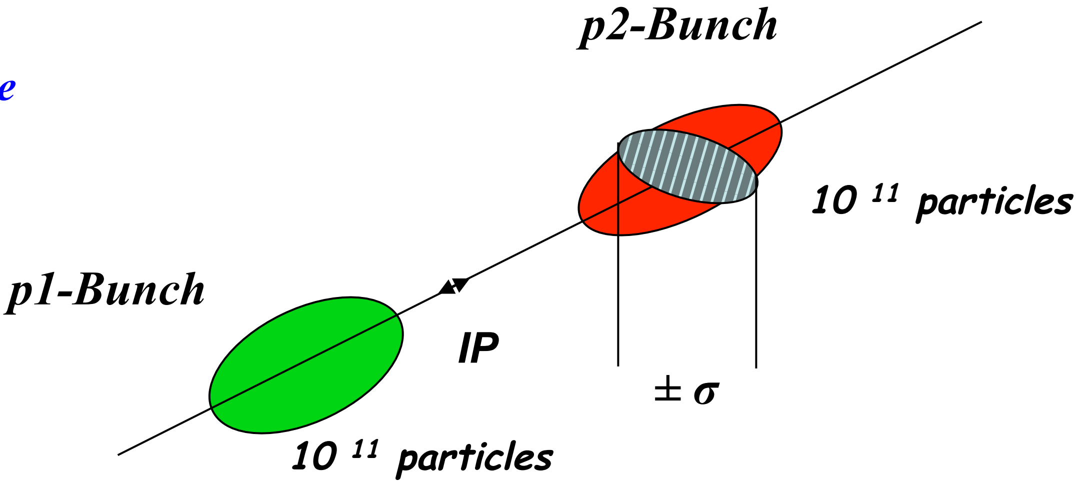


aperture requirements: $r_0 = 17 * \sigma$

5.) Luminosity

Ereignis Rate:“Physik“ pro Sekunde

$$R = L * \Sigma_{react}$$



Example: Luminosity run at LHC

$$\beta_{x,y} = 0.55\ m \qquad f_0 = 11.245\ kHz$$

$$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\ m \qquad n_b = 2808$$

$$\sigma_{x,y} = 17\ \mu m$$

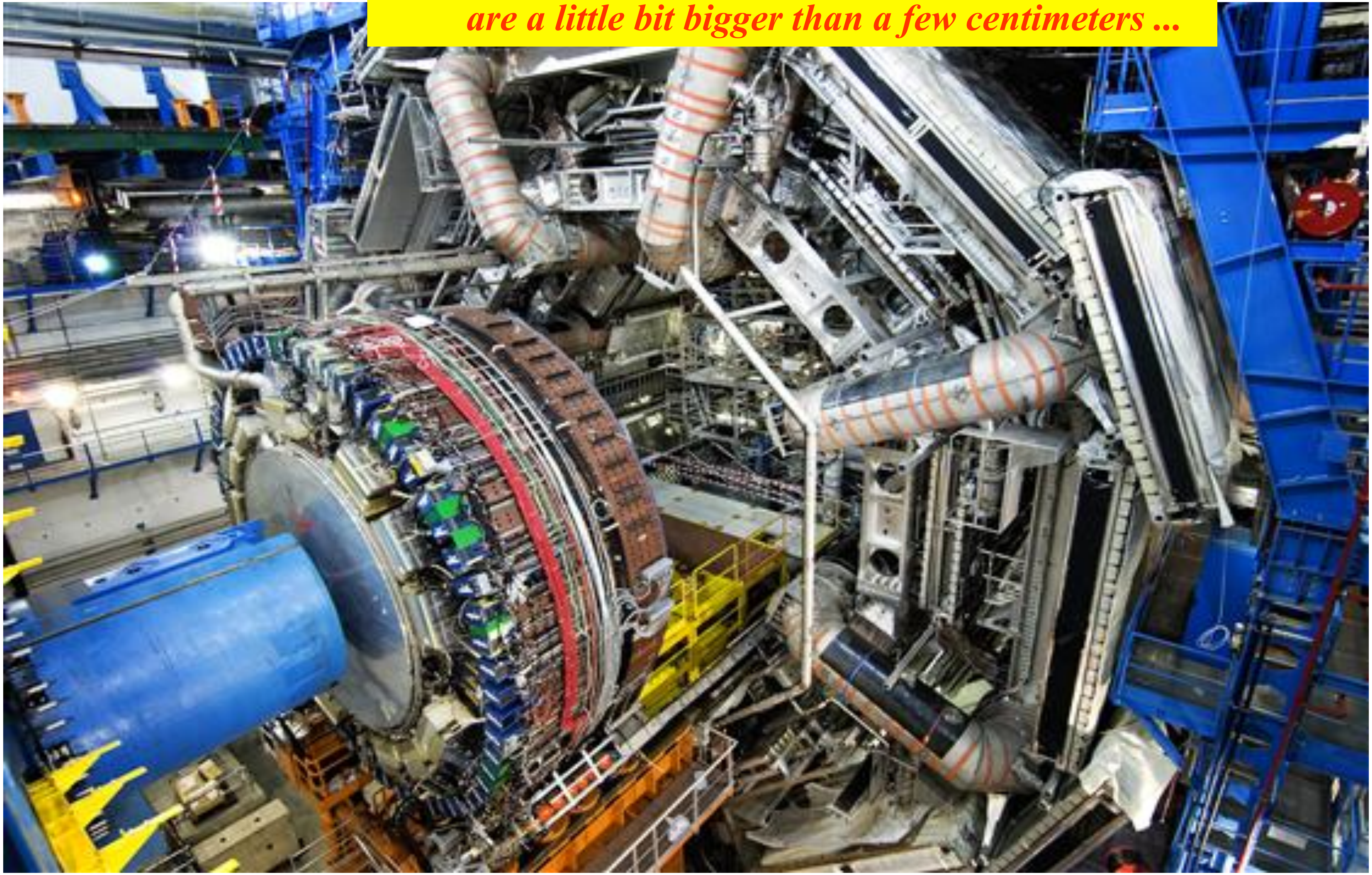
$$I_p = 584\ mA$$

$$\boxed{L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}}$$

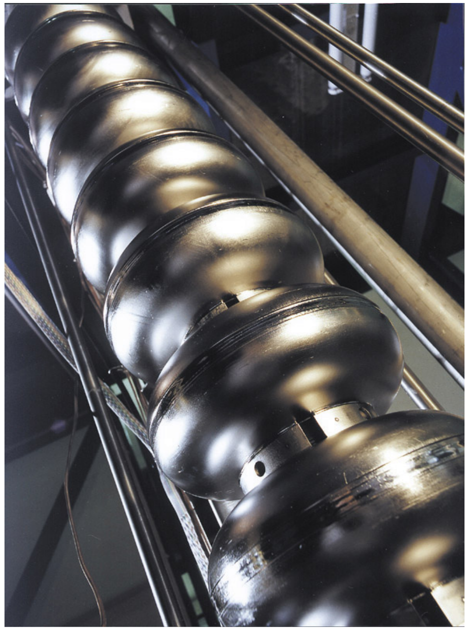
$$L = 1.0 * 10^{34}\ {}^{1}\!/_{cm^2 s}$$

... clearly there is an

... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...

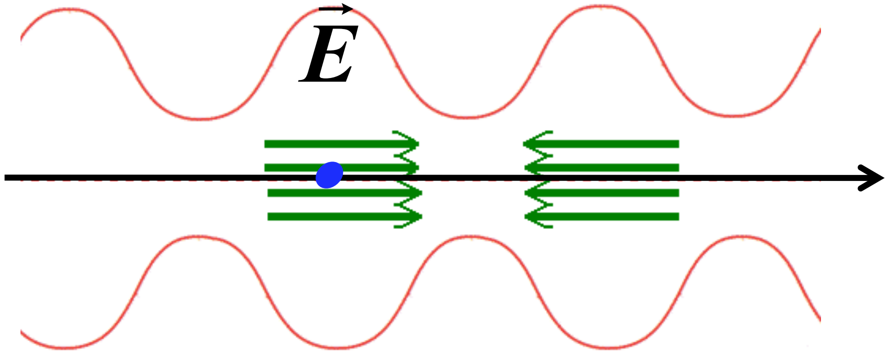


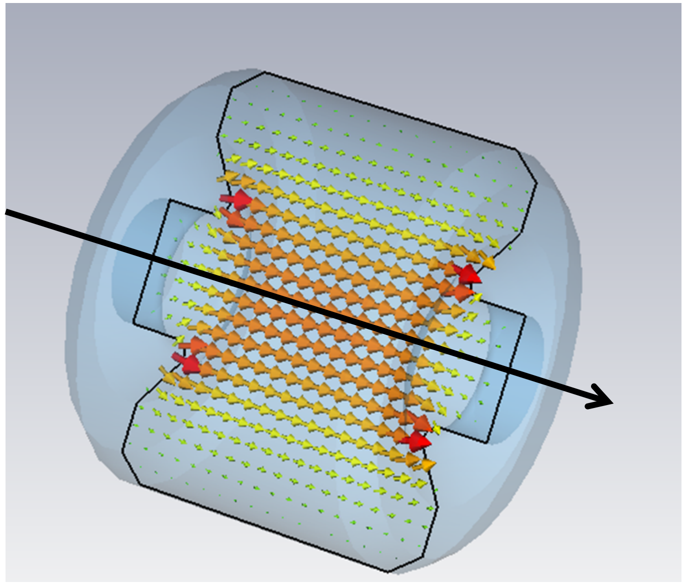
III. The Acceleration



Where is the acceleration?

Install an RF accelerating structure in the ring:





B. Salvant
N. Biancacci

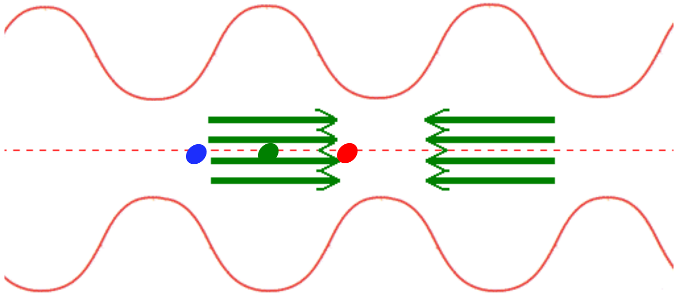
The Acceleration & "Phase Focusing"

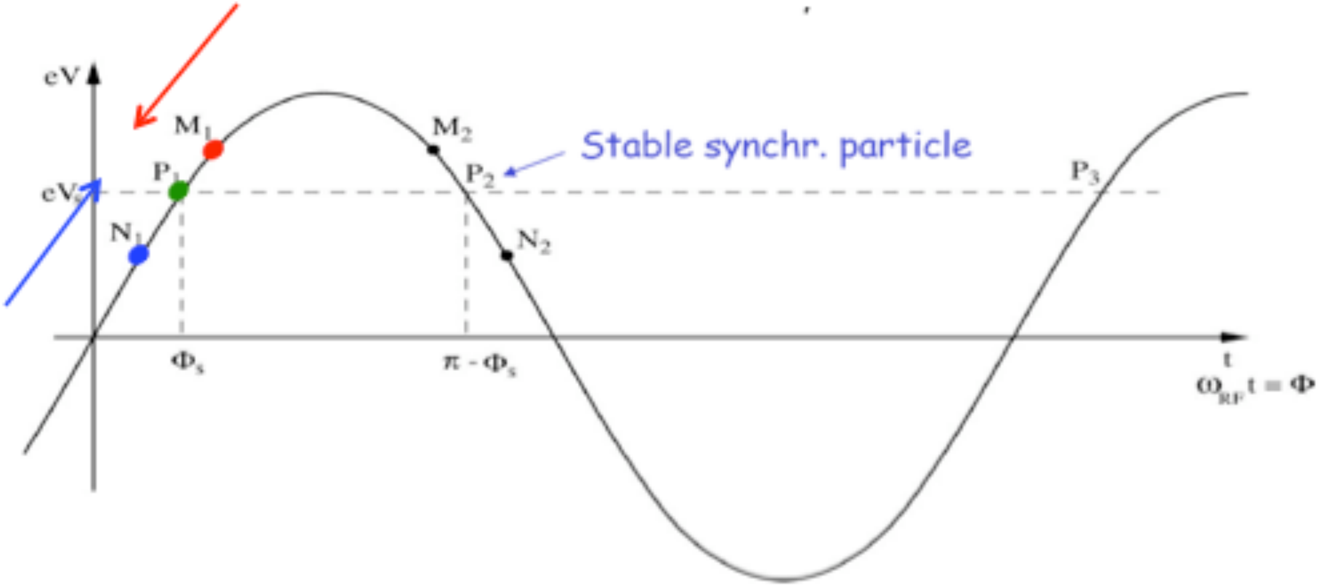
$\Delta p/p \neq 0$ below transition

ideal particle ●

particle with $\Delta p/p > 0$ ● *faster*

particle with $\Delta p/p < 0$ ● *slower*



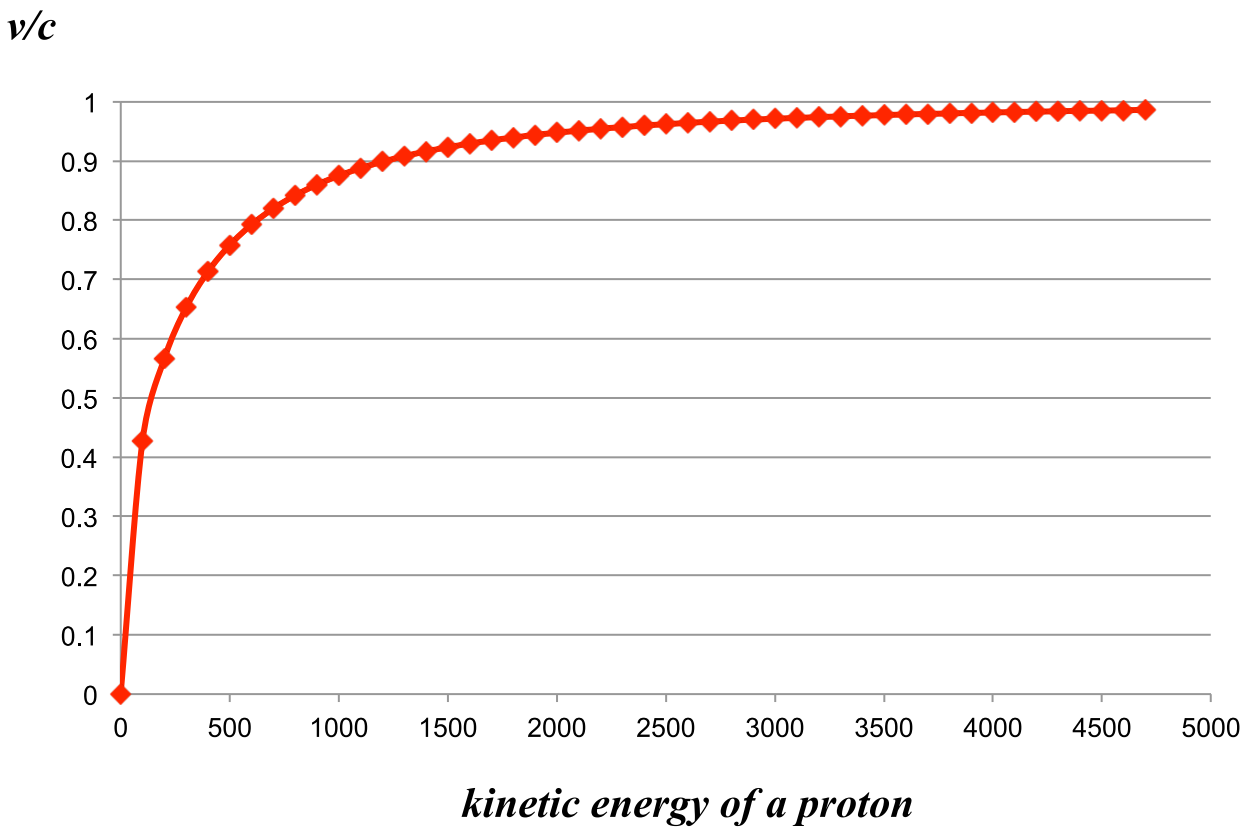


Focussing effect in the longitudinal direction keeping the particles close together ... forming a "bunch"

oscillation frequency: $f_s = f_{rev}\sqrt{-\frac{h\alpha_s}{2\pi} * \frac{qU_0\cos\phi_s}{E_s}}$ *≈ some Hz*

... so sorry, here we need help from Albert:

$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow \frac{v}{c} = \sqrt{1-\frac{mc^2}{E^2}}$$



... some when the particles do not get faster anymore

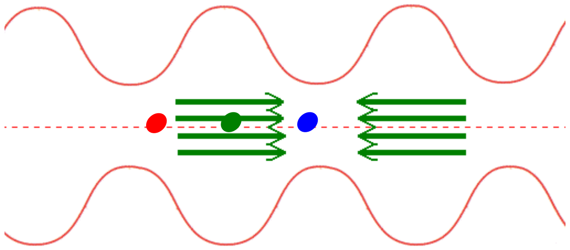
.... but heavier !

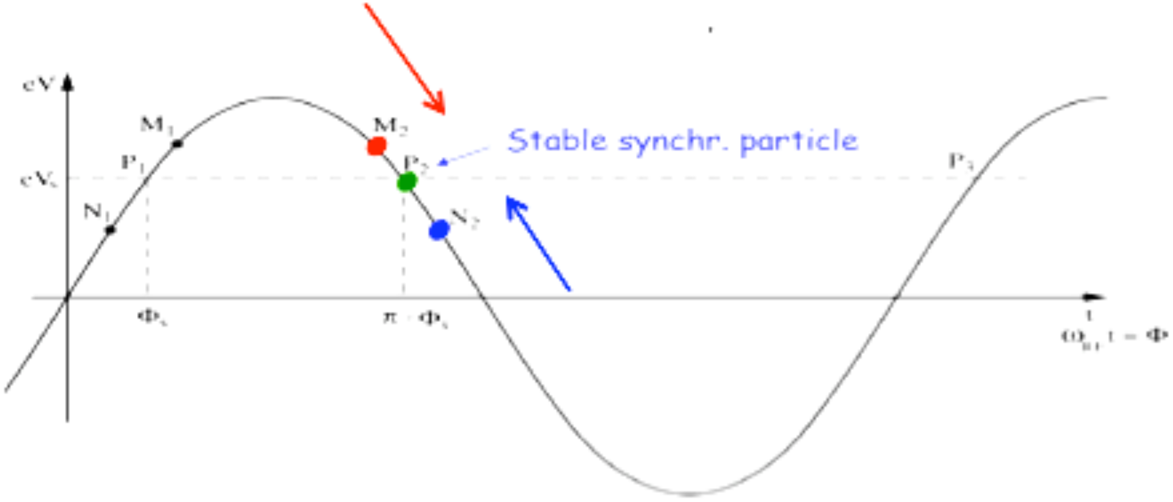
The Acceleration above transition

ideal particle ●

particle with $\Delta p/p > 0$ ● *heavier*

particle with $\Delta p/p < 0$ ● *lighter*



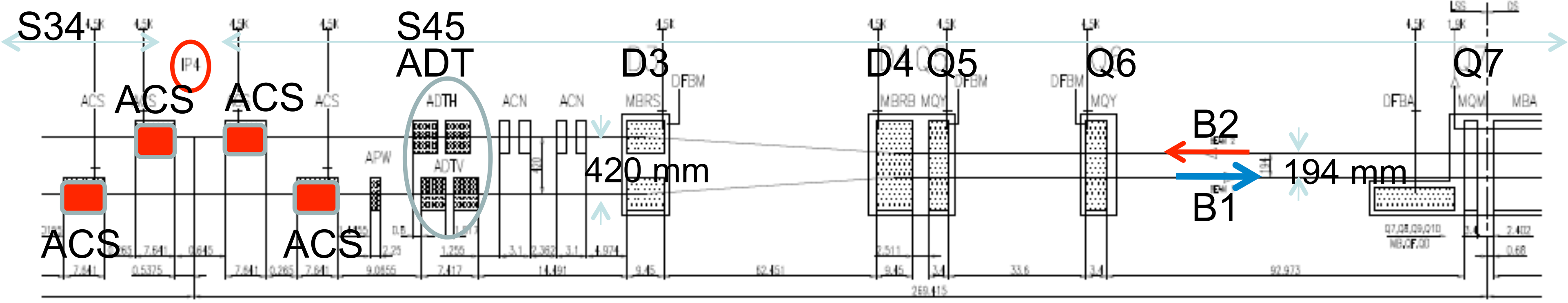


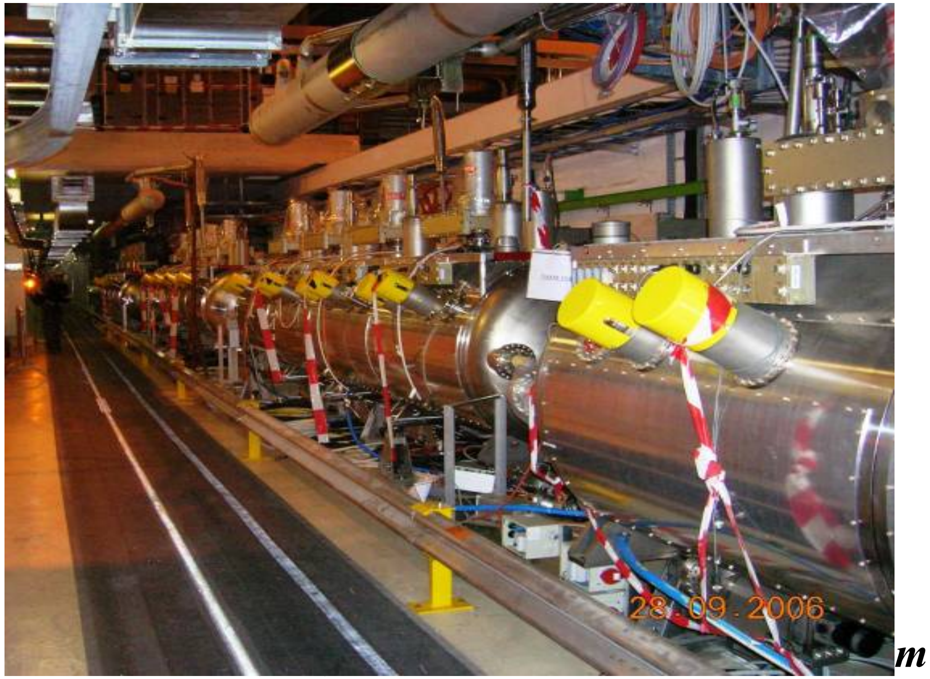
Focussing effect in the longitudinal direction
keeping the particles close together ... forming a "bunch"

... and how do we accelerate now ???

with the dipole magnets !

The RF system: IR4





m

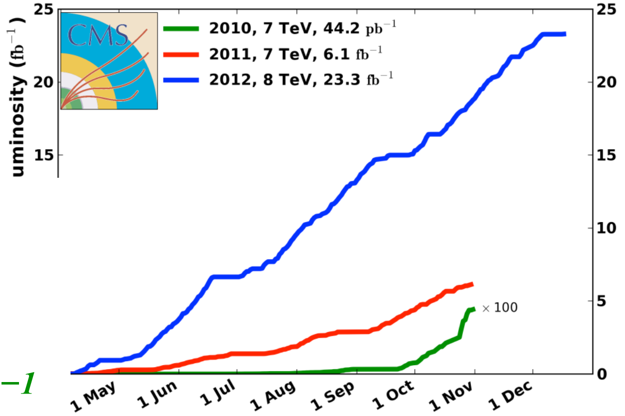
Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm

Bunch length (4σ)	*ns*	*1.06*
Energy spread (2σ)	10^{-3}	*0.22*
Synchr. rad. loss/turn	*keV*	*7*
Synchr. rad. power	*kW*	*3.6*
RF frequency	*M Hz*	*400*
Harmonic number		*35640*
RF voltage/beam	*MV*	*16*
Energy gain/turn	*keV*	*485*
Synchrotron frequency	*Hz*	*23.0*

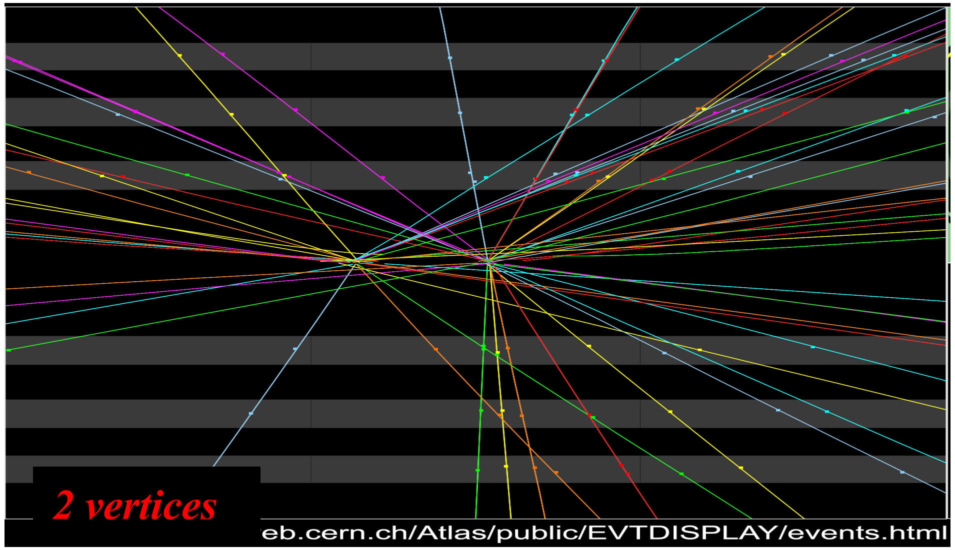
1.) Where are we ?

- Standard Model of HEP
- Higgs discovery

And still...
The LHC Performance in Run 1

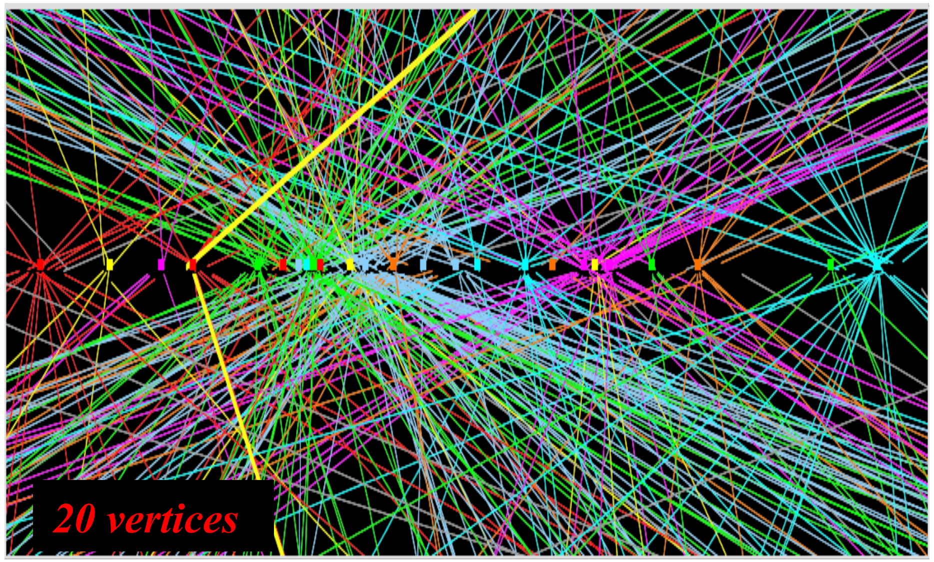


	Design	*2012*
Momentum at collision	*7 TeV /c*	*4 TeV/c*
Luminosity	10^{34} cm $^{-2}$ s^{-1}	$7.7 * 10^{33}$ cm $^{-2}$ s^{-1}
Protons per bunch	1.15×10^{11}	1.50×10^{11}
Number of bunches/beam	*2808*	*1380*
Nominal bunch spacing	*25 ns*	*50ns*
*beta **	*55 cm*	*60 cm*
rms beam size IP	*17 µm*	*20 µm*



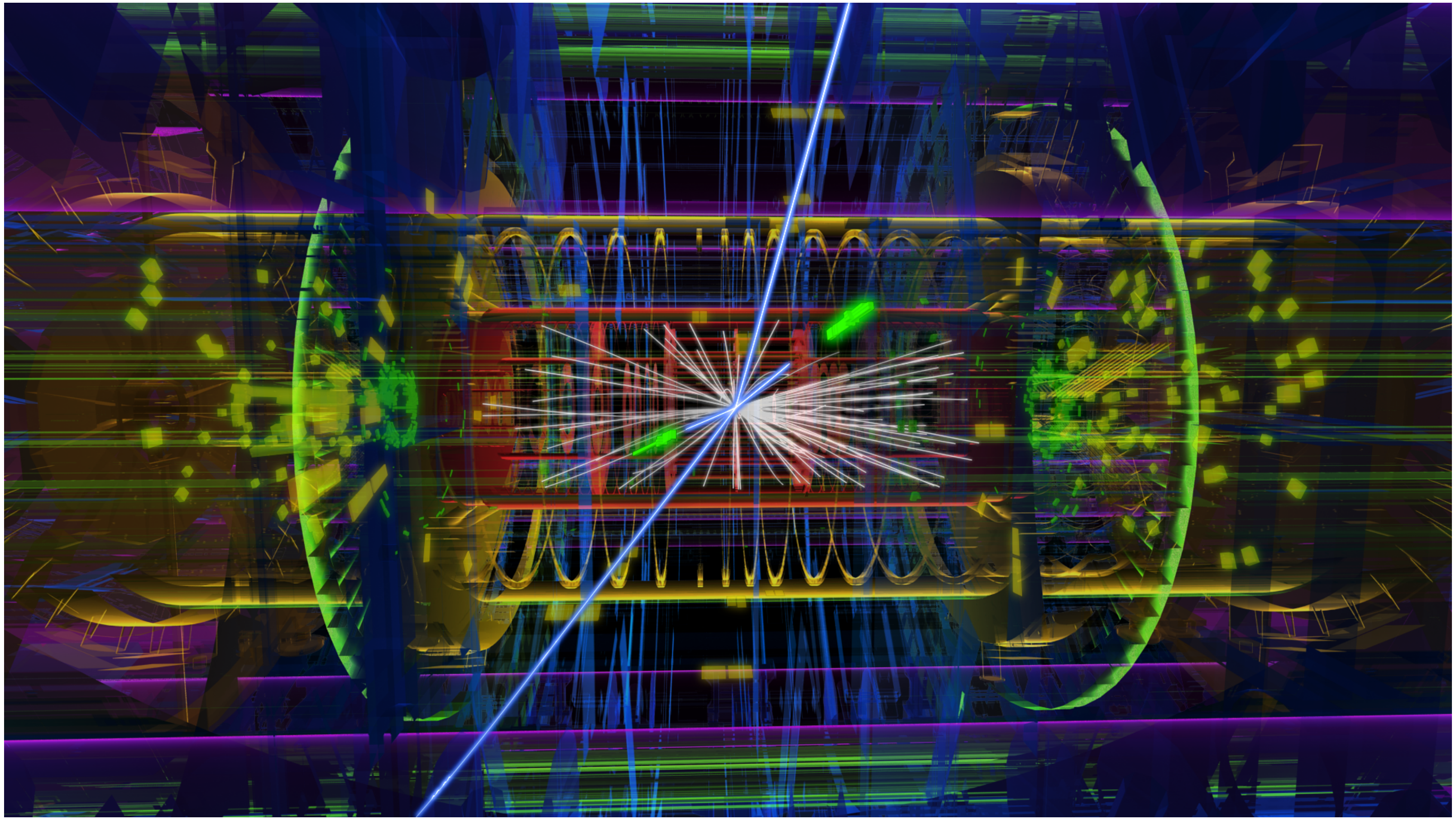
2 vertices

eb.cern.ch/Atlas/public/EVTDISPLAY/events.html



20 vertices

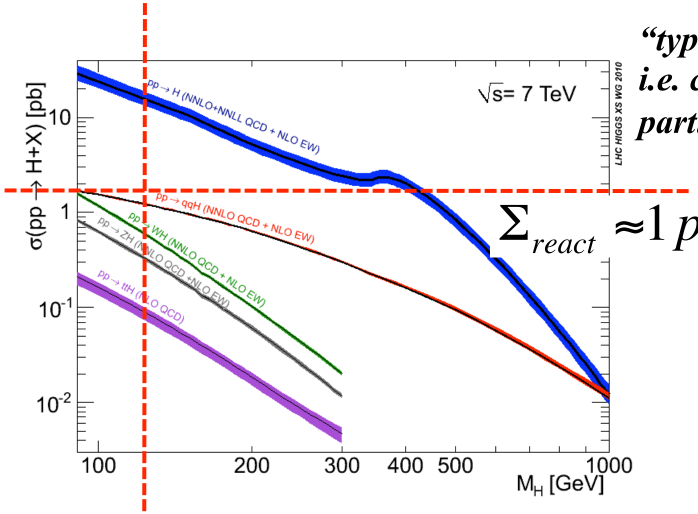
High Light of the HEP-Year 2012 / 13 naturally the HIGGS



ATLAS event display: Higgs => two electrons & two muons

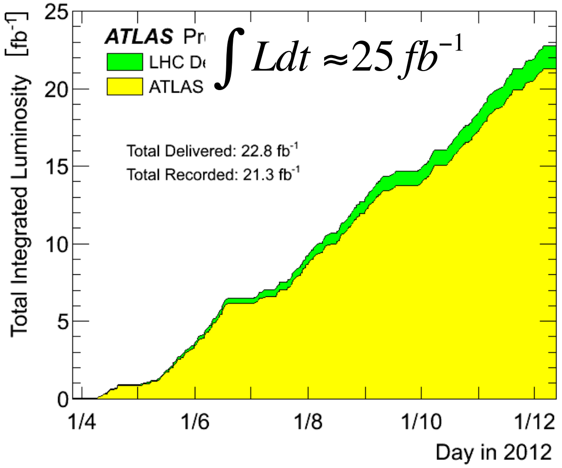
The High light of the year

production rate of events** is **determined by** the **cross section Σ_{react}
and a parameter L that is given by the design of the accelerator:
… the luminosity



$$\Sigma_{react} \approx 1\,pb$$

"typical particle size"
i.e. cross section for
particle production



$$\int L dt \approx 25\, fb^{-1}$$

accumulated
collision rate
in LHC run 1

$$1b = 10^{-24}\,cm^2 \quad = \quad 1/mio * 1/mio * 1/mio * \frac{1}{100}\,mm^2$$

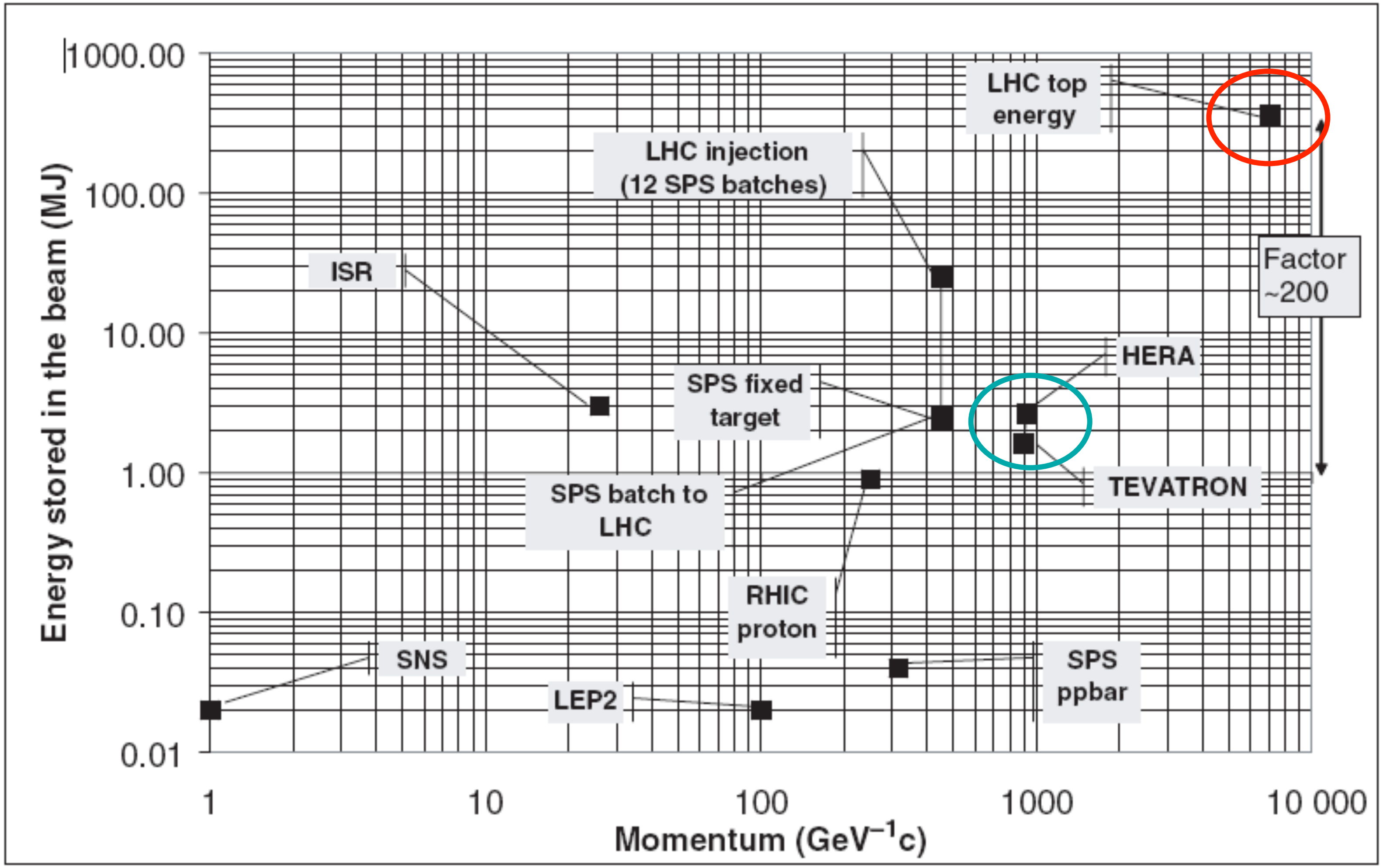
The particles are "very small"

$$R = L * \Sigma_{react} \quad \approx 10^{-12}\,b \cdot 25\,\frac{1}{10^{-15}\,b} = some\,1000\,H$$

During collider run we had in Run 1 …
1400 bunches circulating,
***with 800 Mio proton collisions per second** in the experiments*
and collected only 450 Higgs particles in three years.

LHC Operation:
Machine Protection & Safety

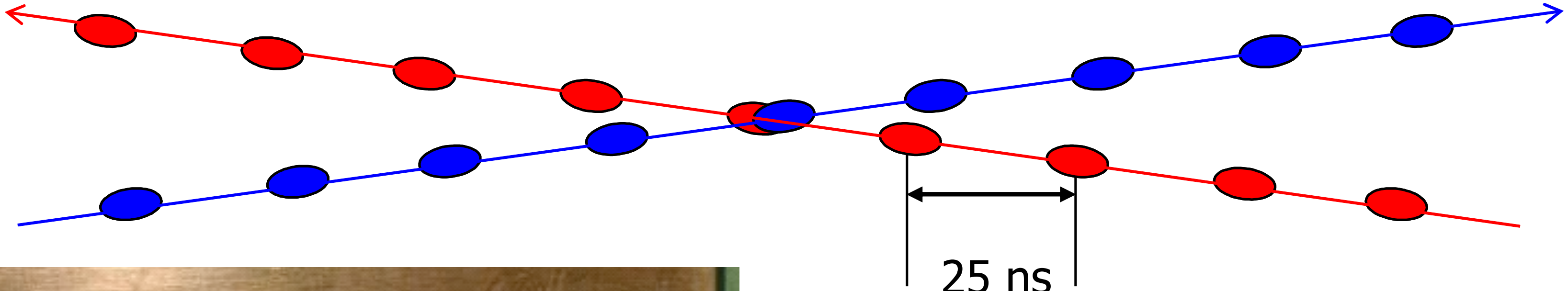
Energy Stored in the Beam of different Storage Rings

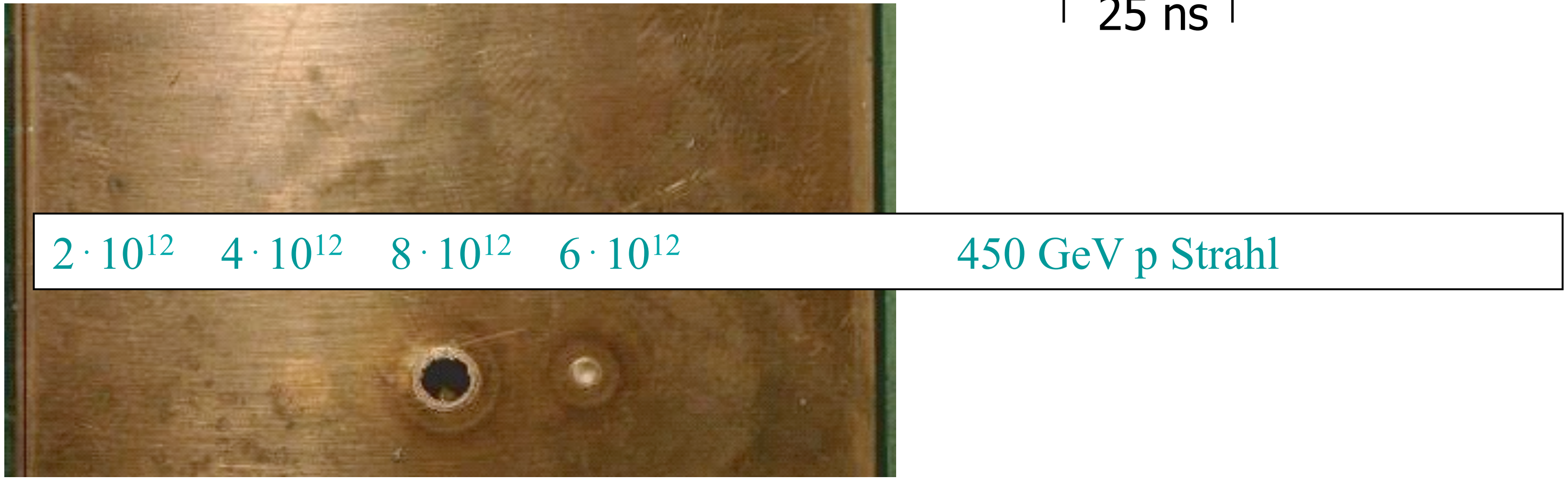


LHC Operation:
Machine Protection & Safety

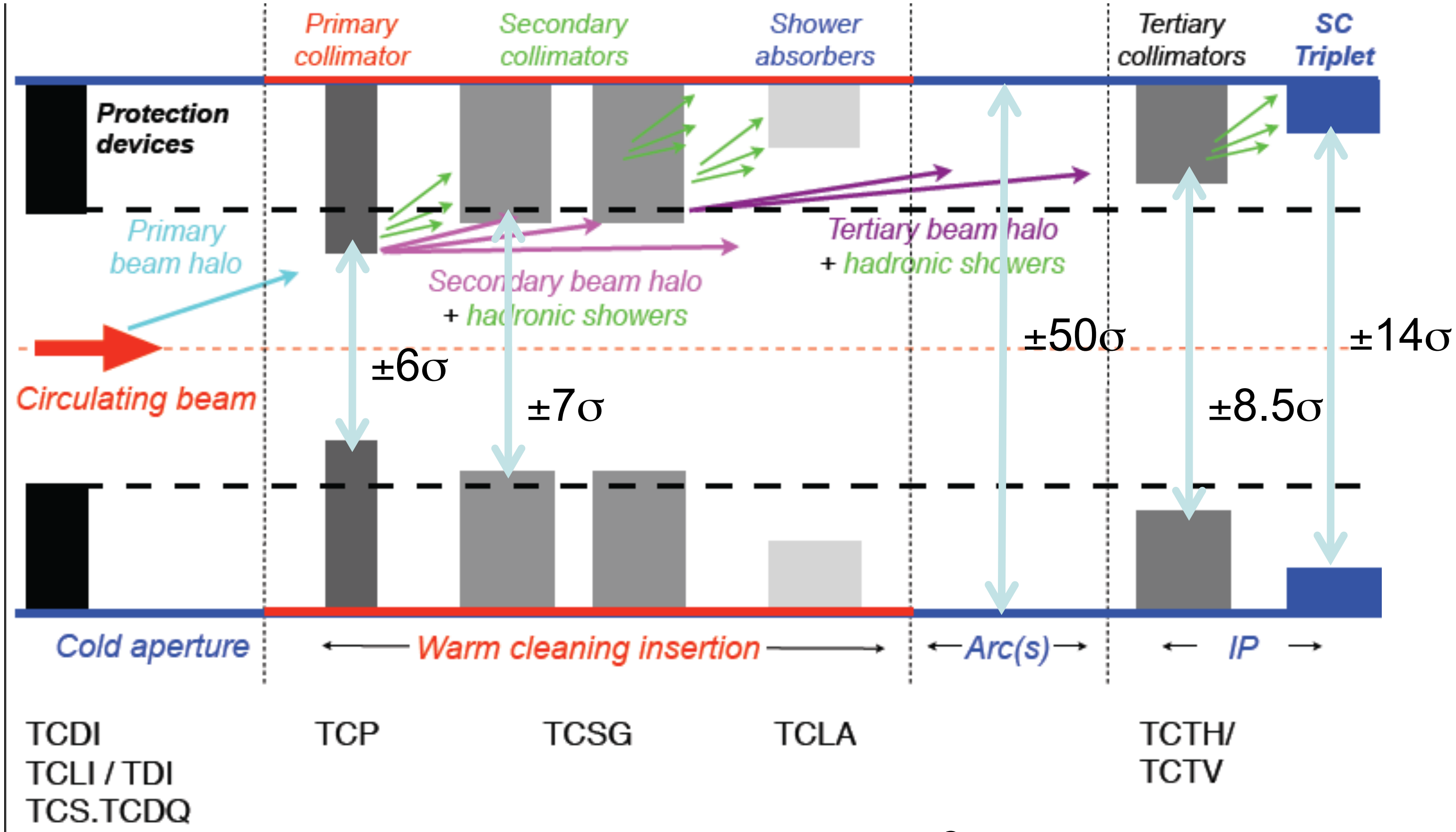
Energy stored in magnet system	10	GJ
Energy stored in one main dipole circuit	1.1	GJ
Energy stored in one beam	362	MJ

Enough to melt 500 kg of copper





LHC Aperture and Collimation



S. Redaelli, OP WG on Checkout, 08-11-2007

Settings @7TeV and $\beta^*=0.55$ m

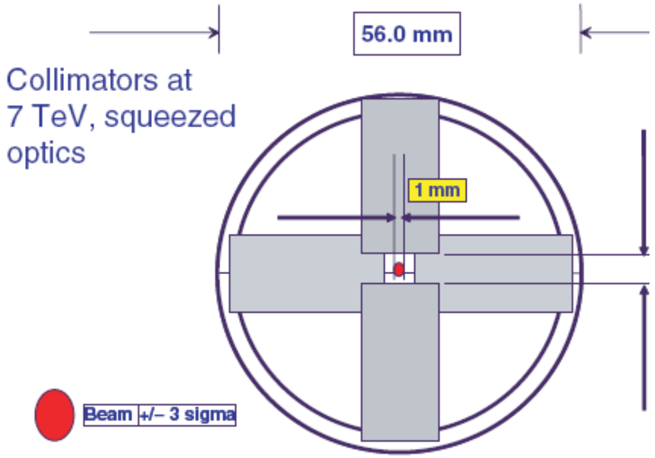
Beam size (σ) = 300 μm (@arc)

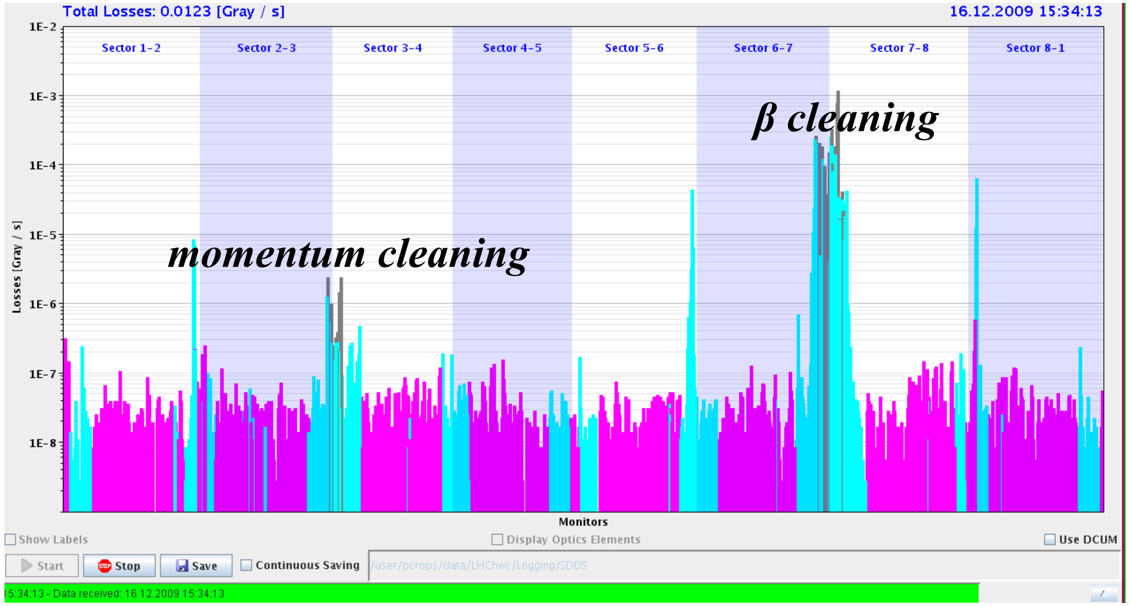
Beam size (σ) = 17 μm (@IR1, IR5)

LHC Operation:
Machine Protection & Safety

... Komponenten des Machine Protection Systems :

beam loss monitors
QPS
permit server
orbit control
power supply control
collimators
online on beam check of all (?)
hardware components
a fast dump
the gaussian beam profile

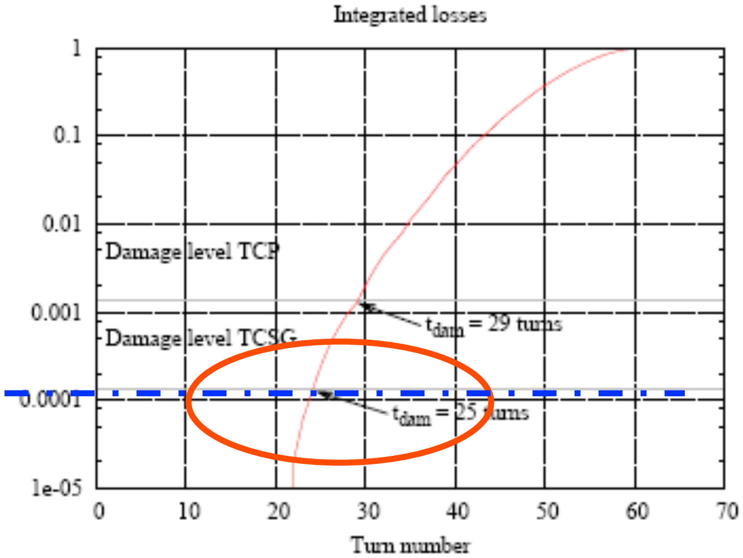




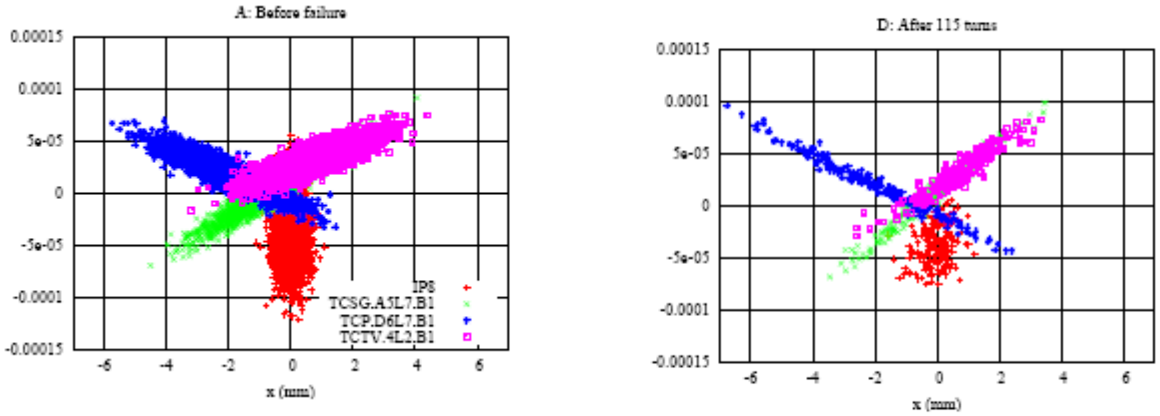
15:34:13 - Data received: 16.12.2009 15:34:13

momentum cleaning

β cleaning



LHC Operation: Machine Protection & Safety



What will happen in
case of Hardware Failure

Phase space deformation in case of failure of RQ4.LR7
(A. Gómez)

Short Summary of the studies:

quench in sc. arc dipoles: τ_{loss} *=20 - 30 ms*
BLM system reacts in time, QPS is not fast enough

quench in sc. arc quadrupoles: τ_{loss} *=200 ms*
BLM & QPS react in time

failure of nc. quadrupoles: $\tau_{det} = 6$ *ms*
$\tau_{damage} = 6.4$ *ms*

failure of nc. dipole: $\tau_{damage} = 2$ *ms*

→ FMCM installed

Energy stored in the magnets

~ 10 Gjoule* (only in the main dipoles) corresponds to …

… an aircraft carrier at battle-speed of 55 km/h

The energy of ~3 Tons TNT
The energy of 370 kg dark chocolate

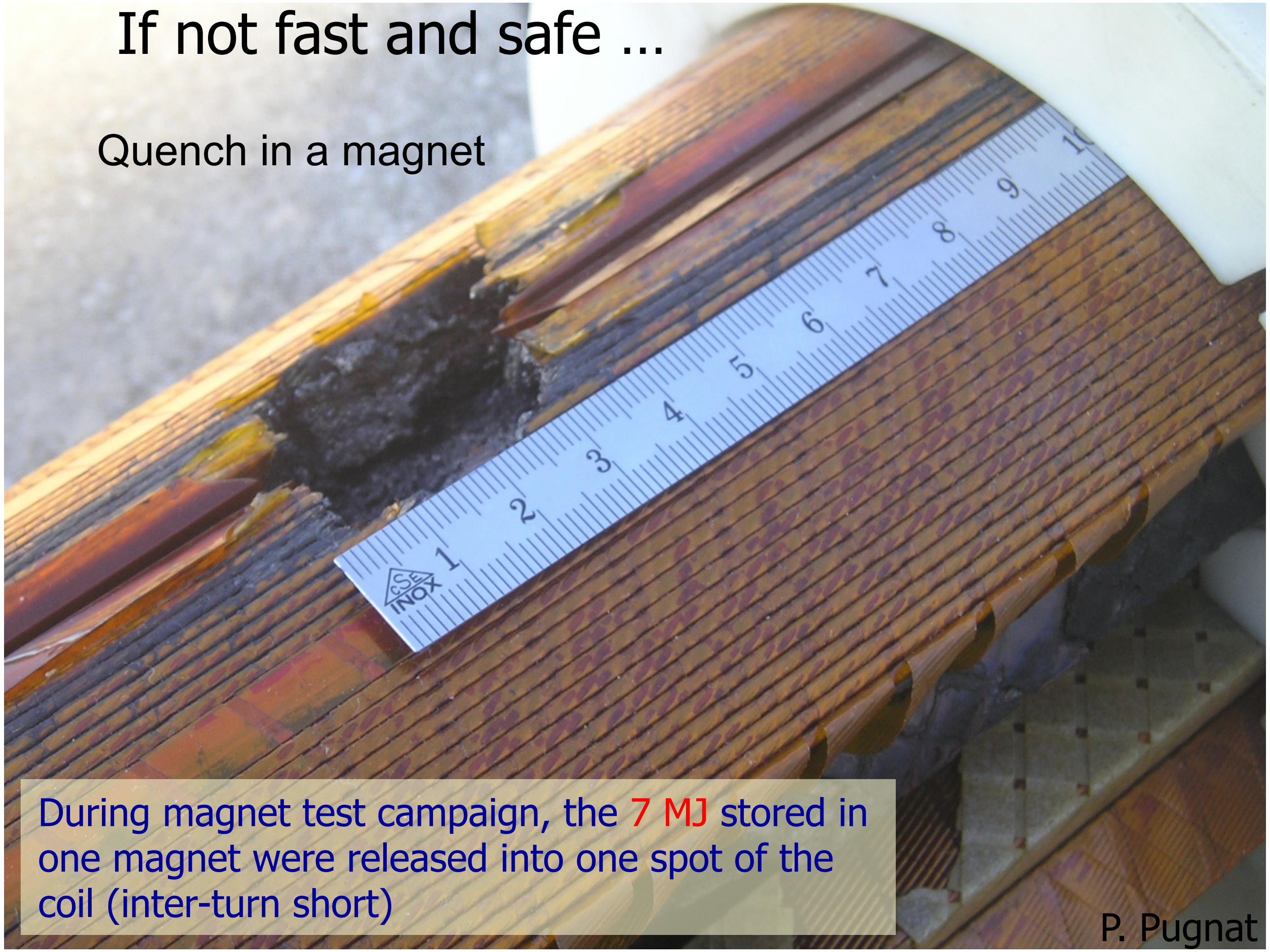
More important than the amount of energy is …

How fast (an safe) can this energy be released?

**$E=1/2LI^2$*

L: inductance ~0.1 Henry for LHC dipoles

Energy stored in the magnets: quench



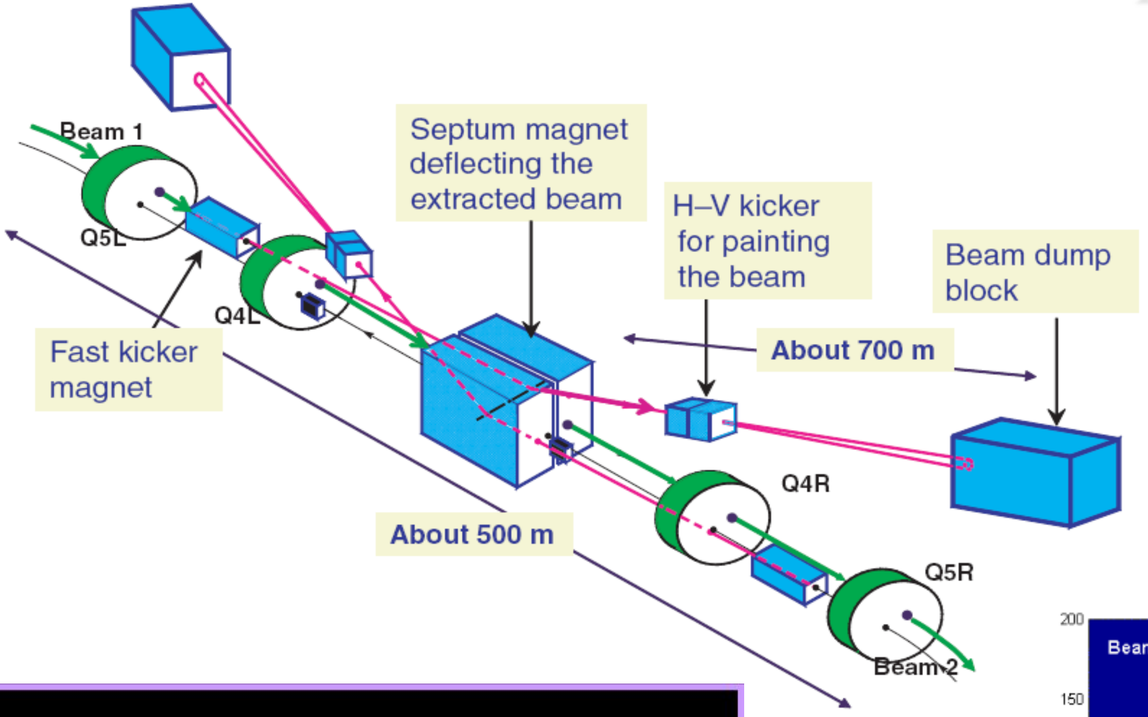
If not fast and safe …

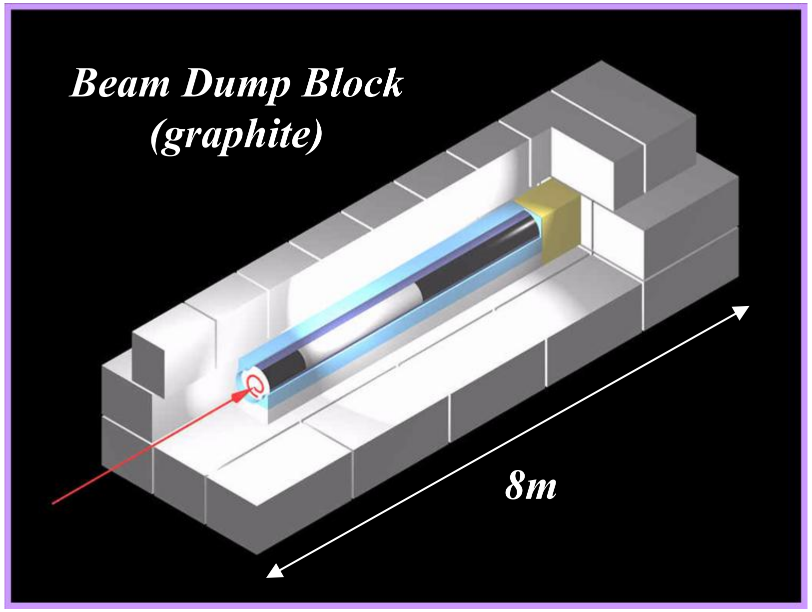
Quench in a magnet

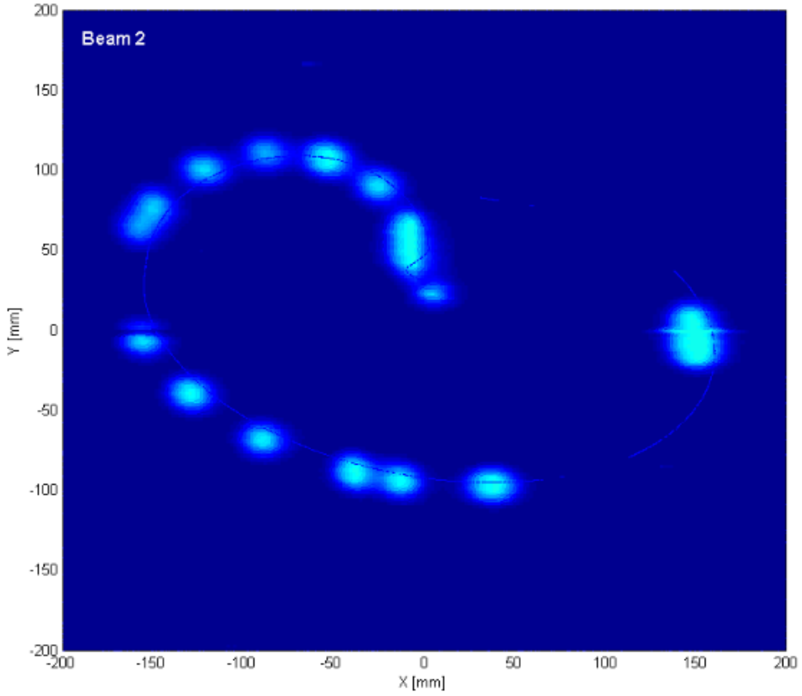
During magnet test campaign, the 7 MJ stored in one magnet were released into one spot of the coil (inter-turn short)

P. Pugnat

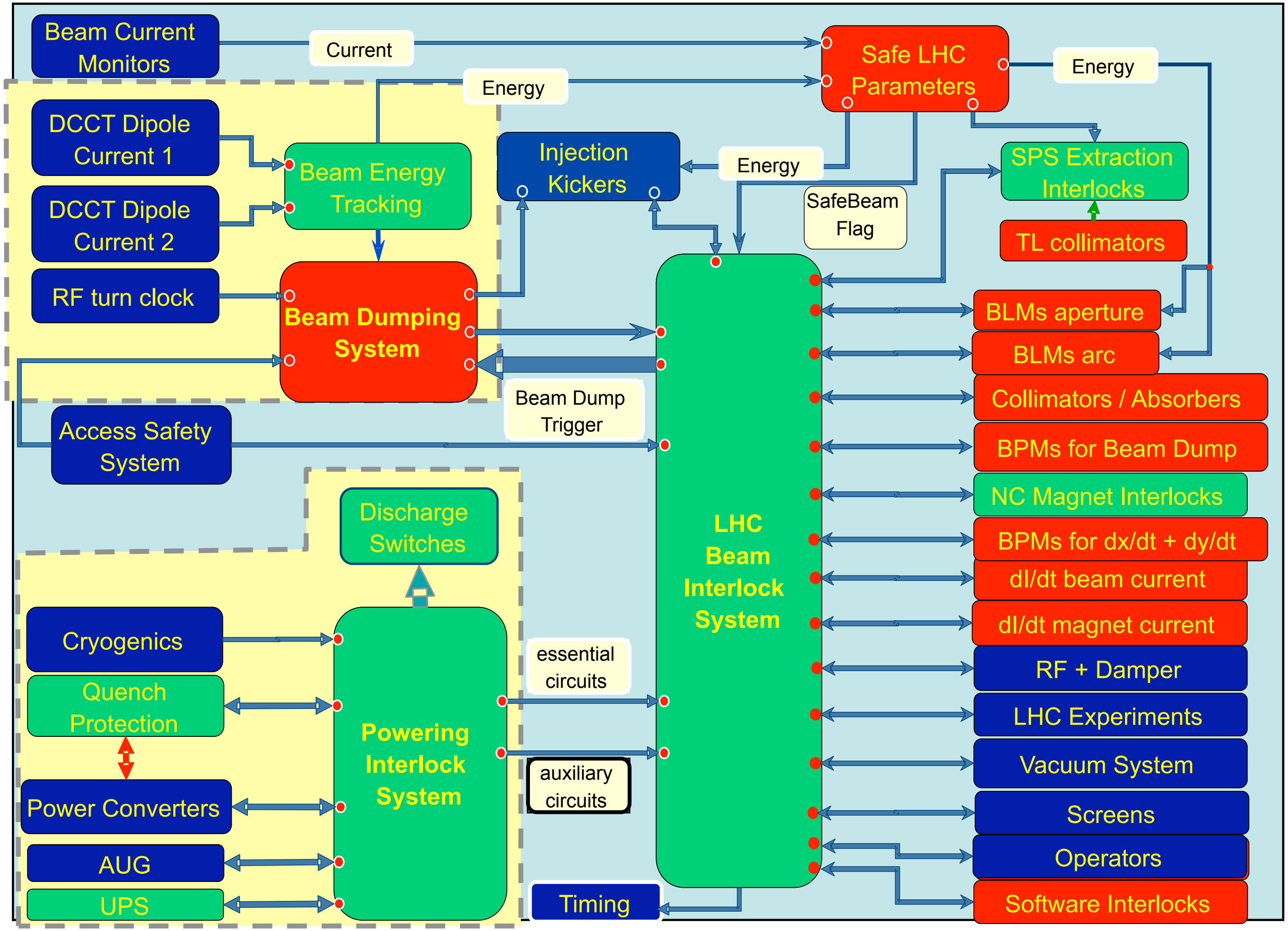
LHC Operation: Dump System




Beam Dump Block
(graphite)



LHC Operation: Machine Protection & Safety



… no comment

2.) Where do we go ?

**** Physics beyond the Standard Model***

**** Dark Matter / Dark Energy***