DGLAP evolution at NLO accuracy in Parton Showers

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Motivation

- QCD radiation omnipresent at the LHC
- enters as signal (and background) in high-$p_\perp$ analyses
  - multi-jet signatures

    $\longrightarrow$ multijet merging & higher-order matching (not the topic today)

  - inner-jet structures e.g. from "fat jets"

    $\longrightarrow$ parton shower algorithms
- begs the question:

  can we improve on parton showers and increase their precision?

  (keep in mind: accuracy vs. precision)

Outline

- how (LO) parton showers work
- including NLO splitting kernels
- looking at flavour changing $1 \to 3$ kernels
- results
- outlook

parton showers @LO: reminder

Sudakov form factor

- parton showers are approximations, based on

  leading colour, leading logarithmic accuracy, spin-average
- parametric accuracy by comparing Sudakov form factors:

$$\Delta = \exp\left\{-\int \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\left[A\log\frac{k_\perp^2}{Q^2}+B\right]\right\},$$

  where A and B can be expanded in $\alpha_S(k_\perp^2)$
- Q_T resummation includes $A_{1,2,3}$ and $B_{1,2}$

  (transverse momentum of Higgs boson etc.)
- showers usually include terms $A_{1,2}$ and B_1

  A = cusp terms ("soft emissions"), $B \sim$ anomalous dimensions γ

Connection to Fragmentation Functions

- DGLAP for FFs:

$$\frac{\mathrm{d}\, xD_a(x,\ t)}{\mathrm{d}\log t} = \sum_{b=q,g} \int_0^1 \mathrm{d}\tau \int_0^1 \mathrm{d}z\, \delta(x-\tau z)\, \frac{\alpha_S}{2\pi}\, [zP_{ab}(z)]_+\, \tau D_b(\tau, t)\,.$$

- rewrite for definition of "+"-function, $[zP_{ab}(z)]_+ = \lim_{\epsilon\to 0} zP_{ab}(z,\epsilon)$:

$$P_{ab}(z,\epsilon) = P_{ab}(z)\,\Theta(1-z-\epsilon) - \delta_{ab} \sum_{c=q,g} \frac{\Theta(1-z-\epsilon)}{\epsilon} \int_0^1 \mathrm{d}\xi\, \xi P_{ac}(\xi)$$

$$\frac{\mathrm{d}\log D_a(x,t)}{\mathrm{d}\log t} = \underbrace{-\sum_{c=q,g} \int_0^{1-\epsilon} \mathrm{d}\xi\, \frac{\alpha_S}{2\pi}\, \xi P_{ac}(\xi)}_{\text{derivative of Sudakov}} + \sum_{b=q,g} \int_x^{1-\epsilon} \frac{\mathrm{d}z}{z}\, \frac{\alpha_S}{2\pi}\, P_{ac}(z)\, \frac{D_b(\frac{x}{z}, t)}{D_a(x,t)}$$

- re-introduce Sudakov form factor

$$\Delta_a(t, t_0) = \exp\left\{-\int\limits_{t_0}^{t}\frac{\mathrm{d}t'}{t'}\sum_{c=q,g}\int\limits_0^{1-\epsilon}\mathrm{d}\xi\,\frac{\alpha_S}{2\pi}\,\xi P_{ac}(\xi)\right\}$$

to express equation above through generating functional $\mathcal{D}_a(x, t, \mu^2) = D_a(x, t)\Delta_a(\mu^2, t)$:

$$\frac{\mathrm{d}\,\log\mathcal{D}_a(x, t, \mu^2)}{\mathrm{d}\,\log t} = \sum_{b=q,g}\int\limits_x^{1-\epsilon}\frac{\mathrm{d}z}{z}\frac{\alpha_S}{2\pi}\,P_{ac}(z)\,\frac{D_b(\frac{x}{z}, t)}{D_a(x, t)}$$

- add initial states (PDFs) & arrive at argument(s) for Sudakov form factors when jets not measured

$$\sum_{i\in IS}\sum_{b=q,g}\int\limits_{x_i}^{1-\epsilon}\frac{\mathrm{d}z}{z}\frac{\alpha_S}{2\pi}\,P_{ba_i}(z)\,\frac{f_b(\frac{x_i}{z}, t)}{f_{a_i}(x, t)} + \sum_{j\in FS}\sum_{b=q,g}\int\limits_{x_i}^{1-\epsilon}\mathrm{d}z\,z\frac{\alpha_S}{2\pi}\,P_{a_jb}(z)\,.$$

Symmetry factors

- observations for LO PS in final state:
  - only $P_{qq}^{(0)}$ used but not $P_{qg}^{(0)}$
  - $P_{gg}^{(0)}$ comes with “symmetry factor” 1/2
- challenge this way of implementing symmetry through:

(Jadach & Skrzypek, hep-ph/0312355)

$$\sum_{i=q,g} \int_0^{1-\epsilon} \mathrm{d}z\, z\, P_{qi}^{(0)}(z) = \int_\epsilon^{1-\epsilon} \mathrm{d}z\, P_{qq}^{(0)}(z) + \mathcal{O}(\epsilon)$$

$$\sum_{i=q,g} \int_0^{1-\epsilon} \mathrm{d}z\, z\, P_{gi}^{(0)}(z) = \int_\epsilon^{1-\epsilon} \mathrm{d}z\, \left[\frac{1}{2} P_{gg}^{(0)}(z) + n_f P_{gq}^{(0)}(z)\right] + \mathcal{O}(\epsilon)$$

- net effect: replace symmetry factors by parton marker z

Implementation in DIRE

- evolution and splitting parameter $((ij)+k \to i+j+k)$:

$$\kappa^2_{j,ik} = \frac{4(p_ip_j)(p_jp_k)}{Q^4} \quad \text{and} \quad z_j = \frac{2(p_jp_k)}{Q^2}\,.$$

- splitting functions including IR regularisation

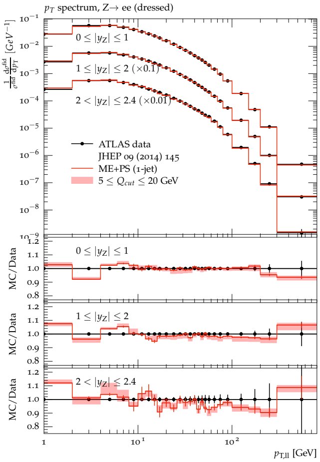
(a la Curci, Furmanski & Petronzio, Nucl.Phys. B175 (1980) 27-92)

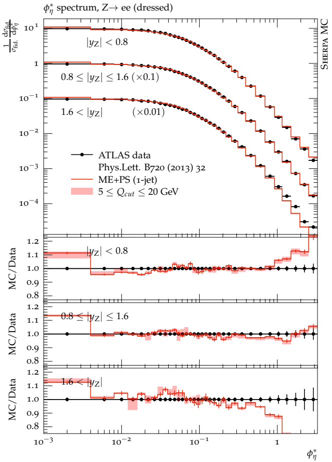
$$\begin{aligned}
P^{(0)}_{qq}(z,\kappa^2) &= 2C_F\left[\frac{1-z}{(1-z)^2+\kappa^2}-\frac{1+z}{2}\right]\,,\\
P^{(0)}_{qg}(z,\kappa^2) &= 2C_F\left[\frac{z}{z^2+\kappa^2}-\frac{2-z}{2}\right]\,,\\
P^{s(0)}_{gg}(z,\kappa^2) &= 2C_A\left[\frac{1-z}{(1-z)^2+\kappa^2}-1+\frac{z(1-z)}{2}\right]\,,\\
P^{(0)}_{gq}(z,\kappa^2) &= T_R\left[z^2+(1-z)^2\right]
\end{aligned}$$

- renormalisation/factorisation scale given by $\mu = \kappa^2 Q^2$
- combine gluon splitting from two splitting functions with different spectators k $\to$ accounts for different colour flows

LO results for Drell-Yan

(example of accuracy in description of standard precision observable)





including NLO splitting kernels

Including NLO splitting kernels

(Hoeche, FK & Prestel, 1705.00982, and Hoeche & Prestel, 1705.00742)

- expand splitting kernels as

$$P(z,\,\kappa^2)\;=\;P^{(0)}(z,\,\kappa^2)\,+\,\frac{\alpha_S}{2\pi}\,P^{(1)}(z,\,\kappa^2)$$

- aim: reproduce DGLAP evolution at NLO
  include all NLO splitting kernels
- three categories of terms in $P^{(1)}$:
  - cusp (universal soft-enhanced correction) (already included in original showers)
  - corrections to $1 \to 2$
  - new flavour structures (e.g. $q \to q'$), identified as $1 \to 3$
- new paradigm: two independent implementations

Implementation details: $1 \to 2$ splittings

- problem: new pole structure $1/z$ appears
- in final-state shower: symmetrisation yields extra factor z

  (such a factor is present in IS shower)
- this factor accounts for $1/2$ typically applied to $g \to gg$
- include also $q \to gq$ splitting
- physical interpretation:
  - "unconstrained" (without) vs. "constrained" evolution

    (DGLAP evolution for fragmentation functions)
  - factor z explicitly guarantees (momentum) sum rules
  - it also identifies final state particle

- symmetry factors not so clear at NLO $\longrightarrow$ more care needed

$$\sum_{b=q,g,b\neq a} \int\limits_0^{1-\epsilon} \mathrm{d}z_1 \int\limits_0^{1-\epsilon} \mathrm{d}z_2 \, \frac{z_1 z_2}{1-z_1} \, \Theta(1-z_1-z_2)$$

$$\times \left[P_{a\to ba\bar{b}}(z_1, z_2, \dots) + P_{a\to b\bar{b}a}(z_1, z_2, \dots) \right]$$

$$= \sum_{b=q,g,b\neq a} \int\limits_0^{1-\epsilon} \mathrm{d}z_1 \int\limits_0^{1-z_1} \mathrm{d}z_2 \, \frac{1}{\prod\limits_{i=q,g} n_i!} \, P_{a\to ba\bar{b}}(z_1, z_2, \dots) + \mathcal{O}(\epsilon)$$

implementing $1 \to 3$ kernels
(flavour changing only)

$2 \to 4$, FF case as example

(Hoeche & Prestel, 1705.00742)

- start with triple-collinear splitting functions

  (Campbell & Glover, hep-ph/9710255 & Catani & Grazzini, hepph/9908523)

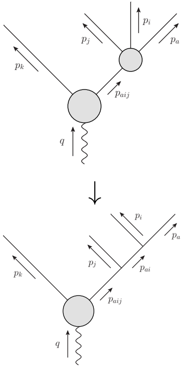
- re-interpret splitting as sequential:
  $(aij)+k \to (ai)+j+k \otimes (ai)+k \to a+i+k$
- kinematic mappings from CDST

  (Catani, Dittmaier, Seymour & Trocsanyi, hep-ph/0201036)

- evolution and splitting parameters:

$$t = \frac{4(p_j p_{ai})(p_{ai} p_k)}{q^2 - m_{aij}^2 - m_k^2}\,, \quad z_a = \frac{2 p_a p_k}{q^2 - m_{aij}^2 - m_k^2}$$

$$s_{ai} = 2 p_a p_i + m_a^2 + m_i^2\,, \quad x_a = \frac{p_a p_k}{p_{ai} p_k}$$



- phase space factorised by successive s-channels:

(Dittmaier, hep-ph/9904440)

$$\mathrm{d}\Phi_{+2} = \left[\frac{1}{4(2\pi)^3}\frac{\mathrm{d}t}{t}\mathrm{d}z_a\mathrm{d}\phi_j J_{FF}^{(1)}\right]\left[\frac{1}{4(2\pi)^3}\mathrm{d}s_{ai}\frac{\mathrm{d}x_a}{x_a}\mathrm{d}\phi_i J_{FF}^{(2)}\,(2p_{ai}p_j)\right]$$

- combine with ME in coll. limit $\longrightarrow$ diff. branching probability:

$$\frac{\mathrm{d}\,\log\Delta^{1\to 3}_{(aij)a}}{\mathrm{d}\,\log t} = \int \mathrm{d}z_a \int \mathrm{d}s_{ai} \int \frac{\mathrm{d}x_a}{x_a} \int \frac{\mathrm{d}\phi}{2\pi}\left(\frac{\alpha_S}{2\pi}\right)^2 \frac{z_a z_i}{1-z_a}\frac{P_{(aij)a}(p_a,p_i,p_j)}{s_{aij}^2/(2p_a p_j)}$$

with $P_{(aij)a}$ = triple-collinear splitting function

Subtractions

- must subtract spin-correlated iterated $1 \to 2$ splittings

$$\frac{\mathrm{d}\,\log\Delta^{(1\to2)^2}_{(aij)a}}{\mathrm{d}\,\log t} = \int \mathrm{d}z_a \int \frac{\mathrm{d}s_{ai}}{s_{ai}} \int \frac{\mathrm{d}\xi}{\xi} \left(\frac{\alpha_S}{2\pi}\right)^2 \frac{z_a z_i}{1-z_a} \frac{P^{(0)}_{(aij)(ai)}(\xi)\, P^{(0)}_{(ai)a}(\frac{z_a}{\xi})}{s_{aij}/(2p_a p_j)}$$

- must subtract convolution of one-loop matching coefficient with fixed-order renormalisation of fragmentation function, $\mathcal{I}$

$$\mathcal{I}_{qq'}(z) = 2C_F \int_z \frac{\mathrm{d}x}{x} \left(\frac{1+(1-x)^2}{x} \log[x(1-x)] + x\right) P^{(0)}_{gq'}(\frac{z}{x})$$

(this is the finite part of convoluting $1 \to 2$ in D dimensions with another $1 \to 2$ in 4 dimensions)

Final result

- arrive at final expression, ready for MC implementation

$$P_{qq'}(z) = \left(I + \frac{1}{\epsilon}\mathcal{P} - \mathcal{I}\right)_{qq'}(z) + \int \mathrm{d}\Phi_{+1} \left(R - S\right)_{qq'}(z, \Phi_{+1}),$$

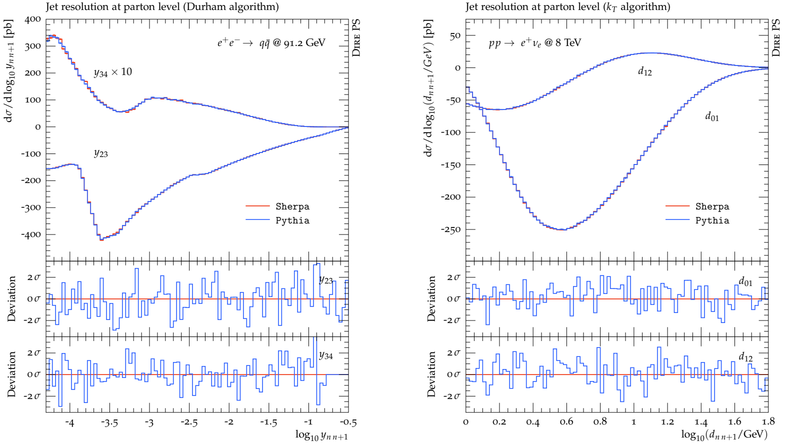
where

$$\begin{aligned}
\left(I + \frac{1}{\epsilon}\mathcal{P}\right)_{qq'}(z) &= \int \mathrm{d}\Phi_{+1}\, S_{qq'}(z, \Phi_{+1})\Big|_{\mathrm{finite}} \\
R_{qq'}(z, \Phi_{+1}) &= P^{1\to 3}_{qq'}(z, \Phi_{+1}) \\
S_{qq'}(z, \Phi_{+1}) &= \frac{s_{aij}}{s_{ai}} \left(P^{(0)}_{qg} \otimes P^{(0)}_{gq'}\right)(z, \Phi_{+1})
\end{aligned}$$

- this looks like MC@NLO inside the Sudakov exponent

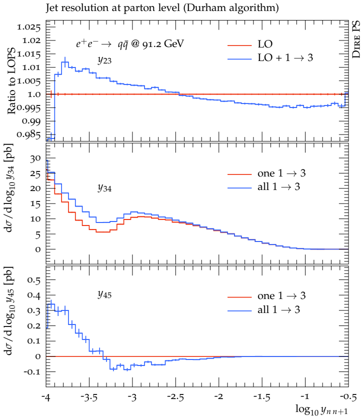
Validation of $1 \to 3$ splittings

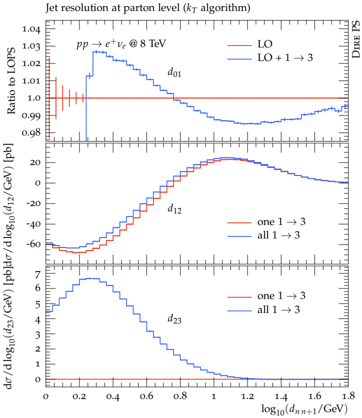
(Hoeche & Prestel, 1705.00742)



Impact of $1 \to 3$ splittings

(Hoeche & Prestel, 1705.00742)

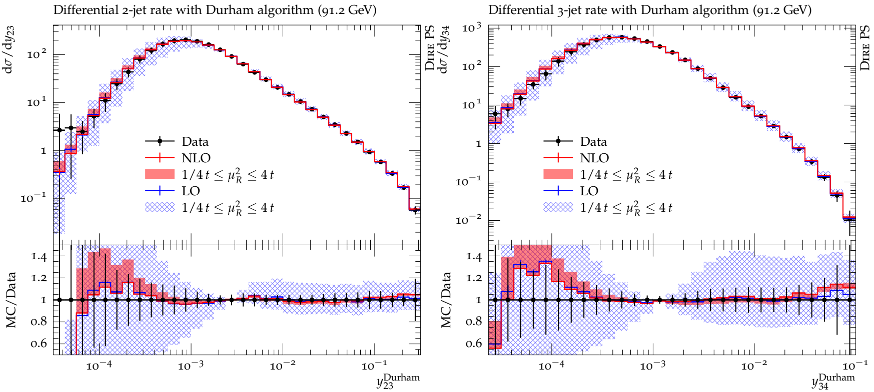




results

Physical results: $e^- e^+ \to$ hadrons

(Hoeche, FK & Prestel, 1705.00982)



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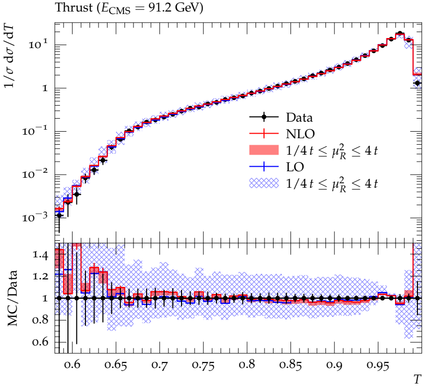
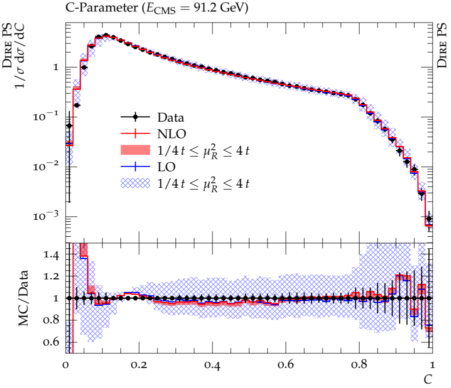
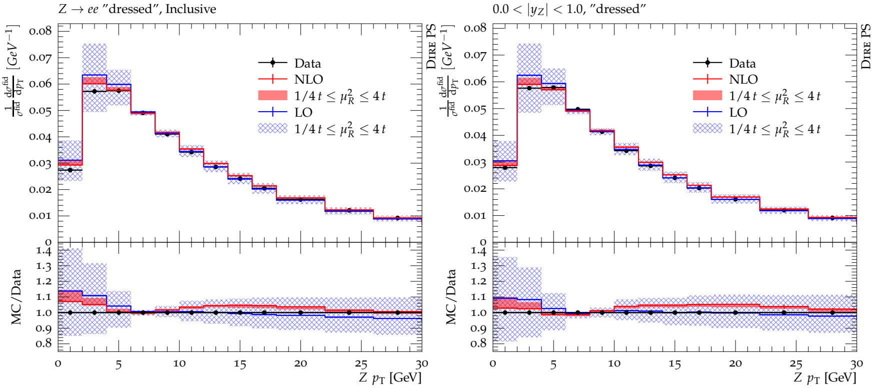




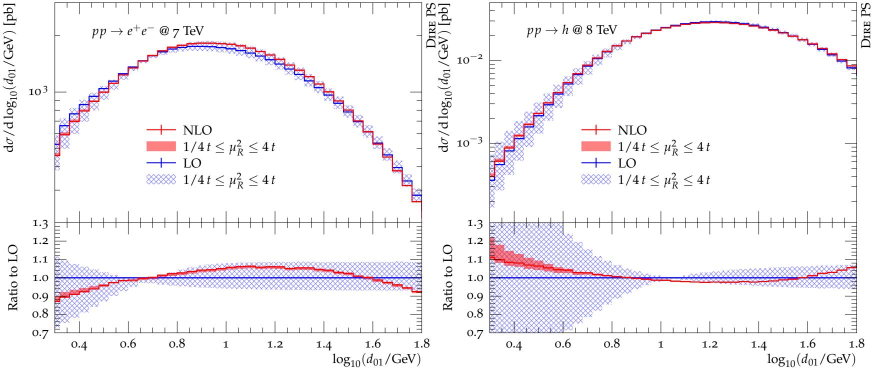
Physical results: DY at LHC

(Hoeche, FK & Prestel, 1705.00982)



Physical results: diff. jet rates at LHC

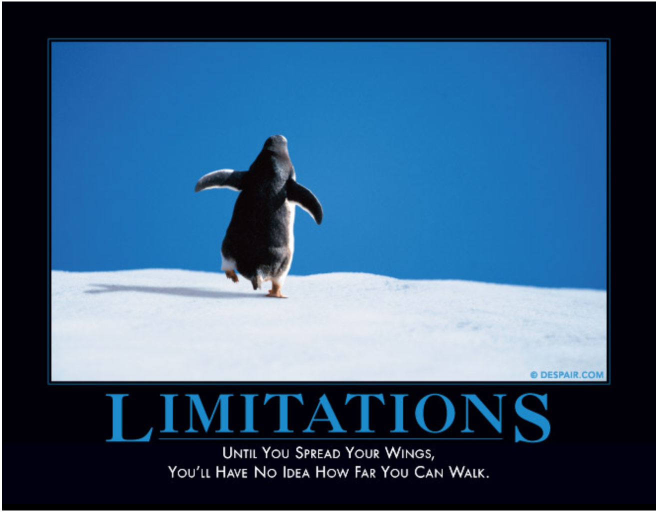
(Hoeche, FK & Prestel, 1705.00982)



outlook

Summary Outlook

- implemented NLO DGLAP kernels into two independent showers
- cross-validated implementations PYTHIA $\longleftrightarrow$ SHERPA
- extension to include loop-corrections to $1to2$ straightforward
  will allow to use triple-collinear splitting functions throughout
- future plans: soft-gluon emissions and non-trivial colour correlations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LIMITATIONS
Until You Spread Your Wings,
You'll Have No Idea How Far You Can Walk.