Associated Higgs Boson Production with a Top-quark Pair at NNLL+NLO at the LHC

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DFG Deutsche Forschungsgemeinschaft

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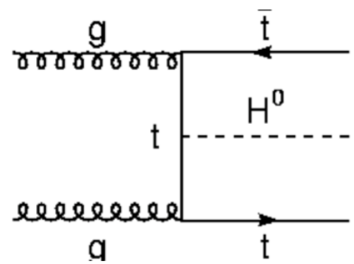
Associated Higgs Production with Top Quarks

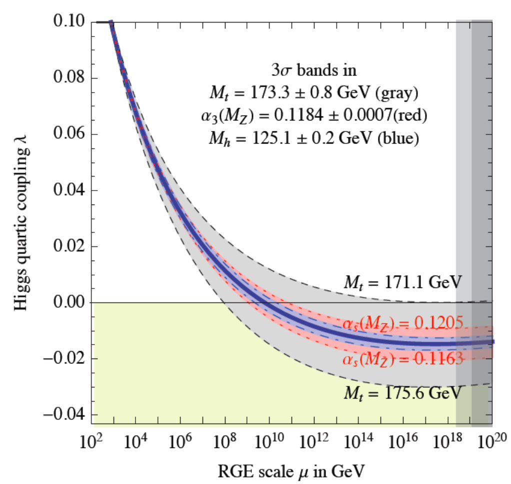
$$pp \to t\bar{t}H$$



- Direct probe of the strength of the top-Yukawa coupling without making any assumptions regarding its nature

$$V = -\frac{m^2}{2}|H|^2 + \lambda|H^4|$$

$$4\pi^2 \frac{d\lambda}{d\ln\mu^2} \simeq 3y_t^4 + 6\lambda y_t^2 + 12\lambda^2 + \ldots$$



[Buttazzo et al.'13]

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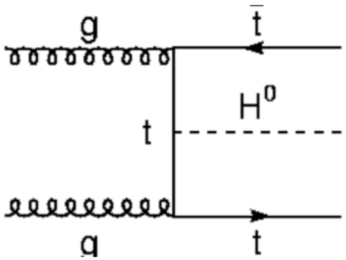
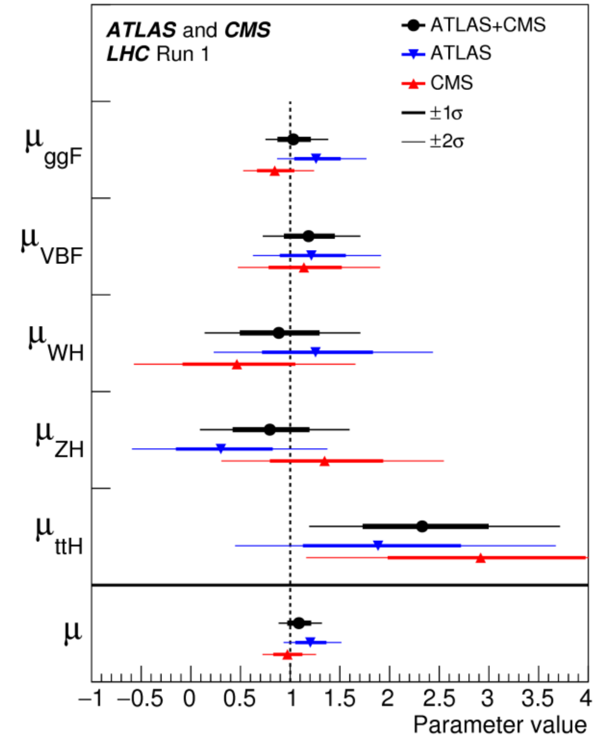


- Direct probe of the strength of the top-Yukawa coupling without making any assumptions regarding its nature
- An example of a 2⟶3 process for which full NNLO results are not available yet
- ca. 10 % NLO QCD scale uncertainty @13-14 TeV
- One of the most eagerly awaited measurements at Run 2

[Buttazzo et al.'13]

TTH @ THE LHC

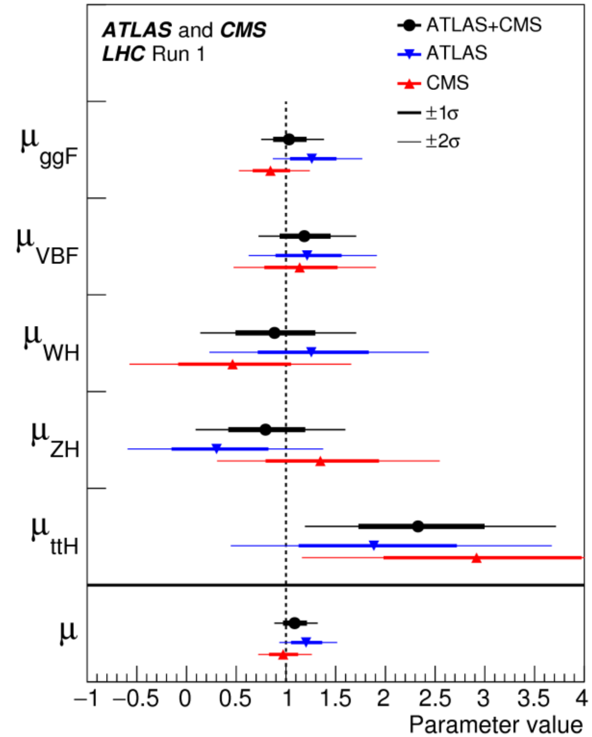
- Both ATLAS and CMS observed 2σ excesses in Run 1 in the signal strength $\mu_i = \sigma_i / \sigma_{i,SM}$



TTH @ THE LHC

- Both ATLAS and CMS observed 2σ excesses in Run 1 in the signal strength $\mu_i = \sigma_i / \sigma_{i,SM}$

- Run 2: sophisticated analysis methods (event categories, multi-variant analysis techniques) for many challenging final states:
  - multi-leptons + jets ($H \rightarrow WW/ZZ/\tau\tau$)
  - leptons + multiple *b*-jets ($H \rightarrow bb$)
  - di-photons + leptons and/or *b*-jets ($H \rightarrow \gamma\gamma$)



μ	Multi-lepton	bb	γγ
ATLAS	$2.5^{+1.3}_{-1.1}$	$2.1^{+1.0}_{-0.9}$	$-0.25^{+1.26}_{-0.99}$
CMS	$2.0^{+0.8}_{-0.7}$	$-0.19^{+0.80}_{-0.81}$	$1.9^{+1.5}_{-1.2}$

so far, inconclusive

Theory Status

- NLO QCD available since some time *[Beenakker, Dittmaier, Krämer, Plumper, Spira, Zerwas '01-'02][Reina, Dawson'01][Reina, Dawson, Wackeroth'02][Dawson,Orr,Reina,Wackeroth'03] [Dawson, Jackson, Orr, Reina, Wackeroth'03]*
  - ~ 20% correction to the total cross section
- NLO interfaced to parton showers in aMC@NLO *[Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli'11]* and POWHEG-Box *[Garzelli, Kardos,Papadopoulos,Trocsanyi'11] [Hartanto, Jäger,Reina, Wackeroth'15]* as well as SHERPA *[Gleisberg, Höche, Krauss, Schönherr, Schumann, Siegert, Winter]*
  - improved description of distribution shapes
- NLO EW (+QCD) corrections *[Frixione, Hirschi, Pagani,Shao,Zaro'14-'15][Zhang, Ma, Zhang, Chen, Guo'14]*
  - percent level corrections
- Top and Higgs off-shell effects: NLO QCD and EW for $W^+ W^- bb H$ *[Denner, Feger'15][Denner,Lang, Pellen, Uccirati'16]*
  - small effects for total cross sections, NLO EW corrections can be important in distributions

STATUS: RESUMMATION

- NLL+NLO resummation in the absolute threshold limit, $\hat{s} \to M^2 = (m_t + m_{\bar{t}} + m_H)^2$ obtained using direct QCD (Mellin space) approach *[AK, Motyka, Stebel, Theeuwes'15]*

- "Approximated" NNLO based on the SCET approach to resummation in the invariant mass threshold limit $\hat{s} \to Q^2 = (p_t + p_{\bar{t}} + p_H)^2$ *[Broggio, Ferroglia, Pecjak, Signer, Yang'15]*

Status discussed in the LHC HXSWG YR4 (2016)

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- NNLL+NLO resummation in the invariant mass threshold limit, hybrid SCET/Mellin space method *[Broggio, Ferroglia, Pecjak, Yang'16]*

- Here: NNLL+NLO resummation in the invariant mass threshold limit, direct QCD method

General Formalism: 2→2 Dijets Example

[Contapanagos, Laenen, Sterman'96] [Kidonakis, Oderda, Sterman'98]

Factorization principle

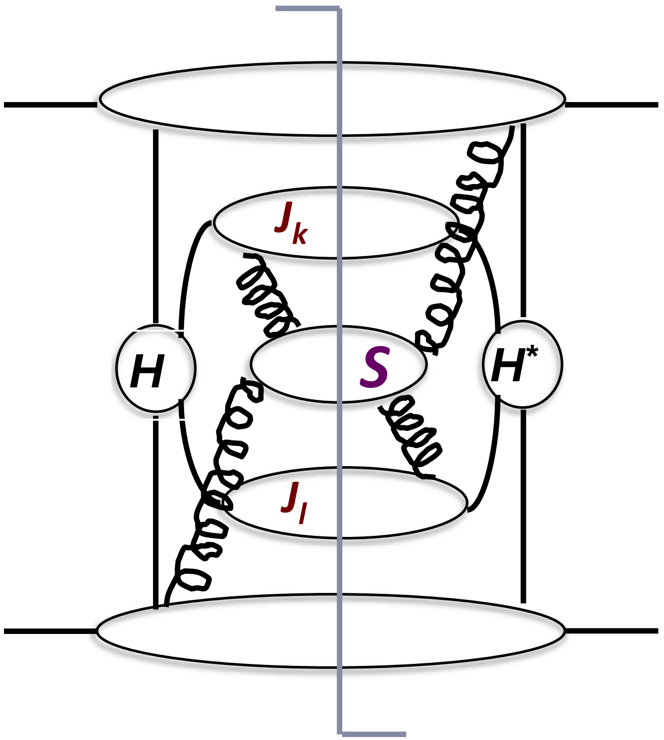
$$\hat{s}_{ab\to kl} = H_{IJ} \otimes E_a \otimes E_b \otimes S_{JI} \otimes J_k \otimes J_l$$

realized via (valid near threshold)

$$W = w_a c_a + w_b c_b + w_s + w_k + w_l$$

total weight

individual weights for each of the factorized functions, vanish at threshold



PIM: $c_a = c_b = 1$

1PI: $c_a = \dfrac{u}{t+u}$ $\quad c_b = \dfrac{t}{t+u}$

General Formalism

$$\frac{d\sigma_{AB\to kl}}{d\Pi} = \sum_{a,b}\int dx_a dx_b\, f_{a/A}(x_a^2,\mu^2) f_{b/B}(x_b^2,\mu^2)\; \Omega_{ab\to kl}(w,\hat{\Pi},\mu^2,\{m^2\})$$

$$\frac{d\sigma_{ab\to kl}}{d\hat{\Pi}} = H_{ab\to kl,IJ}\int dw_a\, dw_b\, dw_s\, dw_k\, dw_l\, \delta(W - c_a w_a - c_b w_b - w_s - w_k - w_l)$$
$$\psi_{a/a}(w_a, Q/\mu,\ldots)\psi_{b/b}(w_b, Q/\mu,\ldots) S_{ab\to kl,JI}(w_s, Q/\mu,\ldots) J_k(w_k, Q/\mu,\ldots) J_l(w_l, Q/\mu,\ldots)$$

Laplace transform: $\tilde{F}(N) = \int_0^{\infty} dw\, e^{-Nw} F(w)$

incoming collinear radiation

$$\tilde{\Omega}_{ab\to kl}(N, Q/\mu,\ldots) = H_{ab\to kl,IJ}\frac{\tilde{\psi}_{a/a}(c_a N, Q/\mu,\ldots)\tilde{\psi}_{b/b}(c_b N, Q/\mu,\ldots)}{\tilde{f}_{a/a}(c_a N, Q/\mu,\ldots)\tilde{f}_{b/b}(c_b N, Q/\mu,\ldots)}$$
$$\tilde{S}_{ab\to kl,JI}(N, Q/\mu,\ldots) J_k(N, Q/\mu,\ldots) J_l(N, Q/\mu,\ldots)$$

soft wide-angle emission

outgoing collinear radiation

General Formalism For 2→3

- Factorization principle holds for any number of jets/particles in the final state *[Kidonakis, Oderda, Sterman'98][Bonciani, Catani, Mangano, Nason'03]* but adding one more particle/jet requires adjusting for
  - colour structure of the underlying hard scattering: affects hard H and soft S functions

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right)S^{(N)}_{JI}=-\Gamma^{\dagger}_{JK}S(N)_{KI}-S(N)_{JL}\Gamma_{LI}$$

→ soft anomalous dimension

  - more complicated kinematics: affects H, S (anomalous dimension) and the arguments of incoming jet functions (coefficients c_a, c_b)

The Case of Associated Heavy Boson-Heavy quark Pair Production

$$pp \to Q\bar{Q}B$$

- Final state with only 2 massive colored particles
  - no final state jets
  - color structure same as in the QQbar production

-

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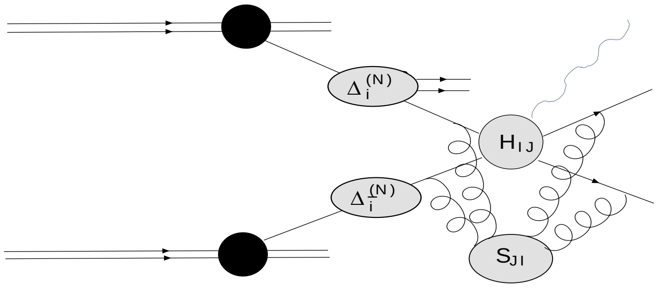

$$\frac{d\tilde{\hat{\sigma}}^{(\mathrm{res})}_{ij\to klB}}{dQ^2}(N, Q^2, \{m^2\}, \mu_F^2, \mu_R^2) =$$
$$= \mathrm{Tr}\left[\mathbf{H}_{ij\to klB}(Q^2, \{m^2\}, \mu_F^2, \mu_R^2)\mathbf{S}_{ij\to klB}(N+1, Q^2, \{m^2\}, \mu_F^2, \mu_R^2)\right]$$
$$\times\, \Delta^i(N+1, Q^2, \mu_F^2, \mu_R^2)\Delta^j(N+1, Q^2, \mu_F^2, \mu_R^2),$$

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- Threshold definitions:
  - absolute threshold limit

$$w = b^2 = 1 - \frac{(m_3 + m_4 + m_5)^2}{\hat{s}} \qquad \hat{s} \to M^2 = (m_3 + m_4 + m_5)^2$$

  - invariant mass threshold limit ("Triple Invariant Mass kinematics", TIM)

$$w = 1 - \frac{(p_3 + p_4 + p_5)^2}{\hat{s}} \qquad \hat{s} \to Q^2 = (p_3 + p_4 + p_5)^2$$

  - 1PI, 2PI kinematics…

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- Threshold definitions:
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simplest

$$w = b^2 = 1 - \frac{(m_3 + m_4 + m_5)^2}{\hat{s}} \qquad \hat{s} \to M^2 = (m_3 + m_4 + m_5)^2$$

  - invariant mass threshold limit ("Triple Invariant Mass kinematics", TIM)

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  - 1PI, 2 PI kinematics…

One-Loop Soft Anomalous Dimension

$$\Gamma_{q\bar{q}\to klB} = \frac{\alpha_s}{\pi}\begin{bmatrix} -C_F(L_{\beta,kl}+1) & \frac{C_F}{C_A}\Omega_3 \\ 2\Omega_3 & \frac{1}{2}(C_A-2C_F)(L_{\beta,kl}+1)+C_A\Lambda_3+(8C_F-3C_A)\Omega_3 \end{bmatrix}$$

singlet-octet colour basis

$$L_{\beta,kl} = \frac{\kappa^2+\beta_{kl}^2}{2\kappa\beta_{kl}}\left(\log\left(\frac{\kappa-\beta_{kl}}{\kappa+\beta_{kl}}\right)+i\pi\right)$$

$$\beta_{kl} = \sqrt{1-\frac{(m_k+m_l)^2}{s_{kl}}}$$

$$T_i(m) = \frac{1}{2}\left(\ln((m^2-\hat{t}_i)^2/(m^2\hat{s}))-1+i\pi\right)$$

$$U_i(m) = \frac{1}{2}\left(\ln((m^2-\hat{u}_i)^2/(m^2\hat{s}))-1+i\pi\right)$$

$$\Lambda_3 = (T_1(m_k)+T_2(m_l)+U_1(m_l)+U_2(m_k))/2$$

$$\Omega_3 = (T_1(m_k)+T_2(m_l)-U_1(m_l)-U_2(m_k))/2$$

$$t_1 = (p_i-p_k)^2 \qquad t_2 = (p_j-p_l)^2$$

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Reduces to the 2->2 case in the limit $p_B \to 0$, $m_B \to 0$

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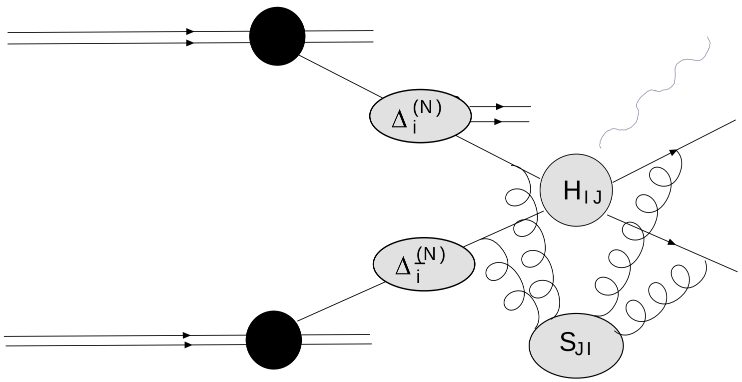
Absolute threshold limit: non-diagonal terms vanish

Coefficients $D^{(1)}_{ab\to kl\,B,I}$ governing soft emission same as for the QQbar process: soft emission at absolute threshold driven only by the color structure

Absolute Threshold Resummation for QQB



Colour space basis in which Γ_{IJ} is diagonal in the threshold limit

$$\hat{\sigma}^{(\mathrm{res},N)}_{ab\to klB} = \sum_I \underbrace{\hat{\sigma}^{(0,N)}_{ab\to klB,I}\, C_{ij\to klB,I}}_{\text{hard function } H_{ab\to kl B,I}}\, \Delta_a^{(N+1)} \Delta_b^{(N+1)} \Delta^{(\mathrm{soft},N)}_{ab\to klB,I}$$

incoming jet factors, known

soft-wide angle emission

$$\log \Delta^{(\mathrm{soft},N)}_{ab\to klB,I} = \int_0^1 dz \frac{z^{N-1}-1}{1-z} D_{ij\to klB,I}(\alpha_s(Q^2(1-z)^2)) \qquad D_{ij\to klB,I} = \lim_{\beta\to 0} \frac{\pi}{\alpha_s} 2\mathrm{Re}(\bar{\Gamma}_{II})$$

At NLL accuracy $C_{ab\to kl\,B,I} = 1$

The Case of Associated Heavy Boson-Heavy quark Pair Production

- Absolute threshold limit

$$\hat{s} \to M^2 = (m_t + m_{\bar{t}} + m_H)^2$$

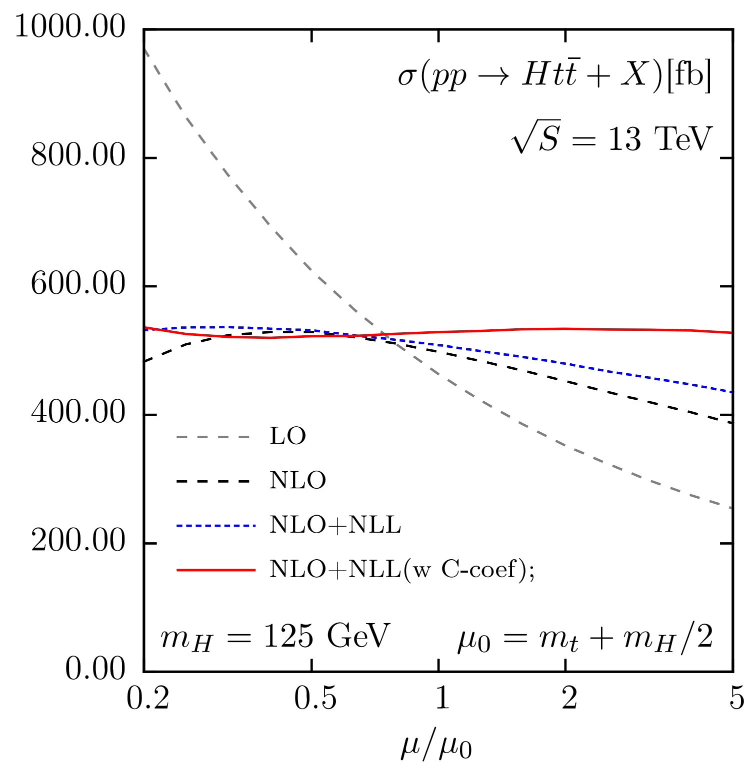
$$b = \sqrt{1 - M^2 / \hat{s}} \to 0$$

but:

- LO cross section suppressed in the limit β→*0* as β^4 due to massive 3-particle phase-space
- Absolute threshold scale *M* away from the region contributing the most

nevertheless:

- Well defined class of corrections which can be resummed *[AK, Motyka, Stebel, Theeuwes'15]*



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Coefficients $D^{(1)}_{ab\rightarrow kl\,B,l}$ governing soft emission same as for the QQbar process: soft emission at absolute threshold driven only by the color structure

Invariant mass threshold: non-diagonal elements present

INVARIANT MASS KINEMATICS

$$\hat{s} \to Q^2 = (p_3 + p_4 + p_5)^2$$

Problem: hard and soft functions are now (and in general) matrices in colour space

$$\frac{d\hat{\sigma}^{(\mathrm{res})}_{ij\to klB}}{dQ^2}(N) = Tr\left[\mathbf{H}_{ij\to klB}\,\bar{\mathbf{U}}_{ij\to klB}(N)\,\tilde{\mathbf{S}}_{ij\to kl}\,\mathbf{U}_{ij\to klB}(N)\right]\,\Delta^i(N+1)\Delta^j(N+1)$$

$$\mathbf{U}_{ij\to klB}(N) = \mathrm{P}\exp\left[\int_\mu^{Q/\bar{N}}\frac{dq}{q}\mathbf{\Gamma}_{ij\to klB}\left(\alpha_{\mathrm{s}}\left(q^2\right)\right)\right]$$

Diagonalization procedure
[Kidonakis, Oderda, Sterman'98]

$$\mathbf{\Gamma}^{(i)}_R = \mathbf{R}^{-1}\mathbf{\Gamma}^{(i)}\mathbf{R}$$
$$\mathbf{H}_{\mathbf{R}} = \mathbf{R}^{-1}\mathbf{H}\left(\mathbf{R}^{-1}\right)^\dagger$$
$$\tilde{\mathbf{S}}_R = \mathbf{R}^\dagger\tilde{\mathbf{S}}\mathbf{R}$$

leads to, at NLL:

$$\tilde{S}_{ij\to kl,R,IJ}(N) = \tilde{S}_{ij\to kl,R,IJ}\exp\left[\int_\mu^{Q/N}\frac{dq}{q}\left\{\lambda^*_{R,I}\left(\alpha_{\mathrm{s}}\left(q^2\right)\right) + \lambda_{R,J}\left(\alpha_{\mathrm{s}}\left(q^2\right)\right)\right\}\right]$$

where λ's are eigenvalues of the one-loop soft-anomalous dimension matrix

Invariant Mass Kinematics Ctnd.

- Extending resummation in this kinematics to NNLL requires:
  - Knowledge of the two-loop soft anomalous dimension
  - Amended treatment of the path-ordered exponential to account for it
  - Knowledge of the one-loop hard-matching coefficient

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$$\mathbf{\Gamma}_{ij\to klB} = \left[\left(\frac{\alpha_s}{\pi}\right) \mathbf{\Gamma}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \mathbf{\Gamma}^{(2)} + \ldots \right]$$

✓ Soft anomalous dimensions known at two loops for any number of legs *[Mert-Aybat, Dixon, Sterman'06] [Becher, Neubert'09] [Mitov, Sterman, Sung'09-'10] [Ferroglia, Neubert, Pecjak, Yang'09] [Beneke, Falgari, Schwinn'09], [Czakon, Mitov, Sterman'09] [Kidonakis'10]*

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Perturbative expansion *[Buchalla, Buras, Lautenbacher'96] [Ahrens, Neubert, Pecjak, Yang'10]*

$$\mathbf{U}_R(N, Q^2, \{m^2\}, \mu_F^2, \mu_R^2) = \left(\mathbf{1} + \frac{\alpha_s\left(Q^2/\bar{N}^2\right)}{\pi}\mathbf{K}\right)\left[\left(\frac{\alpha_s\left(\mu_F^2\right)}{\alpha_s\left(Q^2/\bar{N}^2\right)}\right)^{\frac{\vec{\lambda}^{(1)}}{2\pi b_0}}\right]_D\left(\mathbf{1} - \frac{\alpha_s\left(\mu_F^2\right)}{\pi}\mathbf{K}\right)$$

$$K_{IJ} = \delta_{IJ}\lambda_I^{(1)}\frac{b_1}{2b_0^2} - \frac{\left(\mathbf{\Gamma}_R^{(2)}\right)_{IJ}}{2\pi b_0 + \lambda_I^{(1)} - \lambda_J^{(1)}} \qquad \vec{\lambda}^{(1)} = \left\{\lambda_1^{(1)}, \dots, \lambda_D^{(1)}\right\}$$

eigenvalues of $\Gamma^{(1)}$

INVARIANT MASS KINEMATICS CTND.

- Extending resummation in this kinematics to NNLL requires:
  - Knowledge of the two-loop soft anomalous dimension
  - Amended treatment of the path-ordered exponential to account for it
  - Knowledge of the one-loop hard function H_{IJ}
    - needs access to colour structure of virtual corrections
    - currently approximated by introducing an overall one-loop hard coefficient $C^{(1)}$ with virtual contributions to $C^{(1)}$ extracted numerically using publicly available POWHEG implementation *[Hartanto, Jäger, Reina, Wackeroth'15]*

$$\frac{d\tilde{\hat{\sigma}}^{(\mathrm{NNLL})}_{ij\to klB}}{dQ^2}(N, Q^2, \{m^2\}, \mu_{\mathrm{F}}^2, \mu_{\mathrm{R}}^2) = \left(1 + \frac{\alpha_{\mathrm{s}}(\mu_{\mathrm{R}}^2)}{\pi} \mathcal{C}^{(1)}_{ij\to klB}(Q^2, \{m^2\}, \mu_{\mathrm{F}}^2, \mu_{\mathrm{R}}^2)\right)$$
$$\times \, \mathrm{Tr}\left[\boldsymbol{\sigma}^{(0)}_R(Q^2, \{m^2\}, \mu_{\mathrm{R}}^2)\, \bar{\mathbf{U}}_R(N+1, Q^2, \{m^2\}, Q^2, \mu_{\mathrm{R}}^2)\, \tilde{\mathbf{S}}^{(0)}_R\right.$$
$$\left. \times \, \mathbf{U}_R(N+1, Q^2, \{m^2\}, Q^2, \mu_{\mathrm{R}}^2)\right] \Delta^i(N+1, Q^2, \mu_{\mathrm{F}}^2, \mu_{\mathrm{R}}^2)\Delta^j(N+1, Q^2, \mu_{\mathrm{F}}^2, \mu_{\mathrm{R}}^2)$$

Resummation-improved NNLL+NLO total cross section

- NNLL resummed expression has to be matched with the full NLO result

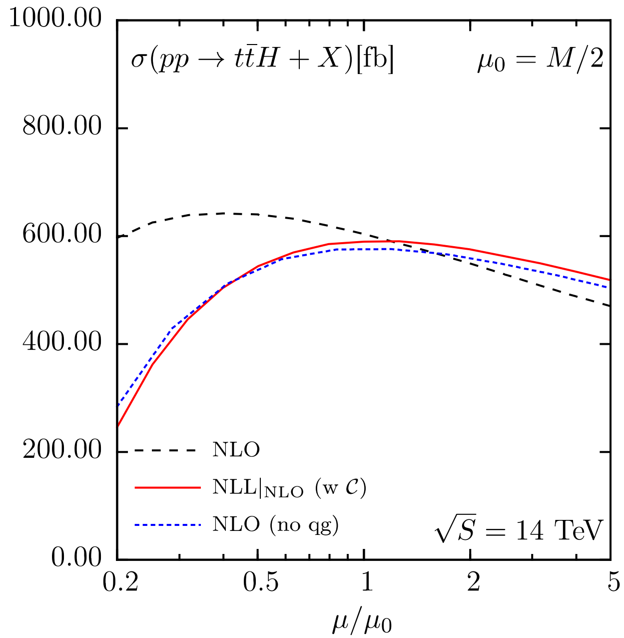
$$\sigma^{(\mathrm{match})}_{h_1 h_2 \to kl}(\rho, \{m^2\}, \mu^2) = \sum_{i,j=q,\bar{q},g} \int_{C_{\mathrm{MP}}-i\infty}^{C_{\mathrm{MP}}+i\infty} \frac{dN}{2\pi i}\, \rho^{-N}\, f^{(N+1)}_{i/h_1}(\mu^2)\, f^{(N+1)}_{j/h_2}(\mu^2)$$
$$\times \left[\hat{\sigma}^{(\mathrm{res},N)}_{ij\to kl}(\{m^2\},\mu^2) - \hat{\sigma}^{(\mathrm{res},N)}_{ij\to kl}(\{m^2\},\mu^2)\Big|_{NLO} \right]$$
$$+ \sigma^{\mathrm{NLO}}_{h_1 h_2 \to kl}(\rho, \{m^2\}, \mu^2)\,,$$

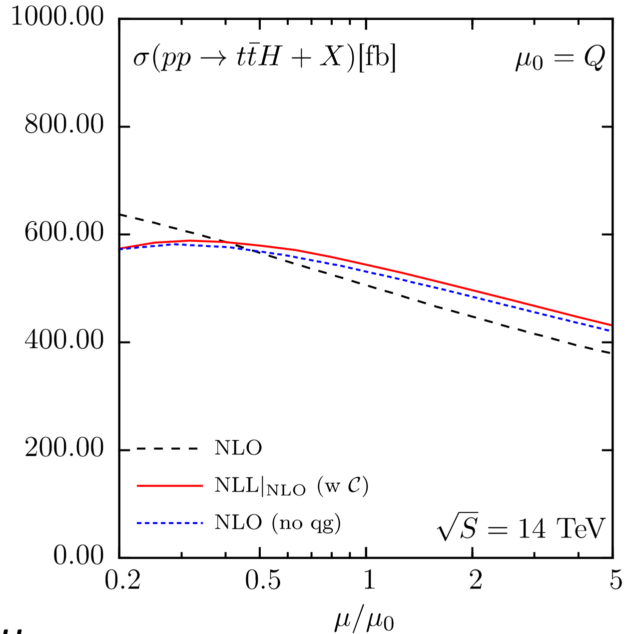
- Inverse Mellin transform evaluated using a contour in the complex *N* space according to 'Minimal Prescription' *[Catani, Mangano, Nason Trentadue'96]*

Numerical Results

- NLO cross sections evaluated with aMC@NLO *[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro]*
- Mellin transform of the hard function, including leading order terms, taken numerically
- Pdfs: PDF4LHC15_100 sets
- Total cross sections obtained by integrating $d\sigma/dQ^2$ resummed in the invariant mass kinematics over Q
- Results for two central scale choices: $\mu_F = \mu_R = \mu_0 = Q$, $\mu_F = \mu_R = \mu_0 = m_t + m_H/2$, covering a span of relevant scales
- Theory error due to scale variation calculated using the 7-point method, based on the minimum and maximum values of results for $(\mu_F/\mu_0, \mu_R/\mu_0)$ $=(0.5,0.5),(0.5,1),(1,0.5),(1,1),(1,2),(2,1),(2,2)$

NLO vs. $NLL_{|NLO}$



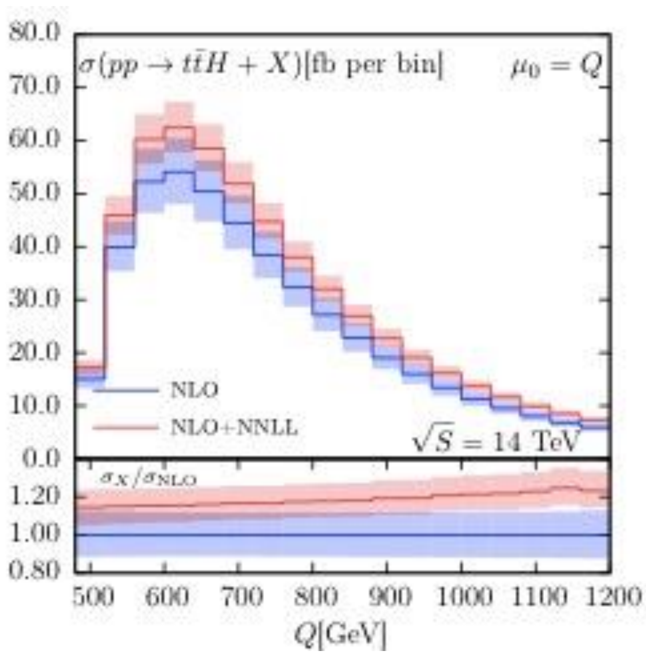


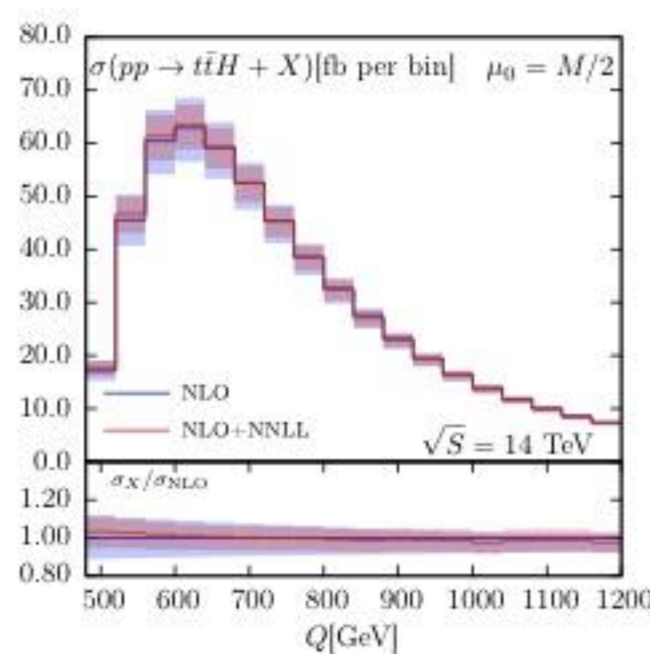
$\mu_F = \mu_R = \mu$

- *qg* channel quite significant numerically, especially in the context of the scale dependence → appears first at NLO, i.e. suppressed w.r.t. other channels, no resummation performed for it

- $NLL_{|NLO}$ agrees well with NLO w/o *qg* channel → approximation of hard-matching coefficient by the colour-channel weighted average works well

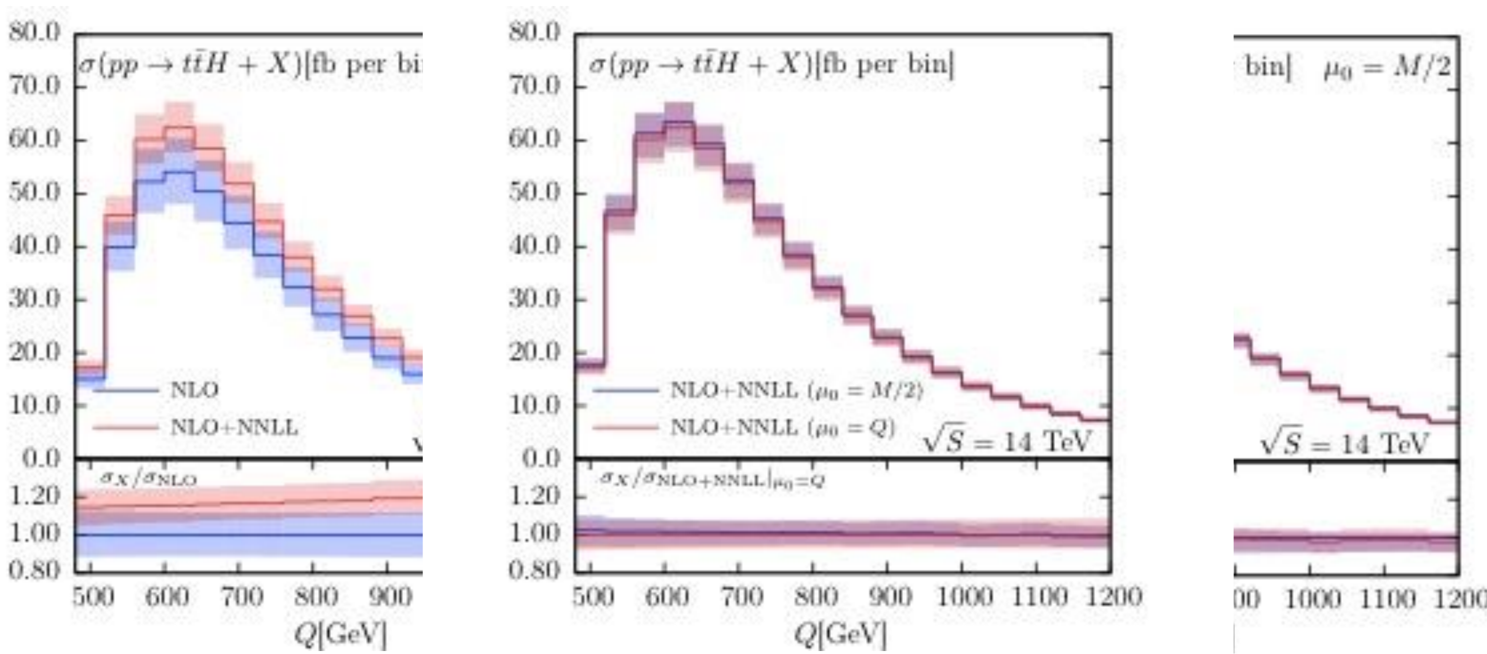
Invariant Mass Distribution





- NNLL+NLO distributions for the two scale choices very close, NLO results differ visibly → K_{NNLL} factors also different
- NNLL+NLO error band slightly narrower than NLO (7-point method)

Invariant Mass Distribution



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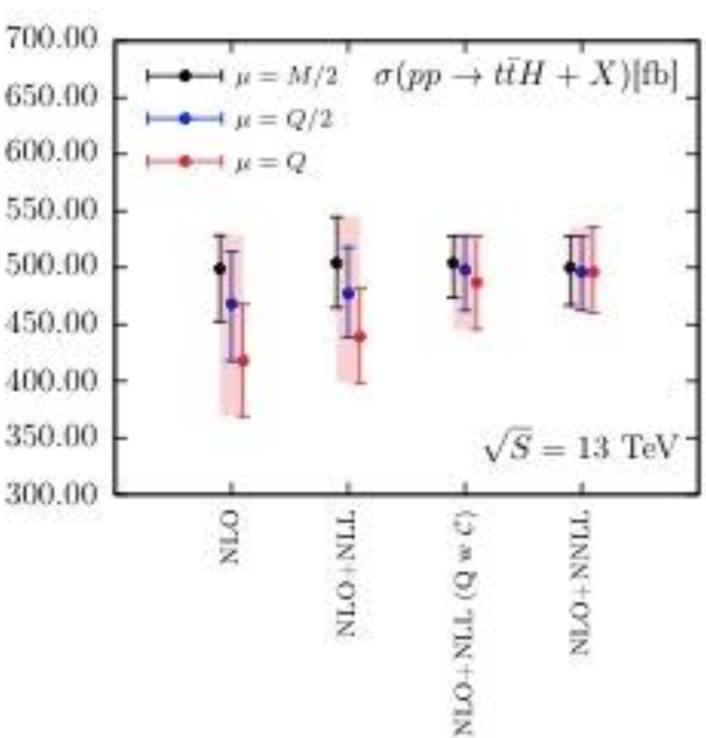
Total Cross Section

$\sqrt{S}$ [TeV]	μ_0	NLO [fb]	NLO+NLL[fb]	NLO+NLL with C [fb]	NLO+NNLL[fb]
13	Q	$418^{+11.9\%}_{-11.7\%}$	$439^{+9.8\%}_{-9.2\%}$	$487^{+8.4\%}_{-8.5\%}$	$496^{+8.0\%}_{-7.3\%}$
	$Q/2$	$468^{+9.8\%}_{-10.7\%}$	$477^{+8.6\%}_{-8.0\%}$	$498^{+6.1\%}_{-7.2\%}$	$496^{+6.4\%}_{-6.8\%}$
	$M/2$	$499^{+5.9\%}_{-9.3\%}$	$504^{+8.1\%}_{-7.8\%}$	$504^{+4.8\%}_{-6.1\%}$	$500^{+5.6\%}_{-6.6\%}$



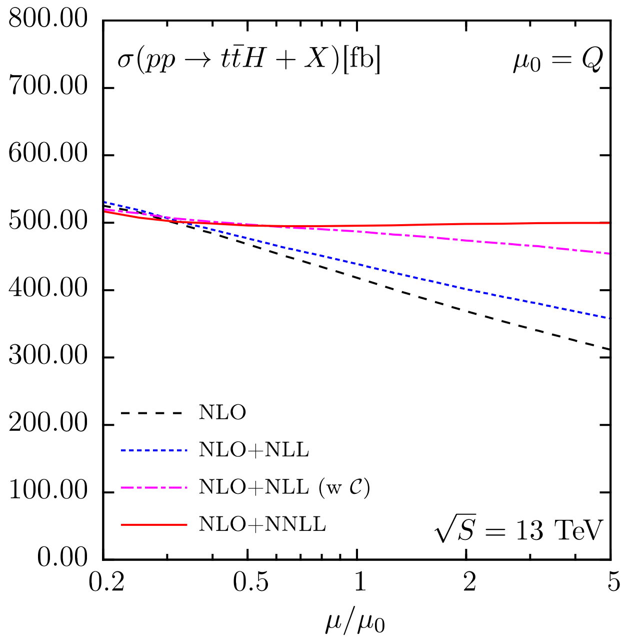
- Compared to NLO, remarkable stability of NLO+NNLL

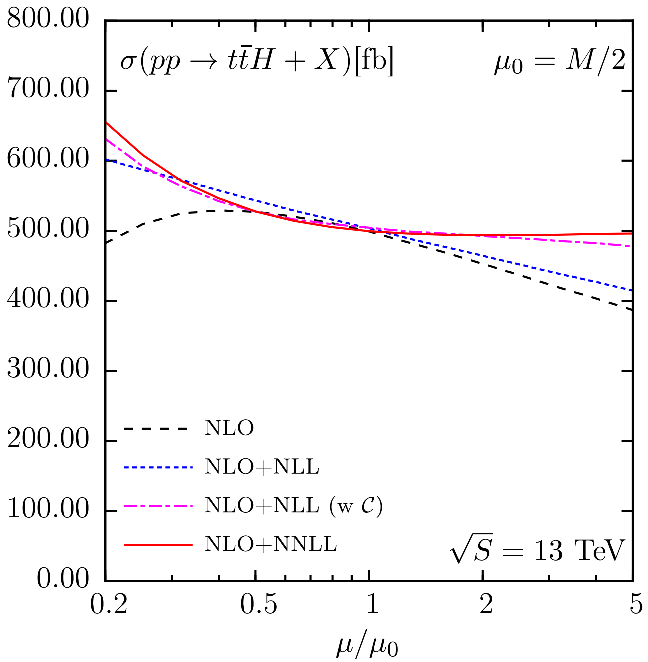
$$\sigma_{\rm NLO+NNLL} = 497^{+7.7\%+3.0\%}_{-7.6\%-3.0\%}\ {\rm fb},$$

- Stability improves with increasing accuracy of resummation
- Reduction of the theory scale error
- "Best" NNLL+NLO prediction in agreement with NLO at $\mu_0 = M/2$
- Significant effect from the hard matching coefficient

SCALE DEPENDENCE OF THE TOTAL CROSS SECTION

$\mu_F = \mu_R = \mu$



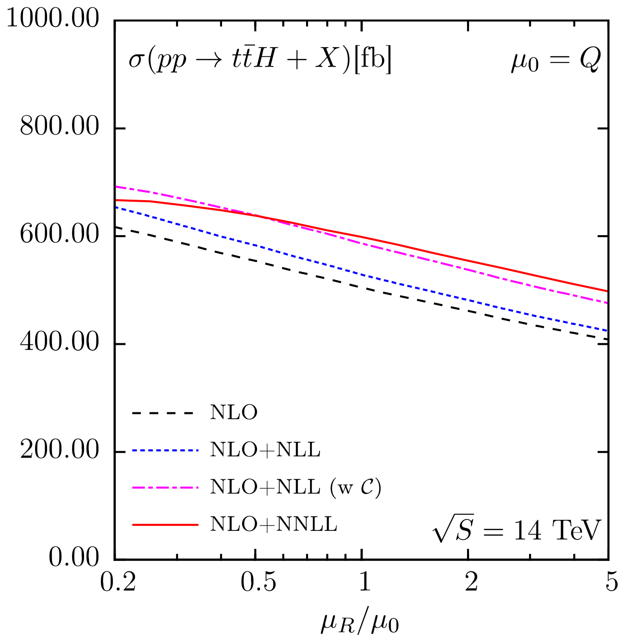


- For μ_0 = *Q,* decrease in scale dependence with increasing accuracy. NLO+NNLL scale dependence in the $\mu_0/2$ - $2\mu_0$ range of order 1%.

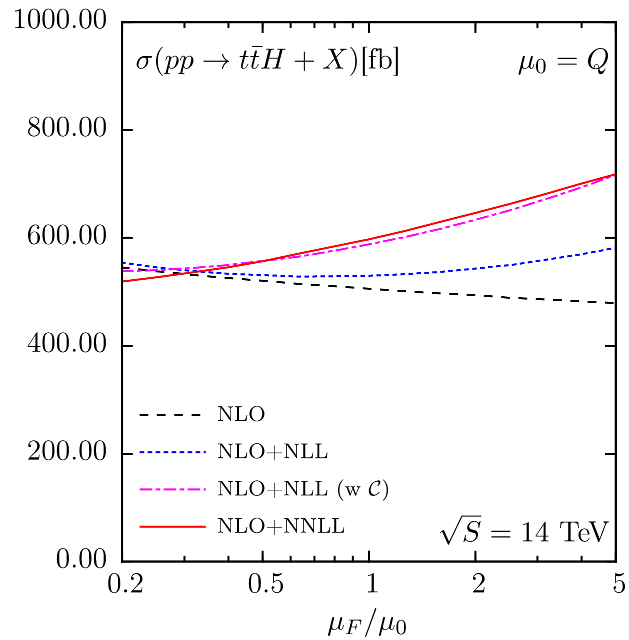
- For μ_0 = *M/2,* mostly similar behaviour. The rise at the small scale due to (unresummed) contributions from the *qg* channel

SCALE DEPENDENCE OF THE TOTAL CROSS SECTION

$\mu_F = \mu_0 = Q$



$\mu_R = \mu_0 = Q$



- Apparent cancellations between μ_F and μ_R scale dependence
- No significant change in dependence on μ_F while increasing the accuracy: α_s running effect
- μ_F dependence modified by the hard-matching coefficient

→ 7-point method

SUMMARY

- Applications of threshold resummation techniques to the class of 2->3 processes expected to lead to interesting phenomenology and improved precision of the predictions
- Current knowledge makes it feasible to develop such applications of the standard resummation approach at NNLL accuracy, at least in the invariant mass and absolute threshold kinematics
- Application: NNLL+NLO predictions for the process $pp \to t\bar{t}H$ calculated in the 3-particle invariant mass kinematics
- Remarkable stability of the NNLL+NLO differential and total cross sections w.r.t. scale variation; improving stability with growing accuracy
- Reduction (albeit small using the 7-point method) of the theory error due to scale variation