

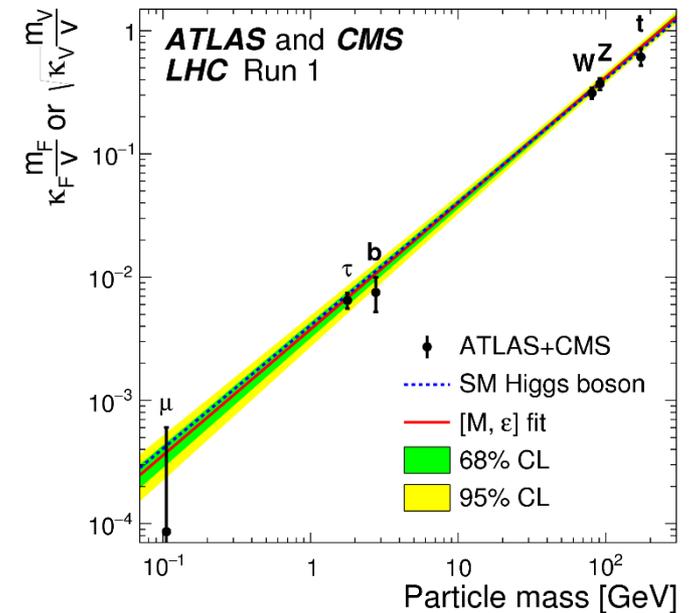
Top-bottom interference effects in Higgs plus jet production at the LHC

Chris Wever (Karlsruhe Institute of Technology)

In collaboration with: J. Lindert , K. Melnikov, L. Tancredi

Introduction

- Studies of Higgs boson properties are a crucial part of LHC physics program
- One important focus is the study of *Higgs couplings* to other particles
- After high-luminosity run it is expected that major Higgs couplings can be constrained to few percent level
- Higgs couplings to light generation quarks practically unconstrained
- Current bounds from global fits to inclusive Higgs production cross section and exclusive Higgs decays



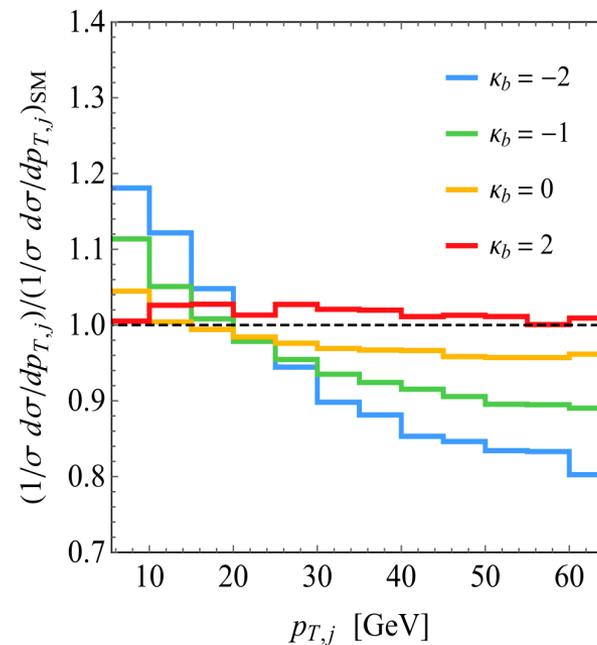
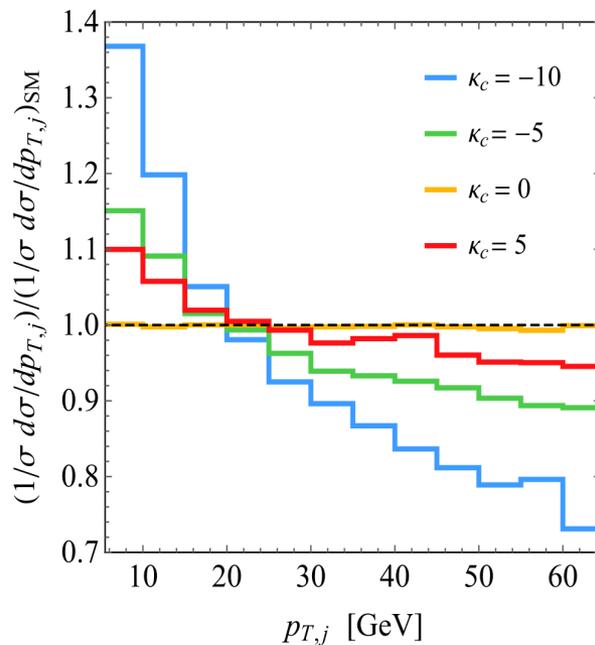
[arXiv:1606.02266]

2

Introduction: $H + j$ production

- Non-trivial Higgs transverse momentum ($p_{T,H}$) distribution generated when extra jet is radiated: $H + j$
- Shape of $p_{T,H}$ distribution may put stronger constraints on light-quark Yukawa couplings

[Bishara, Monni et al '16;
Soreq et al '16]



$$\kappa_j = y_j / y_{j,SM}$$

- Bounds expected from HL-LHC $\kappa_c \in [-0.6, 3.0]$ $\kappa_b \in [0.7, 1.6]$

[Bishara, Monni et al '16]



Reliable theoretical predictions for $H + j$ differential cross section required

3

Bottom corrections

- QCD corrections to Higgs production known to be large, about hundred percent at NLO
- Inclusive production cross section at N3LO to few percent accuracy, using a point-like, top-loop induced ggH coupling (HEFT)

[Anastasiou, Duhr, Furlan, Mistlberger et al'16]



- At $p_{T,H}$ larger than twice the bottom mass, the ggH coupling is not point-like

- Bottom corrections **naively suppressed** compared to top by factor

$$y_t y_b m_b/m_h \sim y_t m_b^2/m_h^2 \sim 10^{-3}$$

- Bottom amplitude contains large Sudakov-like logarithms, **suppressed actually by**

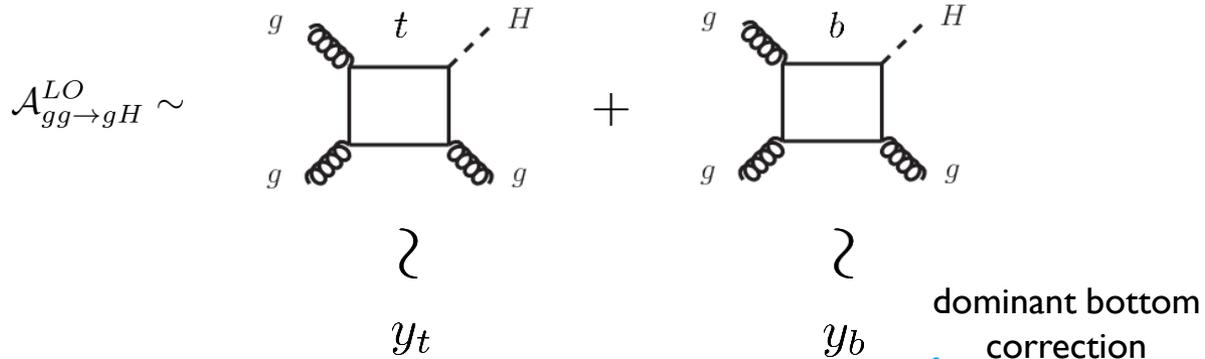
$$m_b^2/m_h^2 (\log^2(m_h^2/m_b^2), \log^2(p_\perp^2/m_b^2)) \sim 10^{-1}$$

- In fact, LO bottom contribution $\sim 5\text{-}10\%$ of LO top contribution at $p_\perp \in [10, 40]$ GeV

4

Top-bottom interference

- Higgs plus jet production at LHC proceeds largely through quark loops



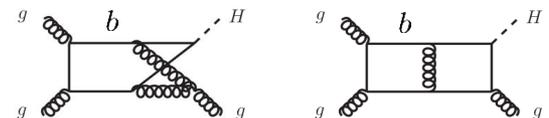
- Differential cross section $d\sigma \sim |\mathcal{A}|^2 \rightarrow d\sigma = d\sigma_{tt} + d\sigma_{tb} + d\sigma_{bb}$, $d\sigma_{ij} \sim \mathcal{O}(y_i y_j)$

$y_j \sim m_j$
 $m_b = 4.5 \text{ GeV}$
 $m_t = 173 \text{ GeV}$

- NLO correction to $d\sigma_{tb}$ may be large, as observed also for top contribution $\sim 40\%$, and relevant for reaching percent accuracy in differential cross section

- Two-loop adds extra factor of

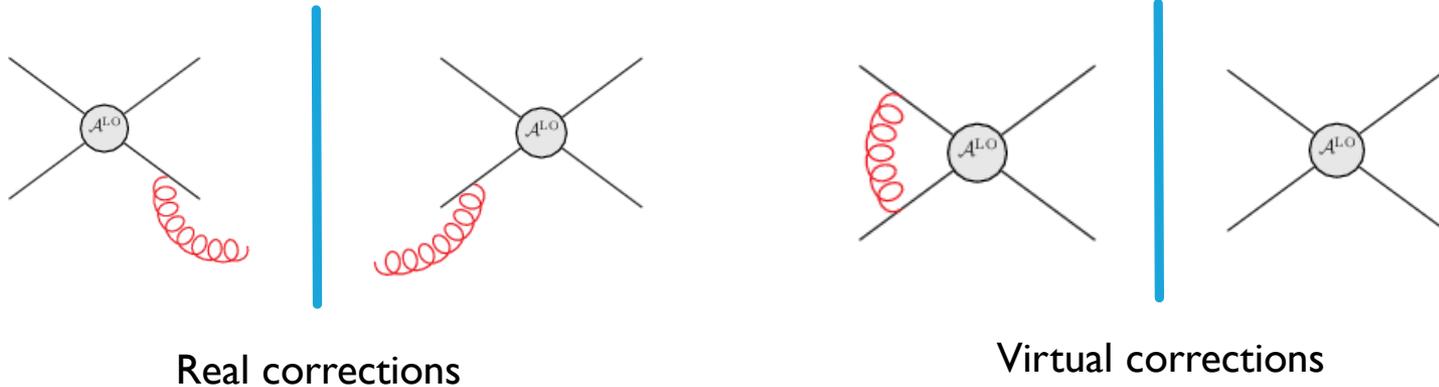
$\log^2(p_{\perp}^2/m_b^2), \log^2(m_h^2/m_b^2)$



➔
Quantitatively, how large are the bottom corrections at NLO?

Calculation at NLO

- Real (2 to 3) and virtual (2 to 2) contributions need to be combined, very well understood at NLO



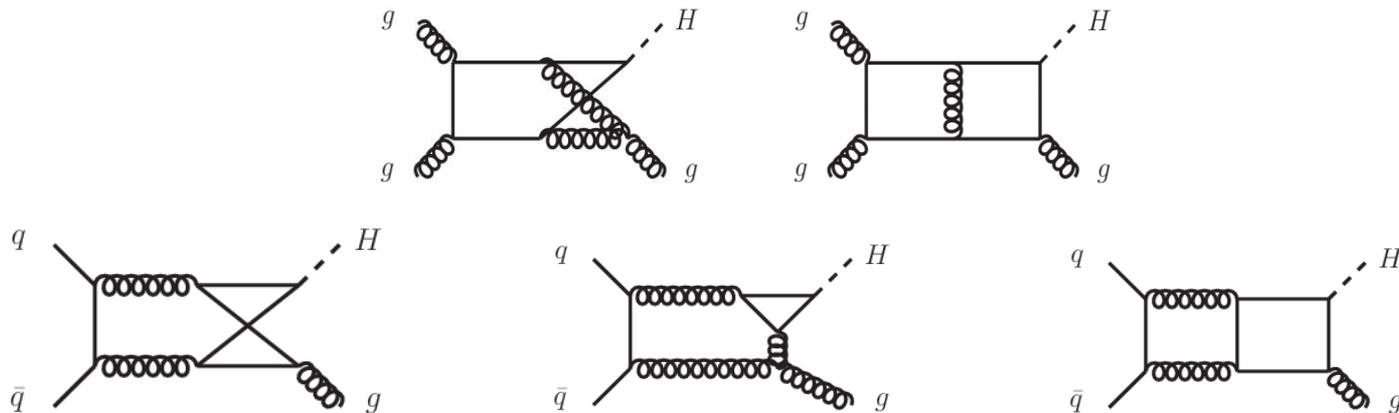
- Peculiarity in this case: LO is already 1-loop
- Real corrections receive contributions from kinematical regions where one parton become soft or collinear to another parton, so a numerically stable approach required
- Real corrections computed in **Openloops** with exact top, bottom mass dependence
- One new ingredient are two-loop virtual corrections

[Cascioli, Lindert,
Pozzorini et al '12-'17;
Denner et al '03-'17]

Virtual corrections

$$d\sigma_{tb}^{\text{virt}} \sim \text{Re} \left[\frac{\alpha_s}{2\pi} (A_t^{\text{NLO}} A_b^{\text{LO}*} + A_t^{\text{LO}} A_b^{\text{NLO}*}) \right]$$

- Typical two-loop Feynman diagrams are:



- Exact mass dependence in two-loop Feynman Integrals currently out of reach [planar diagrams: Bonciani et al '16]

Scale hierarchy: $m_b \ll p_\perp, m_h \ll m_t$ ➔

Top: Infinite top mass limit, well known how to be treated, expanded systematically via effective Lagrangian (HEFT)

Bottom: Small bottom mass expansion is different because loop is resolved ➔ new methods required

Two-loop bottom amplitudes expanded in bottom mass with differential equation method

[Mueller & Ozturk '15;
Melnikov, Tancredi,
CW '16-'17]

Computing virtual bottom amplitudes

- Virtual amplitude made up of complicated two-loop tensor Feynman integrals
- Powerful tool for scalar integrals: IBP reduction to minimal set of *Master Integrals (MI)*

→ project amplitude onto form factors

$$\mathcal{A}_{H \rightarrow ggg}(p_1^{a_1}, p_2^{a_2}, p_3^{a_3}) = f^{a_1 a_2 a_3} \epsilon_1^\mu \epsilon_2^\nu \epsilon_3^\rho (F_1 g^{\mu\nu} p_2^\rho + F_2 g^{\mu\rho} p_1^\nu + F_3 g^{\nu\rho} p_3^\mu + F_4 p_3^\mu p_1^\nu p_2^\rho)$$

- Form factors F_i expressed in terms of scalar integrals

Three families flashing by

$$\mathcal{I}_{\text{top}}(a_1, a_2, \dots, a_8, a_9) = \int \frac{\mathcal{D}^d k \mathcal{D}^d l}{[1]^{a_1} [2]^{a_2} [3]^{a_3} [4]^{a_4} [5]^{a_5} [6]^{a_6} [7]^{a_7} [8]^{a_8} [9]^{a_9}}$$

Prop.	Topology PL1	Topology PL2	Topology NPL
[1]	k^2	$k^2 - m_b^2$	$k^2 - m_b^2$
[2]	$(k - p_1)^2$	$(k - p_1)^2 - m_b^2$	$(k + p_1)^2 - m_b^2$
[3]	$(k - p_1 - p_2)^2$	$(k - p_1 - p_2)^2 - m_b^2$	$(k - p_2 - p_3)^2 - m_b^2$
[4]	$(k - p_1 - p_2 - p_3)^2$	$(k - p_1 - p_2 - p_3)^2 - m_b^2$	$l^2 - m_b^2$
[5]	$l^2 - m_b^2$	$l^2 - m_b^2$	$(l + p_1)^2 - m_b^2$
[6]	$(l - p_1)^2 - m_b^2$	$(l - p_1)^2 - m_b^2$	$(l - p_3)^2 - m_b^2$
[7]	$(l - p_1 - p_2)^2 - m_b^2$	$(l - p_1 - p_2)^2 - m_b^2$	$(k - l)^2$
[8]	$(l - p_1 - p_2 - p_3)^2 - m_b^2$	$(l - p_1 - p_2 - p_3)^2 - m_b^2$	$(k - l - p_2)^2$
[9]	$(k - l)^2 - m_b^2$	$(k - l)^2$	$(k - l - p_2 - p_3)^2$

Computing virtual bottom amplitudes

- Virtual amplitude made up of complicated two-loop tensor Feynman integrals
- Powerful tool for scalar integrals: IBP reduction to minimal set of **Master Integrals (MI)**

→ project amplitude onto form factors

$$\mathcal{A}_{H \rightarrow ggg}(p_1^{a_1}, p_2^{a_2}, p_3^{a_3}) = f^{a_1 a_2 a_3} \epsilon_1^\mu \epsilon_2^\nu \epsilon_3^\rho (F_1 g^{\mu\nu} p_2^\rho + F_2 g^{\mu\rho} p_1^\nu + F_3 g^{\nu\rho} p_3^\mu + F_4 p_3^\mu p_1^\nu p_2^\rho)$$

- Form factors F_i expressed in terms of scalar integrals

- Integration by parts (IBP) identities $\int \left(\prod_i d^d k_i \right) \frac{\partial}{\partial k_j^\mu} (v^\mu I) = \text{Boundary term} \stackrel{DR}{=} 0$

- Reduce to set of *MI* is very difficult, naïve reduction with public codes failed
- Performed in steps: top topology to subtopology reduction with Form+Reduze, then FIRE

$$\mathcal{I}_{a_1 \dots a_n}(s) = \sum_{(b_1 \dots b_n) \in \text{Master Integrals}} \text{Rational}_{a_1 \dots a_n}^{b_1 \dots b_n}(s, d) \text{MI}_{b_1 \dots b_n}(s)$$

MI with DE method for small m_b (1/2)

- System of partial differential equations (**DE**) in m_b, s, t, m_h^2 with IBP relations
$$\frac{\partial}{\partial \tilde{s}_k} \vec{\mathcal{I}}^{MI}(\tilde{s}, \epsilon) \stackrel{\text{IBP}}{=} \overline{\overline{M}}_k(\tilde{s}, \epsilon) \cdot \vec{\mathcal{I}}^{MI}(\tilde{s}, \epsilon)$$

- Interested in m_b expansion of Master integrals I^{MI}



expand homogeneous matrix M_k in small m_b

Step I: solve DE in m_b

- Solve m_b DE with following ansatz

$$\mathcal{I}_i^{MI}(m_b^2, s, t, m_h^2, \epsilon) = \sum_{ijkn} c_{ijkn}(s, t, m_h^2, \epsilon) \left(\frac{m_b^2}{m_h^2} \right)^{j-k\epsilon} \log^n \left(\frac{m_b^2}{m_h^2} \right)$$

- Plug into m_b DE and get constraints on coefficients c_{ijkn}
- c_{i000} is $m_b = 0$ solution (hard region) and has been computed before

[Gehrmann & Remiddi '00]

MI with DE method for small m_b (2/2)

- Ansatz
$$\mathcal{I}_i^{MI}(m_b^2, s, t, m_h^2, \epsilon) = \sum_{ijkn} c_{ijkn}(s, t, m_h^2, \epsilon) \left(\frac{m_b^2}{m_h^2}\right)^{j-k\epsilon} \log^n \left(\frac{m_b^2}{m_h^2}\right)$$

Step 2: solve s, t, m_h^2 DE for $c_{ijkn}(s, t, m_h^2)$

- Solution expressed in extensions of usual polylogarithms: *Goncharov Polylogarithms*
- After solving DE for unknown c_{ijkn} , we are left with unknown boundary constants that only depend on ϵ

Step 3: fix ϵ dependence

- Determination of most boundary constants in ϵ by imposing that unphysical singularities in solution vanish
- Other constants in ϵ fixed by matching solution of DE to Master integrals computed via various methods (Mellin-Barnes, expansion by regions, numerical fits) in a specific point of s, t, m_h^2

Step 4: numerical checks with FIESTA

Numerical setup

[Lindert, Melnikov, Tancredi, CW '17]

- LHC 13 TeV
- PDF set and associated strong coupling constant: NNPDF3.0_lo for LO and NNPDF3.0_nlo for NLO
- Central scale is dynamical:

$$\mu_r = \mu_f = \mu_0 = H_T/2, \quad H_T = \sqrt{m_H^2 + p_\perp^2} + \sum_j p_{\perp,j}$$

$$m_H = 125 \text{ GeV}, \quad m_t = 173.2 \text{ GeV}$$

Theory uncertainties considered

- Scale variation: $\mu = \{1/2, 2\} * \mu_0$
- Large ambiguity in bottom mass scheme: appropriate renormalization scheme for m_b from Yukawa coupling is MSbar scheme at $\mu \sim m_h$, while scheme for m_b from helicity flip might require on-shell bottom mass scheme instead. Two bottom mass schemes considered:

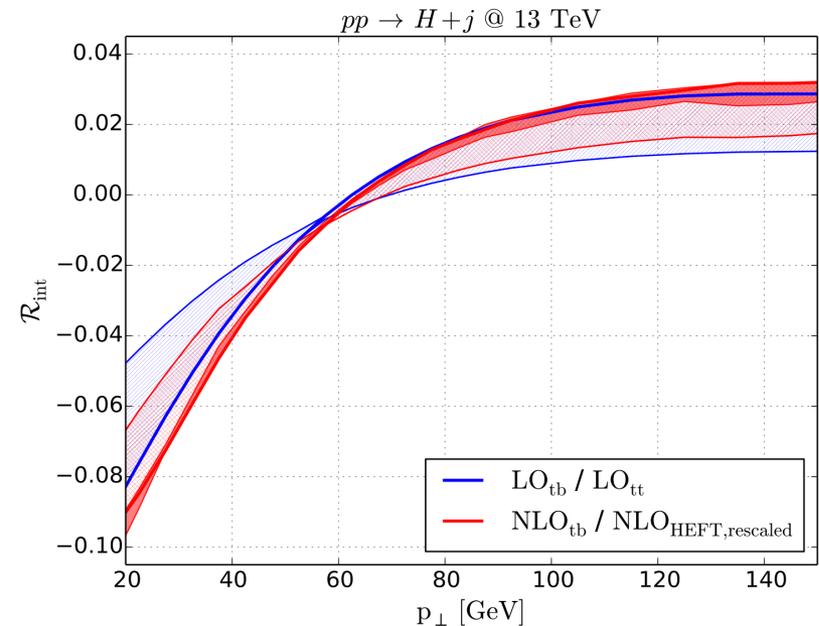
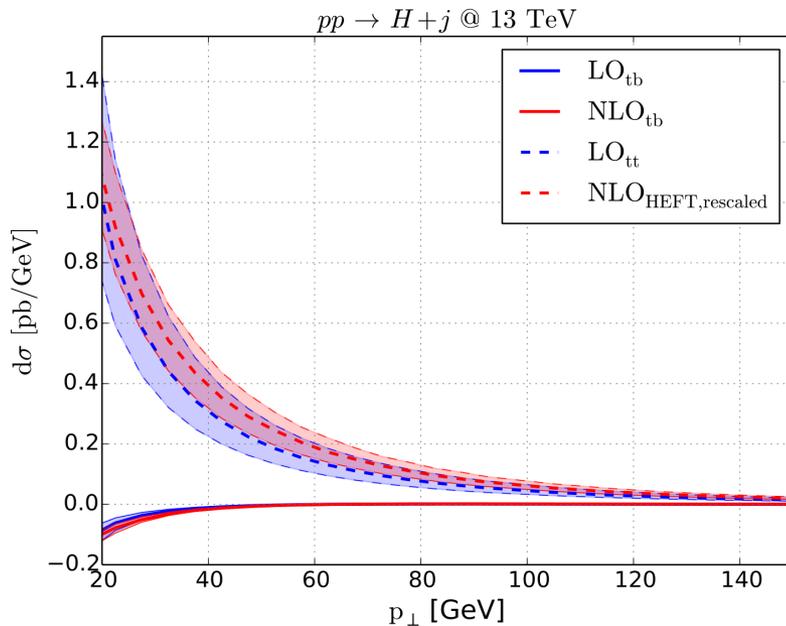
$$m_b^{\text{OS}} = 4.75 \text{ GeV}$$

$$m_b^{\overline{\text{MS}}}(\mu = 100) = 3.07 \text{ GeV}$$

Higgs transverse momentum distribution

II

[Lindert, Melnikov, Tancredi, CW '17]

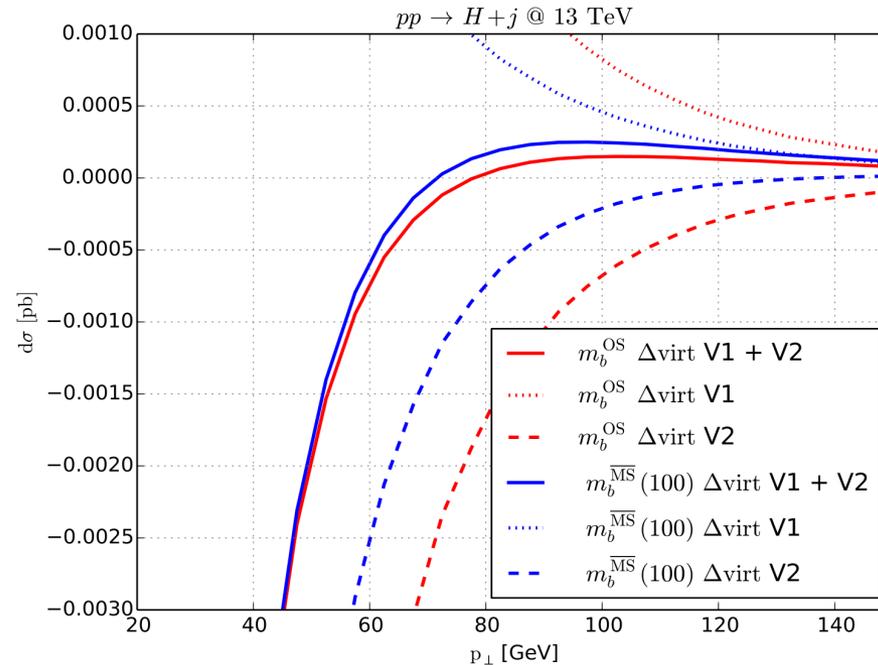


- Top-bottom interference at $p_{T,H}=30 \text{ GeV}$: -6% at LO and -7% at NLO
- Large relative corrections to top-bottom interference \sim relative corrections to top-top $\sim 40\%$
- Large mass renormalization-scheme ambiguity
- At small $p_{T,H}$ the ambiguity is reduced by a factor of two at NLO; less pronounced at larger $p_{T,H}$

V1:NLO(t)xLO(b) vs V2:LO(t)xNLO(b)

12

[Lindert, Melnikov, Tancredi, CW '17]



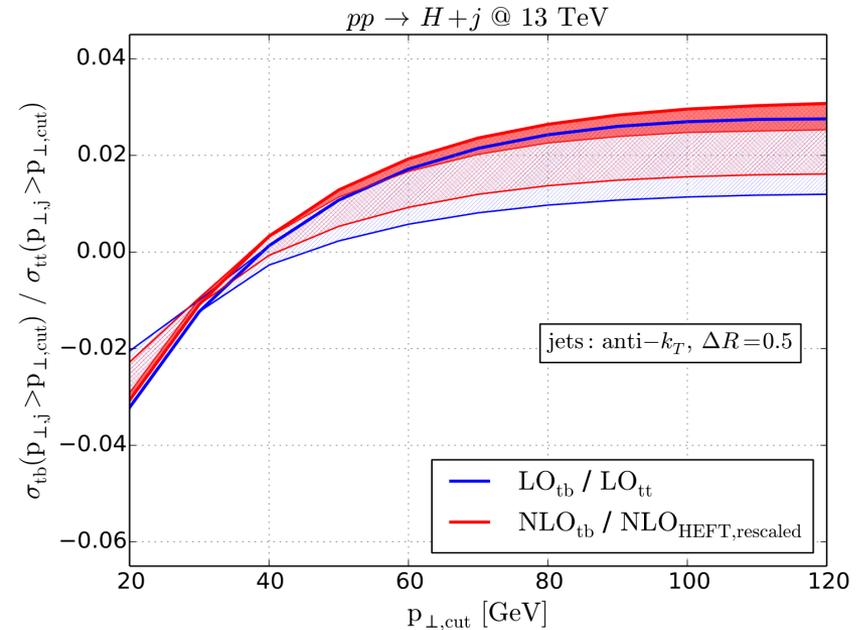
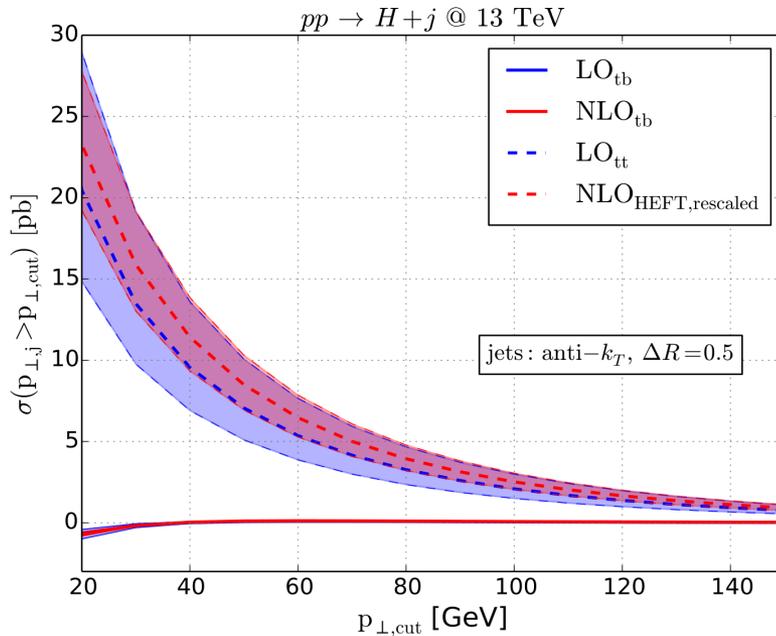
$$d\sigma_{tb}^{\text{virt}} \sim \text{Re} \left[A_t^{\text{LO}} A_b^{\text{LO}*} + \frac{\alpha_s}{2\pi} \left(\underbrace{A_t^{\text{NLO}} A_b^{\text{LO}*}}_{\text{V1}} + \underbrace{A_t^{\text{LO}} A_b^{\text{NLO}*}}_{\text{V2}} \right) \right]$$

- Two contributions enter with opposite signs
- V2 is dominant at low $p_{T,H} \sim 20\text{-}50 \text{ GeV}$ which reduces mass scheme ambiguity
- At large $p_{T,H}$ V1 \sim V2 and V1 represents LO bottom mass scheme ambiguity

Total Higgs plus jet cross section

[Lindert, Melnikov, Tancredi, CW '17]

- Total integrated cross section as function of threshold on jet $p_{T,j}$



$$\sigma_{tb}/\sigma_{tt}(p_{T,j} > 20, 30, 40, 50)_{LO} = -3.2, -1.2, +0.1, 1.1\%$$

$$\sigma_{tb}/\sigma_{tt}(p_{T,j} > 20, 30, 40, 50)_{NLO} = -3.1, -1.1, +0.3, 1.3\%$$

- Total integrated NLO top-bottom interference contributes [-3% , 3%] of NLO top-top contribution
- Strong dependence on jet $p_{T,j}$ cut

Summary

- Fully differential NLO QCD corrections to top-bottom interference first time computed
- Two-loop integrals computed at first order in bottom mass expansion with DE method
- NLO bottom contribution $\sim [-10, -4]$ % of NLO top contribution at lower range of Higgs $p_{T,H}$
- Large relative NLO corrections to top-bottom interference similar to pure top NLO corrections $\sim 40\%$ for Higgs $p_{T,H}$ and rapidity distributions
- On-shell vs MSbar bottom mass: large renormalization scheme ambiguity. Reduced at small $p_{T,H} \sim 20-40$ GeV, but unchanged at larger $p_{T,H} \sim 60-100$ GeV

Outlook

Combine various contributions to get best $H + j$ prediction:

- Low $p_{T,H}$ -resummation
- NNLO HEFT corrections
- NLO top-bottom interference

[Grazzini et al '13; Banfi et al '14,16; Bagnaschi et al '15]

[Boughezal et al '14,15; Gehrmann et al '15,16]

[Lindert et al '17]

Backup slides

IBP reduction

[Melnikov, Tancredi, CW '16-'17]

- IBP reduction to Master Integrals

$$\mathcal{I}_{a_1 \dots a_n}(s) = \sum_{(b_1 \dots b_n) \in \text{Master Integrals}} \text{Rational}_{a_1 \dots a_n}^{b_1 \dots b_n}(s, d) \text{MI}_{b_1 \dots b_n}(s)$$

- Reduction very non-trivial: we were not able to reduce top non-planar integrals with $t = 7$ denominators with FIRE5/Reduze
- Reduction fails because coefficients multiplying MI become too large to simplify \sim hundreds of Mb of text

- Reduction for complicated $t=7$ non-planar integrals performed in two steps:

1) FORM code reduction:

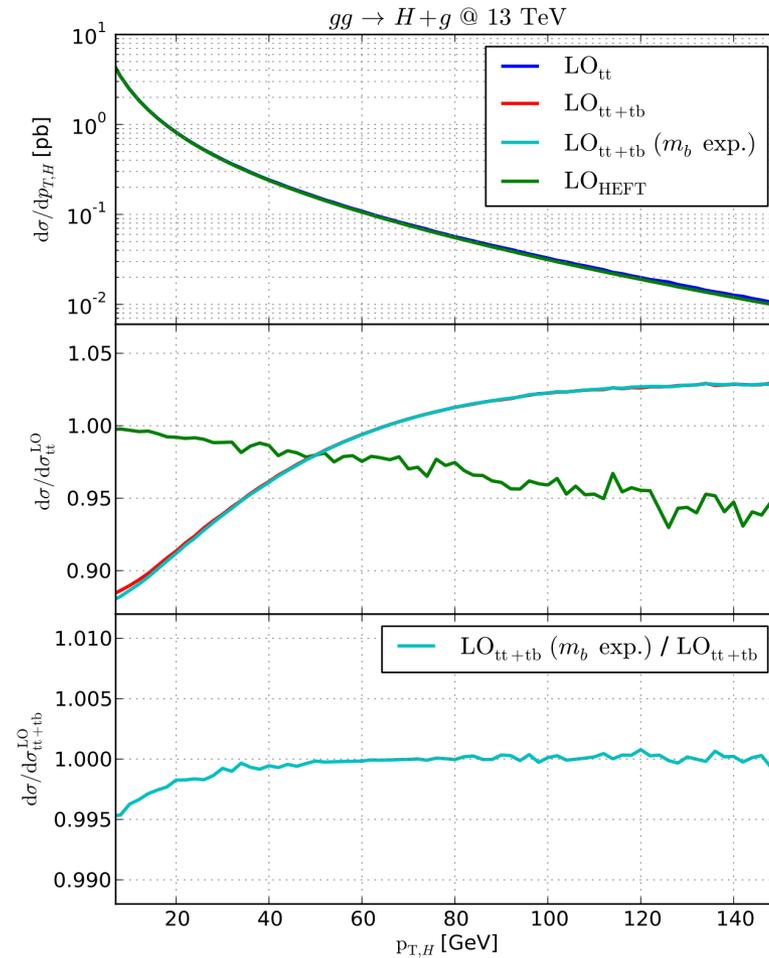
$$\mathcal{I}_{t=7}^{\text{NPL}} = \sum c_i \text{MI}_{t=7}^i + \sum d_i \mathcal{I}_{t=6}^i$$

2) Plug reduced integrals into amplitude, expand coefficients c_i, d_i in m_b

3) Reduce with FIRE/Reduze: $t = 6$ denominator integrals $\mathcal{I}_{t=6}$

- Exact m_b dependence kept at intermediate stages. Algorithm for solving IBP identities directly expanded in small parameter is still an open problem
- Expansion in m_b occurs at last step: solving with Master integrals with *differential equation method*

LO contributions



How useful is m_b expansion?

- NLO amplitudes require computing 2-loop Feynman integrals with massive quark loop
- If these integral are computed exactly in quark mass, results in very complicated functions

$$\begin{aligned}
 & \log(x_3 x_1^2 - x_1^2 + x_2 x_1 - 4x_3 x_1 + R_1(x_1)R_2(x_1)R_7(x)) , \\
 & \log(-x_2^2 + x_1 x_2 - x_1 x_3 x_2 + 2x_3 x_2 + 2x_1 x_3 + R_1(x_2)R_2(x_2)R_7(x)) , \\
 & \log(-x_3^2 x_1^2 + 3x_3 x_1^2 + 4x_3^2 x_1 - 4x_2 x_3 x_1 + R_1(x_3)R_5(x)R_6(x)x_1) , \\
 & \log(x_3 R_1(x_2)R_2(x_2) + x_2 R_1(x_3)R_2(x_3)) , \\
 & \log(x_1 R_1(x_2)R_2(x_2) + x_2 R_1(x_1)R_2(x_1)) , \\
 & \log(x_1 R_1(x_3)R_2(x_3) - R_1(x_1)R_1(x_3)R_5(x)) , \\
 & \log(x_3 R_1(x_1)R_2(x_1) - R_1(x_1)R_1(x_3)R_5(x)) , \\
 & \log(-x_2 R_1(x_1)R_2(x_1) + x_3 R_1(x_1)R_2(x_1) + x_1 R_3(x_3)R_4(x_3)) , \\
 & \log(-x_2 R_1(x_2)R_2(x_2) + x_3 R_1(x_2)R_2(x_2) + x_2 R_3(x_3)R_4(x_3)) , \\
 & \log(-x_2 R_1(x_3)R_2(x_3) + x_1 R_1(x_3)R_2(x_3) + x_3 R_3(x_1)R_4(x_1)) , \\
 & \log(-x_2 R_1(x_2)R_2(x_2) + x_1 R_1(x_2)R_2(x_2) + x_2 R_3(x_1)R_4(x_1)) , \\
 & \log(-x_3^2 x_1^2 + 3x_3 x_1^2 + 4x_3^2 x_1 - 3x_2 x_3 x_1 + R_1(x_1)R_1(x_3)R_5(x)R_7(x)) , \\
 & \log(x_2 R_1(x_1)R_1(x_3)R_5(x) - x_1 x_3 R_1(x_2)R_2(x_2)) , \\
 & \log(-x_2 x_3 + x_1 x_3 + R_1(x_2)R_2(x_2)x_3 - R_1(x_1)R_1(x_3)R_5(x)) .
 \end{aligned}$$

[planar diagrams: Bonciani et al '16]

$$\begin{aligned}
 R_1(x_1) &= \sqrt{-x_1}, R_1(x_3) = \sqrt{-x_3}, R_1(x_2) = \sqrt{-x_2}, \\
 R_2(x_1) &= \sqrt{4-x_1}, R_2(x_3) = \sqrt{4-x_3}, R_2(x_2) = \sqrt{4-x_2}, \\
 R_3(x_1) &= \sqrt{x_2-x_1}, R_3(x_3) = \sqrt{x_2-x_3}, \\
 R_4(x_1) &= \sqrt{x_2-x_1-4}, R_4(x_3) = \sqrt{x_2-x_3-4}, \\
 R_5(x) &= \sqrt{4x_2+x_1x_3-4(x_1+x_3)}, \\
 R_6(x) &= \sqrt{2x_3(-2x_2+x_1+2x_3)-x_1x_3^2-x_1}, \\
 R_7(x) &= \sqrt{2x_1x_3(x_2-x_1)+(x_2-x_1)^2+(x_1-4)x_1x_3^2}.
 \end{aligned}$$

- Starting from weight three not possible to express in terms of usual GPL's anymore
- Expanding in small bottom quark mass results in simple 2-dimensional harmonic polylogs

[Vermaseren,
Remiddi,
Gehrmann]

Real corrections with Openloops

- Channels for real contribution to Higgs plus jet at NLO

$$gg \rightarrow Hgg, gg \rightarrow Hq\bar{q}, qg \rightarrow Hqg, q\bar{q} \rightarrow Hgg, \dots$$

- Receives contributions from kinematical regions where one parton become soft or collinear to another parton
- This requires a delicate approach of these regions in phase space integral
- Openloops algorithm is publicly available program which is capable of dealing with these singular regions in a numerically stable way
- Crucial ingredient is tensor integral reduction performed via expansions in small Gram determinants: Collier

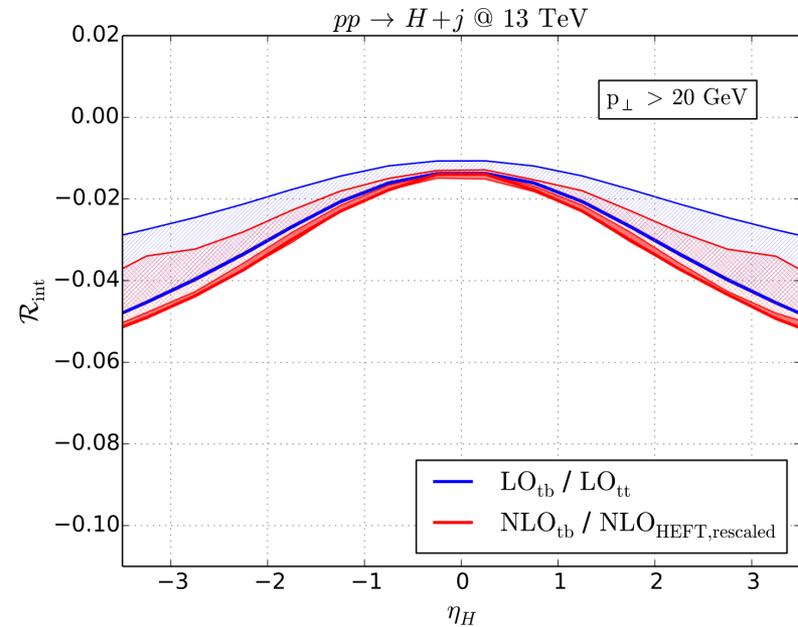
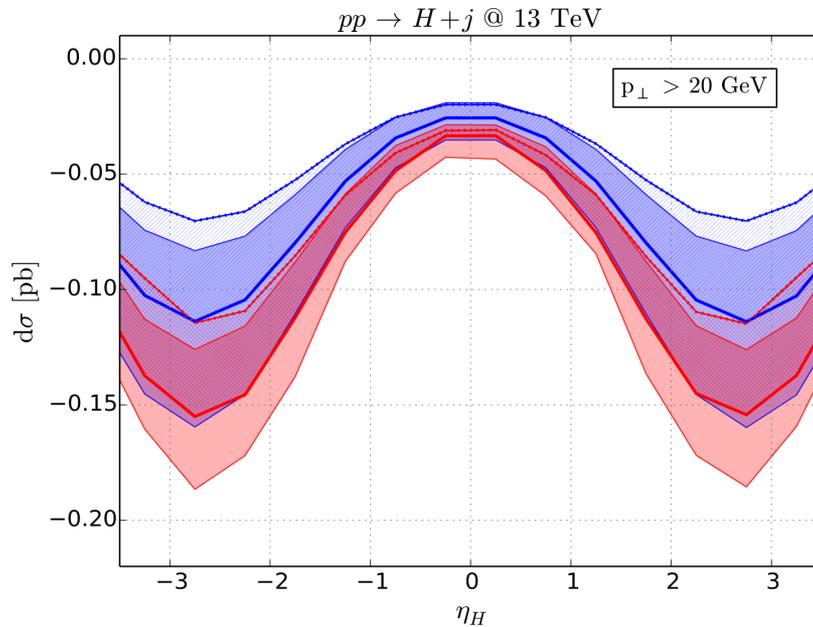
[Cascioli et al '12, Denner et al '03-'17]

- Exact top and bottom mass dependence kept throughout for both top-top and top-bottom contribution to differential cross section

Higgs pseudo-rapidity distribution

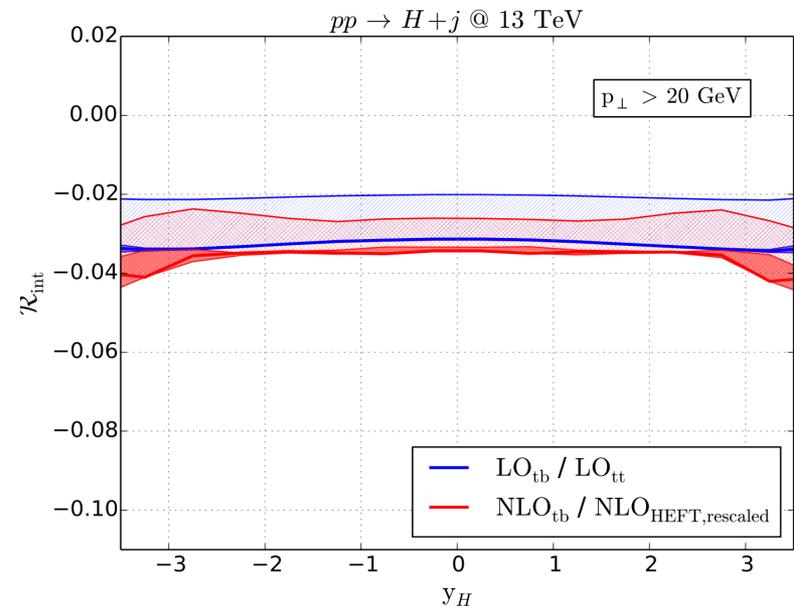
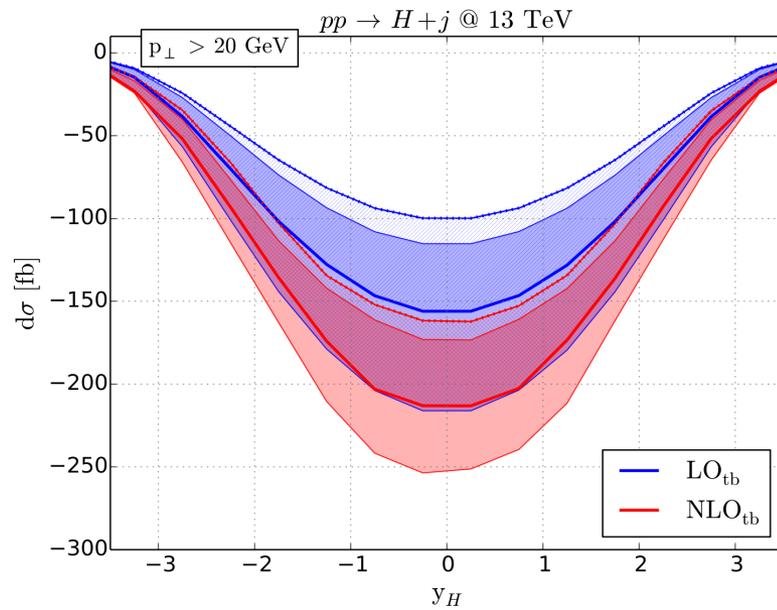
19

[Lindert, Melnikov, Tancredi, CW '17]



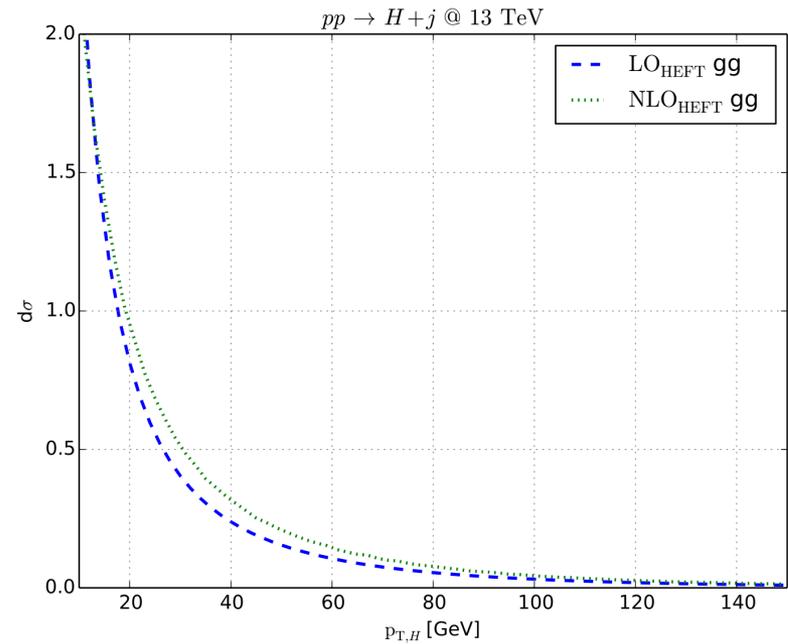
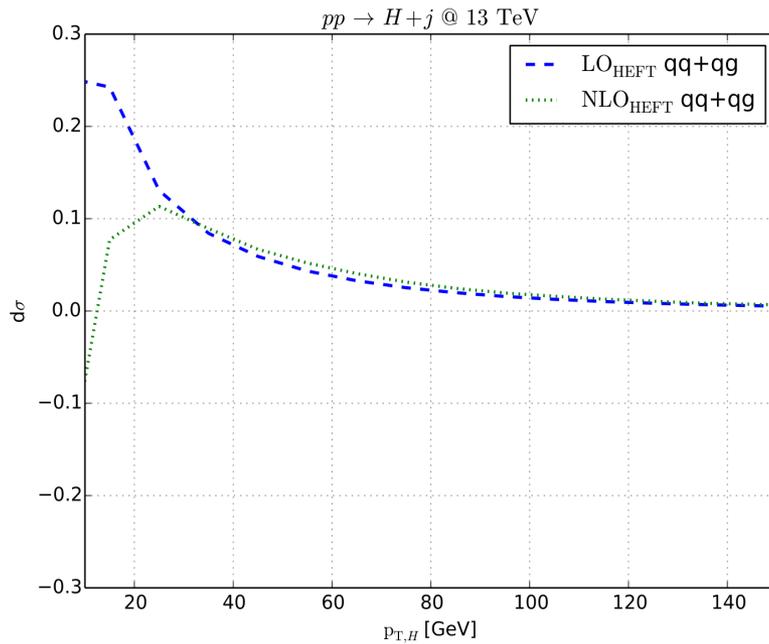
- Relative corrections to top-bottom interference \sim relative corrections to top-top
- At central rapidity (dominated by large $p_{T,H}$) mass scheme ambiguity similar between LO and NLO
- At larger absolute rapidity (dominated by small $p_{T,H}$) the mass scheme variation band is smaller for NLO

Higgs rapidity distribution



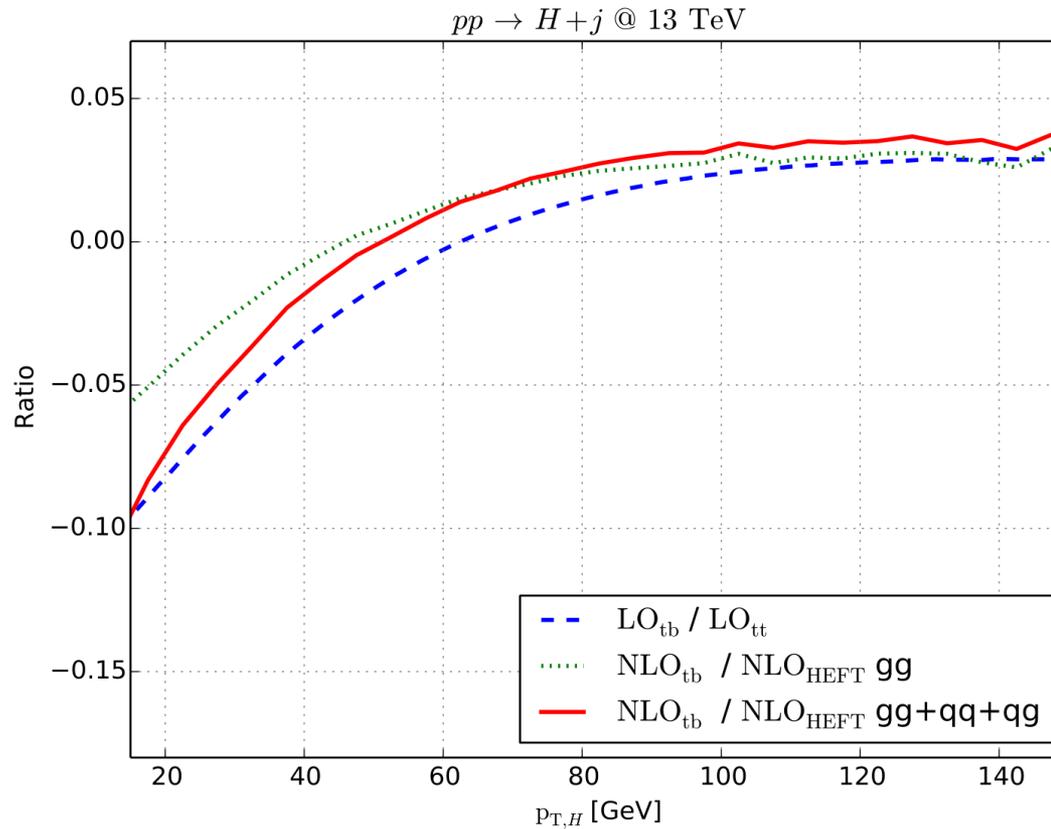
Channel contribution: $t\bar{t}$

21



- gg fusion channel dominates

Channel contribution: tb



- gg fusion channel dominates