Is Nature Natural?

Nathaniel Craig
UC, Santa Barbara

CERN-TH 08.08.18
IS HINCHLIFFE’S RULE TRUE? ·

Hinchliffe has asserted that whenever the title of a paper is a question with a yes/no answer, the answer is always no.

(Craig’s Conjecture: Talks are exempt from Hinchcliffe’s Rule)
The Parable of the Piscine Physicists

Proposed solutions:

Symmetry: Most incidence angles deflected by partner rocks.

Low Cutoff: Rocks only exist in a finite range of angles.

Piscatorial Reasoning: No fish can survive in rivers where rocks enter at higher incidence angles.

Apathy: Nature doesn’t care about finely-tuned rocks.
The Naturalness Strategy

An analogy

Classical E&M: electron + E,B fields

\[ \Delta E_C = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r_e} \]

\( (m_e c^2)_{\text{obs}} = (m_e c^2)_{\text{bare}} + \Delta E_C \)

Experimentally \( r_e \lesssim 10^{-18} \text{ cm} \Rightarrow \Delta E_C \gtrsim 100 \text{ GeV} \)

If so, \( 0.511 = -99999.489 + 100000.000 \text{ MeV} \)

To avoid fine-tuning, i.e. for the theory to be “natural”, need picture to change on scales below \( 2.8 \times 10^{-13} \text{ cm} \)
The Naturalness Strategy

An analogy

Weisskopf (1939)

$\Delta t \sim \frac{\hbar}{\Delta E} \sim \frac{\hbar}{(2m_e c^2)}$

$d \sim c \Delta t \sim 200 \times 10^{-13} \text{ cm}$

$\Delta E = \Delta E_C + \ldots$

$\Delta E = -\Delta E_C + \ldots$

$\Delta E = \Delta E_C - \Delta E_C + \frac{3\alpha}{4\pi} m_e c^2 \log \frac{\hbar}{m_e c r_e}$
The Naturalness Strategy

What about scalars?

Another divergence…

\[ m_{\pi^\pm}^2 - m_{\pi^0}^2 = \frac{3\alpha}{4\pi} \Lambda^2 \]

Consider the pion…

Given observed splitting, \textit{predict} scale of new physics:

\[ m_{\pi^\pm}^2 - m_{\pi^0}^2 = (35.5 \text{ MeV})^2 \Rightarrow \Lambda \lesssim 850 \text{ MeV} \]

Another (more predictive) example: \( K_L - K_S \) mass difference.
The “Hierarchy Problem”

The Higgs is an apparently elementary scalar

Assuming the Standard Model is valid down to some length scale $r_{\text{new}} \equiv \frac{\hbar c}{\Lambda}$ then we have

$$\Delta m_H^2 = \frac{\Lambda^2}{16\pi^2} \left[ -6y_t^2 + \frac{9}{4} g_2^2 + \frac{3}{4} g_Y^2 + 6\lambda + \ldots \right]$$

Expecting NP at $\Lambda$ such that $\Delta m_H^2 \sim m_H^2$ is a strategy.
What are the options?

- Quantum gravity cutoff
- Higgs sector cutoff
- Uninteresting RG flow to IR
- Standard Model (~unique vacuum)
- $m_H$ is not technically natural

$\Rightarrow$ hierarchy problem
The Usual Suspects

Low cutoff

Symmetry

Multiplicity
Thus far...
Not for lack of trying

Projected reach (c. 2012) full LHC run @ 14 TeV

CMS (e.g.) exclusion w/ 1/100 of full LHC data set
OH WOW!
PARADIGM
SHIFT!
The Unusual Suspects

Low cutoff

Symmetry

Multiplicity
Reasoning by analogy: localize graviton zero mode w/ randomly spaced & tensioned branes

But: not obvious that it works in detail

Related: An interesting source of exponential hierarchies for scalars, fermions, vectors in 4 & 5 dimensions [NC, Sutherland '17]

Disorder

[Rothstein '13]

Anderson localization: exponential localization from disorder.

Linear Dilaton / Continuum Clockwork

[Giudice, McCullough '16; Giudice, Kats, McCullough, Torre, Urbano '17; Antoniadis, Dimopoulos, Giveon '01; Antoniadis, Arvanitaki, Dimopoulos, Giveon '11; ....]

\[ ds^2 = e^{\frac{4}{3} k |y|} (\eta_{\mu\nu} dx^\mu dx^\nu + dy^2) \]

\[ S(y) = 2k |y| \]

[Giudice, Kats, McCullough, Torre, Urbano '17]
The Unusual Suspects

- Low cutoff

Symmetry

- Multiplicity
Discrete symmetries

E.g. “Twin Higgs” [Chacko, Goh, Kribs ’05]
Discrete symmetries

Higgs is a pNGB of an accidental SU(4), since $Z_2$-symmetric mass corrections respect accidental SU(4)

$$\Delta V = -\frac{6y_t^2}{16\pi^2}\Lambda^2 \left( |H_A|^2 + |H_B|^2 \right) + \ldots$$

$$\Delta m^2_H = -\frac{6y_t^2}{16\pi^2}\Lambda^2 + \frac{6y_t^2}{16\pi^2}\Lambda^2 - 6\frac{y_t^2}{16\pi^2}(m_T^2 - m_t^2) \log \frac{\Lambda^2}{m_T^2}$$

Still a plethora of new particles, but not interacting via SM forces.
Why Not?

Higgs portal maintains equilibrium down to $T \sim \text{GeV}$

$\Delta N_{\text{eff}} \gg 1$

Options are

Change the cosmology

- RHN decay
- Saxion decay

Signals in CMB

- [Chacko, NC, Fox, Harnik ‘16]
- [NC, Koren, Trott ‘16]

Change the spectrum

Copious new physics at $\sim\text{few TeV}$

Signals @ LHC

- Fraternal Twin Higgs
- Holographic Twin Higgs
- Composite Twin Higgs
- Orbifold Higgs
- …
Change the Cosmology

**The problem**: thermal history of $Z_2$-symmetric theory has too much energy density in twin $\nu$, $\gamma$

Introduce an unstable neutral particle $N$ that

- decouples while relativistic
- decays some time thereafter
- decays primarily to $A$

\[ \Delta N_{\text{eff}} \approx 7.4 \left. \frac{\rho_B}{\rho_A} \right|_{\text{BBN}} \approx 5.6 \]

Signals now in future CMB experiments, e.g. CMB Stage-IV

[NC, Koren, Trott '16]

See also: NNaturalness [Arkani-Hamed, Cohen, D’Agnolo, Hook, Kim, Pinner ’16]
Change the Spectrum

“Fraternal Twin Higgs” [NC, Katz, Strassler, Sundrum ’15]
Exotic Higgs Decays

- Must have twin QCD, confines around QCD scale
- Higgs couples to bound states of twin QCD
- Glueballs most interesting; lightest have same quantum # as Higgs

$$\mathcal{L} \supset \frac{\phi}{f} \frac{h}{f} G'''_{\mu\nu} G''''_{\mu\nu}$$

Produce in rare Higgs decays (BR~$10^{-3}$-$10^{-4}$)

$$gg \rightarrow h \rightarrow 0^{++} + 0^{++} + \ldots$$

Decay back to SM via Higgs

$$0^{++} \rightarrow h^* \rightarrow f \bar{f}$$

Long-lived, length scale ~ LHC detectors

Hidden Valley signature [Strassler, Zurek ’06]
New searches & experiments

To Catch a Long-Lived Particle

Collisions at the Large Hadron Collider could be generating particles that physicists have never seen before — perhaps because they haven’t been looking in the right places. So-called long-lived particles would travel dozens of meters through rock before decaying into ordinary particles. New proposed detectors such as Mathusla, pictured here, would be able to catch these decays.

Not to scale
<table>
<thead>
<tr>
<th></th>
<th>Scalar</th>
<th>Fermion</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strong Direct Production</strong></td>
<td>QCD</td>
<td>SUSY</td>
</tr>
<tr>
<td><strong>DY Direct Production</strong></td>
<td></td>
<td>Composite Higgs/RS</td>
</tr>
<tr>
<td><strong>Higgs Portal Direct Production</strong></td>
<td></td>
<td>Quirky Little Higgs</td>
</tr>
<tr>
<td>Singlet</td>
<td></td>
<td>Twin Higgs</td>
</tr>
</tbody>
</table>

- Mirror Glueballs
- Higgs portal observables
- Higgs coupling shifts $\sim$ tuning

[Curtin, Verhaaren '15]
Hyperbolic Higgs

[Cohen, NC, Giudice, McCullough ’18]
(Related: Accidental SUSY, [Cheng, Li, Salvioni, Verhaaren ’18])

Instead of accidental SU(4) from $Z_2$, what about “accidental SU(2,2)?”
(NB, not a symmetry of the full quadratic action)

- Take 2 copies of the MSSM, related by exchange:

\[
\begin{array}{c}
\text{MSSM} \\
\xrightarrow{Z_2}
\end{array}
\begin{array}{c}
\text{MSSM}\mathcal{H}
\end{array}
\]

- Introduce SU(2,2) symmetric tree-level potential:

\[
V(H, H_\mathcal{H}) = \lambda \left( |H|^2 - |H_\mathcal{H}|^2 \right)^2
\]

- Lift scalars in MSSM, fermions in MSSM\mathcal{H} (e.g. via 5D SSSB)

\[
\delta V(H, H_\mathcal{H}) = -c\Lambda^2 \left( |H|^2 - |H_\mathcal{H}|^2 \right) + \ldots
\]
Hyperbolic Higgs

Flat direction ("goldstone" of spontaneously broken SU(2,2))

\[ H = H_0 \sinh \left( \frac{H_{\text{flat}}}{f} \right), \quad H_{\mathcal{H}} = H_0 \cosh \left( \frac{H_{\text{flat}}}{f} \right) \]

Identification w/ SM-like Higgs,

\[ h_{\text{SM}} = h \cos \theta + h_{\mathcal{H}} \sin \theta, \quad \tan \theta = \frac{v}{v_{\mathcal{H}}} \]

Light top partner is SM-neutral stop of MSSM_{\mathcal{H}}

Novel dark sector phenomenology, especially if there are hyperbolic charge- and color-breaking minima
<table>
<thead>
<tr>
<th></th>
<th>scalar</th>
<th>fermion</th>
</tr>
</thead>
<tbody>
<tr>
<td>QCD</td>
<td>SUSY</td>
<td>Composite Higgs/RS</td>
</tr>
<tr>
<td>EW</td>
<td>folded SUSY</td>
<td>Quirky Little Higgs</td>
</tr>
<tr>
<td>singlet</td>
<td></td>
<td>Twin Higgs</td>
</tr>
</tbody>
</table>

- **strong direct production**
- **DY direct production**
- **Higgs portal direct production**

- **Mirror Glueballs**
- **Higgs portal observables**

- **Higgs coupling shifts ~ tuning**
The Unusual Suspects

Low cutoff

Symmetry

Multiplicity
Relaxion

What if the weak scale is selected by scanning?

The idea: couple Higgs to field whose minimum sets $m_H = 0$

The problem: How to make $m_H = 0$ a special point of potential?

The solution: what turns on when $m_H^2$ goes negative?

Vev gives quark masses which give axion potential.

“Relaxion”

[Graham, Kaplan, Rajendran ‘15]

But: immense energy stored in evolving field, need dissipation.
Relaxion

Simplest version: an axion coupled to QCD during inflation.

\[ \Lambda^4(H) \cos(\phi/f) + F(g\phi) + (-M^2 + g\phi)|H|^2 \]

Viable for Higgs + non-compact axion + inflation w/

- Very low Hubble scale (\( \ll \Lambda_{\text{QCD}} \))
- 10 Giga-years of inflation

Various other subtleties regarding technical naturalness, CC, avoidance of fine-tuning to inflationary sector; need to solve strong CP problem.

Forces us to grapple with new UV considerations.

Extensive development, e.g. [Espinosa et al. '15; Hardy '15; Gupta et al '15; Batell, Giudice, McCullough '15; Choi, Im '15; Kaplan, Rattazzi '15; Di Chiara et al. '15; Ibanez et al. '15; Hook, Marques-Tavares '16; Nelson, Prescod-Weinstein '17; ...]
New Signals

Higgs portals

\[ g\phi|H|^2 \quad \Lambda^4(H) \cos(\phi/f) \]

\[ \Lambda^4(H) \cos(\phi/f) \] gives \( \phi - H \) mixing\(^*\) w/ \[ \sin \theta \approx \frac{\Lambda^4}{v f m_h^2} \]

\(^*\)assuming \( \langle \phi \rangle \) breaks CP

[Flacke, Friquele, Fuchs, Gupta, Perez '16]

+5th force for \( m_\phi < \text{eV} \) & cosmology for \( \text{eV} < m_\phi < \text{MeV} \)

[Flacke, Friquele, Fuchs, Gupta, Perez '16]

*assuming \( \langle \phi \rangle \) breaks CP
Particle production relaxion

Alternative possibility: keep bumps across entire potential, turn on dissipation at a special point of potential.

Novel source of dissipation: particle production

Instead of

\[ \frac{\phi}{f} G \tilde{G} \]
+ inflation

\[ \Rightarrow \]

Use coupling to EWK gauge bosons:

\[ \frac{\phi}{f} \left( g^2 W W - g'^2 B \tilde{B} \right) + \Lambda_c^4 \cos \frac{\phi}{f'} \]

Exponential production of EWK gauge bosons around $h \sim v$ slows evolution

Important subtlety: can’t couple to pairs of photons!

(Not a tuning, can be made natural with symmetries, e.g., $SU(2)_L \times SU(2)_R$)
New Signals

Even if tree-level relaxion couplings to SM states are engineered to be

\[ \phi / f \left( g^2 \bar{W}W - g'^2 \bar{B}B \right) \]

in the UV...

...radiative couplings to fermions induced at one loop, photon pairs at one & two loops [Bauer, Neubert, Thamm '17; NC, Hook, Kasko '18]

\[ f_{\gamma} \sim 16\pi^2 \frac{m_W^2}{m_a^2} f_a + (16\pi^2)^2 \frac{m_f^2}{m_a^2} f_a \]

Astrophysical and collider signatures abound; still viable parameter space [Fonseca, Morgante, Servant '18]
Only the beginning...

Asymptotic fragility, agravity, ...

Nontrivial flow (CFT, ...)

Not the SM (Lee-Wick, ...)

Non-locality

UV/IR mixing

Many more ideas under exploration, many to come.
Two examples of UV/IR mixing:
Quantum gravity:...

...and non-commutative QFT.
For example [Minwalla, Seiberg, Van Raamsdonk '99]

$$[x^\mu, x^\nu] = i\Theta^{\mu\nu}$$

$$\sim \int \frac{d^4 k}{k^2} \sim \Lambda^2$$

$$\sim \int \frac{d^4 k}{k^2} e^{ip\Theta_k} \sim \frac{1}{\Theta^2 p^2}$$
Indirect UV/IR: WGC

(Electric) weak gravity conjecture: an abelian gauge theory must contain a state of charge $q$ and mass $m$ satisfying

$$q > \frac{m}{M_{Pl}}$$

[Arkani-Hamed, Motl, Nicolis, Vafa ‘06]

Ride the coattails [Cheung, Remmen ‘14]: Charge SM fermions under weakly gauged (unbroken) $U(1)_{B-L}$ (bounds currently $q \lesssim 10^{-24}$). Cancel anomalies with RHN $\nu_R$

Neutrino mass is

$$y_\nu H \bar{L} \nu_R \rightarrow m_\nu \sim y_\nu v$$

so $m_\nu \sim 0.1$ eV, $q \approx 10^{-29}$

See also: [Ibañez, Martin-Lozano, Valenzuela ’17,…]
Direct UV/IR Mixing

Take the bull by the horns…

*Study field theories with UV/IR mixing*

 Canonical example:

QFT on non-commutative spacetime \[ [\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \]

UV/IR mixing from “uncertainty principle” \[ \Delta \hat{x}^\mu \Delta \hat{x}^\nu \geq \frac{1}{2} |\theta^{\mu\nu}| \]

**Caveats**: Lorentz violating; Minkowski NCQFT unitary only for space-space non-commutativity (i.e. \( \theta^{0i}=0 \)).

Not the theory of our universe, but a useful toy model. (See e.g. [Heckman & Verlinde '14])
NCQFT

Two common approaches:

1. QFT on commutative coordinates w/ star product:

\[ f(x) \star g(x) = \exp \left( \frac{i}{2} \theta_{\mu\nu} \partial_\mu \partial_\nu \right) f(y)g(z) \bigg|_{y=z=x} \]

2. Seiberg-Witten map [Seiberg, Witten ’99]:

i.e., \[ f \star g = f \cdot g + \frac{i}{2} \theta^{\mu\nu} \partial_\mu \partial_\nu g + \mathcal{O}(\theta^2) \]

\[ \hat{A}_\mu[A] = A_\mu + \frac{1}{4} \theta^{\rho\sigma} \{ A_\sigma, \partial_\rho A_\mu \} + \frac{1}{4} \theta^{\rho\sigma} \{ F_{\rho\mu}, A_\sigma \} + \mathcal{O}(\theta^2) \]

Equivalent to any finite order in \( \theta \) (i.e., option (2) defines a low-energy effective action), but UV/IR mixing only apparent in (1).
NCQFT: $\phi^4$

Consider just $\phi^4$ in Euclidean $d=4$:

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{4!} g^2 \phi \ast \phi \ast \phi \ast \phi$$

*Quadratic terms identical to commutative theory*

Interactions associated w/ additional phases:

$$V(k_1, k_2, k_3, k_4) = e^{-\frac{i}{2} \sum_{i<j} k_{i\mu} \theta^{\mu\nu} k_{j\nu}}$$

*Not invariant under arbitrary permutations of $k$*

*Planar graphs*: reduces to an overall phase involving external momenta

*Nonplanar graphs*: additional phases from crossing lines

Feynman rules as usual modulo phases in nonplanar diagrams:
NCQFT: $\phi^4$

Compute one-loop radiative corrections to scalar 2-pt function. Both “planar” and “non-planar” diagrams at one loop:

**Planar Diagram**:

\[
\sim \int \frac{d^4 k}{k^2} \Gamma_1^{2,p} = \frac{g^2}{48\pi^2} \left( \Lambda^2 - m^2 \log \left( \frac{\Lambda^2}{m^2} \right) + \ldots \right)
\]

**Non-Planar Diagram**:

\[
\sim \int \frac{d^4 k}{k^2} e^{ip\theta k} \sim \frac{1}{p^2} \Gamma_1^{2,np} = \frac{g^2}{96\pi^2} \left( \Lambda_{\text{eff}}^2 - m^2 \log \left( \frac{\Lambda_{\text{eff}}^2}{m^2} \right) + \ldots \right)
\]

Appearance of a new “scale”:

\[
\Lambda_{\text{eff}}^2 = \frac{1}{1/\Lambda^2 + p \circ p}
\]

\[
p \circ k = |p^\mu \theta_{\mu\nu}^2 k^\nu|
\]

\[
\Lambda_{\text{eff}}^2 \rightarrow \left\{ \begin{array}{ll}
\frac{\Lambda^2}{p^\circ p} & p \circ p \rightarrow 0 \\
\Lambda & \Lambda \rightarrow \infty
\end{array} \right.
\]

[Minwalla, Seiberg, Van Raamsdonk ’99]
New poles

1-loop 1PI quadratic effective action:
\[
\frac{1}{2} \left( p^2 + M^2 + \frac{g^2}{96\pi^2 (p \circ p + 1/\Lambda^2)} + \ldots \right) \phi(p)\phi(-p)
\]

w/ renormalized mass \( M \):
\[
M^2 = m^2 + \frac{g^2 \Lambda^2}{48\pi^2} - \frac{g^2 m^2}{48\pi^2} \ln \frac{\Lambda^2}{m^2}
\]

Action @ infinite cutoff:
\[
\frac{1}{2} \left( p^2 (p^2 + M^2 + \frac{g^2}{96\pi^2 (p \circ p + \ldots)} \right) \phi(p)\phi(-p))
\]

Two poles in \( \Lambda \to \infty \) action:

1. Usual one (\( \phi \) quanta) at \( p^2 + m^2 = O(g^2) \)

2. New one at \( p \circ p = -\frac{g^2}{96\pi^2} \frac{1}{p_c^2 + m^2} + \ldots \)

Second pole signals existence of new light “particle” arising from high-momentum modes of \( \phi \)

[Minwalla, Seiberg, Van Raamsdonk ’99
Alvarez-Gaume, Vazquez-Mozo ‘03]
Wilsonian interpretation

Normally require renormalizable Wilsonian action to satisfy

1. Correlation functions well-defined as $\Lambda \to \infty$

2. Correlation functions at finite $\Lambda$ differ from limiting value by $O(1/\Lambda)$ at all momenta

Badly violated here at small $p$.

$$\frac{1}{2} \left( p^2 + M^2 + \frac{g^2}{96\pi^2(p \circ p + 1/\Lambda^2)} + \ldots \right) \phi(p)\phi(-p)$$

Restore Wilsonian interpretation w/ new particle $\chi$:

$$\delta \mathcal{L} = \frac{1}{2} \partial \chi \circ \partial \chi + \frac{1}{2} \Lambda^2 (\partial \circ \partial \chi)^2 + i \frac{1}{\sqrt{96\pi^2}} g\chi\phi$$

Quadratic, so integrate out:

$$+ \frac{1}{2} \frac{1}{96\pi^2} \left( \frac{g^2}{p \circ p} - \frac{g^2}{p \circ p + 1/\Lambda^2} \right) \phi(p)\phi(-p)$$

[Minwalla, Seiberg, Van Raamsdonk '99]
What have we learned?

High-momentum modes of massive fields in a non-commutative scalar theory are “dual” to additional (peculiar) light fields

4d fields in case of quadratic divergences, 5d for linear divergences, 6d for logarithmic divergences

In a fantasy application to the hierarchy problem, apparently light scalars are the $\chi$ fields, not the $\phi$ fields

$$\delta \mathcal{L} = \frac{1}{2} \partial \chi \circ \partial \chi + \frac{1}{2} \Lambda^2 (\partial \circ \partial \chi)^2 + i \frac{1}{\sqrt{96\pi^2}} g \chi \phi$$

Just a fantasy in this setting, but worth understanding basic features & trying to extract lessons [NC, Koren, in progress]

Other controlled QFTs with similar features?
Conclusions

• Naturalness a compelling strategy for physics beyond the Standard Model within experimental reach.

• Robust program of LHC searches for conventional approaches based on continuous symmetries…

• …and entirely new approaches now being explored, based on exotic cutoffs, discrete symmetries, dynamics, etc.

• New signals abound: LLPs, Higgs properties, diverse ALPs,…

• Next step: apparent failure of naturalness strategy might signal breakdown of EFT. Opportunity/challenge to make this concrete.
Thank you!