QED+QCD NNLO corrections to Drell-Yan

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Several observables measured with high accuracy TH precision

QCD corrections can be rather large: NNLO needed

\[ \mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2) \] suggests NLO EW \sim NNLO QCD

Enhanced

• by photon emission kinematical effects, mass-singular log’s \( \propto \alpha \ln(m_\mu/Q) \)

• at high energies
  EW Sudakov log’s \( \propto (\alpha/s_W^2) \ln^2(M_W/Q) \)

Requiere perturbative and non-perturbative effort

• \( \mathcal{O}(\alpha) \) corrections to all PDFs
typical impact: \( \Delta(PDF) \lesssim 0.3\% \) (1\%) for \( x \lesssim 0.1 \) (0.4)

• LUXqed : precise determination of photon content of the proton
  Manohar, Nason, Salam, Zanderighi (2016)

• QEDxQCD splitting functions DdeF, Rodrigo, Sborlini (2016)
Big effort to obtain EW/QED perturbative corrections for DY

Full results at NLO
- QED NLO: Baur, Keller, Sakumoto (1997)
- EW NLO: Baur, Brein, Hollik, Schappacher, Wackeroth (2001)

Partial results at NNLO EWxQCD for inclusive cross section
- Real corrections: Bonciani, Buccioni, Mondini, Vicini (2017)

Mixed EWxQCD corrections in the resonance region

Cross section dominated by factorizable terms: production x decay

- Rely on assumption that missing initial-initial (production) terms are negligible
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Here: NNLO QED+QCD corrections to Z production
NNLO in QCD+QED

general expansion in both couplings

\[ d\sigma = \sum_{i,j} \alpha_s^i \alpha^j d\sigma^{(i,j)} \]

“Full NNLO” means \( i + j = 2 \)

(2,0) QCD\(^2\)

(1,1) QED\times QCD

(0,2) QED\(^2\)

QCD NNLO for (inclusive) DY has been available for quite some time

\textbf{NNLO in QCD+QED}

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(2,0) QCD

(1,1) QEDxQCD

(0,2) QED

\textbf{QCD NNLO for (inclusive) DY has been available for quite some time}


\textbf{It is possible to use the NNLO QCD result to obtain the QEDxQCD mixed terms and the QED}^2 : Abelianization procedure

\begin{align*}
\text{QCD} & \quad \text{QED} \\
\text{color factors} & \quad \text{color factors}
\end{align*}


\[ \alpha_s^2 \]

\[ C_F^2 \]

\[ 2e_q^2 C_F \]

\[ e_q^4 \]

\[ C_F \ C_A \]

\[ \frac{C_F \ C_A}{2} \]

\[ n_F \ C_F T_R \]

\[ e_q^2 \left[ N_C \sum_{k \in Q} e_k^2 + \sum_{k \in L} e_k^2 \right] \]
As stated above, we consider the Born coupling between the quark factors (including electric charges of both quarks and leptons that also the colour factor). The same occurs for other channels, after treating carefully the abelian limit. Here, by turning two gluons into photons from the initial flux factor, which depends on the colour properties of initial state particles. For instance, since the result for corrections in Fig. 1. This strategy can be extended for all the topologies in the products of the type

\[ \sigma = \tau \sigma_Z (M_Z^2) W_Z (\tau, M_Z^2) \]

\[ \sigma_Z (M_Z^2) = \frac{\pi^2 \alpha}{4M_Z^2 N_C \sin^2 \theta_W \cos^2 \theta_W} \]

Considered as effective “Weak” coupling (“not QED”)
\[ \sigma = \tau \sigma_{Z} \left( M_{Z}^{2} \right) \mathcal{W}_{Z}(\tau, M_{Z}^{2}) \]

\[ \sigma_{Z}(M_{Z}^{2}) = \frac{\pi^2 \alpha}{4 M_{Z}^{2} N_{C} \sin^2 \theta_{W} \cos^2 \theta_{W}} \]

Considered as effective “Weak” coupling (“not QED”)

\[
w^{(1,1)}_{Z} = \sum_{i \in Q, \bar{Q}} q_{i}(x_{1}) \bar{q}_{i}(x_{2}) e_{i}^{2} C_{F} \Delta^{(2)C_{F}}_{q \bar{q}}(x) + \sum_{i \in Q, \bar{Q}} q_{i}(x_{1}) q_{i}(x_{2}) e_{i}^{2} C_{F} \Delta^{(2)id}_{q q}(x) + \sum_{i \in Q, \bar{Q}} [2 C_{A} C_{F} (q_{i}(x_{1}) \gamma(x_{2}) + \gamma(x_{1}) q_{i}(x_{2})) + (q_{i}(x_{1}) g(x_{2}) + g(x_{1}) q_{i}(x_{2}))] \times e_{i}^{2} \Delta^{(2)C_{F}}_{q g}(x) + (g(x_{1}) \gamma(x_{2}) + \gamma(x_{1}) g(x_{2})) 2 C_{A} \left( \sum_{k \in \bar{Q}} c_{k} e_{k}^{2} \right) \Delta^{(2)}_{g g}(x) \]
\[ \sigma = \tau \sigma_Z (M_Z^2) W_Z (\tau, M_Z^2) \quad \text{and} \quad \sigma_Z (M_Z^2) = \frac{\pi^2 \alpha}{4M_Z^2 N_C \sin^2 \theta_W \cos^2 \theta_W} \]

Considered as effective “Weak” coupling (“not QED”)

\[
w_{Z}^{(1,1)} = \sum_{i \in Q, \bar{Q}} q_{i} (x_1) \bar{q}_{i} (x_2) c_i \frac{2e^2}{N_C} F_{q} \Delta^2 (x) + \sum_{i \in Q, \bar{Q}} q_{i} (x_1) q_{i} (x_2) c_i \frac{2e^2}{N_C} F_{q} \Delta^2 (x) \\
+ \sum_{i \in Q, \bar{Q}} \left[ 2C_A F_{q} (q_{i} (x_1) \gamma (x_2) + \gamma (x_1) q_{i} (x_2)) + (q_{i} (x_1) g (x_2) + g (x_1) q_{i} (x_2)) \right] \times \frac{e^2}{N_C} \Delta_{qq}^2 (x) \\
+ \left( g (x_1) \gamma (x_2) + \gamma (x_1) g (x_2) \right) 2C_A \left( \sum_{k \in Q} c_k e^2 \right) \Delta_{gg}^2 (x)
\]

Parameters set-up

\[ M_Z = 91.187 \text{ GeV} \quad \sin^2 \theta_W = 0.23 \]

Default scales choice \[ \mu_R = \mu_F = M_Z \]

Running couplings \[ \alpha (M_Z) \sim \frac{1}{128} \]

PDF: LUXqed NNLO set (LHAPDF) \quad \text{Manohar, Nason, Salam, Zanderighi(2016)}
\[ K_{\text{QED}}^{\text{NLO}} = \frac{\sigma^{(0,0)} + \alpha \sigma^{(0,1)}}{\sigma^{(0,0)}} \]

\[ K_{\text{QCD}}^{\text{NNLO}} = \frac{\sigma^{(0,0)} + \alpha_s \sigma^{(1,0)} + \alpha_s^2 \sigma^{(2,0)}}{\sigma^{(0,0)} + \alpha_s \sigma^{(1,0)}} \]

\[ K_{\text{QED}}^{\text{NNLO}} = \frac{\sigma^{(0,0)} + \alpha \sigma^{(0,1)} + \alpha^2 \sigma^{(0,2)}}{\sigma^{(0,0)} + \alpha \sigma^{(0,1)}} \]

\[ K_{\text{QCDxQED}}^{\text{NNLO}} = \frac{\sigma^{(0,0)} + \alpha \sigma^{(0,1)} + \alpha_s \sigma^{(1,0)} + \alpha \alpha_s \sigma^{(1,1)}}{\sigma^{(0,0)} + \alpha \sigma^{(0,1)} + \alpha_s \sigma^{(1,0)}} \]

- \(\alpha_s^2 \sim \alpha\) QED NLO \~\ QCD NNLO around 5 per-mille
- Mixed QEDxQCD below the per-mille level (max. \~\ 2 TeV)
- At 14 TeV QCD NNLO \~\ 3.5 mixed QEDxQCD (not \~\ 15)
- QED\(^2\) \~\ \mathcal{O}(10^{-5})
Approx. based on “factorization” for QED+QCD

- additive  no mixed terms
- multiplicative  \( K \approx [K_{QED} \times K_{QCD}] \)

\[
\kappa_{\text{fact}} = \left[ K_{QED}^{\text{NLO}} \times K_{QCD}^{\text{NLO}} \right] \mathcal{O}(\alpha \alpha_s) = \alpha \alpha_s \frac{\sigma^{(0,1)} \sigma^{(1,0)}}{\sigma^{(0,0)} \sigma^{(0,0)}} \\
\kappa_{\text{mixed}} = \alpha \alpha_s \frac{\sigma^{(1,1)}}{\sigma^{(0,0)}}
\]

\[
R = \frac{\kappa_{\text{mixed}}}{\kappa_{\text{fact}}} = \frac{\sigma^{(0,0)} \sigma^{(1,1)}}{\sigma^{(0,1)} \sigma^{(1,0)}}
\]

- Factorization approach fails by more than a factor of 2
- Effect in cross section small (because QED small)
- Might be worse for some distributions (lepton qt sensitive to IS radiation)
- Mixed QEDxQCD contribution from different channels

- Tiny photon initiated contribution
- Dominated by qq and qg
- qg and qq with different sign: 50% cancellation
- qg contribution might be suppressed in exclusive distributions (cuts)

Enhance QEDxQCD by 2

for inclusive XS $0.5\%$ effect $\rightarrow$ $1\%$ effect
Scale dependence

LO ($\sigma^{(0,0)}$)
NLO ($\sigma^{(0,0)} + \alpha \sigma^{(0,1)} + \alpha_s \sigma^{(1,0)}$)
NNLO ($\sigma^{(0,0)} + \alpha \sigma^{(0,1)} + \alpha_s \sigma^{(1,0)} + \alpha\alpha_s \sigma^{(1,1)} + \alpha^2 \sigma^{(0,2)} + \alpha_s^2 \sigma^{(2,0)}$)

- Clear improvement in stabilization at higher orders
- Mostly QCD dominated but small QED effect
Conclusions

- Full QED+QCD NNLO corrections to DY (on-shell Z production)
- QED NLO ~ QCD NNLO around 5 per-mille
- Mixed QEDxQCD below the per-mille level for inclusive XS
  - Cancellation between qq and qg channels
- At 14 TeV QCD NNLO ~ 3.5 mixed QEDxQCD (QCD cancellation)
- Factorization approach for mixed QEDxQCD fails by factor of 2
- Very stable under scale variations at NNLO

Future

- Fully differential cross section (observables sensitive to IS radiation)
- Add final state QED radiation
- EW corrections
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