NNLO computations: status, limitations and prospects

Fabrizio Caola, Frank Petriello, Frank Tackmann

CERN TH Institute, Physics at the LHC and beyond, Jul. 19th 2018
Outline

• Introduction: current situation, future prospects

• Problem I: loop amplitudes
  • Numerical evaluation of loop integrals [S. Borowka]

• Problem II: infra-red subtraction
  • Subtraction: nested soft-collinear subtraction [FC]
  • Subtraction: antennas [A. Huss]
  • Slicing: N-jettiness [FT]

This is meant to stimulate discussion, and not as a comprehensive review of all recent developments
NNLO (relative $\alpha_s^2$) is becoming today’s state of the art

NNLO explosion: a closer look

timeline of calculations as of mid June

[123x528]NNLO (relative $\alpha_s^2$) is becoming today’s state of the art

12

18 months

timeline of calculations as of mid June

[G. Salam, 2016]
NNLO explosion: a closer look

2-loop amplitudes for jj, Vj

2-loop amplitudes for Hj

NNLO (relative $\alpha_s^2$) is becoming today’s state of the art

2-loop amplitudes for jj, Vj

2-loop amplitudes for Hj

timeline of calculations as of mid June

subtraction schemes for color-singlet production

[2-loop amplitudes for jj, Vj]

[2-loop amplitudes for Hj]

NNLO (relative $\alpha_s^2$) is becoming today’s state of the art
NNLO explosion: a closer look

NNLO (relative $\alpha_s^2$) is becoming today’s state of the art

2-loop amplitudes for $jj, Vj$

2-loop amplitudes for $Hj$

new ideas/techniques for multi-loop amplitude calculation: $VV@2\text{loop}$

timeline of calculations as of mid June

subtraction schemes for color-singlet production

subtraction schemes for generic processes
The status now: $2 \rightarrow 2$

Recent development: (detailed) phenomenology for (genuine) $2 \rightarrow 2$ processes ($tt, tH, VBF, Vj, jj, VV \ldots$) now possible

Lessons learned (technical side):
• Computations are still highly non-trivial. Cross-validation, different techniques/ implementations crucial
• By and large, result “as expected”. Good convergence w.r.t. NLO, reduced scale variation, unless good reasons (but there are exceptions)

Pheno opportunity:
• We are past the “we can produce a plot” stage. Detailed pheno studies are now possible. A lot of “theoretical data”
• Interesting real emission/jet dynamics effects [VBF, jj, $V(H \rightarrow bb) \ldots$]
• Matching with resummation
• PDFs…
• Can stress-test our understanding of QCD for collider studies
Example: $Z_{pt} @ NNLO + N^3LL$

- Tiny uncertainties
- At face value, slight data/theory tension
- Underestimate uncertainty, PDFs, non-perturbative, …?

Nice ``test case'' for precision targets for HL/future colliders
Looking ahead

HL/future colliders:

- can achieve high statistics/precision for more complicated processes/observables. Go beyond $2 \rightarrow 2$ ($3j$, $Hjj$, $Vjj$…)
- can explore tails of distributions. Effects that we typically neglect start playing a crucial role (virtual massive particles, EW…)
- both involve highly non trivial (conceptual) developments

**TWO MAIN CHALLENGES**

- Complicated two-loop amplitudes
- Efficient enough subtraction schemes for dealing with extra emission
Loop amplitudes
Loop amplitudes

The 1-loop case:

\[ A^{1-\text{loop}}_n = \sum_i d_i + \sum_i c_i \]

\[ + \sum_i b_i + R_n + O(\varepsilon) \]

- any (physical) amplitude: sum of process-dependent coefficient \( \times \) universal basis integrals
- universal "master" integrals well known
- extraction of process dependent coefficients well-understood (PV, unitarity, OPP). Purely algebraic process
- efficient numerical implementations for (reasonably) arbitrary processes
Loop amplitudes

The 2-LOOP case:

\[ A_{n}^{1\text{-loop}} = \sum_{i} d_{i} + \sum_{i} c_{i} + \sum_{i} b_{i} + R_{n} + O(\varepsilon) \]

• any (physical) amplitude: sum of process-dependent coefficient \( \times \) universal process dependent basis integrals
• universal "master" integrals well known \( \times \)
• extraction of process dependent coefficients well-understood \( \times \). Purely algebraic-process
• efficient numerical implementations \( \times \)
• Explosion in complexity with number of particles / masses
Loop amplitudes

The 2-LOOP case:

\[ A_n^{1\text{-loop}} = \sum_i d_i + \sum_i c_i \]

- only 2→2 results fully available now
- even this is highly non trivial, rapid growth in complexity. For example, pp → γγ: few lines. pp→ZZ: ~10 MB (simplified) analytic expression

- extraction of process dependent coefficients well-understood ✗. Purely algebraic process
- efficient numerical implementations ✗
- Explosion in complexity with number of particles / masses
Towards 2→3 reductions: pp→jjj

A lot of recent progress

• unitarity-inspired ideas, finite field reconstruction…: proof-of-concept numerical evaluation of all the relevant master-integrals coefficients for planar gg→ggg amplitude [Badger et al. (2017); Abreu et al. (2017)]

• integrals for this case are known [Papadopoulos et al (2016); Gehrmann et al (2016)]

<table>
<thead>
<tr>
<th>$\epsilon^{-4}$</th>
<th>$\epsilon^{-3}$</th>
<th>$\epsilon^{-2}$</th>
<th>$\epsilon^{-1}$</th>
<th>$\epsilon^{0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{A}_{----+++}^{(2),[0]}$</td>
<td>12.5</td>
<td>27.7526</td>
<td>-23.773</td>
<td>-168.117</td>
</tr>
<tr>
<td>$P_{----+++}^{(2),[0]}$</td>
<td>12.5</td>
<td>27.7526</td>
<td>-23.773</td>
<td>-168.116</td>
</tr>
<tr>
<td>$\hat{A}_{-+---+}^{(2),[0]}$</td>
<td>12.5</td>
<td>27.7526</td>
<td>2.5029</td>
<td>-35.8094</td>
</tr>
<tr>
<td>$P_{-+++---}^{(2),[0]}$</td>
<td>12.5</td>
<td>27.7526</td>
<td>2.5028</td>
<td>-35.8086</td>
</tr>
</tbody>
</table>

[Badger et al. (2017)]

• new incarnation of "standard" technique [Chawdry, Lim, Mitov (2018)] leads to promising result (though be aware, ~GB analytic expression)

Still far from something one can use, but pp→3j amplitude on the horizon

How effective this would be for more complicated processes (Hjj…)?
Computing the integrals


**IN THE PAST:** most of results in terms of *very well known functions* (*Goncharov polylogarithms*)

- Extremely powerful methods to handle/simplify them (symbol, coproduct…)
- Fast and reliable numerical evaluation

**THIS IS NO LONGER THE CASE**

- Interesting physics requires loop amplitudes with generic (internal) mass assignment (e.g.: boosted Higgs, ttH, single-top → virtual top quark effects, tails of distributions → mixed QCD/EW…)
- Moving away from the massless case → new structures (*``elliptic``”). Much less understood, now crucial. A lot of recent progress, a lot still to be done
- A possible alternative way: direct numerical integration. Particularly suited for low multiplicity processes (e.g. 2→2: \(A = A(s,t,\{m_i\})\) → can tabulate)
Subtraction
Real emission

Apart from complicated two-loop amplitudes, the big problem of NNLO computations is how to consistently handle IR singularities.

Extracting singularities arising from phase space integration in a fully exclusive way (i.e. arbitrary process / observable) non trivial.
The solution: two philosophies

Same problem at NLO. Two different approaches have been developed

Phase space slicing

\[ \int |M|^2 F_J d\phi_d = \int_0^\delta [ |M|^2 F_J d\phi_d ]_{s.c.} + \int_\delta^1 |M|^2 F_J \phi_4 + \mathcal{O}(\delta) \]

- conceptually simple, straightforward implementation
- must be very careful with residual \( \delta \) dependence (esp. in diff. distr.)
- highly non-local → severe numerical cancellations

Subtraction

\[ \int |M|^2 F_J d\phi_d = \int ( |M|^2 F_J - S ) d\phi_4 + \int S d\phi_d \]

- in principle can be made fully local → less severe numerical problems
- requires the knowledge of subtraction terms, and their integration
The solution: two philosophies

Both methods have proven useful for $2 \rightarrow 2$ computations

Phase space slicing

$$\int |M|^2 F_J d\phi_d = \int_0^\delta [ |M|^2 F_J d\phi_d ]_{s.c.} + \int_\delta^1 |M|^2 F_J \phi_4 + \mathcal{O}(\delta)$$

- $q_t$ subtraction [Catani, Grazzini] $\rightarrow$ H, V, $V(H \rightarrow \text{bb})$, VV, HH
- N-jettiness [Boughezal et al; Gaunt et al] $\rightarrow$ H, V, $\gamma\gamma$, VH, Vj, Hj, single-top

Subtraction

$$\int |M|^2 F_J d\phi_d = \int (|M|^2 F_J - S)d\phi_4 + \int Sd\phi_d$$

- antenna [Gehrmann-de Ridder, Gehrmann, Glover] $\rightarrow$ jj, Hj, Vj, DIS
- Sector-decomposition+FKS [Czakon; Boughezal, Melnikov, Petriello; Czakon, Heymes; FC, Röntsch, Melnikov] $\rightarrow$ ttbar, single-top, Hj, V, V($H \rightarrow \text{bb}$)
- P2B [Cacciari, Dreyer, Karlberg, Salam, Zanderighi] $\rightarrow$ VBF$_H$, single-top
- Colorful NNLO [Del Duca, Somogyi, Tocsanyi, Duhr, Kardos]: only $e^+e^-$ so far
- +other being developed [Herzog(2018); Magnea et al (2018)]
The solution: two philosophies

Some of these techniques are quite generic

**In Principle,** they allow for **Arbitrary Computations**

**In Practice:** `genuine' $2 \rightarrow 2$ reactions, with big computer farms. $O(100,000)$ CPU hours or more. Going beyond $2 \rightarrow 2$ (or NNLO) challenging for current frameworks.

No matter the technique, calculations require one-loop amplitudes in highly singular kinematics. Non obvious, even after `one-loop’ revolution.
Nested soft-collinear subtraction

[FC, Melnikov, Röntsch (2017-18+work in progress), based on Czakon (2011)]

The guiding principles for a "good subtraction"

• generic (arbitrary process, observable)

• numerically stable (avoid large cancellations at intermediate stages)

• minimal (avoid proliferation of subtraction terms)

• good control over IR structures

• flexible (allow exploration of different solutions)

Important for efficient high multiplicity/reliable higher orders
Nested soft-collinear subtraction

[FC, Melnikov, Röntsch (2017-18+work in progress), based on Czakon (2011)]

The guiding principles for a "good subtraction"

• generic (arbitrary process, observable) ✔
• numerically stable (avoid large cancellations at intermediate stages)
  fully local subtraction, can easily control amount of subtraction ✔
• minimal (avoid proliferation of subtraction terms) ✔
  maximal use of QCD symmetry (color coherence) to disentangle soft/collinear singularities
  * soft: natural splitting in correlated/uncorrelated
  * collinear: natural splitting according to rapidity separation
• good control over IR structures ✔
  final IR structure: very transparent. Some integrals evaluated numerically.
  Most non-trivial class: recently computed analytical [Delto et al (2018)]
• flexible ✔ (so far implementation based on Czakon’s sector improved residue subtraction)
**Test case:** $pp \rightarrow \gamma^* \rightarrow l^+l^-$

Stress-test: can we obtain *arbitrarily* accurate results?

\[
\sigma = \sigma^{\text{LO}} + d\sigma^{\text{NLO}} + d\sigma^{\text{NNLO}}
\]

\[
d\sigma^{\text{NNLO}} = 14.471(4) \text{ pb}
\]

\[
d\sigma^{\text{NNLO}}_{\text{analytic}} = 14.470 \text{ pb}
\]

- better than per-mill agreement on NNLO correction
- control over 5 orders of magnitude
- per-mill precision on physical result ($\sigma$): ~10 CPU hours
Test case: \( pp \rightarrow \gamma^* \rightarrow l^+l^- \)

Differential distributions

\[
\begin{align*}
\frac{d\sigma}{dy_v} \quad \text{[pb]} \\
\text{NLO} & \quad \text{NNLO}
\end{align*}
\]

\[
\begin{align*}
\frac{d\sigma}{dy_l} \quad \text{[pb]} \\
\text{NLO} & \quad \text{NNLO}
\end{align*}
\]

\[
\begin{align*}
\frac{d\sigma}{dy_e} \quad \text{[pb]} \\
\text{NLO} & \quad \text{NNLO}
\end{align*}
\]

\[
\begin{align*}
\frac{\text{NNLO}}{\text{NLO}}
\end{align*}
\]

\( O(10 \text{ CPU hours}) \) \hspace{1cm} \( O(100 \text{ CPU hours}) \)

Figure 3: Upper panes: Rapidity distribution of the vector boson, rapidity distribution of the lepton and \( p_T \) distribution of the lepton at different orders of perturbation theory. Lower panes: the ratio of NNLO/NLO prediction for a given observable. Plots on the left: the runtime of \( O(10) \) CPU hours; plots on the right: the runtime of \( O(100) \) CPU hours. Note that the dip in the ratio of NNLO/NLO lepton \( p_T \) distribution at \( p_T \approx 25 \text{ GeV} \) is a physical feature and not a fluctuation.

The state-of-the-art comparison of this and other observables in Drell-Yan production between different NNLO codes was presented in Ref. \[51\].
Application: VH, $H \rightarrow bb$ decay@NNLO

- NNLO corrections for decay recently computed
- **Large Corrections in the Exp. Fiducial region** ($p_{T,W} > 150$ GeV)

**Where do large corrections come from?**

[Ferrera, Somogyi, Tramontano (2017)]

**In Principle, Problematic! Where do large corrections come from?**
The source of large corrections

- Large corrections in regions not populated at LO
- Everywhere else: very stable NNLO!
- Dominated by extra parton emission
- Captured by parton shower?
VH, H→bb decay@NNLO: PS simulation

- Compare NNLO calculation with "typical" simulation
- Out-of-the-shelf PS simulation: HWJ MiNLO, decay from Pythia8

- Bulk of the effect correctly captured by PS → simulations used in analysis are reasonably good!
- Similar trend seen in more sophisticated NNLOPS simulations [Astill et al (2018)]
- We can now investigate structure of higher order emission. Detailed comparison fixed-order/(merged) PS now makes sense (similar considerations for VBF,jj…)

![Graphs showing comparison between NNLO and PS simulation](image)