NNLO computations: status, limitations and prospects

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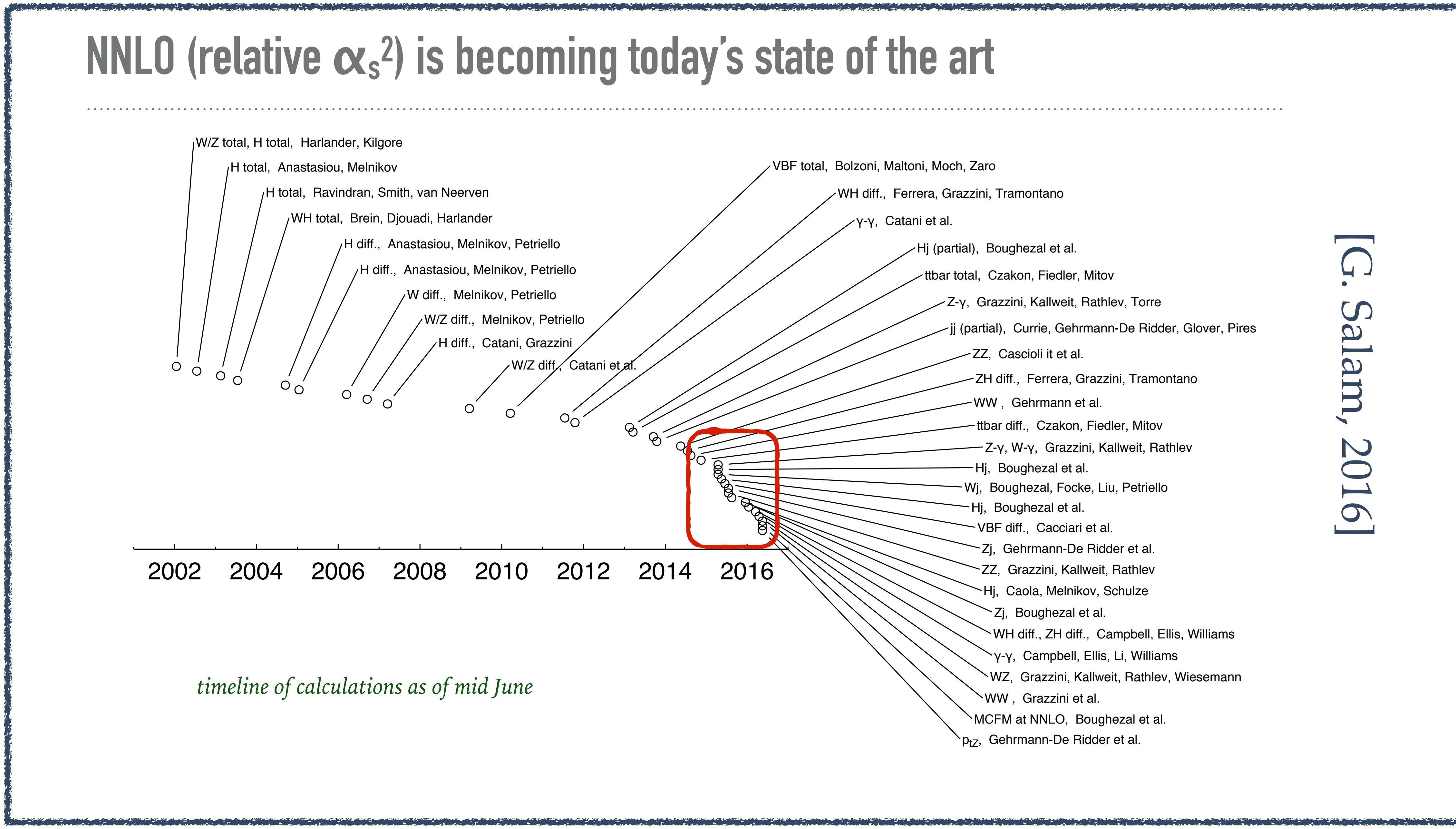
Outline

- Introduction: current situation, future prospects
- PROBLEM I: loop amplitudes
  - *Numerical evaluation of loop integrals [S. Borowka]*
- PROBLEM II: infra-red subtraction
  - *Subtraction: nested soft-collinear subtraction [FC]*
  - *Subtraction: antennas [A. Huss]*
  - *Slicing: N-jettiness [FT]*

This is meant to stimulate discussion, and not as a comprehensive review of all recent developments

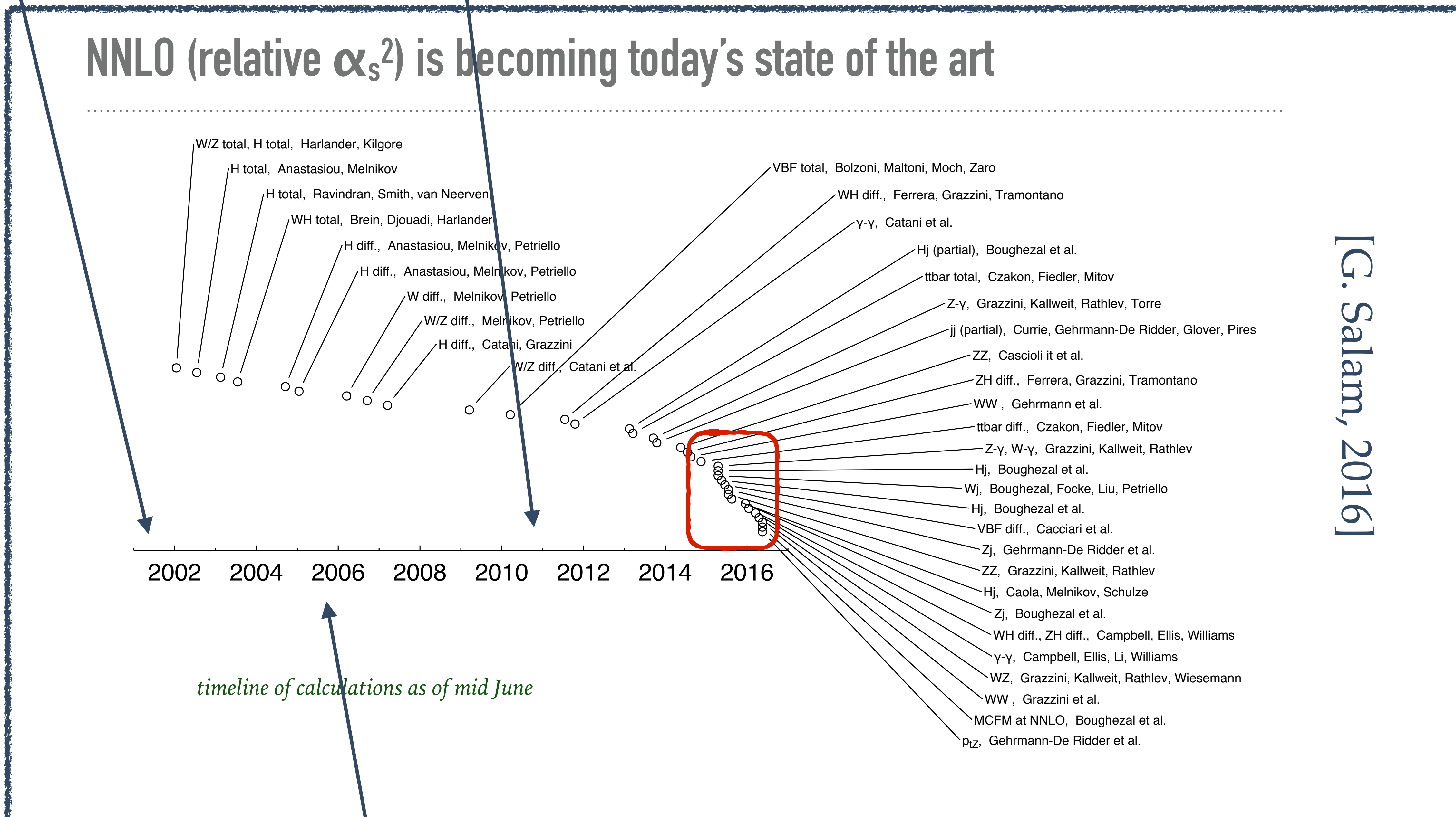
NNLO explosion: a closer look



NNLO explosion: a closer look

2-loop amplitudes for jj,Vj

2-loop amplitudes for Hj



timeline of calculations as of mid June

[G. Salam, 2016]

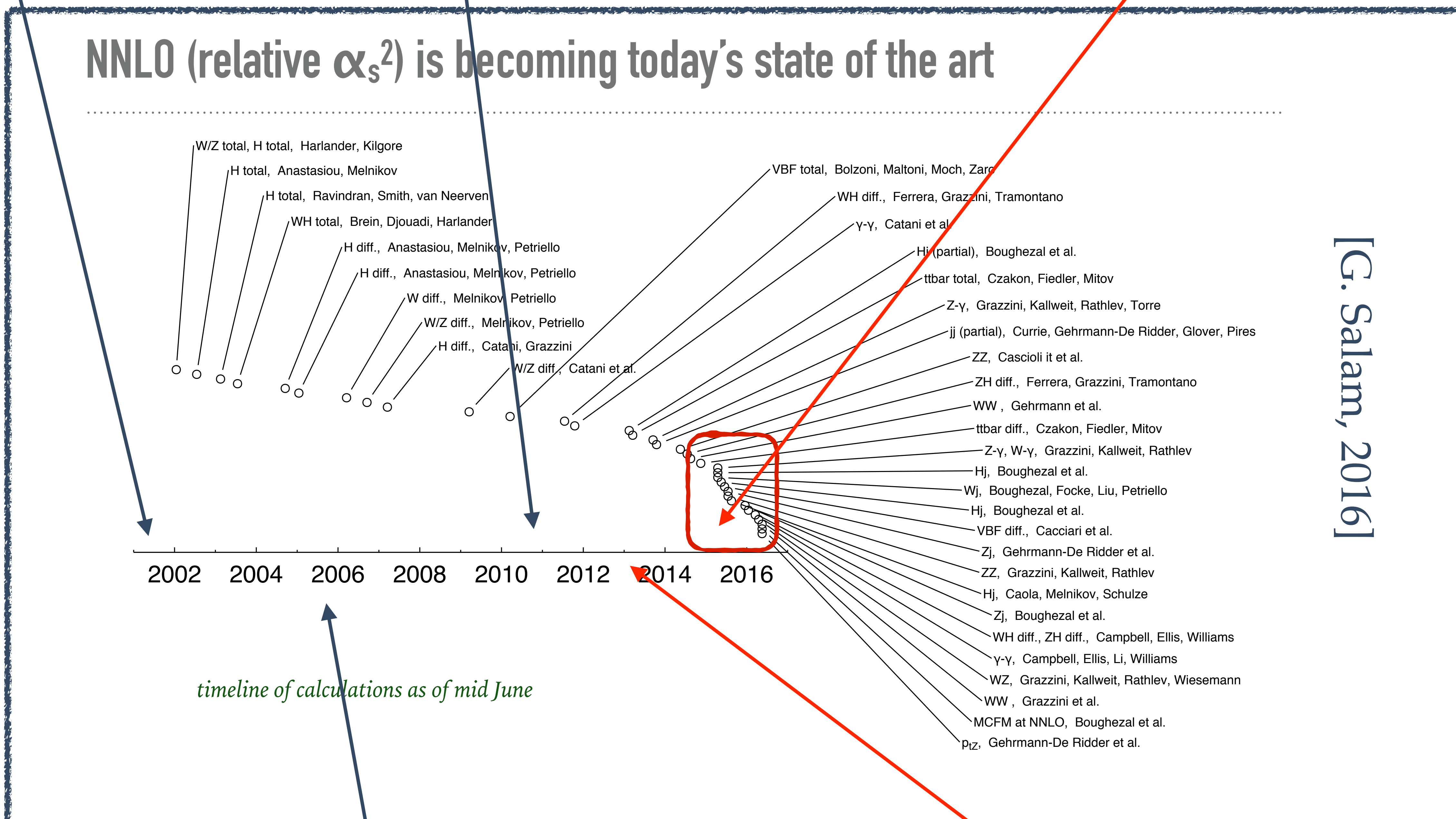
subtraction schemes for color-singlet production

NNLO explosion: a closer look

2-loop amplitudes for jj,Vj

2-loop amplitudes for Hj

new ideas/techniques for multi-loop amplitude calculation: VV@2loop



subtraction schemes for color-singlet production

subtraction schemes for generic processes

The status now: 2→2

Recent development: (detailed) phenomenology for (genuine) 2→2 processes (tt,t,Hj,VBF,Vj,jj,VV…) now possible

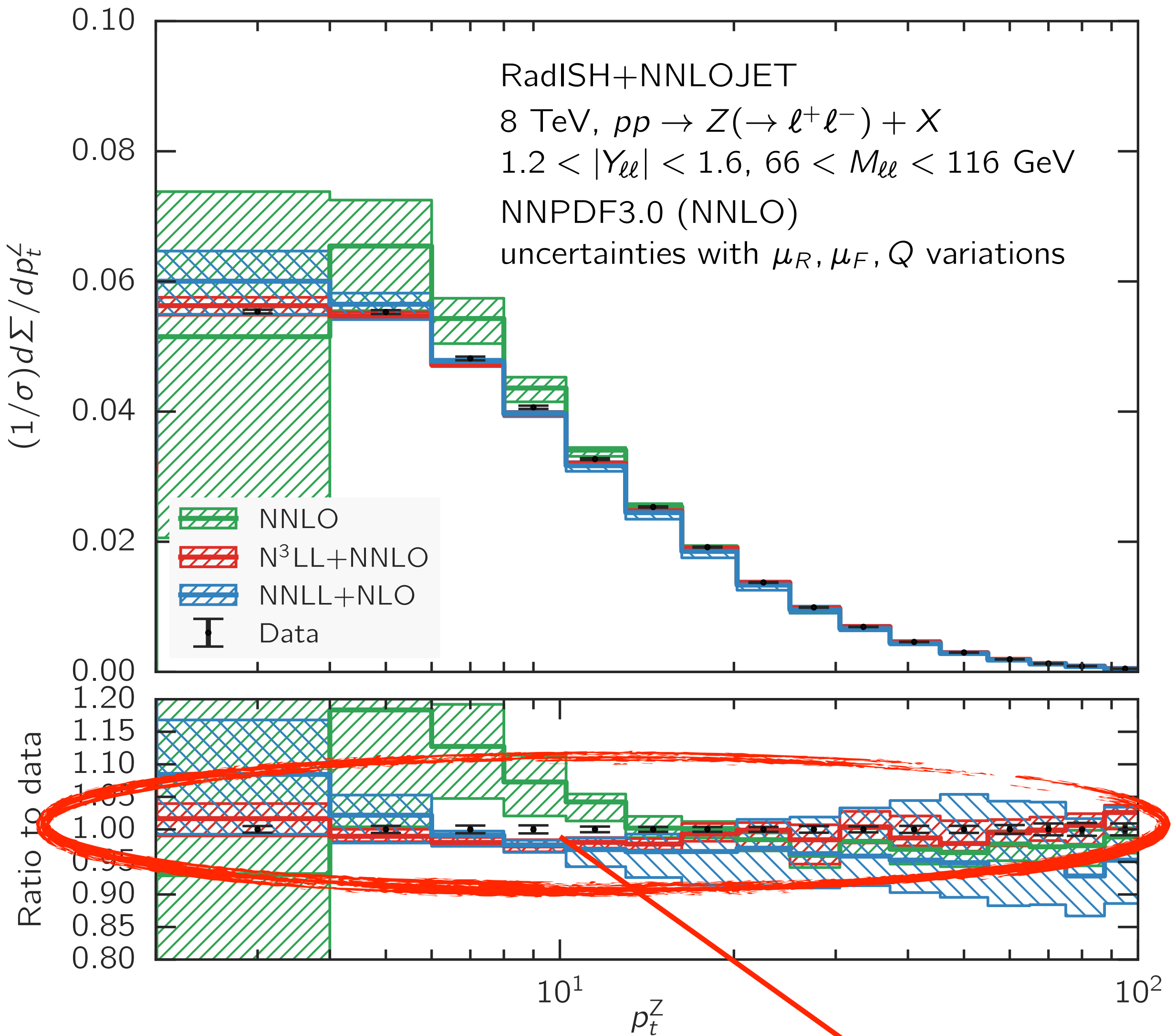
Lessons learned (technical side):

- Computations are still highly non-trivial. Cross-validation, different techniques/implementations crucial
- *By and large*, result ``as expected''. Good convergence w.r.t. NLO, reduced scale variation, unless good reasons (but there are exceptions)

Pheno opportunity:

- We are past the ``we can produce a plot'' stage. Detailed pheno studies are now possible. A lot of ``theoretical data''
- Interesting real emission/jet dynamics effects [VBF, jj, V(H→bb)…]
- Matching with resummation
- PDFs…
- Can stress-test our understanding of QCD for collider studies

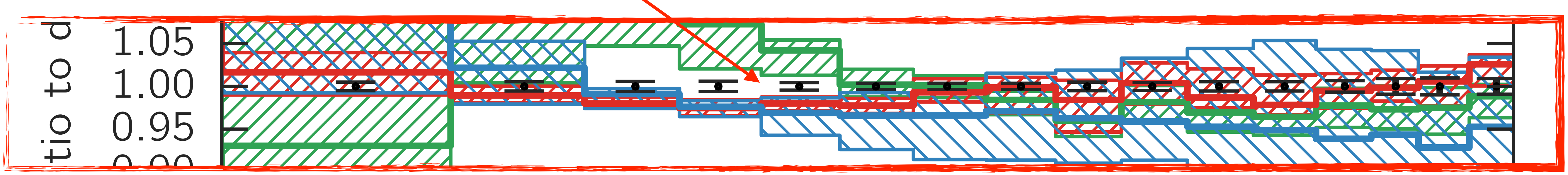
Example: Z p_t@NNLO+N^{3}LL



[Bizon et al, 2018]

- Tiny uncertainties
- At face value, slight data/theory tension
- Underestimate uncertainty, PDFs, non perturbative, …?

Nice ``test case'' for precision targets for HL/future colliders

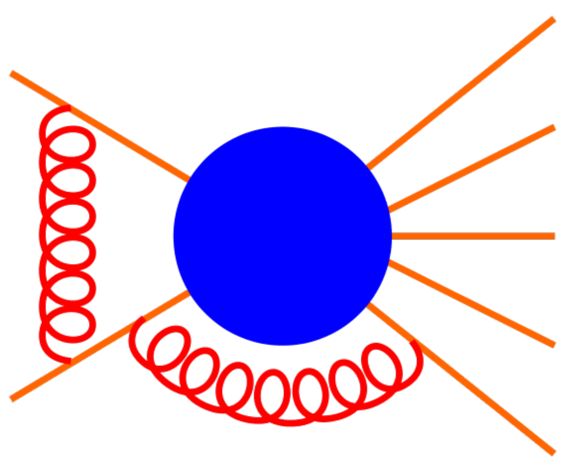


Looking ahead

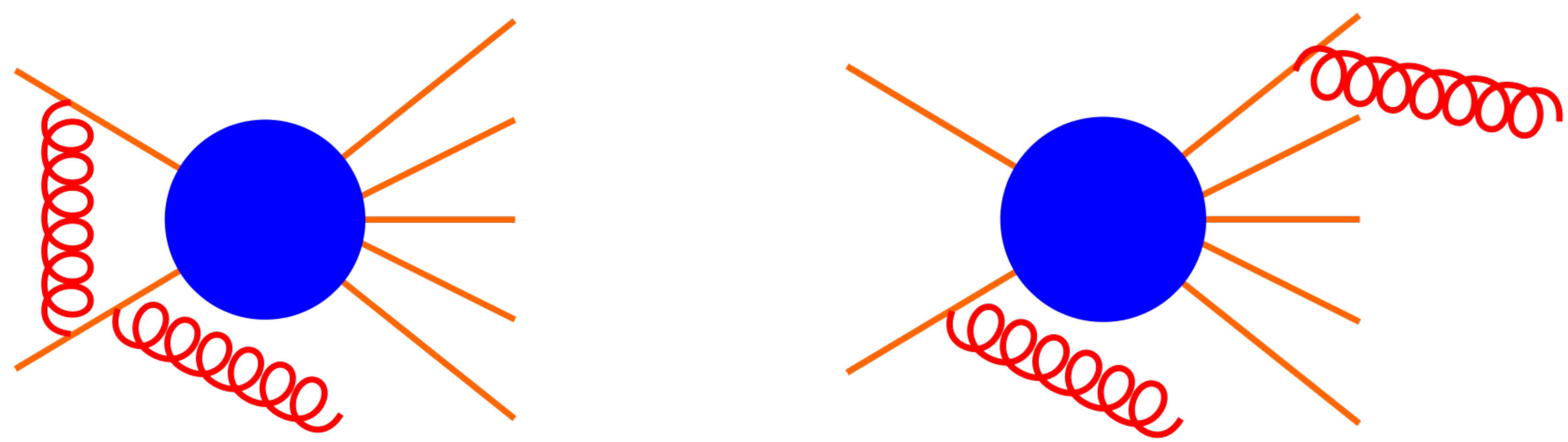
HL / future colliders:

- can achieve high statistics / precision for more complicated processes / observables. Go beyond $2 \to 2$ (3j, Hjj, Vjj…)
- can explore tails of distributions. Effects that we typically neglect start playing a crucial role (virtual massive particles, EW…)
- both involve highly non trivial (conceptual) developments

TWO MAIN CHALLENGES



Complicated two-loop amplitudes

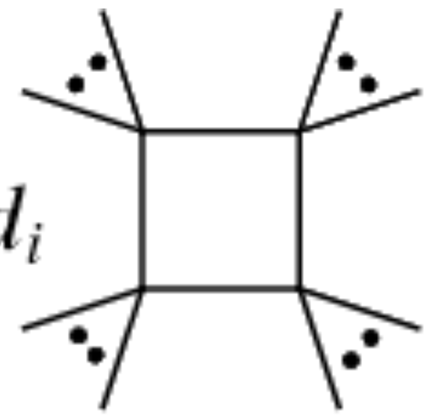
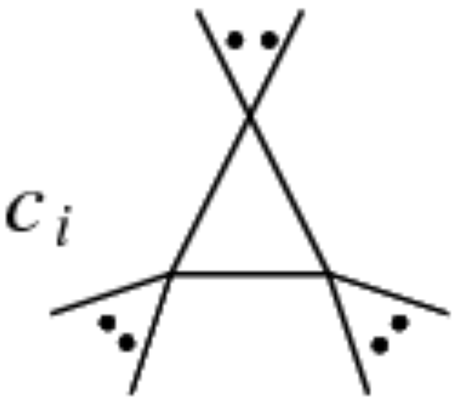


Efficient enough subtraction schemes for dealing with extra emission

Loop amplitudes

Loop amplitudes

The 1-loop case:

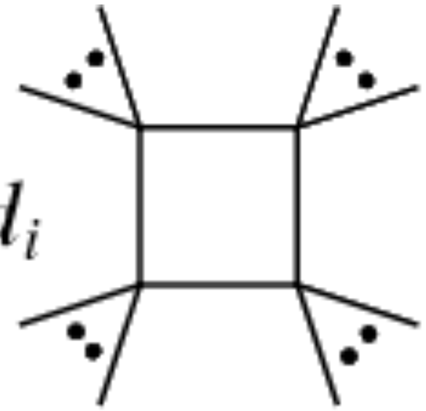
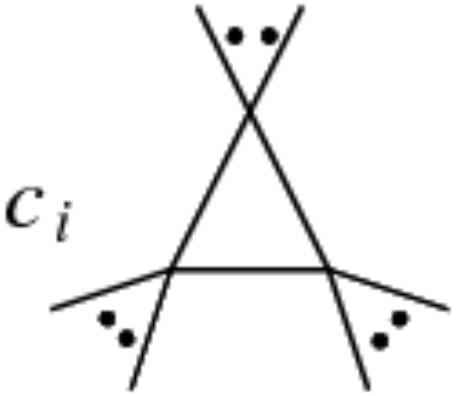
$$A_n^{1\text{–loop}} = \sum_i d_i$$

$$+ \sum_i c_i$$

$$+ \sum_i b_i$$

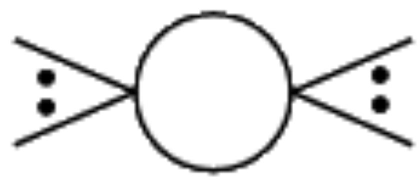
$$+ \; R_n \; + \; O(\varepsilon)$$

- any (physical) amplitude: sum of *process-dependent coefficient* x *universal basis integrals*
- universal ``master'' integrals well known
- extraction of process dependent coefficients well-understood (PV, unitarity, OPP). Purely algebraic process
- efficient numerical implementations for (reasonably) arbitrary processes

Loop amplitudes

The 2-LOOP case:

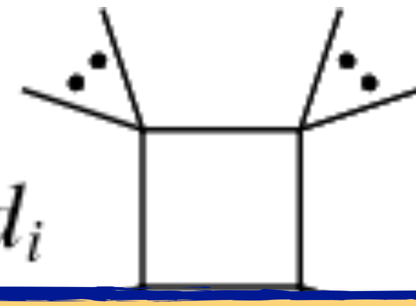
$$A_n^{\text{1-loop}} = \sum_i d_i$$  $$+ \sum_i c_i$$ 

$$+ \sum_i b_i$$  $$+ \; R_n \; + \; O(\varepsilon)$$

- any (physical) amplitude: sum of *process-dependent coefficient* x ~~*universal*~~ *process dependent basis integrals*
- universal ``master” integrals well known ✘
- extraction of process dependent coefficients well-understood ✘. ~~Purely algebraic process~~
- efficient numerical implementations ✘
- Explosion in complexity with number of particles / masses

Loop amplitudes

The 2-LOOP case:

$A_n^{1\text{–loop}} = \sum d_i$  $+ \sum c_i$ 

- only 2→2 results fully available now
- even this is highly non trivial, rapid growth in complexity. *For example,* pp → γγ: few lines. pp→ZZ: ~10 MB (simplified) analytic expression

- extraction of process dependent coefficients well-understood ✘. ~~Purely algebraic process~~
- efficient numerical implementations ✘
- Explosion in complexity with number of particles / masses

Towards 2→3 reductions: pp→jjj

A lot of recent progress

- unitarity-inspired ideas, finite field reconstruction…: *proof-of-concept numerical evaluation* of all the relevant master-integrals coefficients for planar gg→ggg amplitude [Badger et al. (2017); Abreu et al. (2017)]
- integrals for this case are known [Papadopoulos et al (2016); Gehrmann et al (2016)]

Prediction for poles

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$\widehat{A}^{(2),[0]}_{--+++}$	12.5	27.7526	-23.773	-168.117	-175.207±0.004
$P^{(2),[0]}_{--+++}$	12.5	27.7526	-23.773	-168.116	—
$\widehat{A}^{(2),[0]}_{-+-++}$	12.5	27.7526	2.5029	-35.8094	69.661±0.009
$P^{(2),[0]}_{-+-++}$	12.5	27.7526	2.5028	-35.8086	—

Numerical result

[Badger et al. (2017)]

- new incarnation of ``standard'' technique [Chawdry, Lim, Mitov (2018)] leads to promising result (though be aware, ~GB analytic expression)

Still far from something one can use, but pp→3j amplitude on the horizon

How effective this would be for more complicated processes (Hjj…)?

Computing the integrals

New incarnations of old ideas (*differential equations*) allow to tackle very complicated integrals [Kotikov (1991), Remiddi (1997), Henn (2013), Papadopoulos (2014)]

- IN THE PAST: most of results in terms of very well known functions (*Goncharov polylogarithms*)
- Extremely powerful methods to handle/simplify them (symbol, coproduct…)
- Fast and reliable numerical evaluation

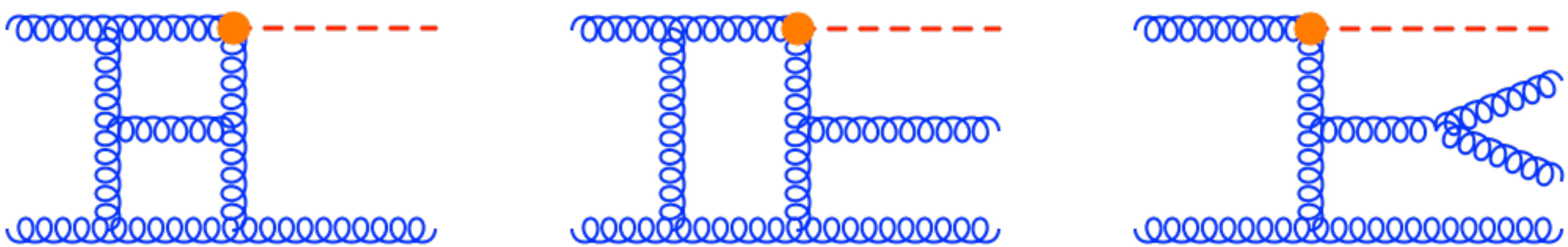
- THIS IS NO LONGER THE CASE
- Interesting physics requires loop amplitudes with generic (internal) mass assignment (eg.: boosted Higgs, ttH, single-top → virtual top quark effects, tails of distributions → mixed QCD/EW…)
- Moving away from the massless case → new structures (``*elliptic*"). Much less understood, now crucial. A lot of recent progress, a lot still to be done
- A possible alternative way: direct numerical integration. Particularly suited for low multiplicity processes (e.g. 2→2: $A = A(s,t,\{m_i\})$ → can tabulate)

Subtraction

Real emission

Apart from complicated two-loop amplitudes, the big problem of NNLO computations is how to consistently handle IR singularities

VV RV RR



$$\int \left[\frac{\mathrm{vv}_4}{\epsilon^4} + \frac{\mathrm{vv}_3}{\epsilon^3} + \frac{\mathrm{vv}_2}{\epsilon^2} + \frac{\mathrm{vv}_1}{\epsilon} + \mathrm{vv}_0\right] d\phi_2$$

$$\int \left[\mathrm{rr}_0\right] d\phi_4$$

$$\int \left[\frac{\mathrm{rv}_2}{\epsilon^2} + \frac{\mathrm{rv}_1}{\epsilon} + \mathrm{rv}_0\right] d\phi_3$$

Extracting singularities arising from phase space integration in a fully exclusive way (i.e. arbitrary process/observable) non trivial

The solution: two philosophies

Same problem at NLO. Two different approaches have been developed

Phase space slicing

$$\int |M|^2 F_J d\phi_d = \int_0^\delta \left[|M|^2 F_J d\phi_d\right]_{s.c.} + \int_\delta^1 |M|^2 F_J \phi_4 + \mathcal{O}(\delta)$$

- conceptually simple, straightforward implementation
- must be very careful with residual δ dependence (esp. in diff. distr.)
- highly non-local → severe numerical cancellations

Subtraction

$$\int |M|^2 F_J d\phi_d = \int (|M|^2 F_J - \mathcal{S}) d\phi_4 + \int \mathcal{S} d\phi_d$$

- in principle can be made fully local → less severe numerical problems
- requires the knowledge of subtraction terms, and their integration

The solution: two philosophies

Both methods have proven useful for 2→ 2 computations

Phase space slicing

$$\int |M|^2 F_J d\phi_d = \int_0^\delta \left[|M|^2 F_J d\phi_d\right]_{s.c.} + \int_\delta^1 |M|^2 F_J \phi_4 + \mathcal{O}(\delta)$$

- q_t subtraction [Catani, Grazzini] → H, V, V(H→bb), VV, HH
- N-jettiness [Boughezal et al; Gaunt et al] → H, V, $\gamma\gamma$, VH, Vj, Hj, single-top

Subtraction

$$\int |M|^2 F_J d\phi_d = \int (|M|^2 F_J - \mathcal{S}) d\phi_4 + \int \mathcal{S} d\phi_d$$

- antenna [Gehrmann-de Ridder, Gehrmann, Glover] → jj, Hj, Vj, DIS
- Sector-decomposition+FKS [Czakon; Boughezal, Melnikov, Petriello; Czakon, Heymes; FC, Röntsch, Melnikov] → ttbar, single-top, Hj, V, V(H→bb)
- P2B [Cacciari, Dreyer, Karlberg, Salam, Zanderighi] → VBF_H, single-top
- *Colorful NNLO [Del Duca, Somogyi, Tocsanyi, Duhr, Kardos]: only e^+e^- so far*
- *+other being developed [Herzog(2018); Magnea et al (2018)]*

The solution: two philosophies

Some of these techniques are quite generic

IN PRINCIPLE, they allow for ARBITRARY COMPUTATIONS

IN PRACTICE: `genuine' 2→2 REACTIONS, with big computer farms. O(100.000) CPU hours or more. Going beyond 2→2 (or NNLO) challenging for current frameworks

No matter the technique, calculations require one-loop amplitudes in highly singular kinematics. Non obvious, even after ``one-loop'' revolution

Nested soft-collinear subtraction

[FC, Melnikov, Röntsch (2017-18+work in progress), based on Czakon (2011)]

The guiding principles for a ``good subtraction''

- generic (arbitrary process, observable)
- numerically stable (avoid large cancellations at intermediate stages)
- minimal (avoid proliferation of subtraction terms)
- good control over IR structures
- flexible (allow exploration of different solutions)

Important for efficient high multiplicity/reliable higher orders

Nested soft-collinear subtraction

[FC, Melnikov, Röntsch (2017-18+work in progress), based on Czakon (2011)]

The guiding principles for a ``good subtraction''

- generic (arbitrary process, observable) ✔
- numerically stable (avoid large cancellations at intermediate stages)
  fully local subtraction, can easily control amount of subtraction ✔
- minimal (avoid proliferation of subtraction terms) ✔
  maximal use of QCD symmetry (color coherence) to disentangle soft/collinear singularities
  ✴*soft: natural splitting in correlated/uncorrelated*
  ✴*collinear: natural splitting according to rapidity separation*
- good control over IR structures ✔
  final IR structure: very transparent. Some integrals evaluated numerically. Most non-trivial class: recently computed analytical [Delto et al (2018)]
- flexible ✔ *(so far implementation based on Czakon's sector improved residue subtraction)*

Test case: $pp \to \gamma^* \to l^+ l^-$

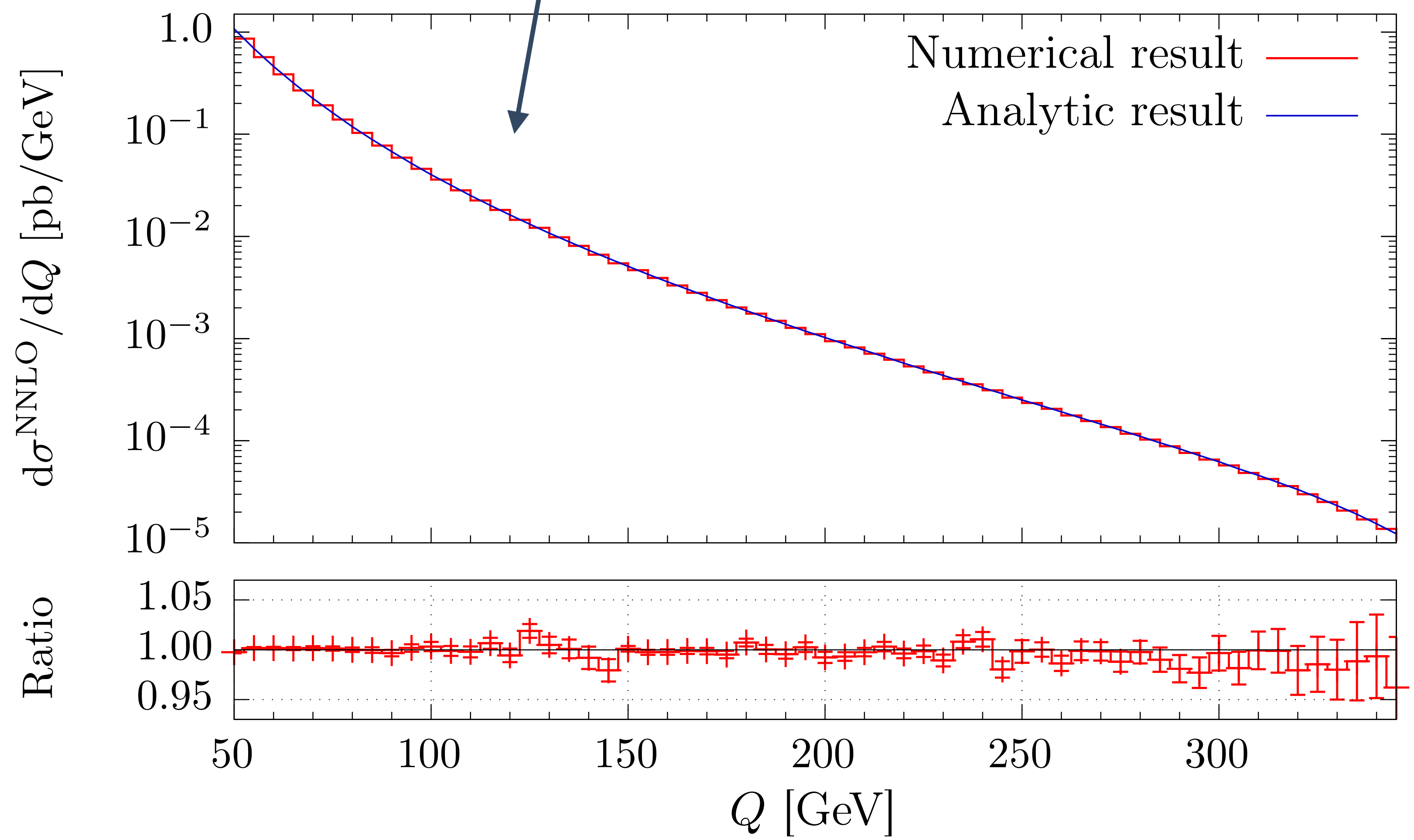
Stress-test: can we obtain *arbitrarily accurate results?*

$$\sigma = \sigma^{\mathrm{LO}} + \mathrm{d}\sigma^{\mathrm{NLO}} + \mathrm{d}\sigma^{\mathrm{NNLO}}$$

$$\mathrm{d}\sigma^{\mathrm{NNLO}} = 14.471(4)\ \mathrm{pb}$$

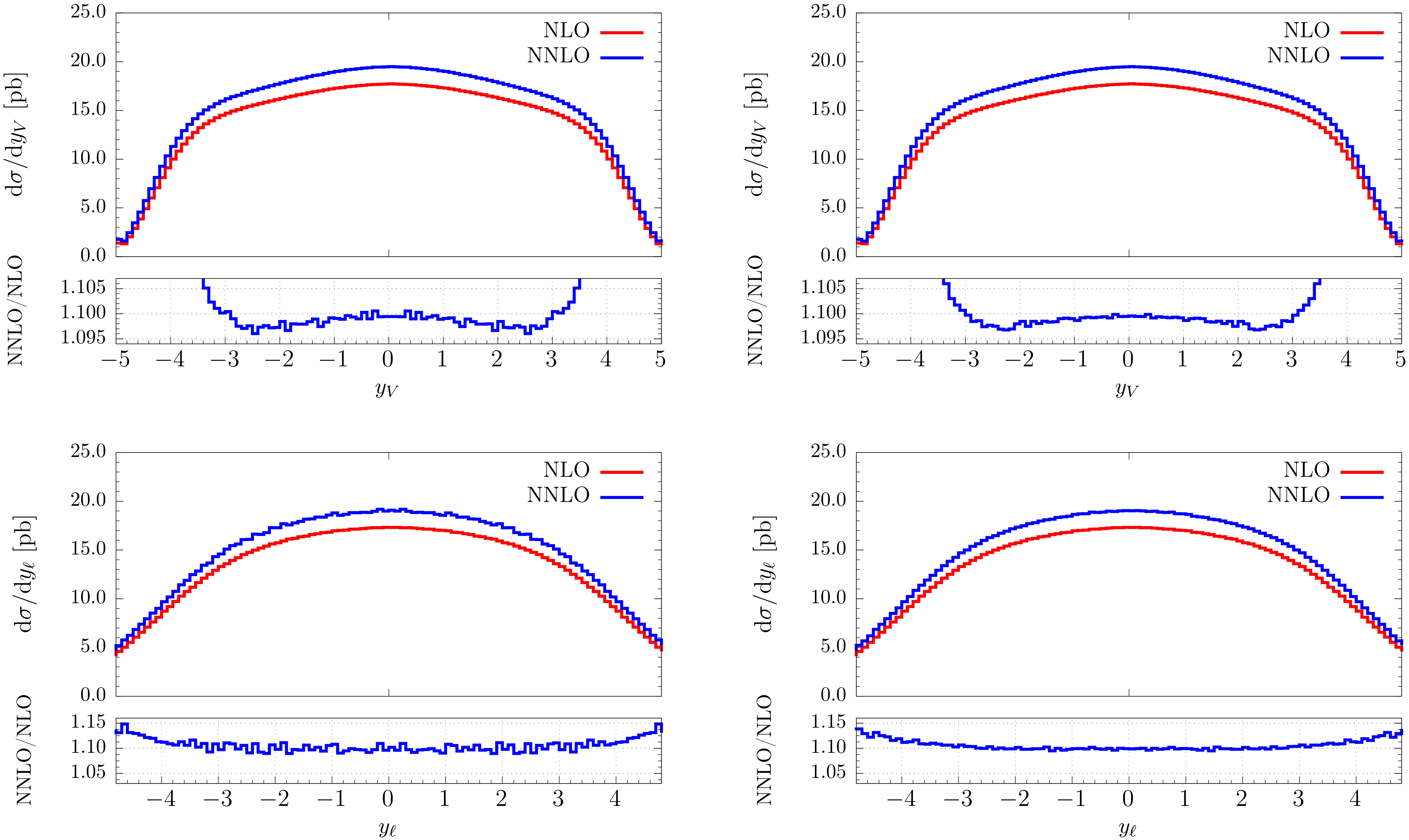
$$\mathrm{d}\sigma^{\mathrm{NNLO}}_{\mathrm{analytic}} = 14.470\ \mathrm{pb}$$

- better than per-mill agreement on *NNLO correction*
- control over 5 orders of magnitude
- per-mill precision on physical result (σ): ~10 CPU hours



Test case: $pp \rightarrow \gamma^* \rightarrow l^+ l^-$

Differential distributions



y_V

y_{l^+}

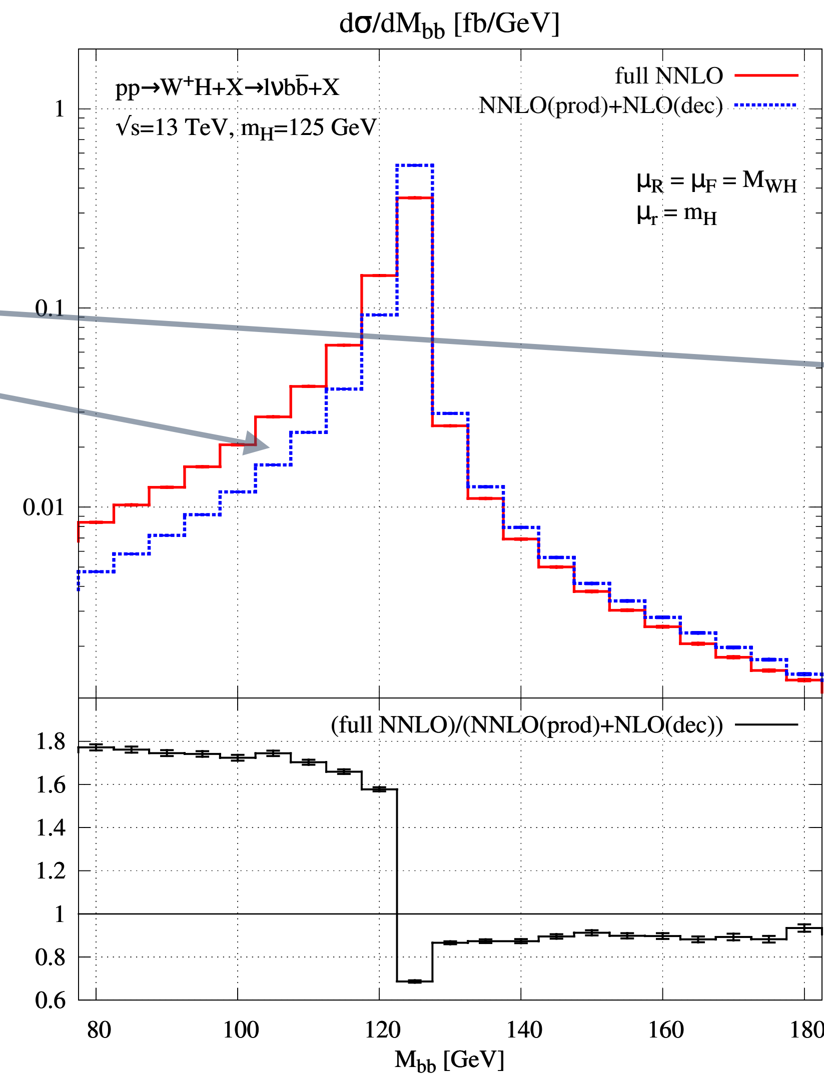
O(10 CPU hours)

O(100 CPU hours)

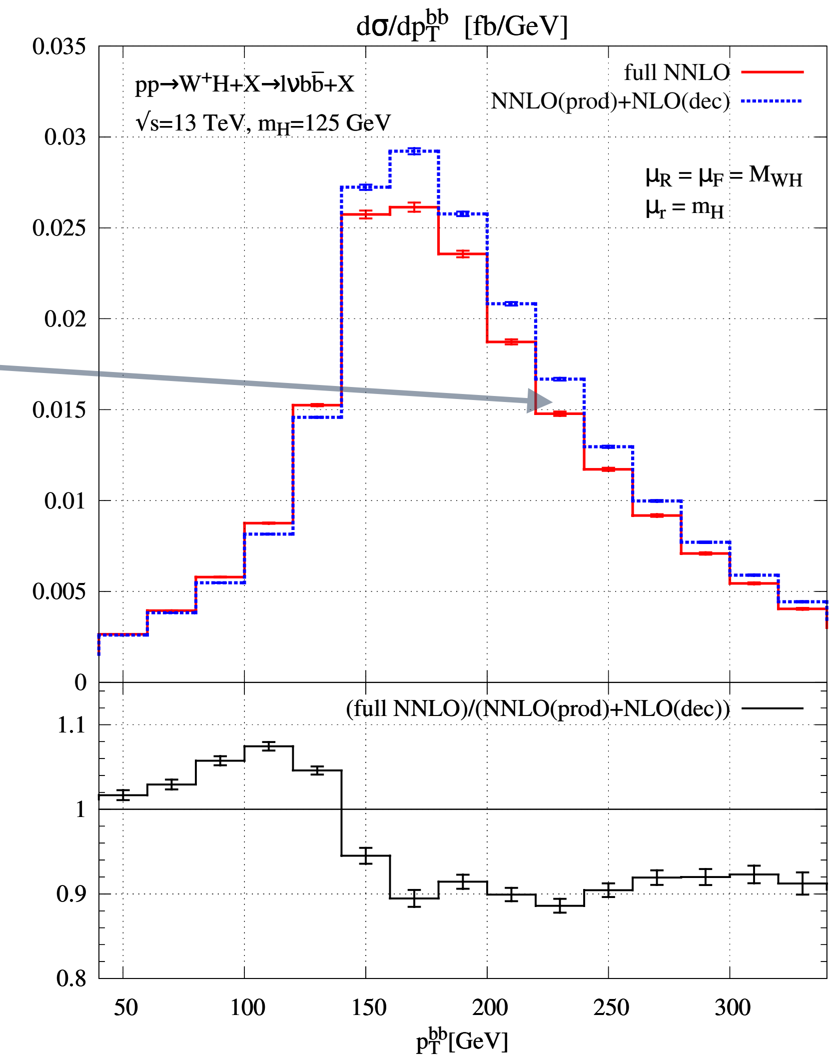
Application: VH, H→bb decay@NNLO

- NNLO corrections for decay recently computed
- LARGE CORRECTIONS IN THE EXP. FIDUCIAL REGION ($p_{T,W} > 150$ *GeV*)

Large corrections from radiation in the decay



Reconstructed Higgs mass

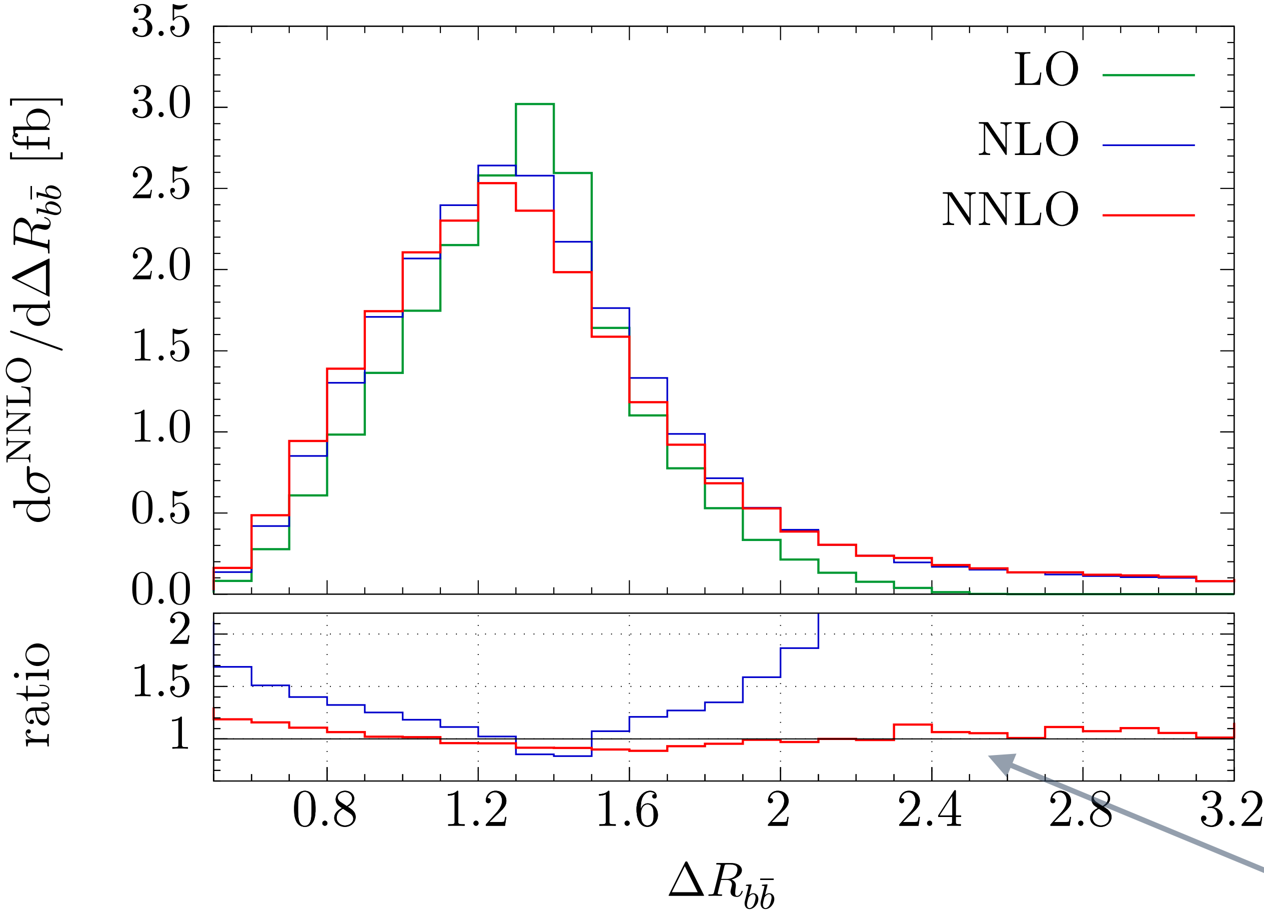


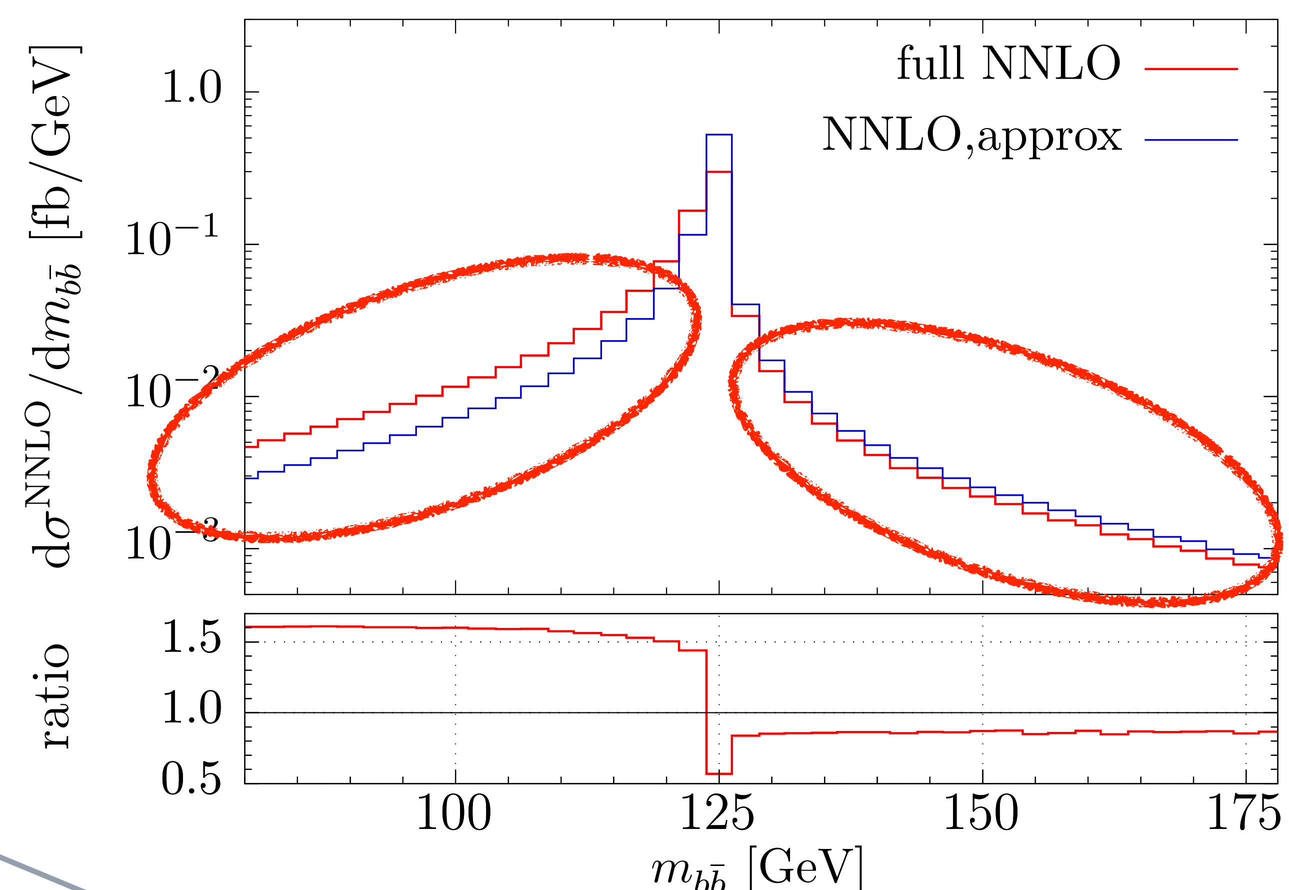
Reconstructed Higgs p_T

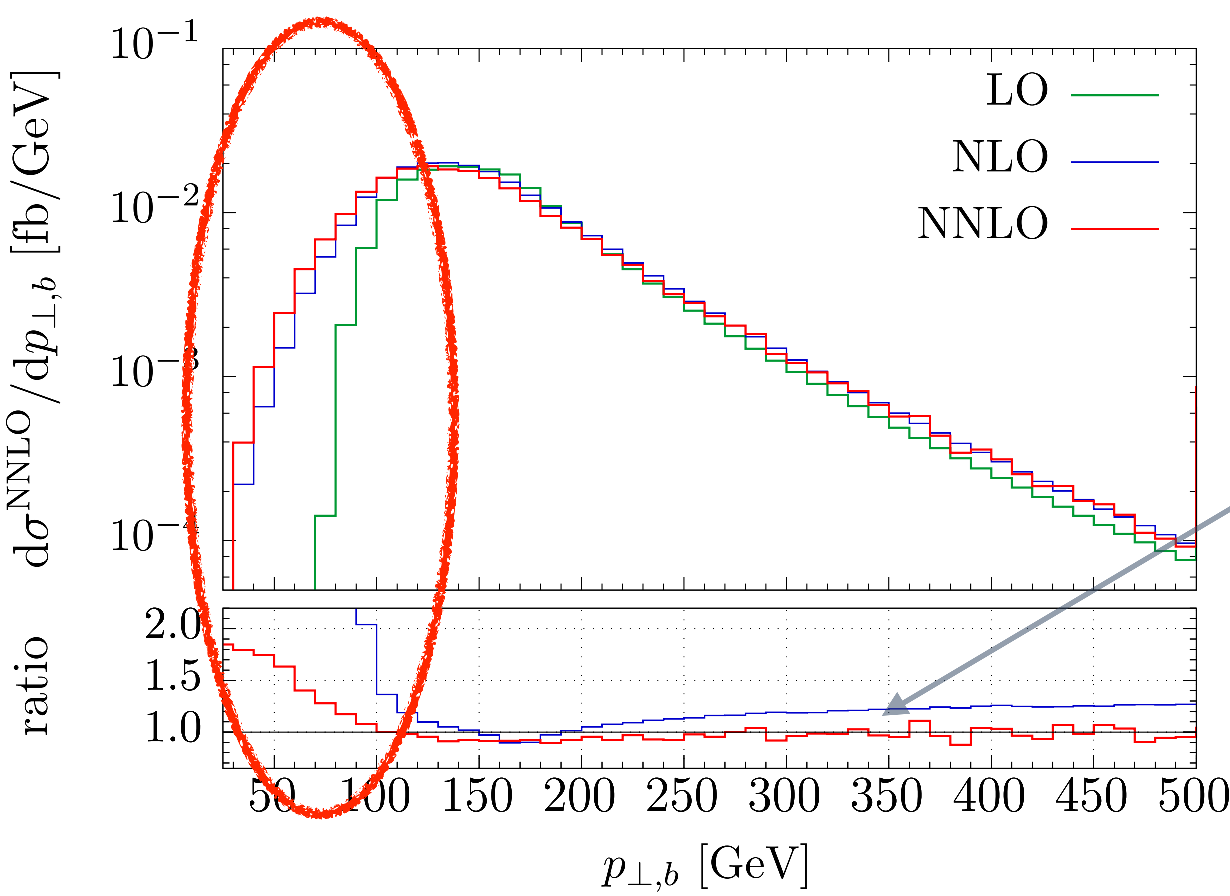
[Ferrera, Somogyi, Tramontano (2017)]

IN PRINCIPLE, PROBLEMATIC! *Where do large corrections come from?*

The source of large corrections



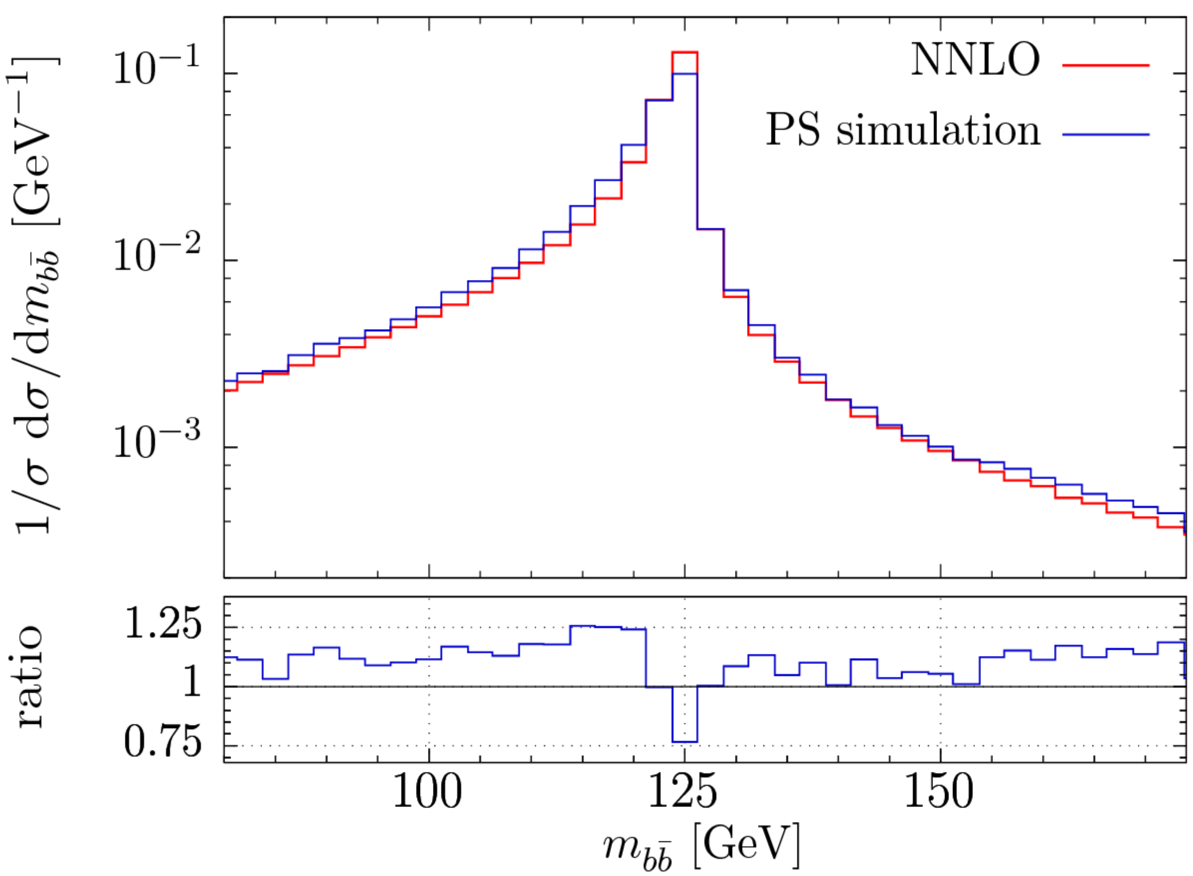


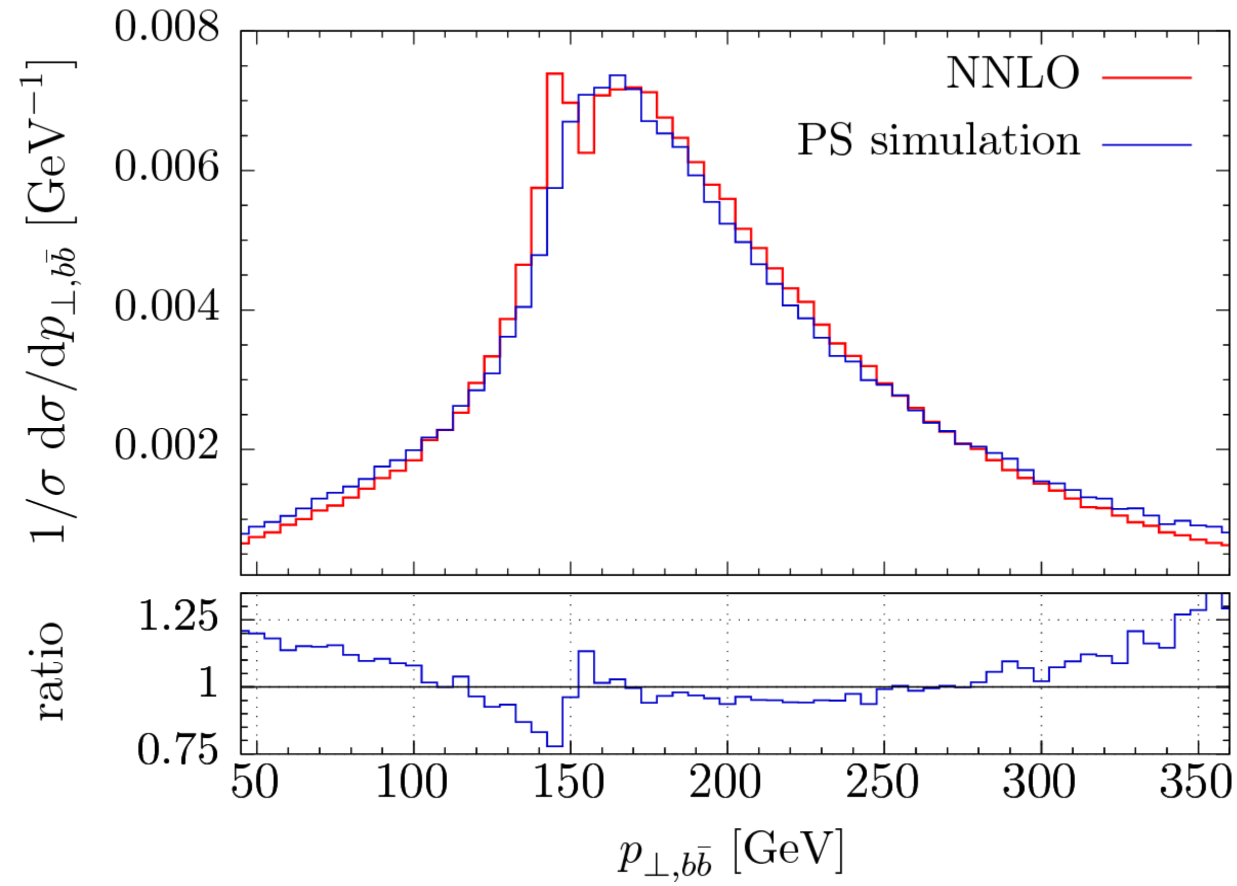


- Large corrections in regions not populated at LO
- Everywhere else: very stable NNLO!
- Dominated by extra parton emission
- *Captured by parton shower?*

VH, H→bb decay@NNLO: PS simulation

- Compare NNLO calculation with ``typical” simulation
- Out-of -the-shelf PS simulation: HWJ MiNLO, decay from Pythia8





- Bulk of the effect correctly captured by PS → *simulations used in analysis are reasonably good!*
- Similar trend seen in more sophisticated NNLOPS simulations [Astill et al (2018)]
- *We can now investigate structure of higher order emission. Detailed comparison fixed-order/(merged) PS now makes sense (similar considerations for VBF,jj…)*