Resummation

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Refs

Long history of fixed order calculations:

M. Ciafaloni, P. Ciafaloni, Comelli, Fadin, Lipatov, Martin, Melles, Kühn, Penin, Smirnov, Jantzen, Denner, Pozzorini, and many more

EFT approach — Chiu, Golf, Kelley, AM:

Phys. Rev. Lett. **100** (2008) 021802

Phys. Rev. D **77** (2008) 053004

Phys. Rev. D **78** (2008) 073006

Phys. Rev. D **80** (2009) 094013

Phys. Rev. D **81** (2010) 014023

Perturbation Series

$\mathsf{L} = \log Q^2/M^2$:

$$\mathbf{A} = \begin{pmatrix} 1 & & & & \\ \alpha \mathsf{L}^2 & \alpha \mathsf{L} & \alpha & & \\ \alpha^2 \mathsf{L}^4 & \alpha^2 \mathsf{L}^3 & \alpha^2 \mathsf{L}^2 & \alpha^2 \mathsf{L} & \alpha^2 \\ \alpha^3 \mathsf{L}^6 & & \ldots & & \\ \vdots & & & & \end{pmatrix}$$

$2n + 1$ terms at order n.

The logarithm of the scattering amplitude has an expansion of the form

$$\log \mathbf{A} = \begin{pmatrix} \alpha \mathsf{L}^2 & \alpha \mathsf{L} & \alpha & & \\ \alpha^2 \mathsf{L}^3 & \alpha^2 \mathsf{L}^2 & \alpha^2 \mathsf{L} & \alpha^2 & \\ \alpha^3 \mathsf{L}^4 & \alpha^3 \mathsf{L}^3 & \alpha^3 \mathsf{L}^2 & \alpha^3 \mathsf{L} & \alpha^3 \\ \alpha^4 \mathsf{L}^5 & & \ldots & & \\ \vdots & & & & \end{pmatrix}$$

$n + 2$ terms at order n.

Perturbation Series

Using L $\sim 1/\alpha$ (10^{19} TeV using α_2/π)

$$\mathbf{A} = \begin{pmatrix} 1 & & & & \\ \frac{1}{\alpha} & 1 & \alpha & & \\ \frac{1}{\alpha^2} & \frac{1}{\alpha} & 1 & \alpha & \alpha^2 \\ \frac{1}{\alpha^3} & & & \ldots & \\ \vdots & & & & \end{pmatrix}.$$

Clearly the fixed order perturbation expansion breaks down.
The terms in the exponentiated form are

$$\log \mathbf{A} = \begin{pmatrix} \frac{1}{\alpha} & 1 & \alpha & & \\ \frac{1}{\alpha} & 1 & \alpha & \alpha^2 & \\ \frac{1}{\alpha} & 1 & \alpha & \alpha^2 & \alpha^3 \\ \frac{1}{\alpha} & & & \ldots & \\ \vdots & & & & \end{pmatrix}.$$

Perturbation Series

If we write $f_n(\alpha L)$ for the N^{n}LL contribution:

$$\log \mathbf{A} = \frac{1}{\alpha} f_0 + f_1 + \alpha f_2 + \ldots$$
$$= \frac{1}{\alpha}\left[f_0 + \alpha f_1 + \alpha^2 f_2 + \ldots\right]$$

so that f_1 and f_2 are corrections to $\log \mathbf{A}$. However,

$$\mathbf{A} = \exp\left[\frac{1}{\alpha} f_0 + f_1 + \alpha f_2 + \ldots\right]$$
$$= e^{\frac{1}{\alpha} f_0} \times e^{f_1} \times e^{\alpha f_2} \times \ldots$$

and $\exp f_1$ can make a large change in $\mathbf{A}$. Only f_2 and higher can be considered as corrections to $\mathbf{A}$.

Perturbation Series

Leading-log-squared (LL^2) regime with $\alpha \mathsf{L}^2 \sim 1$ (10 TeV using α_2/π)

$$\mathbf{A} = \begin{pmatrix} 1 & & & & \\ 1 & \alpha^{1/2} & \alpha & & \\ 1 & \alpha^{1/2} & \alpha & \alpha^{3/2} & \alpha^2 \\ 1 & & & \ldots & \\ & & \vdots & & \end{pmatrix}$$

$$\log \mathbf{A} = \begin{pmatrix} 1 & \alpha^{1/2} & \alpha & & \\ \alpha^{1/2} & \alpha & \alpha^{3/2} & \alpha^2 & \\ \alpha & \alpha^{3/2} & \alpha^2 & \alpha^{5/2} & \alpha^3 \\ \alpha^{3/2} & & & \ldots & \\ \vdots & & & & \end{pmatrix}.$$

Only a few terms are needed for exponentiated form.

EFT Method

SCET (Bauer, Fleming, Luke, Pirjol, Stewart) applied to EW.

Hard scattering process with two scales, Q and M and $Q \gg M$.

1. Match onto hard scattering amplitude at $\mu_h \sim Q$
2. Evolve using RGE from μ_h to μ_l
3. Integrate out EW bosons at μ_l
4. Below μ_l, have the usual QCD complications for jets, PDFs, etc.

Simplest to run to some common scale μ_l and integrate out t, h, W, Z. This does not resum logs of the form

$$\left[\frac{\alpha_2}{4\pi}\log\frac{m_t^2}{M_W^2}\right]^n$$

These are not large, and do not need to be resummed.

EFT Method

$$\mathbf{A} = \exp\left[D_0(\alpha(M)) + D_L(\alpha(M))\log\frac{Q^2}{M^2}\right] \qquad \text{low-scale matching}$$

$$\times \exp\left\{-\int_M^Q \frac{\mathrm{d}\mu}{\mu}\left[A(\alpha(\mu))\log\frac{\mu^2}{Q^2} + B(\alpha(\mu))\right]\right\} \qquad \text{RG evolution}$$

$$\times \exp C(\alpha(Q)) \qquad \text{high-scale matching}$$

- A is the cusp anomalous dimension — Sudakov double logs.
- Anomalous dimension is linear in $\log \mu^2/Q^2$ to all orders
- D is linear in $\log Q^2/M^2$ to all orders. new feature in EW case.
- D_L and A proportional to $\mathbb{1}$.
- can have dependence on conformal ratios at four loops and beyond

$$\frac{(n_1 \cdot n_2)(n_3 \cdot n_4)}{(n_1 \cdot n_4)(n_2 \cdot n_3)}$$

Chiu et al.: General factorization theorem

EFT Method

Chiu et al.

Universal collinear functions which depend on the particle, but are process independent.

Soft functions which are of the form

$$\sum_{\langle ij \rangle} \mathbf{T}_i \cdot \mathbf{T}_j \log \frac{(-n_i \cdot n_j - i0^+)}{2}$$

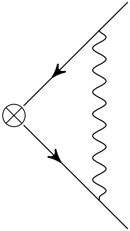
Everything is known expect for the high-scale matching (known for $2 \to 2$).

EFT Method

Series	A	$\mathbf{B}$	D_L	$\mathbf{D}_0$	$\mathbf{c}$
LL	$1\surd$	$-$	$-$	$-$	$-$
NLL	$2\surd$	$1\surd$	$1\surd$	$-$	$-$
NNLL	$3\surd*$	$2\surd*$	2	$1\surd$	$1\surd$

$\surd*$ means the term is known except for the scalar contribution.

Sudakov Form Factor



$$\log F_E(Q) = D(M) + \int_Q^M \frac{\mathrm{d}\mu}{\mu}\gamma(\mu) + C(Q)$$

Use

$$a = \frac{\alpha}{4\pi} \qquad A = A^{(1)}a + A^{(2)}a^2 + \ldots, \text{etc.}$$

Convert all α to a common scale using β-function.
$F_E(Q)$ using these values agrees with Jantzen and Smirnov, Eur. Phys. J. C47, 671 (2006), hep-ph/0603133.

Sudakov Form Factor

$$
\begin{aligned}
\log F_E(Q) = 1 + a(M)&\left[\frac{1}{4}A^{(1)}\mathsf{L}^2 + \left(D_L^{(1)} - \frac{1}{2}B^{(1)}\right)\mathsf{L} + C^{(1)} + D_0^{(1)}\right] \\
&+ a(M)^2\left[-\frac{1}{12}A^{(1)}\beta_0\mathsf{L}^3 + \left(\frac{1}{4}A^{(2)} + \frac{1}{4}B^{(1)}\beta_0\right)\mathsf{L}^2\right. \\
&\left. + \left(D_L^{(2)} - \frac{1}{2}B^{(2)} - C^{(1)}\beta_0\right)\mathsf{L} + C^{(2)} + D_0^{(2)}\right] \\
&+ a(M)^3\left[-\frac{1}{24}A^{(1)}\beta_0^2\mathsf{L}^4 + \ldots\right] + \ldots
\end{aligned}
$$

In LL^2 regime:

1 :	$A^{(1)}$	one loop cusp
$\alpha^{1/2}$:	$B^{(1)}, D_L^{(1)}, \beta_0$	one loop non-cusp, one-loop D_L
α :	$A^{(2)}, D_0^{(1)}, C^{(1)}$	two loop cusp, one-loop D_0, one loop C

Sudakov Form Factor

Fixed order expression much more complicated.

$$
\begin{aligned}
F_E(Q) &= 1 + a(M)\left[\frac{1}{4}A^{(1)}\mathsf{L}^2 + \left(D_L^{(1)} - \frac{1}{2}B^{(1)}\right)\mathsf{L} + C^{(1)} + D_0^{(1)}\right] \\
&+ a(M)^2\left[\frac{1}{32}\left[A^{(1)}\right]^2\mathsf{L}^4 + \left(\frac{1}{4}D_L^{(1)}A^{(1)} - \frac{1}{8}A^{(1)}B^{(1)} - \frac{1}{12}A^{(1)}\beta_0\right)\mathsf{L}^3\right. \\
&+ \left(\frac{1}{8}\left[B^{(1)}\right]^2 + \frac{1}{4}\beta_0 B^{(1)} - \frac{1}{2}D_L^{(1)}B^{(1)} + \frac{1}{2}\left[D_L^{(1)}\right]^2 + \frac{1}{4}A^{(2)} + \frac{1}{4}A^{(1)}C^{(1)}\right. \\
&+ \left.\frac{1}{4}A^{(1)}D_0^{(1)}\right)\mathsf{L}^2 + \frac{1}{2}\left(-B^{(2)} - B^{(1)}C^{(1)} - B^{(1)}D_0^{(1)} + 2C^{(1)}D_L^{(1)}\right. \\
&+ \left.2D_0^{(1)}D_L^{(1)} + 2D_L^{(2)} - 2C^{(1)}\beta_0\right)\mathsf{L} \\
&+ \left.\frac{1}{2}\left(\left[C^{(1)}\right]^2 + 2D_0^{(1)}C^{(1)} + \left[D_0^{(1)}\right]^2 + 2C^{(2)} + 2D_0^{(2)}\right)\right] + \ldots \\
&+ a(M)^3\left[\frac{1}{6144}A_1^4\mathsf{L}^8 + \ldots\right] + \ldots
\end{aligned}
$$

Sudakov Form Factor

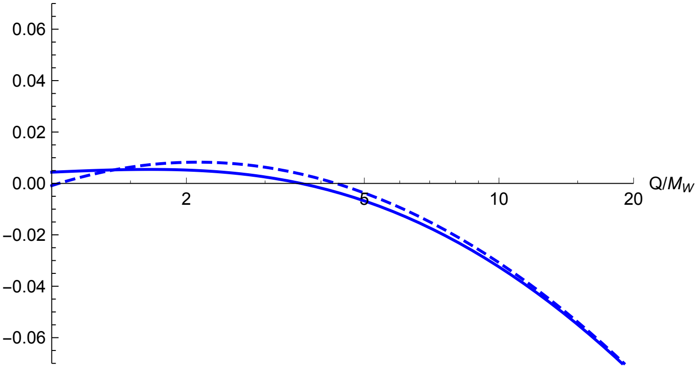
$a^2 L^4, a^2 L^3$ in terms of one-loop quantities

$a^2 L^2$ needs two-loop cusp

$a^2 L$ needs two-loop non-cusp and two-loop D_L.

a^2 needs two-loop matching. [not done in Jantzen and Smirnov]

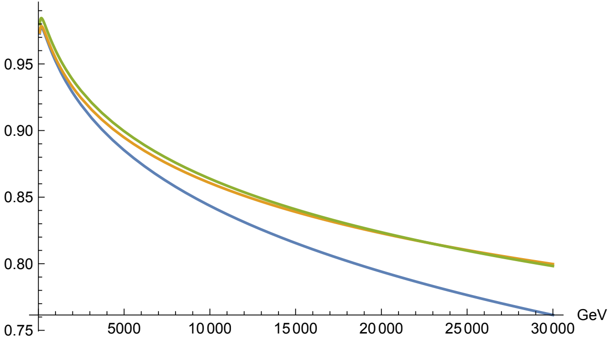
Sudakov Form Factor: Power Corrections



solid: exact
dashed: neglect M^2/Q^2 corrections

Plot of $F_E - 1$ with $\alpha = \alpha_2$.

Sudakov Form Factor: Q

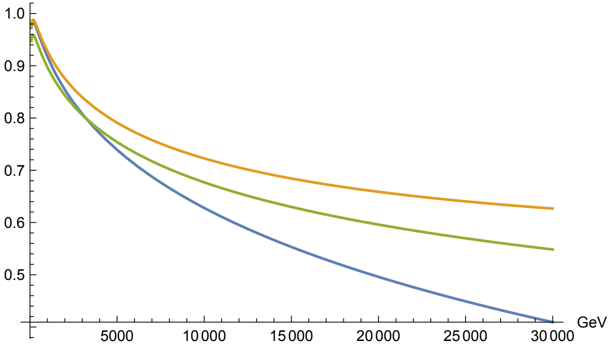


blue: order α
orange: order α^2
green: resummed

Square to get cross section

Initial state also contributes.

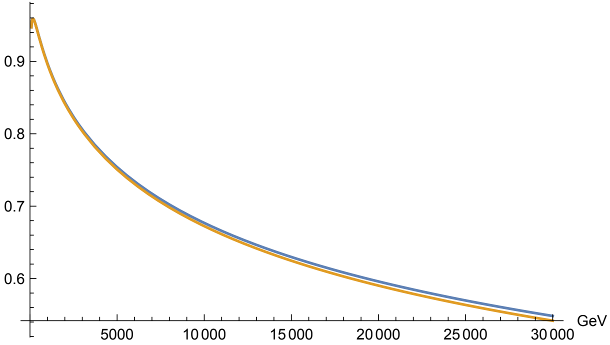
Sudakov Form Factor: W



blue: order α
orange: order α^2
green: resummed

$(\alpha/\pi)^2 \sim 0.001$ at 30 TeV.

Sudakov Form Factor: Two-loop Cusp for W

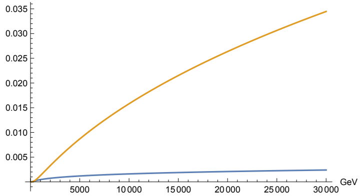


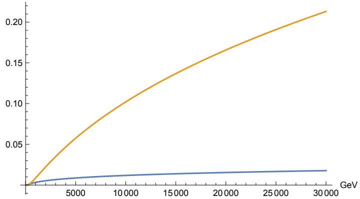
blue: resummed with one-loop cusp
orange: resummed with two-loop cusp

Size of Resummation

Quark(upper) and W(lower): difference between resummed form and fixed order. The cusp anomalous dim needs to be resummed.

$$e^{\alpha x} - (1 + \alpha x) \sim \mathcal{O}(\alpha^2)$$





orange=cusp anomalous dim contribution

blue = non-cusp anomalous dim contribution