

Neutral Naturalness

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Physics at the LHC and beyond

CERN-Korea TH Institute

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Neutral Naturalness

- Practical definition: symmetry-based solutions to the little hierarchy problem with **color-less top partners**
- Stabilize weak scale up to $\Lambda \sim 5 - 10 \text{ TeV}$, large hierarchy cured by SUSY / compositeness / clockwork /...
- Phenomenology radically non-standard, poses challenges to collider-based experiments
- Important interplay with other areas, especially cosmo

About this discussion

- I will cover a few selected aspects of Neutral Naturalness, **focusing mostly on very recent work**

- Not a comprehensive review.

I expect **you** will contribute more topics to the discussion

- No relaxion & related theories, although they may broadly fit in NN
See dedicated sessions later in this workshop

Plan

- (1) SM-singlet scalar top partners**
 - Tripled Top
 - Hyperbolic Higgs
- (2) Z_N models**
- (3) Probing UV completions at colliders**
- (4) Signatures in cosmology**

Plan

(1) SM-singlet scalar top partners

- **Tripled Top**
- **Hyperbolic Higgs**

(2) Z_N models

(3) Probing UV completions at colliders

(4) Signatures in cosmology

Singlet scalar top partners

- The top partner zoo

Curtin, Verhaaren 2015

		scalar	fermion
strong direct production {	<i>QCD</i>	SUSY	Composite Higgs/ RS
<i>DY</i> direct production {	<i>EW</i>	folded SUSY	Quirky Little Higgs
Higgs portal direct production {	<i>singlet</i>	?	Twin Higgs

Mirror Glueballs
Higgs portal observables

Higgs coupling shifts
~ tuning

Singlet scalar top partners

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Mirror Glueballs
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Higgs coupling shifts
 \sim tuning

$$\mathcal{L}_{\text{FSUSY}} \sim y_t q_A H u_A^c + y_t^2 |\tilde{q}_B H|^2 + y_t^2 |\tilde{u}_B^c|^2 |H|^2$$

In Folded SUSY, folded stops **carry SM electroweak charges**

Singlet scalar top partners

- The top partner zoo

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Models with singlet scalar top partners?

From zero to two:

- The Triple Top
- The Hyperbolic Higgs

Cheng, Li, Salvioni, Verhaaren 2018

Cohen, Craig, Giudice, McCullough 2018

Plan

(1) SM-singlet scalar top partners

- **Tripled Top**
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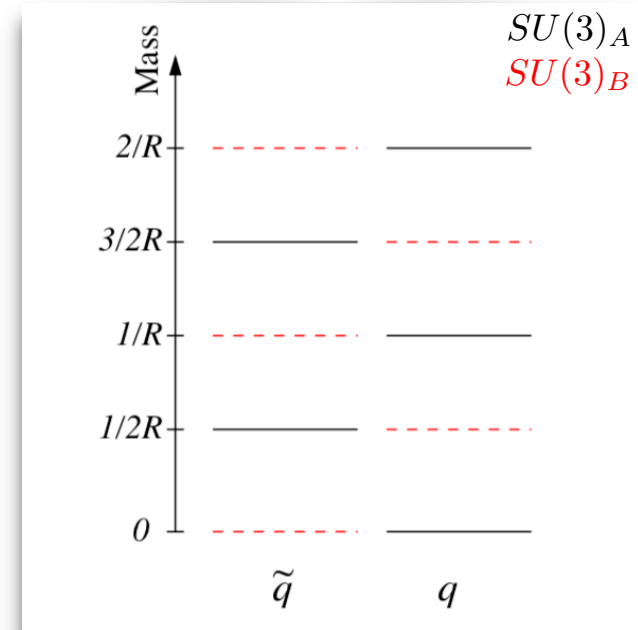
Folded Supersymmetry

- **Folded SUSY**

$$SU(3)_A \times \textcolor{red}{SU(3)}_B \times SU(2) \times U(1)$$

$$Z_2$$

Burdman, Chacko,
Goh, Harnik 2016


- Orbifold extra dimension with Scherk-Schwarz SUSY breaking, only SM fermions + folded scalars have **zero modes**
- An **accidental SUSY** is preserved
- Contribution of top sector to Higgs mass vanishes *exactly* at 1-loop



Folded Supersymmetry

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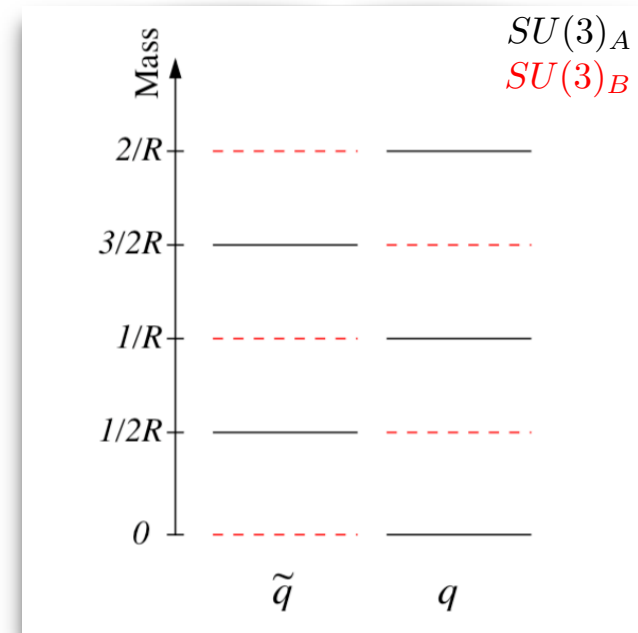

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- Contribution of top sector to Higgs mass vanishes *exactly* at 1-loop
- Protection of Higgs mass is **“too effective:”**
Gauge/gaugino 1-loop term dominates,
vacuum preserves EW symmetry

Cohen, Craig, Lou,
Pinner 2015

$$\delta m_H^2 \approx + \frac{21\zeta(3)g^2}{64\pi^4 R^2}$$



Folded Supersymmetry

Can we build a model with accidental SUSY in pure 4D?

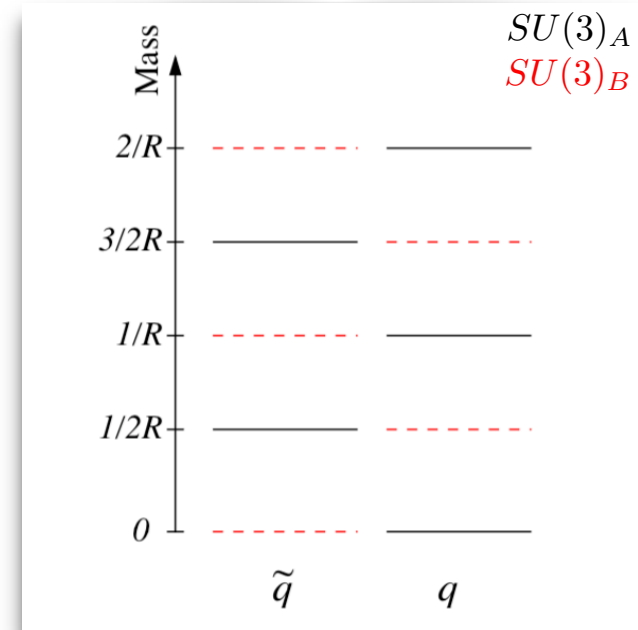
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A Tripled Top model

Cheng, Li, Salvioni,
Verhaaren 2018

- Add **two** copies of the MSSM top sector,

$$SU(3)_A \times SU(3)_B \times SU(3)_C \times SU(2) \times U(1)$$

- Superpotential

few TeV

$$W = y_t (Q_A H u_A^c + Q_B H u_B^c + Q_C H u_C^c) + M(u'_B u_B^c + u'_C u_C^c)$$

Z_3

Z_2

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Z_3

Z_2

- Leading soft masses

$$V_s = +\tilde{m}^2 \left(|\tilde{Q}_A|^2 + |\tilde{u}_A^c|^2 \right) - \tilde{m}^2 \left(|\tilde{u}_B^c|^2 + |\tilde{u}_C^c|^2 \right)$$

raise SM-colored stops

lower $SU(2)$ -singlet
hidden stops

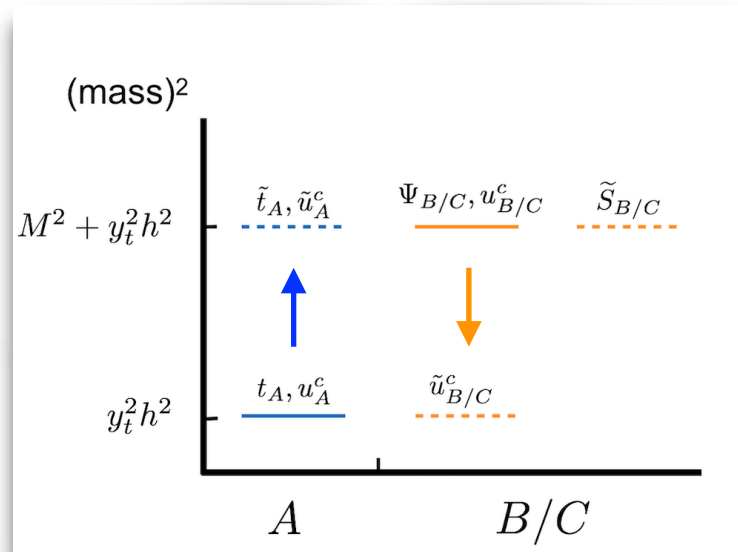
A Tripled Top model

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accidental SUSY

for

$$\tilde{m} \rightarrow M$$



- Superpotential

few TeV

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A Tripled Top model

- Departures from accidental SUSY limit: $\tilde{m} \neq M$

+ SUSY mass for doublets, $\omega(Q_B Q'_B{}^c + Q_C Q'_C{}^c) \in W$

Both OK as long as $\sqrt{M^2 - \tilde{m}^2}$, $\omega \ll \text{TeV}$, for example

$$\delta m_H^2 \approx -\frac{N_c y_t^2}{8\pi^2} \omega^2 \ln \frac{M^2}{\omega^2}$$

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also: **Kats, McCullough, Perez, Soreq, Thaler 2017**

- Hypercharge assignments for hidden fields are **free**,
only requirement is invariance of Yukawas

$$W = y_t (Q_A H u_A^c + Q_B H u_B^c + Q_C H u_C^c)$$



We can choose $Q_{B,C} \sim 2_{-1/2}$

$$u_{B,C}^c \sim 1_0$$

**SM-singlet scalar
top partners**

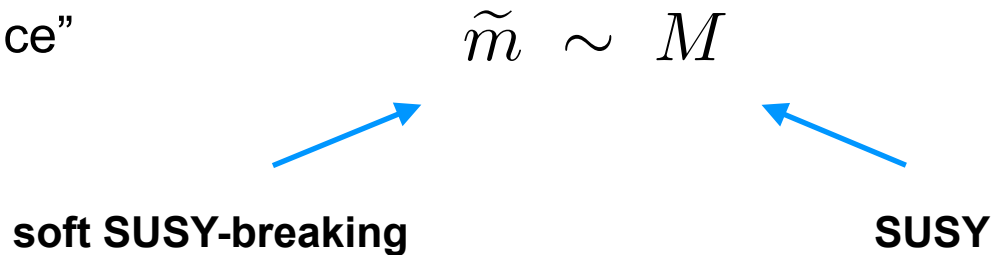
Necessary ingredients

- A particular structure for the soft masses

$$V_s = +\tilde{m}^2 \left(|\tilde{Q}_A|^2 + |\tilde{u}_A^c|^2 \right) - \tilde{m}^2 \left(|\tilde{u}_B^c|^2 + |\tilde{u}_C^c|^2 \right)$$

Possible origins in next slide

- A “coincidence”



If no mechanism can explain it, **tuning** $\sim \frac{\Delta^2}{M^2} \sim \text{few } \%$

$$(\Delta = \sqrt{M^2 - \tilde{m}^2})$$

$$M \sim \text{few TeV}$$

$$\Delta \sim \text{few} \times (100 \text{ GeV})$$

The soft masses?

- Soft masses of equal size and opposite sign?

$$V_s = +\tilde{m}^2 \left(|\tilde{Q}_A|^2 + |\tilde{u}_A^c|^2 \right) - \tilde{m}^2 \left(|\tilde{u}_B^c|^2 + |\tilde{u}_C^c|^2 \right)$$

1. First guess: D -term of an extra $U(1)$, charges +1 and -1

But then, Yukawas are not invariant $W \ni y_t (Q_A H u_A^c + Q_B H u_B^c + Q_C H u_C^c)$

Insertions of $U(1)$ -breaking field will spoil the Z_3

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2. **Working model:** exploit properties of strongly coupled SUSY gauge theories

Top fields are **composite mesons** $P_i \bar{P}_j$ of s-confining SQCD

$$SU(N), \quad F = N + 1$$

Arkani-Hamed, Rattazzi 1998

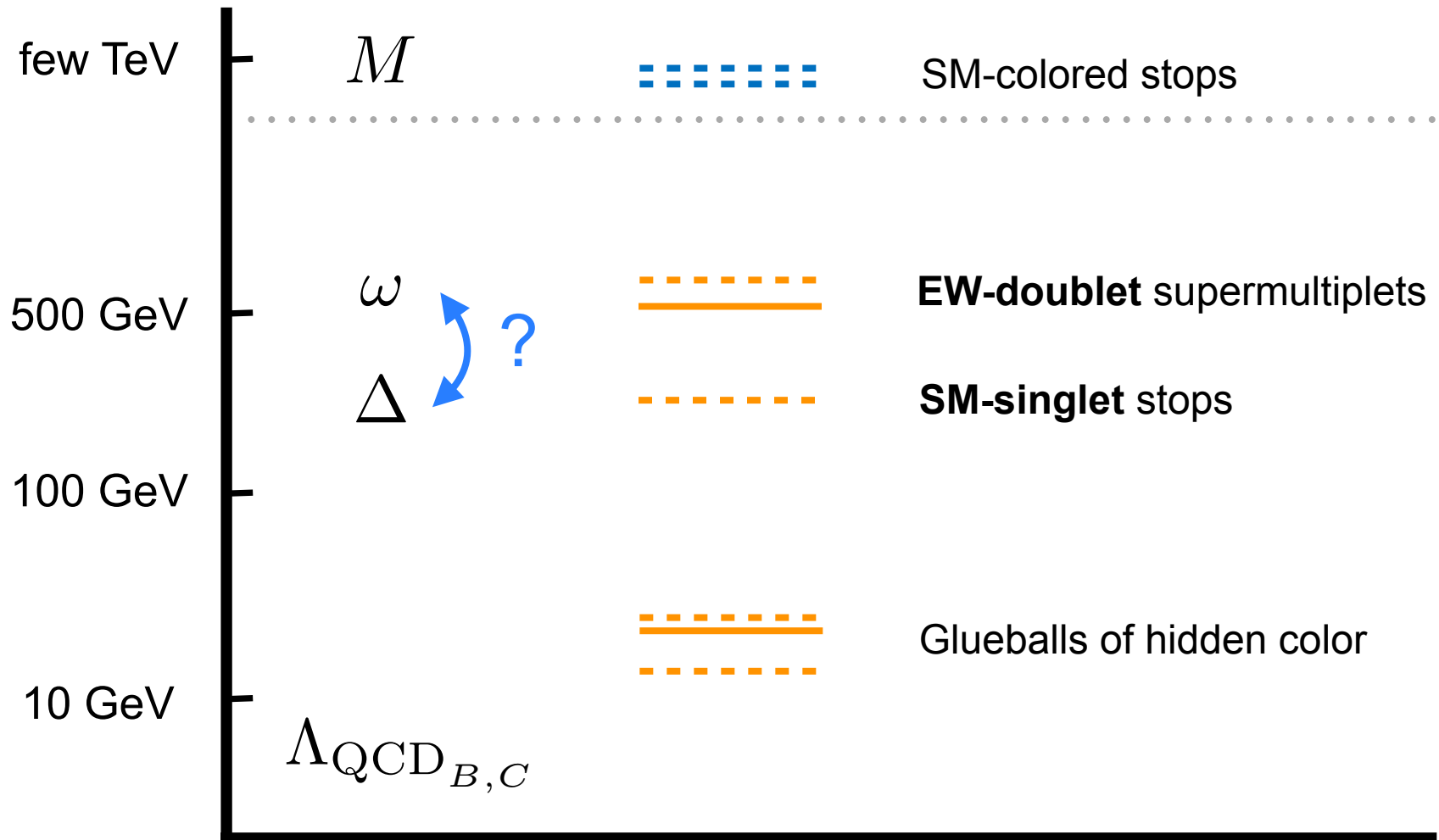
$$m_{ij}^2 = m_{P_i}^2 + m_{\bar{P}_j}^2 - \frac{2}{b} \sum_k T_{r_k} (m_{P_k}^2 + m_{\bar{P}_k}^2)$$

soft masses of IR composites

soft masses of UV constituents

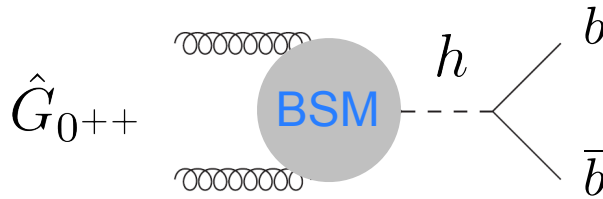
Spectrum of BSM states

mass



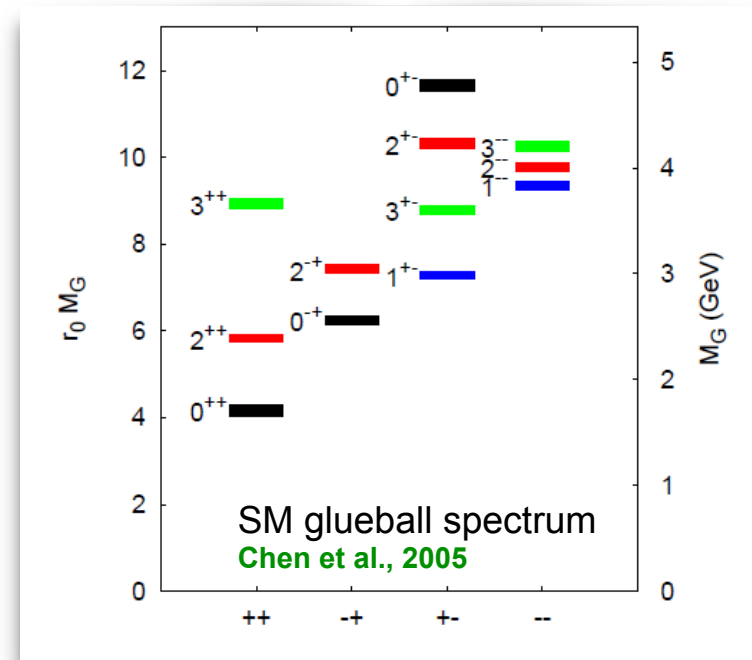
Hidden sector confinement

- Hidden QCD confines at few GeV
- No light matter, low-energy spectrum is made of **glueballs**
- Lightest glueball has $J^{PC} = 0^{++}$, decays to SM via mixing with the Higgs



$$c\tau_{0^{++}} \sim 1.2 \text{ m} \left(\frac{5 \text{ GeV}}{\Lambda_{\text{QCD}_{B,C}}} \right)^7 \left(\frac{\omega}{500 \text{ GeV}} \right)^4 \left(\frac{\Delta}{300 \text{ GeV}} \right)^4 \left(\frac{100 \text{ GeV}}{\delta m} \right)^4$$

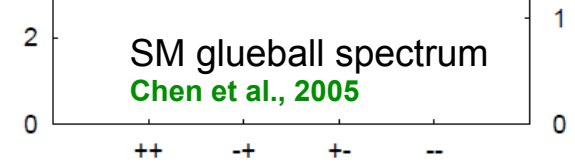
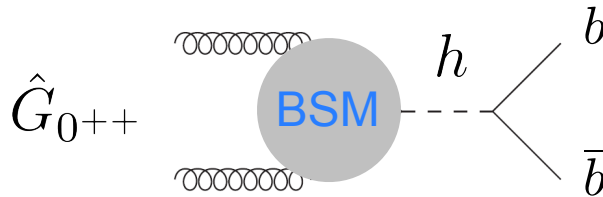
- Lifetime is much **longer** than e.g. in Folded SUSY ($\sim \text{mm}$)
- **Large uncertainty** because depends on **subleading soft masses**



Hidden sector confinement

Assume hidden glueballs escape LHC detectors

Look for other, more robust signatures

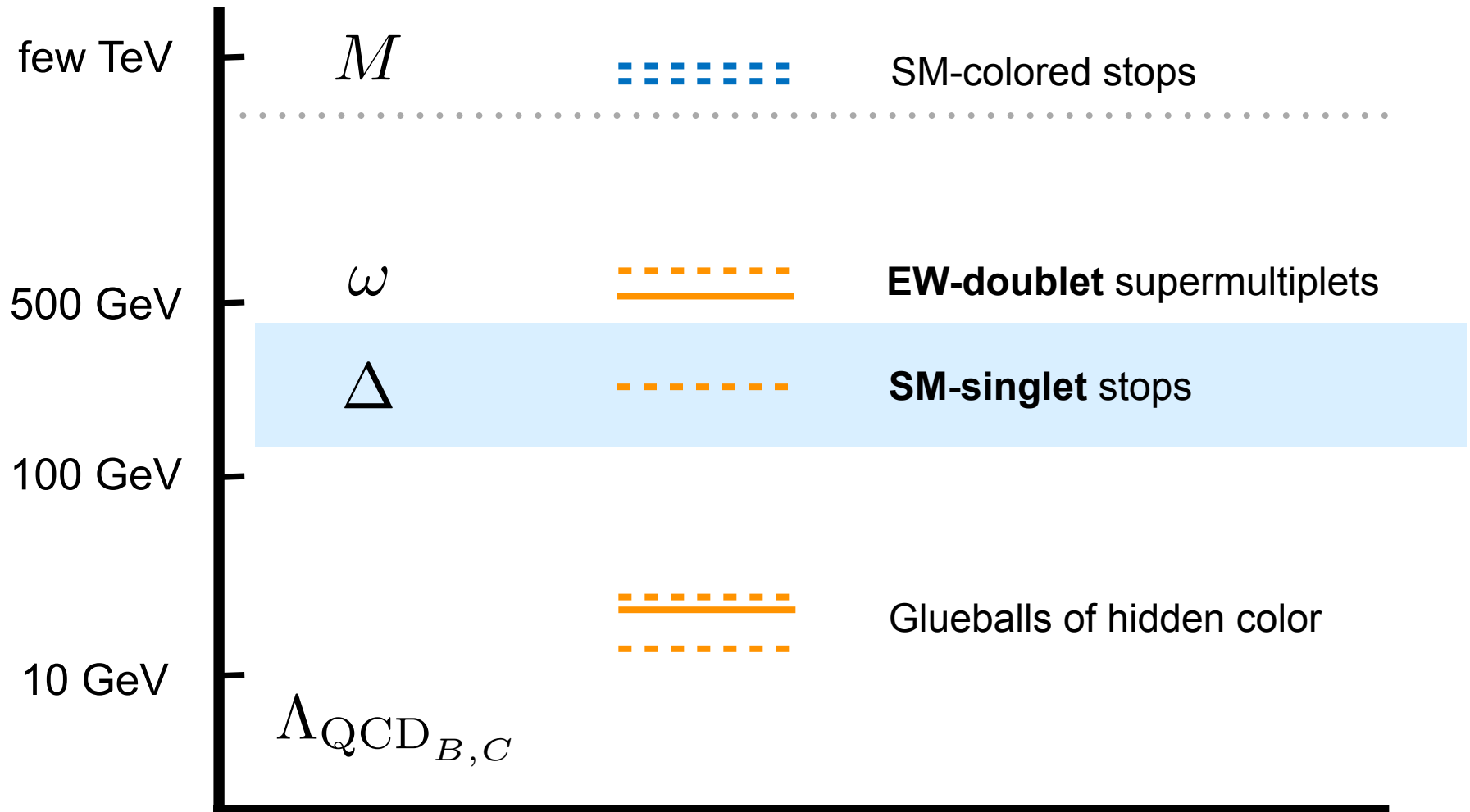


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Spectrum of BSM states: $\Delta < \omega$

mass



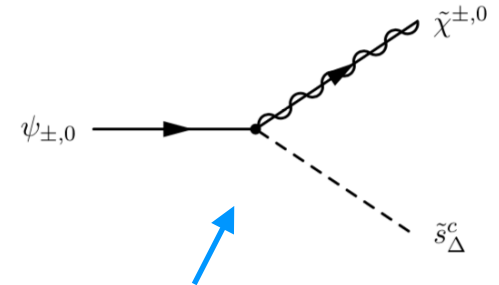
$\Delta < \omega$: light singlet scalars

- If $\Delta < \omega$, the **singlet scalars** are at the bottom of matter spectrum in hidden sectors
- Dominant production of heavier EW-doublet states, they decay down to light scalar \tilde{s}_Δ^c



typical LHC event results

in formation of $\tilde{s}_\Delta^c \tilde{s}_\Delta^{c*}$ “squirky” pair



*decay of fermions
requires light neutralino*

How does the $\tilde{s}_\Delta^c \tilde{s}_\Delta^{c*}$ system de-excite?

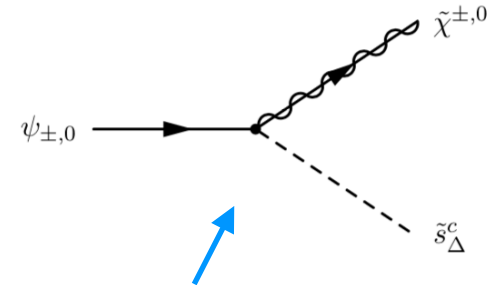
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How does the $\tilde{s}_\Delta^c \tilde{s}_\Delta^{c*}$ system de-excite?

Glueball radiation is prompt, but does not complete de-excitation

Residual kinetic energy

$$K \lesssim m_0 \simeq 7\Lambda_{\text{QCD}_{B,C}} \longleftrightarrow n \sim 10$$

$\Delta < \omega$: light singlet scalars

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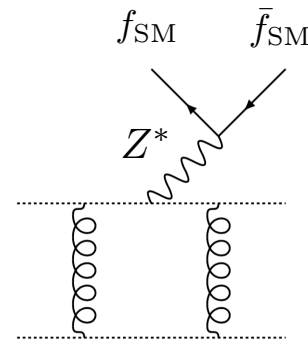


typical LHC event results

in formation of $\tilde{s}_\Delta^c \tilde{s}_\Delta^{c*}$ “squirky” pair

How does the $\tilde{s}_\Delta^c \tilde{s}_\Delta^{c*}$ system de-excite?

The Higgs VEV gives a **small mass mixing** of singlet and doublet scalars, \tilde{s}_Δ^c inherits **coupling to the Z**



$$t_{\text{de-excite}}^Z \sim \frac{32}{27\pi^4} \frac{\cos^4 \theta_w}{\alpha_W^2 \sin^4 \phi_R N_f} \frac{m_Z^4 m_{\tilde{s}_\Delta^c}^4 m_0^3}{\sigma^6} \sim 4 \cdot 10^{-13} \text{ s} \left(\frac{5 \text{ GeV}}{\Lambda_{\text{QCD}_{B,C}}} \right)^9 \left(\frac{m_{\tilde{s}_\Delta^c}}{300 \text{ GeV}} \right)^4$$

$\sim 0.1 \text{ mm}$, still prompt

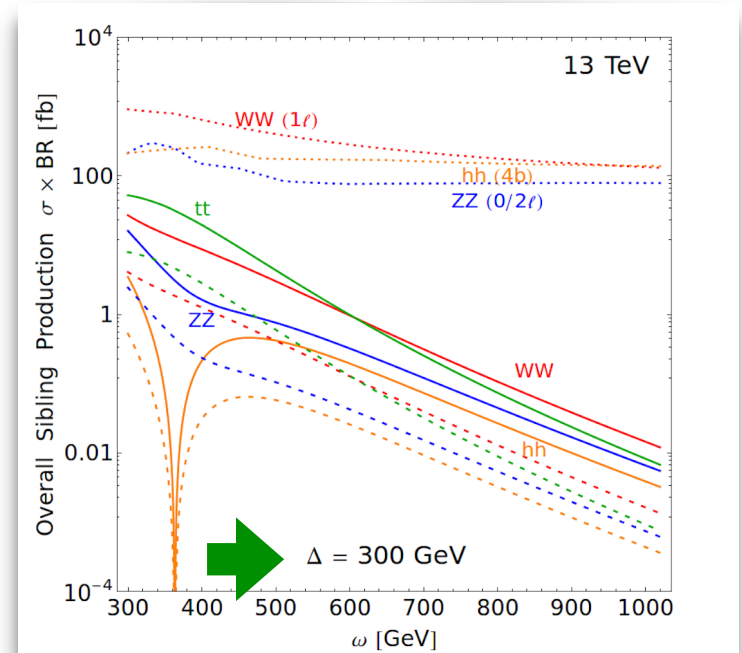
$\Delta < \omega$: light singlet scalars

- Lowest-lying bound state is 0^{++}
- Annihilates dominantly to hidden glueballs, $\text{BR}(\text{SM}) \sim \%$ level

see also: [Burdman, Lichtenstein 2018](#)

➡ Resonant signals well below current sensitivity

➡ **Very light singlets are allowed**

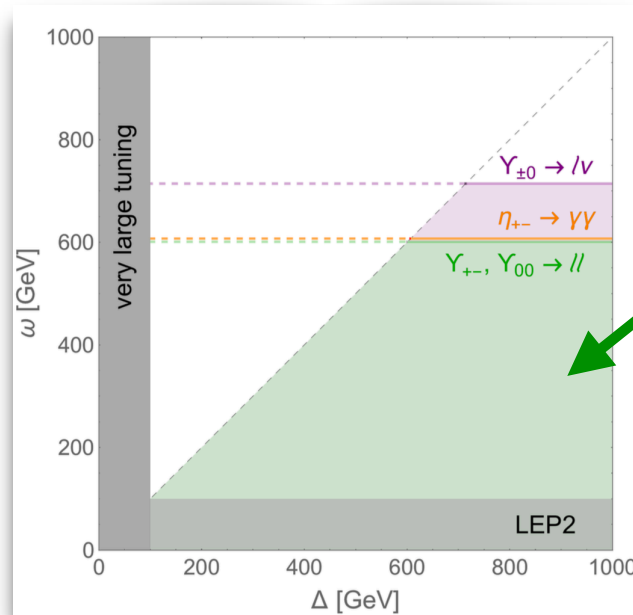


- Extra particles from cascade decays may give further constraints

**Cheng, Li, Salvioni,
Verhaaren, in progress**

Tripled Top parameter space

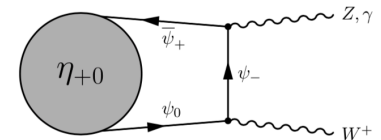
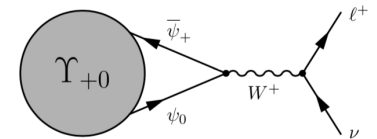
EW-charged
supermultiplets



singlet scalar top partners

$$\Delta > \omega$$

**Drell-Yan
+
resonant signals**



Cheng, Li, Salvioni,
Verhaaren 2018

Plan

(1) SM-singlet scalar top partners

- Tripled Top
- **Hyperbolic Higgs**

(2) Z_N models

(3) Probing UV completions at colliders

(4) Signatures in cosmology

The Hyperbolic Higgs

Cohen, Craig, Giudice,
McCullough 2018

- Tree-level potential with flat direction

$$V = \lambda (|H_{\mathcal{H}}|^2 - |H|^2 - f^2)^2$$

Accidentally $U(2,2)$ symmetric [not a symmetry of full theory]

- Each Higgs charged under its own $SU(2) \times U(1)$

One massless mode,

$$h_{\text{SM}} = \cos \theta h + \sin \theta h_{\mathcal{H}} \qquad \tan \theta = \frac{v}{v_{\mathcal{H}}}$$

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Cohen, Craig, Giudice,
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$$V = \lambda (|H_{\mathcal{H}}|^2 - |H|^2 - f^2)^2$$

Accidentally $U(2,2)$ symmetric [not a symmetry of full theory]

- Couplings to matter

Z₂ symmetry


$$\mathcal{L} = (y_t H \psi_Q \psi_{U^c} + \text{h.c.}) + y_t^2 \left(|H_{\mathcal{H}} \cdot \tilde{Q}_{\mathcal{H}}|^2 + |H_{\mathcal{H}}|^2 |\tilde{U}_{\mathcal{H}}^c|^2 \right)$$

quadratic 1-loop correction



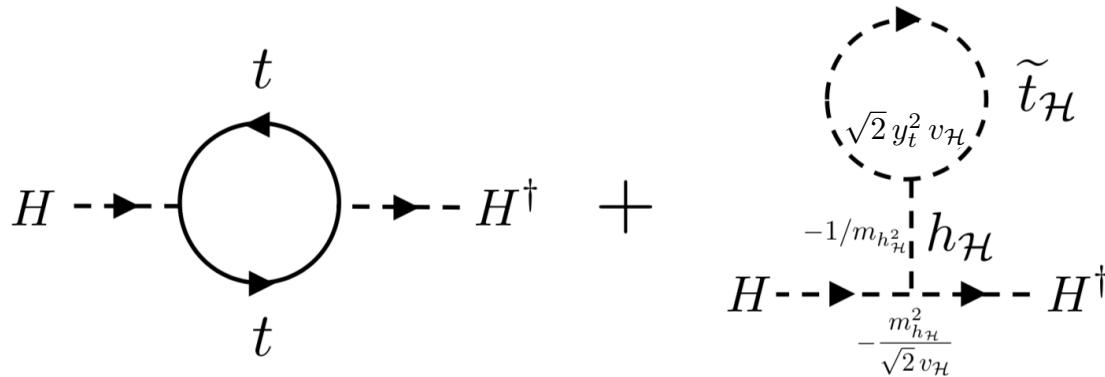
$$\delta V \sim \frac{N_c y_t^2}{16\pi^2} \Lambda^2 (|H_{\mathcal{H}}|^2 - |H|^2)$$

respects $U(2,2)$

The Hyperbolic Higgs

Cohen, Craig, Giudice,
McCullough 2018

- Diagrammatic cancellation

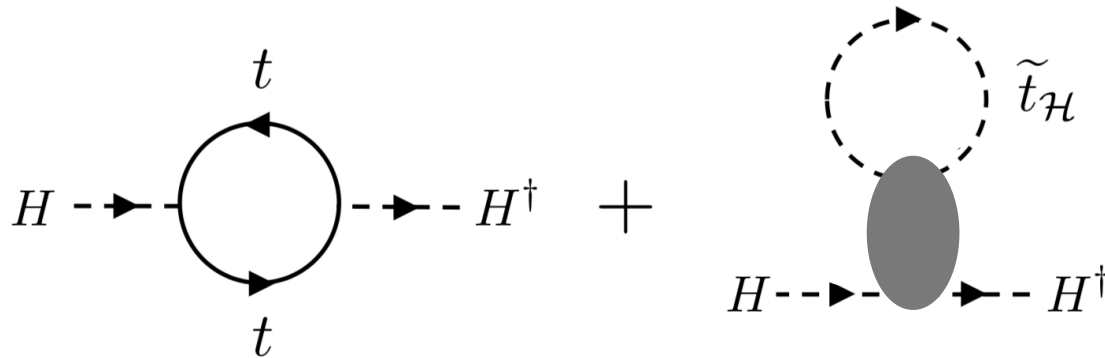


$$\mathcal{L} = (y_t H \psi_Q \psi_{U^c} + \text{h.c.}) + y_t^2 \left(|H_{\mathcal{H}} \cdot \tilde{Q}_{\mathcal{H}}|^2 + |H_{\mathcal{H}}|^2 |\tilde{U}_{\mathcal{H}}^c|^2 \right)$$

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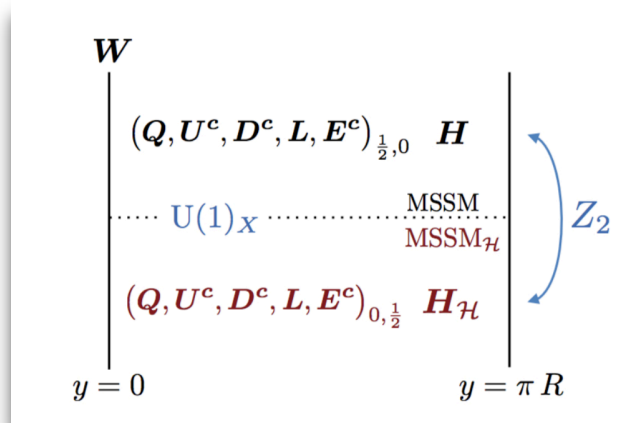
Integrate out heavy radial mode

$$\mathcal{L}_{\text{eff}} = (y_t H \psi_Q \psi_{U^c} + \text{h.c.}) + y_t^2 |H|^2 (|\tilde{t}_{\mathcal{H}}^L|^2 + |\tilde{t}_{\mathcal{H}}^R|^2)$$



$SU(2)_{\mathcal{H}}$ broken below $v_{\mathcal{H}}$

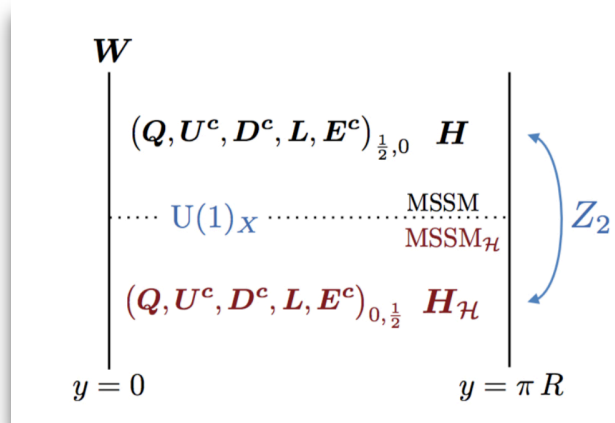
A 5D SUSY completion



$$\Lambda \sim 1/R$$

- $U(1)_X$ D-term potential $V_X = \frac{g_X^2}{2} \xi (|H_{\mathcal{H}}|^2 - |H|^2 - f_X^2)^2$

A 5D SUSY completion



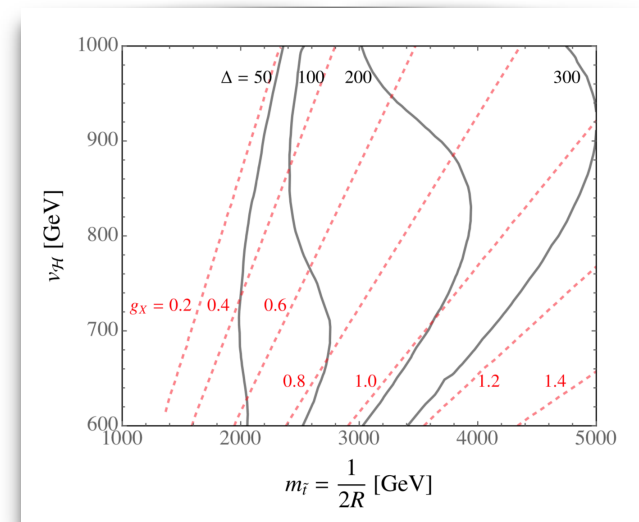
$$\Lambda \sim 1/R$$

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- SUSY breaking gives at 1-loop

$$V_{U(2,2)} \sim \frac{g_X^2 M_X^2}{16\pi^2} (|H|^2 + |H_{\mathcal{H}}|^2)$$

T parameter $\rightarrow \frac{M_X}{g_X} \gtrsim 8.6 \text{ TeV}$



Phenomenology

- SM and hyperbolic Higgses mix, **universal coupling modification**

$$\frac{y_{hPP}}{y_{hPP}^{\text{SM}}} = \cos \theta \simeq 1 - 1.5\% \omega^2 \left(\frac{\text{TeV}}{v_{\mathcal{H}}} \right)^2$$

+ non-universal correction for the top

$$m_t(H) = \frac{1}{\pi R} \arctan(\pi R y_t |H|) \quad \rightarrow \quad -\pi^2 R^2 y_t^2 v^2 \simeq -1.2\% \left(\frac{5 \text{ TeV}}{1/R} \right)^2$$

- Higgs decays to hyperbolic glue,

$$\text{BR}(h_{\text{SM}} \rightarrow g_{\mathcal{H}} g_{\mathcal{H}}) \sim 2 \times 10^{-5} \omega^2 \left(\frac{\text{TeV}}{v_{\mathcal{H}}} \right)^4$$

Spontaneous breaking of hidden color?

- What if the hyperbolic stops get VEVs? $\langle \tilde{t}_{\mathcal{H}}^{L,R} \rangle \neq 0$

Cohen, Craig, Giudice,
McCullough 2018

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Cohen, Craig, Giudice,
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- 8 dofs eaten by massive $SU(3)_{\mathcal{H}}$ gluons
- radial modes mix with the Higgs

→ *Higgs is partly its own top partner*

- No hidden confinement, **collider pheno strongly altered**

Source: 45worlds.com



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Cohen, Craig, Giudice,
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- For tripled top: only **one** light singlet stop in each sector, expect

$$SU(3)_B \xrightarrow{\langle \tilde{u}_B^c \rangle} SU(2)_B$$

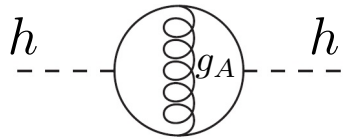
depending on VEV size, $SU(2)$ glueballs may still be at bottom of the spectrum

Spontaneous breaking of hidden color?

Craig, Katz, Strassler,
Sundrum 2015

- **Broader question:**

Gauged and unbroken hidden $SU(3)$ motivated by 2-loop naturalness,



$$\Delta^{-1} \sim 10\% \text{ for } \Lambda = 5 \text{ TeV}$$

(numerically, \sim weak gauge)

$$\delta m_h^2 \sim \frac{3y_t^2 g_s^2}{4\pi^4} \Lambda^2$$

Yields **very rich phenomenology**, hidden hadron signatures

- Does relaxing this (motivated) assumption lead to *different* signatures, as opposed to just *subtracting* some?

If so, may be worthwhile to pursue...

A Twin Higgs example

Batell, McCullough 2015

- Top partners = right-handed neutrinos N

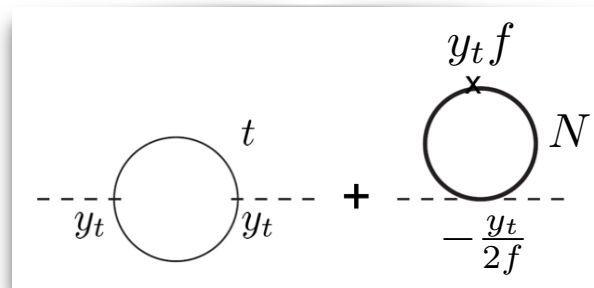
$SU(3)_N$ cannot be exact, broken by Yukawas $\sim y_\nu \ell H N$

- Take e.g. $SU(6) \times SU(4)$ invariant theory, $SU(4) \xrightarrow{\langle \Sigma \rangle} SU(3)$

$$Q = \begin{pmatrix} q_A \\ \begin{pmatrix} N_i \\ E_i \end{pmatrix} \end{pmatrix} \sim (\mathbf{6}, \mathbf{4})$$

$$Q^c = (t_A^c \quad N_i^c) \sim (\bar{\mathbf{6}}, \mathbf{1})$$

$$\mathcal{L}_t = y_t \Sigma Q Q^c \quad \rightarrow \quad y_t q_A H t_A^c + y_t f \left(1 - \frac{H^\dagger H}{2f^2} \right) N_i N_i^c + \dots$$



Chacko, Goh, Harnik 2005

A Twin Higgs example

Batell, McCullough 2015

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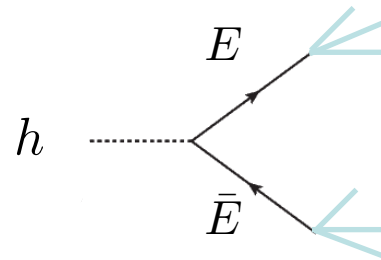
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and can mix with the SM neutrinos,

example $\sim \frac{\phi}{M} E^c (H_A \cdot \ell)$

“sterile neutrino,”
decays to 3 SM fermions



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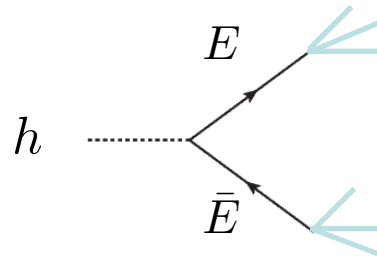
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$SU(3)_N$ cannot be exact, broken by Yukawas $\sim y_\nu \ell H N$

- If $SU(3)_N$ spontaneously broken at $\sim f$, can we access massive gluons at (future) colliders?
- Recent work with “minimal ingredients:” $SO(6)/SO(5)$ Twin Higgs

Serra, Torre 2017

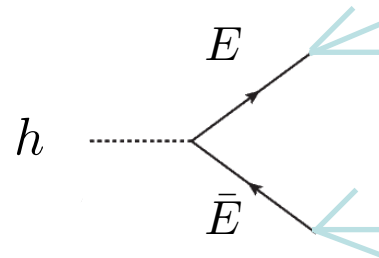
Csáki, Ma, Shu 2017

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Plan

(1) SM-singlet scalar top partners

- Tripled Top
- Hyperbolic Higgs

(2) Z_N models

(3) Probing UV completions at colliders

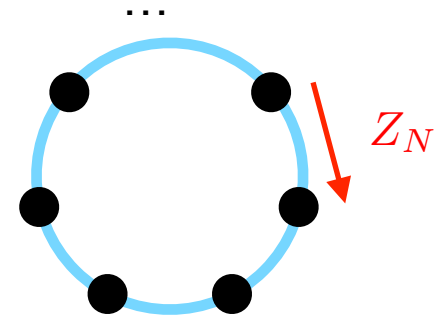
(4) Signatures in cosmology

Z_N - symmetric quantum corrections

Hook 2018

- A scalar coupled in Z_N - invariant way can have its scalar potential **exponentially suppressed** in N
- Concretely: ϕ periodic, shift symmetry broken to $\phi \rightarrow \phi + 2\pi f$ by $\epsilon \sin\left(\frac{\phi}{f} + \theta\right)$
 Z_N acts as $\phi \rightarrow \phi + \frac{2\pi f}{N}$

$$\mathcal{L} = \sum_{k=1}^N \left[m_\psi + \epsilon \sin\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right) \right] \psi_k \psi_k^c$$



- A Z_N - invariant potential for ϕ requires at least N insertions, since for $m < N$

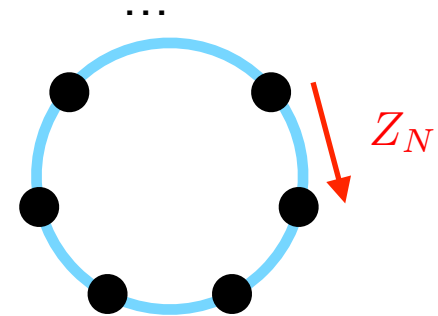
$$\sum_{k=1}^N \sin^m\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right) = \text{constant}$$

Z_N - symmetric quantum corrections

Hook 2018

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- A Z_N - invariant potential for ϕ requires at least N insertions, so

$$V \sim \frac{\epsilon^N}{\Lambda^{N-4}} \sum_{k=1}^N \sin^N\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right) \sim \frac{N\epsilon^N}{\Lambda^{N-4}} \cos\left(\frac{N\phi}{f}\right) \quad \Rightarrow \quad m_\phi^2 \sim \frac{N^3 \epsilon^N}{\Lambda^{N-4} f^2}$$

[frequencies must match, $e^{iN\phi/f} = (e^{i\phi/f})^N$]

Z_N - symmetric quantum corrections

Hook 2018

- TH cancellation is a version of $N = 4$, but **2** copies are enough (gauge invariance forbids odd powers of h)

$$\sum_{k=0}^{N-1} \sin^2 \left(\frac{h}{f} + \frac{2\pi k}{N} \right) = \sin^2 \left(\frac{h}{f} \right) + \sin^2 \left(\frac{h}{f} + \frac{\pi}{2} \right) + \cancel{\sin^2 \left(\frac{h}{f} + \pi \right)} + \cancel{\sin^2 \left(\frac{h}{f} + \frac{3\pi}{2} \right)}$$

- First non-vanishing term is familiar quartic, $\sim \sin^4 \left(\frac{h}{f} \right) + \cos^4 \left(\frac{h}{f} \right)$

- A Z_N - invariant potential for ϕ requires at least N insertions, so

$$V \sim \frac{\epsilon^N}{\Lambda^{N-4}} \sum_{k=1}^N \sin^N \left(\frac{\phi}{f} + \frac{2\pi k}{N} \right) \sim \frac{N\epsilon^N}{\Lambda^{N-4}} \cos \left(\frac{N\phi}{f} \right) \quad \rightarrow \quad m_\phi^2 \sim \frac{N^3 \epsilon^N}{\Lambda^{N-4} f^2}$$

[frequencies must match, $e^{iN\phi/f} = (e^{i\phi/f})^N$]

Solving the hierarchy problem discretely

Hook 2018

- Exponential suppression holds as long as ϕ does not hit a phase transition
- V can be unsuppressed on other side of transition



exploit this to address little hierarchy problem

Solving the hierarchy problem discretely

Hook 2018

- ϕ as modulus of the Higgs mass. Take N copies of the SM, for example $N = 3$

$$V = \sum_k m_{H,k}^2(\phi) |H_k|^2 + \lambda |H_k|^4, \quad m_{H,k}^2(\phi) = -m_H^2 + \epsilon^2 \cos\left(\frac{\phi}{f} + \frac{2\pi k}{N}\right)$$

where $m_H^2 = \frac{N_c y_t^2}{8\pi^2} \Lambda^2$

and

$$m_H^2 \lesssim \epsilon^2 \lesssim 2m_H^2$$

across ϕ field space, **one** of the masses flips sign

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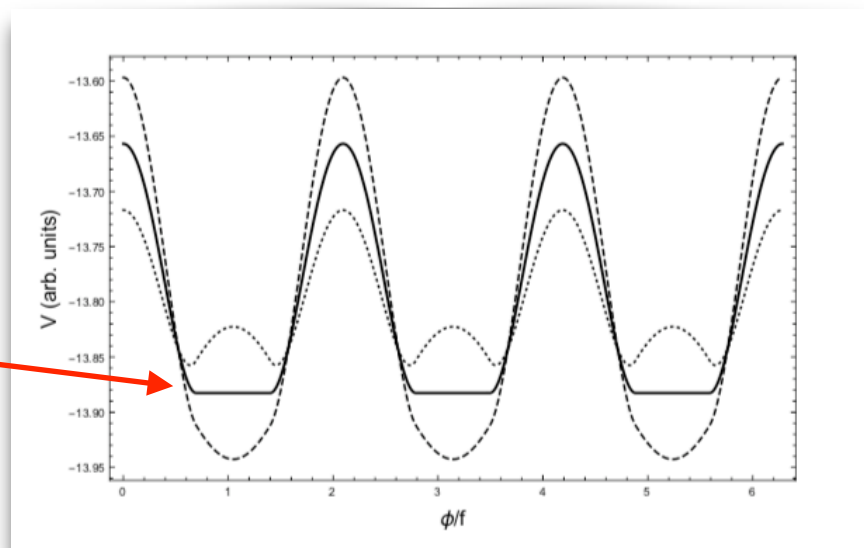
across ϕ field space, **one** of the masses flips sign

- Integrate Higgses out classically

$$V_H = - \sum_k \frac{m_{H,k}^4(\phi)}{4\lambda} \Theta[-m_{H,k}^2(\phi)]$$

ϕ - independent if **all masses negative**

(would need at least N insertions)



Solving the hierarchy problem discretely

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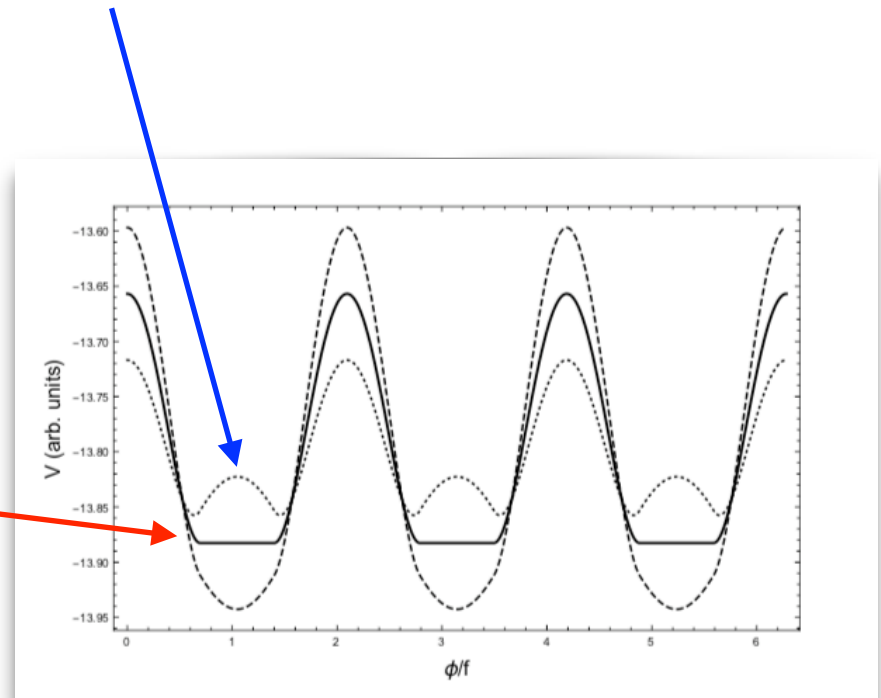
$$V_{UV} \sim \frac{\epsilon^{2N}}{\Lambda^{2N-4}} \cos\left(\frac{N\phi}{f} + \theta\right) \quad \theta = 0$$

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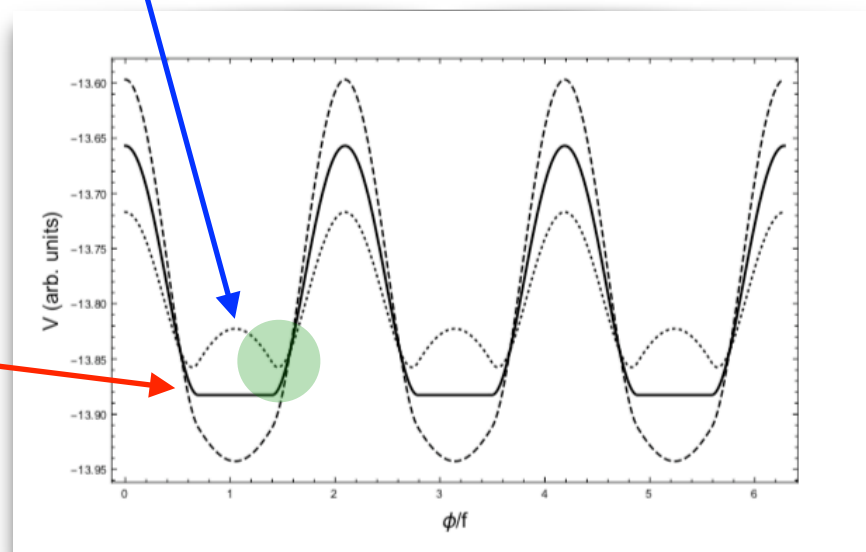
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ϕ - independent if **all masses negative**

(would need at least N insertions)

at minimum, one of the masses is small (and positive)



Questions/comments

- Wrong sign of SM Higgs mass can be fixed, e.g. by adding $\delta V \sim \cos\left(\frac{M\phi}{f}\right)$, $M \gg N$
- Larger N faces two issues:

$$m_H^2 \lesssim \epsilon^2 \lesssim \frac{m_H^2}{\cos(\pi/N)}$$

tuning $\sim 1/N^2$

$$V_{\text{CW}} \sim \frac{\beta}{16\pi^2} \sum_k |H_k|^4 \log \frac{|H_k|^2}{\Lambda^2}$$

dominates over UV potential,
too large for light Higgs

- Our sector is reheated preferentially, à la Naturalness
- Cross-quartics lead to Higgs mixing, decays to hidden sectors
- Constraints on ϕ need to be worked out
- A model where the other masses are large and *positive*,
so only the Higgs is copied N times?

Arkani-Hamed, Cohen,
D'Agnolo, Hook, Kim, Pinner 2016

Plan

- (1) SM-singlet scalar top partners
 - Tripled Top
 - Hyperbolic Higgs
- (2) Z_N models
- (3) **Probing UV completions at colliders**
- (4) Signatures in cosmology

Probing UV completions

- In neutral naturalness, low-energy theory is elusive by construction
Furthermore, signatures (e.g. coupling modifications) are not unique to NN
- UV completion at multi-TeV can be key to characterisation, even **discovery**
Assess reach of **future colliders**

- Example: **Twin Higgs**

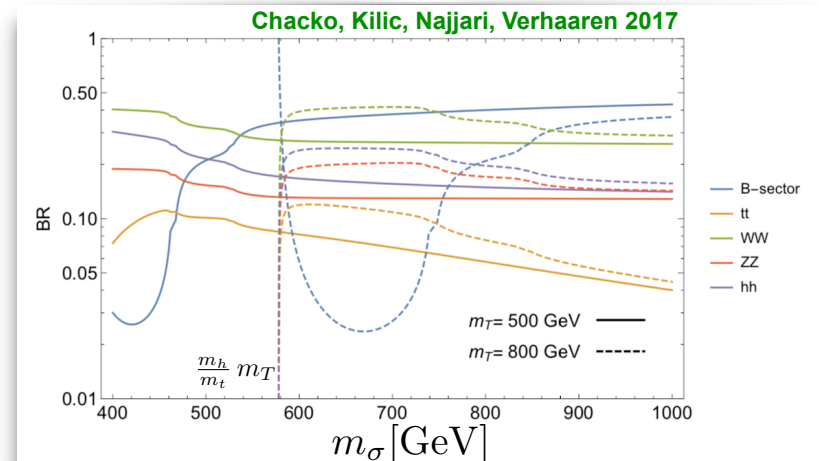
In weakly coupled completions, **scalar radial mode** is robust feature

$$SO(8) \xrightarrow{f} SO(7)$$

$$V \ni \lambda |H|^4 \rightarrow \lambda f \sigma \left(\sum_i \pi_i^2 \right)$$



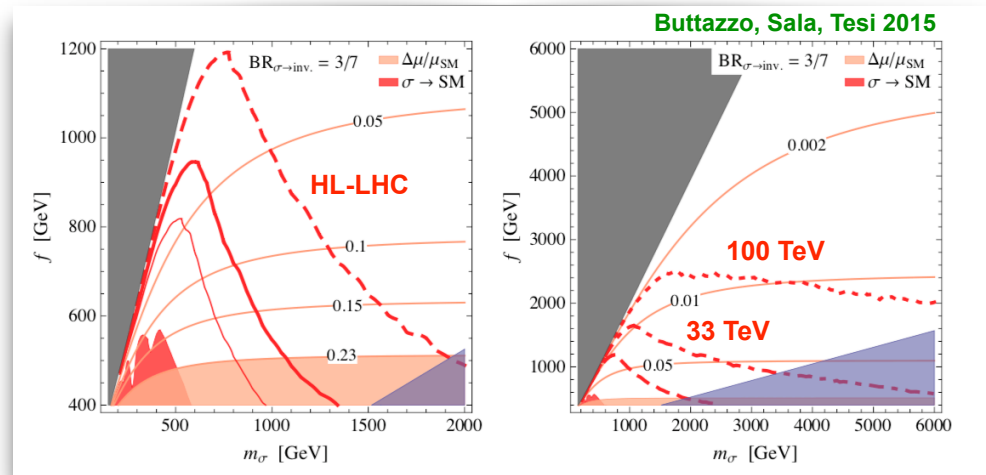
$$\text{BR}(\sigma \rightarrow WW) \simeq 2 \text{BR}(\sigma \rightarrow hh) \simeq \frac{2}{7}$$



Radial mode at colliders

- At hadron colliders σ produced in gluon fusion

Best reach in $\sigma \rightarrow VV$



also: Katz, Mariotti, Pokorski, Redigolo, Ziegler 2016

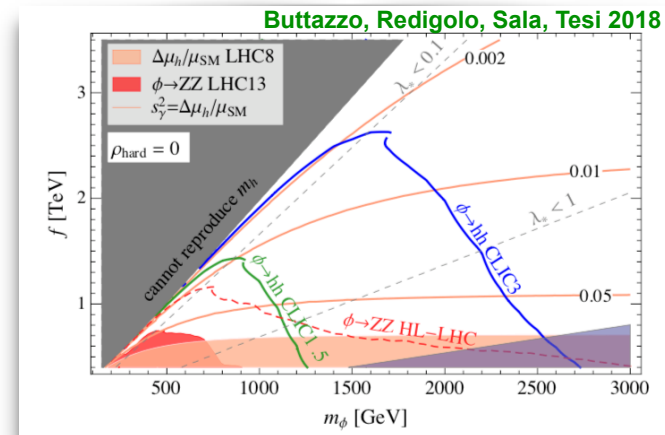
- At high-energy lepton colliders dominant production in WW fusion

Best reach in $\sigma \rightarrow hh \rightarrow 4b$

- If Z_2 is broken **only softly**, $\{G_F, m_h, g_{hVV}/g_{hVV}^{\text{SM}}, m_\sigma\}$ completely fix potential

➡ $\sigma \rightarrow \text{SM SM}$ is genuine **test** of TH mechanism

Chacko, Kilic, Najjari, Verhaaren 2017



Strongly coupled UV completions

- Radial mode may be too broad to observe
- Expect new **vector-like fermions** charged under **both SM and Twin**
- Simplest way to introduce them: $SU(6) \times SU(4)$ invariant top Yukawa

$$Q^c = \begin{pmatrix} t_A^c & t_B^c \end{pmatrix} \sim (\bar{\mathbf{6}}, \mathbf{1})$$

$$Q = \begin{pmatrix} q_A & \tilde{q}_B \\ \tilde{q}_A & q_B \end{pmatrix} \sim (\mathbf{6}, \mathbf{4})$$

$$\mathcal{L}_t = y_t \Sigma Q Q^c + \widetilde{M}(\bar{\tilde{q}}_A \tilde{q}_A + \bar{\tilde{q}}_B \tilde{q}_B)$$

Chacko, Goh, Harnik 2005

- Their mass \widetilde{M} cuts off logarithmic divergences in Higgs potential
- More in general, they appear as composite resonances / KK modes

Geller, Telem 2014

Barbieri, Greco, Rattazzi, Wulzer 2015

Low, Tesi, Wang 2015

Strongly coupled UV completions

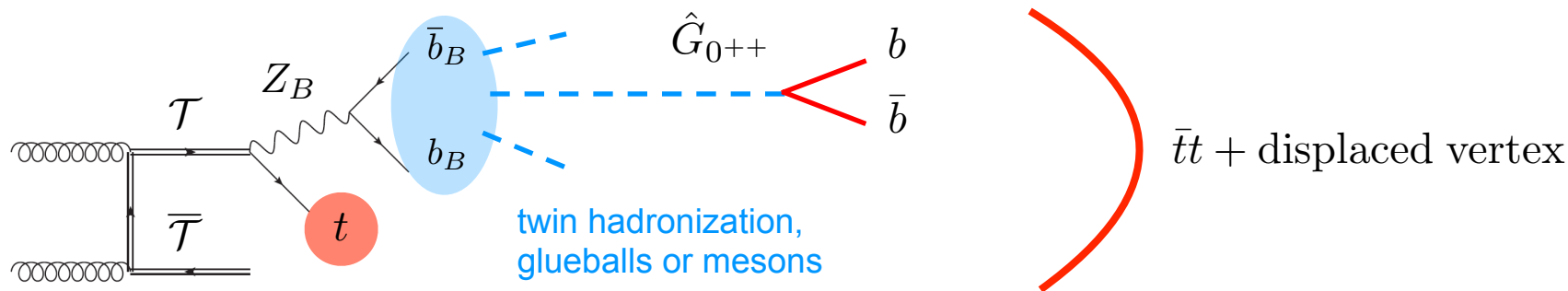
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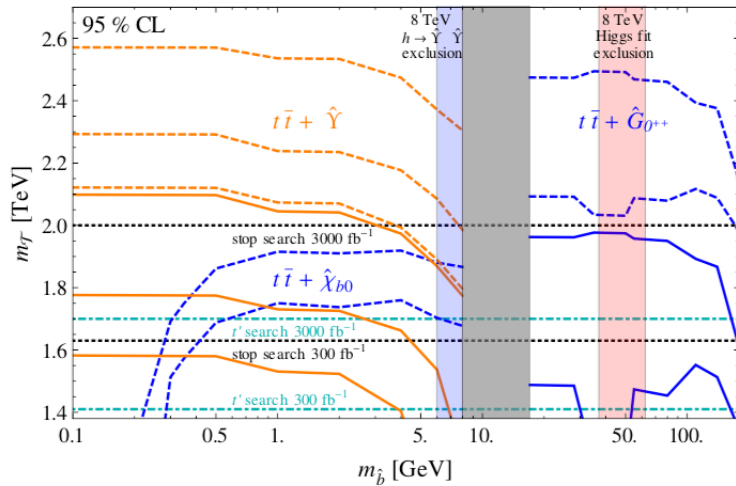
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SM color *and* twin weak \rightarrow high-energy portal at hadron colliders

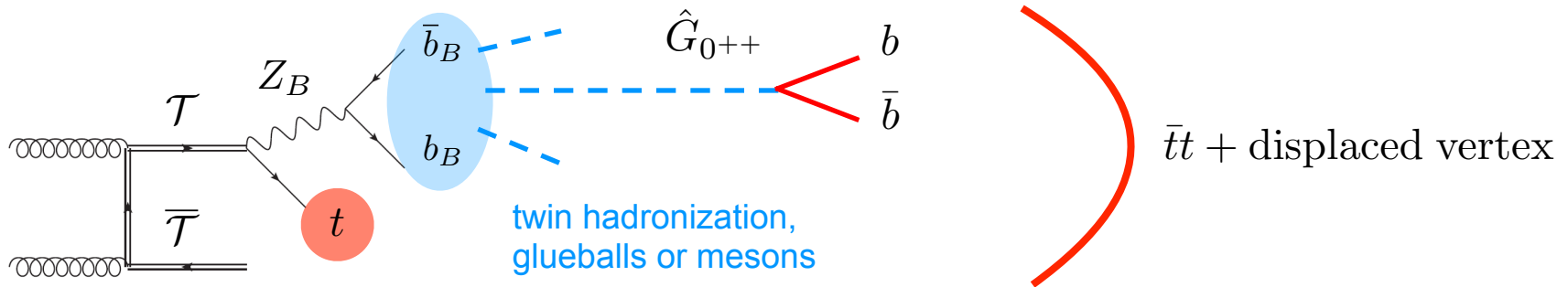
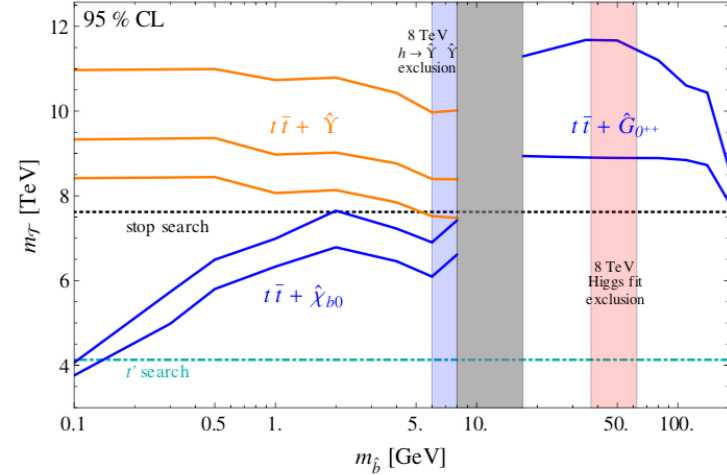


Strongly coupled UV completions

LHC 13 TeV, 300 and 3000 fb⁻¹. $\Lambda = 5$ GeV, $f = 1$ TeV



100 TeV, 1 ab⁻¹. $\Lambda = 5$ GeV, $f = 1$ TeV



Cheng, Jung, Salvioni, Tsai 2015

[assume Fraternal TH setup]

Plan

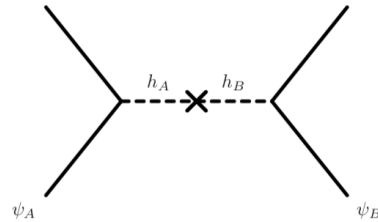
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Cosmology of Mirror Twin Higgs

- The Mirror TH (= Z_2 broken only softly) has well-known conflict with cosmology: too much dark radiation in the form of twin photon and neutrinos

$$\Delta N_{\text{eff}} \approx 5.6$$

$$(\Delta N_{\text{eff}} \lesssim 0.6 \text{ at } 95\% \text{ CL})$$



$$T_{\text{dec}} \sim 3 \text{ GeV}$$

- **Several solutions proposed in last ~ 2 years**

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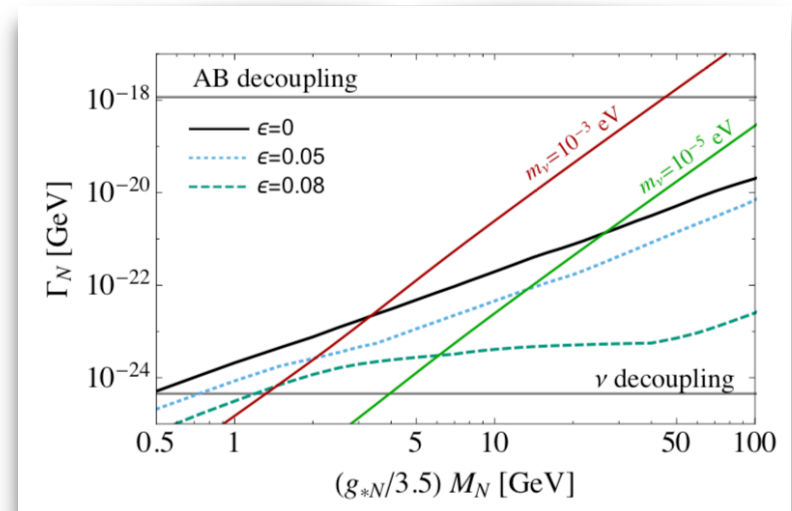
- **Asymmetric reheating** of the two sectors after decoupling, for example by decays of right-handed neutrinos

$$\Delta N_{\text{eff}} \approx 7.4 \frac{v^2}{f^2}$$

- Does not require hard Z_2 breaking

also: **Craig, Koren, Trott 2016**

Chacko, Craig, Fox, Harnik 2016



Cosmology of Mirror Twin Higgs

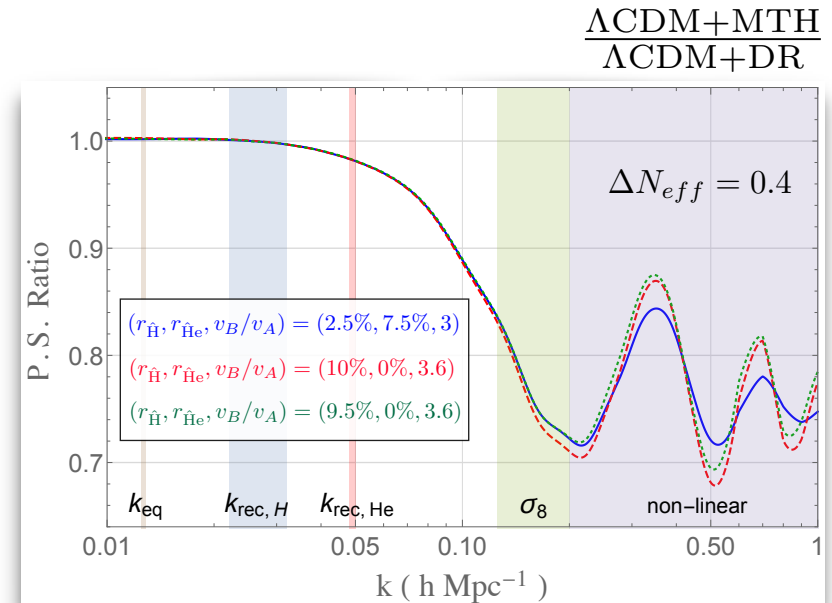
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- Cosmological signatures predicted both
in large scale structure (twin BAO)

Chacko, Curtin, Geller, Tsai 2018



and in CMB: different effects from twin photon / neutrinos
(scattering vs free stream), testable prediction

$$\frac{\Delta N_{\text{eff}}^{\nu B}}{\Delta N_{\text{eff}}^{\gamma B}} = \frac{3}{\frac{8}{7} \left(\frac{11}{4} \right)^4} \approx 0.68$$

Cosmology of Mirror Twin Higgs

- “Minimal Mirror TH”, **hard Z_2 breaking from Yukawas**

➡ raise temperature of twin QCD phase transition to $T_{\text{QCD}}^B > T_{\text{dec}} > T_{\text{QCD}}^A$

Barbieri, Hall, Harigaya 2016

- **Neutrino portal** can also help, delaying decoupling to < 1 GeV

Csáki, Kuflik, Lombardo 2017

- Z_2 - breaking mass in Higgs potential can be generated radiatively
- In all cases, the extra radiation should be detectable in future CMB experiments:
at Stage-IV

$$\Delta N_{\text{eff}} \lesssim 0.04 \text{ (95\% CL)}$$

(Snowmass, 1309.5383)

More...

- Fine tuning analyses

Katz, Mariotti, Pokorski, Redigolo, Ziegler 2016
Contino, Greco, Mahbubani, Rattazzi, Torre 2017

- Electroweak precision

Contino, Greco, Mahbubani, Rattazzi, Torre 2017

- Tadpole-induced EWSB

Harnik, Howe, Kearney 2016

- Twin SIMPs

Hochberg, Kuflik, Murayama 2018

Thanks!

Backup

The soft masses

Cheng, Li, Salvioni,
Verhaaren, 1803.03651

$$SU(2) \quad F = 3$$

$$\begin{matrix} \tilde{m}_{\bar{P}_2}^2 \\ \tilde{m}_{\bar{P}_2}^2 \\ \tilde{m}_{\bar{P}_1}^2 \end{matrix} \begin{pmatrix} \tilde{m}_P^2 & \tilde{m}_P^2 & \tilde{m}_P^2 \\ & Q_A & \\ \hline & & \end{pmatrix}$$

$$\begin{matrix} \tilde{m}_{\bar{P}_2}^2 \\ \tilde{m}_{\bar{P}_2}^2 \\ \tilde{m}_{\bar{P}_1}^2 \end{matrix} \begin{pmatrix} \tilde{m}_P^2 & \tilde{m}_P^2 & \tilde{m}_P^2 \\ & u_A^c & \\ \hline & & \end{pmatrix}$$

$$\begin{matrix} \tilde{m}_{\bar{P}_1}^2 \\ \tilde{m}_{\bar{P}_1}^2 \\ \tilde{m}_{\bar{P}_2}^2 \end{matrix} \begin{pmatrix} \tilde{m}_P^2 & \tilde{m}_P^2 & \tilde{m}_P^2 \\ & u_{B,C}^c & \\ \hline & & \end{pmatrix}$$

$$m_{ij}^2 = m_{P_i}^2 + m_{\bar{P}_j}^2 - \frac{2}{b} \sum_k T_{r_k} (m_{P_k}^2 + m_{\bar{P}_k}^2)$$



(e.g.: $m_{\bar{P}_2}^2 > 0$, $m_{\bar{P}_1}^2 = 0$)

$$V_s = +\tilde{m}^2 \left(|\tilde{Q}_A|^2 + |\tilde{u}_A^c|^2 \right) - \tilde{m}^2 \left(|\tilde{u}_B^c|^2 + |\tilde{u}_C^c|^2 \right)$$

- Z_3 - symmetric Yukawas

$$W \ni \frac{g_t}{\Lambda_{UV}^2} P \bar{P} P \bar{P} H \quad \longrightarrow \quad y_t \sim g_t \frac{\Lambda_G^2}{\Lambda_{UV}^2}$$

Soft masses of composite mesons

- s-confinement = smooth confinement without chiral symmetry breaking and with non-vanishing confining superpotential

Arkani-Hamed, Rattazzi
hep-th/9804068

- In the UV, from $P \rightarrow \sqrt{Z} P$

$$\frac{1}{4} \int d^2\theta S(\mu_{UV}) W^2 + \text{h.c.} + \int d^4\theta Z F \left(S(\mu_{UV}) + S^\dagger(\mu_{UV}) - \frac{T}{4\pi^2} \ln Z \right) P^\dagger e^V P$$

- Anomalous $U(1)$ symmetry $Z \rightarrow Z\chi\chi^\dagger$, $P \rightarrow P/\chi$, $S(\mu_{UV}) \rightarrow S(\mu_{UV}) + \frac{T}{4\pi^2} \ln \chi$
 Z is promoted to background vector superfield

- Only invariant object is $I = \Lambda_h^\dagger Z^{2T/b} \Lambda_h$ ($\Lambda_h = \mu_{UV} e^{-8\pi^2 S/b}$)

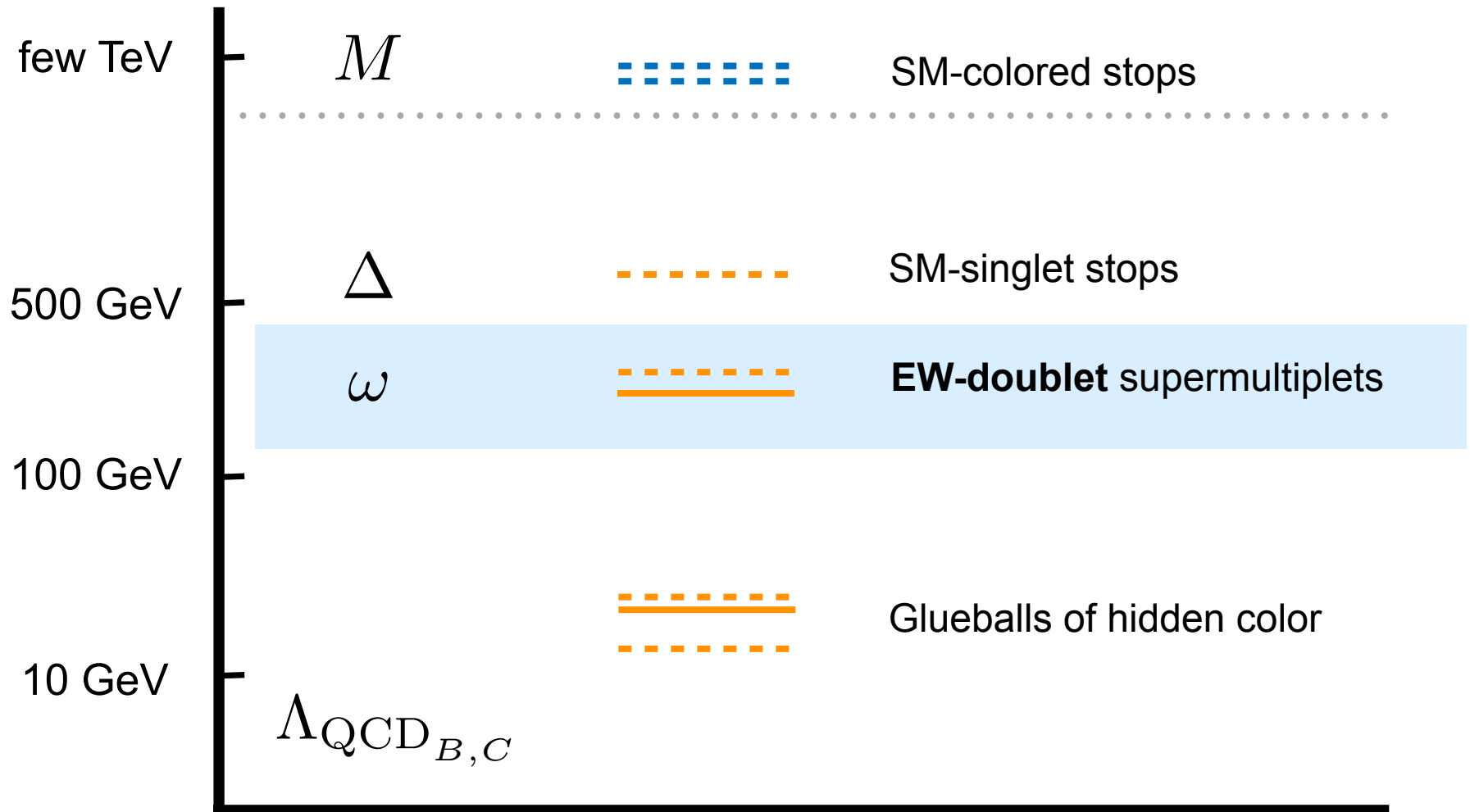
and
$$m_P^2(\mu_{UV}) = -[\ln Z]_{\theta^2\bar{\theta}^2} - [\ln F(\mu_{UV})]_{\theta^2\bar{\theta}^2} \xrightarrow{\mu_{UV} \rightarrow \infty} -[\ln Z]_{\theta^2\bar{\theta}^2}$$

- In the IR, effective Kähler potential for mesons starts with

$$K \supset c_{M_{ij}} \frac{M_{ij}^\dagger Z_i Z_{\bar{j}} M_{ij}}{I} + \dots \quad \Rightarrow \quad m_{M_{ij}}^2 \Big|_{\mu_{IR} \rightarrow 0} = - \left[\ln \frac{Z_i Z_{\bar{j}}}{I} \right]_{\theta^2\bar{\theta}^2} = - [\ln Z_i]_{\theta^2\bar{\theta}^2} - [\ln Z_{\bar{j}}]_{\theta^2\bar{\theta}^2} + [\ln I]_{\theta^2\bar{\theta}^2} = m_{P_i}^2 + m_{P_j}^2 - \frac{2}{b} \sum_k T_{r_k} \left(m_{P_k}^2 + m_{\bar{P}_k}^2 \right)$$

Spectrum of BSM states: $\Delta > \omega$

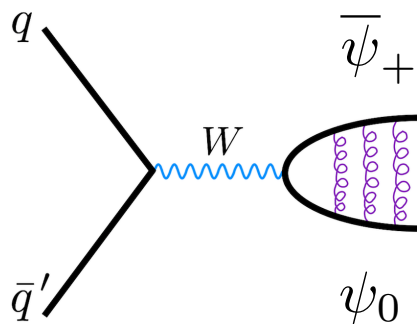
mass



$\Delta > \omega$: quirk phenomenology

- If $\Delta > \omega$, then target are the EW-doublet supermultiplets with mass $\sim \omega$
- Fermions have larger Drell-Yan production than scalars,

$$Q_{B,C} \sim \mathbf{2}_{-1/2} \sim \begin{pmatrix} \psi_0 \\ \psi_- \end{pmatrix}$$

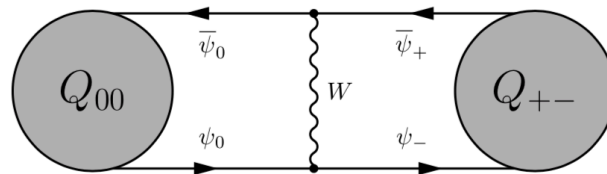


“quirky”
bound state



de-excites down to ground state
via emission of **soft photons**

$$\hat{s} > 4m_\psi^2$$

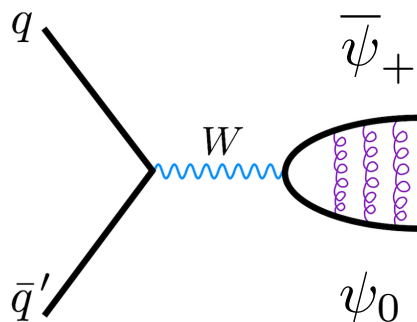


(electrically-neutral pairs too,
via mass mixing)

$\Delta > \omega$: quirk phenomenology

- If $\Delta > \omega$, then target are the EW-doublet supermultiplets with mass $\sim \omega$
- Fermions have larger Drell-Yan production than scalars,

$$Q_{B,C} \sim \mathbf{2}_{-1/2} \sim \begin{pmatrix} \psi_0 \\ \psi_- \end{pmatrix}$$



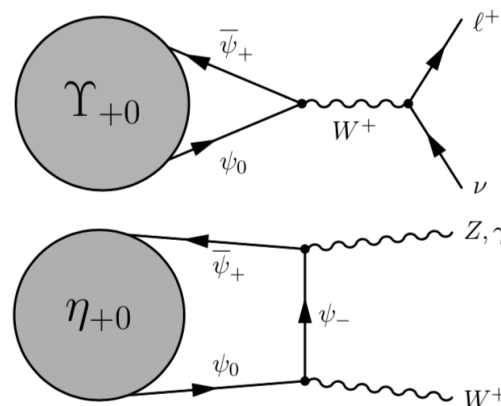
“quirky”
bound state



de-excites down to ground state
via emission of **soft photons**



annihilation of $n = 1$ states



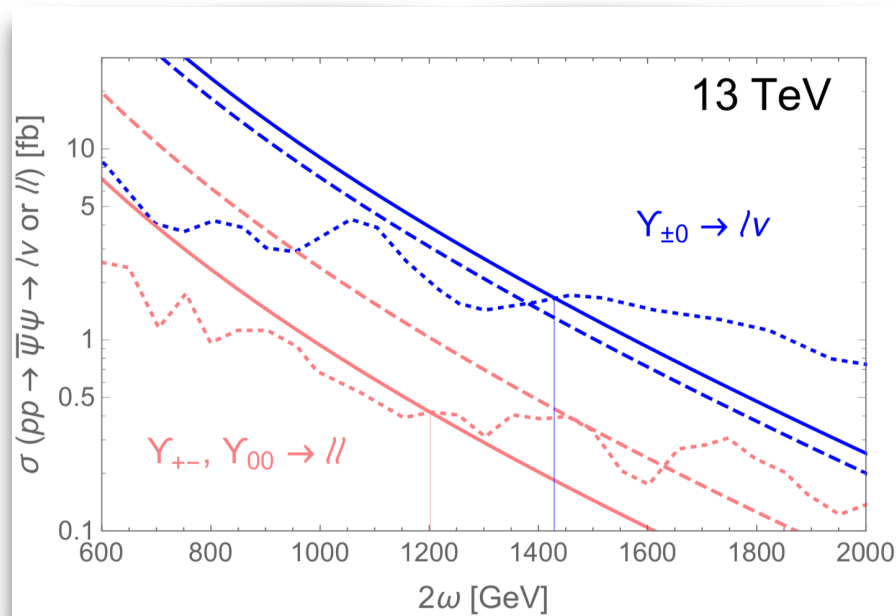
**resonant
signals**

$\Delta > \omega$: quirk phenomenology

- Strongest bounds come from **charged channel**
(decays to pure hidden gluons forbidden)

$$\omega \gtrsim 700 \text{ GeV}$$

$$\text{from } \Upsilon_{+0} \rightarrow \ell \nu$$



- Neutral channels give $\omega \gtrsim 600 \text{ GeV}$ from $\eta_{+-} \rightarrow \gamma\gamma$
 $\Upsilon_{+-,00} \rightarrow \ell\ell$

Higgs quartic and T parameter

- Higgs quartic: for example
$$\lambda \simeq \frac{N_c y_t^4}{16\pi^2} \left(\frac{3}{2} + \log \frac{\omega^2}{m_t^2} \right) + \frac{m_Z^2}{2v^2} \cos^2(2\beta)$$
$$(\Delta \ll \omega)$$

Numerically,

$$M = 2 \text{ TeV}, \quad \Delta = 300 \text{ GeV}, \quad \omega = 500 \text{ GeV} \quad \rightarrow \quad \lambda \lesssim 0.14$$

but, 2-loop corrections important...

- T parameter: leading contribution comes from light scalars,

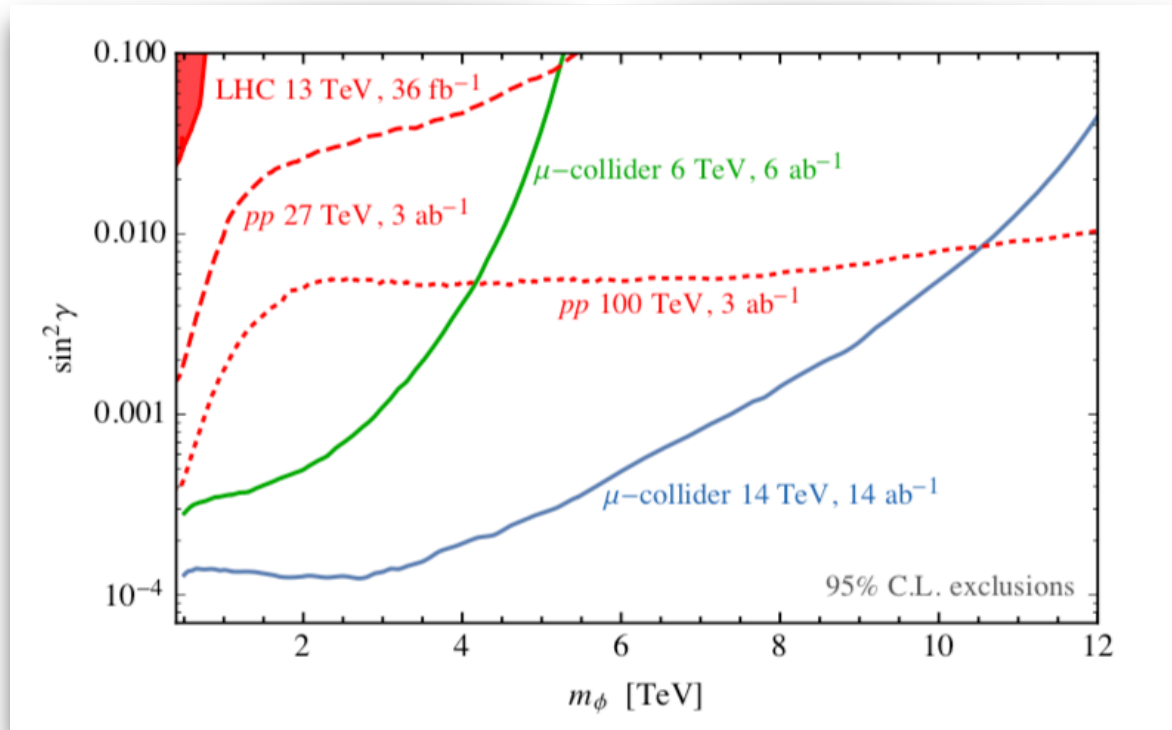
$$\hat{T}_{sc,B+C} \approx \frac{1}{3} \frac{N_c y_t^2}{16\pi^2} m_t^2 \omega^2 \left[\frac{\omega^6 + 9\omega^4 \Delta^2 - 9\omega^2 \Delta^4 - 6\omega^2 \Delta^2 (\omega^2 + \Delta^2) \log \frac{\omega^2}{\Delta^2} - \Delta^6}{(\omega^2 - \Delta^2)^5} \right]$$

Numerical example:

under control (even welcome)

$$\Delta = 300 \text{ GeV}, \quad \omega = 500 \text{ GeV} \quad \rightarrow \quad \hat{T}_{sc,B+C} \approx 4 \times 10^{-4}$$

Reach on scalar singlets at future colliders



hadron: $\phi \rightarrow ZZ$, lepton: $\phi \rightarrow hh \rightarrow 4b$

$$\text{BR}(\phi \rightarrow ZZ, hh) = 1/4$$