Higgscitement: Cosmological Dynamics of Fine Tuning

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CERN theory workshop

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Status of the Higgs Fine-Tuning

What we have learned from the LHC so far: both direct and indirect searches seem to hint at least a factor of 10 (or worse) fine tuning in the Higgs potential.

There is still very active and interesting research aiming to fill loopholes of LHC searches or to develop new natural models (neutral naturalness, relaxion….).

In the talk, I will take a different view point: Nature is probably tuned.

Test fine-tuning

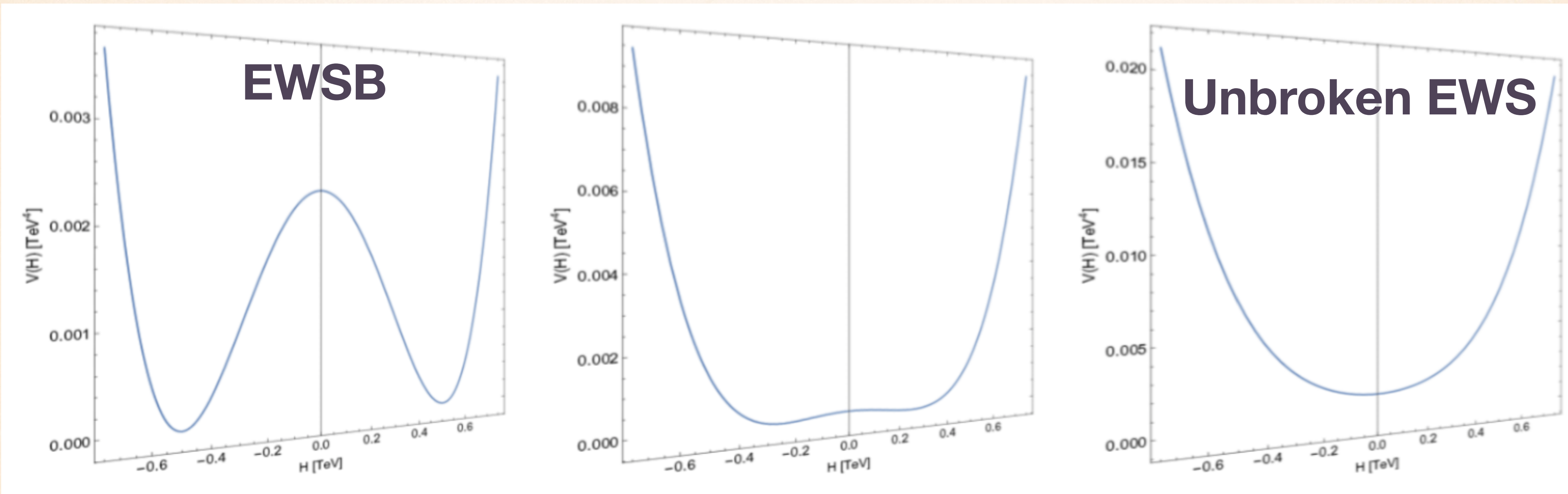
Usually, we test the naturalness of the Higgs in two ways:

1. Look for deviations of the Higgs's properties from SM predictions.
2. Come up with a concrete natural theory, like SUSY or composite Higgs, and look for the new particles it predicts.

These both give evidence for fine-tuning in a ***negative*** way, that is, we look for deviations and don't find them.

In this talk: can we find a ***positive*** signal of fine-tuning?

Fine-tuning: if we could change SM parameters, the electroweak physics could be changed dramatically.



Surely the SM parameters are fixed in our Universe. We don't go back and forth between different electroweak theories.

Or can we? **Couplings depend on VEVs.**

In the early universe, various scalar fields could have had large field range and the Higgs could couple to them. So effective couplings (mass) of the Higgs could be different.

Could have had unbroken electroweak symmetry or much more badly broken electroweak symmetry.

Even better, could have *dynamics — oscillations between different electroweak phases*, fine-tuning in *time*.

Well motivated theories supply lots of good candidates of scalars with large field range: saxions, moduli, D-flat directions.

Let's explore what can happen!

Based on work with Mustafa A. Amin (Rice), Kaloian D. Lozano (Max-Planck) and Matt Reece (Harvard), 1802.00444

Start with Higgs potential today

$$V(h) = \left(-\mu^2 + \frac{M^2}{16\pi^2}\right) h^\dagger h + \lambda (h^\dagger h)^2 = -m_h^2 h^\dagger h + \lambda (h^\dagger h)^2$$

bare mass

quantum correction from, e.g., top loop;
M: natural Higgs mass scale

SM Higgs mass^2
~$(125\ \mathrm{GeV})^2$

Fine tuning ~ $\dfrac{M^2}{m_h^2}$

$$M \gg m_h \Rightarrow \mathrm{fine-tuned!}$$

In the early Universe, Higgs coupling to a modulus (a scalar with a large field range), ϕ

$$V(h,\phi) = \left(-\mu^2 + \frac{M^2}{16\pi^2}\right) h^\dagger h + \lambda (h^\dagger h)^2 + \frac{M^2}{f}\phi h^\dagger h + \cdots$$

$$= -m_h^2 h^\dagger h + \lambda (h^\dagger h)^2 + \frac{M^2}{f}\phi h^\dagger h + \cdots$$

Same size as they come from the same UV physics.
Easiest to realize in SUSY: M^2 ~ soft SUSY breaking mass squared

Fine tuning ~ $\dfrac{M^2}{m_h^2}$

More on the moduli coupling: a spurion analysis

Modulus superfield: $\boldsymbol{X} \supset X + F_X\theta^2$

$$\langle \boldsymbol{X} \rangle = X_0 + F_{X,0}\theta^2, \quad \text{where} \quad X_0 \sim m_{\rm pl}, \quad F_{X,0} \sim m_{3/2} m_{\rm pl}.$$

$$\int d^4\theta \frac{\xi_{XZ}}{m_{\rm pl}^2} \boldsymbol{X}^\dagger \boldsymbol{X} \boldsymbol{Z}^\dagger \boldsymbol{Z}$$

Z: generic chiral superfield

$$\xi_{XZ} \frac{|F_X|^2}{m_{\rm pl}^2} Z^\dagger Z, \qquad \frac{2\xi_{XZ}\mathrm{Re}(F_{X,0} m_X)}{m_{\rm pl}^2} \mathrm{Re}(X) Z^\dagger Z.$$

soft mass: $m_{3/2}^2$

trilinear coupling: $m_{3/2}^2/m_{\rm pl}$

$$V(h,\phi) = \left(-\mu^2 + \frac{M^2}{16\pi^2}\right) h^\dagger h + \frac{M^2}{f}\phi h^\dagger h + \lambda (h^\dagger h)^2 + m_\phi^2 \phi^2$$

$$= -m_h^2 h^\dagger h + \frac{M^2}{f}\phi h^\dagger h + \lambda (h^\dagger h)^2 + m_\phi^2 \phi^2$$

~(125 GeV)^2

Modulus mass

Modulus field range (e.g, ~ Planck scale)

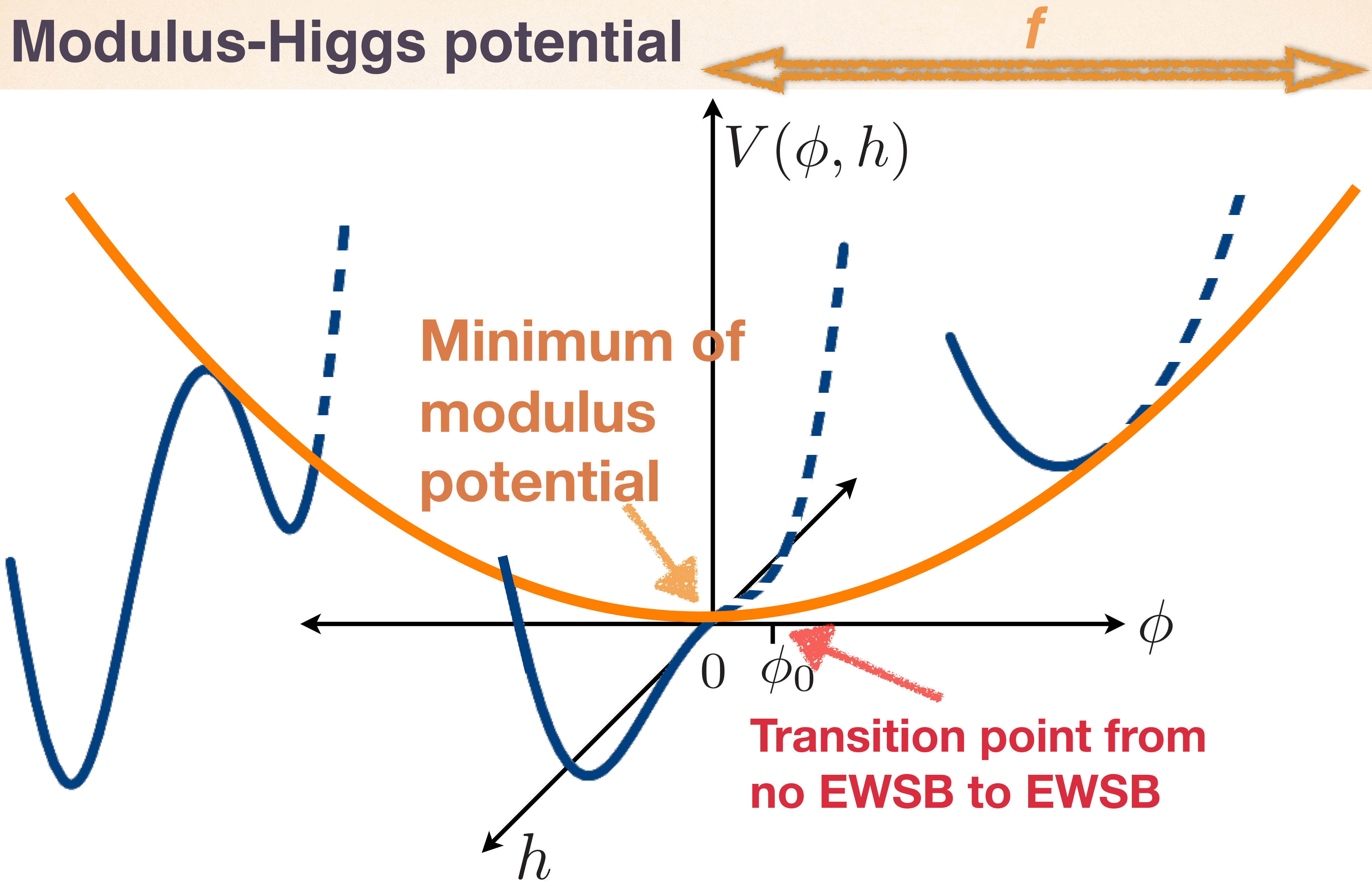
Possible hierarchies: $m_h \ll m_\phi \lesssim M \ll f \sim M_{pl}$

(other variations are possible too)

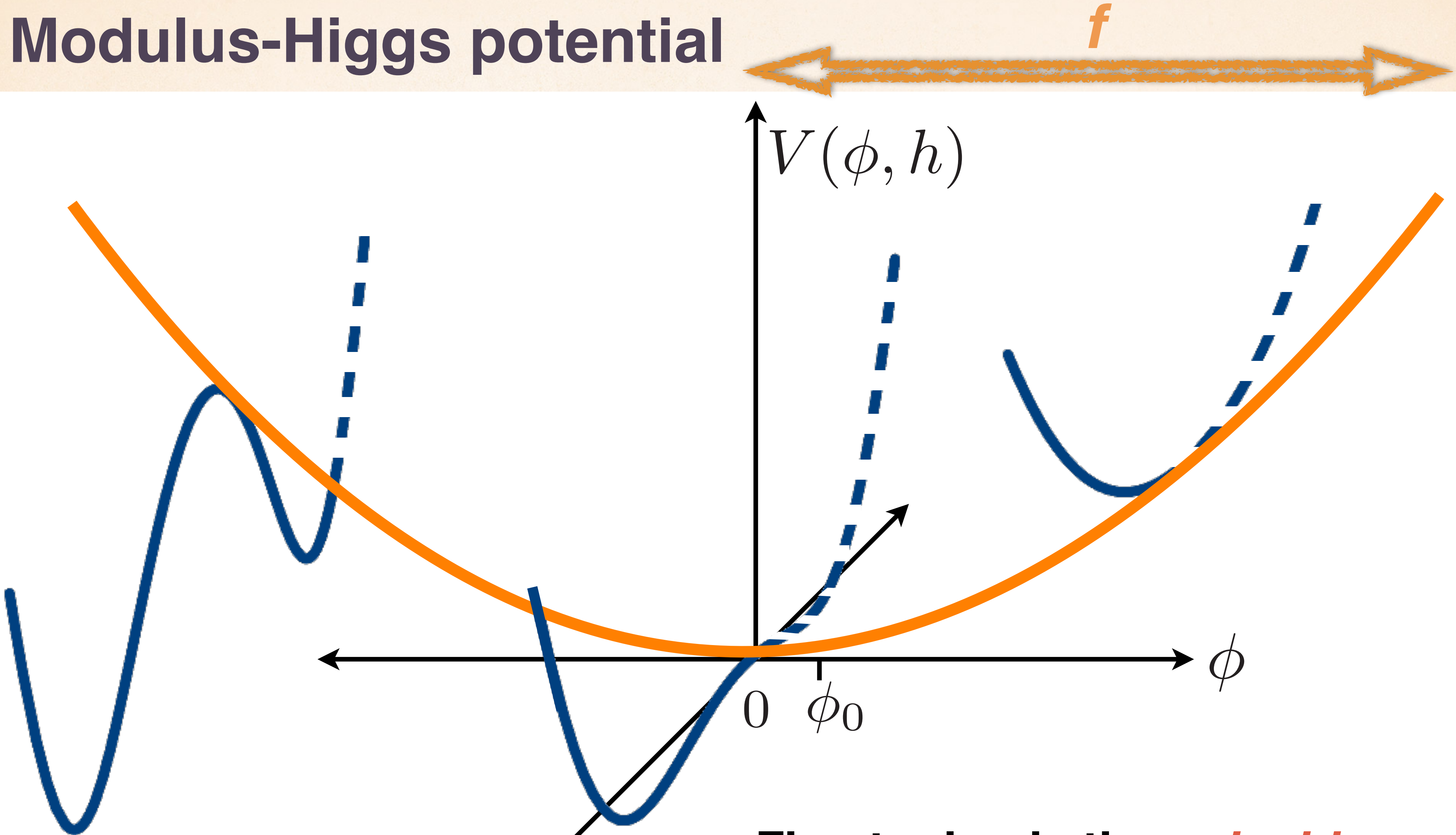
Effective Higgs mass: $-m_h^2 + \frac{M^2}{f}\phi$

At $\phi_0 = \frac{m_h^2}{M^2} f$, **Higgs mass changes sign!**

Modulus-Higgs potential



Modulus-Higgs potential



Measure of fine-tuning:

$$\frac{M^2}{m_h^2} \sim \left| \frac{\phi_0}{f} \right|$$

Fine tuning is the *coincidence* between the minimum of the ϕ potential and the point of marginal EWSB.

Oscillating between no EWSB and EWSB

Initially the modulus is stuck at a random point in the field space (~ f) due to Hubble friction.

The modulus starts oscillating when Hubble is below its mass. For a modulus-dominated universe,

red-shifted amplitude

$$\phi(t) \approx \frac{\xi_\phi f}{m_\phi t} \cos(m_\phi (t - t_0))$$

ξ_ϕ **: O(1) number**

the Higgs will flip between tachyonic and not tachyonic if $|\phi(t)| > \phi_0$

This flipping stops when

$$m_\phi t \gtrsim \xi_\phi \frac{f}{\phi_0}$$

f/ϕ_0 **is a measure of tuning!**

The number of EW-flipping oscillations probes fine tuning.

Tachyonic particle production

As the modulus oscillates, if mϕ is at least a little bit small compared to *M*, the Higgs has time to respond to the change of its potential in an oscillation period of the modulus.

When the Higgs mass flips sign, there is a tachyonic instability:

$$\ddot{h}_k + \omega_k^2 h_k = 0, \quad \text{with} \quad \omega_k(t)^2 = k^2 + m_{\text{eff}}^2(\phi)$$

When $\omega_k^2 < 0$, the Higgs modes grow exponentially.

That is, there is a tachyonic particle production process when the modulus flips to the tachyonic side, converting modulus energy into the Higgs energy.

Tachyonic resonance efficiency parameter: $q \equiv \dfrac{M^2}{m_\phi^2} \gg 1$

The problem of backreaction

But: once many Higgs particles are created, they backreact and fragment the modulus field.

Simple estimate: the particle production will be stalled once

$$\rho_h \sim \rho_\phi$$

Crudely, can think of this as the quartic

$$\lambda h^4 \sim \lambda \langle h^2 \rangle h^2$$

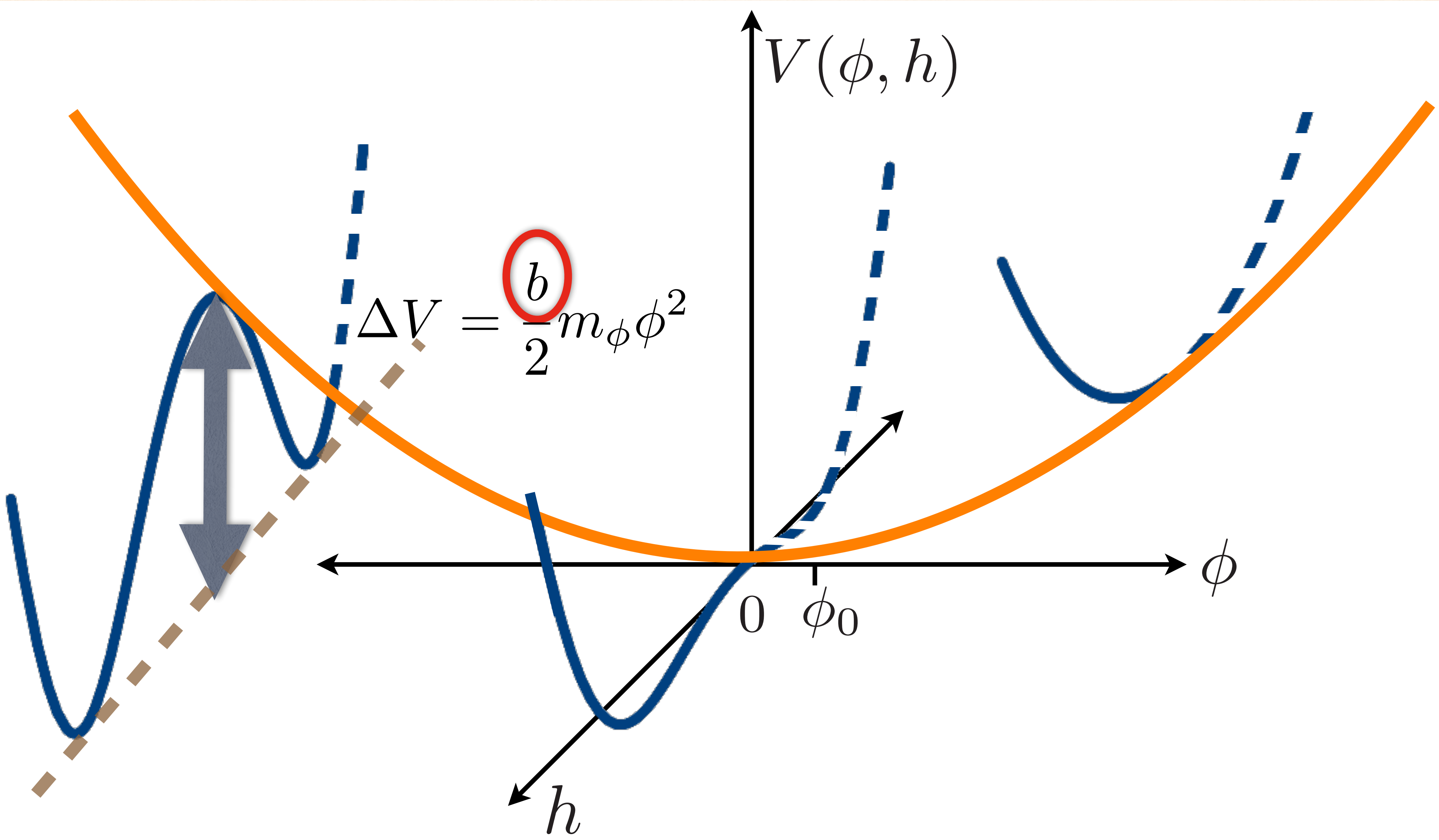
turning into a positive mass for the Higgs.

Since $h^2 \sim \dfrac{M^2}{\lambda}$**,** $\qquad \rho_h \sim \rho_\phi \Rightarrow \dfrac{M^4}{\lambda} \sim m_\phi^2 f^2$

$$\Rightarrow \underbrace{b}_{\text{back-reaction parameter}} \equiv \frac{M^4}{2\lambda f^2 m_\phi^2} \sim 1$$

b<1, otherwise run-away direction

back-reaction parameter



$V(\phi, h)$
$\Delta V = \frac{b}{2} m_\phi \phi^2$
ϕ
0
ϕ_0
h

Numerics

Saying what happens after backreaction occurs analytically is difficult. Turn to numerical simulations.

Use a modified version of LatticeEasy (Felder, Tkachev ’00).

These are *classical field theory* calculations on a lattice with stochastic initial conditions.

They are valid only for a limited range of times. Power transferred to small scales eventually invalidates the calculation.

Still, we can learn at least a couple of useful parametric statements from the results (which are not in early literature).

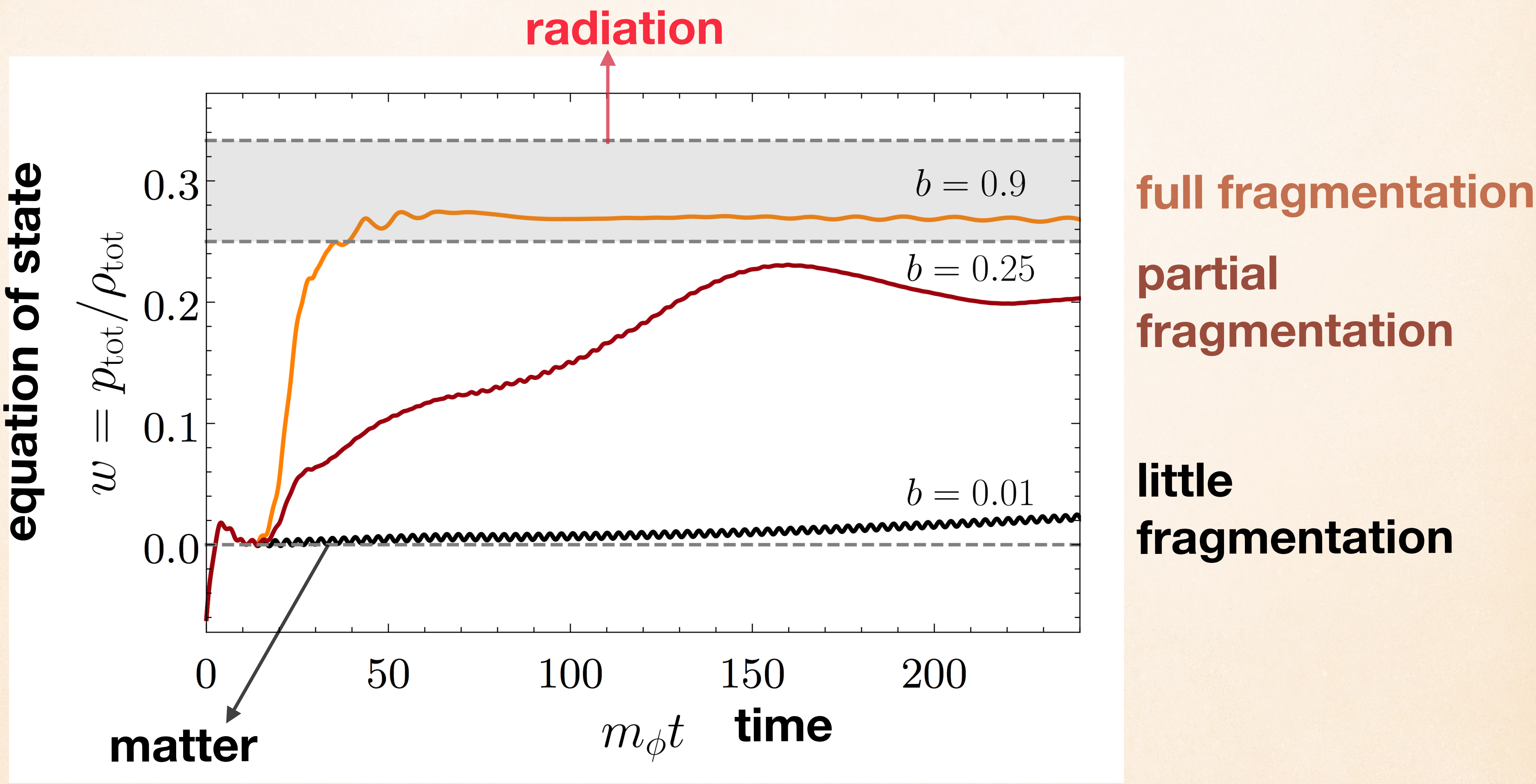
For some parameters, the dynamics are violent, the modulus fragments, and we get an interesting *interacting phase*.

This scenario is similar to “tachyonic preheating”: Dufaux, Felder, Kofman, Peloso, Podolsky, hep-ph/0602144.

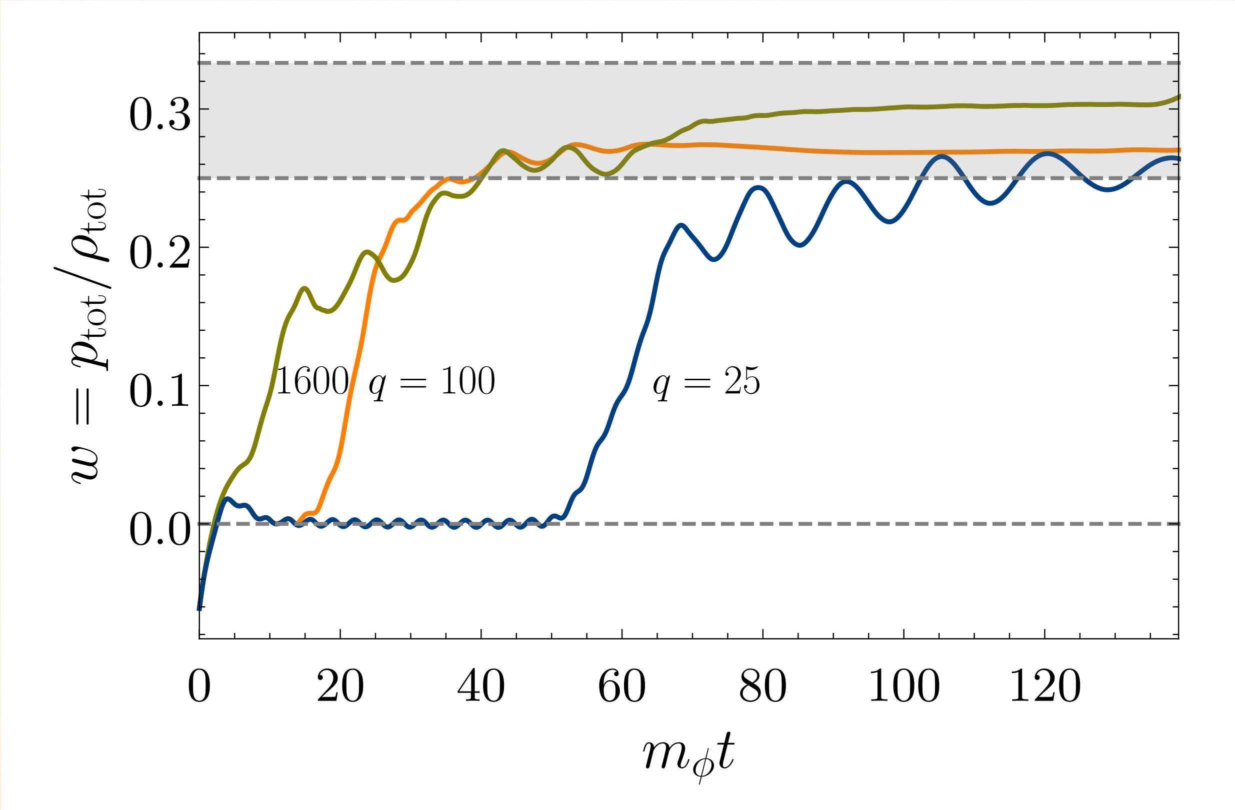
Results: fragmentation and equation of state

Fragmentation of the modulus due to back-reaction is controlled by $b \equiv \frac{M^4}{2\lambda f^2 m_\phi^2} < 1$

b>1, run-away direction in the potential

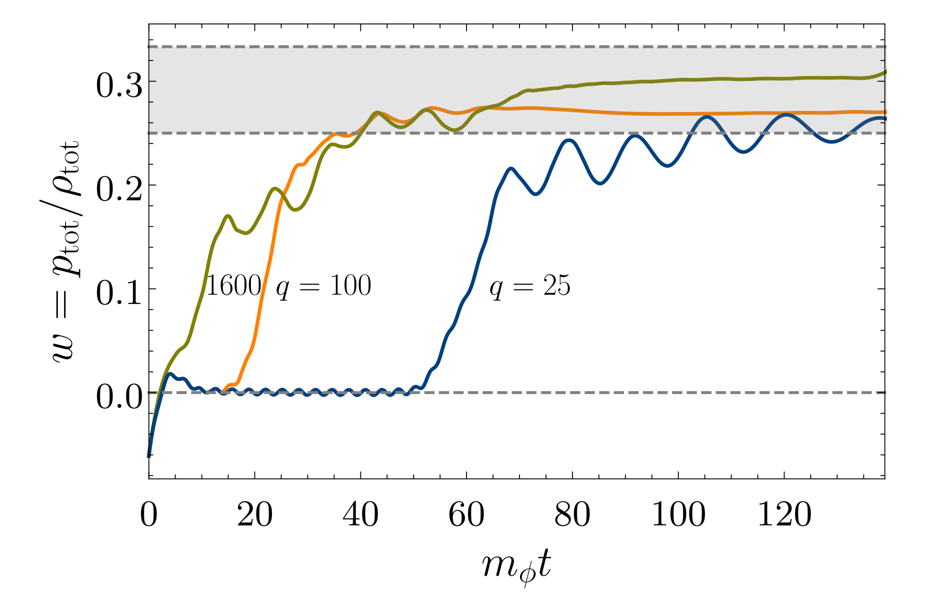


Fixing b = 0.9, varying $q = \dfrac{M^2}{m_\phi^2}$



q controls the particle production efficiency.

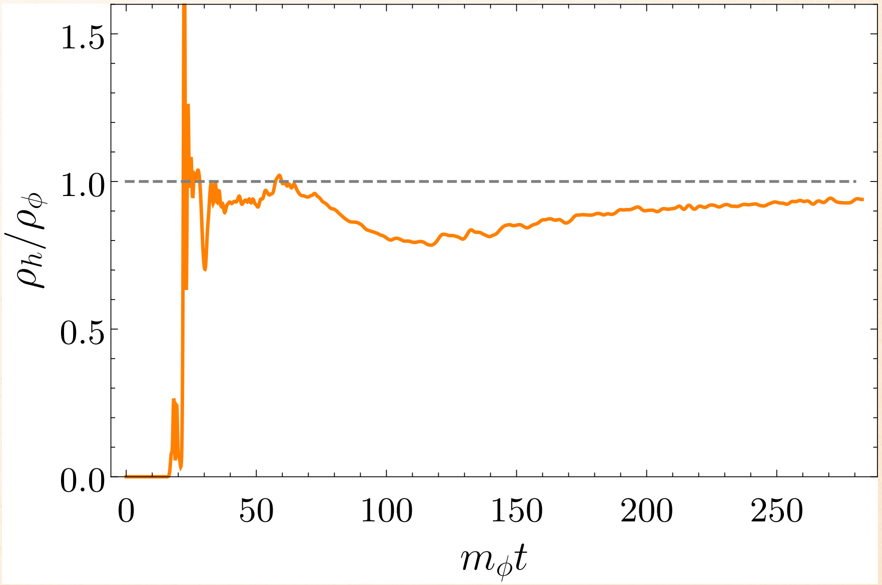
Full fragmentation (b ~ 1)



***Coupled phase*: neither matter domination nor radiation domination.**

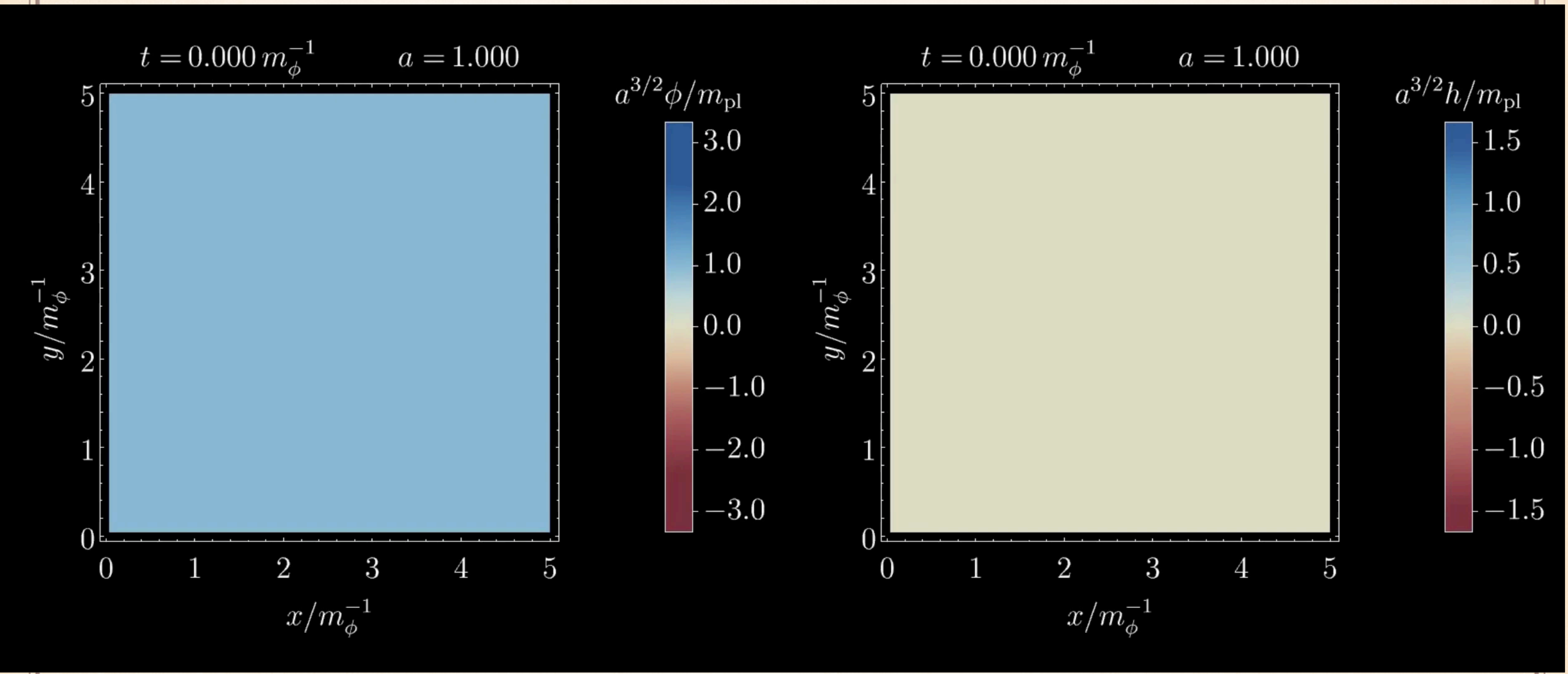
The modulus and the lighter field remain at comparable energy density.

$$\rho(h)/\rho(\phi) \approx 1$$



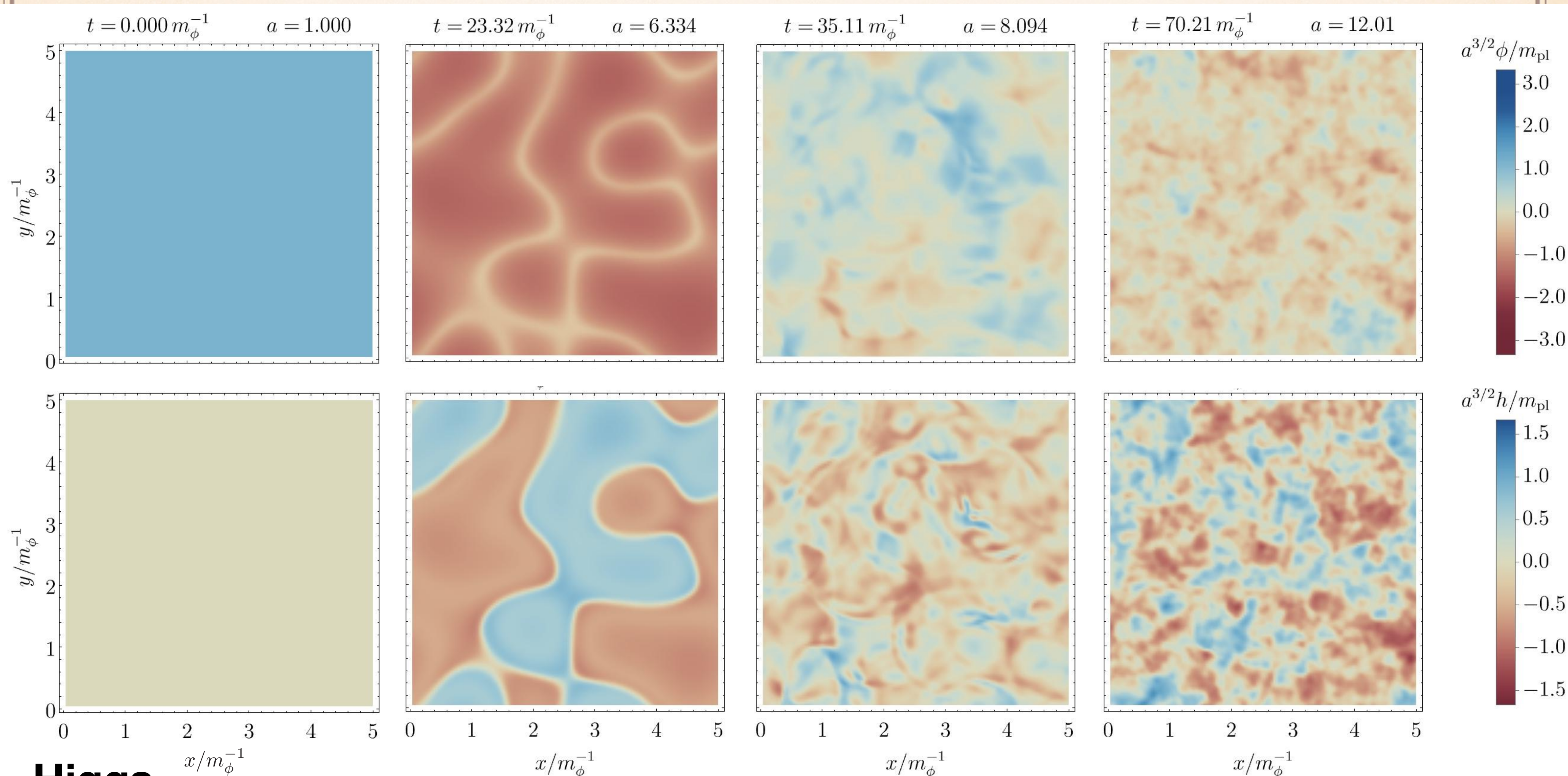
Field evolutions (full fragmentation)

Modulus **Higgs**



Snapshots of the field evolutions (full frag.)

Modulus



Higgs

transient domain walls

Comments on thermalization

- We imagine that there is no SM thermal bath when modulus starts to oscillate. This may be achieved when inflaton decays to hidden sector dominantly or modulus is the inflaton.

- We don’t consider the decays of Higgs particles. The Higgs VEVs are large in most regions and thus SM particles are more heavy than the Higgs. More detailed studies and numerical simulations are needed.

Summary of the numerical results

Backreaction efficiency parameter: $$b \equiv \frac{M^4}{2\lambda f^2 m_\phi^2} \leq 1$$

Tachyonic resonance efficiency parameter: $$q \equiv M^2/m_\phi^2$$

$$b \sim 1, q \gg 1 : \quad w \approx 1/3$$

$\downarrow$

Efficient conversion of modulus energy into Higgs (radiation)

Gravitational Wave Production

Easther, Lim ’06; Amin, Hertzberg, Kaiser, Karouby ’14

Violent dynamics, like fragmenting the modulus field, produces GW background with amplitude

$$\Omega_{\rm gw}(f_0) \sim \Omega_{r0}\delta_\pi^2\beta^2,$$

***IF* the universe remains radiation dominated after GW production until the usual matter-radiation equality**

δ_π **: fraction of energy in quadrupoles ~ O(0.1)**

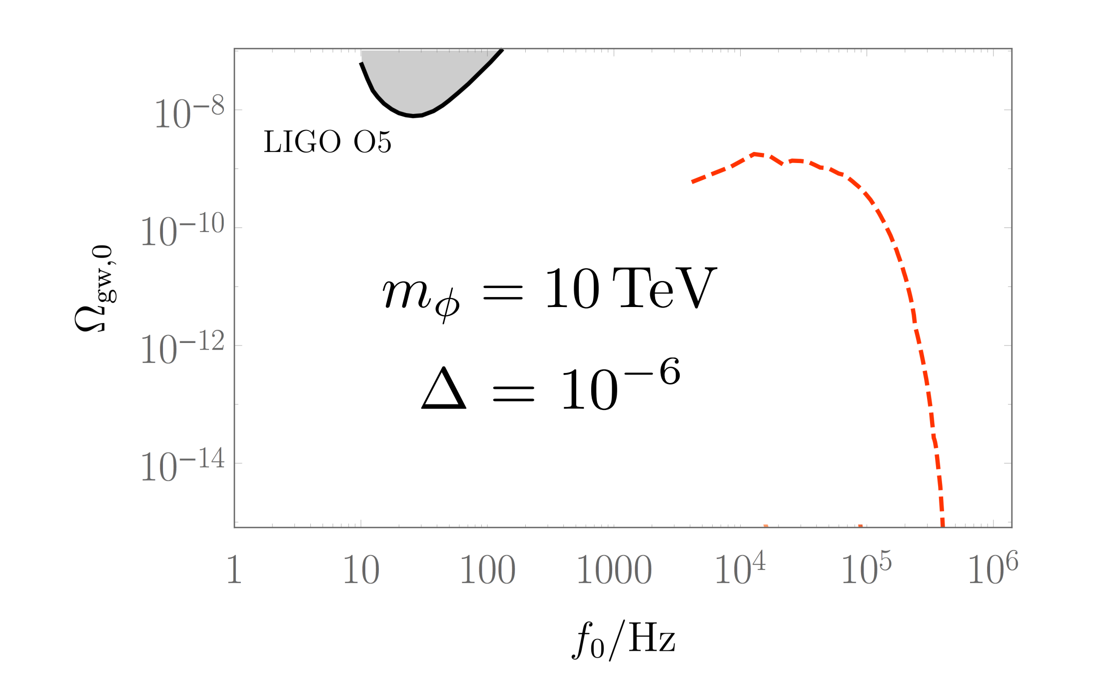
β **: relation between GW peak wavelength and Hubble (~10^{-1} for q ~ 100; $\beta \sim q^{-1/2}$)**

Gravitational Waves from Moduli fragmentation

If **the out-of-equilibrium dynamics immediately converts all of the moduli to radiation, these simple estimates yield (β~10^{-1}):**

$$f_0 \sim \frac{a_{\rm osc}}{a_0}\beta^{-1}H_{\rm osc} \sim \text{kHz} \times \beta^{-1}\sqrt{\frac{m_\phi}{10\,\text{TeV}}}\,,$$

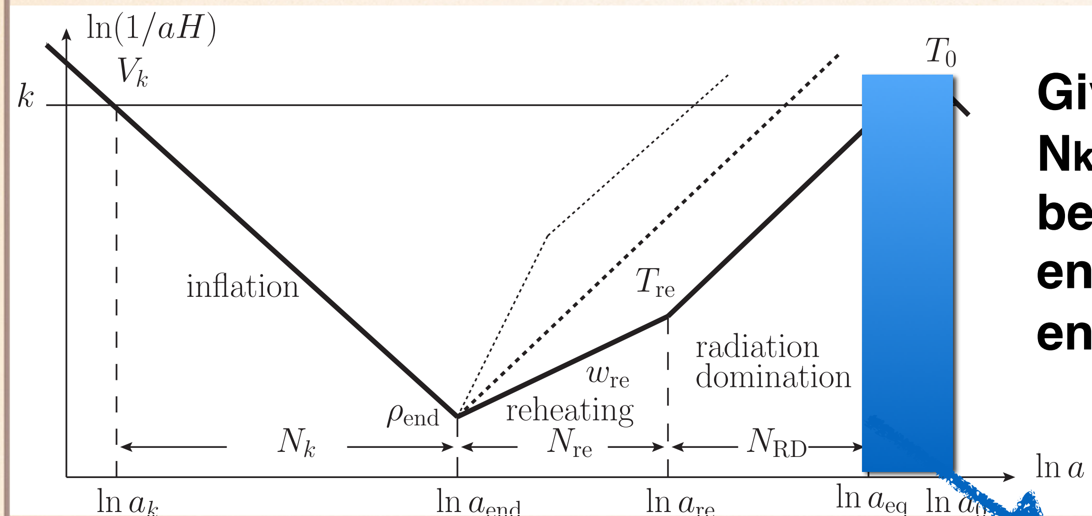
$$\Omega_{gw} \sim \Omega_{r,0}\delta_\pi^2\beta^2 \sim 10^{-6}\beta^2$$



This frequency is above the LIGO band. Need new technologies (Akutsu et. al '08; Arvanitaki and Geraci '12; Goryachev, Tobar '14).

The *amplitude* isn't terrible, and astrophysical backgrounds are low at high frequencies.

(n_s, r) and the Time Interval After Inflation

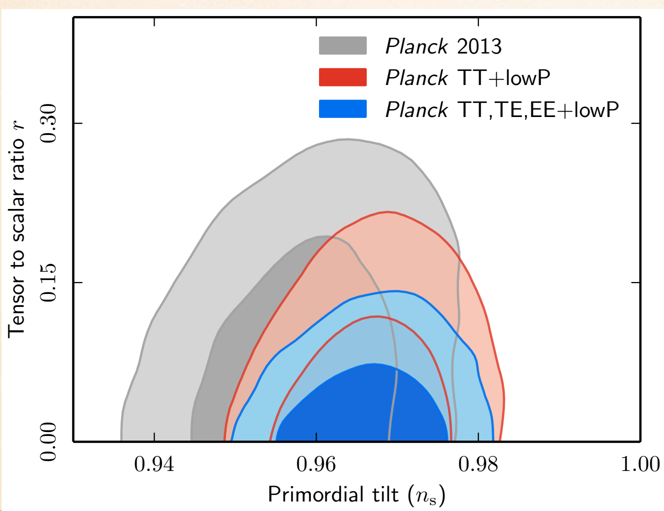


Given a cosmological history,
N_k related to the total number of e-folds between end of inflation and today;
energy density during inflation related to energy density today.

early-time matter domination

Liddle, Leach '03
Dai, Kamionkowski, Wang '14

Inflationary constraints (n_s, r)



Constraints on after-inflation history, e.g., modulus mass

$$\frac{m_\phi^2}{M_{\rm Pl}^2} \gtrsim \exp\left[\frac{-6(1+w_{\rm mod})}{1-3w_{\rm mod}}\left(57 - N_k + \ln\left(\frac{r\rho_k}{\rho_{\rm end}}\right)^{\frac{1}{4}}\right)\right]$$

Connection to inflationary parameters

Constraints on after-inflation history, e.g., modulus mass

modulus mass ←
$$\frac{m_\phi^2}{M_{\rm Pl}^2} \gtrsim \exp\left[\frac{-6(1+w_{\rm mod})}{1-3w_{\rm mod}}\left(57 - N_k + \ln\left(\frac{r\rho_k}{\rho_{\rm end}}\right)^{\frac{1}{4}}\right)\right]$$

$w_{\rm mod}$: **equation of state during the modulus epoch**

For some inflation models, (n_s, r) *disfavors* extended period of matter domination and sets a (much) stronger constraint on modulus mass compared to the well-known cosmological modulus bound (Dutta, Maharana '14)

$0 < w_{\rm mod} < 1/3$ **bound relaxed considerably compared to** $w_{\rm mod} = 0$

Early matter domination with non-linear dynamics

Early matter domination without non-linear dynamics

Parametrics: Can We Get an Effect?

What the numerics are showing is that to get a significant period of coupled, out-of-equilibrium modulus/Higgs dynamics, we need

$$M^4 \sim \lambda m_\phi^2 f^2 \qquad \left(M^2 \frac{\phi}{f} H^\dagger H\right)$$

This could be satisfied in:

$$a) m_\phi \lesssim M \ll f \sim M_{\rm pl}, \lambda \ll 1$$
$$b) m_\phi \ll M \ll f \sim M_{\rm pl}, \lambda \sim 1$$

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$$b) m_\phi \ll M \ll f \sim M_{\rm pl}, \lambda \sim 1$$

For a), ***small quartics can arise along D-flat directions in SUSY.***

More realistic model: SUSY

How to achieve small Higgs quartic? $m_\phi \lesssim M \ll f \sim M_{\rm pl}, \lambda \ll 1$

Reminder:

The tree-level MSSM has a Higgs quartic coupling from D-terms, completely fixed by the Higgs' electroweak representations:

$$V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (bH_u^0 H_d^0 + \text{c.c.})$$
$$+ \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2$$

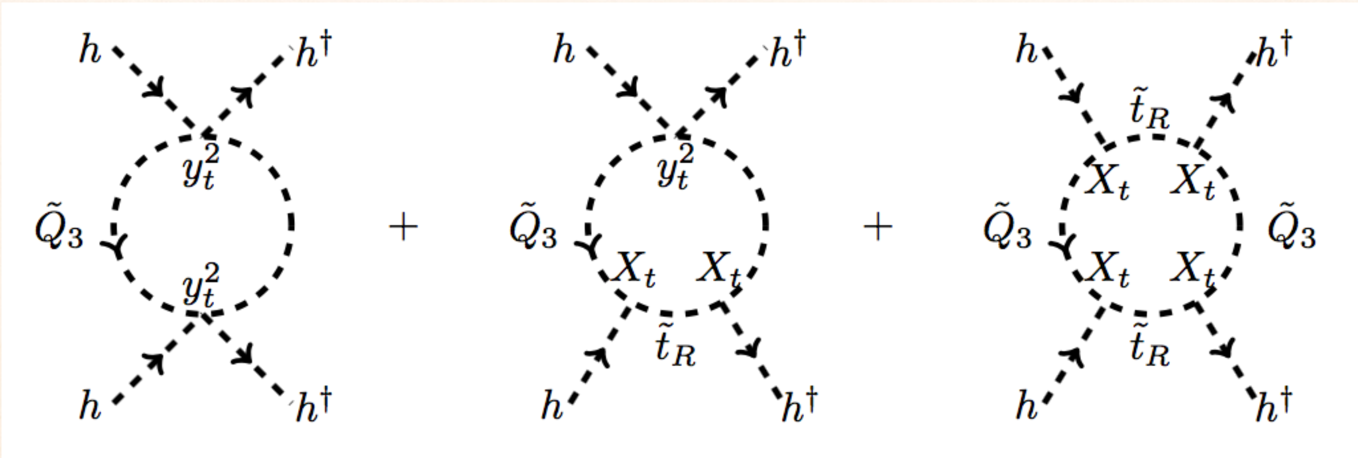
Notice the D-flat direction: $|H_u^0| = |H_d^0|$

The Higgs quartic coupling

In addition to the tree-level potential,

$$V = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (bH_u^0 H_d^0 + \text{c.c.})$$
$$+ \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2$$

a SUSY-breaking contribution to the Higgs quartic comes from loops of stops:



$$V_{1-\text{loop}} \approx \frac{3y_t^4}{16\pi^2}(H_u^\dagger H_u)^2 \left[\log\frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2}\left(1 - \frac{1}{12}\frac{X_t^2}{m_{\tilde{t}}^2}\right)\right]$$

Non-vanishing along the D-flat direction. Does it stop us?

EWSB Along the Flat Direction

Suppose there is a tachyonic direction pointing along the flat direction, that is, that we have

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} |\mu|^2 + m_{H_u}^2 & -b \\ -b & |\mu|^2 + m_{H_d}^2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2 - 2b < 0$$

How large will the Higgs VEV be? At first, you would expect to be stopped by the loop-level quartic coupling:

$$V_{1-\mathrm{loop}} \approx \frac{3y_t^4}{16\pi^2}(H_u^\dagger H_u)^2 \left[\log \frac{m_{\tilde{t}}^2}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}}^2}\left(1 - \frac{1}{12}\frac{X_t^2}{m_{\tilde{t}}^2}\right)\right]$$

But importantly, the stop mass here is the geometric mean of the *physical* stop masses,

$$m_{\tilde{t}}^2 \approx m_{Q_3,\bar{u}_3}^2 + y_t^2 |H_u^0|^2$$

and as we move far out along the flat direction the stop and top become degenerate:

$$\langle H_u^0 \rangle \gg M_{\mathrm{soft}} \Rightarrow m_{\tilde{t}} \approx m_t$$

Approximate SUSY suppresses the quartic by a factor of M_{soft}^2/H^2, allowing Higgs VEVs much larger than soft masses!

Summary

Cosmology could allow us to see the effects of fine-tuning directly.

Time-dependent VEVs of moduli explore regions where the Higgs potential can be very different than in our late-time universe.

This can lead to a *coupled dynamical evolution* of the modulus and the Higgs, with exotic equation of state w close to 1/3.

The modulus can fragment and produce *gravitational waves*.

The non-linear dynamics also affects *the time elapsed from inflation to the CMB*, influencing fits of inflationary models or the early matter domination before BBN.

However, that may require unusual parameter choices, for instance *tiny quartic couplings*. In SUSY, such tiny quartics occur when venturing out along the *D-flat directions*! The fact that our universe is tuned might make it easy to access such regions of field space.

Thank you!

Backup

The problem of backreaction

But: once many Standard Model particles are created, they backreact.

Simple estimate: the particle production will be stalled once

$$\rho_{\mathrm{SM}} \sim \rho_\phi$$

Crudely, can think of this as the quartic

$$\lambda h^4 \sim \lambda \langle h^2 \rangle h^2$$

turning into a positive mass for the Higgs.

Back-reaction parameter

$$b \equiv \frac{M^4}{2\lambda f^2 m_\phi^2}$$

A toy model: Coupling a modulus to the Higgs

Consider a coupling linear in the modulus, ϕ :

$$\frac{1}{2}m_\phi^2\phi^2 + \frac{M^2}{f}(\boxed{\phi} - \boxed{\phi_0})\left(h^\dagger h - \frac{v^2}{2}\right) + \lambda(h^\dagger h)^2.$$

Higgs mass term depends on the modulus value.

SM Higgs mass

Global minimum at H/√2 = v, ϕ = 0
($v^2 = M^2\phi_0/(\lambda f).$)

Modulus: scalar fields with Planck-suppressed couplings. Ubiquitous in string theory constructions and low energy (SUSY) models. They could have very large field range. Here I just use it as a very weakly-coupled scalar field with a large field range.

A toy model: Coupling a modulus to the Higgs

Consider a coupling linear in the modulus:

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Higgs mass term depends on the modulus value.

SM Higgs mass

Scales:

$\mu^2 = M^2\phi_0/f$: **Standard Model Higgs mass squared param**

f : **Modulus field range (e.g, ~ Planck)**

M : **"Natural" Higgs mass param (e.g., ~ 100s TeV)**

m_ϕ : **Modulus mass (e.g., ~ 100s TeV)**

Possible hierarchies: $\mu \ll m_\phi \lesssim M \ll f$

(Worth considering other variations too)

A toy model: Coupling a modulus to the Higgs

Consider a coupling linear in the modulus:

$$\frac{1}{2}m_\phi^2\phi^2 + \frac{M^2}{f}\left(\boxed{\phi} - \boxed{\phi_0}\right)\left(h^\dagger h - \frac{v^2}{2}\right) + \lambda(h^\dagger h)^2.$$

Higgs mass term depends on the modulus value. Natural Higgs mass:

$$\frac{M^2}{f}\phi \sim M^2$$ **when $\phi \sim f$**

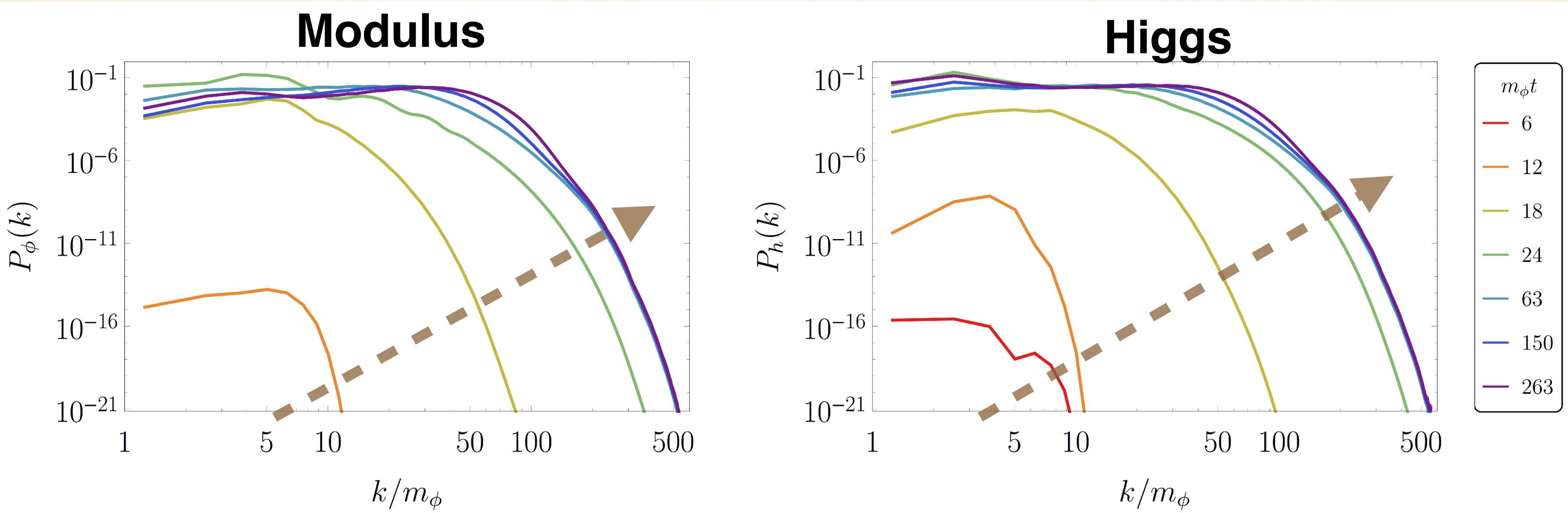
SM Higgs mass

$$\mu^2 = \frac{M^2}{f}\phi_0$$

Measure of fine-tuning: $$\frac{M^2}{\mu^2} \sim \left|\frac{\phi_0}{f}\right|$$

Power spectra

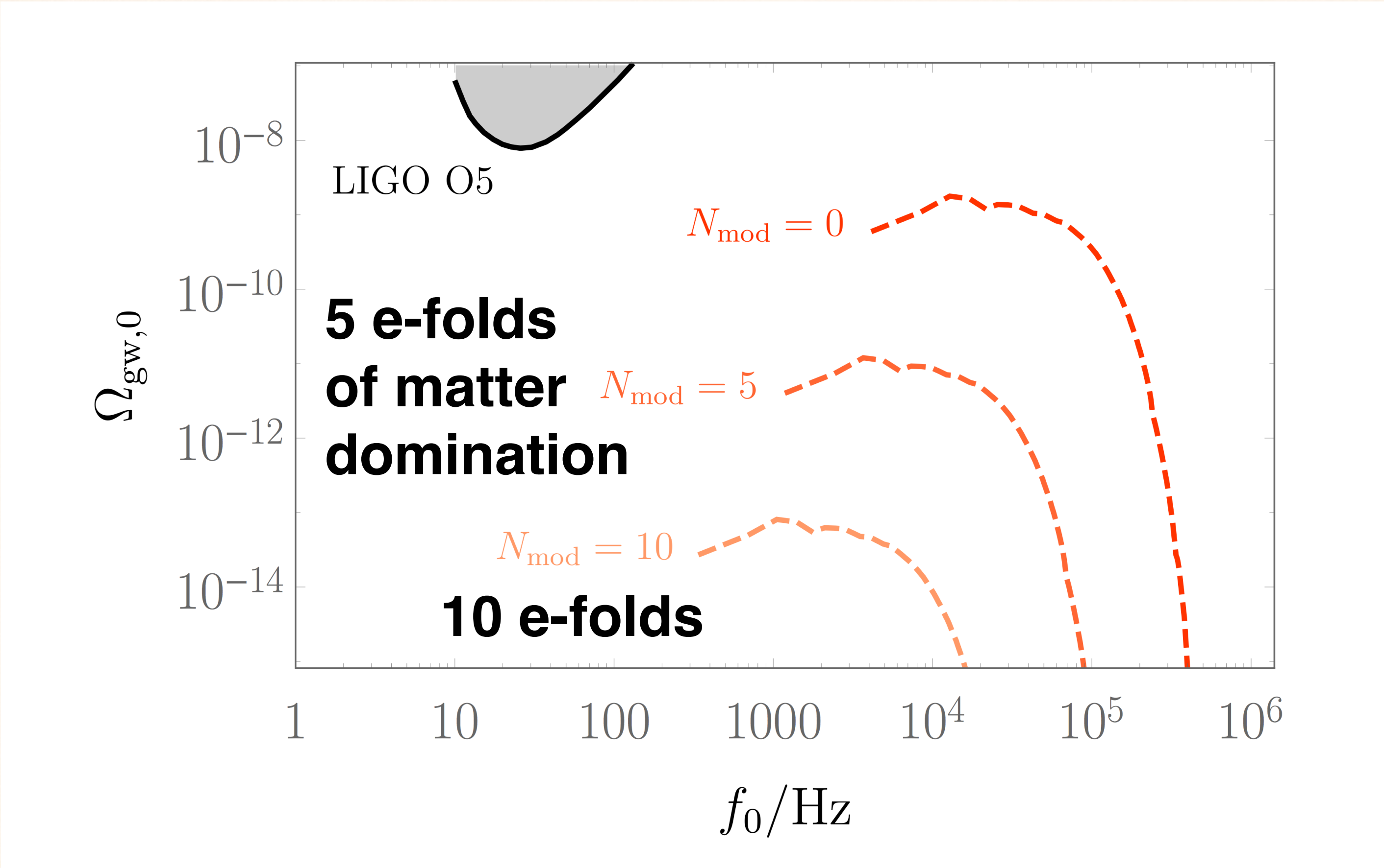
$$P_F(k) \equiv \phi_{\rm osc}^{-2}(d/d\ln k)\overline{F^2(\mathbf{x})},$$



As time grows (the dashed arrow), modulus field fragments (P ~ O(1)) and power propagates to higher comoving modes.

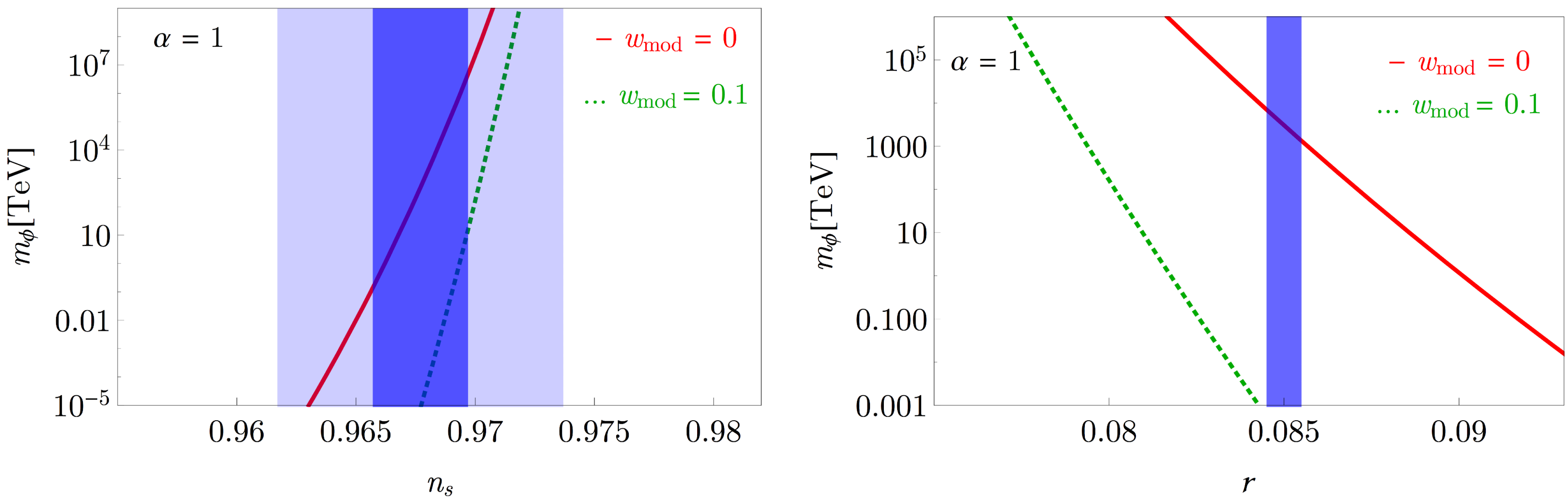
Possible complication

Assumption: a radiation-like equation of state till the perturbative decay of the modulus (final radiation domination). Yet the very long-term dynamics is unclear…



For example, $$V_{\rm inf} = \frac{1}{2} m^{4-\alpha} \phi_{\rm inf}^{\alpha},$$

For α = 1,



Higher-Dimension Operators Lifting the Flat Direction

Flat directions should always be lifted at very large field values.

Kähler corrections are compatible with VEVs of order the cutoff:

$$\int d^4\theta \frac{X^\dagger X}{\Lambda^4}(H_u^\dagger H_u)^2 \to \frac{m_{\rm soft}^2}{\Lambda^2}(H_u^\dagger H_u)^2$$

Superpotential terms at first glance appear more dangerous.

$$\int d^2\theta \left(\mu H_u \cdot H_d + \frac{1}{M}(H_u \cdot H_d)^2\right)$$

gives rise to quartics:

$$\frac{\mu^\dagger}{M}(H_u^\dagger H_u)(H_u \cdot H_d) + \ldots \Rightarrow \langle h \rangle \sim \sqrt{\mu M}$$

but given that some spurion forbids the mu term we expect

$$\frac{1}{M} \lesssim \frac{\mu}{\Lambda^2} \Rightarrow \langle h \rangle \sim \Lambda$$

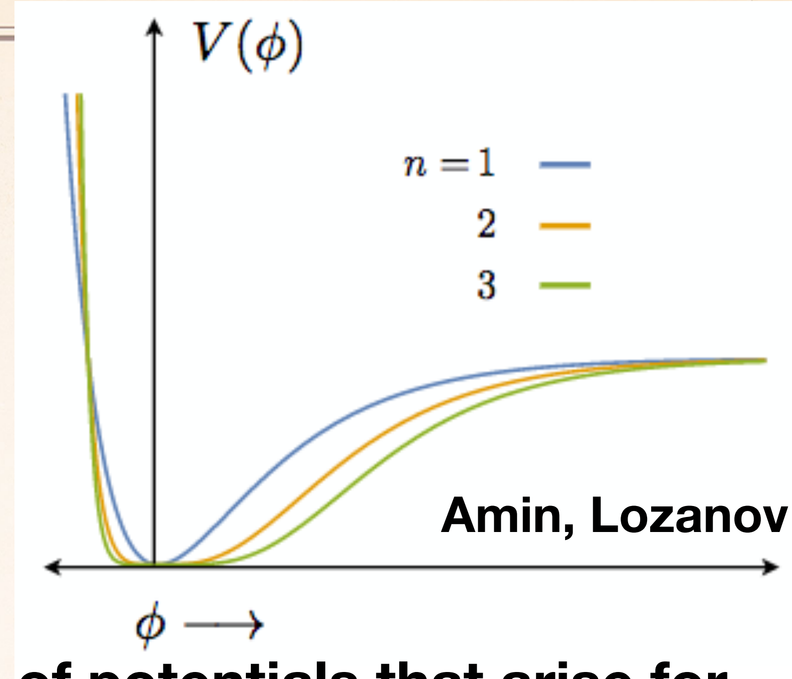
Possible future directions

Model building
- saxion, D-flat direction
- Different hierarchies of parameters

Non-linear dynamics
- More powerful simulation
- Analytical understanding

Signals
- High frequency GW
- Other consequences: phase transitions?

Other possible dynamics: Oscillons

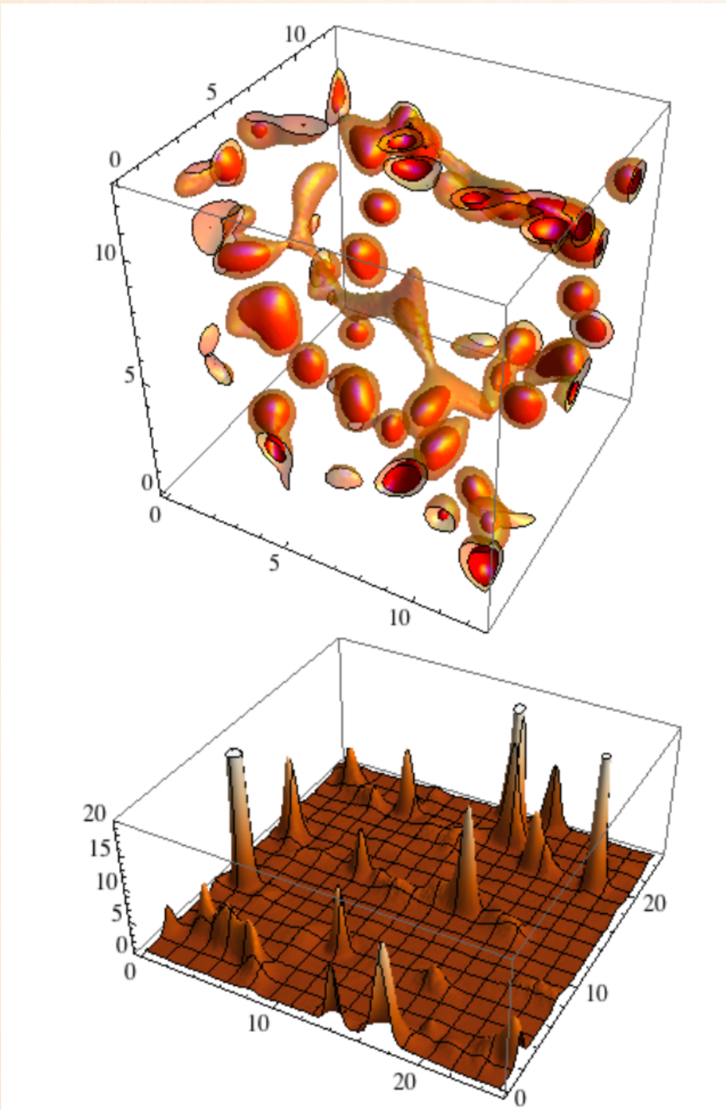


Amin, Lozanov ’17

The shapes of potentials that arise for moduli can lead to formation of “oscillons”—localized lumps of oscillating field.

This could change our story in interesting ways, as the modulus doesn’t redshift inside the oscillon. More mass sign flipping and less backreaction?

No conclusions yet! Need more studies.



Amin, Easther, Finkel, Flauger, Hertzberg ’11