Theoretical calculations at the NNLO frontier

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Precision QCD at the LHC

- No spectacular new physics appeared so far. Extremely good control on many different key observables may highlight (small) deviations from SM behavior → indication of new physics

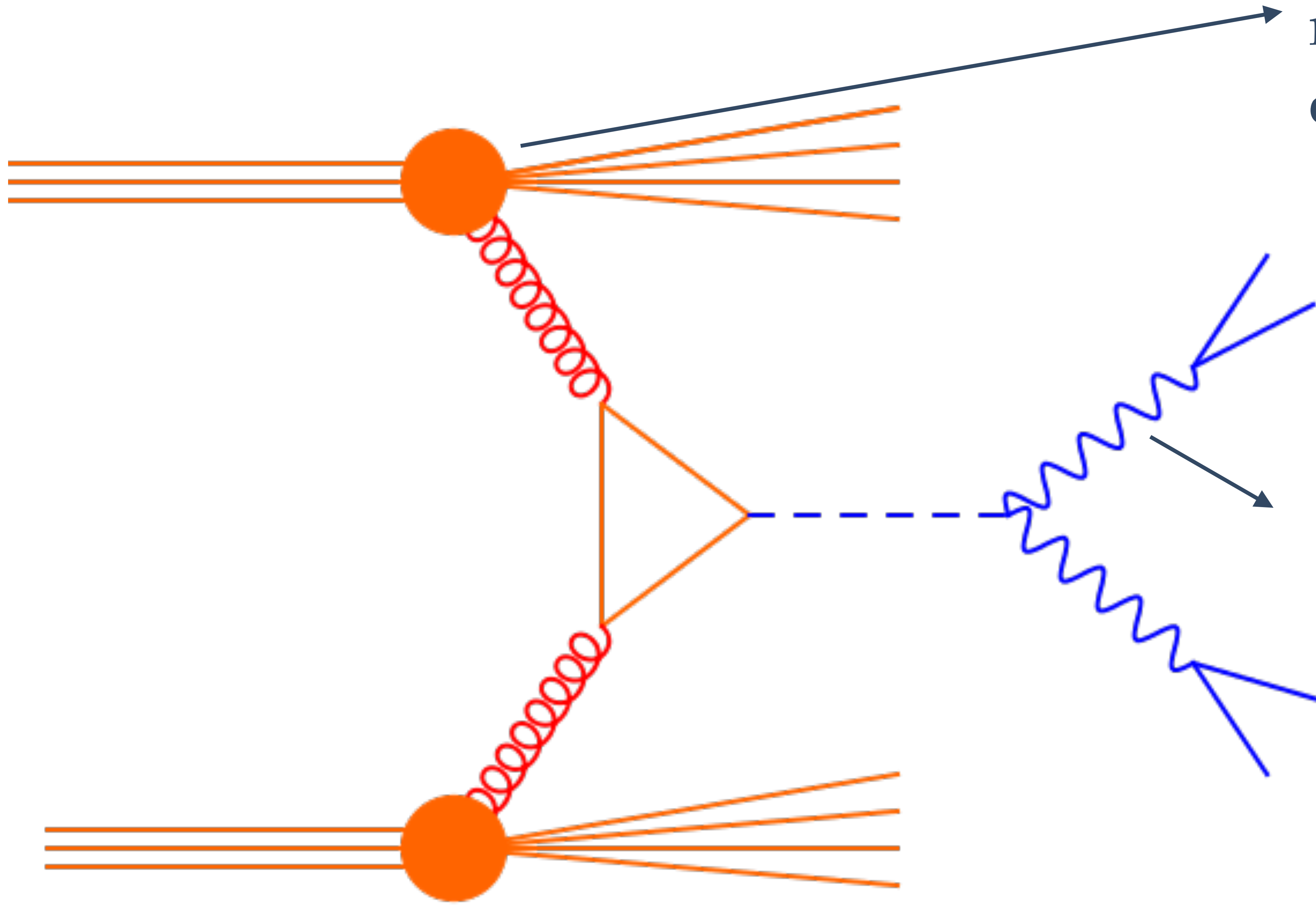
Imagine to have new physics at a scale Λ

- if Λ small → should see it directly, bump hunting
- if Λ large, typical modification to observable w.r.t. standard model prediction: $\delta O \sim Q^2/\Lambda^2$
- standard observables at the EW scale: to be sensitive to ~ TeV new physics, we need to control δO to few percent
- *high scale processes (large p_T, large invariant masses…): sensitive to ~TeV if we control δO to 10-20%*

The LHC machine and experimental program are running extremely well. *These levels or precision are within reach*

"*Few percent*": the theory side

QCD at hadron colliders: FACTORIZATION



Short distance non perturbative effects (PDFs)

Interesting high-Q phenomena

$$d\sigma = \int dx_1 dx_2 f(x_1) f(x_2) d\sigma_{\text{part}}(x_1, x_2) F_J (1 + \mathcal{O}(\Lambda_{\text{QCD}}/Q))$$

"*Few percent*": the theory side

$$d\sigma = \int dx_1 dx_2 f(x_1) f(x_2) d\sigma_{\text{part}}(x_1, x_2) F_J (1 + \mathcal{O}(\Lambda_{\text{QCD}}/Q))$$

Input parameters: ~few percent.

In principle improvable

NP effects: ~ few percent

No good control/understanding of them at this level. LIMITING FACTOR FOR FUTURE DEVELOPMENT

[m_t, m_W…]

HARD SCATTERING MATRIX ELEMENT

- large Q → most interesting and theoretically clean
- α_s ~ 0.1 → For TYPICAL PROCESSES, we need NLO for ~ 10% and NNLO for ~ 1 % accuracy. Processes with large color charges (Higgs): $\alpha_s C_A$~ 0.3 → N^3LO
- Going beyond that is neither particularly useful (exp. precision) NOR POSSIBLE GIVEN OUR CURRENT UNDERSTANDING OF QCD

Where can we achieve high accuracy?

Focus on simple [*clean exp/th comparison, good control*] processes, high Q [*little non pert. contamination*] observables. Typical examples:

- H / H+j(j) / VH → Higgs couplings / characterization
- V / V+j(j) → PDFs, backgrounds
- tt, single top → gluon and b PDF, V_{tb}, backgrounds…
- VV → anomalous couplings, (Higgs) backgrounds…
- jj(j) → PDFs, jet dynamics, α_S…

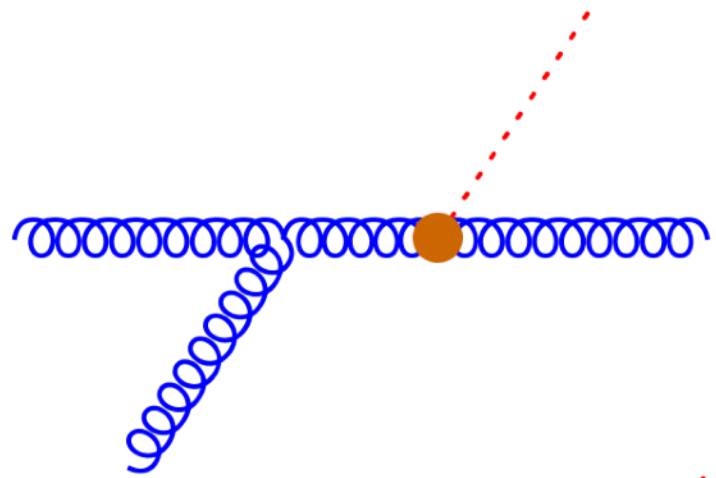
Why fixed order?

- Able to provide HIGH PRECISION while PROPERLY ACCOUNT FOR EXPERIMENTAL SETUP (cuts, fiducial region…)
- At high Q, typically processes are a multi-scale problem. However, no huge scale hierarchies → fixed (high enough) order predictions correctly capture all the relevant logs

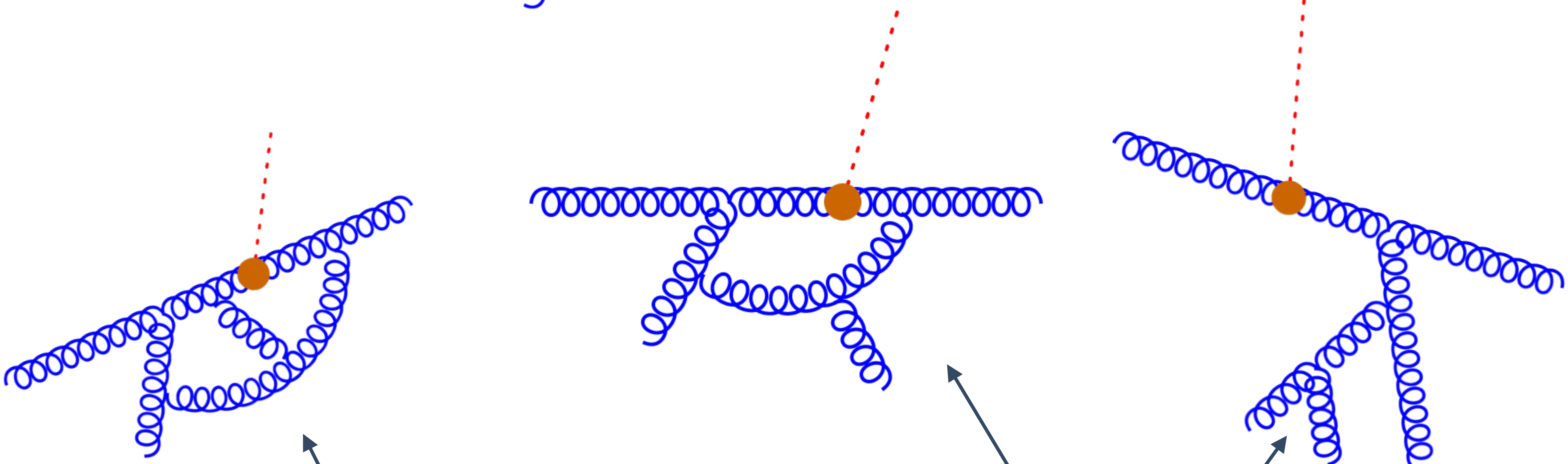
NNLO computations: challenges

$O(\alpha_s^2)$ corrections: two-loop (VV), one-loop+j (RV), tree+jj (RR)

E.g. Higgs p_t: LO



NNLO →



Two Big Problems:

loop amplitudes / IR structure of extra emission

Loop amplitudes: status

- Amplitude COMPLEXITY GROWS VERY FAST with the number of scales: invariants (~# legs) and particle masses

$$
\begin{aligned}
F^{\rm L}_{--++} = & -(x^2+y^2)\Big[4\mathrm{Li}_4(-x)+\frac{1}{48}Z_+^4 \\
& +(\tilde{Y}-3\tilde{X})\mathrm{Li}_3(-x)+\Xi\mathrm{Li}_2(-x) \\
& +i\frac{\pi}{12}Z_+^3+i\frac{\pi^3}{2}X-\frac{\pi^2}{12}X^2-\frac{109}{720}\pi^4\Big] \\
& +\frac{1}{2}x(1-3y)\Big[\mathrm{Li}_3(-x/y)-Z_-\mathrm{Li}_2(-x/y) \\
& -\zeta_3+\frac{1}{2}Y\tilde{Z}\Big]+\frac{1}{8}\left(14(x-y)-\frac{8}{y}+\frac{9}{y^2}\right)\Xi \\
& +\frac{1}{16}(38xy-13)\tilde{Z}-\frac{\pi^2}{12}-\frac{9}{4}\left(\frac{1}{y}+2x\right)\tilde{X} \\
& +\frac{1}{4}x^2\Big[Z_-^3+3\tilde{Y}\tilde{Z}\Big]+\frac{1}{4}+\Big\{t\leftrightarrow u\Big\},
\end{aligned}
$$

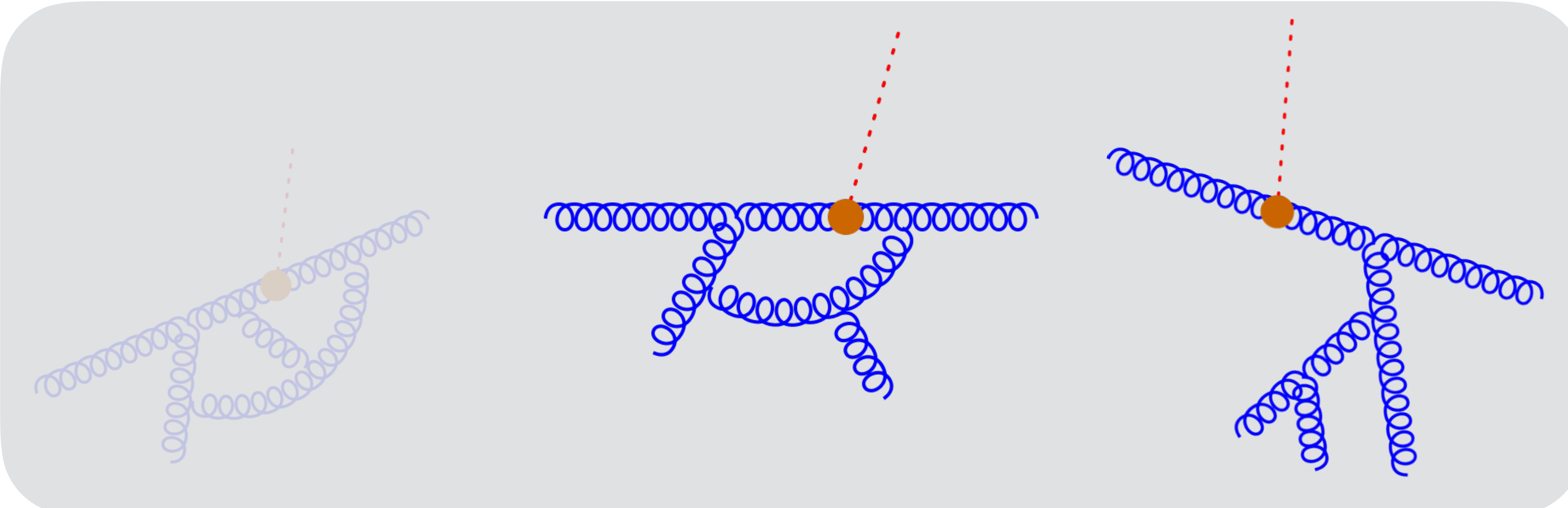
gg→γγ

[Bern, De Freitas, Dixon [2002]

gg→VV: ~ 10 MB expression

- Despite a lot of recent progress still pretty limited knowledge. State of the art:
  - Analytically: 2 -> 2, external masses (pp->VV*) [FC, Henn, Melnikov, Smirnov, Smirnov (2014-15); Gehrmann, Manteuffel, Tancredi (2014-15)]
  - Numerically: 2->2, internal/external masses (pp-> tt, pp->HH) [Czakon; Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke (2016)]
  - Lot of recent progress: towards 2->3 [Badger et al (2016)]; numerical unitarity [Abreu et al (2017)], many-scales integrals [Gehrmann, Henn, Lo Presti (2015); Papadopoulos, Tommasini, Wever (2016); Tancredi, Remiddi (2016); Weinzierl et al (2017); Bonciani et al (2016)]

IR structure of real emission



- IR divergences hidden in PS integrations of extra real emission(s)
- After integrations, all singularities are manifest and cancel against two-loop (KLN)
- We are interested in realistic setup (arbitrary cuts, arbitrary observables) → we need fully differential results, we are not allowed to integrate over the PS
- The challenge is to EXTRACT PS-INTEGRATION SINGULARITIES WITHOUT ACTUALLY PERFORMING THE PS-INTEGRATION

Dealing with real emission

A lot of conceptual progress in the last few years → we are now able to tackle this issue

``q_t'' [Catani, Grazzini]

``Antenna'' [Gehrmann-de Ridder, Gehrmann, Glover]

``N-jettiness'' [Boughezal et al, Gaunt et al]

``Sector decomposition+FKS'' [Binoth, Heinrich; Anastasiou, Melnikov, Petriello; Czakon; Czakon, Heymes; FC, Melnikov, Röntsch]

``Projection to Born'' [Cacciari, Dreyer, Karlberg, Zanderighi, Salam]

``Colorful NNLO'' [Del Duca, Duhr, Kardos, Somogyi, Trocsanyi]

Dealing with real emission

A lot of conceptual progress in the last few years → we are now able to tackle this issue

Some of these techniques are quite generic

- IN PRINCIPLE, they allow for ARBITRARY COMPUTATIONS
- IN PRACTICE: `genuine' 2→2 REACTIONS, with big computer farms

A typical example:

- Inclusive DY (2→1 inclusive): ~ few second
- DY, precise lepton observables (2→1 exclusive): ~ 100 CPU hours
- V+jet, diff. distribution (2→2 exclusive): ~ 100.000 CPU hours

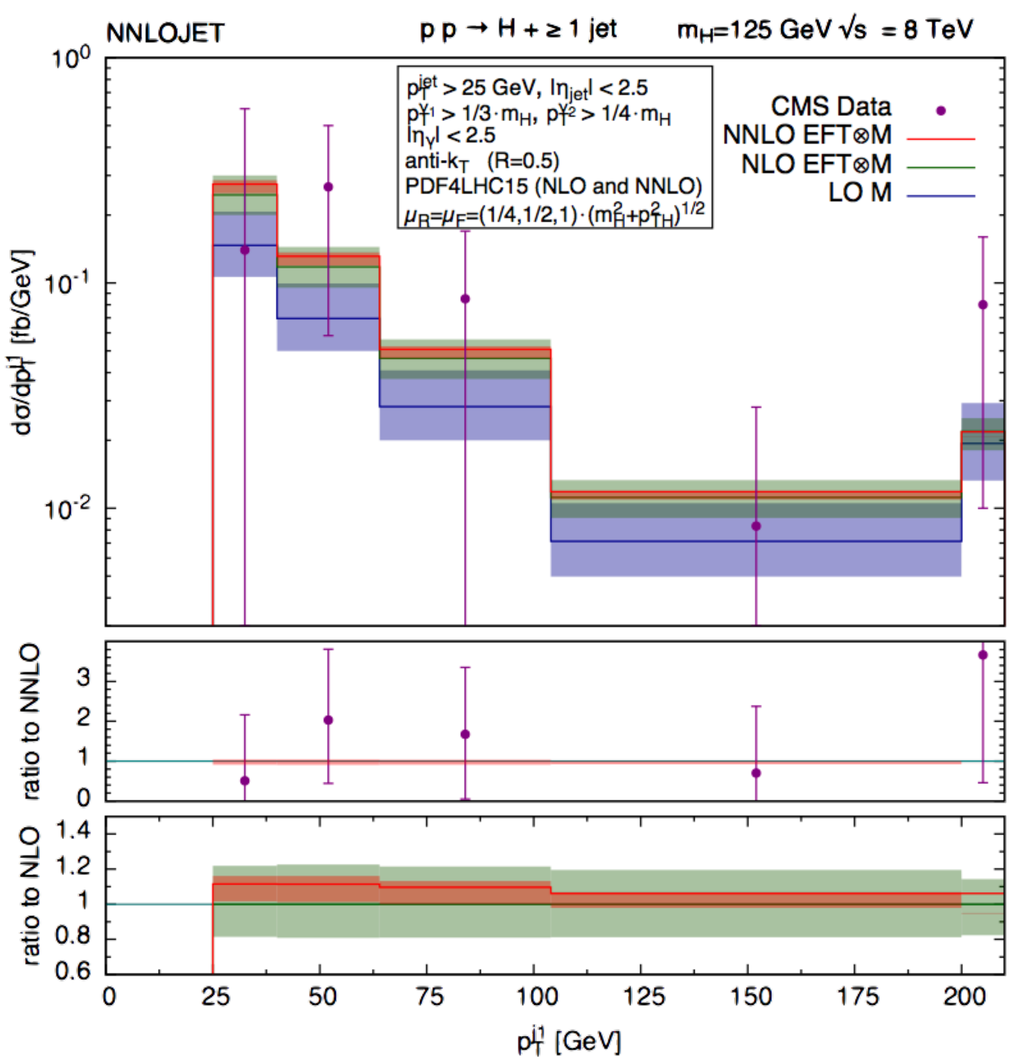
$2 \rightarrow 2$ phenomenology at NNLO: the global picture

2→2 pheno @ NNLO: the global picture

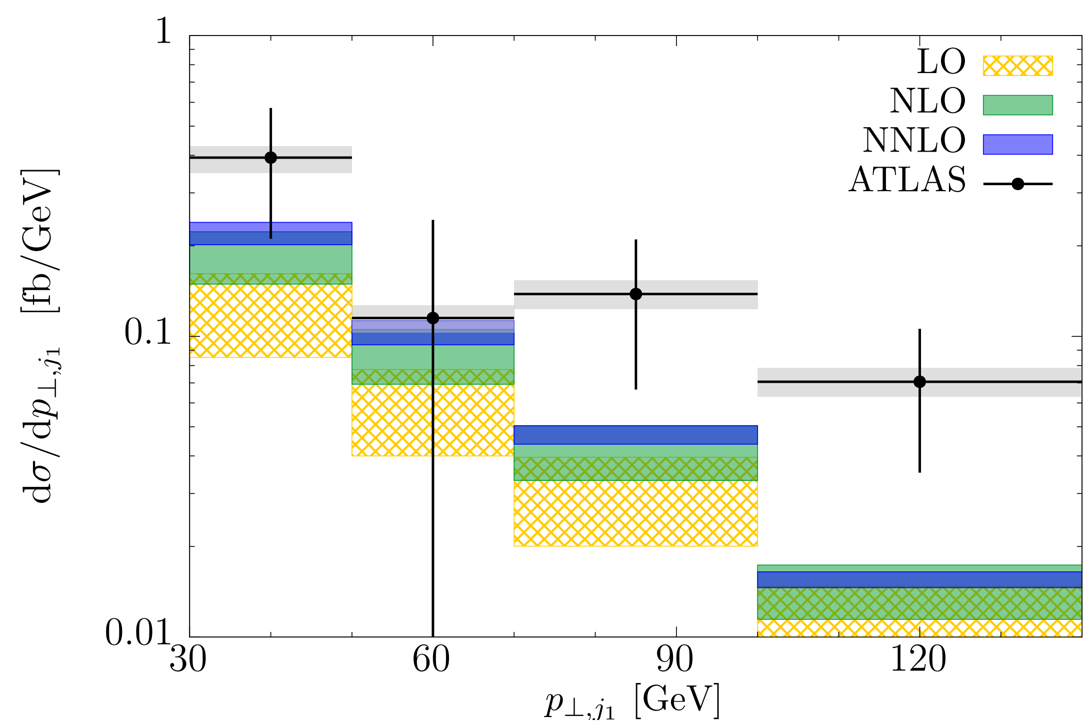
Greatly reduced theoretical uncertainties, perturbative convergence established

$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\ (\text{theory}) \pm 1.56\,\text{pb}\,(3.20\%)\ (\text{PDF}+\alpha_s)\,.$$

Inclusive H@N^{3}LO
[Anastasiou, Duhr, Dulat, Herzog, Mistlberger]

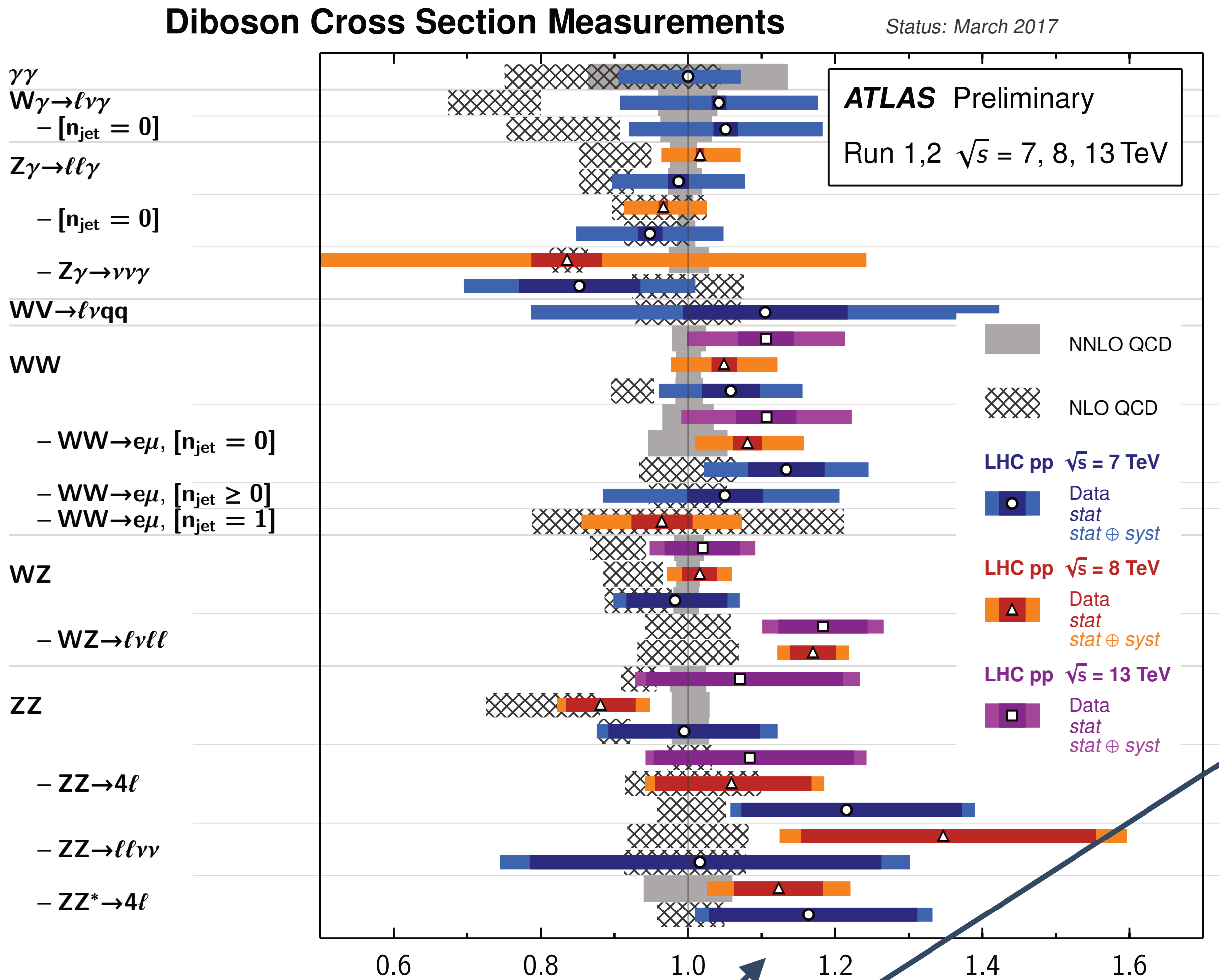


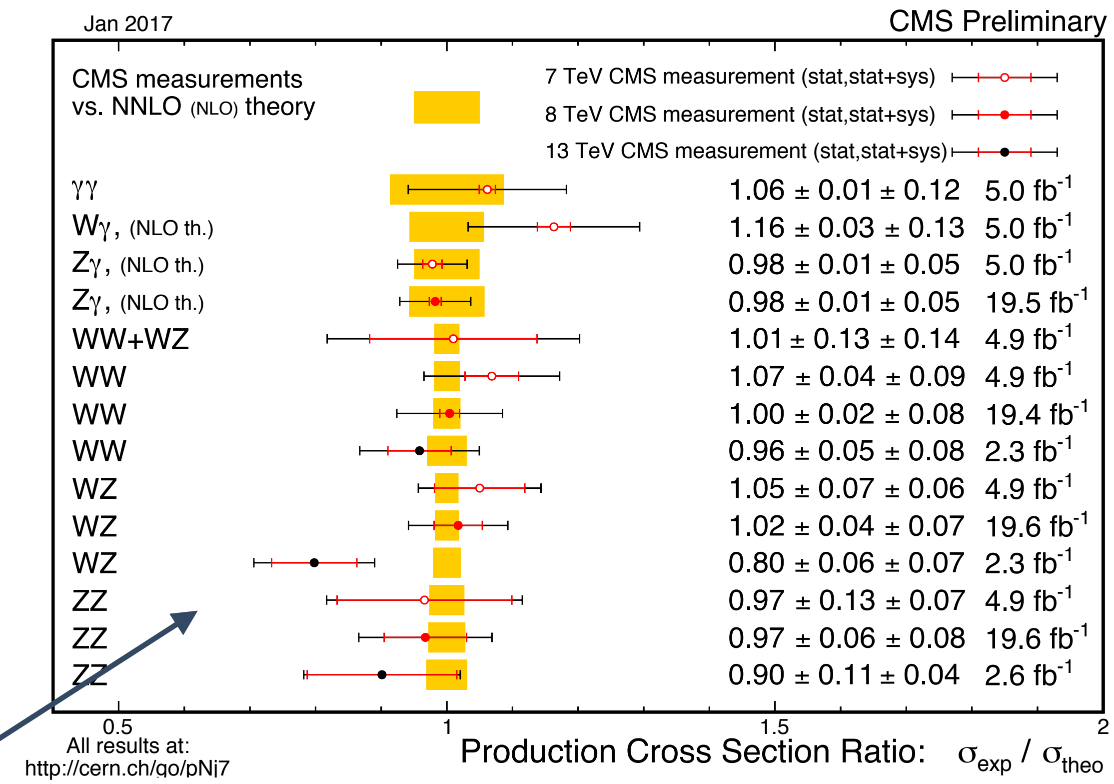
Exclusive Higgs + jet
[Boughezal, et al; Chen et al; FC, Melnikov, Schulze]

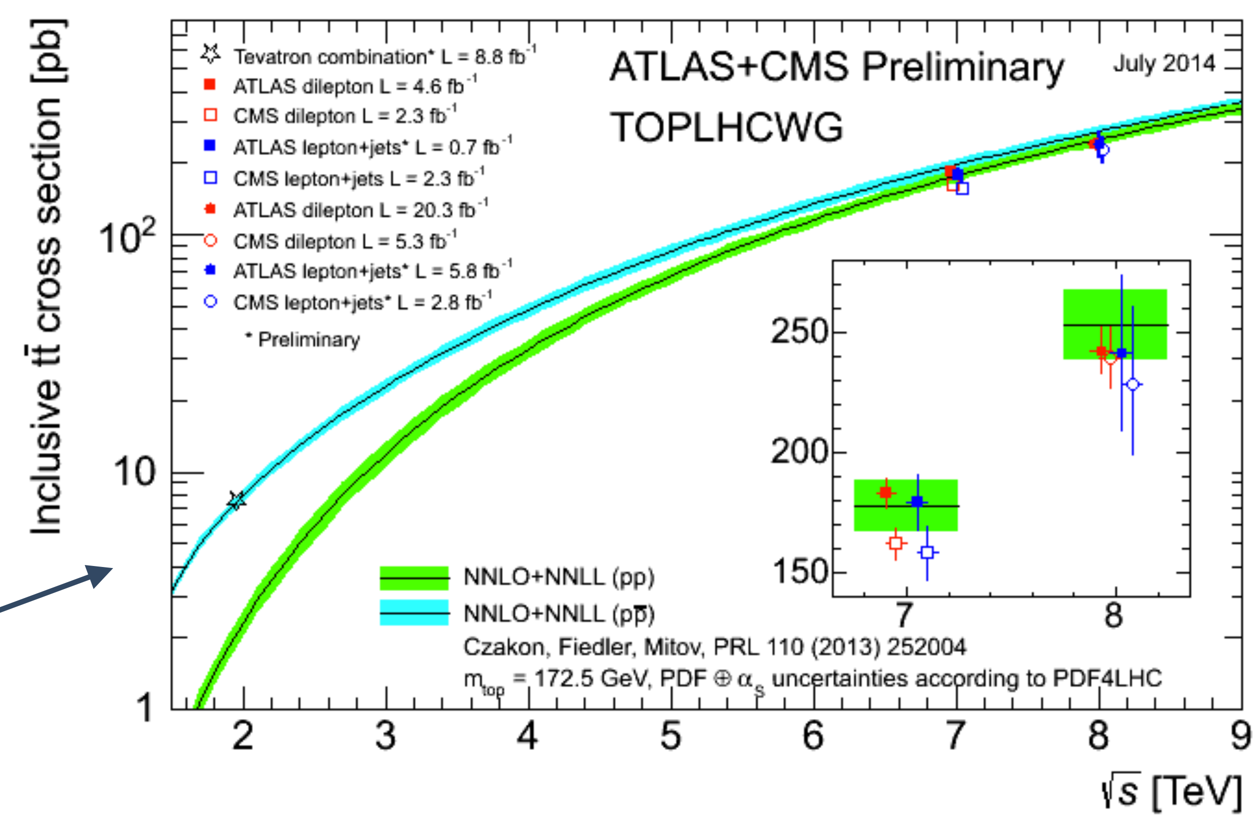


2→2 pheno @ NNLO: the global picture

Very good / improved data-theory comparison





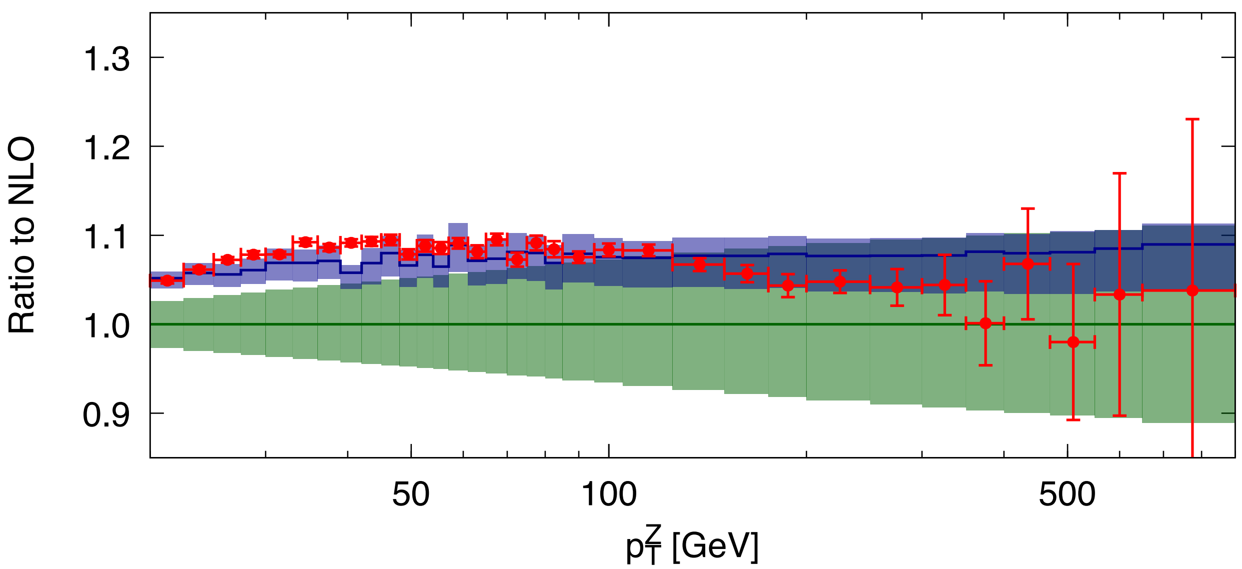


Di-bosons
[Catani, Grazzini; Campbell, Ellis, Williams; Grazzini et al (2015-2017)]

Top pairs [Czakon, Fiedler, Mitov]

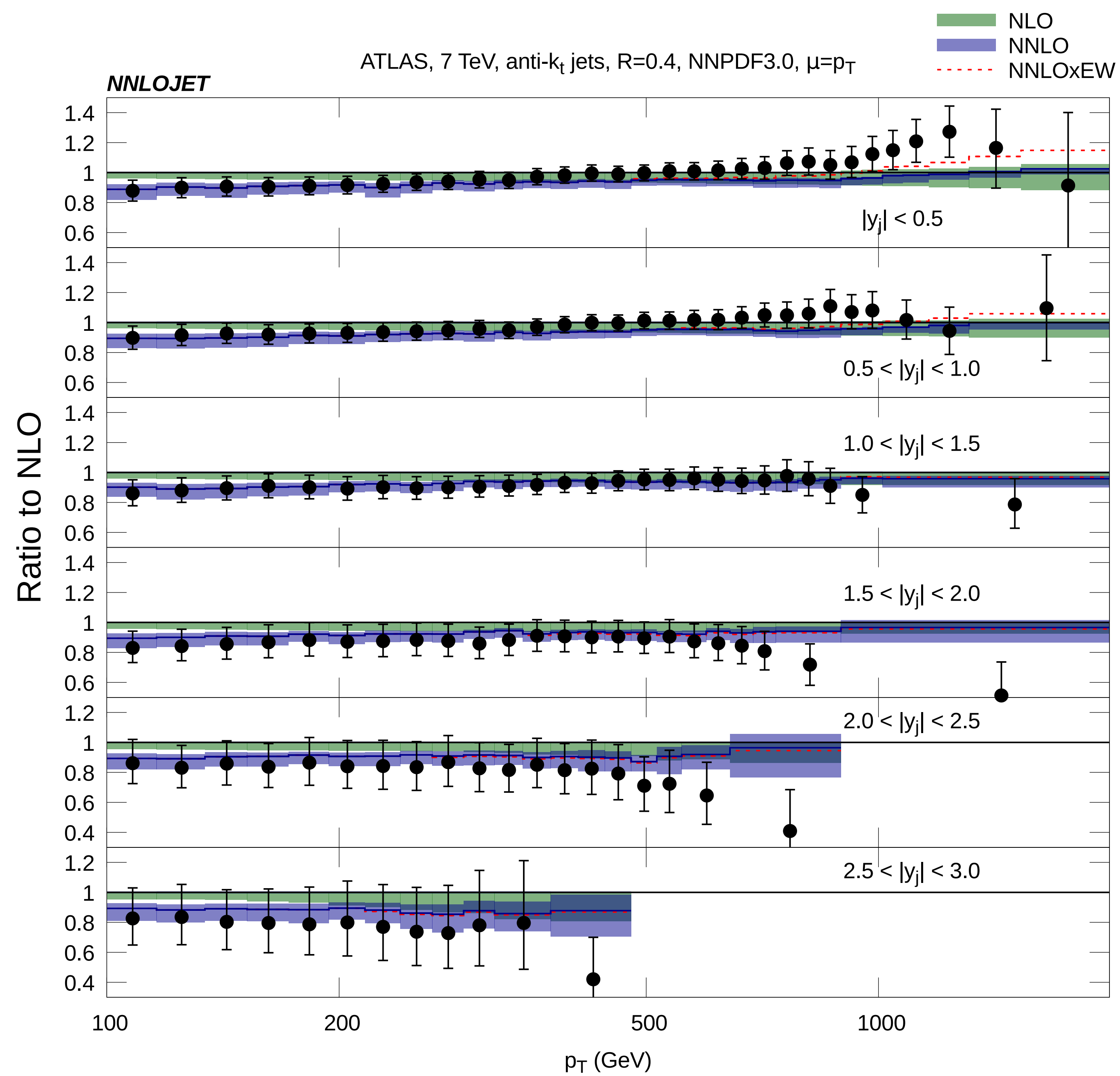
2→2 pheno @ NNLO: the global picture

Very good / improved data-theory comparison



Z+J/Z p_T *shape*

[Gehrmann-de Ridder et al; Boughezal et al]



Inclusive jet production

For a particular scale choice

[Currie, Glover, Gehrmann, Gehrmann-de Ridder, Huss, Pires (2017)]

$2 \to 2$ phenomenology at NNLO: what have we learned so far?

NNLO: what have we learned so far?

- At this level of precision, basically everything becomes relevant

[Mistlberger, QCD@LHC2016]

$$\sigma = 48.58\,\text{pb}^{+2.22\,\text{pb}\,(+4.56\%)}_{-3.27\,\text{pb}\,(-6.72\%)}\ (\text{theory}) \pm 1.56\,\text{pb}\,(3.20\%)\ (\text{PDF}+\alpha_s)\,.$$

$$\begin{aligned}
48.58\,\text{pb} = \quad & 16.00\,\text{pb} & (+32.9\%) \quad & (\text{LO, rEFT}) \\
& +20.84\,\text{pb} & (+42.9\%) \quad & (\text{NLO, rEFT}) \\
& -\ \ 2.05\,\text{pb} & (-4.2\%) \quad & ((t,b,c),\ \text{exact NLO}) \\
& +\ \ 9.56\,\text{pb} & (+19.7\%) \quad & (\text{NNLO, rEFT}) \\
& +\ \ 0.34\,\text{pb} & (+0.7\%) \quad & (\text{NNLO},\ 1/m_t) \\
& +\ \ 2.40\,\text{pb} & (+4.9\%) \quad & (\text{EW, QCD-EW}) \\
& +\ \ 1.49\,\text{pb} & (+3.1\%) \quad & (\text{N}^3\text{LO, rEFT})
\end{aligned}$$

Recent progress: see C. Wever's talk

- Todo List:
  - Full mass dependent NNLO
  - Mixed $\mathcal{O}(\alpha\alpha_S)$ corrections
  - N3LO PDFs
  - ....

δ(scale)	δ(trunc)	δ(PDF-TH)	δ(EW)	$\delta(t,b,c)$	$\delta(1/m_t)$
$^{+0.10\text{ pb}}_{-1.15\text{ pb}}$	±0.18 pb	±0.56 pb	±0.49 pb	±0.40 pb	±0.49 pb
$^{+0.21\%}_{-2.37\%}$	±0.37%	±1.16%	±1%	±0.83%	±1%

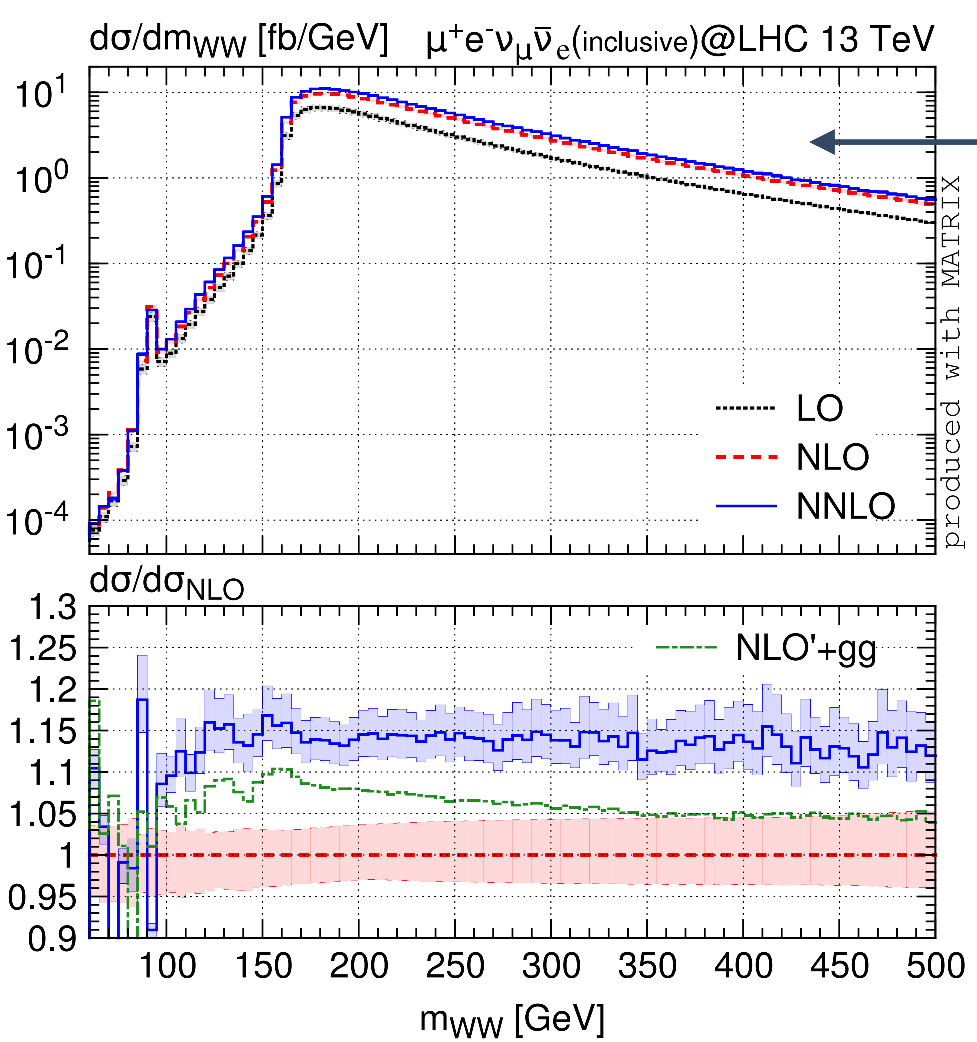
(inclusive VBF@N^{3}LO: [Dreyer, Karlberg (2016)])

NNLO: what have we learned so far?

- Properly modeling the actual experimental setup is crucial *(especially for cuts constraining QCD radiation)*

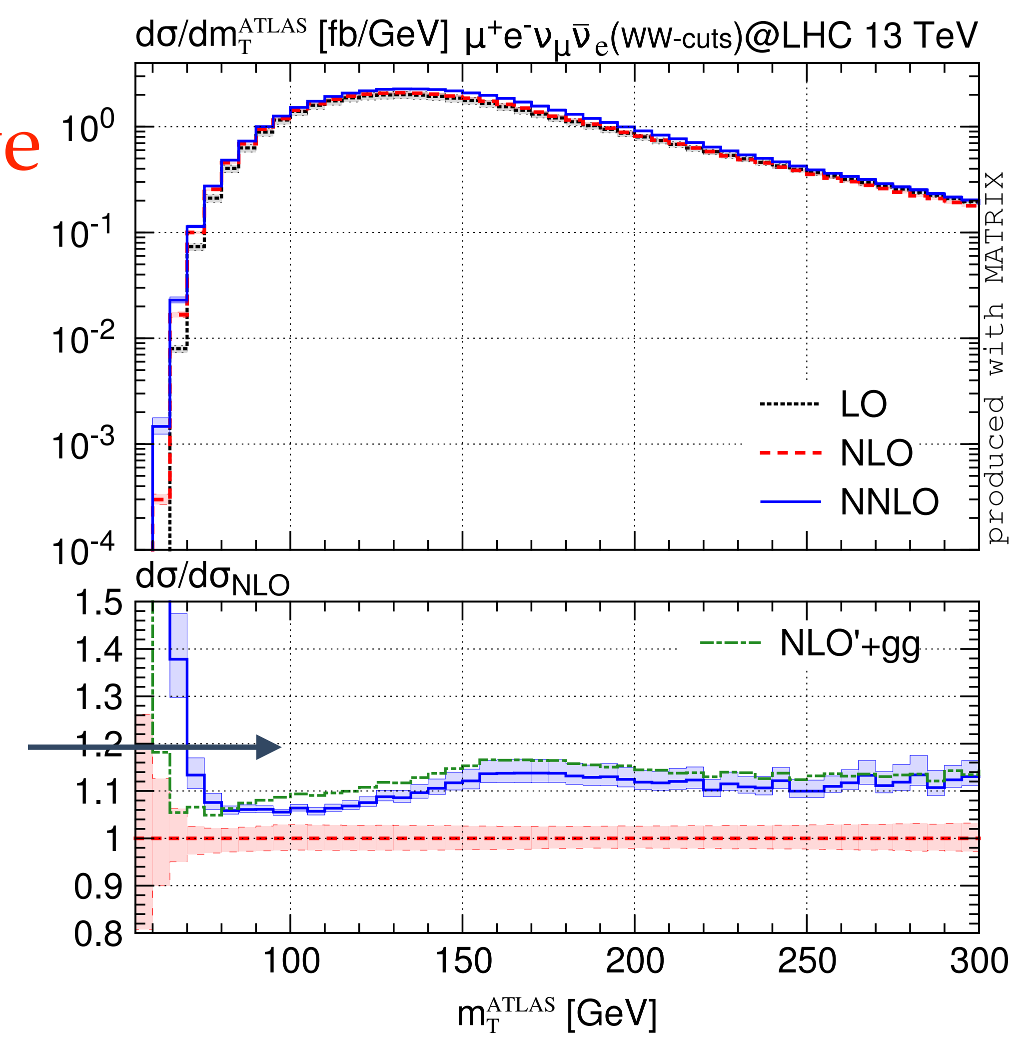
Example: WW, 13 TeV: qq- vs gg-initiated sub-processes

- full inclusive [*unobservable*]: qq@NNLO +7%, gg + 4%
- WW fiducial region: qq@NNLO -2%, gg +9% (similar result for Higgs-cuts)



Inclusive

Fiducial



[Grazzini, Kallweit, Pozzorini, Rathlev, Wiesemann (2016)]

[higher order corrections to gg component: FC, Dowling, Melnikov, Röntsch, Tancredi (2016)]

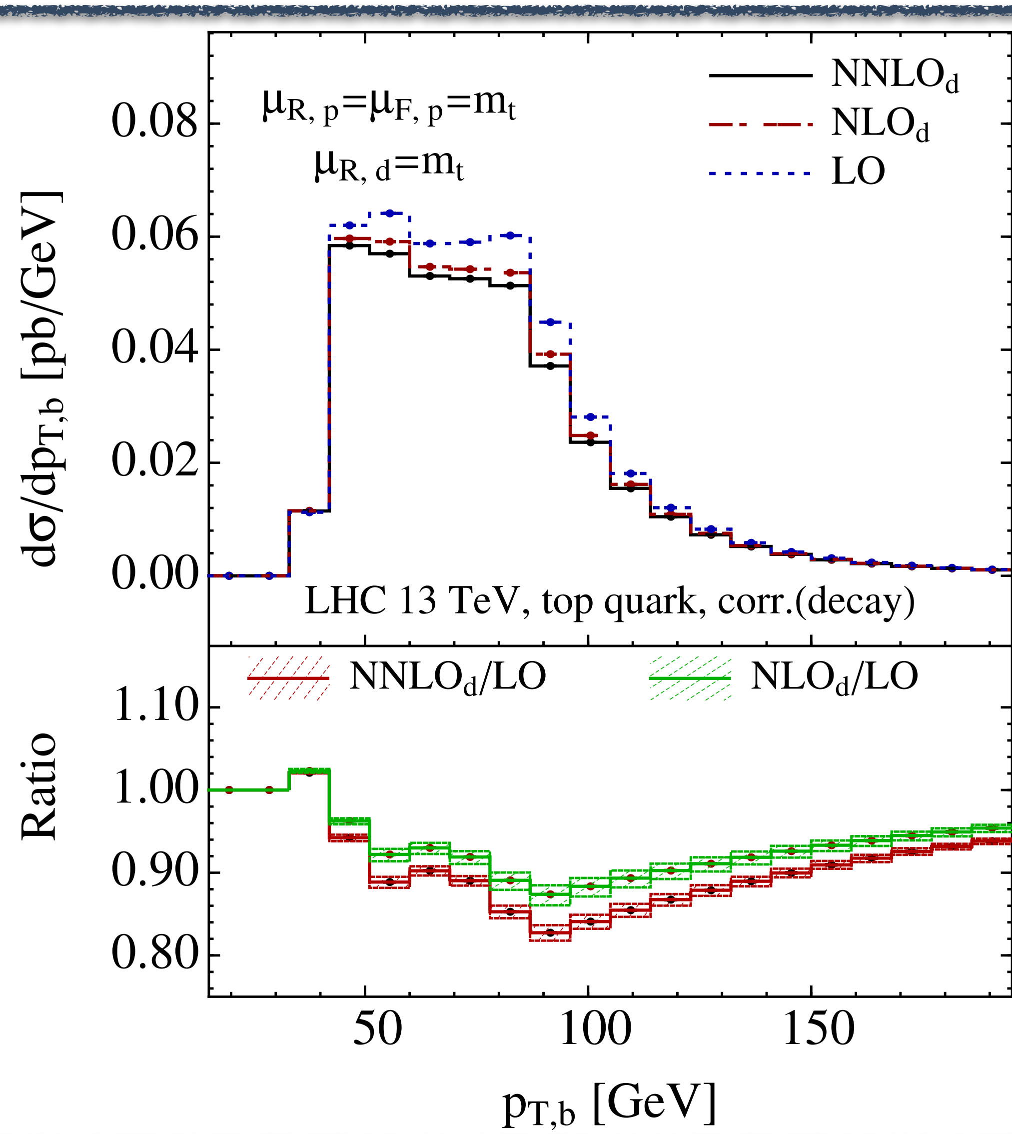
NNLO: what have we learned so far?

Example: modeling TOP decay

tt, approx $NNLO_{prod}$ *(carefully benchmarked against [Czakon et al])* x $NNLO_{decay}$

CMS setup, $e^{\pm}\mu^{\mp}, e^+e^-, \mu^+\mu^-$ channel [25], 2 b-jets required (anti-k_t algorithm [66], $R = 0.5$)						
energy	fiducial volume	LO [pb]	NLO [pb]	$\hat{\text{N}}$NLO [pb]	$\delta_{\text{dec.}}$	CMS [pb]
8 TeV	$p_T(l^{\pm}) > 20$ GeV, $\lvert\eta(l^{\pm})\rvert < 2.4$, $p_T(J_b) > 30$ GeV, $\lvert\eta(J_b)\rvert < 2.4$	$3.780^{+37.4\%}_{-25.3\%}$	$4.483^{+9.0\%}_{-11.5\%}$	$4.874^{+2.5\%}_{-6.8\%}$	-8.0%	$4.73^{+4.7\%}_{-4.7\%}$

[Papanastasiou, Gao (2017)]



t-channel single-top plus top-decay (NWA)

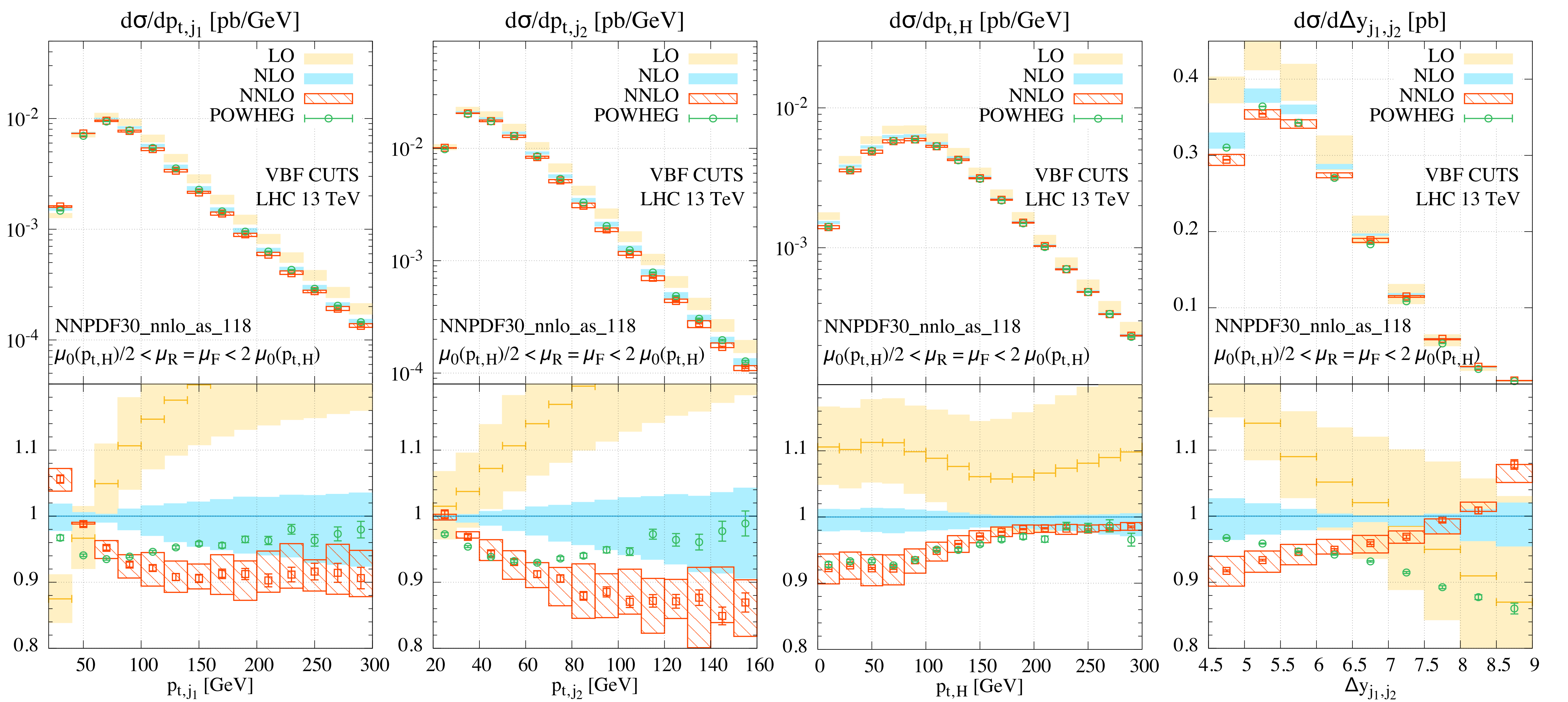
[Berger, Gao, Yuan, Zhu (2016)]

- Small inclusive corrections
- LARGE CORRECTIONS in exclusive region

NNLO: what have we learned so far?

Example: VBF

In the fiducial region: ~5-10% corrections, i.e. one order of magnitude larger than for the inclusive cross-section. Non trivial shapes



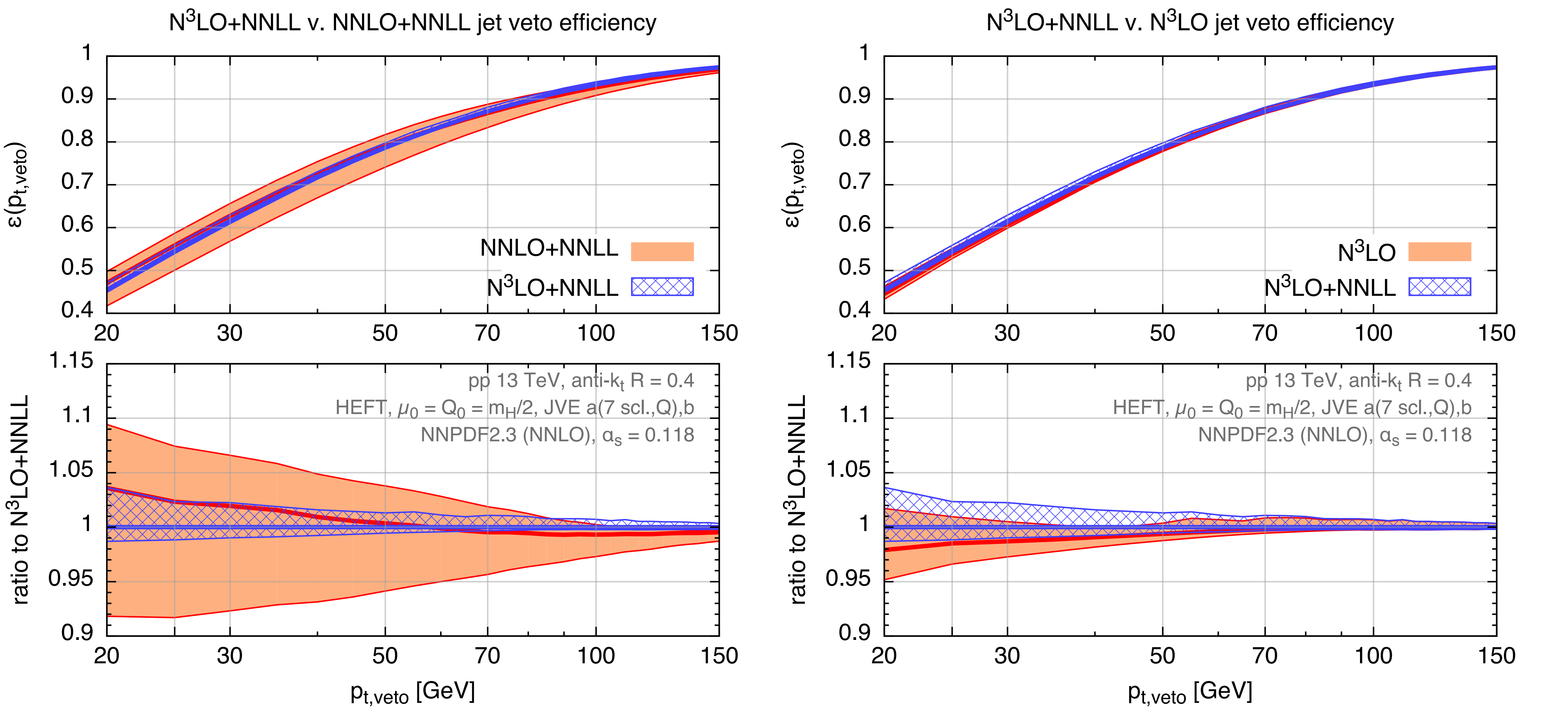
[Cacciari, Dreyer, Karlberg, Salam, Zanderighi (2016)]

towards N^{3}LO differential in ggF → see B. Mistlberger's talk

NNLO: what have we learned so far?

- *Can we trust NNLO results in the fiducial region?* Harsh cuts could introduce largish logs → perturbative breakdown…

Example: Higgs production with jet veto (H→WW…)



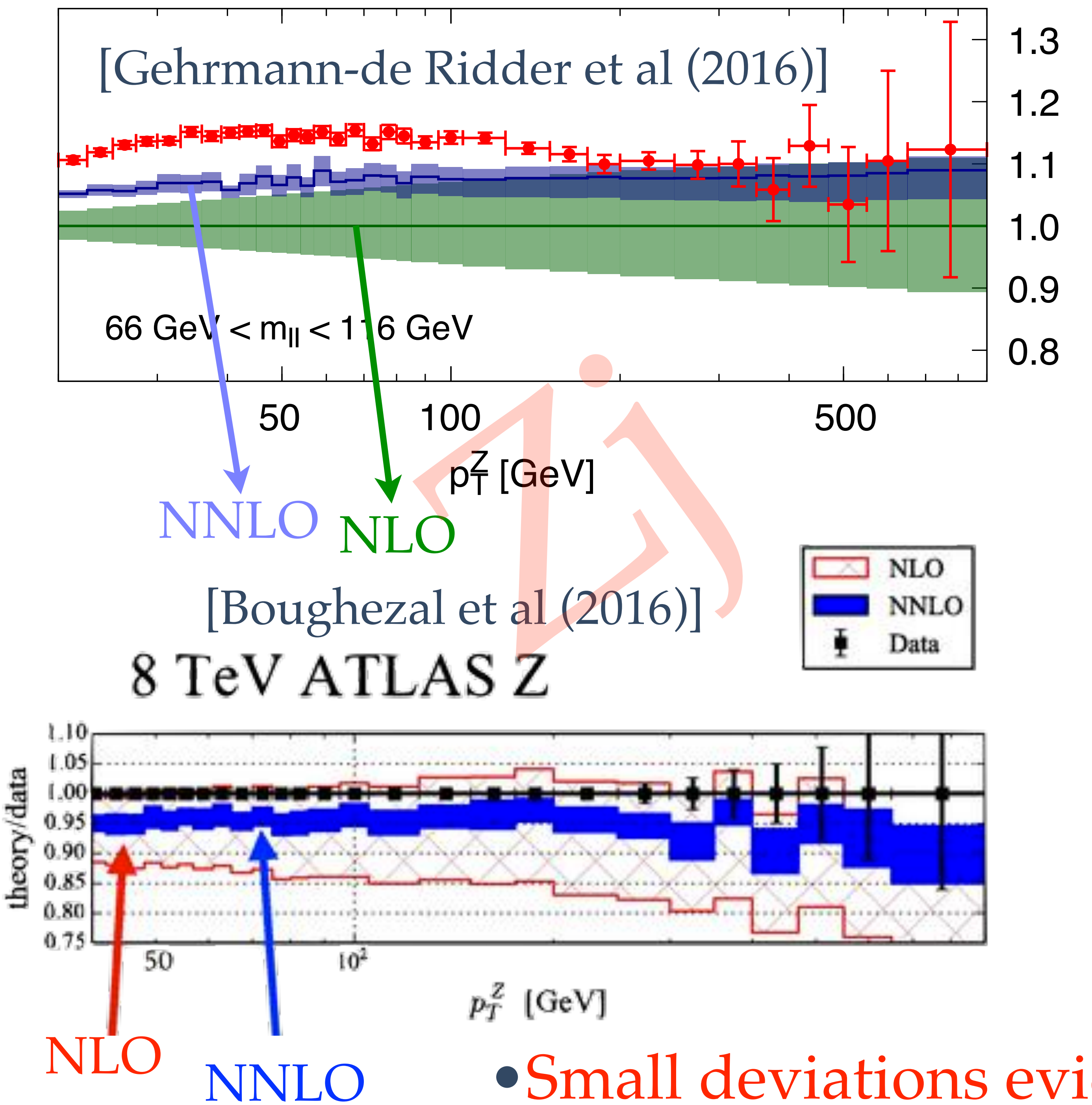
[Banfi, FC, Dreyer, Monni, Salam, Zanderighi, Dulat (2015)]

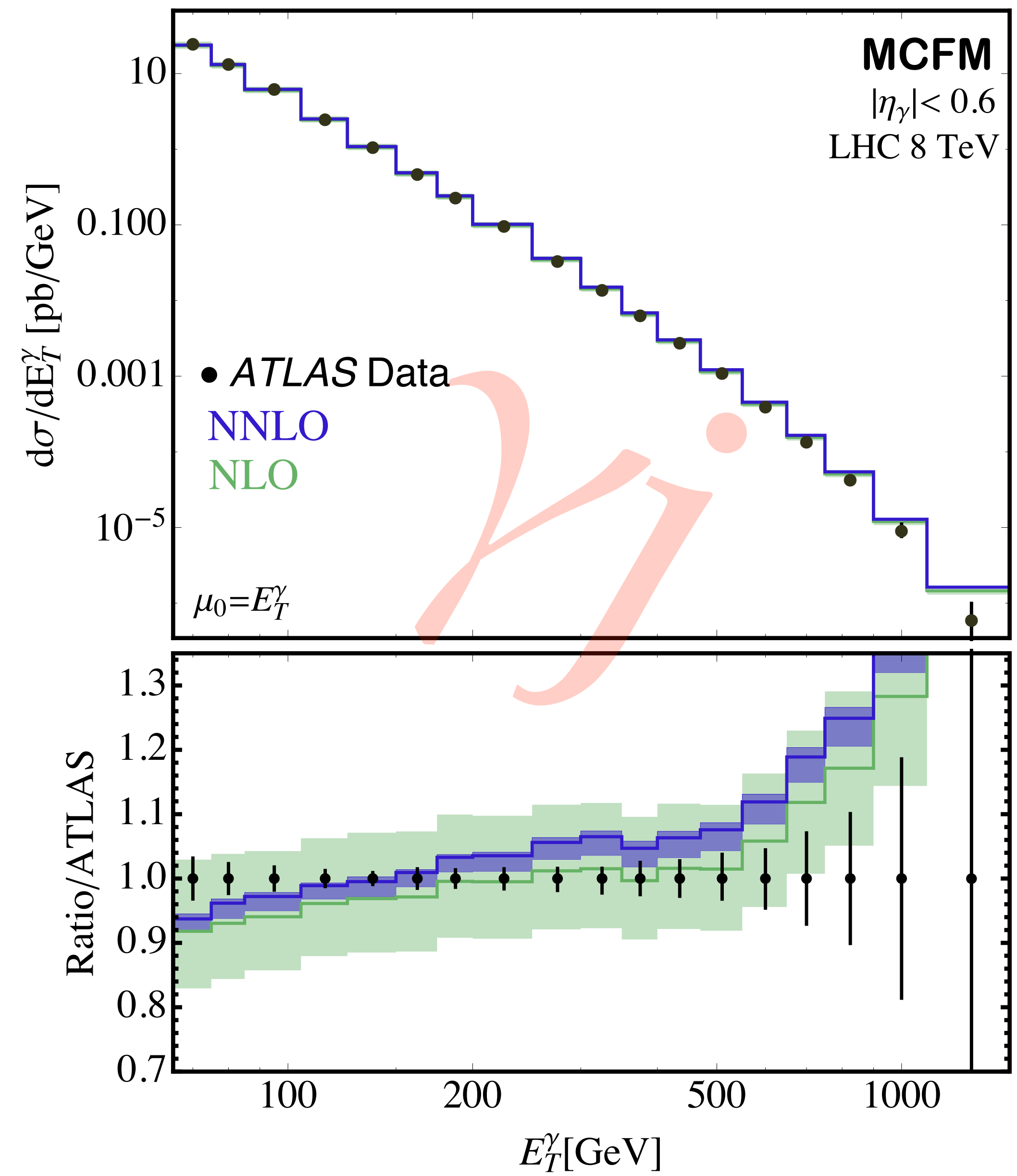
- No breakdown of fixed (high) order till very low scales
- Fixed (higher) order captures bulk of the effect

$2 \to 2$ phenomenology at NNLO: unsolved puzzles

NNLO: open puzzles

- V+j: unexpected disagreement even with high precision / clean data



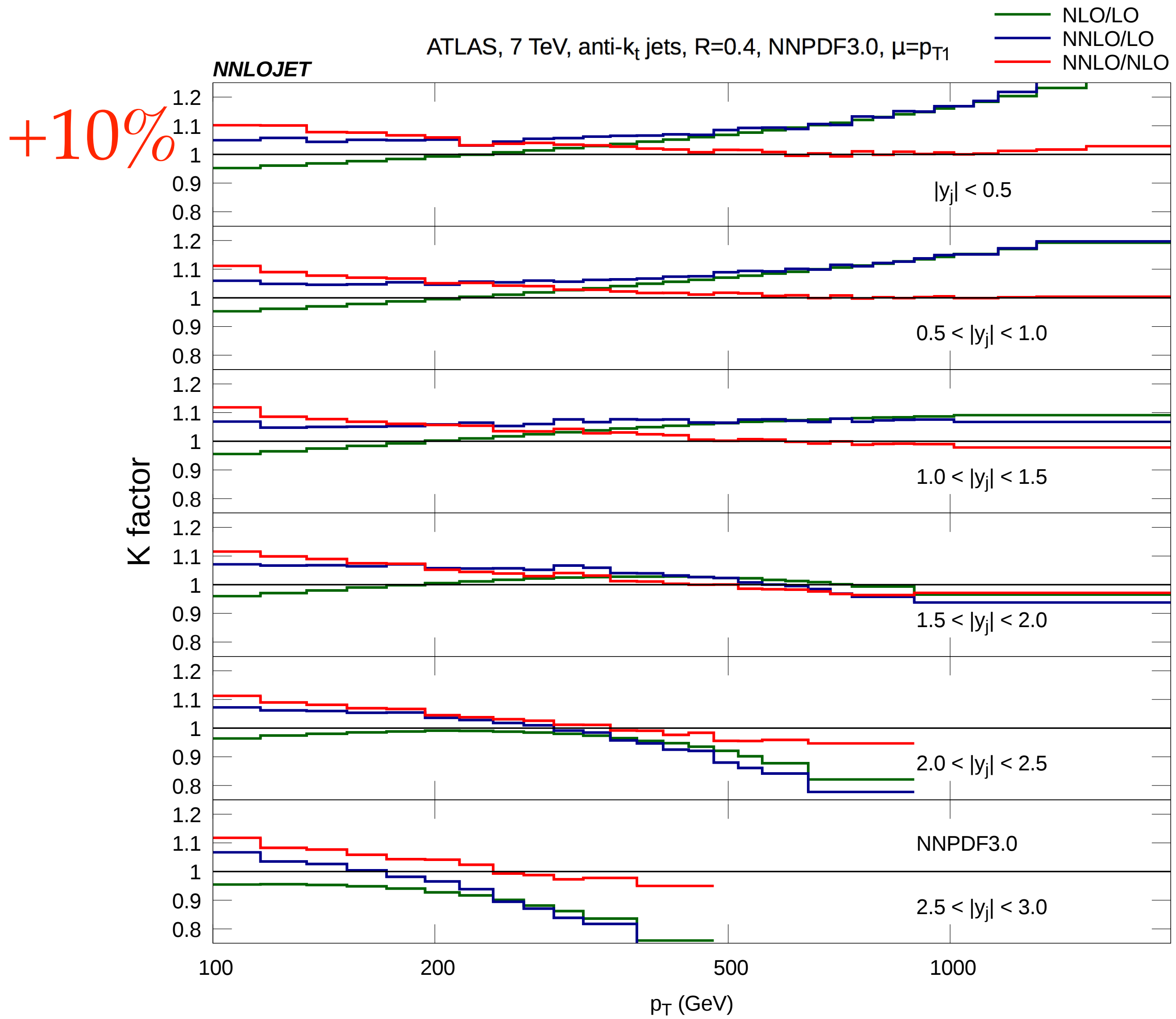


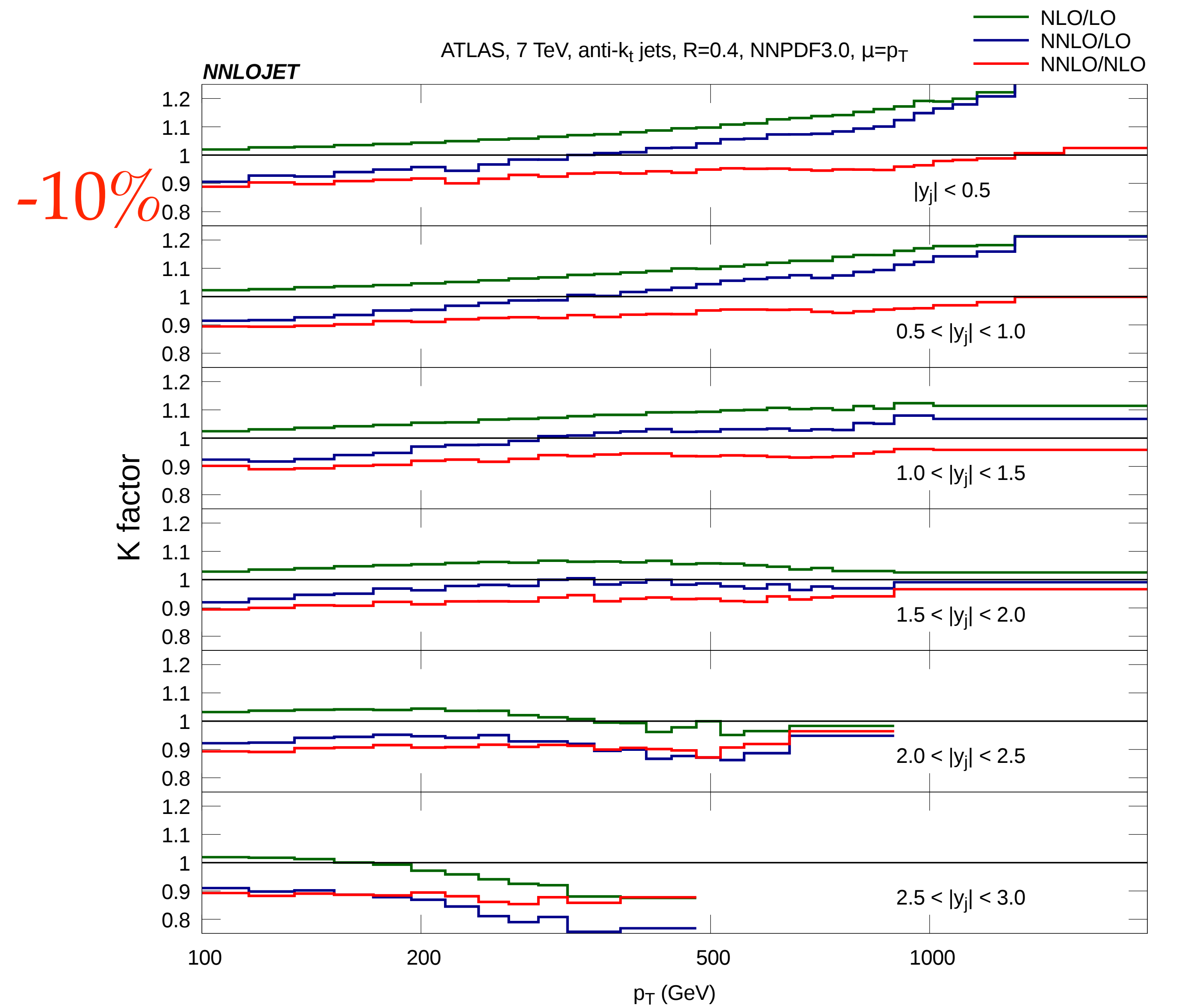
[Campbell, Ellis, Williams (2016)]

- Small deviations evident in the overall normalization (Z p_t) and shape (γ E_T). Calibration? PDFs? *Non pert*?

NNLO: open puzzles

- Inclusive jet spectrum: $\mu = p_{t,L}$ vs p_t





[Currie, Glover, Gehrmann, Gehrmann-de Ridder, Huss, Pires (2017)]

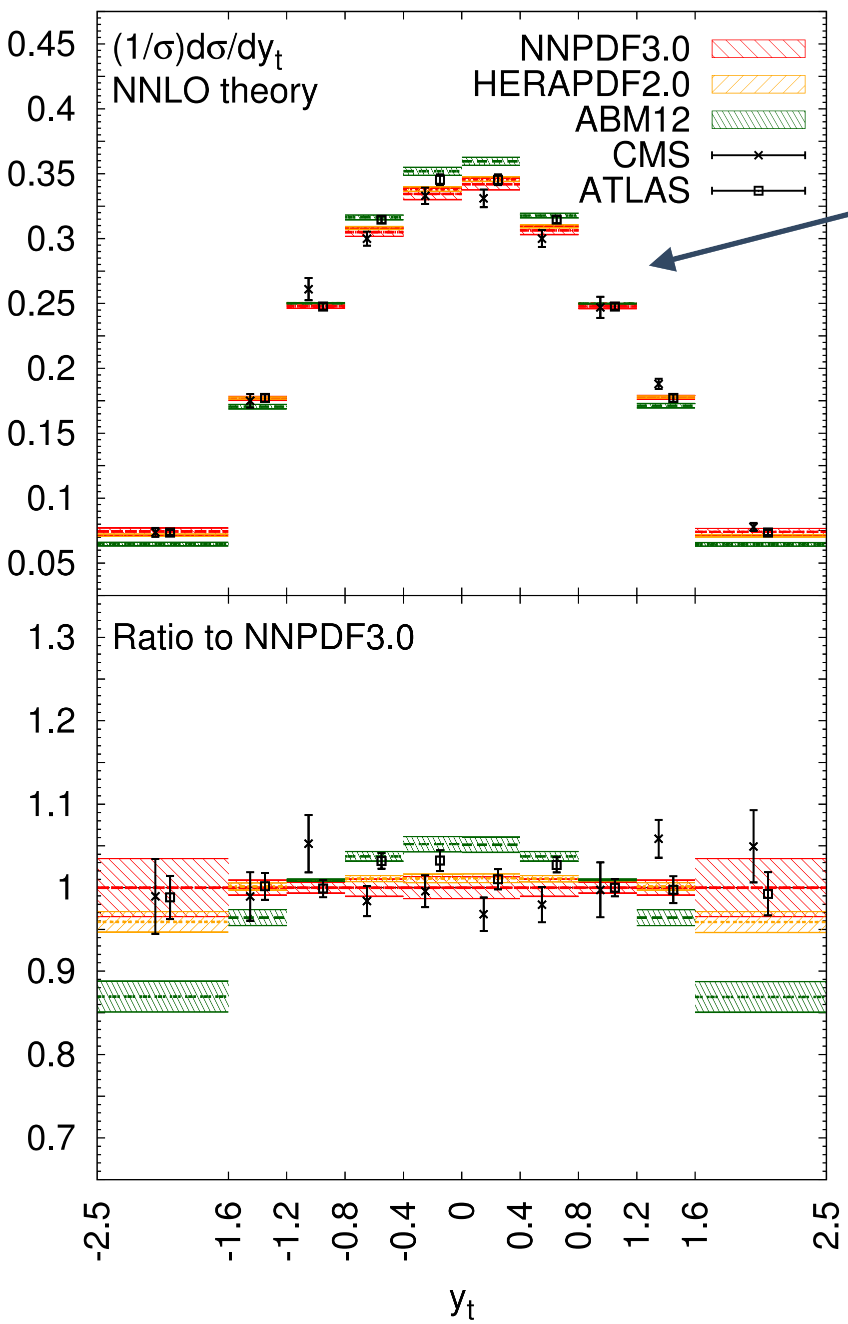
- Despite small scale variation, very large dependence on scale choice (hardest jet in the event vs individual jet). Non trivial jet dynamics to be understood

$2 \rightarrow 2$ phenomenology at NNLO: applications

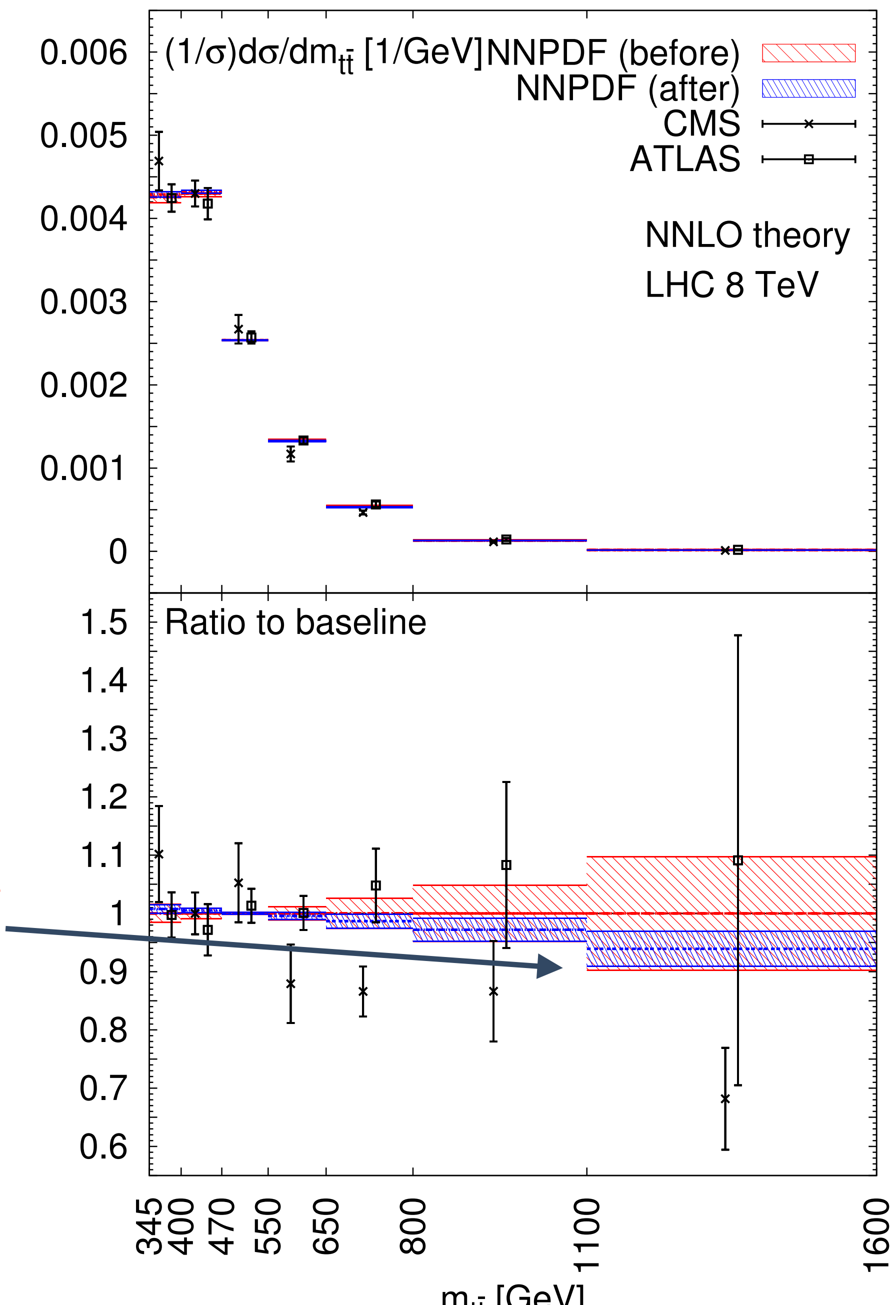
NNLO: applications

High precision data on top, di-jet and Z p_T give DIRECT SENSITIVITY TO THE GLUON PDF. Can now be systematically included in NNLO PDF fits *(coming very soon, NNPDF3.1 sets already on LHAPDF)*

A key player: TOP DIFFERENTIAL DISTRIBUTIONS



Discriminating power between different PDF sets

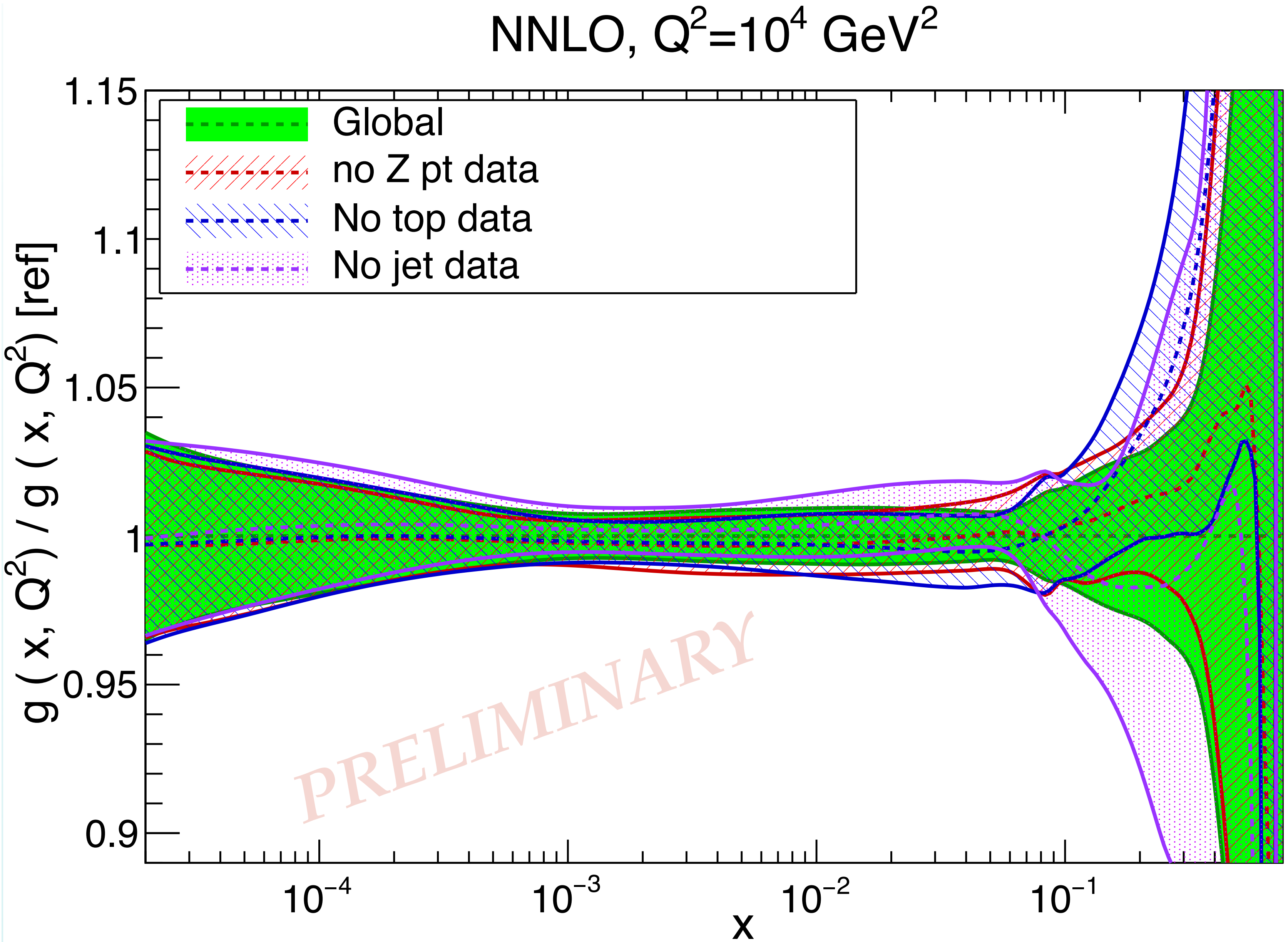


Better control on high mass tail (BSM…)

[Czakon, Hartland, Mitov, Nocera, Rojo (2017)]

NNLO: applications

Are data/predictions globally compatible: IT SEEMS SO
(with the caveats mentioned before…)

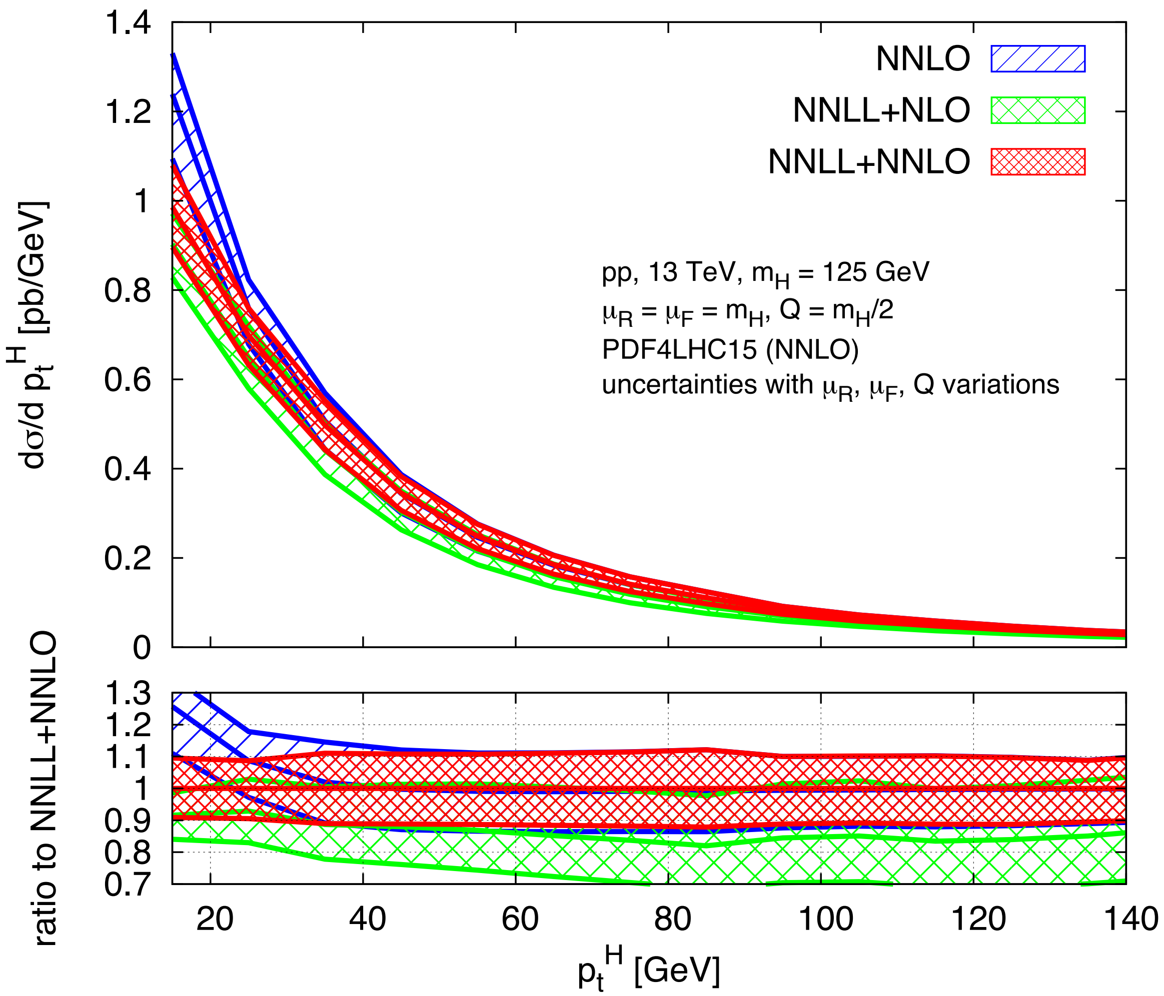


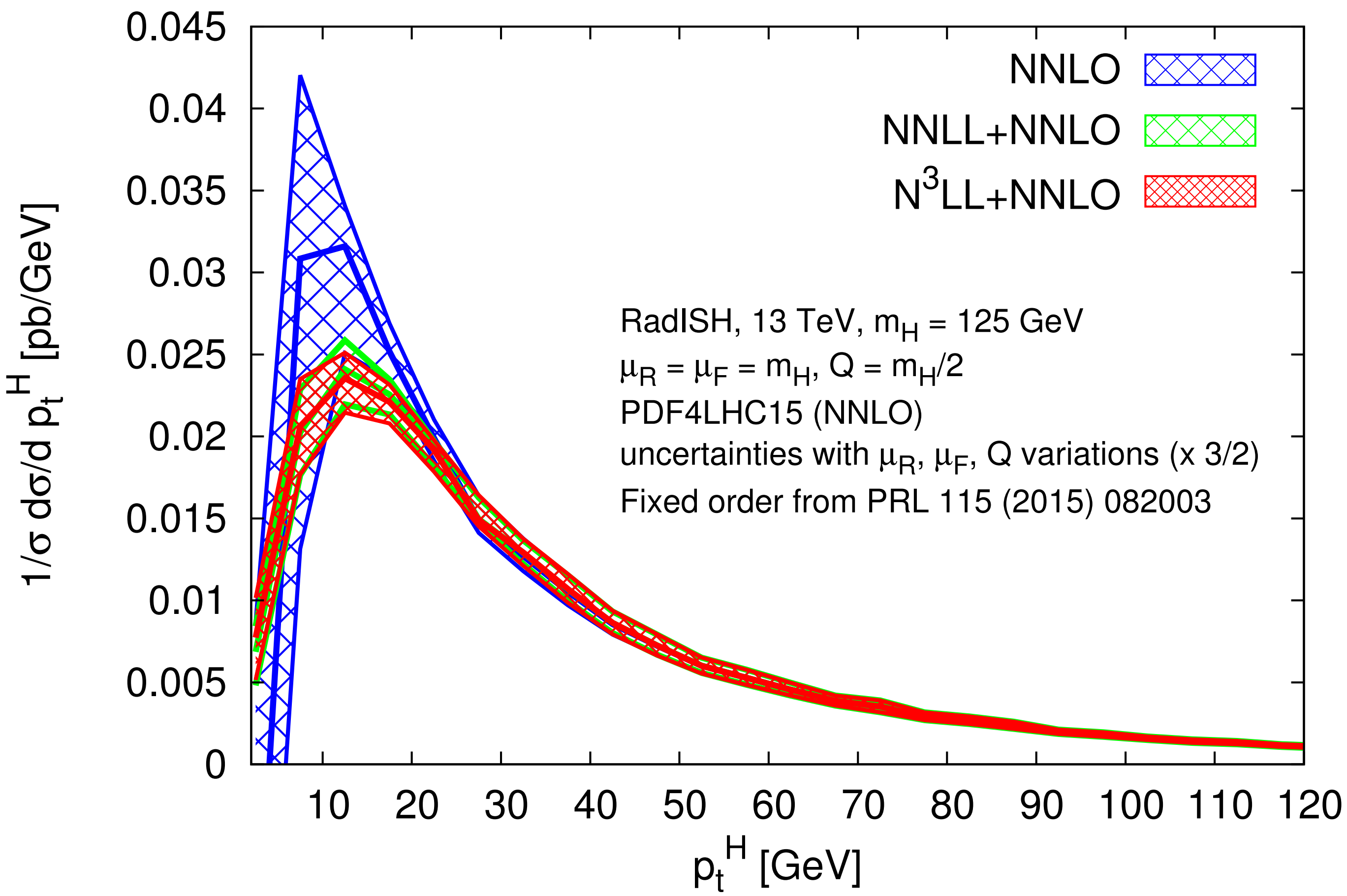
NNPDF3.1 *(preliminary?)*

[J. Rojo, PDF4LHC (2017)]

Application of NNLO results: H p_T

[Bizon, Monni, Re, Rottoli, Torrielli (2017)]





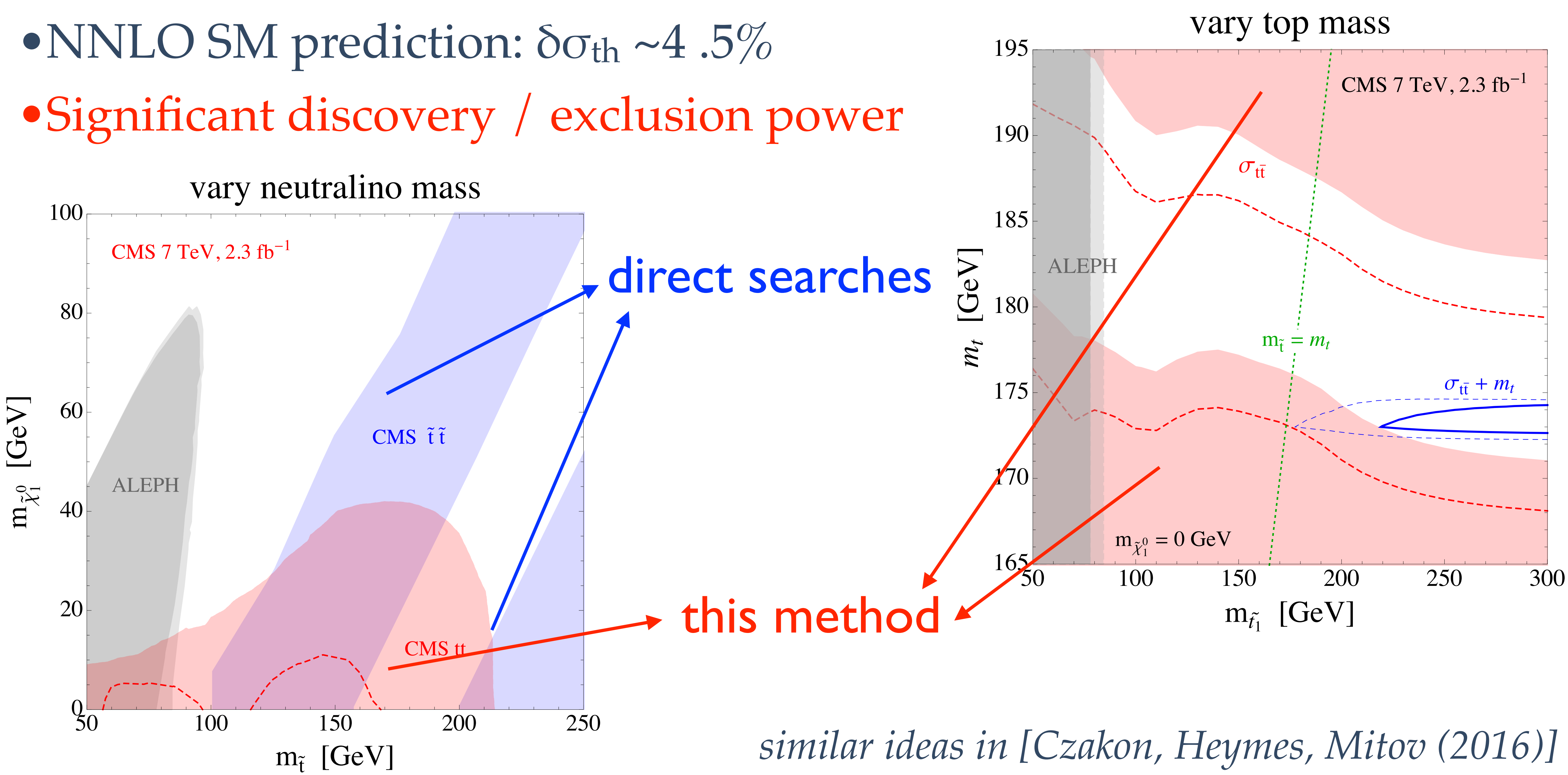
- Matching of NNLO H+J with N^3LL Higgs p_T resummation
- Significant reduction of perturbative uncertainties from NLO+NNLL to NNLO+NNLL, no large N^3LL effect
- Again, no breakdown of perturbation theory until very low scales (resummation effects: 25% at p_T = 15 GeV, ~0% at p_T = 40 GeV)

Application: hunting elusive BSM signals

[Czakon, Mitov, Papucci, Ruderman, Weiler (2014)]

$$\tilde{t} \to t + \chi_1^0/\tilde{G}, \quad m_{\tilde{t}} \sim m_t \gg m_{\chi_1^0,\tilde{G}}, \quad \sigma_{\tilde{t}} \approx 0.15 \cdot \sigma_t$$

- Hunting for stealthy stop
- CMS di-lepton analysis: $\delta\sigma_{exp}$ ~4 .5%
- NNLO SM prediction: $\delta\sigma_{th}$ ~4 .5%
- Significant discovery / exclusion power



similar ideas in [Czakon, Heymes, Mitov (2016)]

NNLO: status and future

- A lot of theoretical progress in the recent past
- This lead to realistic $2\to 2$ *PHENOMENOLOGY AT NNLO*
- Many interesting features
  - Greatly reduced th. uncertainties (expected)
  - Stability w.r.t. logarithmic corrections (not so obvious) → fiducial region
- And a few surprises
  - Non trivial jet dynamics (larger than naively expected corrections)
  - Curious data/theory discrepancies (PDFs? NonPert?)
- A lot more to explore
  - More pheno: e.g. jet dynamics @ NNLO vs mergedPS…
  - $2\to 2$ in ``extreme'' kinematics (boosted/off-shell H+j and $pp\to VV$)
  - better understanding of jet dynamics: $pp \to 3j$. Also: α_s, maybe some extra handle to understand NP effects?
  - Important backgrounds / precision tests: Hjj (VBF contamination, jet-bin correlations…), Vjj, ttj

NNLO: status and future

- This will require significant improvement on stat-of-the art
- Breaking the 2 → 2 barrier highly non trivial
  - 2-loop amplitudes
  - 1-loop: stable/fast 2→4 loop amplitudes in the soft/collinear region
  - more efficient IR subtraction
  - even if the goal is ≠ from NLO, at least some degree of automation
- Beyond NNLO?
  - Exclusive Higgs at N^3LO
  - N^3LO beyond the Higgs?

THE LHC PROVIDES CONSTANT MOTIVATION AND INSPIRATION

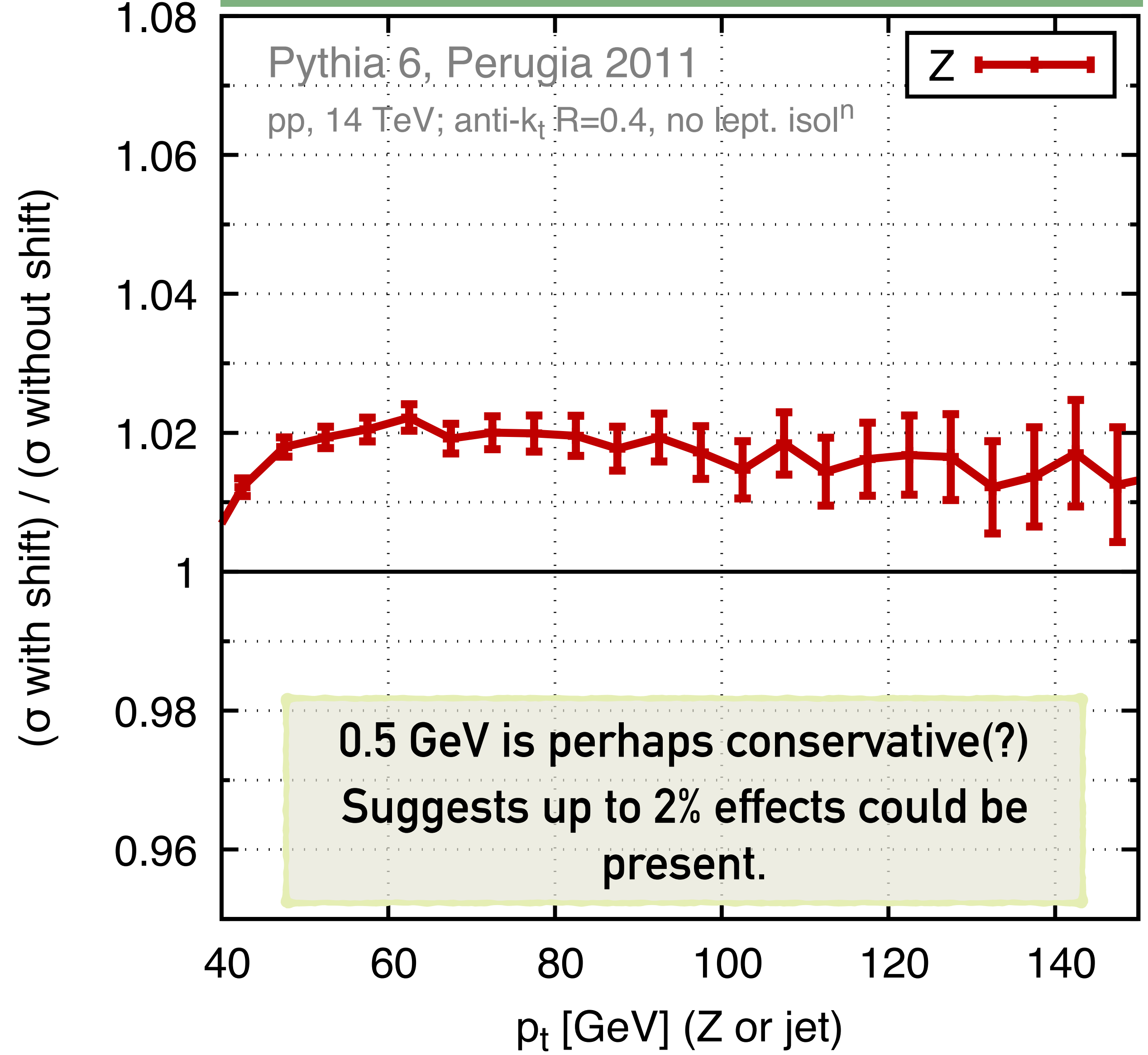
EXCITING TIMES AHEAD!

Thank you
very much for
your attention!

Non-perturbative effects in Z p_T

- Inclusive Z cross section should have $\sim\Lambda^2/M^2$ corrections ($\sim 10^{-4}$?)
- Z p_T is **not inclusive** so corrections can be $\sim\Lambda/M$.
- Size of effect can't be probed by turning MC hadronisation on/off
  [maybe by modifying underlying MC parameters?]
- Shifting Z p_T by a finite amount illustrates what could happen

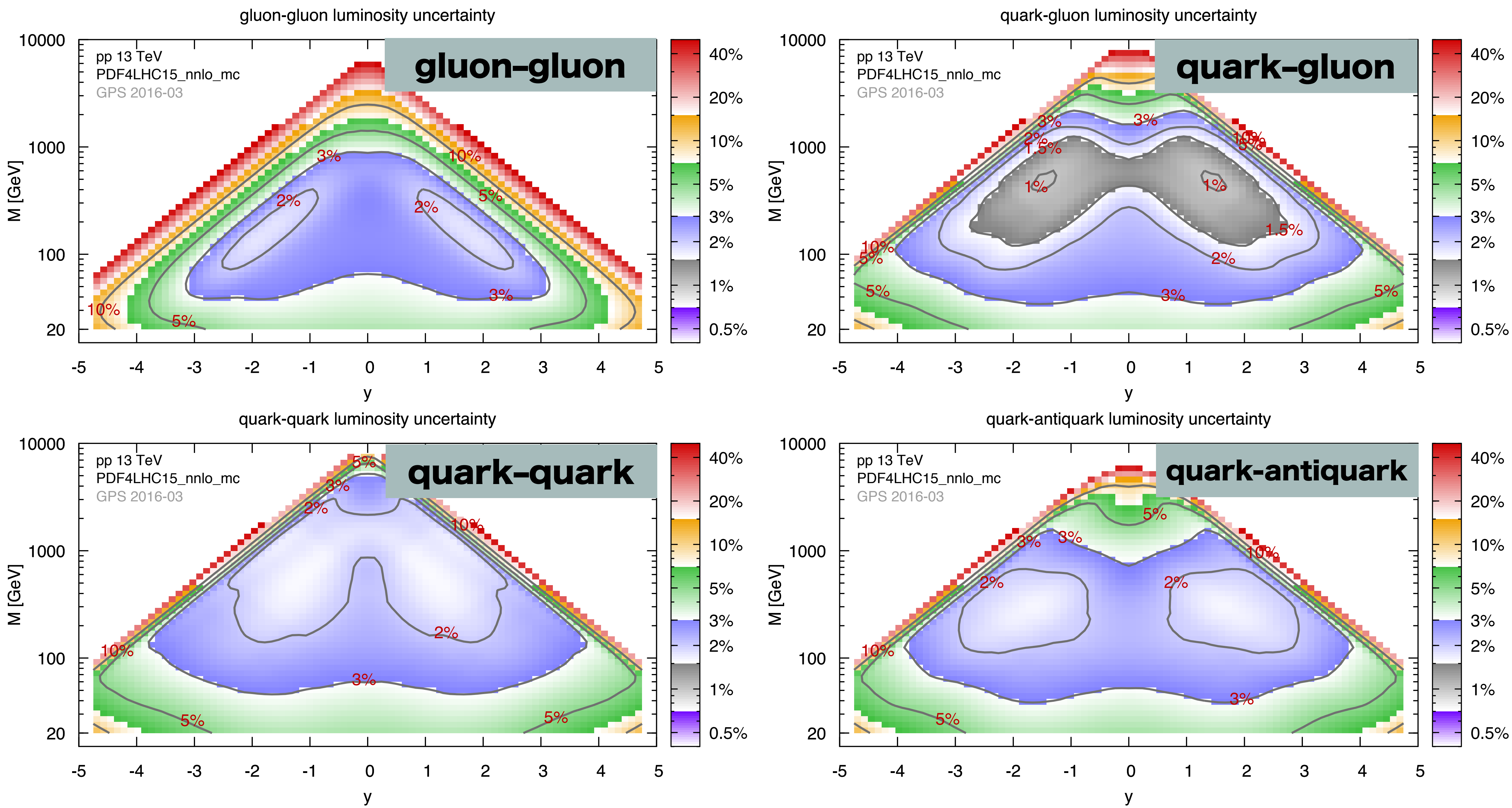
impact of 0.5 GeV shift of Z p_T



A conceptually similar problem is present for the W momentum in top decays

[G. Salam, ``Future challenges for precision QCD'']

Parton distribution functions circa 2016



[plots by G. Salam]

- Big improvement w.r.t. few years ago [better handling on fit, larger data coverage (LHC)]. Reasonable consensus among different groups
- FOR CENTRAL EW PRODUCTION: 2/3% PRECISION
- *Going below may require some rethinking of PDF uncertainty*