Domain Walls in the Early Universe and Matter-Antimatter Domains

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Outline

- The model
- Bounds on parameters
- Evolution of fields during inflation
- Generation of BAU
- Domain walls in expanding Universe

Model

Lagrangian:

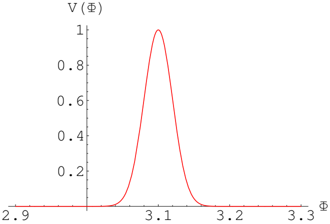
$$L = L_\Phi + L_\chi + L_{int},$$

where

$$\begin{aligned} L_\Phi &= \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}M^2\Phi^2, \\ L_\chi &= \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m^2\chi^2 - \frac{1}{4}\lambda_\chi\chi^4, \\ L_{int} &= \mu^2\chi^2 V(\Phi). \end{aligned}$$

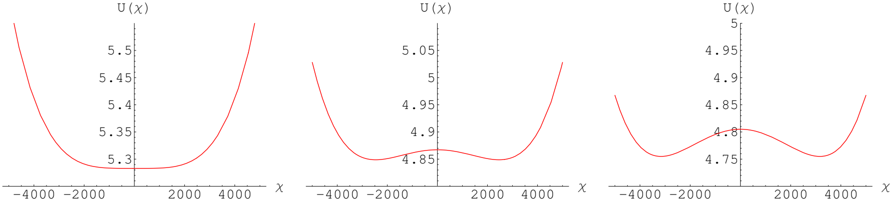
Potential shape:

$$V(\Phi) = \exp\left[-\frac{(\Phi - \Phi_0)^2}{2\Phi_1^2}\right],$$



Evolution of the potential

$$U(\Phi,\chi) = \left(\frac{1}{2}m^2 - \mu^2 V(\Phi)\right)\chi^2 + \frac{1}{4}\lambda_\chi \chi^4 + \frac{1}{2}M^2\Phi^2$$



$$\Phi = \Phi_0 + 2\Phi_1\sqrt{\ln\left(\sqrt{2}\mu/m\right)}$$
$$m^2/2 - \mu^2 V(\Phi) = 0$$

$$\Phi = \Phi_0 + \Phi_1$$

$$\Phi = \Phi_0$$

$\Phi_0 = 3.1\, m_{Pl}$, $\Phi_1 = 0.02\, m_{Pl}$, $\mu = 10^{-4} m_{Pl}$, and $m = 10^{-10} m_{Pl}$.
Field χ is measured in units of M, $U(\Phi,\chi)$ is in units $10^{-12}\, m_{Pl}^4$.

Equations of motion:

$$
\begin{aligned}
\ddot{\Phi} + 3H\dot{\Phi} + M^2\Phi + \mu^2\chi^2\frac{\Phi - \Phi_0}{\Phi_1^2}V(\Phi) &= 0, \\
\ddot{\chi} + 3H\dot{\chi} + m^2\chi + \lambda_\chi\chi^3 - 2\mu^2\chi V(\Phi) &= 0,
\end{aligned}
$$

where $H = \dot{a}/a$ is the Hubble parameter, $a(t)$ is a scale factor, which enters the FLRW metric

$$ds^2 = dt^2 - a^2(t)\,d\mathbf{x}^2.$$

The Hubble parameter is defined by energy density ρ

$$H = \sqrt{\frac{8\pi\rho}{3m_{Pl}^2}} = \sqrt{\frac{8\pi}{3m_{Pl}^2}\left(\frac{\dot{\Phi}^2}{2} + \frac{M^2\Phi^2}{2} + \frac{\dot{\chi}^2}{2} + \frac{m^2\chi^2}{2} + \frac{\lambda_\chi\chi^4}{4} - \mu^2\chi^2 V(\Phi)\right)}\,,$$

where $m_{Pl} \approx 1.2 \cdot 10^{19}$ GeV is the Planck mass.

Bounds on model parameters

- We do not want to break common inflation scenario:
  $\Phi_{in} > 3.3 m_{Pl}$, $10^{-7} m_{Pl} < M < 10^{-6} m_{Pl}$.
- The size of a domain should be large enough (10 Mpc):
  $\Phi_0 \approx 3.1 m_{Pl}$
- χ should not noticeably affect the inflaton field:

$$M^2 \Phi_0^2 \gg \mu^2 \left. \chi^2 \right|_{\Phi=\Phi_0} \sim \frac{\mu^4}{\lambda_\chi},$$

$$\mu^4 \ll M^2 \Phi_0^2 \lambda_\chi.$$

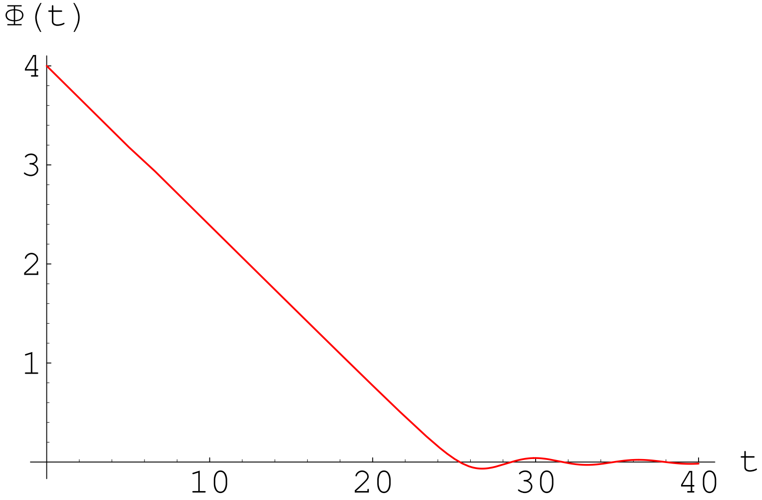
  For $M = 10^{-6} m_{Pl}, \ \Phi_0 = 3.1\, m_{Pl}$ we obtain $\mu \ll 1.8 \cdot 10^{-3} m_{Pl} \sqrt[4]{\lambda_\chi}$.
- χ should be able to reach the minimum:
  $\chi \propto \exp(\mu t)$ for $\mu \gg H = \sqrt{4\pi/3}\, M/m_{Pl}\, \Phi \sim 6 \cdot 10^{-6} m_{Pl}$

$$\mu \tau = \mu \frac{8\sqrt{3\pi}\Phi_1}{M m_{Pl}} \gtrsim \ln \frac{\eta_{max}}{\chi_{in}} = \frac{1}{2} \ln \frac{2\mu^2}{\lambda_\chi \chi_{in}^2},$$

$$\mu \gtrsim \frac{M m_{Pl}}{16\sqrt{3\pi}\Phi_1} \ln \frac{2\mu^2}{\lambda_\chi \chi_{in}^2}.$$

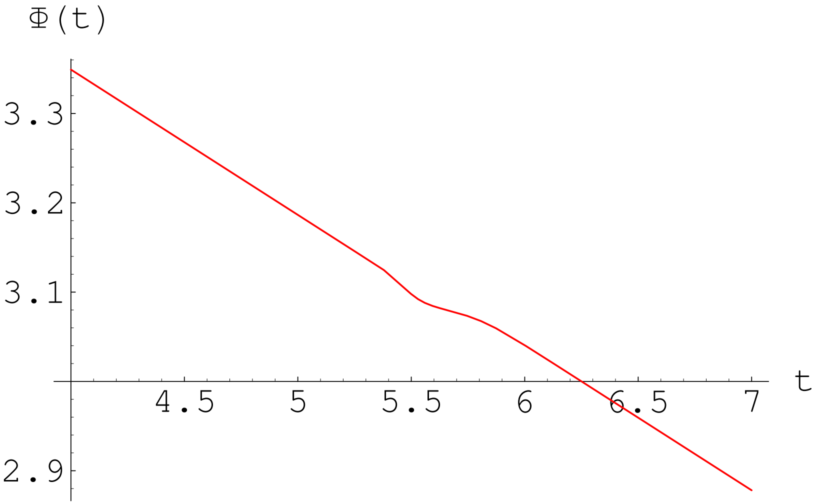
- Field χ should slowly decrease with time after after vanishing of $V(\Phi)$:
  If $\lambda_\chi \chi^3$ dominates in equations of motion then $\chi = \sqrt{\frac{3H}{2\lambda_\chi}} \frac{1}{\sqrt{t-C}}$

$\Phi_{in} = 4, \Phi_0 = 3.1, \Phi_1 = 0.02, M = 10^{-6}, \chi_{in} = 10^{-6}, m = 10^{-10}, \lambda_\chi = 2{\cdot}10^{-3}, \mu = 10^{-4}.$



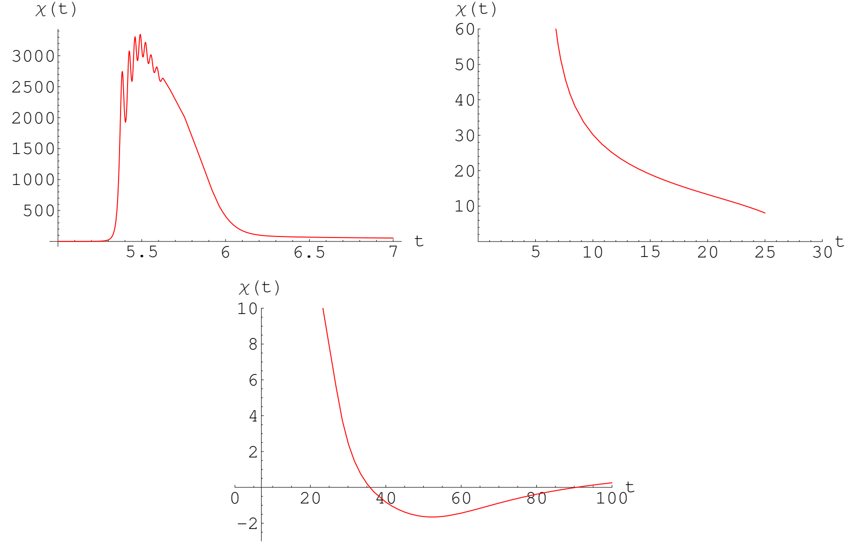
Inflaton field evolution

$\Phi_{in} = 4, \Phi_0 = 3.1, \Phi_1 = 0.02, M = 10^{-6}, \chi_{in} = 10^{-6}, m = 10^{-10}, \lambda_\chi = 2{\cdot}10^{-3}, \mu = 10^{-4}.$



Evolution of χ

$$\Phi_{in} = 4, \Phi_0 = 3.1, \Phi_1 = 0.02, M = 10^{-6}, \chi_{in} = 10^{-6}, m = 10^{-10}, \lambda_\chi = 2{\cdot}10^{-3}, \mu = 10^{-4}.$$



$$L_{free} = \bar{\psi}^k i\hat{\partial}\psi^k - m_{\psi kl}\bar{\psi}^k\psi^l = \bar{\psi}^k_R i\hat{\partial}\psi^k_R + \bar{\psi}^k_L i\hat{\partial}\psi^k_L - m_{\psi kl}(\bar{\psi}^k_R\psi^l_L + \bar{\psi}^k_L\psi^l_R).$$

$$L_{\chi\psi\psi} = g_{kl}\chi\bar{\psi}^k i\gamma_5\psi^l = g_{kl}\chi(\bar{\psi}^k_R i\gamma_5\psi^l_L + \bar{\psi}^k_L i\gamma_5\psi^l_R) = ig_{kl}\chi(\bar{\psi}^k_L\psi^l_R - \bar{\psi}^k_R\psi^l_L).$$

$$L_{free} + L_{\chi\psi\psi} = \bar{\psi}_R i\hat{\partial}\psi_R + \bar{\psi}_L i\hat{\partial}\psi_L - (\bar{\psi}_R M_\psi\psi_L + \bar{\psi}_L M^\dagger_\psi\psi_R),$$

where $M_\psi = m_\psi + ig\chi$.
With two unitary transformations, $\psi_R \to \psi'_R = U_R\psi_R$ and $\psi_L \to \psi'_L = U_L\psi_L$, it is always possible to diagonalize mass matrix:

$$L'_{free} = \bar{\psi}^a i\hat{\partial}\psi^a - m'_{\psi ab}\bar{\psi}^a\psi^b,$$

If there is an interaction with a vector boson X:

$$g_{Rkl}X_\mu\bar{\psi}^k_R\gamma^\mu\psi^l_R + g_{Lkl}X_\mu\bar{\psi}^k_L\gamma^\mu\psi^l_L \to g'_{Rab}X_\mu\bar{\psi}^a_R\gamma^\mu\psi^b_R + g'_{Lab}X_\mu\bar{\psi}^a_L\gamma^\mu\psi^b_L.$$

Asymmetry:

$$\Delta_B \sim \delta\frac{h}{g_X}\left(\frac{m_{th}}{m_{Pl}}\right)^{1/2} \Rightarrow \text{for } h/g_X \sim 1,\ m_{th} \sim M \text{ we get } \delta \sim 10^{-7}$$

$$ds^2 = dt^2 - e^{2Ht}\left(dx^2 + dy^2 + dz^2\right).$$

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi - \frac{\lambda}{2}\left(\varphi^2 - \eta^2\right)^2.$$

$H = 0$, one-dimensional case ($\varphi = \varphi(z)$):

$$\frac{d^2\varphi}{dz^2} = 2\lambda\varphi\left(\varphi^2 - \eta^2\right).$$

Solution (wall at $z = 0$):

$$\varphi(z) = \eta \tanh \frac{z}{\delta_0},$$

where $\delta_0 = 1/(\sqrt{\lambda}\eta)$ is the width.

$H > 0$, stationary solutions (φ depends only on $za(t)$):

Basu, Vilenkin, Phys. Rev. D 50 (1994) 7150

$$\varphi = \eta \cdot f(u), \qquad \text{where} \quad u = Hze^{Ht}.$$

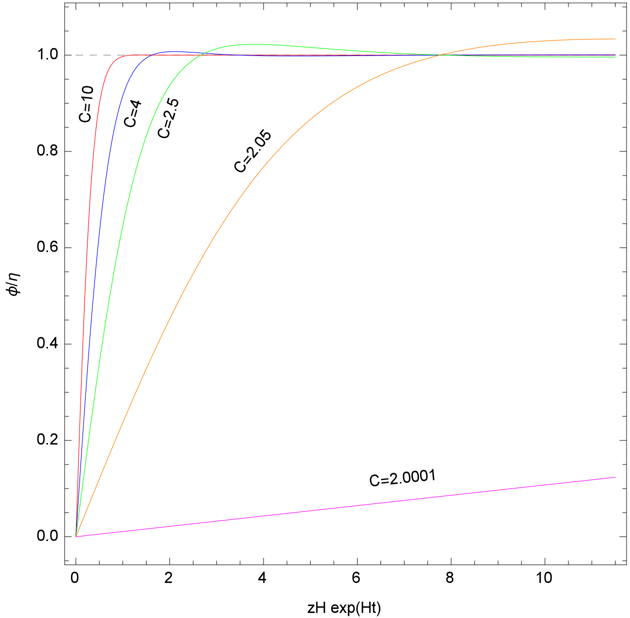
Equation of motion:

$$\left(1 - u^2\right) f'' - 4uf' = -2Cf\left(1 - f^2\right),$$

where $C = 1/(H\delta_0)^2 = \lambda\eta^2/H^2 > 0$.

Boundary conditions: $\quad f(0) = 0, \; f(\pm\infty) = \pm 1.$

Stationary solutions



$$\frac{\partial^2\varphi}{\partial t^2} + 3H\frac{\partial\varphi}{\partial t} - e^{-2Ht}\frac{\partial^2\varphi}{\partial z^2} = -2\lambda\varphi\left(\varphi^2 - \eta^2\right).$$

With the dimensionless variables $\tau = Ht$, $\zeta = Hz$, $f(\zeta,\tau) = \varphi(z,t)/\eta$:

$$\frac{\partial^2 f}{\partial\tau^2} + 3\frac{\partial f}{\partial\tau} - e^{-2\tau}\frac{\partial^2 f}{\partial\zeta^2} = 2Cf\left(1 - f^2\right),$$

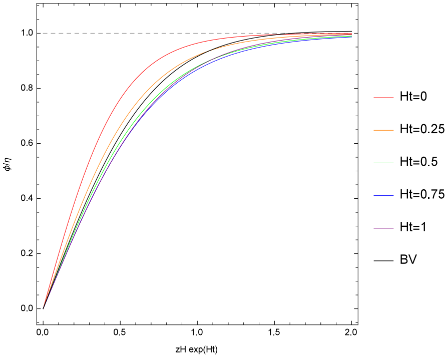
where $C = \lambda\eta^2/H^2 = 1/(H\delta_0)^2 > 0$.
Boundary conditions:

$$f(0,\tau) = 0, \qquad f(\pm\infty,\tau) = \pm 1,$$

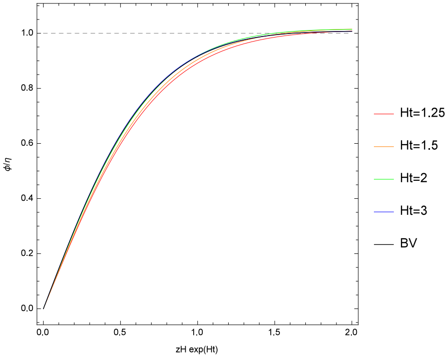
Starting conditions:

$$f(\zeta,0) = \tanh\frac{z}{\delta_0} = \tanh\sqrt{C}\zeta, \qquad \left.\frac{\partial f(\zeta,\tau)}{\partial\tau}\right|_{\tau=0} = 0.$$

$C = 4$

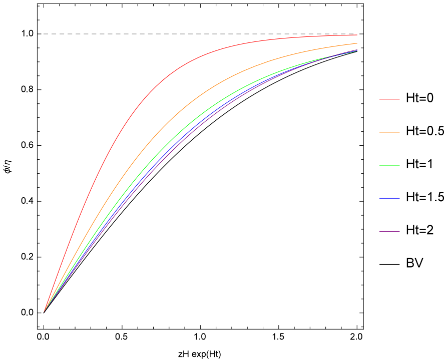


ϕ/η
zH exp(Ht)
Ht=0
Ht=0.25
Ht=0.5
Ht=0.75
Ht=1
BV

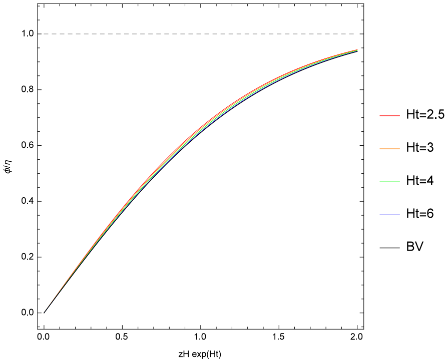


ϕ/η
zH exp(Ht)
Ht=1.25
Ht=1.5
Ht=2
Ht=3
BV

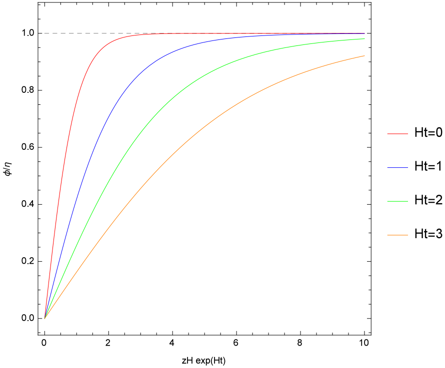
$C = 2.5$



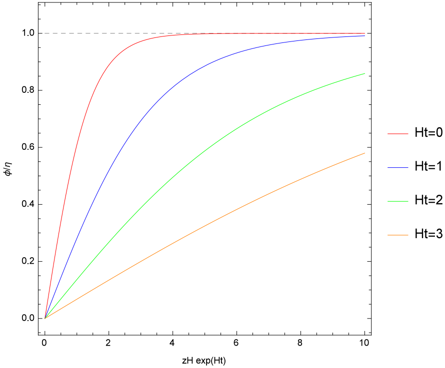
Ht=0
Ht=0.5
Ht=1
Ht=1.5
Ht=2
BV
ϕ/η
zH exp(Ht)



Ht=2.5
Ht=3
Ht=4
Ht=6
BV
ϕ/η
zH exp(Ht)

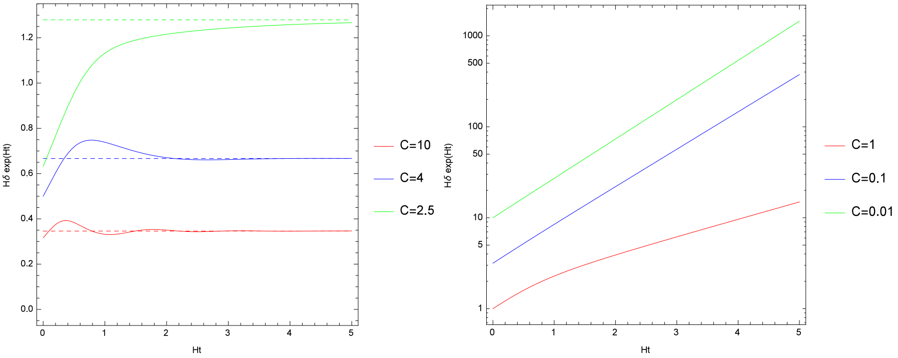

1.0
0.8
0.6
0.4
0.2
0.0
ϕ/η
0
2
4
6
8
10
zH exp(Ht)
Ht=0
Ht=1
Ht=2
Ht=3

$C = 1$


1.0
0.8
0.6
0.4
0.2
0.0
ϕ/η
0
2
4
6
8
10
zH exp(Ht)
Ht=0
Ht=1
Ht=2
Ht=3

$C = 0.5$

Wall width



Conclusions

- The scenario for generation of matter-antimatter domains (separated by cosmologically large distances) is suggested:
  - We found bounds on parameters at which this scenario can be realized.
  - The numerical simulation was performed to demonstrate that this scenario is possible.

- The evolution of a domain wall in the de Sitter space was studied:
  - In case $C = \lambda\eta^2/H^2 = 1/(H\delta_0)^2 > 2$ the solution tends to the stationary one.
  - In case $C = \lambda\eta^2/H^2 = 1/(H\delta_0)^2 < 2$ the solution is quickly expands. For $C \lesssim 0.1$ the growth of the width becomes almost exponential, i.e. the wall expands with the Universe.