Charged lepton Flavour Violation and m_ν

?footprints of New Physics in the leptons?

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Thanks to:
V. Cirigliano, A. Crivellin, M. Elmer, M. Gorbahn, G. Isidori, Y. Kuno, M. Pruna, A.Signer, ...

1. Introduction
   - $m_\nu \Rightarrow$ CFV
   - to calculate: models and EFT
2. EFT recipe for CFV (selon me)
3. What do we know (experimentally)?
4. lessons from $\mu \leftrightarrow e$:
   - do we care about SM loops?
   - sensitivity vs exclusions
   - do we need dimension 8?
   - wee details/devils
   - ...

Two simple ways to add m_ν to the SM

1. add $\{\nu_R\}$ + Yukawa couplings: $\mathcal{L}_{SM} \to \mathcal{L}_{SM} + Y_\nu \bar{\ell} \tilde{H} \nu_R + h.c.$
   - renormalisable, L-conserving-m_ν ≡ "Dirac"
   - $\mathcal{A}_{CLV} \propto \frac{m_\nu^2}{m_W^2}$, mutiplicative GIM suppression ...? how to obtain log-GIM in the lepton sector?

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2. add "Weinberg operator": $\mathcal{L}_{SM} \to \mathcal{L}_{SM} + \frac{\ell H \ell H}{\Lambda} + h.c.$
   — non-renormalisable (dim 5), $\not{L}$-$m_\nu \equiv$ "Majorana"
   — $\mathcal{A}_{CLV}$ diverges...(need maaaaany counterterms; not predictive)

3. could add non-renormalisable but L-conserving, or higher-dim $\not{L}$.

Since "Dirac" masses give suppressed CLV, lets consider case of heavy NP in the lepton sector.

Assume heavy NP in lepton sector; to obtain a finite CLFV rate:

1. replace non-renormalisable operator by a renorm. model
   - predict all observables
   - identify correlations $\leftrightarrow$ distinguish models?

   $+ \approx$ scientific method $=$ construct hypothesis then test

   $-$ extensively pursued: invent models faster than exclude models...

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2. *add* the counterterms.

   $\Rightarrow$ add all dim 6 CLV operators to $\mathcal{L}_{SM}$ with arbitrary coefficients $\Leftrightarrow$ **EFT**

   (allows to subtract divs, impose exptal constraints on coefficients)

   $+$ separate what I know (SM $+$ data), from what I don't (BSM)

   $-$ No predictions? What to do? ??

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$\Rightarrow$ Study SM loop corrections to operator coefficients

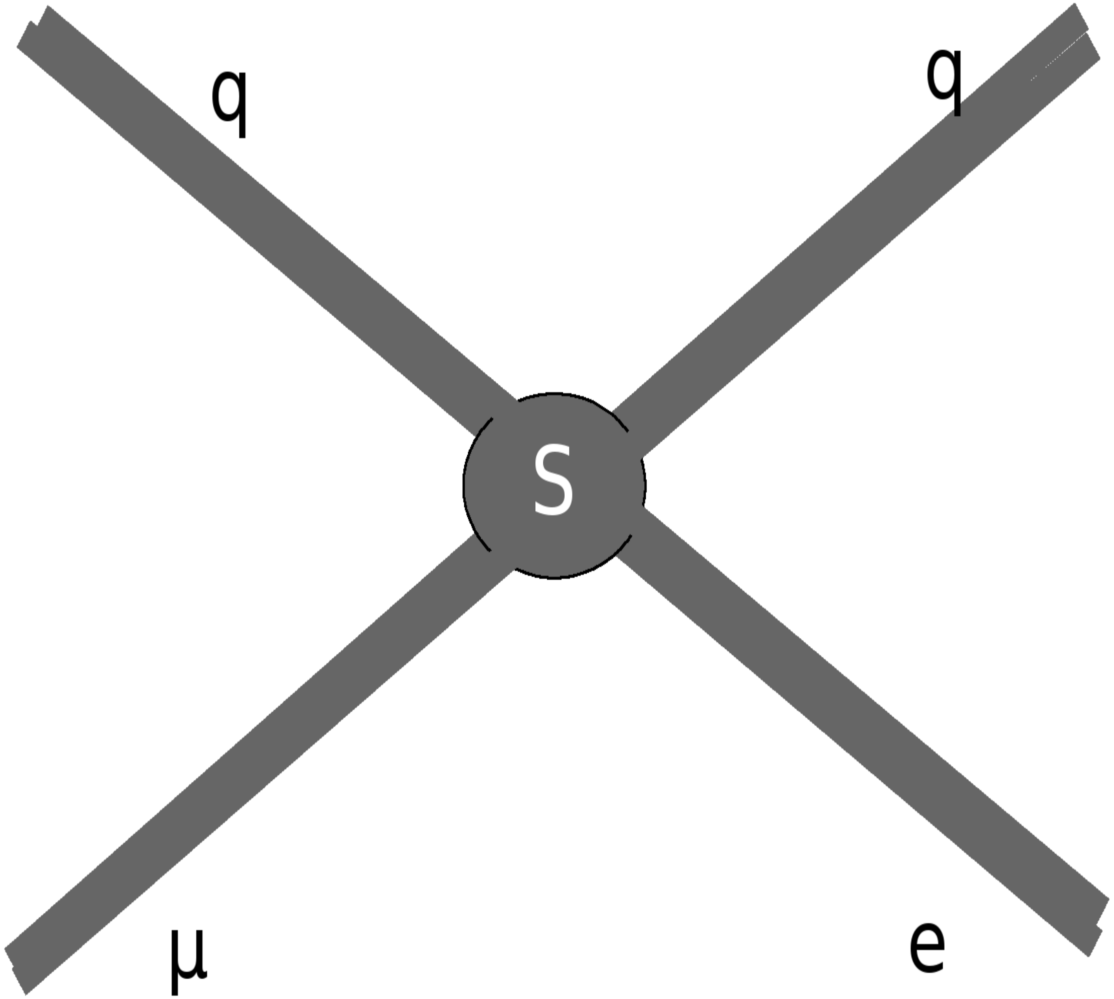
$\Leftrightarrow$ take exptal constraints to NP scale

(? and then model-build? maybe gives new perspective?)

Rather than gaze at mountain-tops from valley-bottom and hypothesize about the NP who lives there, ask SM to carry me and exptal constraints as far up as possible...

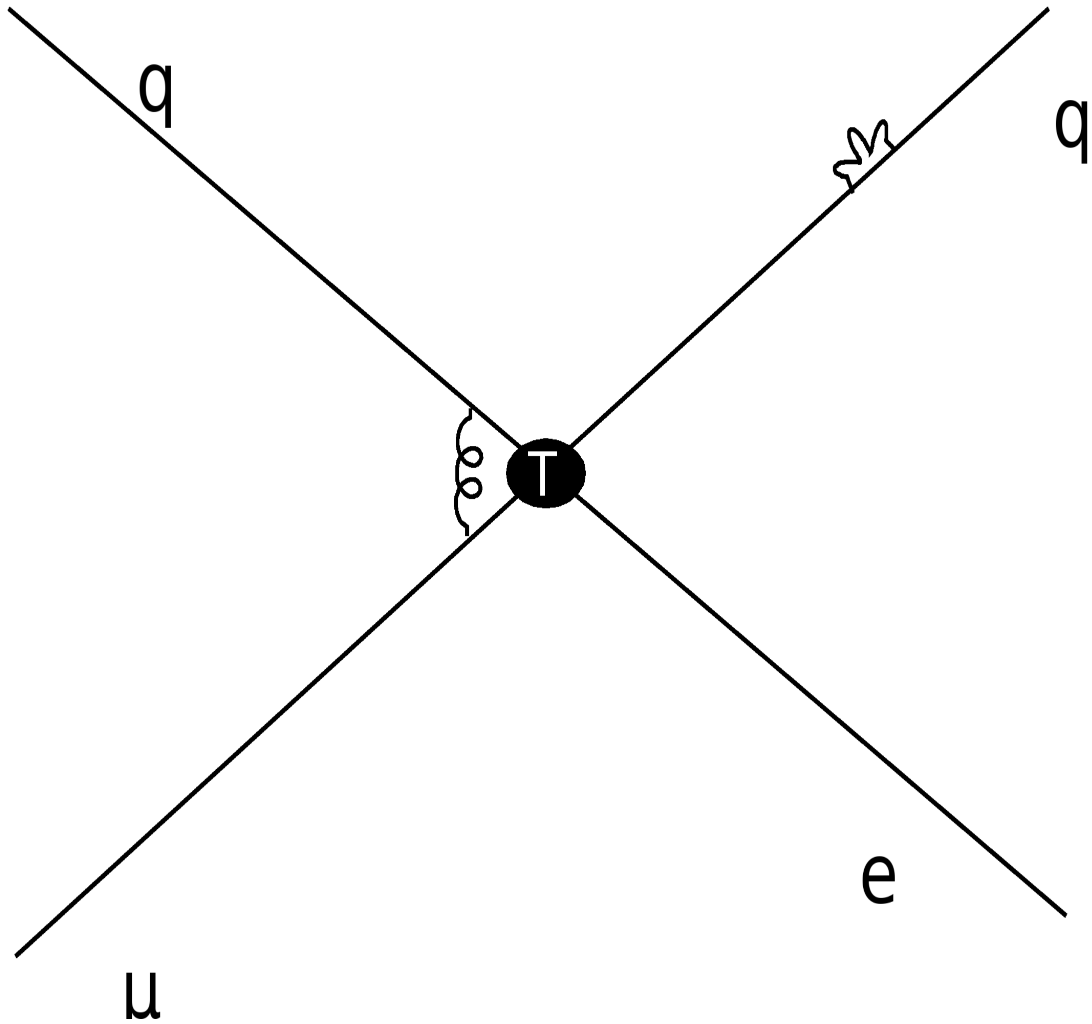
Want to "peel off" SM coating of loop corrections

expt measures operator coefficient $c(\mu_{exp})$ at exptal energy scale $\sim \mu_{exp} \sim m_\tau$



Peeling off SM loops

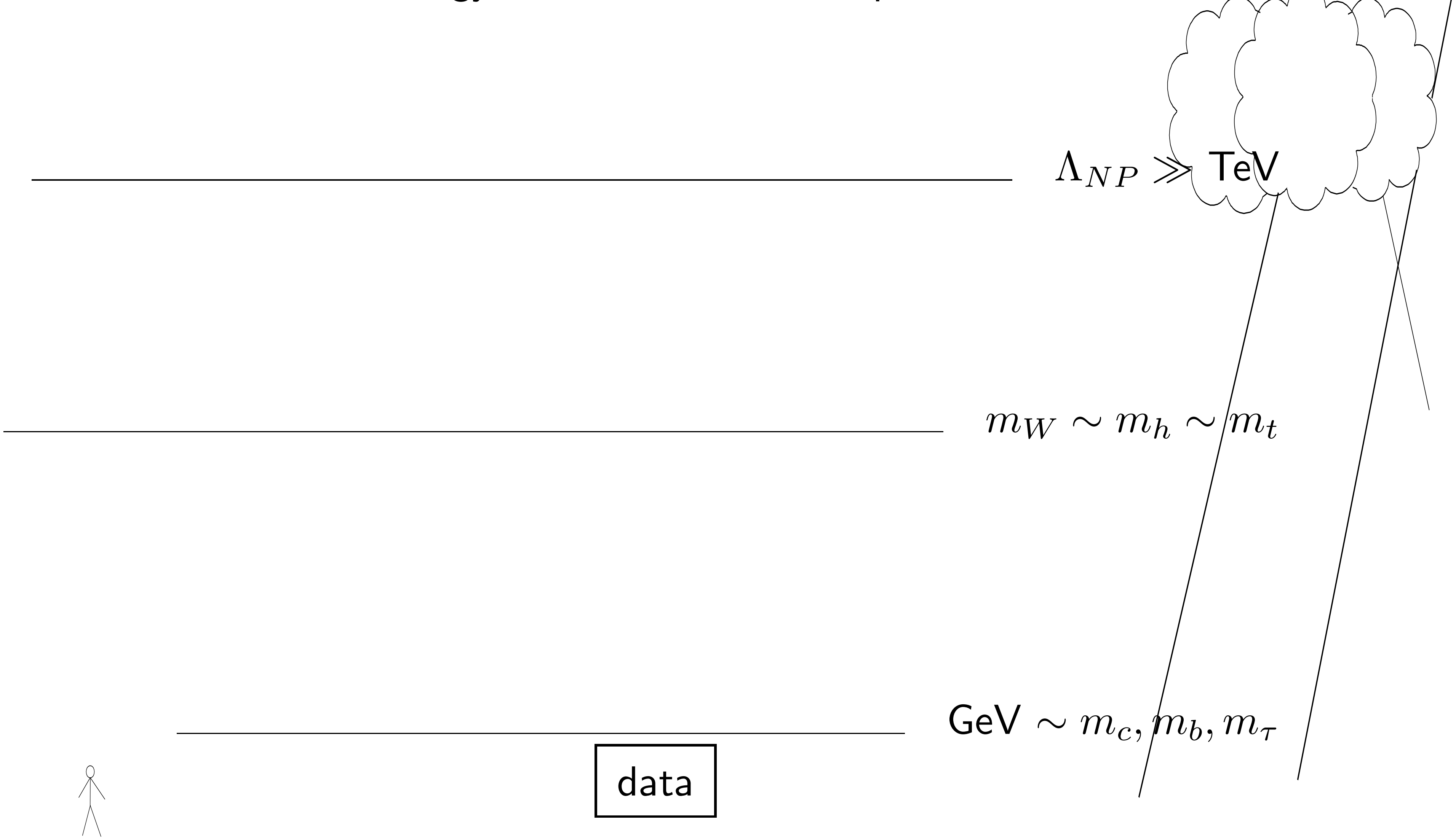
But if I look on shorter distance scale ($\sim 1/m_W$) I might see



Georgi, EFT, ARNPP 43(93) 209 (one of my all-time favourite papers)

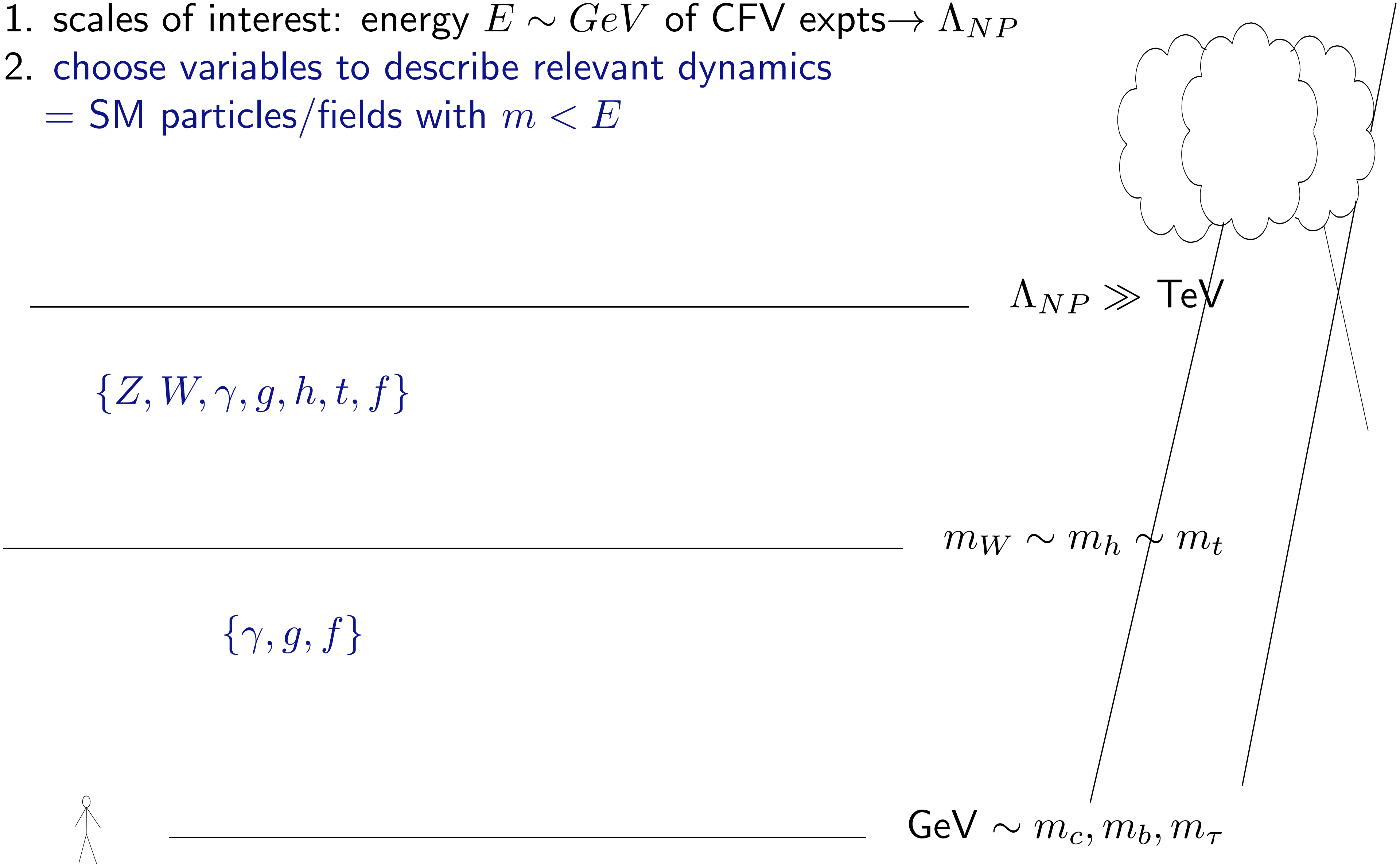
EFT for CLV induced by heavy NP ($\Lambda_{NP} \gg m_W$)

1. scales of interest: energy $E \sim GeV$ of CFV expts$\to \Lambda_{NP}$



EFT for CLV induced by heavy NP ($\Lambda_{NP} \gg m_W$)

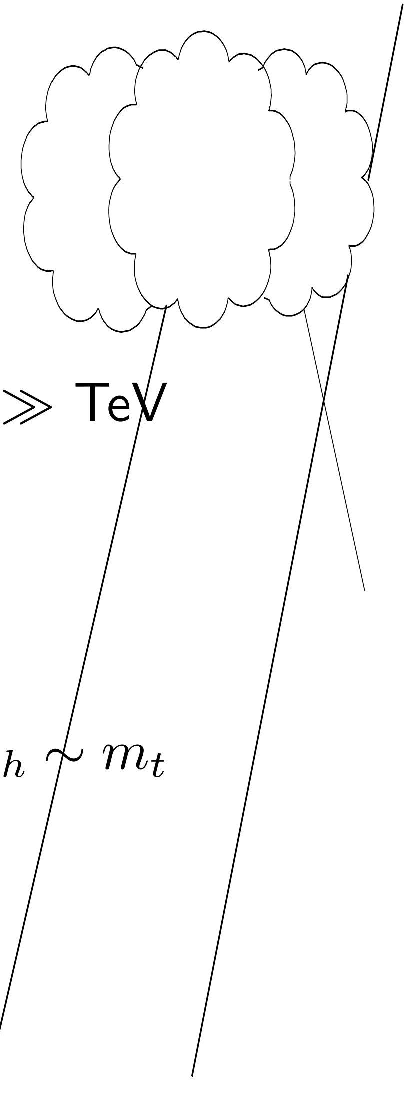
1. scales of interest: energy $E \sim GeV$ of CFV expts$\to \Lambda_{NP}$
2. choose variables to describe relevant dynamics
   $=$ SM particles/fields with $m < E$



EFT for CLV induced by heavy NP ($\Lambda_{NP} \gg m_W$)

1. choose the scale of interest: energy E of CFV expts
2. dynamical variables = SM particles/fields with $m < E$
3. 0^{th} order theory = renormalisable interactions

   (send $\to \infty$ all $M \gg E$, send $\to 0$ all $m \ll E$)



$\Lambda_{NP} \gg$ TeV

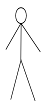
$\{Z, W, \gamma, g, h, t, f\}$

$$\mathcal{L}_{SU(3)*SU(2)*U(1)}$$

$m_W \sim m_h \sim m_t$

$\{\gamma, g, f\}$

$$\mathcal{L}_{QED \times QCD}$$



GeV $\sim m_c, m_b, m_\tau$

EFT for CLV induced by heavy NP ($\Lambda_{NP} \gg m_W$)

1. choose the scale of interest: energy E of CFV expts
2. dynamical variables = SM particles/fields with $m < E$
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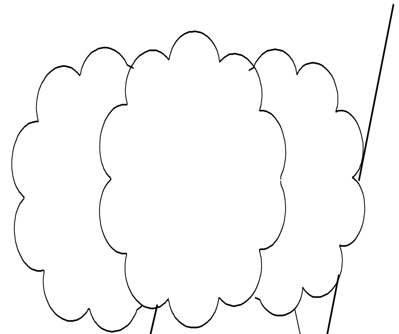
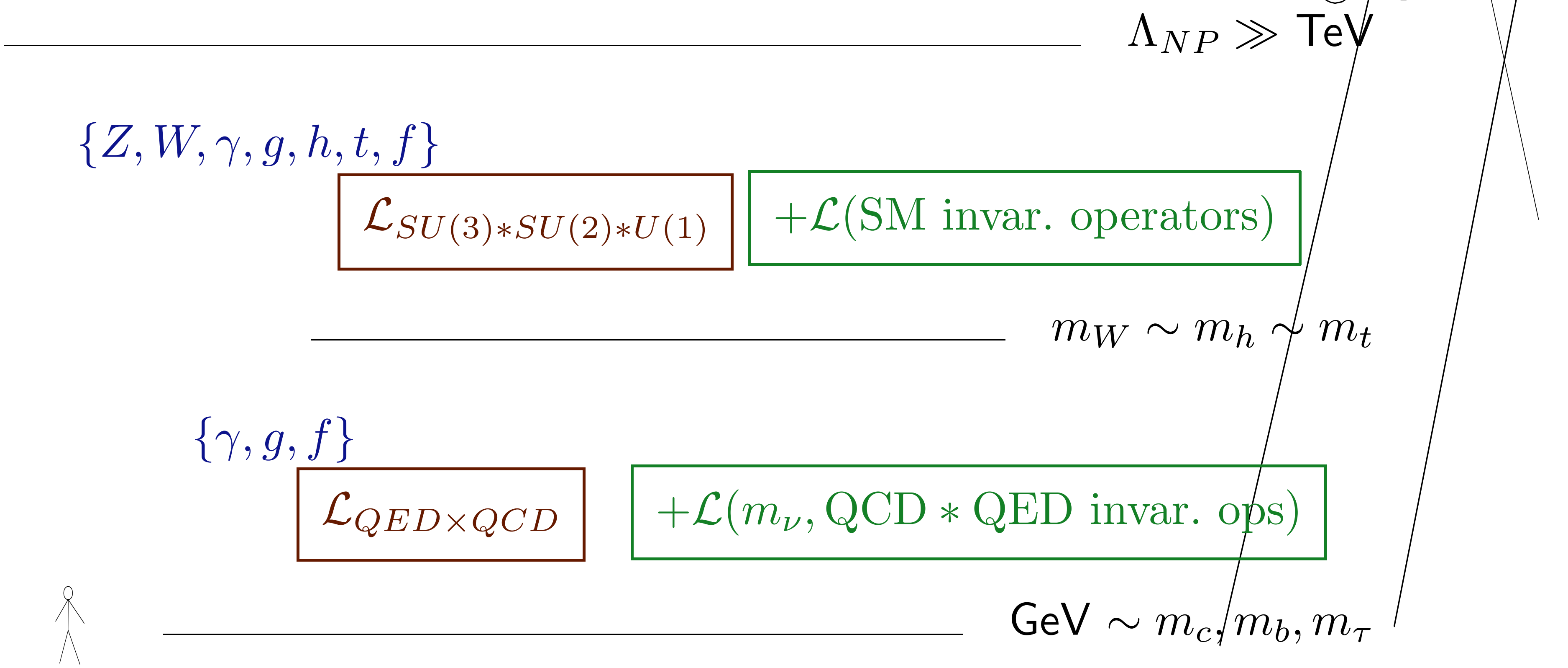
4. **perturb in scale ratios:** $\frac{m_\nu}{E}, \frac{E}{\Lambda}, \frac{v}{\Lambda}$





To implement in practise, need operator basis + recipe to change scale

A basis...is a boring tool? Of doubtful physical significance?

(?? Is there anything like "Jarlskog invariants" for EFT ??)

$\Rightarrow$ choose convenient basis(and not change during calculation)

Most CLV operators induce processes absent in the SM $\Rightarrow$ no contributions to SM observables $\Rightarrow$ basis choice simpler than eg for Higgs-EFT.

For $\mu \to e$ processes at scale $\sim m_\mu$:

three and four-point interactions involving e and μ, and 1 or 2 gauge fields, or 2(same-flavour) fermions $\in u, d, s, e$ correspond to the $QED * QCD$ invariant operators:

$$e m_\mu (\overline{e} \sigma^{\alpha\beta} P_Y \mu) F_{\alpha\beta} \qquad dim\ 5$$

$$(\overline{e}\gamma^\alpha P_Y \mu)(\overline{e}\gamma_\alpha P_Y e) \qquad (\overline{e}\gamma^\alpha P_Y \mu)(\overline{e}\gamma_\alpha P_X e)$$

$$(\overline{e} P_Y \mu)(\overline{e} P_Y e) \qquad dim\ 6$$

$$(\overline{e}\gamma^\alpha P_Y \mu)(\overline{u}\gamma_\alpha u) \qquad (\overline{e}\gamma^\alpha P_Y \mu)(\overline{u}\gamma_\alpha \gamma_5 u)$$

$$(\overline{e}\gamma^\alpha P_Y \mu)(\overline{d}\gamma_\alpha d) \qquad (\overline{e}\gamma^\alpha P_Y \mu)(\overline{d}\gamma_\alpha \gamma_5 d)$$

$$(\overline{e} P_Y \mu)(\overline{u} u) \qquad (\overline{e} P_Y \mu)(\overline{u}\gamma_5 u)$$

$$(\overline{e} P_Y \mu)(\overline{d} d) \qquad (\overline{e} P_Y \mu)(\overline{d}\gamma_5 d)$$

$$(\overline{e}\sigma P_Y \mu)(\overline{d}\sigma d)$$

$$(\overline{e}\sigma P_Y \mu)(\overline{u}\sigma u)$$

$$\frac{1}{m_t}(\overline{e} P_Y \mu) G_{\alpha\beta} G^{\alpha\beta} \qquad dim\ 7$$

...zzz...

(plus operators with $d \leftrightarrow s$). $\mu \to e\gamma$, $\mu \to e\bar{e}e$, and $\mu - e$ conv. are *sensitive* to most new 3 or 4-point μ-e interactions. $(P_X, P_Y = (1 \pm \gamma_5)/2)$

Some more operators for $\mu \to e$ at all scales $< m_W$

(That was only operators with one μ and lighter fermions...). At higher scales there are also operators containing μ, τ, c, b bilinears: :

$$
\begin{array}{rcl}
(\overline{e}\gamma^\alpha P_Y \mu)(\bar{l}\gamma^\alpha P_Y l) & , & (\overline{e}\gamma^\alpha P_Y \mu)(\bar{l}\gamma^\alpha P_X l) \\
(\overline{e} P_Y \mu)(\bar{l} P_Y l) & & (\overline{e} P_Y \mu)(\overline{\tau} P_X \tau) \\
(\overline{e}\sigma P_Y \mu)(\overline{\tau}\sigma P_Y \tau) & &
\end{array}
$$

$$
\begin{array}{rcl}
(\overline{e}\gamma^\alpha P_Y \mu)(\overline{q}\gamma^\alpha P_Y q) & , & (\overline{e}\gamma^\alpha P_Y \mu)(\overline{q}\gamma^\alpha P_X q) \\
(\overline{e} P_Y \mu)(\overline{q} P_Y q) & , & (\overline{e} P_Y \mu)(\overline{q} P_X q) \\
(\overline{e}\sigma P_Y \mu)(\overline{q}\sigma P_Y q) & &
\end{array}
$$

where $l \in \{\mu, \tau\}$, $q \in \{c, b\}$, $X, Y \in \{L, R\}$, and $X \neq Y$.

(notice: only lepton tensors with τ bilinear, and $(\overline{e}\sigma P_L \mu)(\overline{\tau}\sigma P_R \tau) = 0$)

Then **more operators if allow flavour non-diagonal quark bilinears...**
eg mediate $K \to \bar{\mu}e$....

And **different operators above m_W...**

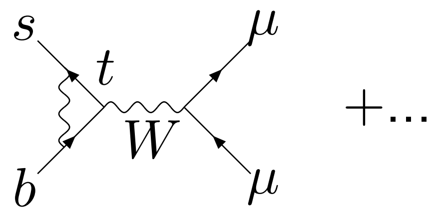
BuchmullerWyler
GrzadkowskiIMR

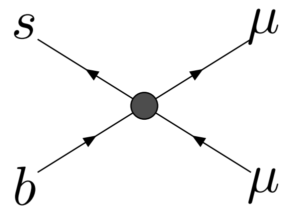
To implement in practise, need operator basis + recipe to change scale

need a recipe to relate EFTs at different scales

1. when change EFTs (eg at m_W):
   match (= set equal) Greens functions
   in both EFTs at the matching scale

$$\Rightarrow C_V \sim \frac{V_{ts}}{16\pi^2}$$





$G_F C_V (\bar{s}\gamma b)(\bar{\mu}\gamma\mu)$

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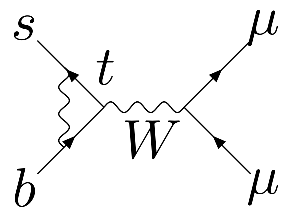
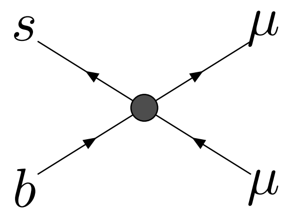


$+ ...$



$G_F C_V (\bar{s}\gamma b)(\bar{\mu}\gamma\mu)$

2. Within an EFT: Lagrangian parameters $(\alpha_s(\mu), \phi(\mu), C_I(\mu), ...)$ evolve with scale (due to loops).Described by Renormalisation Group Eqns. For $\{C_I\}$ below m_W:

Davidson,CrivellinDPS

$$\mu \frac{\partial}{\partial \mu}(C_I, ...C_J, ...) = \frac{\alpha_s}{4\pi}\vec{C}\mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi}\vec{C}\mathbf{\Gamma}^e$$

line up operator coefficients in $\vec{C}$, $\mathbf{\Gamma}$ = anomalous dimension matrix :
$\mathbf{\Gamma}^s \leftrightarrow$ rescales coefficients, $\mathbf{\Gamma}^e \leftrightarrow$ transform one coeff to another...

Above m_W : $\mathbf{\Gamma}$ for $SU(3) \times SU(2) \times U(1)$

JenkinsManoharTrott

Where are we?

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3. **What do we know experimentally?**
4. lessons from $\mu \leftrightarrow e$:
   - need SM loops?
   - sensitivity vs exclusions
   - do we need dim 8?
   - wee details + devils
   - does one need to match at m_W, and what goes wrong?

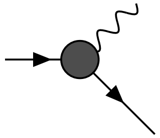
What do we know? (experimentally)

some processes	current constraints	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	2×10^{-14} (MEG)
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$(SINDRUM)	10^{-16} (2018, Mu3e)
$\mu A \to eA$	$< 7 \times 10^{-13}$ Au, (SINDRUM)	10^{-16} (Mu2e,COMET)
		10^{-18} (PRISM/PRIME)
$\overline{K_L^0} \to \mu\bar{e}$	$< 4.7 \times 10^{-12}$ (BNL)	
$K^+ \to \pi^+\bar{\mu}e$	$< 1.3 \times 10^{-11}$ (E865)	10^{-12} (NA62)
$\tau \to \ell\gamma$	$< 3.3, 4.4 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to 3\ell$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to e\phi$	$< 3.1 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)

$\mu A \to eA \equiv \mu$ bound in $1s$ state of nucleus A converts to e

Loop effects...is there sensitivity?

Two dipole operators contribute to $\mu \to e\gamma$:



$$\delta\mathcal{L}_{meg} = -\frac{4G_F}{\sqrt{2}} e\, m_\mu \left(c_L^D \overline{\mu_R}\sigma^{\alpha\beta} e_L F_{\alpha\beta} + c_R^D \overline{\mu_L}\sigma^{\alpha\beta} e_R F_{\alpha\beta}\right)$$

$$BR(\mu \to e\gamma) = 384\pi^2(|c_R^D|^2 + |c_L^D|^2) < 4.2 \times 10^{-13}$$

$$\Rightarrow |c_X^D| \lesssim 10^{-8}$$

MEG expt, PSI

How big does one expect c to be?

Is there sensitivity to loop effects ?

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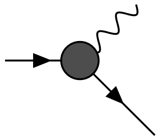

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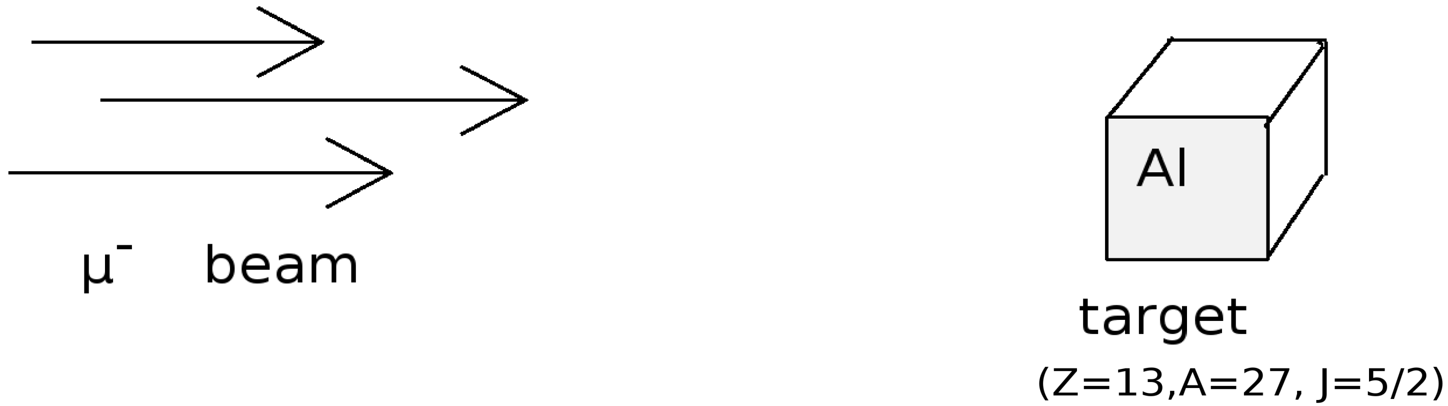
MEG expt, PSI

How big does one expect c to be? Suppose operator coefficient

			$n = 1$	$n = 2$
$ec\frac{m_\mu}{v^2} \sim \frac{ev}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 3000$ TeV	300 TeV
$ec\frac{m_\mu}{v^2} \sim \frac{em_\mu}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 100$ TeV	10 TeV

$\Rightarrow \mu \to e$ expts probe multi-loop effects in NP theories with $\Lambda_{NP} \gg$ reach of LHC

RGEs, mixing and all that... does it matter? Consider $\mu \to e$ conversion



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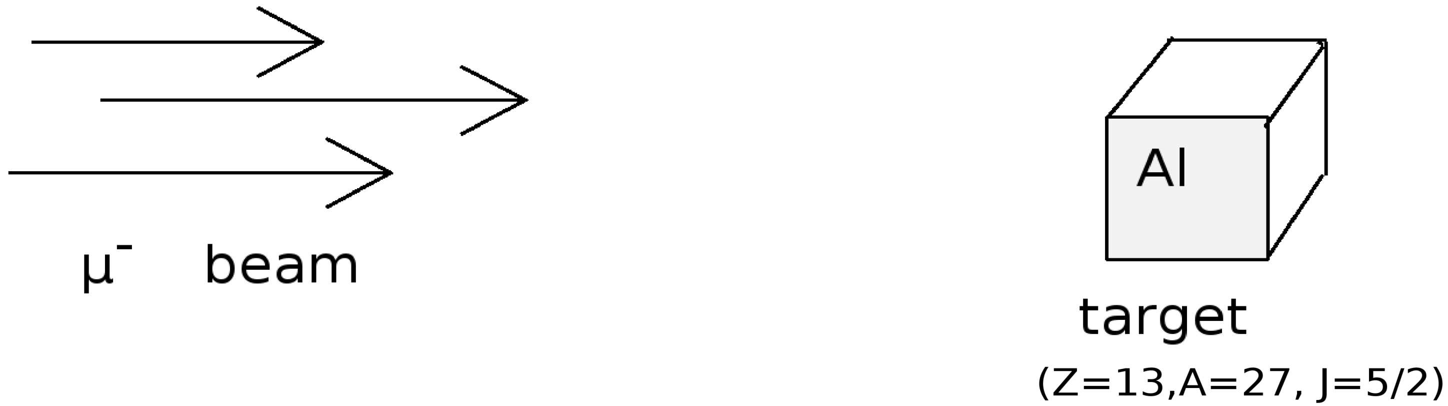

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$$\delta\mathcal{L} = C_T^{uu}(m_W)(\overline{e}\sigma P_R\mu)(\overline{u}\sigma u) + C_A^{uu}(m_W)(\overline{e}\gamma P_L\mu)(\overline{u}\gamma\gamma_5 u)$$

- nuclear expectation value of quark currents like for WIMP scattering (at $q^2 = 0$):

V,S quark currents $\longrightarrow$ Spin-Indep, A,T quark currents $\longrightarrow$ Spin-Dep conversion.

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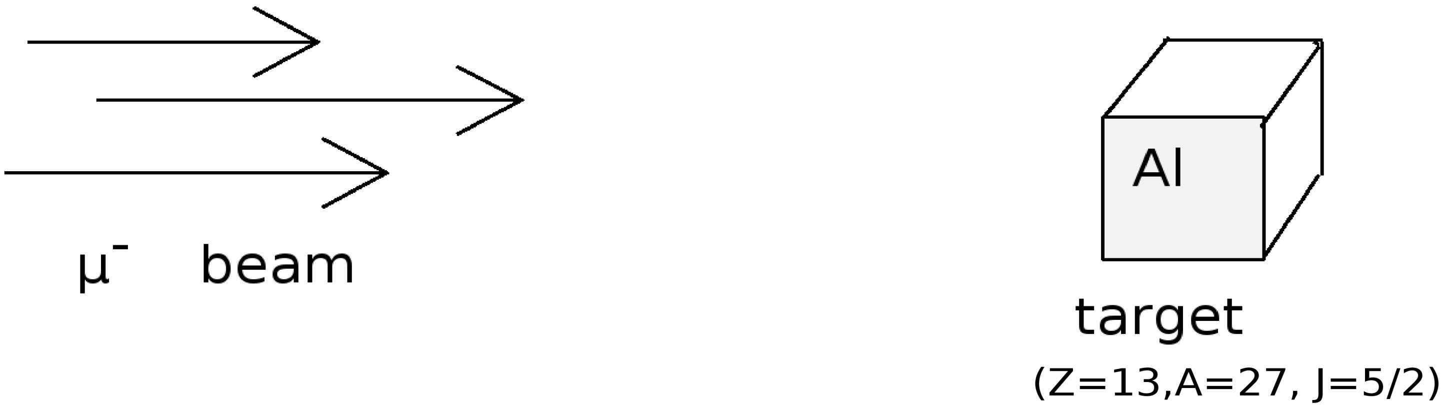

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- Neglecting RG loops, get

CiriglianoDavidsonKuno

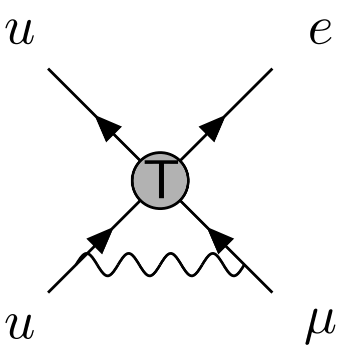
$$BR(\mu Al \to eAl)_{SD} \sim 8B\frac{J_{Al}+1}{J_{Al}}S_p^2|C_A^{uu} + 2C_T^{uu}|^2$$

$S_p \equiv \langle Al|\vec{S}_p|Al\rangle \sim .3,\ B \sim .33$

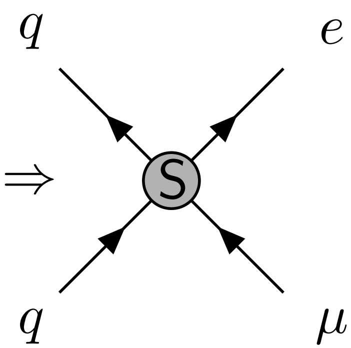
EngelRTO, KlosMGS

Include QED loops between $m_W \leftrightarrow m_\mu$

$C_T(\overline{u}\sigma u)(\overline{e}\sigma P_Y \mu)$



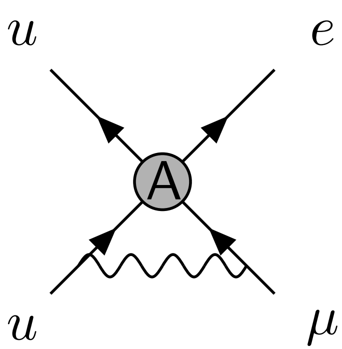
$+\ldots \quad \Rightarrow$



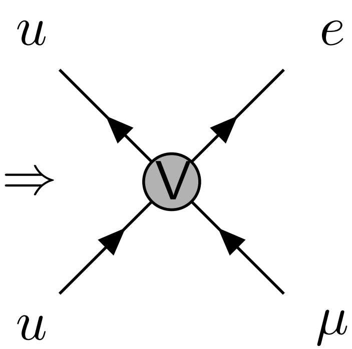
$$64\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\; C_T\;(\overline{u}u)(\overline{e}P_Y\mu)$$

$$\Delta C_S(m_\tau) \sim \tfrac{1}{7}C_T(m_W)$$

$C_A(\overline{u}\gamma\gamma_5 u)(\overline{e}\gamma P_Y \mu)$



$+\ldots \quad \Rightarrow$



$$8\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}C_A(\overline{u}\gamma u)(\overline{e}\gamma P_Y\mu)$$

$$\Delta C_V(m_\tau) \sim \tfrac{1}{50}C_A(m_W)$$

Including the loop effects...

Recall $\Delta C_S^{uu} \sim 1/7 C_T^{uu}$ from RG mixing,
then $\langle p|\bar{u}u|p\rangle \sim 10\langle p|\bar{u}\sigma u|p\rangle$, so $\widetilde{C}_S^{pp} \gtrsim \widetilde{C}_T^{pp}$, and

$$BR(\mu Al \to e Al)_{SI} \sim 0.33(27)^2 |.03 C_A^{uu} + 2C_T^{uu}|^2$$

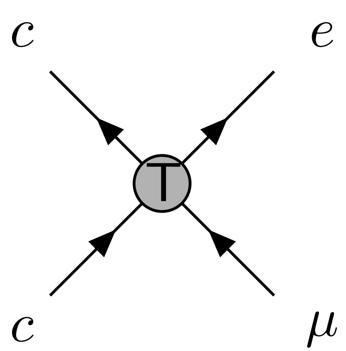
(A = 27 for Al)
(Recall that the BR_{SD} induced directly was $BR(\mu Al \to e Al)_{SD} \sim 0.1|C_A^{uu} + 2C_T^{uu}|^2$)

$\Rightarrow$ loop effects change $BR(\mu Al \to e Al)$ by $\left\{ \begin{array}{ll} \mathcal{O}(10^3) & \text{for } u, d \text{ tensor} \\ \mathcal{O}(\text{few}) & \text{for axial} \end{array} \right.$

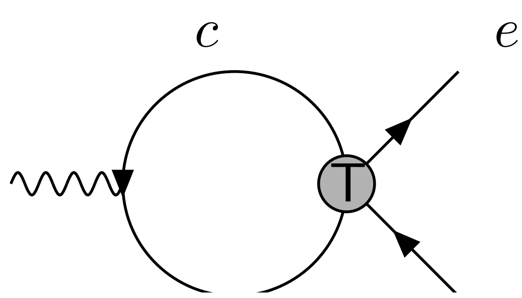
Does one need the loops, part 3? Of the tensor and the dipole...

suppose at $\sim m_W$: $\delta\mathcal{L} \supset C_T^{cc}(\bar{c}\sigma^{\alpha\beta}P_L c)(\bar{e}\sigma_{\alpha\beta}P_L\mu) + \ldots$
(eg from doublet leptoquark S with interactions $\lambda_L(\overline{\nu}s_L^c - \overline{\mu}c_L^c)S + \lambda_R\overline{e}c_R^c S$)

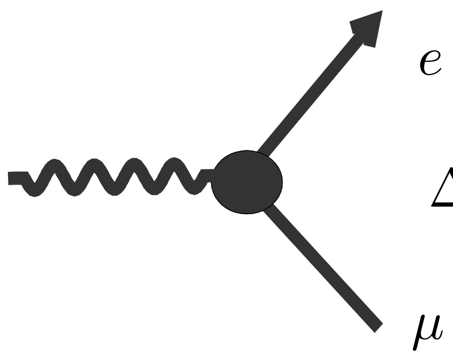
?How to observe that operator at tree level??



$\Rightarrow$



$\frac{16m_c}{em_\mu}\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\ C_T^{cc}m_\mu(\overline{e}\sigma\cdot FP_L\mu)$



$\Delta c_{D,L} \sim 1.2C_T^{cc}m_\mu(\overline{e}\sigma\cdot FP_L\mu)$

recall MEG bound : $c_{D,Y} \lesssim 10^{-8}$ at m_μ

at m_W : $|C_{D,L} - C_{T,L}^{cc} + C_{T,L}^{\tau\tau} + 1.8C_{T,L}^{bb} + \mathcal{O}(10^{-3})C| \lesssim 10^{-8}$

excellent sensitivity of $\mu \to e\gamma$ to mid-weight-fermion tensor operators

How small can we see *vs* How big could it be?

$sensitivity \equiv$ how small a coefficient could one see?
$\Leftrightarrow$ "setting bounds one operator at a time"

1. put a coefficient, eg C_T^{uu} at m_W
2. compute observables, obtain:

$$C_T^{uu} \lesssim \epsilon$$

$\Leftrightarrow$ can't see C_T^{uu} if its smaller than ϵ.

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constraints and exclusions $\equiv$ what values of a coefficient are excluded by the data?

- models induce numerous operators
- observables often depend on linear combinations of operators coefficents...

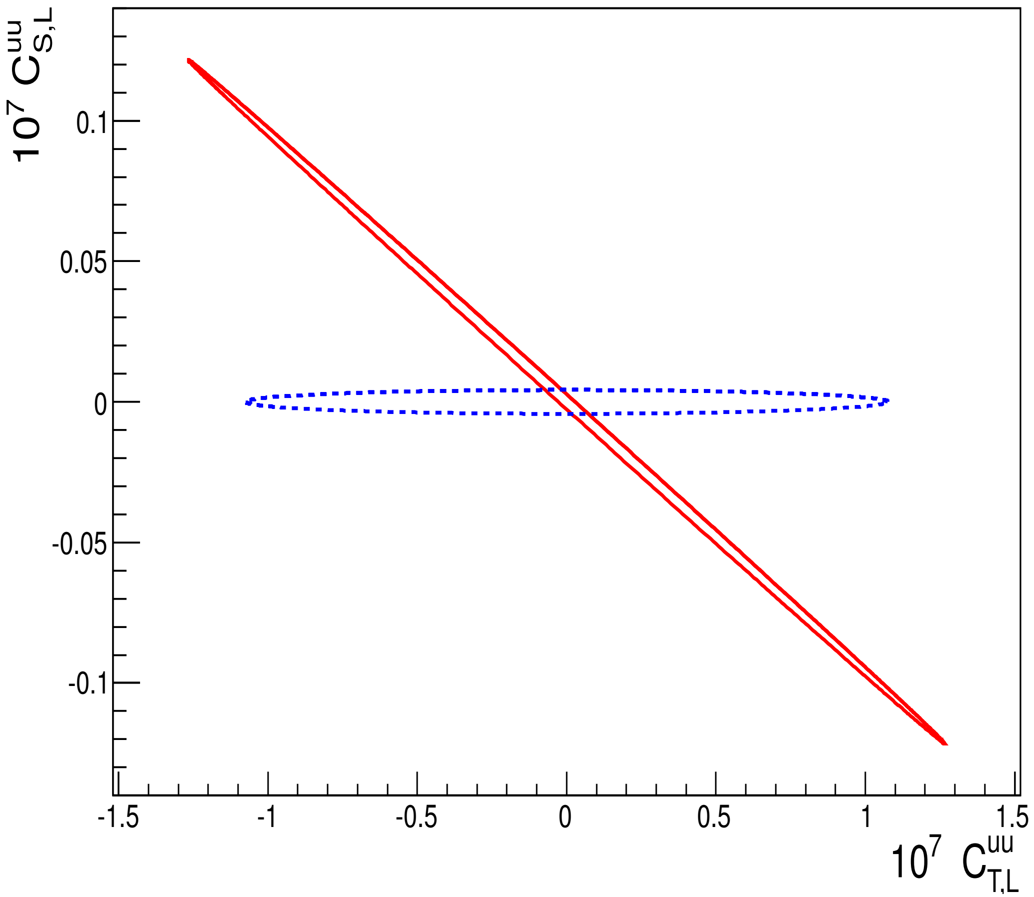
... all coefficients run and mix with scale

$\Rightarrow$ a given expt constrains a linear combination of coefficients

How small can we see *vs* How big could it be?

Suppose that $BR(\mu Al \to eAl) \lesssim 10^{-14}$.
To find exclusions, should allow all operator coefficients at m_W; lets put

$$\delta\mathcal{L}(m_W) = C_T^{uu}(\overline{e}\sigma P_Y \mu)(\overline{u}\sigma u) + C_S^{uu}(\overline{e}P_Y\mu)(\overline{u}u)$$



Outside blue (red) ellipse excluded for C_T^{uu}, C_S^{uu} at exptal scale (at m_W).

Example 2: should the LHC look for $h \to \mu^{\pm} e^{\mp}$?

At Λ_{NP}: $\mathcal{L}_{SM} \quad + \quad \frac{C_h}{\Lambda_{NP}^2} H^\dagger H \overline{\ell_\mu} H e \qquad + \quad \frac{C_{meg}}{\Lambda_{NP}^2} \overline{\ell_\mu} H \sigma \cdot F e$

At m_h: h decays to $\mu^{\pm} e^{\mp}$; LHC *excludes* $\sim \frac{C_h v^2}{\Lambda_{NP}^2} \lesssim 10^{-3}$ (at 1-loop $C_h(m_h) \approx C_h(\Lambda_{NP})$)

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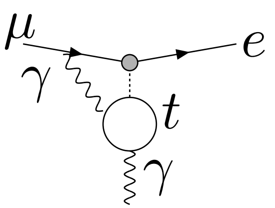
At m_h: h decays to $\mu^{\pm} e^{\mp}$; LHC *excludes* $\sim \frac{C_h v^2}{\Lambda_{NP}^2} \gtrsim 10^{-3}$ ($C_h(m_h) \approx C_h(\Lambda_{NP})$).

At m_μ:



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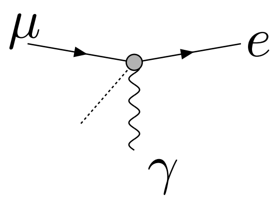
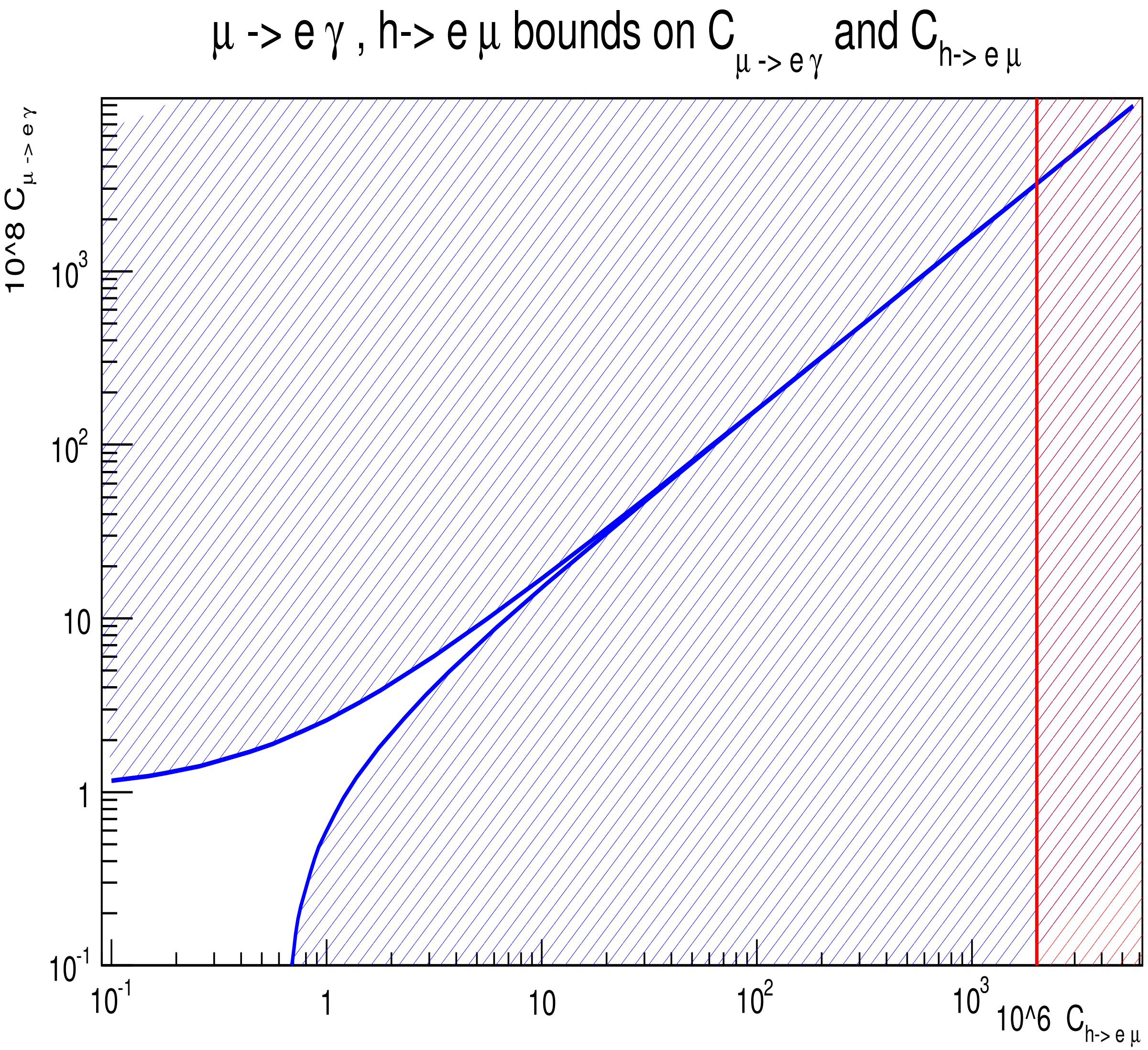


$$BR(\mu \to e\gamma) \Rightarrow \left| \frac{e\alpha}{8\pi^3 Y_\mu} C_h + C_{meg} \right| \lesssim 10^{-8} \frac{\Lambda^2}{v^2} \quad , \quad \frac{e\alpha}{8\pi^3 Y_\mu} \sim 10^{-2}$$

$\mu \to e\gamma$ *sensitive* to $C_h v^2/\Lambda^2 \gtrsim 10^{-6}$...



μ -> e γ , h-> e μ bounds on C_{μ -> e γ} and C_{h-> e μ}
10^8 C_{μ -> e γ}
10^6 C_{h-> e μ}
10^{-1}
1
10
10^2
10^3

(Parenthese...so are there as many constraints as operators?)

1. $\mu \to e\gamma$ mediated by 2 non-interfereing dipoles $\bar{e}\sigma P_Y \mu F \leftrightarrow$ 2 bds

2. $\mu \to e\bar{e}e$ mediated by 6 4f operators + 2 dipoles, 6 bds.

3. $\mu - e$ conv. mediated by 2 dipoles,2 GG operators and 20 4f operators...
   exptal bds in 2 nuclei (Ti, Au) $\Rightarrow$ 4 bds (if independent?)
   (or maybe 6, if allow for spin-dep scattering in Ti).

$\Rightarrow$ 16 - 20 "flat directions" in operator basis made with $\{\gamma, g, u, d, s, e\}$

What about dimension eight?

Exptal bounds on LFV are restrictive $\Leftrightarrow$ probe high scales or many loops: $\mu \to e\gamma$ sensitive to scales $\lesssim 10^3$ TeV at one-loop, to 2 or 3-loops effects at $\Lambda \sim 10$ TeV. *Does that mean that dimension eight operators are negligeable?*

Consider 2HDM in decoupling limit, where mass scale M of heavy doublet $\sim 10v$. Allow LFV Yukawas. Predictive model: Yukawa couplings of heavy Higgses controlled by $\tan\beta$.

One and two-loop (electroweak) contributions to $\mu \to e\gamma$ are known. Extract and compare $1/M^2$ (= dim6) and $1/M^4$ (= dim8) parts:

$$\frac{\text{dim8}}{\text{dim6}} \sim \lambda_i \tan\beta \frac{v^2}{M^2} \quad , \quad \frac{m_W^2}{M^2} \ln^2 \frac{m_W^2}{M^2}$$

$\Rightarrow$ For reasonable Higgs potential parameters $\{\lambda_i\}$, and $\cot\beta$, $\tan\beta \lesssim 50$, the $1/M^2$ parts are larger than the $1/M^4$ terms.

But: would need dimension 8 to get numerically reliable result?

?Accidentally? only the 2-loop, dim8 W contribution is log^2 enhanced, but does not profit from $\tan\beta$. (NB: $z \ln^2 z \sim 0.2 for z \sim 0.01!$)

Summary: its not just about "a sufficiently high scale"; also need "sufficiently non-hierarchical coefficients", and cooperative logs.

Wee technical details and other nightmares

EFT that learn in kindergarten: match at tree level, run at one loop

(the wee problem: at higher order, can appear terms depending on renorm scheme for the operators...these must cancel, because operators are just an approx to the renormalisable NP theory. But do they cancel?)

...but in SM, several expansions: $\left\{ \begin{array}{l} \text{loops} \\ \alpha_s, \alpha_2, \alpha_{em} \\ y_q, y_\ell \end{array} \right\} \frac{y_t}{(16\pi^2)^2} \gg \frac{y_\mu}{(16\pi^2)}$

SM is part of what we $know$, in the EFT calculation: there is only one right answer. When dominant contributions come from loop matching, multi-loop running, need to include....

So what to do?

?? Full calculation at 2 or three loop?

...or want numerically largest contribution of every operator to every observable?

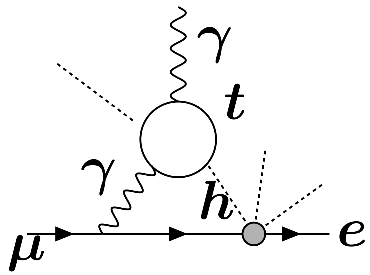
(tbc if is gauge invar and scheme indep...)

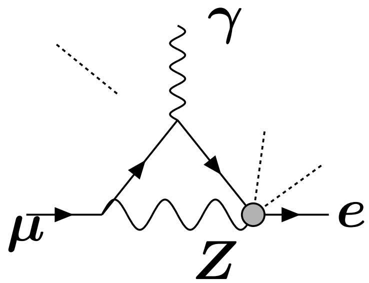
What goes wrong at m_W?

Operator dimensions change at m_W (Higgs field becomes vev)
rule of thumb: if run with 1-loop RGEs, then match at tree
reasonable if same diagram gives matching and running
...but... Dim 6 LFV Higgs and Z vertices:

$$H^\dagger H \overline{L}_\mu H E_e \quad , \quad i(\overline{L}_e \gamma^\alpha L_\mu)(H^\dagger \overleftrightarrow{D}_\alpha H) \quad , \quad i(\overline{E}_e \gamma^\alpha E_\mu)(H^\dagger \overleftrightarrow{D}_\alpha H)$$

contribute in loops to *dim 8* dipole $H^\dagger H(\overline{L}_e H \sigma \cdot F E_\mu)$, so not mix in RG running above m_W to the dim6 dipole, but do contribute in matching at m_W.





Why bother to match at m_W to QED$\times$QCD invar theory?

Why not use SMEFT everywhere?
Could work in full SM all the way down to m_μ with SM-invar operators?
Then only have to match operators to observables.

Answer 1:Because its more difficult.
Quark flavour people use EFT below m_W because replacing EW dynamics with contact interactions allows to focus on the complexities of QCD.

Answer 2: Using SMEFT everywhere doesn't simplify anything.
All the curiosities and difficulties of matching at m_W still arise; just now appear when match to observables.

Answer 3: Does SMEFT-everywhere give the right logs?
EFT is supposed to be a simple recipe to get the right answer. Its simple to regularise with dim reg, but $\overline{MS}$ resums the wrong logs (massless renorm scheme:doesn't know how many quark flavours in the QCD β-fn...)
EFT recipe for "matching out" puts the right logs back!

Summary

The observed neutrino masses imply that Charged lepton Flavour Violation is New Physics that occurs — we just don't know the rate.

Experiments under construction will improve the sensitivity to CLV by four$\to$six orders of magntiude for $\mu \leftrightarrow e$ and at least one for $\tau \leftrightarrow \ell$.

If we see CLV, it tells us something about NP in the lepton sector.

What could data tell us?

I don't know (yet).

T'is an interesting question, maybe EFT can shed some light.

Cannot completely reconstruct a fundamental Lagrangian from an effective Lagrangian (counterexample = number of steriles); that does *not* mean we can't learn anything...

BackUp

Why to do EFT

EFT $\Leftrightarrow$ add (yet more) perturbative expansions(in SM, already loops, gauge cplgs, Yukawas...).

Two perspectives in EFT:
top-down: EFT as the simple way to get the answer to desired accuracy
know the high-scale theory = can calculate operator coeffs
EFT simplifies (loop) calculations: expand in scale ratios ($eg\ m_B/m_W$)
rather than calculate dynamics at different scales

bottom-up: EFT as a parametrisation of ignorance
unknowable accuracy...

So in practise, EFT ...
1) gives a parametrisation of NP $\Leftrightarrow$ an operator basis
2) reorganises SM loop calculations involving those operators

need a basis, and need a recipe to include loops

EFT recipe to translate bounds from expt to Λ_{NP}

$\mathcal{L}_{SM+NP}$

$\Lambda_{NP} \gg$ TeV

$\{Z, W, \gamma, g, h, t, f\}$ $SU(3) \times SU(2) \times U(1)$ RGEs

$\mathcal{L}_{SM}$ $+\mathcal{L}(\text{SM invar. operators})$

(data) $m_W \sim m_h \sim m_t$

$\{\gamma, g, f\}$ RGEs of QCD $\times$ QED

$\mathcal{L}_{QED \times QCD}$ $+\mathcal{L}(\text{QCD} * \text{QED invar. ops})$

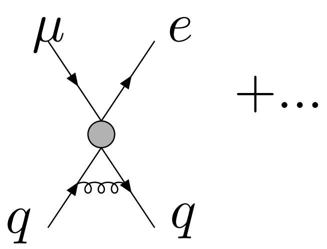
GeV $\sim m_c, m_b, m_\tau$

data ($\mu \to e\gamma$,...)

Step 3: Run up to m_W with *one-loop* RGEs of QCD+QED

$$\mu \frac{\partial}{\partial \mu} \vec{C} = \frac{\alpha_s}{4\pi} \vec{C} \mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi} \vec{C} \mathbf{\Gamma}$$

Step 3: Run up to m_W with *one-loop* RGEs of QCD+QED



$$\mu \frac{\partial}{\partial \mu} \vec{C} = \frac{\alpha_s}{4\pi} \vec{C} \mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi} \vec{C} \mathbf{\Gamma}$$

QCD: not mix ops, should resum ⇒ multiplicative renorm S,T ops
QED:

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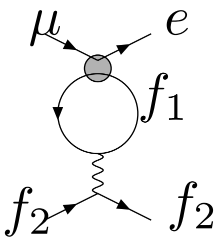
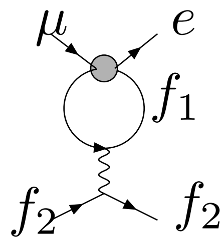
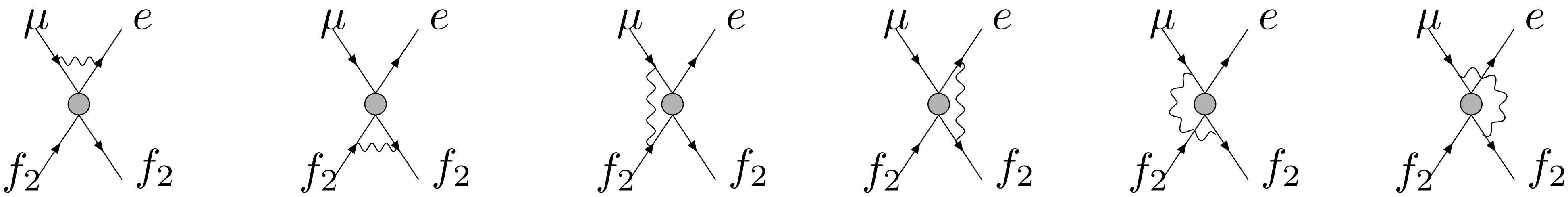
$$C_A(m_W) \left[\frac{\alpha_s(m_W)}{\alpha_s(m_\tau)} \right]^{\frac{\gamma_A^s}{2\beta_0}} \left(\delta_{AB} - \frac{\alpha_{em}}{4\pi} [\Gamma]_{AB} \log \frac{m_W}{m_\tau} + \frac{\alpha_{em}^2}{32\pi^2} [\Gamma\Gamma]_{AB} \log^2 \frac{m_W}{m_\tau} + .. \right) = C_B(m_\tau)$$



3: Run up to m_W with *one-loop* RGEs of QED





$$\mu\frac{\partial}{\partial\mu}\vec{C} = \frac{\alpha_s}{4\pi}\vec{C}\mathbf{\Gamma}^s + \frac{\alpha_{em}}{4\pi}\vec{C}\mathbf{\Gamma}$$

QCD: not mix ops, should resum $\Rightarrow$ multiplicative renorm S,T ops
QED: *does* mix ops, $\alpha_{em} \ll \Rightarrow$ mixing in pert theory, neglect renormalisation:

$$C_A(m_W)\left[\frac{\alpha_s(m_W)}{\alpha_s(m_\tau)}\right]^{\frac{\gamma^s_A}{2\beta_0}}\left(\delta_{AB} - \frac{\alpha_{em}}{4\pi}[\Gamma]_{AB}\log\frac{m_W}{m_\tau} + \frac{\alpha^2_{em}}{32\pi^2}[\Gamma\Gamma]_{AB}\log^2\frac{m_W}{m_\tau} + ..\right) = C_B(m_\tau)$$

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NB: at one loop: $\mathbf{\Gamma} = \left[\begin{array}{cc} \mathbf{\Gamma}_V & 0 \\ 0 & \mathbf{\Gamma}_{STD} \end{array}\right]$ ... $V \rightarrow$dipole mixing arises at 2-loop

(neglect vectors in this talk! Better bounds from $\mu \to e\bar{e}e$,$\mu - e$ conv.... but thats not a reason!)