Neutrino kinetic equations

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In collaboration with Mikko Laine

Neutrinos: the quest for a new physics scale
CERN, March 30 2017

Overview

- Right-handed neutrinos might play a role over a very wide temperature range in the early universe
- Leptogenesis can happen either through decays of very heavy RHNs or through oscillations (ARS…) of lighter ones (T>130 GeV , T_{EW}~160 GeV)
- O(keV) RHNs may be DM. If a lepton asymmetry survives to T~1 GeV then resonant production possible Shi Fuller
- Important to have rates (equilibration, washout, …) and kinetic equations from $T \gg T_{EW}$ to $T \lesssim$GeV

Outline

- Brief introduction
- Evolution equations to order h^2
- Computing the rates entering those eqs. at LO in the SM
    - In the symmetric phase
    - In the broken phase
- Not in this talk: resonant production of keV RHNs
  Asaka Laine Shaposhnikov (2006), Laine Shaposhnikov (2008), JG Laine (2015), Venumadhav Cyr-Racine Abazajian Hirata (2015)

General approach

- Factor the system into "fast" and "slow" modes, and integrate out the former to obtain evolution eqs. for the latter

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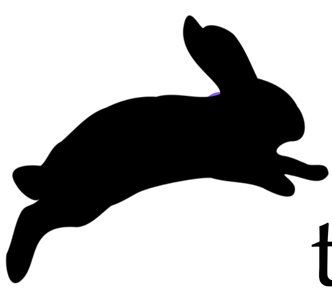
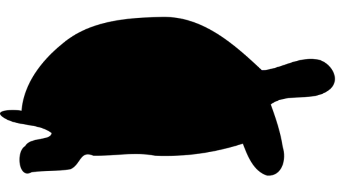
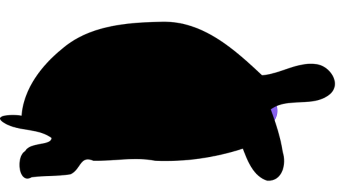
- For instance


for 130 GeV$\lesssim T \lesssim 10^5$ GeV, all SM interactions are in thermal equilibrium


O(GeV) RHNs have ~10^{-7} Yukawas: non-eq. ensemble


Lepton (and baryon) densities also evolve slowly

Constructing the equations

- From this a coupled set of evolution eqs can be obtained rigorously, as in JG Laine **1703.06087**, following the evolutions of these slow variables and keeping track of flavour and helicity effects, as well as backreactions
- Previous results can be obtained in the appropriate limits (Hernandez et al **1606.06719**, Bödeker Sangel Wörmann **1510.06742**, Bödeker Laine **1403.2755**)
- Similar approach at lower T (broken phase) in Eijima Shaposhnikov **1703.06085**
- CTP-based approach (with similar equations) Drewes Garbrecht Gueter Klaric **1606.06690**

Constructing the equations

- The starting point is

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\sum_I \bar{N}_I (i\gamma^\mu \partial_\mu - M_I) N_I - \sum_{I,a} (\bar{N}_I\, h_{Ia} j_a + \bar{j}_a\, h^*_{Ia} N_I)$$

$$j_a = \tilde{\phi}^\dagger a_L\, l_a$$

- We want to follow the evolution of n_a -$n_{\rm B}/3$ (because sphalerons) and of the sterile neutrino density matrix (keeping track of helicity and flavor)

$$\hat{\rho}_{\tau I;\sigma J} \equiv \frac{a^\dagger_{\vec{k}\tau I}\, a_{\vec{k}\sigma J}}{V}$$

- In general $\langle \dot{O}(t) \rangle \equiv {\rm Tr}\left[\dot{O}(t) \rho_{\rm full}(t)\right]$ with $\rho_{\rm full} = \rho^{\rm EQ}_{\rm SM} \otimes \rho_N$

Constructing the equations

- In general $\langle \dot{O}(t) \rangle \equiv \mathrm{Tr}\left[\dot{O}(t)\rho_{\mathrm{full}}(t)\right]$ with $\rho_{\mathrm{full}} = \rho_{\mathrm{SM}}^{\mathrm{EQ}} \otimes \rho_N$
- The two sectors talk through H_{int}, linear in h (Yukawa).

$$H_{\mathrm{int}}(t) = \int_{\mathbf{x}} \sum_{I,a} \left[\bar{j}_a(\mathcal{X})\, h^*_{Ia}\, N_I(\mathcal{X}) \;+\; \bar{N}_I(\mathcal{X})\, h_{Ia}\, j_a(\mathcal{X})\right] , \quad \mathcal{X} = (t, \mathbf{x})$$

Since $\dot{O}$ is linear in h and

$$\rho_{\mathrm{full}}(t) = \rho_{\mathrm{full}}(0) - i \int_0^t \mathrm{d}t' \left[H_{\mathrm{int}}(t'), \rho_{\mathrm{full}}(0)\right] + \mathcal{O}(h^2)$$

then

$$\langle \dot{O}(t) \rangle = \mathrm{Tr}\left\{\left[\dot{O}(t), -i \int_0^t \mathrm{d}t'\, H_{\mathrm{int}}(t')\right] \rho_{\mathrm{full}}(0)\right\} \;+\; \mathcal{O}(h^3)$$

Constructing the equations

- Thanks to the factorization between slow and hard modes, the latter only enter in the spectral function of the SM current $j_a = \tilde{\phi}^\dagger a_L \, l_a$

$$\langle j_a(\mathcal{X})\, \bar{j}_b(\mathcal{Y})\rangle = \delta_{ab} \int_{\mathcal{P}} e^{-i\mathcal{P}\cdot(\mathcal{X}-\mathcal{Y})}\, \Pi_a^{>}(\mathcal{P}) \; ,$$

$$\langle \bar{j}_b(\mathcal{Y})\, j_a(\mathcal{X})\rangle = -\delta_{ab} \int_{\mathcal{P}} e^{-i\mathcal{P}\cdot(\mathcal{X}-\mathcal{Y})}\, \Pi_a^{<}(\mathcal{P})$$

$$\rho_a(P) = \frac{1}{2}\big(\Pi_a^{>}(\mathcal{P}) - \Pi_a^{<}(\mathcal{P})\big)$$

Constructing the equations

$$j_a = \tilde{\phi}^\dagger a_L \, l_a \qquad \rho_a(P) = \frac{1}{2}\left(\Pi_a^{>}(\mathcal{P}) - \Pi_a^{<}(\mathcal{P})\right)$$

- RHN density matrix (in helicity and flavor) evolves as

$$\langle \dot{\hat{\rho}} \rangle = \frac{1}{2}\sum_a \Big\{ \hat{\Gamma}_a^+(t)\left[\mathbf{I}\, n_{\mathrm{F}}(\omega^k - \mu_a) \; - \; \langle\hat{\rho}\rangle\right]$$

$$+ \hat{\Gamma}_a^-(t)\left[\mathbf{I}\, n_{\mathrm{F}}(\omega^k + \mu_a) \; - \; \langle\hat{\rho}\rangle\right] + \mathrm{h.c.} \Big\} + \mathrm{O}(h^3)$$

where

$$\hat{\Gamma}^+_{(a\tau)IJ}(t) \equiv \frac{h^*_{Ia} h_{Ja}}{\sqrt{\omega_I^k \omega_J^k}} \, \bar{u}_{\mathbf{k}\tau J}\, a_{\mathrm{L}}\, \rho_a(\mathcal{K}_J)\, a_{\mathrm{R}}\, u_{\mathbf{k}\tau I}\, e^{i(\omega_J^k - \omega_I^k)t} \, ,$$

$$\hat{\Gamma}^-_{(a\tau)IJ}(t) \equiv \frac{h_{Ia} h^*_{Ja}}{\sqrt{\omega_I^k \omega_J^k}} \, \bar{u}_{\mathbf{k}\tau J}\, a_{\mathrm{R}}\, \rho_a(-\mathcal{K}_J)\, a_{\mathrm{L}}\, u_{\mathbf{k}\tau I}\, e^{i(\omega_J^k - \omega_I^k)t}$$

$\hat{\Gamma}_a^{\pm}(t) n_{\mathrm{F}}(\omega_J^k \mp \mu_a)$ scattering off (anti)leptons

Constructing the equations

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- Similarly

$$\left\langle \dot{n}_a - \frac{\dot{n}_\mathrm{B}}{3} \right\rangle = \frac{1}{2}\int_\mathbf{k} \mathrm{Tr}_{f,h}\Big\{ \left[\hat{\Gamma}_a^{+}(t) + \hat{\Gamma}_a^{+\dagger}(t)\right] \left[\langle\hat{\rho}\rangle \; - \; \mathbf{I}\, n_\mathrm{F}(\omega^k - \mu_a)\right]$$

$$- \left[\hat{\Gamma}_a^{-}(t) + \hat{\Gamma}_a^{-\dagger}(t)\right] \left[\langle\hat{\rho}\rangle \; - \; \mathbf{I}\, n_\mathrm{F}(\omega^k + \mu_a)\right] \Big\} + \mathrm{O}(h^3)$$

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$\hat{\Gamma}_a^\pm(t) n_\mathrm{F}(\omega_J^k \mp \mu_a)$ scattering off (anti)leptons

Intermediate summary

- These equations are valid to order h^2 in the Yukawas and in principle to all orders in the SM couplings. In practice ρ_a known to leading order in the SM

Intermediate summary

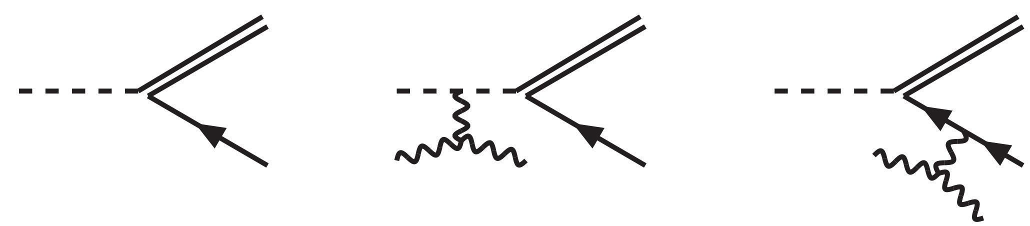
- It is sensible to expand to first order in the chemical potentials. ρ_a is also a function of the chemical potentials μ_a and μ_Y

$$\Gamma^{+}_{(a\tau)IJ} = h^{*}_{Ia} h_{Ja} \left[Q_{(\tau)IJ} + \bar{\mu}_a R_{(\tau)IJ} + \bar{\mu}_Y S_{(\tau)IJ} \right] + \mathcal{O}(\bar{\mu}^2) \,,$$
$$\Gamma^{-}_{(a\tau)IJ} = h_{Ia} h^{*}_{Ja} \left[Q_{(-\tau)IJ} - \bar{\mu}_a R_{(-\tau)IJ} - \bar{\mu}_Y S_{(-\tau)IJ} \right] + \mathcal{O}(\bar{\mu}^2)$$

- $Q_{(+)}$, $R_{(+)}$, $S_{(+)}$ are helicity-flipping (for instance $\nu_L \to \phi N_+$) and thus they survive for vanishing Majorana masses, whereas $Q_{(-)}$, $R_{(-)}$, $S_{(-)}$ are helicity conserving (fermion number violating) and thus require M insertion

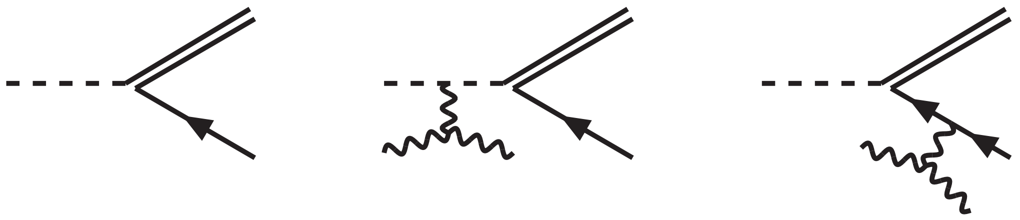
Computing ρ_a

- In the **symmetric phase** T>160 GeV, with $g=(g_1,g_2,h_t,\lambda^{1/2})$ parametrically equivalent and for $M{\sim}gT{\ll}T$, there exist two kinds of processes at leading order

1↔2	 and others, and crossings
2↔2	 and others, and crossings

Besak Bödeker **1202.1288**, Ghisoiu Laine **1411.1765**, Garbrecht Glowna Schwaller **1303.5498**, Hernandez et al **1606.06719**, JG Laine **1703.06087**

1↔2 processes



- Since all masses are $O(gT)$, tree level processes (if possible) are $\sim M^2 \sim g^2T^2$ and collinear
- Long formation times $O(1/g^2T)$) imply that soft scatterings, at rate g^2T, need to be resummed to all orders ⇒ Landau-Pomeranchuk-Migdal (LPM) effect
  Long QCD history (BDMPS, AMY). Introduced for RHNs in the *symmetric phase* in Anisimov Besak Bödeker **JCAP03** (2011),
- Helicity-conserving contributions are suppressed for M~GeV

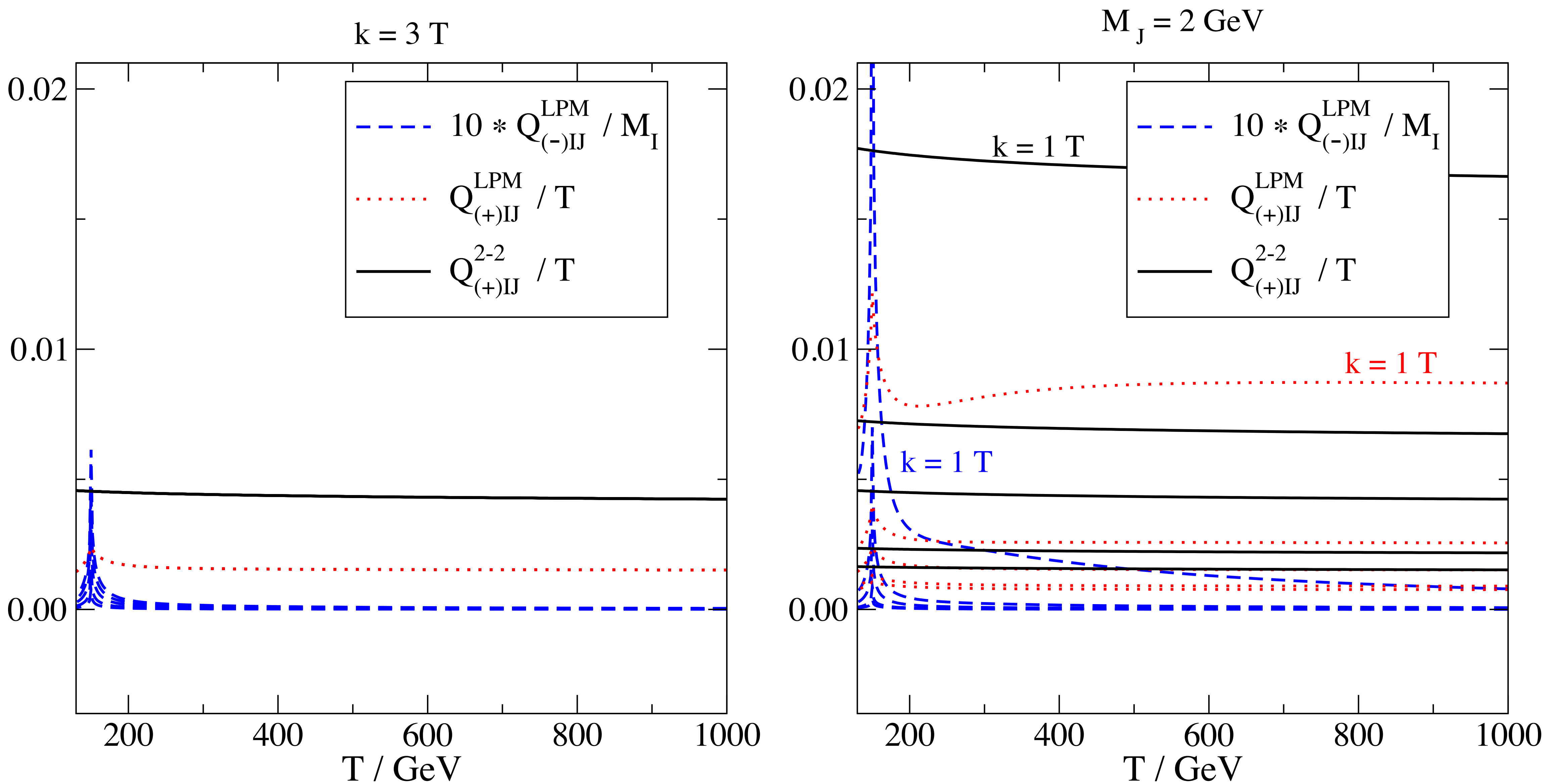
2↔2 processes

- As long as all external state masses are $O(gT)$ they can be neglected at leading order ($O(g^2T^2)$). Helicity-conserving amplitudes absent at this order
  Besak Bödeker **JCAP03** (2012)

$$\int_{\text{ph. space}} f(p)f(p')(1 \pm f(k'))|\mathcal{M}|^2\delta^4(P+P'-K-K')$$

- Phase space convolution of statistical functions (with chemical potentials) and matrix elements. HTL resummation needed for soft fermion exchange.

Symmetric phase results



$M_J/\mathrm{GeV} \in \{0.5, 1, 2, 3, 4\}$ $k/T \in \{1, 2, 3, 6, 9\}$

JG Laine **1703.06087**

Symmetric phase results

- Outlook:
  - solve the evolution equations with these rates and the full effects of helicity and chemical potentials, in the cosmological background, as in Hernandez et al **1606.06719**
  - Get to the physically interesting $O(h^4)$ (and higher) effects from the coupled dynamics of slow modes Shuve Yavin **1401.2459**

Computing ρ_a

- In the **broken phase** the Higgs e.v. $v>0$. We consider the parametric range $T \gtrsim v$, so that thermal masses ($O(gT)$) and Higgs mechanism masses ($O(gv)$) are of the same order. In practice

$$30\,\mathrm{GeV} \lesssim T \lesssim 160\,\mathrm{GeV}$$

where $g=(g_1, g_2, h_t, \lambda^{1/2})$ (parametrically equivalent)

  - In this region $M_I \lesssim gT$

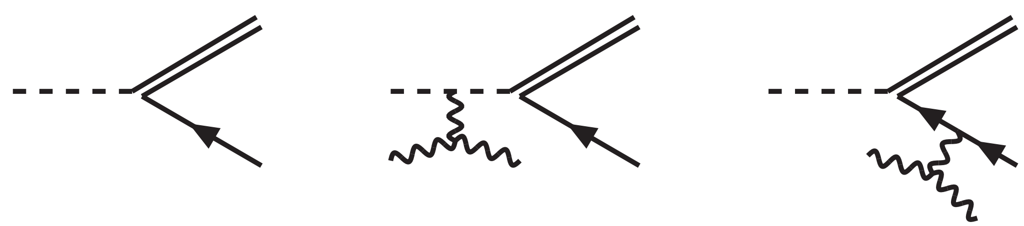
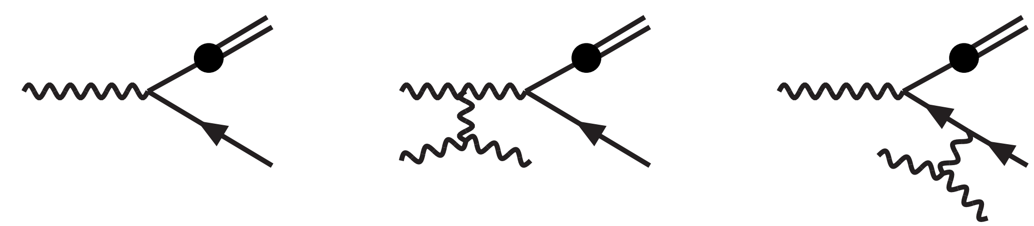
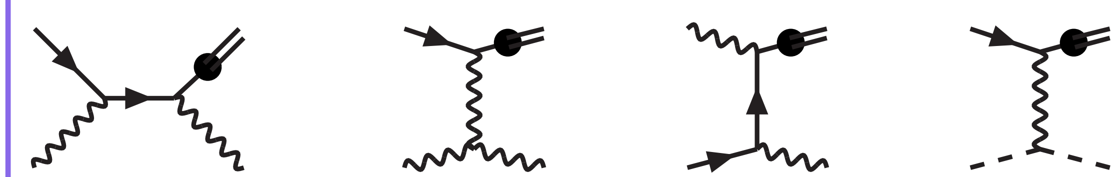
- We also consider $m_W \gtrsim \pi T$ to cover the low-temperature region down to 5 GeV

- Chemical potentials and helicity effects not included

JG Laine **1605.07720**

$j_a = \tilde{\phi}^\dagger a_L\, l_a \qquad\qquad \rho_a(P) = \frac{1}{2}\left(\Pi_a^{>}(\mathcal{P}) - \Pi_a^{<}(\mathcal{P})\right)$

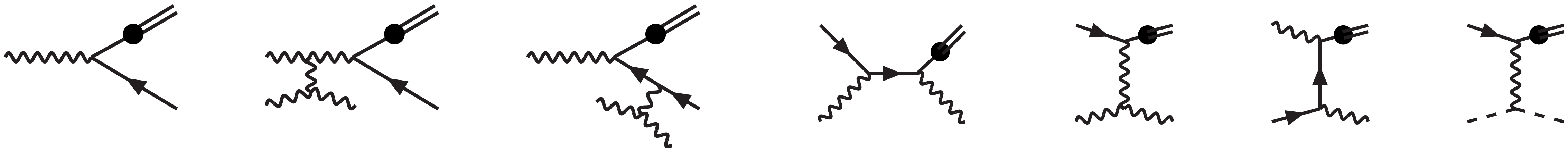
- The Higgs doublet can be a propagating d.o.f. (Higgs or Goldstone) or an expectation value insertion. Distinction into direct and indirect processes

	Direct	Indirect
1↔2	 and others, and crossings	 and others, and crossings
2↔2	 and others, and crossings	 and others, and crossings

- The direct 1↔2 and 2↔2 processes can be extended into the broken phase (more complicated LPM)

- Direct processes give $\varrho \sim g^2 T^2$. Indirect processes can have a near-resonant enhancement (hold on)

Indirect processes

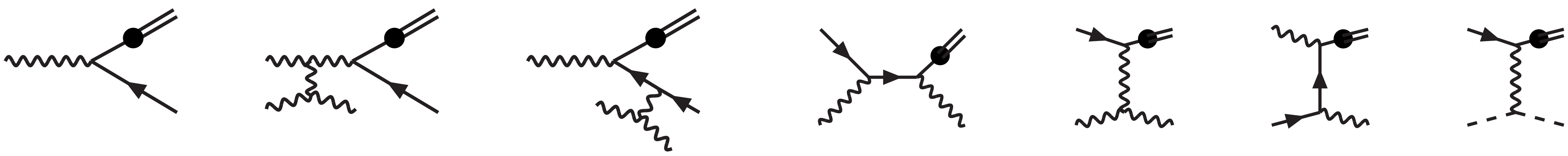


- In the indirect case ρ is directly proportional to the spf of active neutrinos, i.e.

$$\mathrm{Tr}\left[\not{K}\rho(K)_a^{\mathrm{indir.}}\right] = \frac{v^2}{2}\frac{M^2\, 2K\cdot \mathrm{Im}\,\Sigma}{(M^2 + 2K\cdot\mathrm{Re}\,\Sigma)^2 + 4(K\cdot\mathrm{Im}\,\Sigma)^2}$$

  - M^2 dependence: this is an helicity conserving contribution. As temperature drops, this becomes very relevant, also because of resonant-like behavior

Indirect processes



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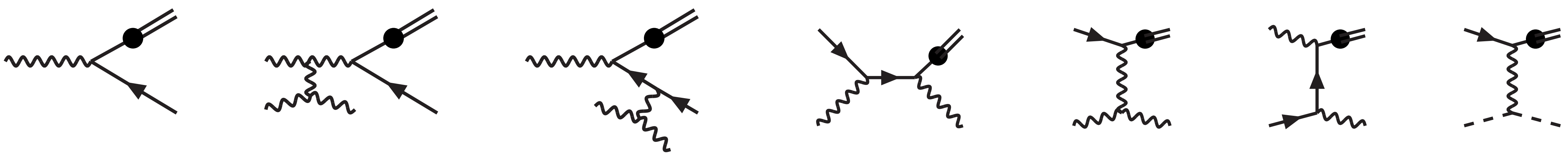
- Real part of the active neutrino self- energy
  - At high T $\;2K\cdot\mathrm{Re}\,\Sigma = -m_l^2 \sim g^2T^2$
  - At low T (positive) matter potential
  - (Broad) resonance

Indirect processes



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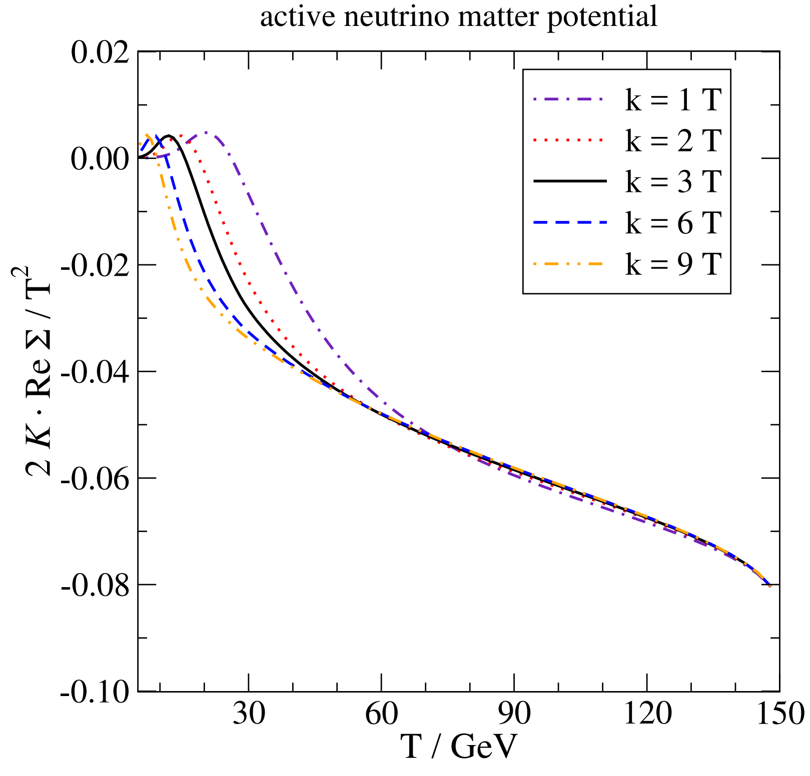
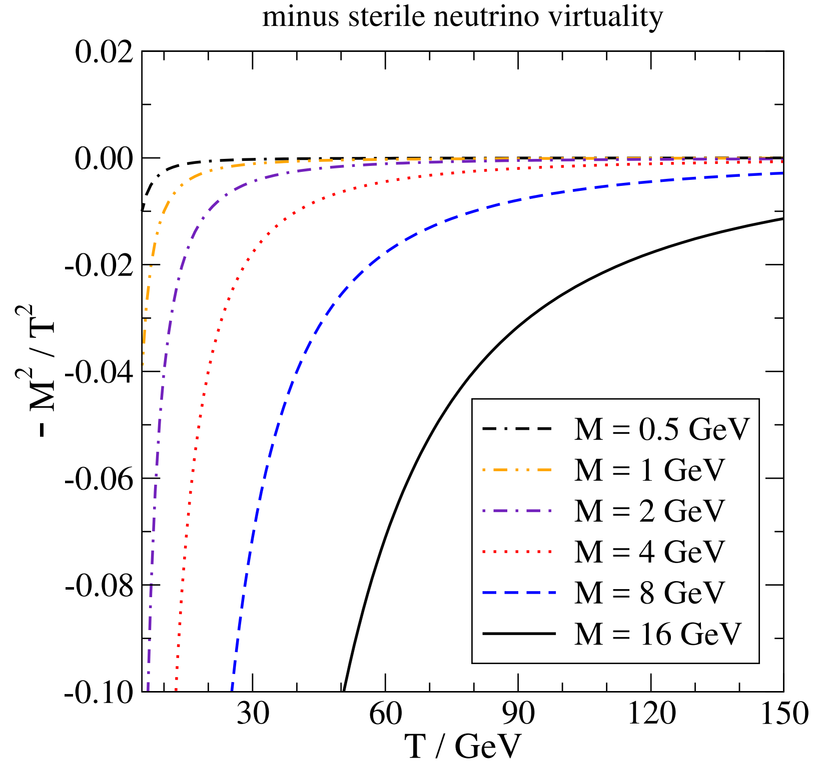
$$\mathrm{Tr}\left[\not{K}\rho(K)_a^{\mathrm{indir.}}\right] = \frac{v^2}{2}\frac{M^2\,{\color{red}2K\cdot\mathrm{Im}\,\Sigma}}{(M^2+{\color{blue}2K\cdot\mathrm{Re}\,\Sigma})^2+4({\color{red}K\cdot\mathrm{Im}\,\Sigma})^2}$$





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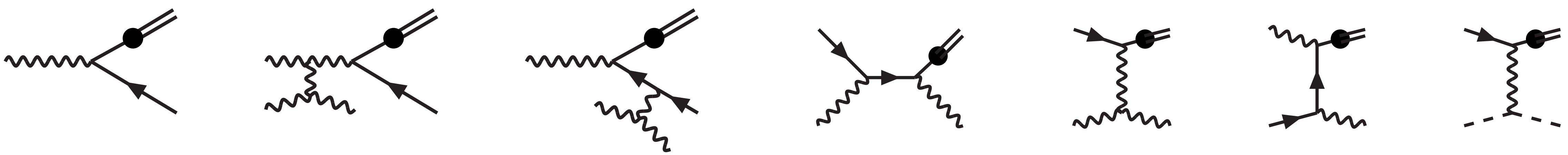

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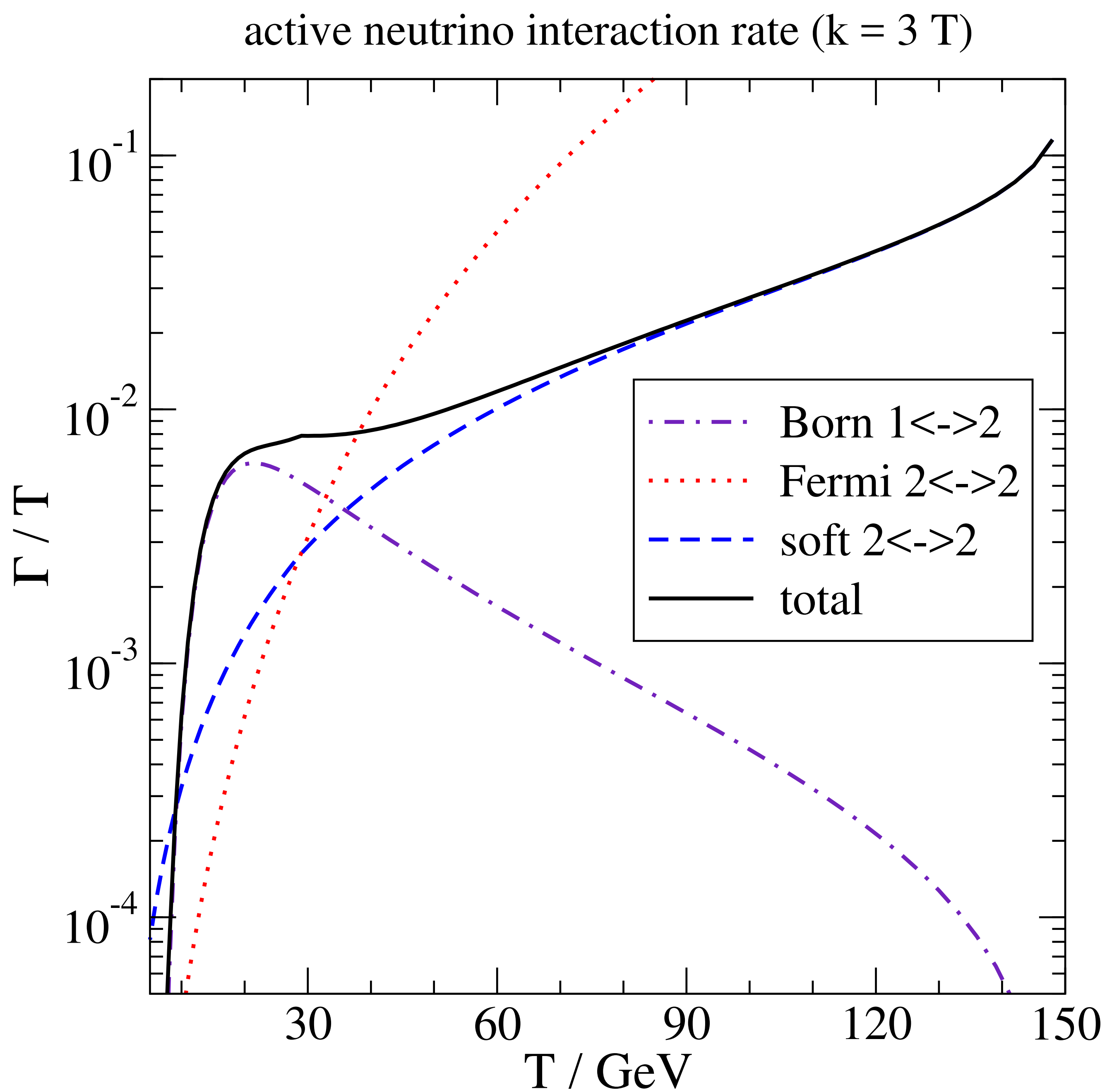
- Imaginay part of the active neutrino self- energy: active neutrino width $2K\cdot\mathrm{Im}\,\Sigma = k_0\Gamma$
    - At high T dominated by soft 2↔2 scatterings. $\Gamma \sim g^2 T$ and thus (for $M\sim gT$) $\rho\sim v^2$
    - At low T dominated by 1↔2 decays of gauge bosons

Indirect processes

- Soft 2↔2 scatterings, leading at high T
- 2↔2 scatterings in the Fermi limit, accurate but subleading at low T
- 1↔2 decay of gauge bosons, leading at low T, inaccurate but negligible at high T
- **Total:** 1↔2 + the appropriate (smallest) 2↔2



JG Laine **1605.07720**

Cosmological implications

- Compare the equilibration and washout rates to the Hubble rate

$$\dot{f}_{I\mathbf{k}} = \gamma_{I\mathbf{k}} \left(n_{\mathrm{F}}(E_I) - f_{I\mathbf{k}}\right) + \mathcal{O}\left[\left(n_{\mathrm{F}} - f_{I\mathbf{k}}\right)^2, n_a^2\right]$$

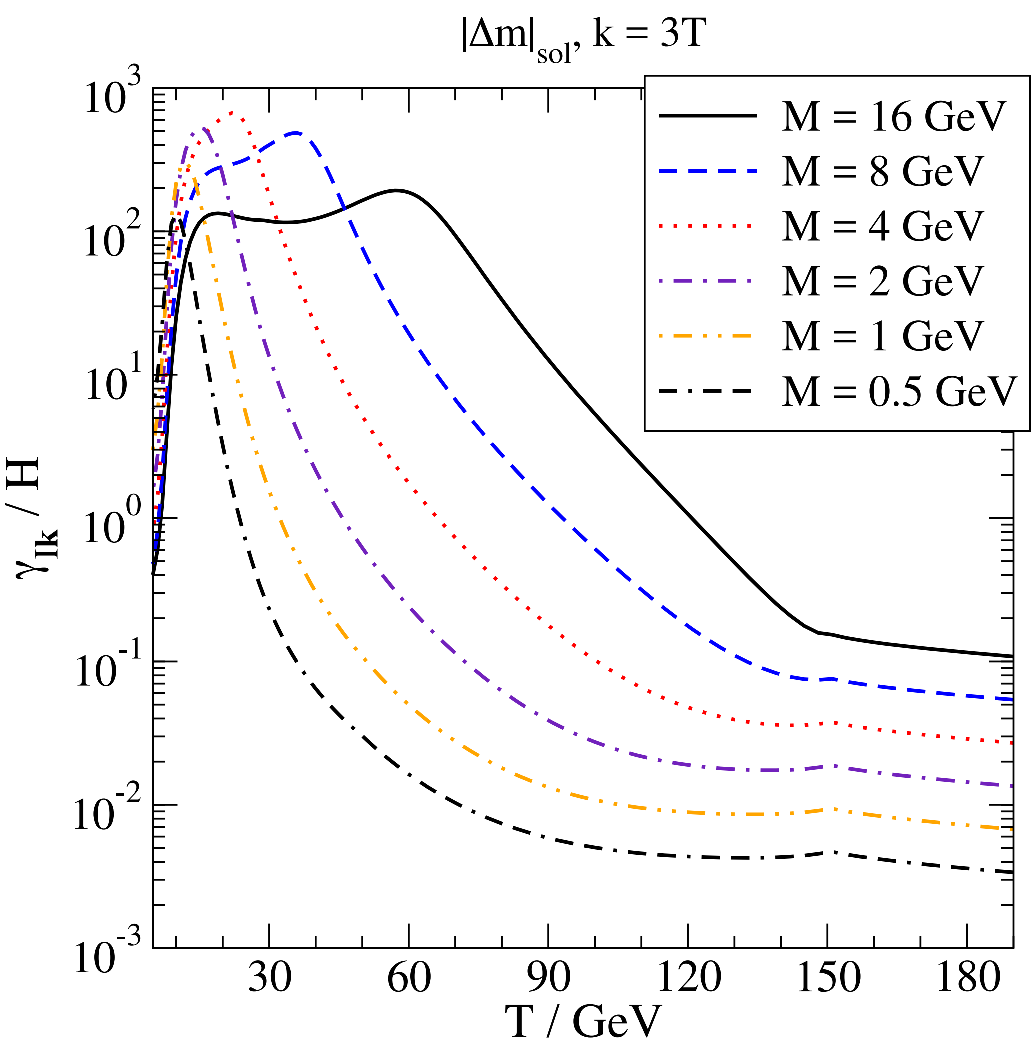
$$\gamma_{I\mathbf{k}} = \sum_a \frac{|h_{Ia}|^2 \mathrm{Tr}[\not{K} \rho_a(K)]}{E_I} + \mathcal{O}(h^4)$$

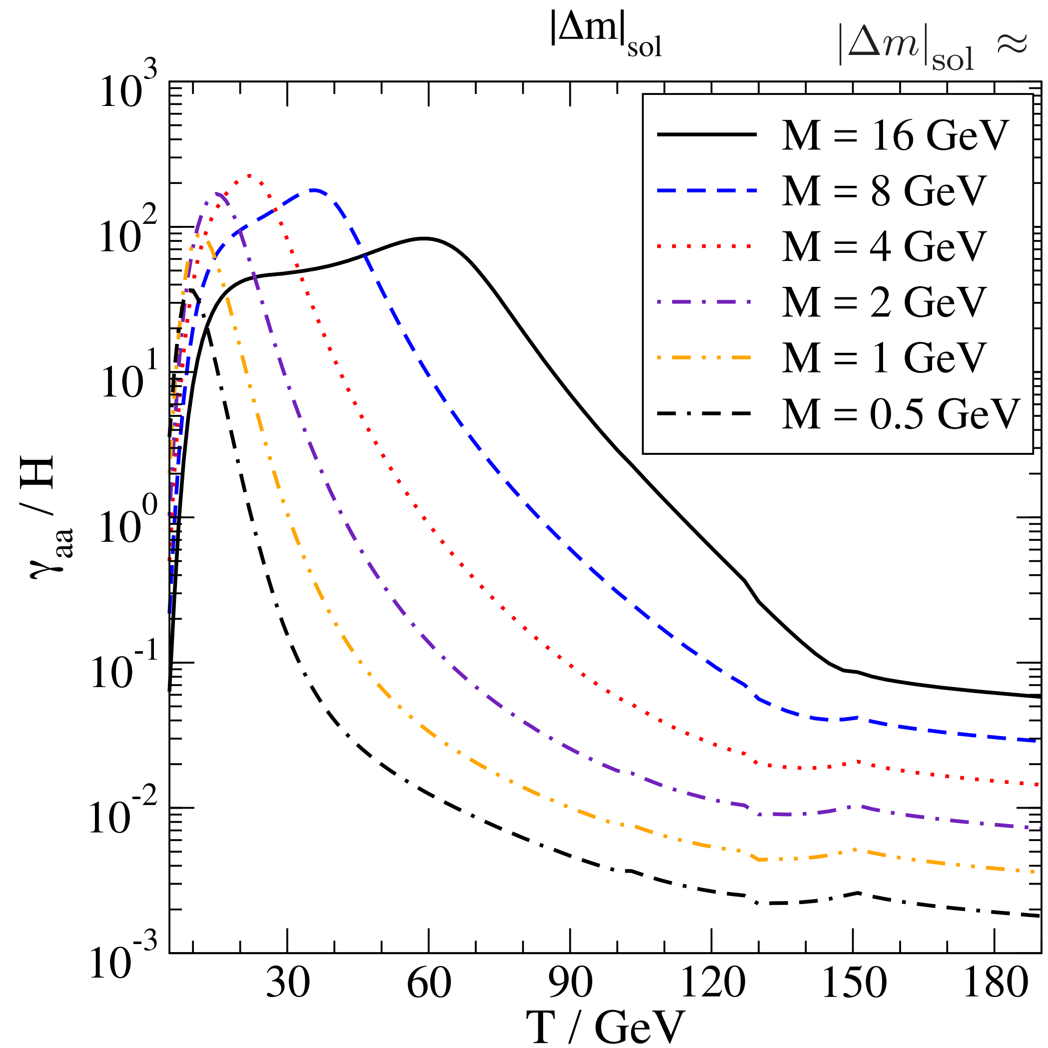
$$\dot{n}_a = -\gamma_{ab} n_b + \mathcal{O}[n_a(n_{\mathrm{F}} - f_{I\mathbf{k}}), n_a^3] \qquad \Xi_{ab} = \partial n_a / \partial \mu_b|_{\mu_b = 0}$$

$$\gamma_{ab} = -\sum_I \int \frac{d^3k}{(2\pi)^3} \frac{2n_{\mathrm{F}}'(E_I)|h_{Ia}|^2 \mathrm{Tr}[\not{K}\rho_a(K)]}{E_I} \Xi_{ab}^{-1} + \mathcal{O}(h^4)$$

- Fix the RHNs Yukawa couplings in a simple seesaw scenario with hierarchical neutrinos, with only one Yukawa coupling contributing to a given mass difference $|\Delta m| = |h_{Ia}|^2 v^2 / (2M)$

Cosmological implications

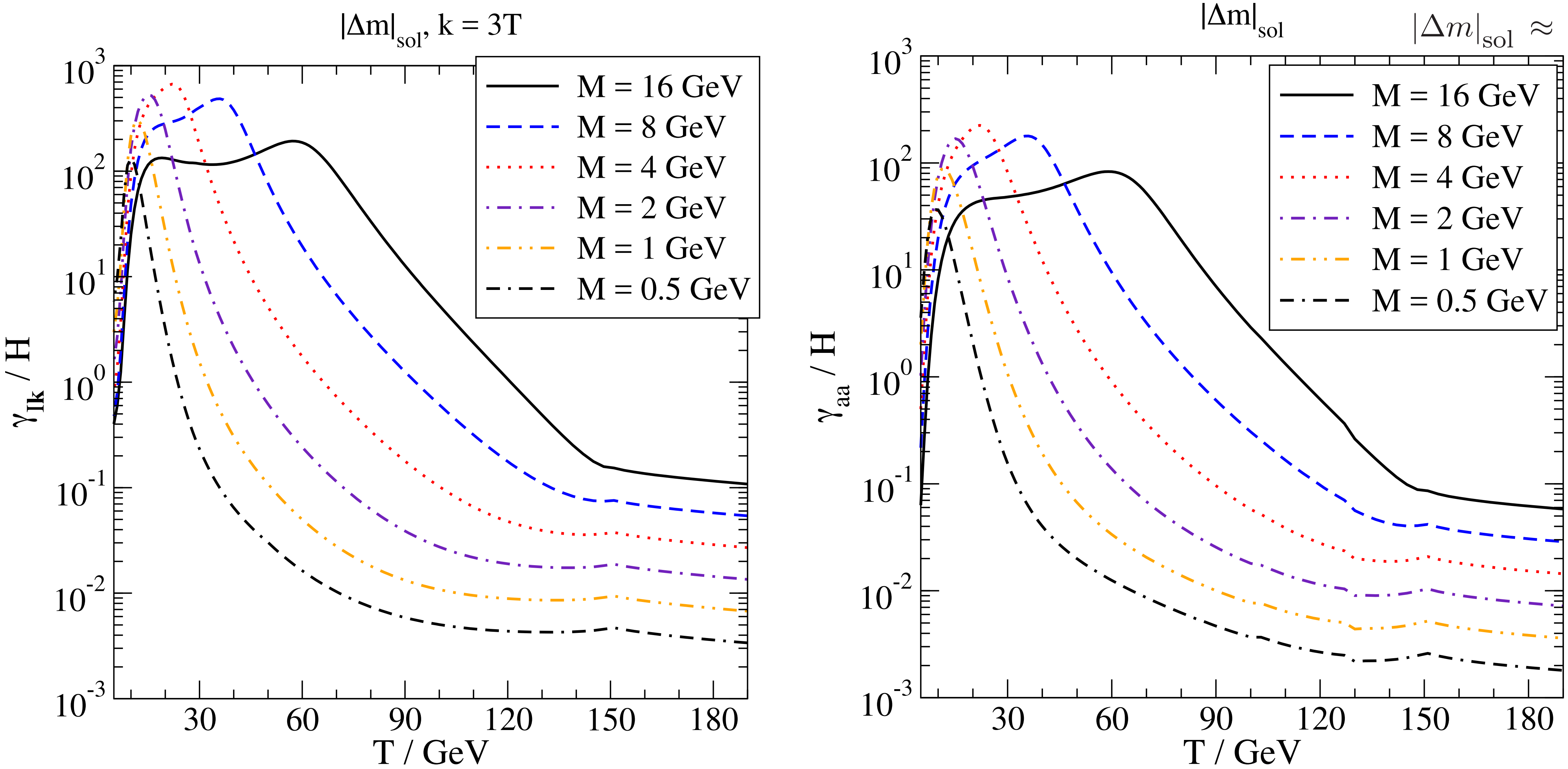




$|\Delta m|_{\rm sol} \approx 8.7 \times 10^{-3}\,{\rm eV}$

- Peaks driven by indirect processes
- Leptogenesis possible because no equilibrium at T≳130 GeV. Resonant generation of keV scale RHNs hindered by washout at T≲30 GeV. Fine-tuned windows still possible

Cosmological implications



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- Helicity asymmetries might be important to this end, since the peak is driven by large helicity-conserving indirect processes, whereas helicity-flipping ones remain small there Eijima Shaposhnikov **1703.06085**

Summary

- I have presented evolution equations for the slow modes, keeping track consistently of flavour and helicity effects, as well as full back-reaction, at order h^2
- The rates entering these evolution equations are known at LO in the SM for small M over a wide temperature range
- In the broken phase **these rates peak at T~10-30 GeV**, due to the **efficient, resonance-like indirect processes**, with consequences for leptogenesis and keV scale dark matter
- Spectra and code available for download at
http://www.laine.itp.unibe.ch/production-{low,mid,high}T/
http://www.laine.itp.unibe.ch/dmpheno/ (resonant prod.)

Backup

Resonant sterile neutrino production

- Resonant production of right-handed neutrinos: a non-zero lepton asymmetry (left) creates a resonance that efficiently converts it into right-handed neutrino (DM) abundance (right) JG Laine **JHEP1511** (2015)

$m_N = 7.1$ keV