



# ARS Leptogenesis – Closed Time Path Approach – Phenomenology

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# Outline

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1.  $CP$  Violation in Boltzmann Equations for Leptogenesis
2. Closed-Time-Path Approach
3. Akhmedov Rubakov Smirnov (ARS) Leptogenesis
4. Conclusions

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1. *CP* Violation in Boltzmann Equations for Leptogenesis
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# *CP* Violation & Quantum Interference

Brief Reminder of the Basics

Squared amplitude for some *CP* violating process:

$$\left| a_1 \cdots a_m |\mathcal{A}_a| e^{i\varphi_a} + b_1 \cdots b_n |\mathcal{A}_b| e^{i\varphi_b} \right|_{\text{x-term}}^2 \supset (a_1 \cdots a_m b_1^* \cdots b_m^* e^{i(\varphi_a - \varphi_b)} + \text{c.c.}) |\mathcal{A}_a \mathcal{A}_b|$$

$a_i, b_i$ : coupling constants

$\mathcal{A}_{a,b}$ : amplitudes stripped of coupling constants

$\arg(a_1 \cdots a_m b_1^* \cdots b_m^*)$ : “weak” phase

$\varphi_{a,b}$ : (“strong”) phases of  $\mathcal{A}_{a,b}$

Rate for *CP* conjugate process:

$$\left| a_1^* \cdots a_m^* |\mathcal{A}_a| e^{i\varphi_a} + b_1^* \cdots b_n^* |\mathcal{A}_b| e^{i\varphi_b} \right|_{\text{x-term}}^2 \supset (a_1^* \cdots a_m^* b_1 \cdots b_m e^{i(\varphi_a - \varphi_b)} + \text{c.c.}) |\mathcal{A}_a \mathcal{A}_b|$$

Difference:  $(a_1 \cdots a_m b_1^* \cdots b_m^* - a_1^* \cdots a_m^* b_1 \cdots b_m) (e^{i(\varphi_a - \varphi_b)} - e^{-i(\varphi_a - \varphi_b)})$

$$= 4 \text{Im}[e^{i(\varphi_a - \varphi_b)}] \text{Im}[a_1^* \cdots a_m^* b_1 \cdots b_m] |\mathcal{A}_a \mathcal{A}_b|$$

# CP Violation & Quantum Interference

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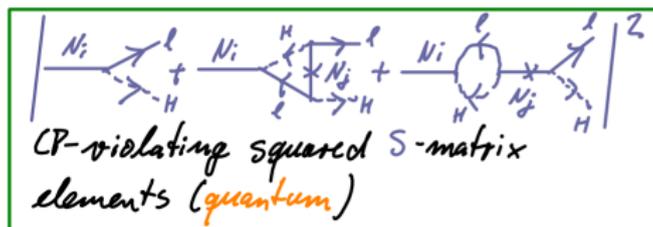
$\text{Im}[e^{i\varphi_{a,b}}]$  comes from *coherent superposition of different quantum states*. For calculable problems, these often correspond

- to **on-shell cuts** in a Feynman diagram (typical view on “standard” leptogenesis)
- or to flavour **mixing and oscillations** (typical view on ARS leptogenesis).

There are parametric regimes where calculations based on the cuts in wave-function diagrams lead to the same answer as those based on the time evolution of mixing and oscillating states.

# Standard Approach to "Standard" Leptogenesis

$$\mathcal{L}_{\text{SM}} \rightarrow \mathcal{L}_{\text{SM}} + \frac{1}{2} \bar{N}_i^c (i\not{\partial} - M_{ij}) N_j - Y_{ia}^* \bar{\ell}_a \phi^\dagger N_i - Y_{ia} \bar{N}_i \phi \ell_a; a = e, \mu, \tau; i = 1, 2, \dots$$



$$L[\psi] = e[\psi]$$

Boltzmann equation (classical)

decay asymmetry:

$$\epsilon_{N_i} = \frac{\Gamma_{N_i \rightarrow \ell H} - \Gamma_{N_i \rightarrow \bar{\ell} H^*}}{\Gamma_{N_i \rightarrow \ell H} + \Gamma_{N_i \rightarrow \bar{\ell} H^*}}$$

Lepton Asymmetry

[Fukugita, Yanagida (1986);  
Covi, Roulet, Vissani (1996)]

"Wave Function" & "Vertex" Contributions:

$$\epsilon_{N_i}^{\text{wf}} = \frac{1}{8\pi} \sum_{j \neq i} \frac{M_i M_j}{M_i^2 - M_j^2} \frac{\text{Im}[(YY^\dagger)_{ij}^2]}{(Y^\dagger Y)_{ii}} \rightarrow \text{resonant enhancement for } M_i \rightarrow M_j$$

$$\epsilon_{N_i}^{\text{vertex}} = \frac{1}{8\pi} \sum_{j \neq i} \frac{M_j}{M_i} \left[ 1 - \left( 1 + \frac{M_j^2}{M_i^2} \right) \log \left( 1 + \frac{M_j^2}{M_i^2} \right) \right] \frac{\text{Im}[(YY^\dagger)_{ij}^2]}{(Y^\dagger Y)_{ii}}$$

- Understand situation when  $|M_i^2 - M_j^2| \gg M_{i,j} \Gamma_{N_{i,j}}$  does *not* hold.

# Real Intermediate State (RIS) Problem

- Interference of tree & loop amplitudes  $\rightarrow CP$  violation.

$$\left| \text{Tree} + \text{Loop} + \text{Box} \right|^2 \quad (*)$$

- $CP$  violating contributions (“strong phase”) from discontinuities  $\rightarrow$  loop momenta where **cut** particles are on shell.
- Is  an extra process or is it already accounted for by  and  ?
- Including (\*) only  $\rightarrow CP$  asymmetry is already generated in equilibrium.  $\Downarrow CPT \Downarrow$  theorem.

# (Inverse) Decays & $CP$ Asymmetry

- Consider the squared matrix elements,  $\varepsilon$  being the decay asymmetry.

$$\begin{array}{cc}
 \begin{array}{c} \text{---} \swarrow \text{---} \\ \text{---} \searrow \text{---} \\ \text{---} \end{array} & |\mathcal{M}_{N \rightarrow l\phi}|^2 \sim 1 + \varepsilon & \begin{array}{c} \text{---} \swarrow \text{---} \\ \text{---} \searrow \text{---} \\ \text{---} \end{array} & |\mathcal{M}_{N \rightarrow \bar{l}\phi^*}|^2 \sim 1 - \varepsilon \\
 \begin{array}{c} \text{---} \swarrow \text{---} \\ \text{---} \searrow \text{---} \\ \text{---} \end{array} & \xrightarrow{\underline{CPT}} & \begin{array}{c} \text{---} \swarrow \text{---} \\ \text{---} \searrow \text{---} \\ \text{---} \end{array} & \xrightarrow{\underline{CPT}} & \begin{array}{c} \text{---} \swarrow \text{---} \\ \text{---} \searrow \text{---} \\ \text{---} \end{array} \\
 \begin{array}{c} \text{---} \swarrow \text{---} \\ \text{---} \searrow \text{---} \\ \text{---} \end{array} & |\mathcal{M}_{\bar{l}\phi^* \rightarrow N}|^2 \sim 1 + \varepsilon & \begin{array}{c} \text{---} \swarrow \text{---} \\ \text{---} \searrow \text{---} \\ \text{---} \end{array} & |\mathcal{M}_{l\phi \rightarrow N}|^2 \sim 1 - \varepsilon
 \end{array}$$

- Naive multiplication\* suggests that an asymmetry is generated already in equilibrium:  $\Gamma_{\bar{l}\phi^* \rightarrow l\phi} \sim 1 + 2\varepsilon$ ,  $\Gamma_{l\phi \rightarrow \bar{l}\phi^*} \sim 1 - 2\varepsilon$

- Ad hoc fix: Subtract real intermediate states (RIS) from [Kolb, Wolfram (1980)].



Fix of the approach by *a posteriori* imposing  $CPT$  theorem

## Better Way Out

- Compute the real time (time dependent perturbation theory), non-equilibrium (statistical physics) evolution of the quantum field theory states of interest.

\*Do not try this at home: The unstable  $N$  are not asymptotic states of a unitary  $S$  matrix  $\rightarrow$  conflict with the  $CPT$  theorem.

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# Closed-Time-Path Approach

- In-in generating functional (in contrast to “in-out” for  $S$  matrix elements): [Schwinger (1961); Keldysh (1965); Calzetta & Hu (1988)]

$$\begin{aligned} Z[J_+, J_-] &= \int \mathcal{D}\phi(\tau) \mathcal{D}\phi_{\text{in}}^- \mathcal{D}\phi_{\text{in}}^+ \langle \phi_{\text{in}}^- | \phi(\tau) \rangle \langle \phi(\tau) | \phi_{\text{in}}^+ \rangle \langle \phi_{\text{in}}^- | \rho | \phi_{\text{in}}^+ \rangle \\ &= \int \mathcal{D}\phi^- \mathcal{D}\phi^+ e^{i \int d^4x \{ \mathcal{L}(\phi^+) - \mathcal{L}(\phi^-) + J_+ \phi^+ - J_- \phi^- \}} \end{aligned}$$

- The Closed Time Path:



Aim: Calculate  $\langle \text{in} | \mathcal{O}(t) | \text{in} \rangle$

- Path-ordered Green functions:

$$i\Delta^{ab}(u, v) = -\frac{\delta^2}{\delta J_a(u) \delta J_b(v)} \log Z[J_+, J_-] \Big|_{J_{\pm}=0} = i \langle \mathcal{C}[\phi^a(u) \phi^b(v)] \rangle$$

$$\text{e.g. } j^\mu(x) = \text{tr}[\gamma^\mu \langle \mathcal{C}[\psi^-(x_1) \bar{\psi}^+(x_2)] \rangle]_{x_1=x_2=x}$$

## Wigner Transformation of Two-Point Functions (Green Function or Self Energy)

$$A(k, x) = \int d^4r e^{ik \cdot r} A(x + r/2, x - r/2) \rightarrow \sim \text{distribution function}$$

$x$ : average coordinate – macroscopic evolution

$r \rightarrow k$ : relative coordinate – microscopic (quantum)

## Path Ordered Green Functions @ Tree Level

- Four propagators (two of which are linearly independent):

$$\left. \begin{aligned} i\Delta^<(u, v) &= i\Delta^{+-}(u, v) = \langle \phi(v)\phi(u) \rangle \\ i\Delta^>(u, v) &= i\Delta^{-+}(u, v) = \langle \phi(u)\phi(v) \rangle \end{aligned} \right\} \text{Wightman functions}$$
$$i\Delta^T(u, v) = i\Delta^{++}(u, v) = \langle T[\phi(u)\phi(v)] \rangle \quad \text{Feynman propagator}$$
$$i\Delta^{\bar{T}}(u, v) = i\Delta^{--}(u, v) = \langle \bar{T}[\phi(u)\phi(v)] \rangle \quad \text{Dyson propagator}$$

- Perturbation theory can be formulated in terms of tree-level Wigner-space propagators:

$$i\Delta^<(p, t) = 2\pi\delta(p^2 + m^2) [\vartheta(p^0)f(\mathbf{p}, t) + \vartheta(-p^0)(1 + \bar{f}(-\mathbf{p}, t))]$$

$$i\Delta^>(p, t) = 2\pi\delta(p^2 + m^2) [\vartheta(p^0)(1 + f(\mathbf{p}, t)) + \vartheta(-p^0)\bar{f}(-\mathbf{p}, t)]$$

$$i\Delta^T(p, t) = \frac{i}{p^2 - m^2 + i\epsilon} + 2\pi\delta(p^2 + m^2) [\vartheta(p^0)f(\mathbf{p}, t) + \vartheta(-p^0)\bar{f}(-\mathbf{p}, t)]$$

$$i\Delta^{\bar{T}}(p, t) = \frac{-i}{p^2 - m^2 - i\epsilon} + 2\pi\delta(p^2 + m^2) [\vartheta(p^0)f(\mathbf{p}, t) + \vartheta(-p^0)\bar{f}(-\mathbf{p}, t)]$$

- $f(\mathbf{p}, t)$ ,  $\bar{f}(\mathbf{p}, t)$ : Particle and antiparticle distribution functions. Carry flavour indices (relevant for leptogenesis: sterile & active flavour).

# Schwinger-Dyson & Kadanoff-Baym Equations

## Feynman Rules

- Vertices either + or -.
- Connect vertices  $a = \pm$  and  $b = \pm$  with  $i\Delta^{ab}$ .
- Factor -1 for each - vertex.

- Schwinger-Dyson equations  $\rightarrow$

- These describe in principle the full time evolution. However, truncations, e.g. perturbation theory, are needed.

- The  $\langle, \rangle \equiv +-, -+$  parts of the Schwinger-Dyson equations are the celebrated Kadanoff-Baym equation:

$$(-\partial^2 - m^2)\Delta^{\langle, \rangle} - \Pi^H \odot \Delta^{\langle, \rangle} - \Pi^{\langle, \rangle} \Delta^H = \underbrace{\frac{1}{2}(\Pi^{\rangle} \odot \Delta^{\langle} - \Pi^{\langle} \odot \Delta^{\rangle})}_{\text{collision term}}$$

- Remaining linear combination gives pole-mass equation:

$$(-\partial^2 - m^2)i\Delta^{R,A} - \Pi^{R,A} \odot i\Delta^{R,A} = i\delta^4, \quad R, A: \text{retarded, advanced,}$$

$$\Pi^H, \Delta^H = \text{Re}[\Pi^R, \Delta^R]$$

- First principle derivation of Boltzmann-like *kinetic equations*.

[Keldysh (1965); Calzetta & Hu (1988)]

$$i\Delta^{ab} = i\Delta^{(a)b} + cd i\Delta^{(c)a} \odot \Pi^{cd} \odot \Delta^{db}$$

$$\text{====} = \text{---} + \text{---} \textcircled{\Pi} \text{---}$$

$$A(x, w) \odot B(w, y) = \int d^4w A(x, w) B(w, y)$$

# Leptogenesis in the CTP Approach

## ■ Schwinger-Dyson equations relevant for leptogenesis:

[Buchmüller, Fredenhagen (2000); De Simone, Riotto (2007); Garny, Hohenegger, Kartatvtsev, Lindner (2009-); Beneke, BG, Herranen, Schwaller (2010-); Anisimov, Buchmüller, Drewes, Mendizabal (2010-)]

$$\frac{d}{dt} \int \frac{dk^0}{2\pi} \text{tr} [\mathcal{P}^0 \text{---}] = \int \frac{dk^0}{2\pi} \text{tr} \left[ \text{---} \overset{N}{\text{---}} \text{---} + \text{---} \text{---} \right]_{\substack{>< \\ <>}}$$

$= \int \frac{dk^0}{2\pi} \text{tr} \left[ \text{---} \overset{N}{\text{---}} \text{---} + \text{---} \text{---} \right]_{\substack{>< \\ <>}}$

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$$\frac{d}{dt} \int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} [\mathcal{P}^0 \text{---}] = \int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} \left[ \text{---} \overset{N}{\text{---}} \text{---} \right]_{\substack{>< \\ <>}} + \dots$$

$= \int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} \left[ \text{---} \overset{N}{\text{---}} \text{---} \right]_{\substack{>< \\ <>}} + \dots$

## ■ Truncation that yields leading asymmetry in non-degenerate regime

$$M_i \Gamma_i, M_j \Gamma_j \ll |M_i^2 - M_j^2|:$$

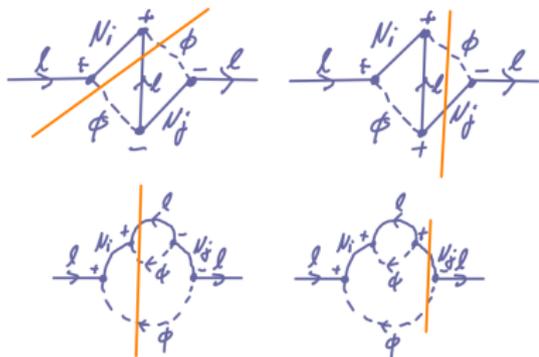
$$\frac{d}{dt} \int \frac{dk^0}{2\pi} \text{tr} [\mathcal{P}^0 \text{---}] = \int \frac{dk^0}{2\pi} \text{tr} \left[ \text{---} \overset{N}{\text{---}} \text{---} + \text{---} \text{---} + \text{---} \text{---} \right]_{\substack{>< \\ <>}}$$

$$\frac{d}{dt} \int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} [\mathcal{P}^0 \text{---}] = \int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} \left[ \text{---} \overset{N}{\text{---}} \text{---} \right]_{\substack{>< \\ <>}}$$

- Non-minimal truncations → e.g. systematic inclusion of thermal corrections.

# Unitarity Restored (without RIS)

- CTP approach readily yields *inclusive* rates for the creation of the charge asymmetry.
- No need to separately remove unwanted/unphysical contribution *a posteriori*.



- Loop insertions in propagator for  $N$  must be resummed unless (typical  $|\mathbf{p}| \sim T$ )

$$|M_i^2 - M_j^2|/\sqrt{\mathbf{p}^2 + M_i^2} \gg \Gamma_i \sim \begin{cases} Y^2/(16\pi)M_i \text{ for } M_i \gg T \\ Y^2 g^2 \log g T \text{ for } M_i \ll T \end{cases} .$$

# Leptogenesis from Oscillations/Resonant Limit

$$\frac{d}{dt} \int \frac{dk^0}{2\pi} \text{tr} [\gamma^0 \underline{\underline{}}] = \int \frac{dk^0}{2\pi} \text{tr} \left[ \text{diagram 1} + \text{diagram 2} \right] \gg \ll$$

$= \int d_e(\vec{k}^0) - \int \bar{d}_e(\vec{k}^0) + \dots$

$$\frac{d}{dt} \int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} [\gamma^0 \underline{\underline{}}] = \int \frac{dk^0}{2\pi} \text{sign} k^0 \text{tr} \left[ \text{diagram 3} \right] \gg \ll$$

$= \int d_\nu(\vec{k}^0)$

Evolution for Matrix-Valued RHN Distributions  $\delta f_{Nh}$  (i.e. deviation from equilibrium form  $f_N^{\text{eq}}$ )

$$\delta f'_{Nh} + \frac{a^2(\eta)}{2k^0} i[M^2, \delta f_{Nh}] + f_N^{\text{eq}'} = -2 \left\{ \text{Re}[Y^* Y^t] \frac{k \cdot \hat{\Sigma}_N^A}{k^0} - i h \text{Im}[Y^* Y^t] \frac{\tilde{k} \cdot \hat{\Sigma}_N^A}{k^0}, \delta f_{Nh} \right\}$$

$\hat{\Sigma}_N^A$ : spectral (cut part) self energy,  $a(\eta), \eta$ : scale factor and conformal time,  $\iota \equiv d/d\eta$ ,

$h$ : helicity,  $\tilde{k} = (|\mathbf{k}|, |k^0| \mathbf{k}/|\mathbf{k}|)$

- $i[M^2, \delta f_{Nh}]_{ij} = i(M_i^2 - M_j^2) \delta f_{Nhij}$  for diagonal  $M^2 \rightarrow$  RHN “flavour” oscillations
- Off-diagonal entries of  $\delta f_{Nhij}$  correspond to interference between the different  $N_i$  that give rise to  $CP$  violation (“strong phases”)
- Note: If  $\delta f'_{Nh}$  and off diagonal elements of the collision term can be neglected, recover result from “standard resonant leptogenesis”  $\rightarrow$  next slide
- Evolution equations are well behaved for  $\Delta M^2 \rightarrow 0$ . Solutions  $\delta f_{Nh}$  enter into the resummed RHN propagators. [BG, Herranen (2010); Iso, Shimada (2014); BG, Gautier, Klaric (2014)]

# Regulator for Resonant Leptogenesis in Strong Washout

Wave-function contribution:

$$\epsilon_{Ni}^{\text{wf}} = \frac{1}{8\pi} \sum_{j \neq i} \frac{M_i M_j (M_i^2 + M_j^2)}{(M_i^2 - M_j^2)^2 + R} \frac{\text{Im}[(YY^\dagger)_{ij}^2]}{(Y^\dagger Y)_{ii}}$$

- In the degenerate limit, this will dominate over the vertex contribution.
- Proposed forms for regulator  $R$  ( $\bar{M} = (M_i + M_j)/2$ ):

- $R = \frac{\bar{M}^4}{64\pi^2} (YY^\dagger)_{jj}^2 = \bar{M}^2 \Gamma_j^2$  [Pilaftsis (1997); Pilaftsis, Underwood (2003)]
- $R = \frac{\bar{M}^4}{64\pi^2} ([YY^\dagger]_{ii} - [YY^\dagger]_{jj})^2$  [Anisimov, Broncano, Plümacher (2005)]
- $R = \frac{\bar{M}^4}{64\pi^2} ([YY^\dagger]_{ii} + [YY^\dagger]_{jj})^2$  [Garny, Hohenegger, Kartavtsev (2011)]
- $R = \frac{M^4}{64\pi^2} \frac{([YY^\dagger]_{11} + [YY^\dagger]_{22})^2}{[YY^\dagger]_{11}[YY^\dagger]_{22}} ((\text{Im}[YY^\dagger]_{12})^2 + \det YY^\dagger)$

Obtained by algebraic solution to oscillation equation, neglecting  $\delta f'_N$

[BG, Gautier, Klaric (2014); Iso, Shimada (2014)]

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# Leptogenesis from Oscillations

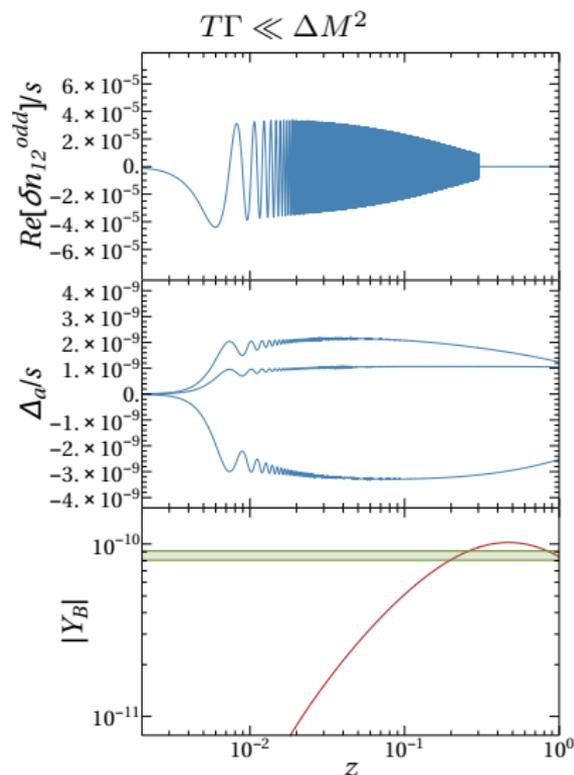
- RHNs perform first oscillation at temperature  $T_{\text{osc}} \sim (|M_i^2 - M_j^2| m_{\text{Pl}})^{1/3}$ 
  - Off-diagonal correlations for the RHNs yield “strong” phase
  - Sizeable asymmetries generated around  $T_{\text{osc}}$  (see plots on next slide)
- In “standard” leptogenesis, “early asymmetries” are washed out by the time  $T \sim M$ . Only asymmetry produced around  $T \sim M$  survives (strong washout).

## Loopholes to Preserve Early Asymmetries

- GeV-scale RHNs ( $\rightarrow T_{\text{osc}} \sim 10^5$  GeV) typically do not equilibrate prior to the electroweak phase transition where  $B$  settles to final value (sphaleron freezeout).  
[Akhmedov, Rubakov, Smirnov (1998); Asaka, Shaposhnikov (2005)].
  - Early asymmetry preserved until baryon number  $B$  freezes in.
- One individual active flavour (typically  $(\nu_e, e_L)$ ) is only weakly washed out, such that early asymmetries survive [BG (2014)].

# Leptogenesis from Oscillation – Dynamics

Generation of the BAU



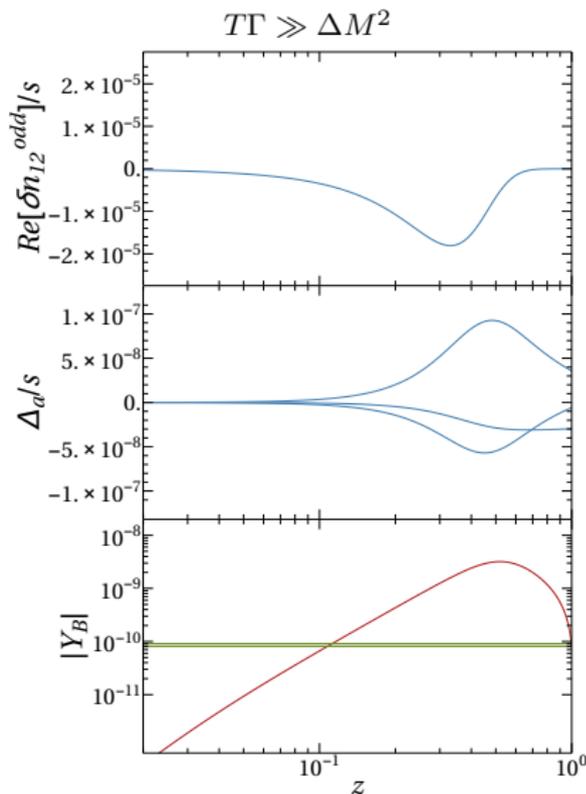
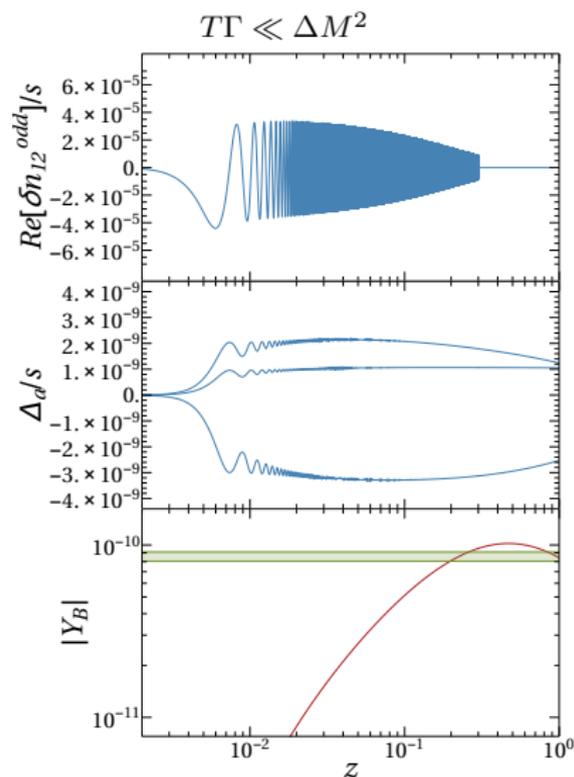
- RHNs perform first oscillation at  $T_{\text{osc}} \sim (|M_i^2 - M_j^2| m_{\text{Pl}})^{1/3}$ .
- Off-diagonal correlations lead to  $CP$  violating source for lepton flavour asymmetries (*purely flavoured*).
- Contributions from subsequent oscillations average out.
- Transfer of asymmetries into helicity asymmetries of RHNs leads to  $Y_L \neq 0$ . Large active-sterile mixing possible if one active flavour is more weakly washed out than the other two.
- Asymmetry frozen in at  $T_{\text{EW}}$ , where sphalerons are quenched by the developing Higgs vev.
- $\delta n = \int d^3k / (2\pi)^3 \delta f_N \rightarrow$  **momentum averaging**

$z = T_{\text{EW}}/T$ ;  $\Delta_a$ : lepton asymmetry in flavour  $a = e, \mu, \tau$ ;  $\delta n$ : number density of RHNs;  
 $Y_B$ : entropy-normalized baryon asymmetry.

[Marco Drewes, BG, Dario Gueter, Juraj Klarić, 1606.06690]

# Leptogenesis from Oscillation – Dynamics

Oscillatory vs Overdamped Regime



$z = T_{\text{EW}}/T$ ;  $\Delta_a$ : lepton asymmetry in flavour  $a = e, \mu, \tau$ ;  $\delta n$ : number density of RHNs;  
 $Y_B$ : entropy-normalized baryon asymmetry.

# Leptogenesis from Oscillation of GeV-scale RHNs

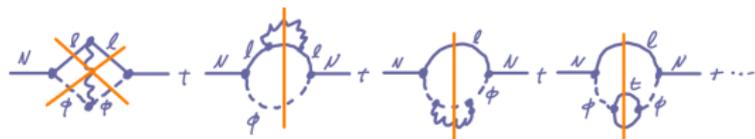
Why it works at mass scales much smaller than for standard leptogenesis

- Source term for flavoured early asymmetries in oscillatory regime around  $T_{\text{osc}}$ : [Drewes, BG (2012)]

$$S_{ab} = \sum_{\substack{c,i,j \\ i \neq j}} \frac{32i}{M_{ii}^2 - M_{jj}^2} \int \frac{d^3k}{(2\pi)^3 2\sqrt{k^2 + M_{ii}^2}}$$

$$\times \left\{ (Y_{ai}^\dagger Y_{ic} Y_{cj}^\dagger Y_{jb}) \left[ (M_{ii}^2 + 2k^2) (\hat{\Sigma}_N^{A02} + \hat{\Sigma}_N^{Ai2}) - 4|\mathbf{k}| \sqrt{k^2 + M_{ii}^2} \hat{\Sigma}_N^{A0} \hat{k}^i \hat{\Sigma}_N^{Ai} \right] \right.$$

$$\left. + (Y_{ai}^\dagger Y_{ic}^* Y_{cj}^\dagger Y_{jb}) M_{ii} M_{jj} \hat{\Sigma}_{N\mu}^A \hat{\Sigma}_N^{A\mu} \right\} \times \delta f_{N\text{hii}}(\mathbf{k}).$$

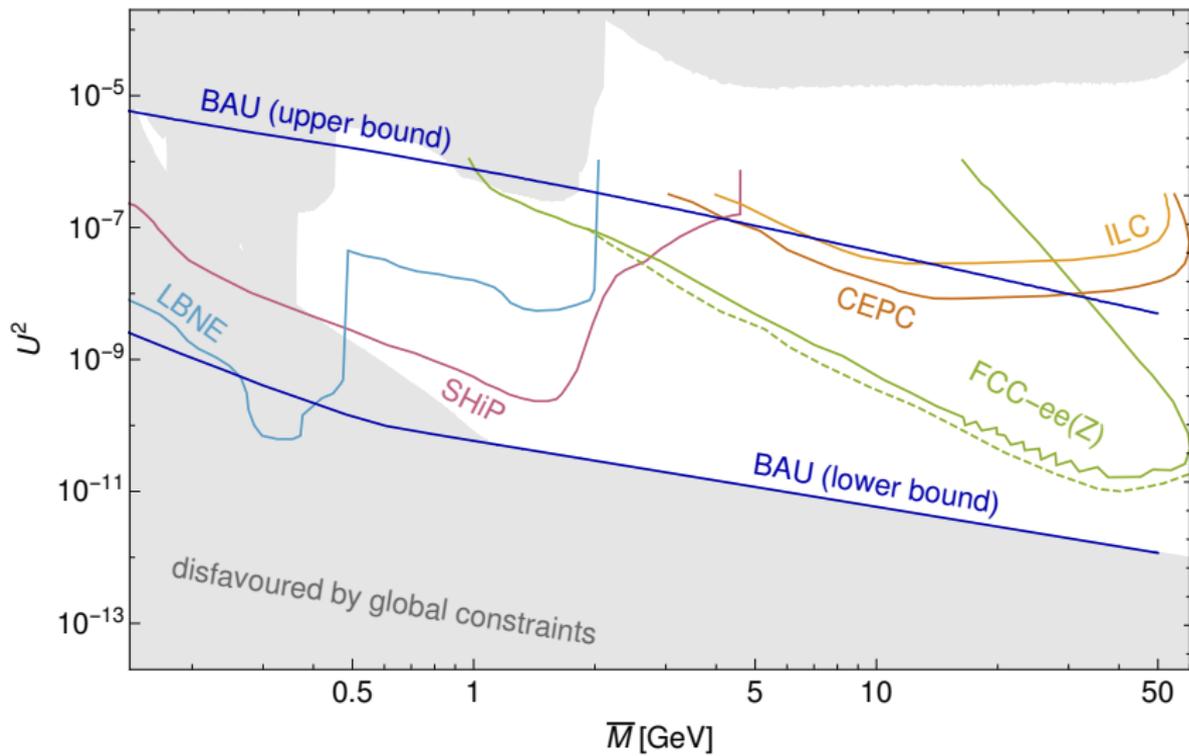
■  $\hat{\Sigma}_N^A =$   [Besak, Bödeker (2012); BG, Glowna, Schwaller (2013)]

RHNs relativistic  $\rightarrow \hat{\Sigma}_N^A$  dominated by thermal effects

- Lepton number violating contribution  $\sim M^2/\Delta M^2$  (Majorana mass insertion) requires  $\Delta M^2/M^2 \rightarrow 0$  for resonant enhancement.
- Lepton flavour violating (lepton number conserving) contribution  $\sim T^2/\Delta M^2 \rightarrow$  large enhancement for  $\Delta M^2 \ll T^2 \rightarrow$  no/less pronounced mass degeneracy needed.
- Leptogenesis is viable with  $n_N = 3$  non-degenerate (in mass) RHNs of the GeV scale [Drewes, BG (2012)] and for masses  $\gtrsim 5 \times 10^3$  GeV [BG (2014)].

# Experimental Prospects

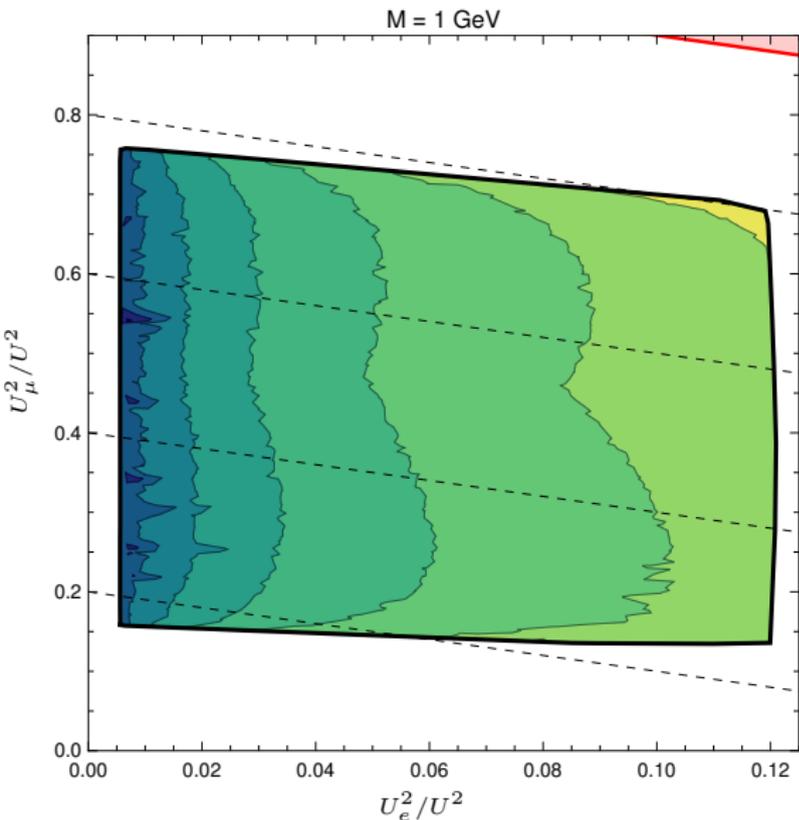
Normal Hierarchy



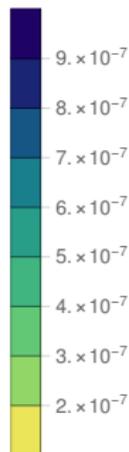
Active-sterile mixing:  $U_{ai} \approx \theta_{ai} = vY_{ai}^\dagger/M_i$ ;  $U^2 = \text{tr}\theta^\dagger\theta$ ;  $n_N = 2$

# GeV-Scale Leptogenesis & Approximate LNC

Normal Hierarchy



$U^2$



- Maximal  $U^2 = \sum_a U_a^2$  for viable leptogenesis shows particular flavour patterns
- Larger  $U^2$  require smaller  $U_e^2/U^2$  in order for the asymmetry stored in  $\ell_e$  to survive washout prior to sphaleron freezeout

[Drewes, BG, Gueter, Klarić,  
1609.09069]

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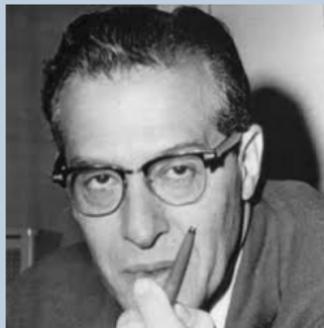
1. *CP* Violation in Boltzmann Equations for Leptogenesis
2. Closed-Time-Path Approach
3. Akhmedov Rubakov Smirnov (ARS) Leptogenesis
4. Conclusions

# Conclusions

## Closed-Time-Path Formalism:

- A *diagrammatic* approach that facilitates to developing a global picture for

*Like the silicon chips of more recent years, the Feynman diagram was bringing computation to the masses.*



- ARS ph
- At the level of approximations used (and presumably required in foreseeable future), CTP approach and density-operator based formalisms should be equivalent.
  - Current status:  $\mathcal{O}(1)$  accuracy. To reach leading order accuracy, need to drop momentum-averaging, ideally in a numerically economic way.
  - In case we are lucky enough to discover RHNs soon, ARS leptogenesis predicts particular flavour patterns that allow to test this mechanism (so far discussions for  $n_N = 2$ ).

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# Conclusions

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## Closed-Time-Path Formalism:

- A *diagrammatic* approach that facilitates to developing a global picture for leptogenesis (*i.e.* in type I seesaw for ultraheavy and GeV-scale RHNs, with or without mass degeneracy)
- Computes the real-time evolution of a statistical QFT state – no RIS necessary
- Beyond leptogenesis application: Formulation in terms of Green functions (no notion of free state necessary), connection with formulation of electroweak baryogenesis, 2PI resummation readily implemented through SD equations, numerical simulation of statistical QFT systems

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Thank you for your attention!