Flavour and CP symmetries/CP violation

with Talk of Mu-Chun Chen

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Neutrinos institute : the quest for a new physics scale
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Discrete Symmetry predicts neutrino mixing angles

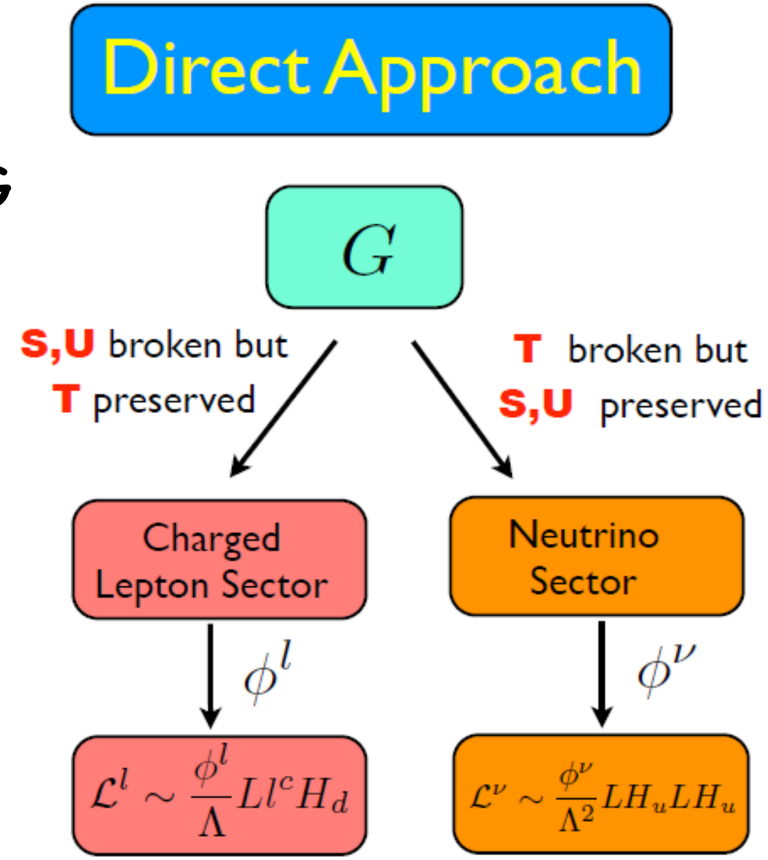
θ_{23} θ_{12} θ_{13}

and CP violating phase δ_{CP}

How to predict the flavour mixing angles based on Flavour Symmetry

Suppose Flavour Symmetry Group G

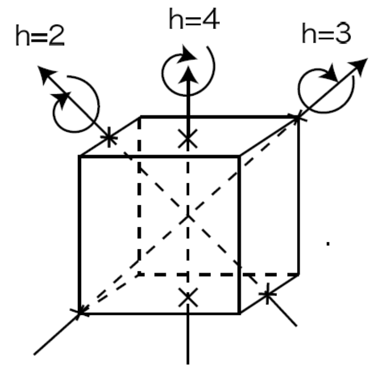
It breaks to different subgroups which are preserved in Neutrino sector and Charged leptons sector, respectively.



S.F.King

arXiv: 1402.4271 King, Merle, Morisi, Simizu, M.T

Simple Examples



Symmetry of a cube

S_4 group

24 elements are generated by S, T and U:

$S^2=T^3=U^2=1$, $ST^3 = (SU)^2 = (TU)^2 = (STU)^4 = 1$

Irreducible representations: 1, 1', 2, 3, 3'

It has subgroups, nine Z_2, four Z_3, three Z_4, four $Z_2 \times Z_2$ (K_4)

Suppose S_4 is spontaneously broken to one of subgroups:

Neutrino sector preserves (1,S,U,SU) (K_4)

Charged lepton sector preserves (1,T,T²) (Z_3)

For 3 and 3'

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}; \quad \omega = e^{2\pi i/3}$$

$$U = \mp \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Then, neutrinos respect S and U
charged leptons respect T, respectively:

$$S^T m^\nu_{LL} S = m^\nu_{LL}, \quad U^T m^\nu_{LL} U = m^\nu_{LL}, \qquad T^\dagger Y_e Y_e^\dagger T = Y_e Y_e^\dagger$$

$$[S, m^\nu_{LL}] = 0, \quad [U, m^\nu_{LL}] = 0, \quad [T, Y_e Y_e^\dagger] = 0$$

Mixing matrices diagonalize mass matrices also diagonalize S,U, and T, respectively !

The charged lepton mass matrix is diagonal because T is diagonal matrix.

$$V_\nu = \begin{pmatrix} 2/\sqrt{6} & /1\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Tri-bimaximal mixing $\theta_{13}=0$

C.S.Lam, PRD98(2008)
arXiv:0809.1185

which digonalizes both S and U.

Independent of mass eigenvalues !

Klein Symmetry can reproduce Tri-bimaximal mixing.

If S_4 is spontaneously broken to another subgroups,
eg. Neutrino sector preserves (1,SU) (Z_2)
Charged lepton sector preserves (1,T,T^2) (Z_3),
Obtained flavour mixing matrix is changed!

$$(SU)^T m^{\nu}_{LL} SU = m^{\nu}_{LL}, \quad T^{\dagger} Y_e Y_e^{\dagger} T = Y_e Y_e^{\dagger}$$

⇓

$$[SU, m^{\nu}_{LL}] = 0, \quad [T, Y_e Y_e^{\dagger}] = 0$$

$$V_{\nu} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -c/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

which digonalizes SU.

Eigenvalues of SU (-1,1,1)

There is a freedom of the rotation between 2nd and 3rd column because a column corresponds to a mass eigenvalue.

$$V_\nu = \begin{pmatrix} 2/\sqrt{6} & c/\sqrt{3} & s/\sqrt{3} \\ -1/\sqrt{6} & c/\sqrt{3} - s/\sqrt{2} & -s/\sqrt{3} - c/\sqrt{2} \\ -1/\sqrt{6} & c/\sqrt{3} + s/\sqrt{2} & -s/\sqrt{3} + c/\sqrt{2} \end{pmatrix}$$

$c = \cos\theta,\ s = \sin\theta$ includes CP phase.

Tri-maximal mixing
TM1

Semi-direct model

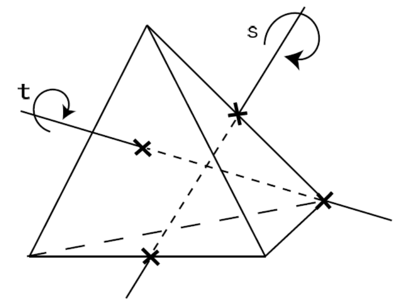
θ is not fixed by the flavor symmetry,

Mixing sum rules

$$\sin^2\theta_{23} = \frac{1}{2}\frac{1}{\cos^2\theta_{13}} \geq \frac{1}{2}\,, \quad \sin^2\theta_{12} \simeq \frac{1}{\sqrt{3}} - \frac{2\sqrt{2}}{3}\sin\theta_{13}\cos\delta_{CP} + \frac{1}{3}\sin^2\theta_{13}\cos 2\delta_{CP}$$

A_4 group

Symmetry of tetrahedron



A_4 has subgroups:

three Z_2, four Z_3, one $Z_2 \times Z_2$ (klein four-group)

Z_2: {1,S}, {1,T²ST}, {1,TST²}

Z_3: {1,T,T²}, {1,ST,T²S}, {1,TS, ST²}, {1,STS,ST²S}

K_4: {1,S,T²ST,TST²}

Suppose A_4 is spontaneously broken to one of subgroups:

Neutrino sector preserves Z_2: {1,S}

Charged lepton sector preserves Z_3: {1,T,T²}

$$S^T m^\nu_{LL} S = m^\nu_{LL}, \quad T^\dagger Y_e Y_e^\dagger T = Y_e Y_e^\dagger$$

$$[S, m^\nu_{LL}] = 0, \quad [T, Y_e Y_e^\dagger] = 0$$

Mixing matrices diagonalise m^ν_{LL}, $Y_e Y_e^\dagger$ also diagonalize S and T, respectively !

Then, we obtain PMNS matrix.

$$V_\nu = \begin{pmatrix} 2c/\sqrt{6} & 1/\sqrt{3} & 2s/\sqrt{6} \\ -c/\sqrt{6}+s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}-c/\sqrt{2} \\ -c/\sqrt{6}-s/\sqrt{2} & 1/\sqrt{3} & -s/\sqrt{6}+c/\sqrt{2} \end{pmatrix}$$

$$c=\cos\theta, \quad s=\sin\theta$$

Tri-maximal mixing : so called TM2

θ is not fixed.

Semi-direct model

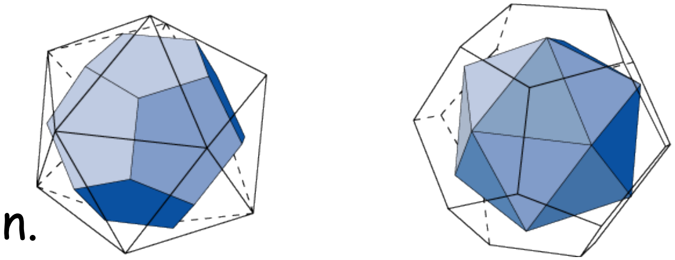
In general, the complex phase is attached.
CP symmetry can predict this phase.

Another Mixing sum rules

$$\sin^2\theta_{12} = \frac{1}{3}\frac{1}{\cos^2\theta_{13}} \geq \frac{1}{3}\,, \qquad \cos\delta_{CP}\tan 2\theta_{23} \simeq \frac{1}{\sqrt{2}\sin\theta_{13}}\left(1-\frac{5}{4}\sin^2\theta_{13}\right)$$

A_5 group

which is isomorphic to the symmetry of regular icosahedron and a regular dodecahedron.



It has subgroups, ten Z_3, six Z_5, five $Z_2 \times Z_2$ (K_4) .

Suppose A_5 is spontaneously broken to one of subgroups:
Neutrino sector preserves S and U (K_4)
Charged lepton sector preserves T (Z_5)

$$S^T m^\nu_{LL} S = m^\nu_{LL}, \quad U^T m^\nu_{LL} U = m^\nu_{LL}, \qquad T^\dagger Y_e Y_e^\dagger T = Y_e Y_e^\dagger$$

$$[S, m^\nu_{LL}] = 0, \quad [U, m^\nu_{LL}] = 0, \quad [T, Y_e Y_e^\dagger] = 0$$

$$S = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -\phi & \frac{1}{\phi} \\ \sqrt{2} & \frac{1}{\phi} & -\phi \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{5}} & 0 \\ 0 & 0 & e^{\frac{8\pi i}{5}} \end{pmatrix} \qquad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

F. Feruglio and Paris, JHEP 1103(2011) 101 **arXiv:1101.0393**

$$U_{GR} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$\theta_{13}=0$

$\tan\theta_{12} = 1/\phi$: $\phi = \frac{1+\sqrt{5}}{2}$ **Golden ratio**

Neutrino mass matrix has μ-τ symmetry.

$$m_\nu = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix}$$ with $z + w = x - \sqrt{2}y$

$\sin^2\theta_{12} = 2/(5+\sqrt{5}) = 0.2763\ldots$

which is rather smaller than the experimental data.

$\sin^2\theta_{12} = 0.306 \pm 0.012$ The model should be modified.

Since simple patterns predict vanishing θ_{13}, larger groups may be used to obtain non-vanishing θ_{13}.

R.de Adelhart Toorop, F.Feruglio, C.Hagedorn, Phys. Lett 703} (2011) 447
G.J.Ding, Nucl. Phys.B 862 (2012) 1
S. F.King, C.Luhn and A.J.Stuart, Nucl.Phys.B867(2013) 203
G.J.Ding and S.F.King, Phys.Rev.D89 (2014) 093020
C.Hagedorn, A.Meroni and E.Molinaro, Nucl.Phys. B 891 (2015) 499

Δ(96) group

$(Z_4 \times Z_4) \ltimes S_3$

Generator S, T and U : $S^2=(ST)^3=T^8=1$, $(ST^{-1}ST)^3=1$

Irreducible representations: 1, 1', 2, 3_1 - 3_6, 6

Subgroup : fifteen Z_2 , sixteen Z_3 , seven K_4 , twelve Z_4 , six Z_8

For triplet 3,

$$S=\frac{1}{2}\begin{pmatrix} 0 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & -1 & 1 \\ \sqrt{2} & 1 & -1 \end{pmatrix} \qquad T=\begin{pmatrix} e^{\frac{6\pi i}{4}} & 0 & 0 \\ 0 & e^{\frac{7\pi i}{4}} & 0 \\ 0 & 0 & e^{\frac{3\pi i}{4}} \end{pmatrix}$$

Neutrino sector preserves
$\{S, ST^4ST^4\}$ $(Z_2 \times Z_2)$
Charged lepton sector preserves
ST (Z_3)

$$U_{TFH1}=\begin{pmatrix} \frac{1}{6}(3+\sqrt{3}) & \frac{1}{\sqrt{3}} & \frac{1}{6}(-3+\sqrt{3}) \\ \frac{1}{6}(-3+\sqrt{3}) & \frac{1}{\sqrt{3}} & \frac{1}{6}(3+\sqrt{3}) \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{pmatrix}$$

$\Theta_{13} \sim 12°$ rather large

In order to build models (effective Lagrangian), flavons are introduced: indirect approach

Flavor symmetry G is (partially, completely) broken by flavon (SU_2 singlet scalors) VEV's.
Flavor symmetry controls couplings among leptons and flavons with special vacuum alignments.

A_4 model	Leptons	flavons	
A_4 triplets	(L_e, L_μ, L_τ)	$\phi_\nu(\phi_{\nu 1}, \phi_{\nu 2}, \phi_{\nu 3})$	couple to neutrino sector
		$\phi_E(\phi_{E1}, \phi_{E2}, \phi_{E3})$	couple to charged lepton sector
A_4 singlets	$e_R : 1 \quad \mu_R : 1" \quad \tau_R : 1'$		

$$\mathbf{3_L} \times \mathbf{3_L} \times \mathbf{3_{flavon}} \to \mathbf{1}$$

G. Altarelli, F. Feruglio, Nucl.Phys. B720 (2005) 64

CG coefficients of flavon couplings are determined by the flavour symmetry.

Flavor symmetry G is broken by VEV of flavons

$3_L \times 3_L \times 3_{flavon} \rightarrow 1$ $\qquad$ $3_L \times 1_R (1_R', 1_R'') \times 3_{flavon} \rightarrow 1$

$$m_{\nu LL} \sim y \begin{pmatrix} 2\langle\phi_{\nu 1}\rangle & -\langle\phi_{\nu 3}\rangle & -\langle\phi_{\nu 2}\rangle \\ -\langle\phi_{\nu 3}\rangle & 2\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle \\ -\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle & 2\langle\phi_{\nu 3}\rangle \end{pmatrix} \qquad m_E \sim \begin{pmatrix} y_e\langle\phi_{E1}\rangle & y_e\langle\phi_{E3}\rangle & y_e\langle\phi_{E2}\rangle \\ y_\mu\langle\phi_{E2}\rangle & y_\mu\langle\phi_{E1}\rangle & y_\mu\langle\phi_{E3}\rangle \\ y_\tau\langle\phi_{E3}\rangle & y_\tau\langle\phi_{E2}\rangle & y_\tau\langle\phi_{E1}\rangle \end{pmatrix}$$

However, specific Vacuum Alingnments preserve the symmetries of S and T.

Take $\langle\phi_{\nu 1}\rangle = \langle\phi_{\nu 2}\rangle = \langle\phi_{\nu 3}\rangle$ and $\langle\phi_{E2}\rangle = \langle\phi_{E3}\rangle = 0$

$\Rightarrow$ $\langle\phi_\nu\rangle \sim (1,1,1)^T$, $\qquad \langle\phi_E\rangle \sim (1,0,0)^T$

$$S\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}, \quad T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

Then, $\langle\phi_\nu\rangle$ preserves S (Z_2) and $\langle\phi_E\rangle$ preserves T(Z_3).

m_E is a diagonal matrix, on the other hand, $m_{\nu LL}$ is

$$m_{\nu LL} \sim 3y \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - y \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

two generated masses and one massless neutrinos !
(0, 3y, 3y)
Flavor mixing is not yet fixed !

Adding A_4 singlet $\xi : \mathbf{1}$ gives flavor mixing matrix.

$\mathbf{3_L} \times \mathbf{3_L} \times \mathbf{1_{flavon}} \rightarrow \mathbf{1}$

$$m_{\nu LL} \sim y_1 \begin{pmatrix} 2\langle\phi_{\nu 1}\rangle & -\langle\phi_{\nu 3}\rangle & -\langle\phi_{\nu 2}\rangle \\ -\langle\phi_{\nu 3}\rangle & 2\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle \\ -\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle & 2\langle\phi_{\nu 3}\rangle \end{pmatrix} + y_2 \langle\xi\rangle \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

A_4 invariant

$\langle\phi_{\nu 1}\rangle = \langle\phi_{\nu 2}\rangle = \langle\phi_{\nu 3}\rangle$, which preserve S symmetry.

$$m_{\nu LL} = 3a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - a \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

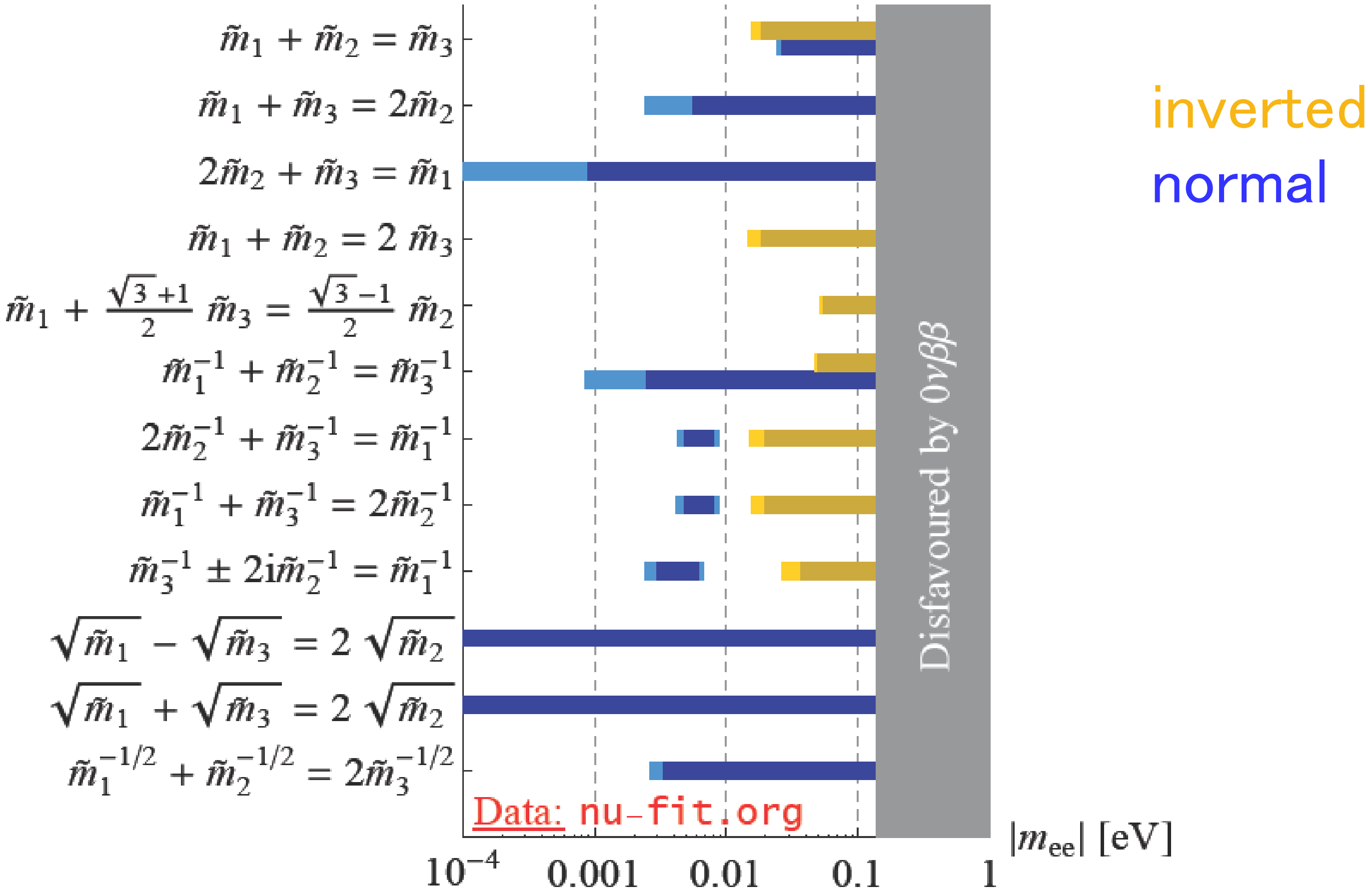
Flavor mixing is determined: Tri-bimaximal mixing. $\theta_{13}=0$

$$m_\nu = 3a+b,\ b,\ 3a-b \ \Rightarrow \boxed{m_{\nu_1} - m_{\nu_3} = 2m_{\nu_2}}$$

Flavons predict Neutrino Mass Sum Rule.

* Additional A_4 singlet $1'$ gives non-vanishing θ_{13} and different masses.

Restrictions by mass sum rules on $|m_{ee}|$



King, Merle, Stuart, JHEP 2013, arXiv:1307.2901

Challenge for CP violation of leptons

Talk of Mu-Chun Chen

CP phase δ_{CP} is related with Flavour Symmetry.

A hint : under μ-τ symmetry $|U_{\mu i}| = |U_{\tau i}| \; i = 1, 2, 3$

$$\cos\theta_{23} = \sin\theta_{23} = \frac{1}{\sqrt{2}}$$

$$\sin\theta_{13}\cos\delta = 0$$

$\delta = \pm\frac{\pi}{2}$ is predicted since we know $\theta_{13} \neq 0$

Ferreira, Grimus, Lavoura, Ludl, JHEP2012,arXiv: 1206.7072

CP violation is constrained by Flavour symmetries !

Generalized CP Symmetry

☆ How will be Flavor Symmetry tested ?

* Mixing angle sum rules

Example: S_4 (TM1) $\sin^2\theta_{23} = \frac{1}{2}\frac{1}{\cos^2\theta_{13}} \geq \frac{1}{2}$, $\sin^2\theta_{12} \simeq \frac{1}{\sqrt{3}} - \frac{2\sqrt{2}}{3}\sin\theta_{13}\cos\delta_{CP} + \frac{1}{3}\sin^2\theta_{13}\cos 2\delta_{CP}$

A_4 (TM2) $\sin^2\theta_{12} = \frac{1}{3}\frac{1}{\cos^2\theta_{13}} \geq \frac{1}{3}$, $\cos\delta_{CP}\tan 2\theta_{23} \simeq \frac{1}{\sqrt{2}\sin\theta_{13}}\left(1 - \frac{5}{4}\sin^2\theta_{13}\right)$

A_5 $\sin^2\theta_{12} = \frac{\sin^2\varphi}{1-\sin^2\theta_{13}} \approx \frac{0.276}{1-\sin^2\theta_{13}}$ $\sin^2\theta_{23} \approx \frac{1}{2}\left(1 \pm (1-\sqrt{5})\sin\theta_{13}\right)$

A.Di Iura, C.Hagedorn and D.Meloni, JHEP1508 (2015) 037

* Neutrino mass sum rules in FLASY ⇔ neutrinoless double beta decays

* Prediction of CP violating phase, which may be correlated with θ_{ij}

☆ Flavon at TeV scale ? Testable at LHC ?

Back up slides

Familiar non-Abelian finite groups

			order
S_n :	$S_2=Z_2$, S_3, S_4 …	Symmetric group	$N\,!$
A_n:	$A_3=Z_3$, $A_4=T$, A_5 …	Alternating group	$(N\,!)/2$
D_n:	$D_3=S_3$, D_4, D_5 …	Dihedral group	$2N$
$Q_{N(\text{even})}$:	Q_4, Q_6 ….	Binary dihedral group	$2N$
$\Sigma(2N^2)$:	$\Sigma(2)=Z_2$, $\Sigma(18)$, $\Sigma(32)$, $\Sigma(50)$ …		$2N^2$
$\Delta(3N^2)$:	$\Delta(12)=A_4$, $\Delta(27)$ …		$3N^2$
$T_{N(\text{prime number})} \cong Z_N \rtimes Z_3$:	T_7, T_{13}, T_{19}, T_{31}, T_{43}, T_{49}		$3N$
$\Sigma(3N^3)$:	$\Sigma(24)=Z_2\times\Delta(12)$, $\Sigma(81)$ …		$3N^3$
$\Delta(6N^2)$:	$\Delta(6)=S_3$, $\Delta(24)=S_4$, $\Delta(54)$ …		$6N^2$
T' : double covering group of $A_4=T$			24

In order to obtain non-zero θ_{13} , A_5 should be broken to other subgroups: for example,

Neutrino sector preserves S or $T^2ST^3ST^2$ (both are K_4 generator)

Charged lepton sector preserves T (Z_5)

$$\begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \times \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$\tan\theta_{12} = 1/\phi \qquad \phi = \frac{1+\sqrt{5}}{2}$$

θ is not fixed, however, there appear testable sum rules:

$$\sin^2\theta_{12} = \frac{\sin^2\varphi}{1-\sin^2\theta_{13}} \approx \frac{0.276}{1-\sin^2\theta_{13}} \qquad \sin^2\theta_{23} \approx \frac{1}{2}\left(1 \pm (1-\sqrt{5})\sin\theta_{13}\right)$$

A.Di Iura, C.Hagedorn and D.Meloni, JHEP1508 (2015) 037

Additional Matrix

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

$$a=\frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{\Lambda},\qquad b=-\frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{3\Lambda},\qquad c=\frac{y^\nu_{\xi}\alpha_\xi v_u^2}{\Lambda},\qquad d=\frac{y^\nu_{\xi'}\alpha_{\xi'} v_u^2}{\Lambda}\qquad a=-3b$$

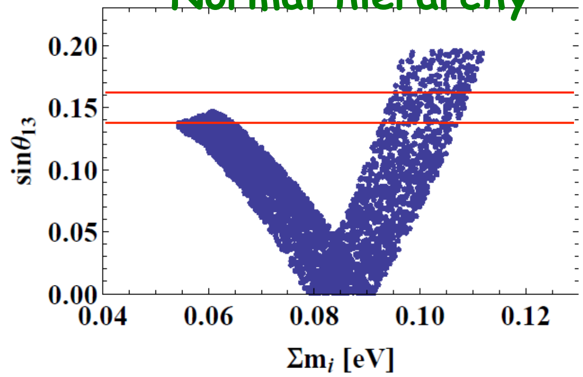
Both normal and inverted mass hierarchies are possible.

$$M_\nu = V_{\text{tri-bi}}\begin{pmatrix}a+c-\frac{d}{2} & 0 & \frac{\sqrt{3}}{2}d\\ 0 & a+3b+c+d & 0\\ \frac{\sqrt{3}}{2}d & 0 & a-c+\frac{d}{2}\end{pmatrix}V^T_{\text{tri-bi}}$$

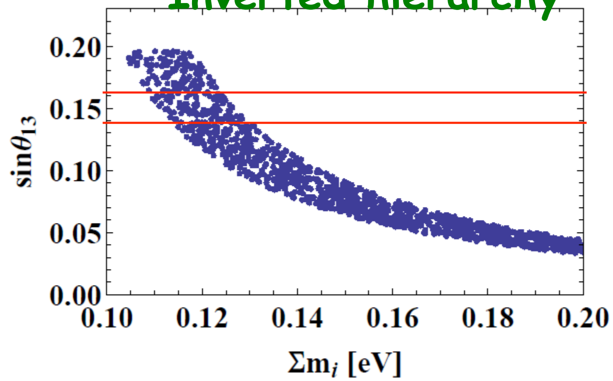
Tri-maximal mixing: TM2

$$\Delta m^2_{31} = -4a\sqrt{c^2+d^2-cd}\,,\qquad \Delta m^2_{21} = (a+3b+c+d)^2-(a+\sqrt{c^2+d^2-cd})^2$$

Normal hierarchy



Inverted hierarchy



Suppose a symmetry including FLASY and CP symmetry:

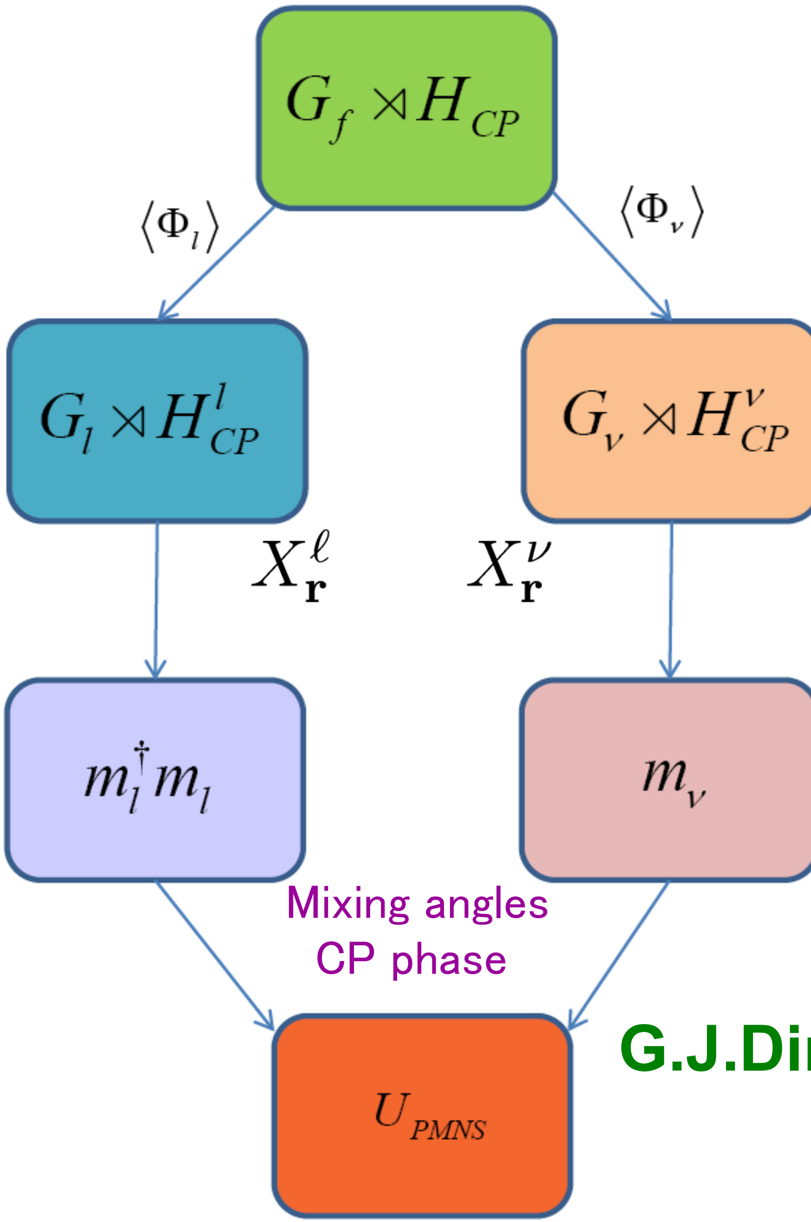
$$G_{CP} = G_f \rtimes H_{CP}$$

is broken to the subgroups in neutrino sector and charged lepton sector.

CP symmetry gives

$$X_{\mathbf{r}}^{\nu T} m_{\nu LL} X_{\mathbf{r}}^{\nu} = m_{\nu LL}^{*}$$

$$X_{\mathbf{r}}^{\ell \dagger} (m_\ell^\dagger m_\ell) X_{\mathbf{r}}^{\ell} = (m_\ell^\dagger m_\ell)^*$$



G.J.Ding

Generalized CP Symmetry

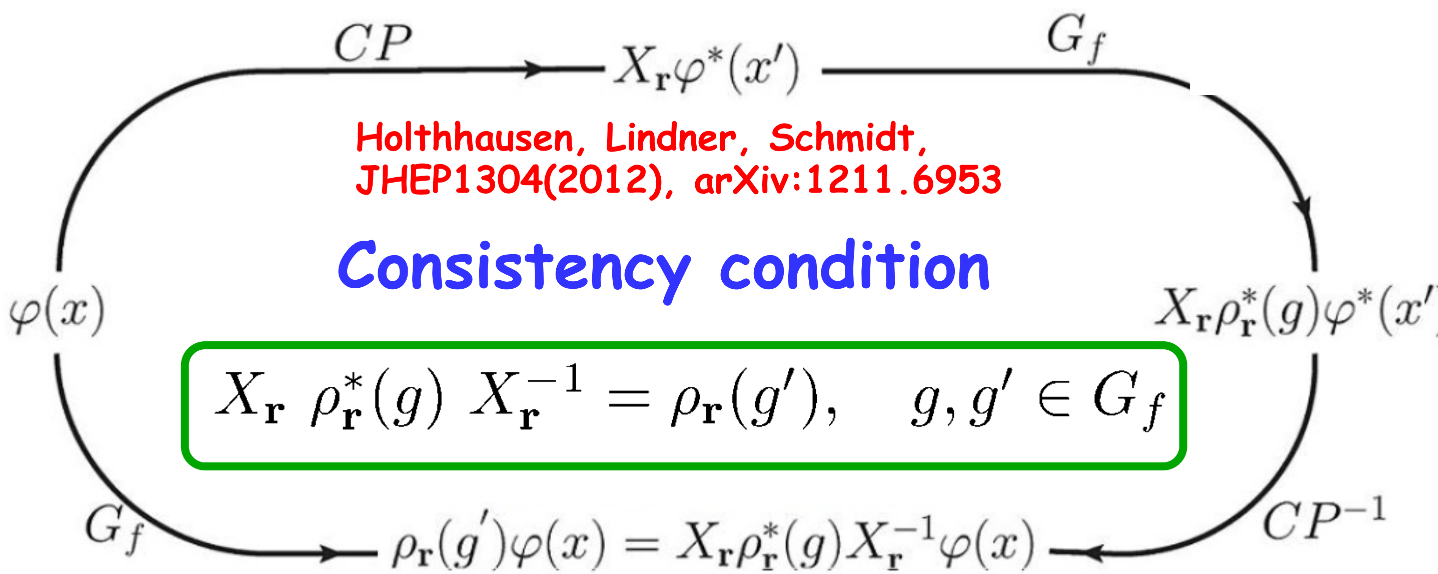
CP Symmetry $\varphi(x) \xrightarrow{\text{CP}} X_{\mathbf{r}} \varphi^*(x'), \quad x' = (t, -\mathbf{x})$

Flavour Symmetry $\varphi(x) \xrightarrow{\text{g}} \rho_{\mathbf{r}}(g) \varphi^*(x), \quad g \in G_f$

$$X_{\mathbf{r}}^{\nu T} m_{\nu LL} X_{\mathbf{r}}^{\nu} = m_{\nu LL}^*$$

$$X_{\mathbf{r}}^{\ell \dagger} (m_\ell^\dagger m_\ell) X_{\mathbf{r}}^{\ell} = (m_\ell^\dagger m_\ell)^*$$

X_r must be consistent with Flavour Symmetry $\rho_{\mathbf{r}}(g)$



Holthhausen, Lindner, Schmidt, JHEP1304(2012), arXiv:1211.6953

Consistency condition

$$X_{\mathbf{r}} \ \rho_{\mathbf{r}}^*(g) \ X_{\mathbf{r}}^{-1} = \rho_{\mathbf{r}}(g'), \quad g, g' \in G_f$$

Mu-Chun Chen, Fallbacher, Mahanthappa, Ratz, Trautner, Nucl.Phys. B883 (2014) 267–305

Origin of Flavor symmetry

Is it possible to realize such discrete symmetres in string theory?
Answer is yes !

Superstring theory on a certain type of six dimensional compact space leads to stringy selection rules for allowed couplings among matter fields in four-dimensional effective field theory.

Such stringy selection rules and geometrical symmetries result in discrete flavor symmetries in superstring theory.

- Heterotic orbifold models (Kobayashi., Nilles, Ploger, Raby, Ratz, 07)
- Magnetized/Intersecting D-brane Model
  (Kitazawa, Higaki, Kobayashi,Takahashi, 06)
  (Abe, Choi, Kobayashi, HO, 09, 10)

Stringy origin of non-Abelian discrete flavor symmetries

T. Kobayashi, H. Niles, F. PloegerS, S. Raby, M. Ratz, hep-ph/0611020

D_4, $\Delta(54)$

Non-Abelian Discrete Flavor Symmetries from Magnetized/Intersecting Brane Models

H. Abe, K-S. Choi, T. Kobayashi, H. Ohki, 0904.2631

D_4, $\Delta(27)$, $\Delta(54)$

Non-Abelian Discrete Flavor Symmetry from T^2/Z_N Orbifolds

A.Adulpravitchai, A. Blum, M. Lindner, 0906.0468

A_4, S_4, D_3, D_4, D_6

Non-Abelian Discrete Flavor Symmetries of 10D SYM theory with Magnetized extra dimensions

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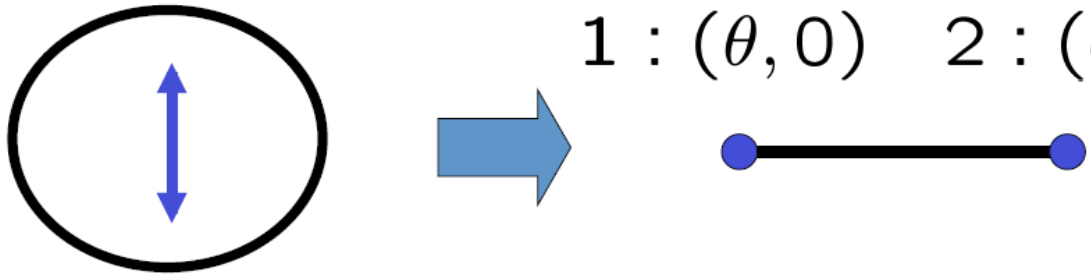
S_3, $\Delta(27)$, $\Delta(54)$

$S^1/\mathbf{Z}_2$ **orbifold** (Kobayashi, Nilles, Ploger, Raby, Ratz, 07)



$1 : (\theta, 0) \quad 2 : (\theta, e_1)$

There are two fixed point under the orbifold twist

These two fixed points can be represented by space group elements which act (θ, v)

$$(\theta, v)\alpha = \theta\alpha + v$$

e_1 : shift vector in one torus $(y \sim y + e_1)$

charge assignment of $\mathbf{Z}_2$: $\begin{pmatrix} 1 \\ 2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(stringy selection rule: Coupling is only allowed in matching of the string boundary conditions)

Discrete flavor symmetry from orbifold $S^1/\mathbf{Z}_2$

This effective Lagrangian also have permutation symmetry of these two fixed point (orbifold geometry).

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \to \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Closed algebra of these transformations $\left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$

$$\Rightarrow D_4 \sim S^2 \cup (\mathbf{Z}_2 \times \mathbf{Z}_2)$$

Two field localized at two fixed points : doublet of D4 $\mathbf{2}$

Bulk mode (untwisted mode) : singlet of D4 $\mathbf{1}$

Thus full symmetry is larger than geometric symmetry

Mass sum rules in A_4, T′, S_4, A_5, $\Delta(96)$ …

(Talk of Spinrath) Barry, Rodejohann, NPB842(2011) arXiv:1007.5217

Different types of neutrino mass spectra correspond to the neutrino mass generation mechanism.

$$\chi\tilde{m}_2 + \xi\tilde{m}_3 = \tilde{m}_1$$

(X=2, ξ=1) (X=-1, ξ=1)

$$\frac{\chi}{\tilde{m}_2} + \frac{\xi}{\tilde{m}_3} = \frac{1}{\tilde{m}_1}$$

M_R structre in See-saw

$$\chi\sqrt{\tilde{m}_2} + \xi\sqrt{\tilde{m}_3} = \sqrt{\tilde{m}_1}$$

M_D structre in See-saw

$$\frac{\chi}{\sqrt{\tilde{m}_2}} + \frac{\xi}{\sqrt{\tilde{m}_3}} = \frac{1}{\sqrt{\tilde{m}_1}}$$

M_R in inverse See-saw

X and ξ are model specific complex parameters

King, Merle, Stuart, JHEP 2013, arXiv:1307.2901
King, Merle, Morisi, Simizu, M.T, arXiv: 1402.4271