Discussion: Quark and lepton flavour and CP symmetry/violation in BSM approaches

Part 1: Within GUTs (in SUSY, 4D)

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Neutrinos: the quest for a new physics scale
CERN, Geneva

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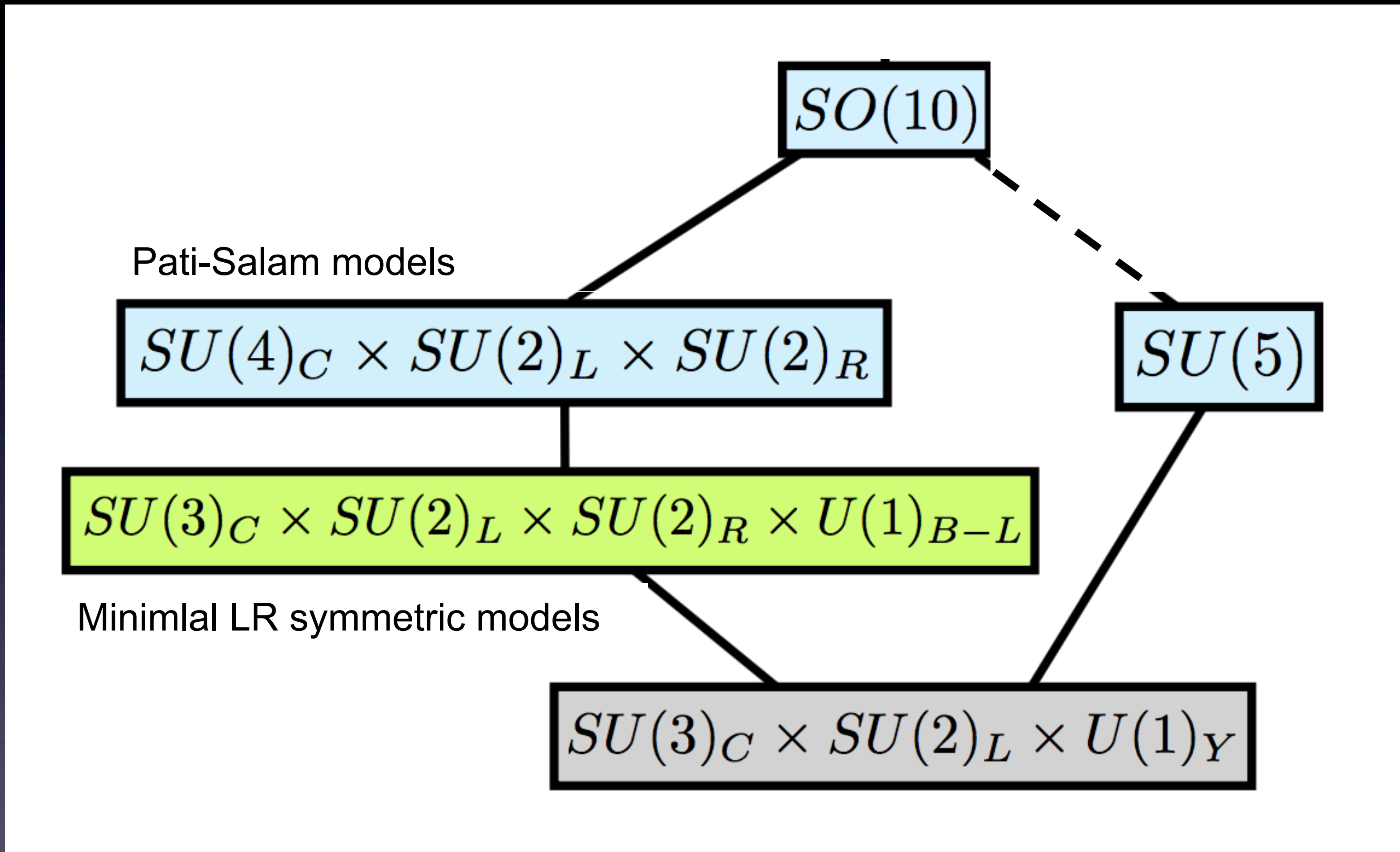
Flavour models in GUTs: two main routes



"Quark-lepton unification": quarks and leptons in joint GUT representations

Flavour models in GUTs: Pati-Salam models

$SU(4)_C$ $\longleftrightarrow$ $SU(4)_C$ $\longleftrightarrow$

$SU(2)_L$ $\updownarrow$

$$f^f_{\rm L} = \begin{pmatrix} u^f_{{\rm L}r} & u^f_{{\rm L}y} & u^f_{{\rm L}b} & \nu^f_{\rm L} \\ d^f_{{\rm L}r} & d^f_{{\rm L}y} & d^f_{{\rm L}b} & e^f_{\rm L} \end{pmatrix}, \quad f^f_{\rm R} = \begin{pmatrix} u^f_{{\rm R}r} & u^f_{{\rm R}y} & u^f_{{\rm R}b} & \nu^f_{\rm R} \\ d^f_{{\rm R}r} & d^f_{{\rm R}y} & d^f_{{\rm R}b} & e^f_{\rm R} \end{pmatrix}$$

$SU(2)_R$ $\updownarrow$

In Pati-Salam models:
$Y_\nu \sim Y_u$ and $Y_d \sim Y_e$

RH neutrinos are predicted (➔ Seesaw)

Note: Neutrino masses from type I and/or type II seesaw mechanism, ...

Towards SO(10) GUTs:

$$\mathbf{16}^f_{\rm SO(10)} = (\mathbf{4},\mathbf{2},\mathbf{1})^f_{G_{422}} + (\overline{\mathbf{4}},\mathbf{1},\mathbf{2})^f_{G_{422}} = f^f_{\rm L} + f^{Cf}_{\rm R}$$

Flavour models in GUTs: SU(5) GUTs

$$\bar{\mathbf{5}}_i = \begin{pmatrix} d_R^{cR} & d_R^{cB} & d_R^{cG} & e_L & -\nu_L \end{pmatrix}_i$$

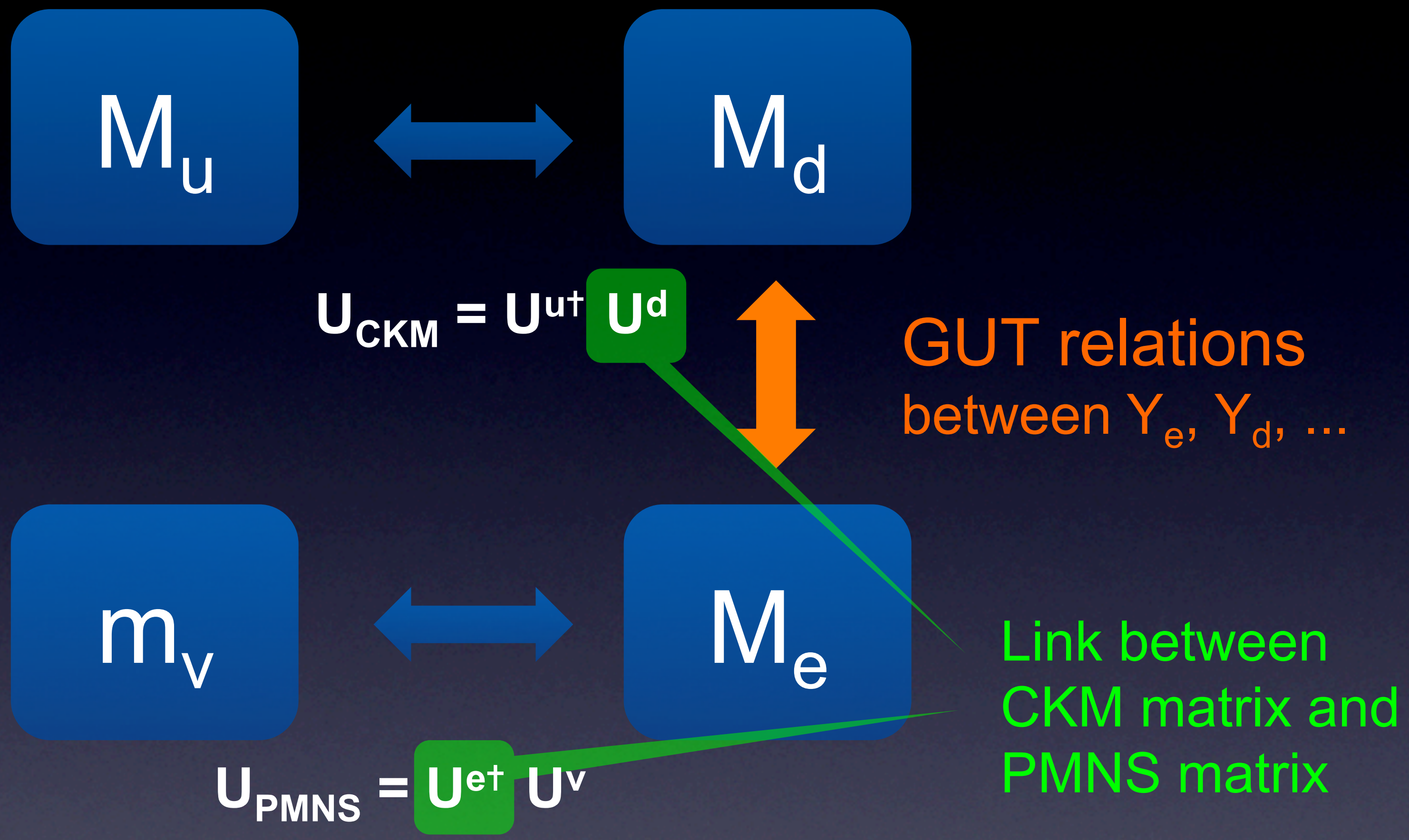
$$\mathbf{10}_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -u_R^{cG} & u_R^{cB} & -u_L^R & -d_L^R \\ u_R^{cG} & 0 & -u_R^{cR} & -u_L^B & -d_L^B \\ -u_R^{cB} & u_R^{cR} & 0 & -u_L^G & -d_L^G \\ u_L^R & u_L^B & u_L^G & 0 & -e_R^c \\ d_L^R & d_L^B & d_L^G & e_R^c & 0 \end{pmatrix}_i$$

In SU(5) GUTs:
$Y_d \sim Y_e^T$

New particles for neutrino mass generation have to be introduced in addition (e.g. RH neutrinos) ...

Note: Neutrino masses from type I and/or type II seesaw mechanism, ...

Links between the quark and lepton flavour



➔ GUT flavour models can be highly predictive: quark-lepton mass ratios, δ^{PMNS} (e.g. linked to $\alpha \approx 90°$), θ_{13}^{PMNS} (e.g. $\theta_{13}^{PMNS} \approx s_{23}^{PMNS}\, \theta_C$), SUSY spectrum, Flavour physics observables, proton decay (smoking gun for GUTs in general), links to cosmology, ...

Extra Slides

Brief: An example GUT flavour model …

Example for a supersymmetric SU(5) GUT flavour model

S. A., C. Gross, V. Maurer, C. Sluka (arXiv:1305.6612)

Symmetries: SU(5) x A_4 x "shaping symmetries"

Charged lepton sector: GUT relations for masses & mixings

$$Y_d = \begin{pmatrix} 0 & \tilde{\epsilon}_2 & 0 \\ \tilde{\epsilon}_{ab} c_{ab} & i\tilde{\epsilon}_{ab} s_{ab} & 0 \\ 0 & \omega^2 \hat{\epsilon}_\chi & \tilde{\epsilon}_3 \end{pmatrix}, \quad Y_e = \begin{pmatrix} 0 & 6\tilde{\epsilon}_{ab} c_{ab} & 0 \\ -\frac{1}{2}\tilde{\epsilon}_2 & i6\tilde{\epsilon}_{ab} s_{ab} & 6\omega^2 \hat{\epsilon}_\chi \\ 0 & 0 & -\frac{3}{2}\tilde{\epsilon}_3 \end{pmatrix}, \quad Y_u = \begin{pmatrix} \epsilon_2^4 & \epsilon_{12}^5 & 0 \\ \epsilon_{12}^5 & \epsilon_{ab}^2 & \epsilon_{23} \\ 0 & \epsilon_{23} & y_t \end{pmatrix}$$

Neutrino sector: TB neutrino mixing (+ charged lepton mixing contr.)

$$Y_\nu = \begin{pmatrix} 0 & \epsilon_{N_2} \\ \epsilon_{N_1} & \epsilon_{N_2} \\ -\epsilon_{N_1} & \epsilon_{N_2} \end{pmatrix}, \quad M_R = \begin{pmatrix} M_{R_1} & 0 \\ 0 & M_{R_2} \end{pmatrix}$$

➔ Normal neutrino mass hierarchy

- $G_{family} = A_4$ is spontaneoulsy broken by 5 flavour Higgs fields (flavons) in representations 3 of A_4 with vacuum expectation values pointing in the following flavour directions (with full flavon superpotential leading to these vevs given in our paper):

- indirect model approach -

$$\frac{\langle\phi_{N_2}\rangle}{\Lambda} \sim \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \varepsilon_{N_2} \qquad \frac{\langle\phi_{N_1}\rangle}{\Lambda} \sim \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \varepsilon_{N_1}$$

"TB Mixing"
(in the neutrino-sector)

hierarchical quark and charged lepton Yukawa matrices

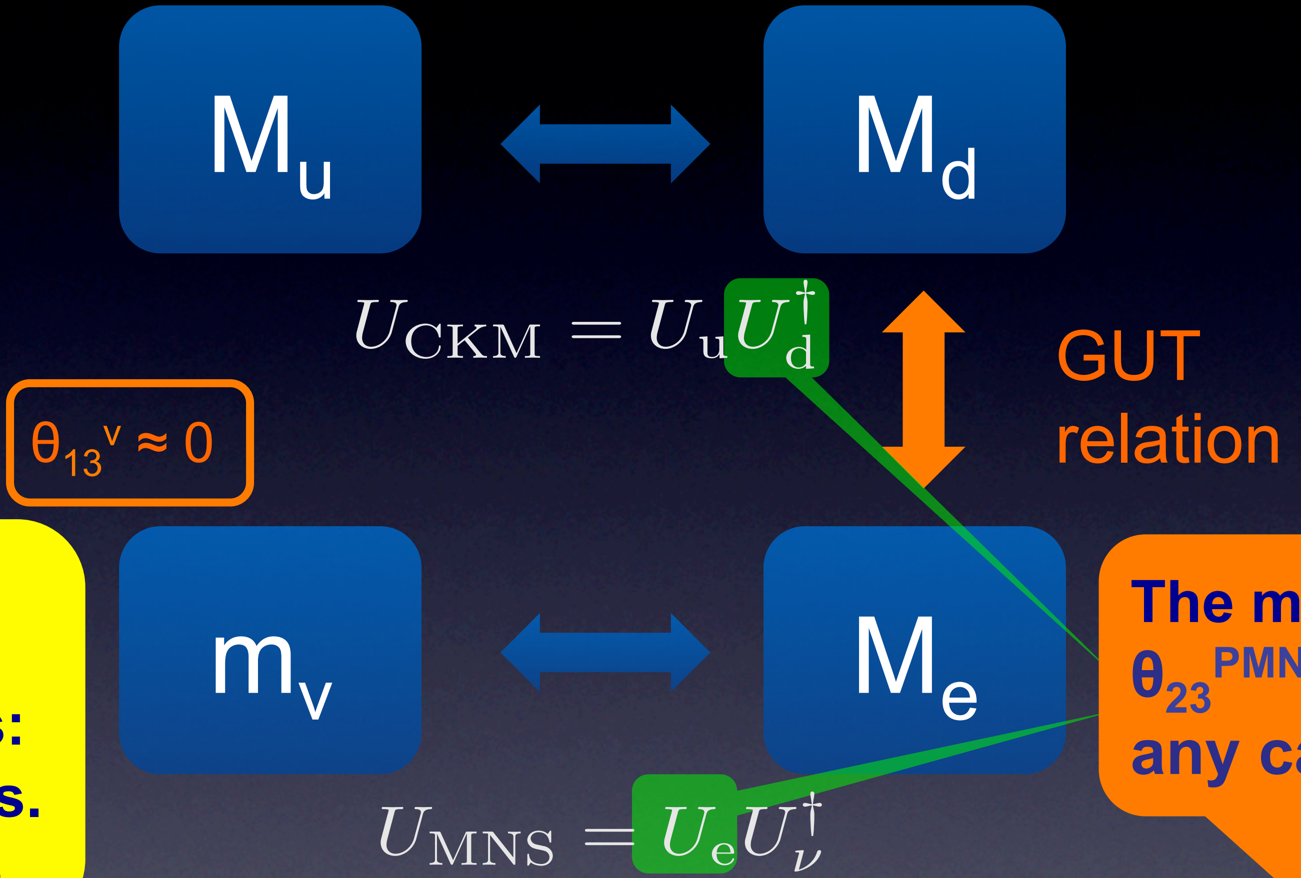
$$\frac{\langle\phi_3\rangle}{\Lambda} \sim \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \varepsilon_3 \qquad \frac{\langle\phi_{ab}\rangle}{\Lambda} \sim \begin{pmatrix} a \\ -ib \\ 0 \end{pmatrix} \varepsilon_{ab}$$

$$\frac{\langle\phi_2\rangle}{\Lambda} \sim \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \varepsilon_2$$

CP violation for the quarks (with $\alpha \approx 90°$) & leptons ($\delta^{PMNS} = -90°$)

→ Overview of predictions:

S. A., C. Gross, V. Maurer, C. Sluka (arXiv:1305.6612)



Best fit point:
χ^2/d.o.f. = 2.0
MCMS analysis:
14 parametrs vs. 20 observables

The model predicts $\theta_{23}^{PMNS} \approx 40°$ (in any case < 45°)

✓ Excellent fit to the 2013 exp. data: 6 predictions, e.g.: $\theta_{13}^{PMNS} \approx s_{23}^{PMNS}\, \theta_C$
$\delta^{PMNS} \sim 270°$, ... , (plus: constraints on SUSY spectrum)

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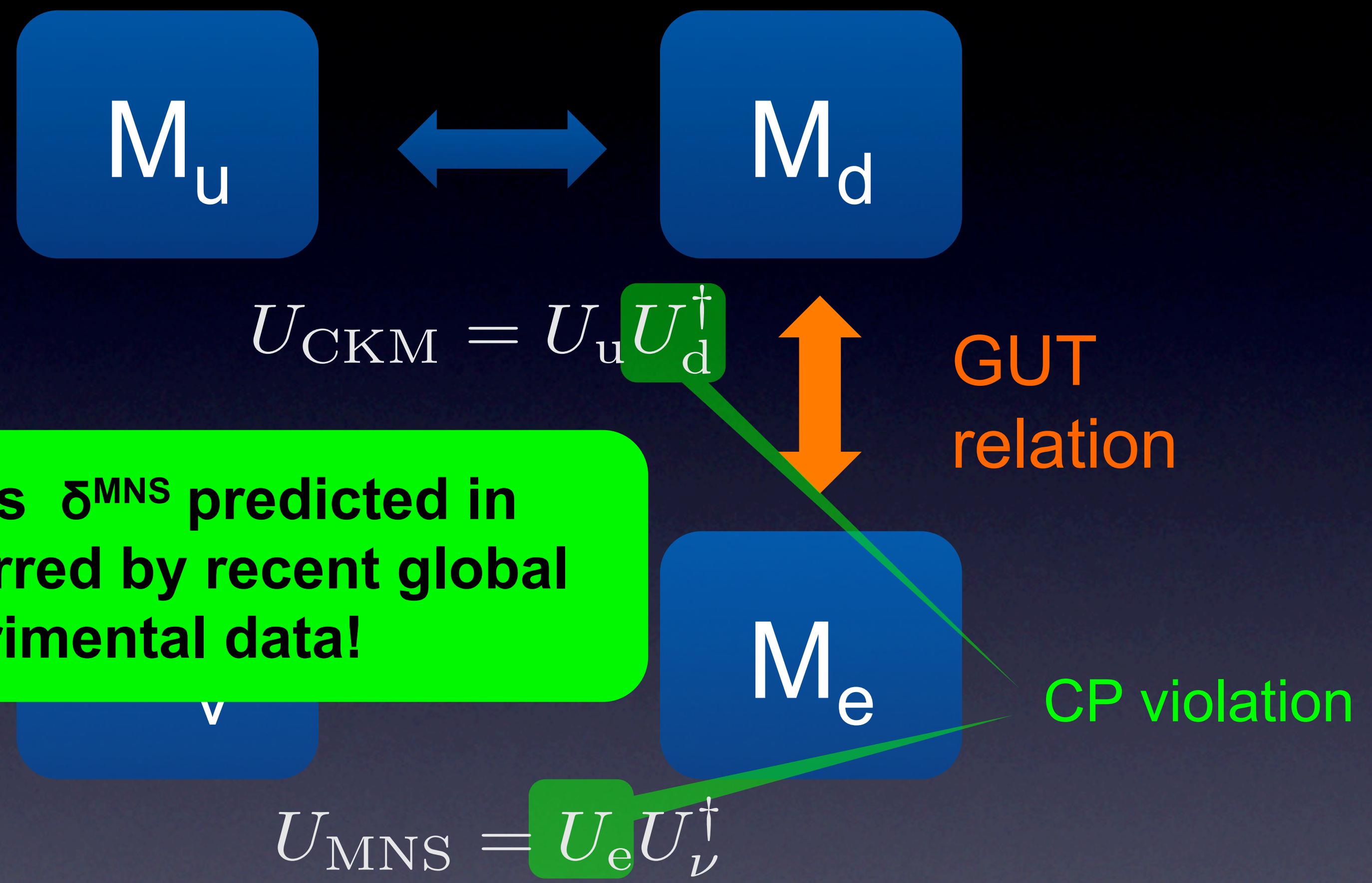

Dirac CP phases δ^{MNS} predicted in the range preferred by recent global fits to the experimental data!

✓ Excellent fit to the 2013 exp. data: 6 predictions, e.g.: $\theta_{13}^{PMNS} \approx s_{23}^{PMNS}\, \theta_C$, $\delta^{PMNS} \sim 270°$, ... , (plus: constraints on SUSY spectrum)

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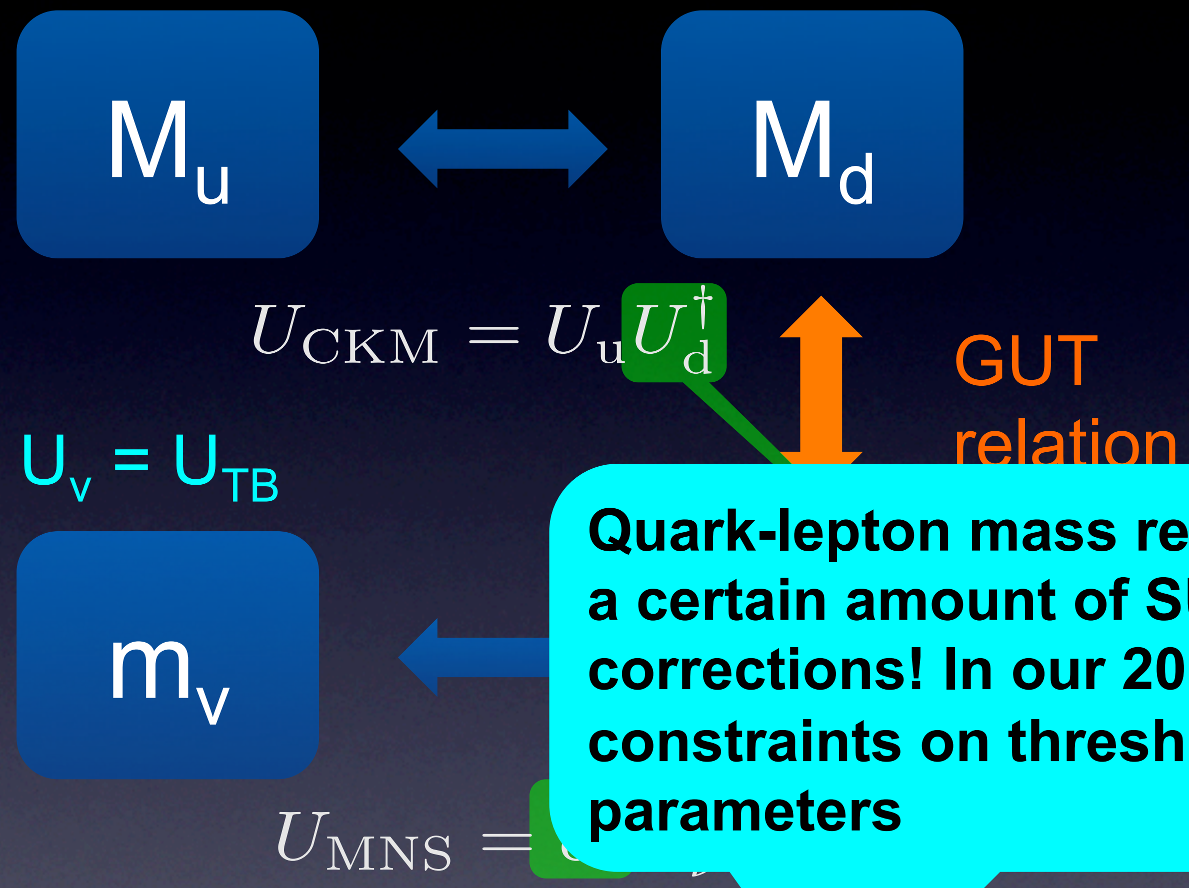

Quark-lepton mass relations require a certain amount of SUSY threshold corrections! In our 2013 paper: constraints on threshold correction parameters

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Prediction for the SUSY spectrum ...

- We consider predictions for the quark lepton mass ratios as in the example flavour GUT model (after diagonalization), i.e. $m_\tau/m_b = 3/2$, $m_\mu/m_s \approx 6$, $m_e/m_d \approx 1/2$

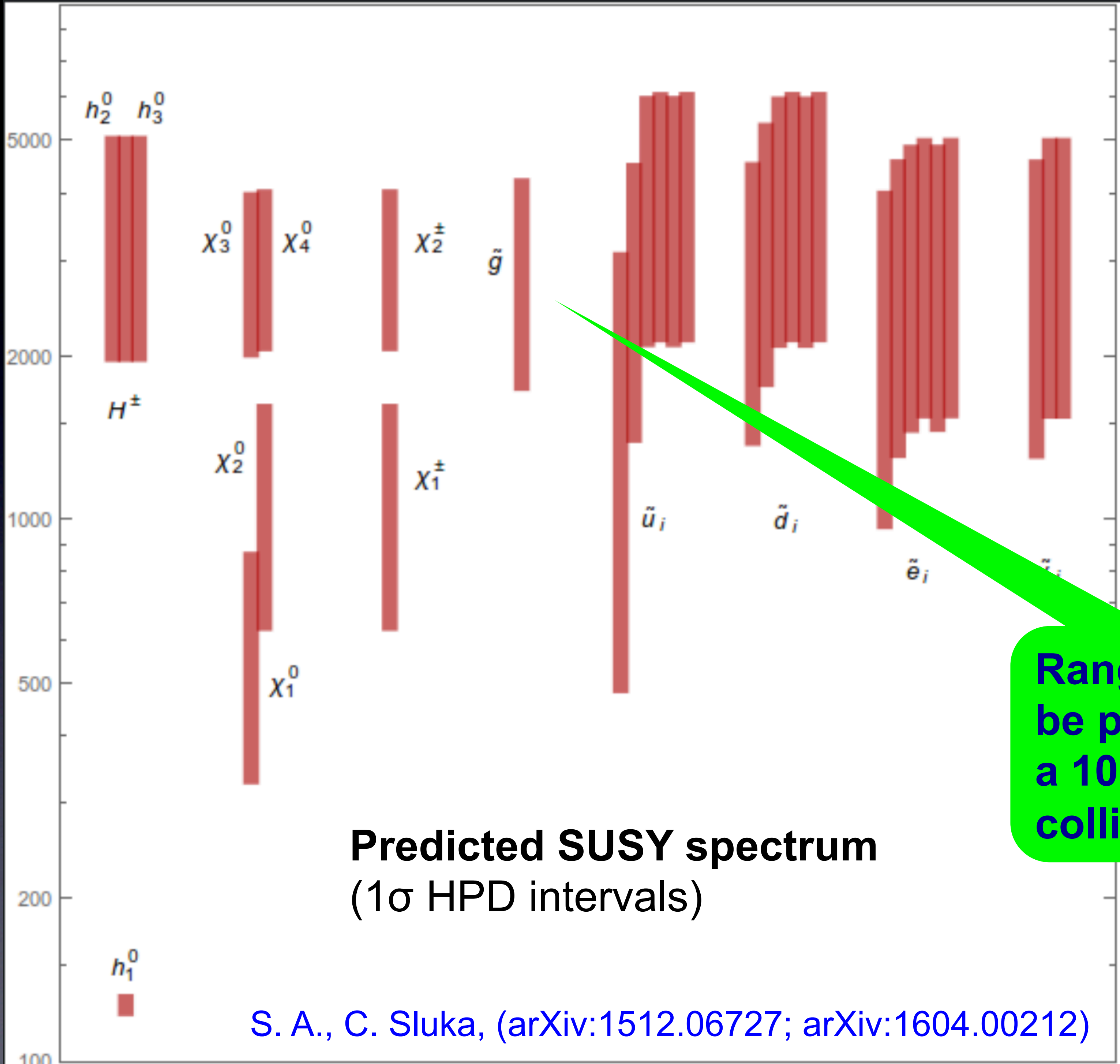
Approximate, in the mass basis of Y_d and Y_e: (CKM mixing now from Y_u)

$$Y_d = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}, \; Y_e = \begin{pmatrix} -\frac{1}{2}y_d & 0 & 0 \\ 0 & 6y_s & 0 \\ 0 & 0 & -\frac{3}{2}y_b \end{pmatrix}, \; Y_u = \begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{12} & y_{22} & y_{23} \\ y_{13} & y_{23} & y_{33} \end{pmatrix}$$

SusyTC: S. A., C. Sluka, arXiv:1512.06727

- We consider CMSSM boundary conditions from the soft terms at the GUT scale
- Using REAP with SusyTC 1.0, we fit the parameters to the experimental data on quark and lepton masses as well as on the mass m_h of the SM-like Higgs boson (using *FeynHiggs 2.11.2*)

FeynHiggs: Heinemeyer, Hahn, Rzehak, Weiglein, Hollik



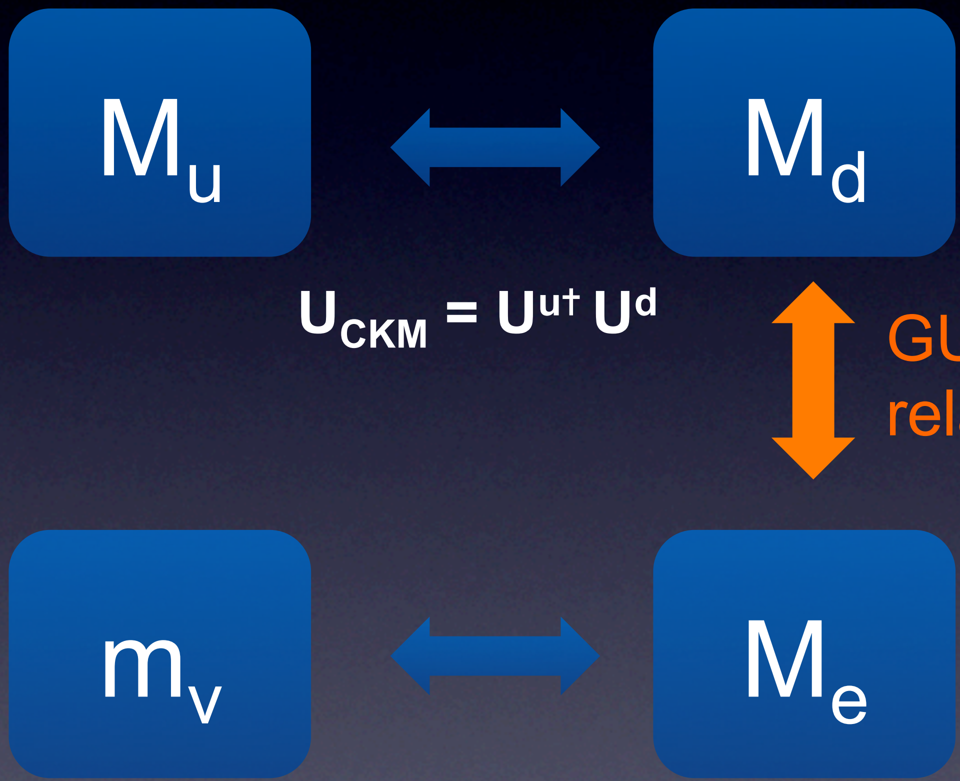
Predicted SUSY spectrum
(1σ HPD intervals)

S. A., C. Sluka, (arXiv:1512.06727; arXiv:1604.00212)

Range can be probed at a 100 TeV pp collider

Overview:
GUT relations

Quark-lepton mass relations



$U_{CKM} = U^{u\dagger} U^d$

GUT relations

$U_{PMNS} = U^{e\dagger} U^\nu$

Example: SU(5)

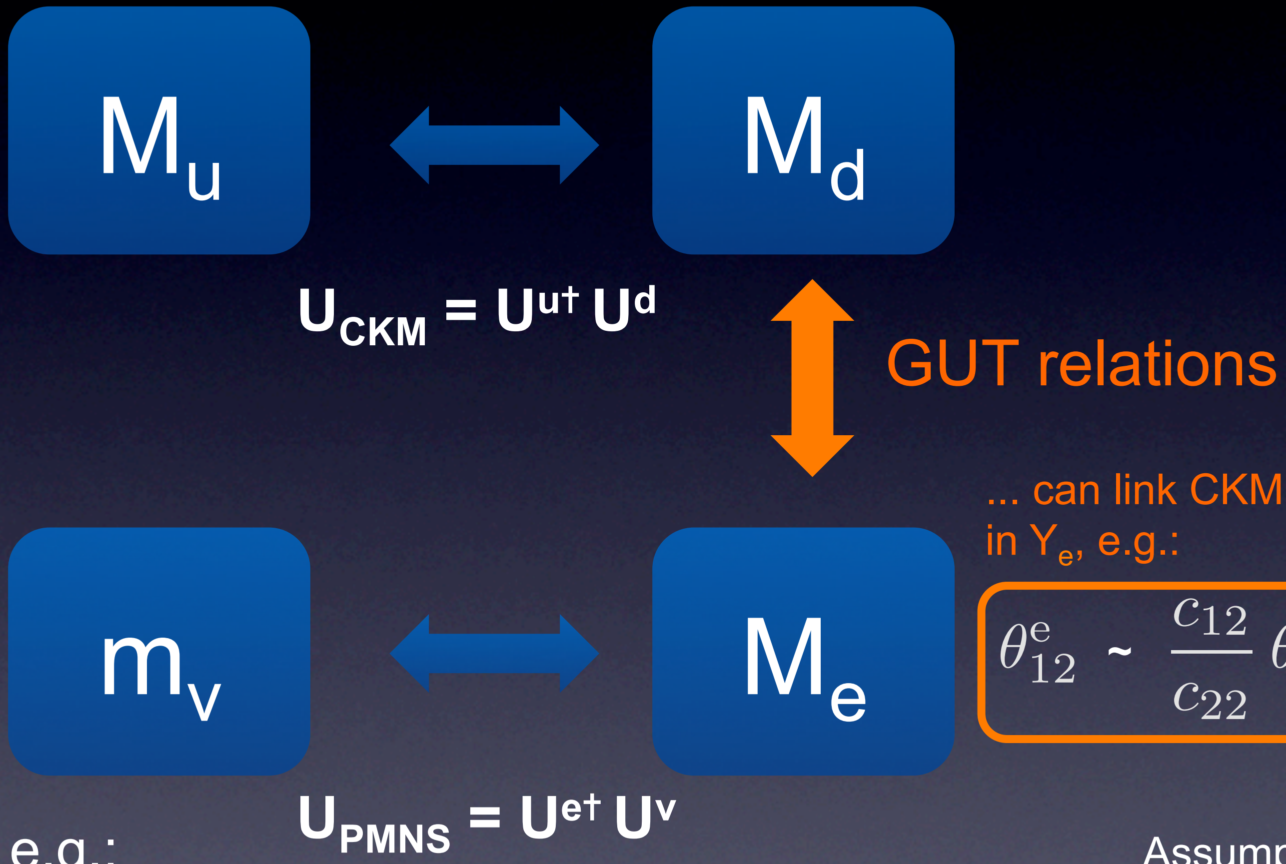
Mass relations* @ M_{GUT}, e.g.:

$$\frac{m_\tau}{m_b} = c_{33} = 1, \frac{3}{2}$$

$$\frac{m_\mu}{m_s} = c_{22} = 3, \frac{9}{2}, 6$$

*) More precise: Yukawa coupling relations

GUT mixing relations and “sum rules”



When we consider U^ν with:

$\theta_{13}^{\nu} \approx 0$

... can link CKM mixing to mixing in Y_e, e.g.:

$$\theta_{12}^{\mathrm{e}} \sim \frac{c_{12}}{c_{22}}\, \theta_{12}^{\mathrm{d}} \sim \frac{c_{12}}{c_{22}}\, \theta_{\mathrm{C}}$$

➔ Consequences are e.g.:

Assumption: $\theta_{13}^{e}, << \theta_{12}^{e}$

1) $\theta_{13}^{\mathrm{PMNS}} \approx \theta_{12}^{\mathrm{e}} s_{23}^{\mathrm{PMNS}}$

2) $\theta_{12}^{\mathrm{PMNS}} - \theta_{13}^{\mathrm{PMNS}} \cot(\theta_{23}^{\mathrm{PMNS}}) \cos(\delta^{\mathrm{PMNS}}) \approx \theta_{12}^{\nu}$

Lepton mixing sum rule: Specific neutrino mixing pattern (θ_{12}^{ν}) linked to δ^{PMNS}

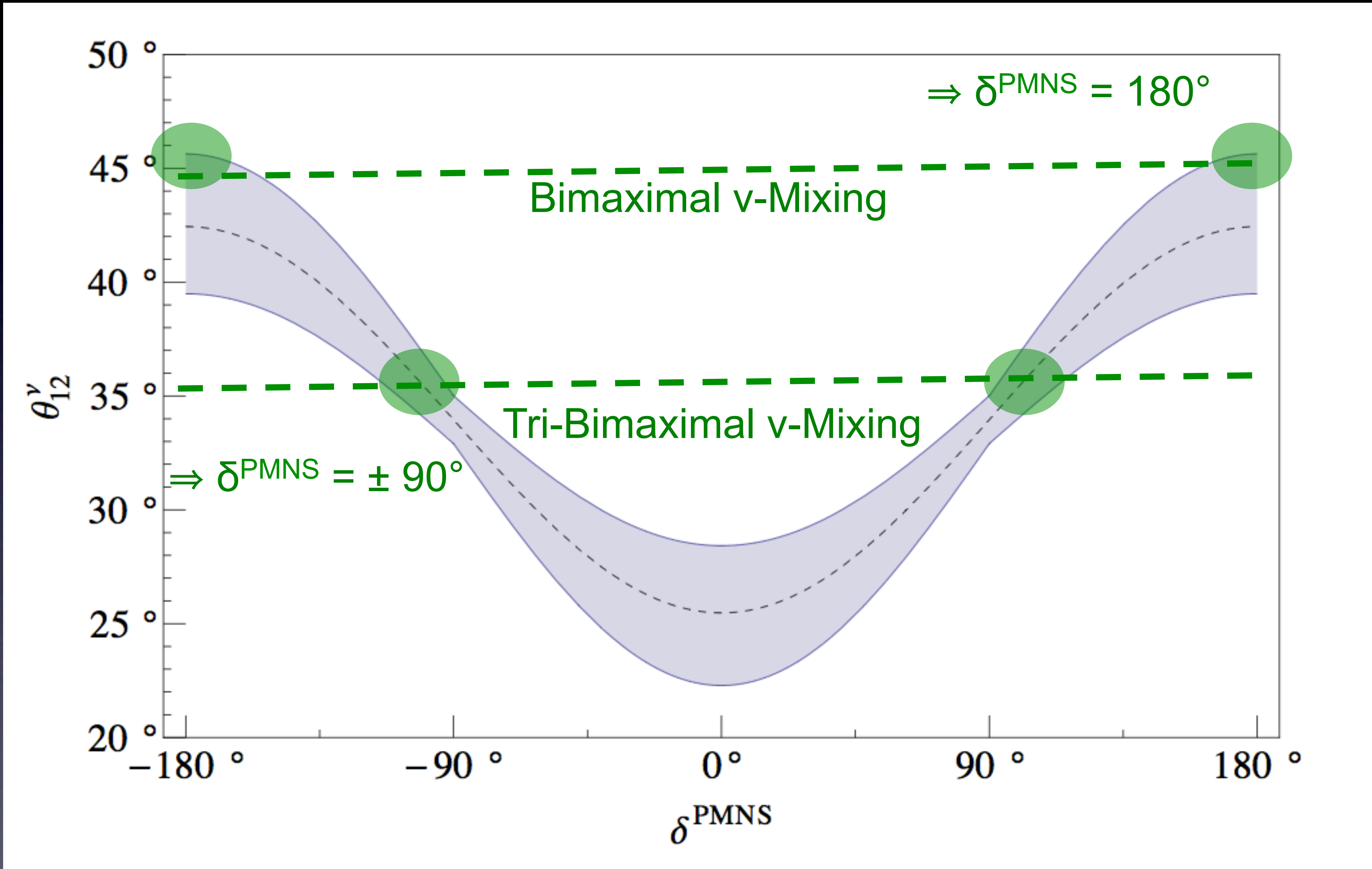
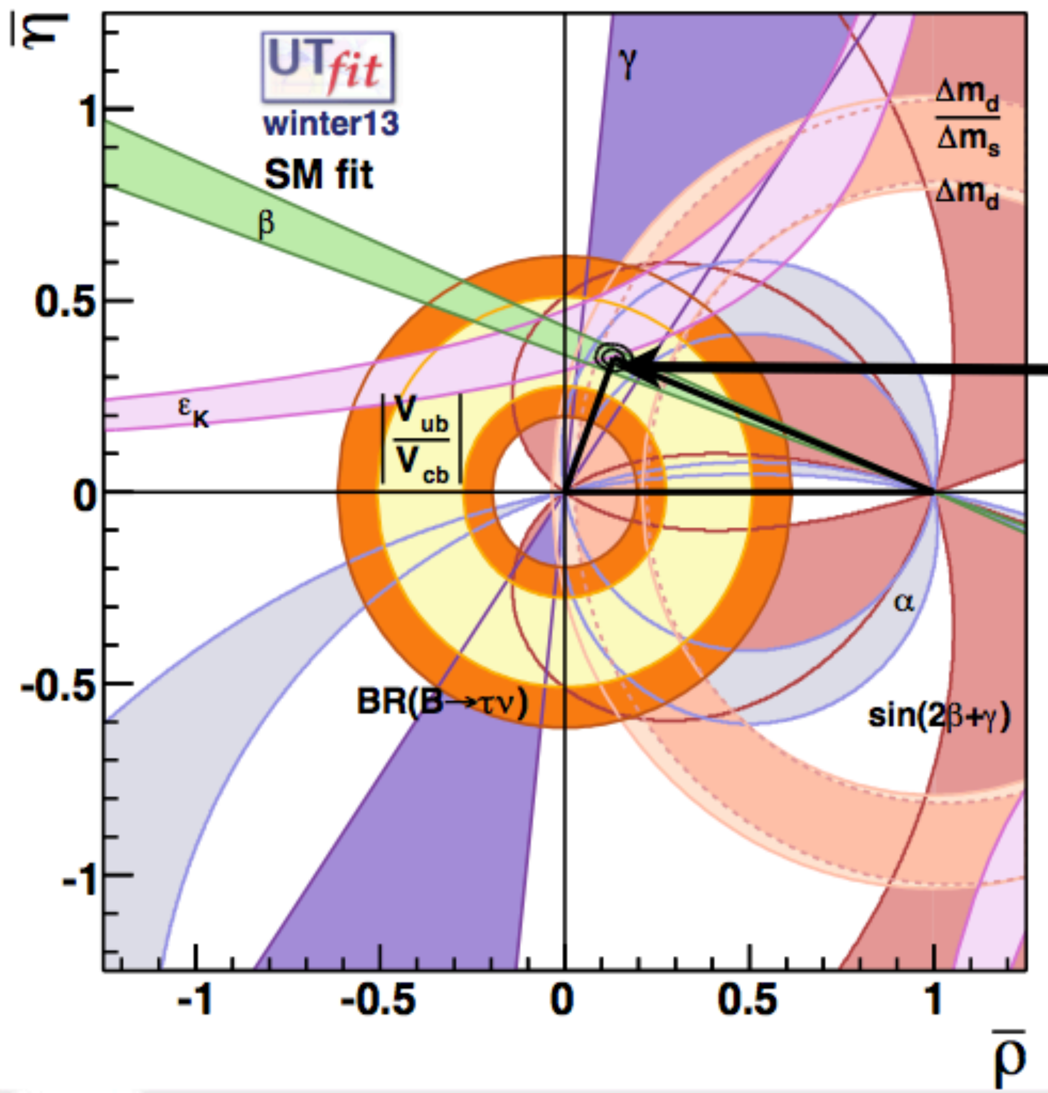


figure from S.A., Gross, Maurer, Sluka ('12)

The right-angled quark unitarity triangle from spontaneous CP breaking and discrete symmetries



$$(V_{\rm CKM}^\dagger V_{\rm CKM})_{bd} = 0$$

Fit result:

$$\alpha = (88.7 \pm 3.1)^\circ$$

[UTFit Winter '13 SM Fit]

Accident or sign of spontaneous CP violation?

- How can α ≈ 90° emerge from the quark mass matrices?

  → Simple ansatz: Assume that the 1-3 elements of the mass matrices are = 0 and that the mass matrices are hierarchical:

$$M_u = \begin{pmatrix} a_u & b_u & 0 \\ * & c_u & d_u \\ * & * & e_u \end{pmatrix} \qquad M_d = \begin{pmatrix} a_d & b_d & 0 \\ * & c_d & d_d \\ * & * & e_d \end{pmatrix}$$

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✓ ... then we obtain for α the "phase sum rule":

$$\boxed{\alpha \approx \delta_{12}^d - \delta_{12}^u} \qquad \delta_{12}^d \sim \arg\left(c_d b_d^*\right), \;\; \delta_{12}^u \sim \arg\left(c_u b_u^*\right)$$

S. A., King, Spinrath, Malinsky ('09)

- The “phase sum rule“ suggests a simple structure of the mass matrices, e.g. (with real a, b, c, d, e):

$$M_u = \begin{pmatrix} a_u & b_u & 0 \\ * & c_u & d_u \\ * & * & e_u \end{pmatrix} \qquad M_d = \begin{pmatrix} a_d & b_d & 0 \\ * & -i\,c_d & d_d \\ * & * & e_d \end{pmatrix}$$

A single purely imaginary element could be the origin of α = 90° and of CP violation in the SM !

Via GUT relations, this can also have an imprint on δ^{PMNS}

Discrete phases from spontaneous CP breaking and discrete symmetries

- Yukawas proportional to flavon vevs, e.g.

$$\langle\phi\rangle \propto (0,0,x)^T \quad \text{or} \quad \langle\phi\rangle \propto (x,x,x)^T$$

- Add term to W compatible with Z_n symmetry

$$\mathcal{W} \supset P\left(\kappa\frac{\phi^n}{\Lambda^{n-2}} \mp \lambda M^2\right)$$

- Solve F-term conditions ($|F_P|=0$) + CP symmetry*

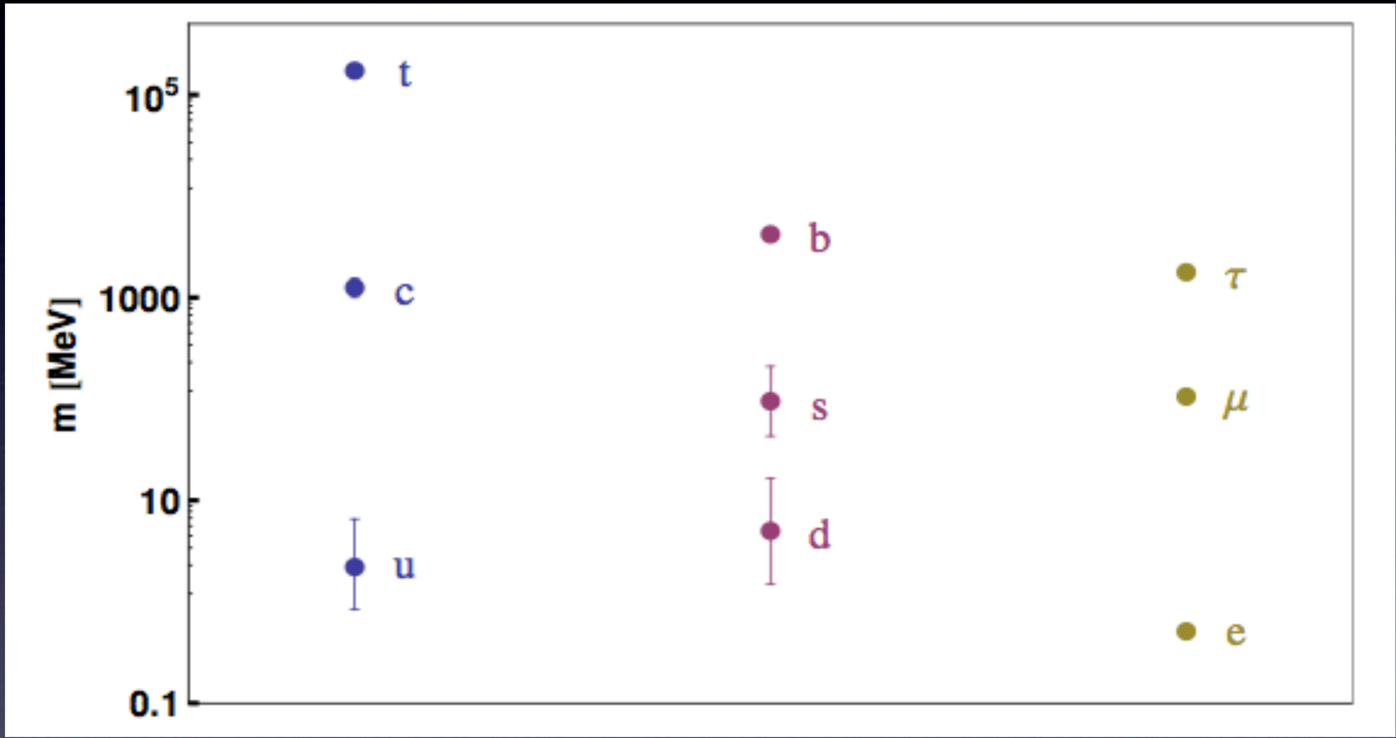
$$\arg(\langle\phi\rangle) = \arg(x) = \begin{cases} \frac{2\pi}{n}q\,, \quad q=1,\ldots,n & \text{for “−”} \\ \frac{2\pi}{n}q+\frac{\pi}{n}\,, \quad q=1,\ldots,n & \text{for “+”} \end{cases}$$

* We assume here that CP enforces κ and λ to be real.

S. A., S.F. King, C. Luhn, M. Spinrath (arXiv:1103.5930)

Quark-lepton mass relations

Masses of quarks and charged leptons



$m_b \leftrightarrow m_\tau$?

$m_s \leftrightarrow m_\mu$?

(running masses at the top-mass scale;
errors are 3 times the 1σ errors ...)

GUT predictions for quark-lepton mass ratios

- GUT predictions from fundamental operators in SU(5)

→ 3rd family masses from

$$y_{33}\ \bar{\mathbf{5}}_3\, \mathbf{10}_3 \,\langle \bar{H}_5 \rangle \ \Rightarrow\ \left.\frac{m_\tau}{m_b}\right|_{M_{GUT}} = 1$$

"b-τ unification"

→ 2nd family masses from

MSSM Higgs H_d in representation $\bar{H}_{45}$

$$y_{22}\ \bar{\mathbf{5}}_2\, \mathbf{10}_2 \,\langle \bar{H}_{45} \rangle \ \Rightarrow\ \left.\frac{m_\mu}{m_s}\right|_{M_{GUT}} = 3$$

Georgi, Jarlskog ('79)

GUT predictions for quark-lepton mass ratios

- New GUT predictions from effective operators, for example:
  - → For the 3rd family relation m_τ / m_b :

$$y_{33}\ \bar{\mathbf{5}}_3 \frac{\langle H_{24} \rangle}{\Lambda} \mathbf{10}_3 \langle \bar{H}_5 \rangle$$

The vev of the GUT-Higgs H_{24} breaks SU(5) to the SM gauge symmetry by $\langle H_{24} \rangle \sim v_{GUT}$ diag(2 , 2 , 2, -3 , -3)

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- **New GUT predictions** from effective operators, for example:
  - → For the 3rd family relation m_τ / m_b :

$$y_{33}\; \bar{\mathbf{5}}_3 \frac{\langle H_{24} \rangle}{\Lambda} \mathbf{10}_3 \, \langle \bar{H}_5 \rangle \;\Rightarrow\; \boxed{\frac{m_\tau}{m_b}\Big|_{M_{GUT}} = \frac{3}{2}}$$

  - → For the 2nd family relation m_μ / m_s:

$$y_{22}\; \bar{\mathbf{5}}_2 \frac{\langle H_{24} \rangle}{\Lambda} \mathbf{10}_2 \, \langle \bar{H}_{45} \rangle \;\Rightarrow\; \boxed{\frac{m_\mu}{m_s}\Big|_{M_{GUT}} = \frac{9}{2}}$$

$$y_{22}\; \bar{\mathbf{5}}_2 \, \langle \overline{H_5} \rangle \, \mathbf{10}_2 \frac{\langle H_{24} \rangle}{\Lambda} \;\Rightarrow\; \boxed{\frac{m_\mu}{m_s}\Big|_{M_{GUT}} = 6}$$

S. A., Spinrath (arXiv:0902.4644)