Photons in a cold axion background

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UZ, 28 October 2016

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EXCELENCIA MARÍA DE MAEZTU

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Introduction

Cold relic axions resulting from vacuum misalignment in the early universe is a popular and viable candidate for dark matter.

Provided that the reheating temperature after inflation is below the Peccei-Quinn transition, in later times the axion field evolves as

$$a(t) = a_0 \sin m_a t, \quad \mathbf{k} = 0 \qquad \rho \simeq a_0^2 m_a^2$$

$$\rho \simeq 10^{-10}\mathrm{eV}^4, \quad \rho^* \simeq 10^{-4}\mathrm{eV}^4 \quad (30 \text{ to } 100 \text{ kpc})$$

The axion background provides a very diffuse concentration of a pseudoscalar condensate that affects the propagation of particles coupled to it such as photons. Can it be detected *directly* ?

In this talk I will discuss several non-standard effects that *might* help.

Outline

- Introduction
- Propagation in a cold axion background
- Three physical effects
  1. Influence on cosmic rays - charged particles radiate spontaneously
  2. Momentum gaps: some photon wavelengths cannot exist
  3. Magnetic field in a cold condensate: changing some characteristics of the Primakoff effect

  If time allows I will briefly discuss another possible manifestation

  4. Bouncing off the axion wall: trapped photons
- Conclusions and outlook

Propagation of photons in a cold axion background

Let us consider electromagnetism in a background where Lorentz symmetry is broken by means of a time-like vector

$$\mathcal{L} = \mathcal{L}_{\rm INV} + \mathcal{L}_{\rm LIV}$$

$$\mathcal{L}_{\rm INV} = -\,\frac{1}{4}\,F^{\alpha\beta}F_{\alpha\beta} \qquad \mathcal{L}_{\rm LIV} = \frac{1}{2}\,m_V^2\,A_\mu\,A^\mu + \,\frac{1}{2}\,\eta_\alpha A_\beta \widetilde{F}^{\,\alpha\beta}$$

E.o.M.:

$$\left\{g^{\,\lambda\nu}\left(k^2 - m_V^2\right) + i\,\varepsilon^{\,\lambda\nu\alpha\beta}\,\eta_\alpha\,k_\beta\right\}\tilde{A}_\lambda(k) = 0$$

We can build two complex and space-like chiral polarization vectors $\varepsilon_\pm^\mu(k)$ which satisfy the orthonormality relations

$$-\,g_{\mu\nu}\,\varepsilon_\pm^{\mu\,*}(k)\,\varepsilon_\pm^\nu(k) = 1 \qquad g_{\mu\nu}\,\varepsilon_\pm^{\mu\,*}(k)\,\varepsilon_\mp^\nu(k) = 0$$

In addition we have

$$\varepsilon_T^\mu(k) \sim k^\mu \qquad \varepsilon_L^\mu(k) \sim k^2\eta^\mu - k^\mu\eta\cdot k$$

$$g_{\mu\nu}\,\varepsilon_A^{\mu\,*}(k)\,\varepsilon_B^\nu(k) \,=\, g_{AB} \qquad g^{\,AB}\,\varepsilon_A^{\mu\,*}(k)\,\varepsilon_B^\nu(k) \,=\, g^{\,\mu\nu}$$

Propagation of photons in a cold axion background

Let us now assume that $\eta_\alpha = \partial_\alpha a(t) = \eta\delta_{\alpha 0}$, $\eta > 0$

The polarization vectors of positive and negative chirality are solutions of the vector field equations if and only if

$$k^\mu_\pm = (\omega_{\mathbf{k}\pm}, \mathbf{k}) \qquad \omega_{\mathbf{k}\pm} = \sqrt{\mathbf{k}^2 + m_V^2 \pm \eta|\mathbf{k}|}$$

In order to avoid problems with causality we want $k^2_\pm \geq 0$.
Photons of positive chirality have no problems with causality
Photons of negative chirality exist as asymptotic states iff

$$|\mathbf{k}| < \frac{m_V^2}{\eta}$$

For $m_V = 0$ they cannot exist as asymptotic states. Changing $\eta \to -\eta$ exchanges the chirality of photons.

Consequences of Lorentz symmetry violation

As is known to everyone processes such as $e^- \to e^- \gamma$ or $\gamma \to e^+ e^-$ cannot occur beause of energy-momentum conservation *and* Lorentz symmetry. But now (for e.g. $\gamma \to e^+ e^-$)

$$\omega_{\mathbf{k}\pm} = \sqrt{\mathbf{k}^2 + m_\gamma^2 \pm \eta\, |\mathbf{k}|} = \sqrt{\mathbf{p}^2 + m_e^2} + \sqrt{(\mathbf{p} - \mathbf{k})^2 + m_e^2}$$

Possible iff

$$|\mathbf{k}| \geq \frac{4 m_e^2}{\eta} \equiv k_{\rm th} \qquad (m_\gamma = 0)$$

The electron-positron pairs will be created with a large momentum. Production of $p\bar{p}$ or $\mu\bar{\mu}$ pairs is largely disfavoured. E.g.

$$\frac{k_{\rm th}(\mu\bar{\mu})}{k_{\rm th}(e\bar{e})} \simeq (\frac{m_\mu}{m_e})^2 \simeq 4 \times 10^4$$

and an even larger threshold for $p\bar{p}$.

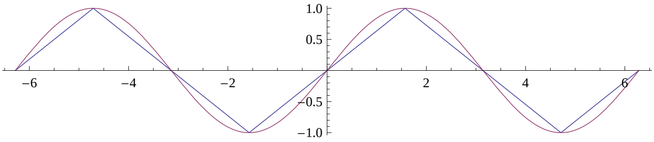
Lorentz violation from a cold axion background

Axion-photon coupling:

$$\Delta\mathcal{L} = -g_{a\gamma\gamma}\frac{\alpha}{\pi}\frac{a_0}{f_a}\cos(m_a t)\,\epsilon^{ijk}A_i F_{jk}$$

where $\partial_\alpha a(t) = (\eta(t), 0, 0, 0)$, and $\eta(t) = \eta_0 \cos m_a t$. Popular models such as DFSZ and KSVZ all give $g_{a\gamma\gamma} \simeq 1$.

If momenta are large $\mathbf{k} >> m_a$ it makes sense to treat the axion background adiabatically with a (quasiconstant) derivative



It makes sense to approximate the sinusoidal variation piecewise by a square profile $\eta(t) = \pm\eta_0$ with period $2\pi/m_a$

Bounds on the axion parameters

- Astrophysics
  Energy loss in stars due to $\gamma e \to ae$

$$f_a > 10^7 \text{ GeV}. \qquad (1)$$

- Cosmology
  Axions must not exceed total dark matter, $\Omega_{\text{dm}} h^2 = 0.112$

$$\Omega_a h^2 = \kappa_a \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{7/6} \Rightarrow f_a < 10^{11} \text{ GeV}$$

  κ_a depends on when inflation happens.
- Acceptable window for axion DM (PQ axions only):

$$10^7 \text{ GeV} < f_a < 10^{11} \text{ GeV} \;\Rightarrow\; 10^{-5} \text{ eV} < m_a < 0.1 \text{ eV}.$$

Astrophysical and experimental bounds

$$\Rightarrow |\eta| \simeq \alpha \frac{\sqrt{\rho_a}}{f_a} \simeq 10^{-20} - 10^{-24}\mathrm{eV}$$

η is the relevant quantity for all the effects discussed in this talk.

We assume $m_\gamma = 0$. This hypothesis could be easily relaxed if dealing with a plasma where $m_\gamma = \omega_p$

Everything is computed at tree level in QED but non-linearities such as the ones described by the Euler-Heisenberg effective lagrangian could be included.

Possible astrophysical consequences?

The polarizations $\varepsilon_+^\mu(k)$, $\varepsilon_-^\mu(k)$ correspond (approximately) to the usual ones of QED. light propagation in an axion background may be subject to modifications (dichroism). For visible light or radiowaves there is no marked separation of scales w.r.t m_a and the time variation of the background cannot be treated adiabatically and the net effect should average to zero over long distances (except for extremely light axions).

We need processes where $|\mathbf{k}| >> m_a$.
Cosmic rays-induced processes such as $p \to p\gamma$ or $e \to e\gamma$ are obvious candidates.
They are prompt processes; intuitively they should not be affected by a slight time variation of the background.
The net effect shall not average to zero.
A detailed calculation reveals that this is correct.

Axion-induced Bremsstrahlung in cosmic rays

$$p(\mathbf{p}) \to p(\mathbf{p}-\mathbf{k})\gamma(\mathbf{k})$$

Energy conservation:

$$\sqrt{E^2+k^2-2pk\cos\theta}+\sqrt{k^2\pm\eta k+m_\gamma^2}-E=0, \qquad \eta>0$$

Kinematical constraints:

Let us first consider the case $m_\gamma = 0$ (but note that η is small).

$$p_{th}=0$$

$$k_{min}=\eta, \qquad \text{for} \cos\theta = -\eta/2p$$

$$k_{max}=\frac{E^2}{p+\frac{m_p^2}{\eta}}, \qquad \text{for} \cos\theta = 1$$

$$k_{max} \simeq E \text{ for } E \gg m_p^2/|\eta| \qquad k_{max} \simeq |\eta| E^2/m_p^2 \text{ for } E \ll m_p^2/|\eta|$$

Let us now consider $m_\gamma > 0$

$$p_{th} \simeq \frac{2m_\gamma m_p}{\eta}$$

$$k(\theta_{max}) \simeq \frac{2m_\gamma^2}{\eta}(1 - 3\frac{pm_\gamma^2}{E^2\eta}) \xrightarrow{p>>p_{th}} \frac{2m_\gamma^2}{\eta}, \qquad \sin^2\theta_{max} \to \frac{\eta^2}{4m_\gamma^2}$$

θ_{max} is small, photons are emitted in a narrow cone

In the opposite extreme, for zero angle there are two solutions

$$k_+(0) \simeq \frac{E^2\eta + pm_\gamma^2 + E\sqrt{E^2\eta^2 - 4m_p^2m_\gamma^2 + 2p\eta m_\gamma^2}}{2p\eta + 2m_p^2} \xrightarrow{p>>p_{th}} \frac{E^2}{p + \frac{m_p^2}{\eta}}$$

which is the same result obtained before, and

$$k_-(0) \simeq \frac{E^2\eta + pm_\gamma^2 - E\sqrt{E^2\eta^2 - 4m_p^2m_\gamma^2 + 2p\eta m_\gamma^2}}{2p\eta + 2m_p^2} \xrightarrow{p>>p_{th}} \frac{m_\gamma^2}{\eta}$$

$$k_-(0) < k(\theta_{max}) < k_+(0)$$

Differential emission rate:

$$d\Gamma(Q) = (2\pi)^4 \delta^{(4)}(q + k - p) \frac{1}{2E} \overline{|M|^2} dQ$$

$$d\Gamma(Q) = \frac{\alpha}{2} \frac{|\mathbf{k}|}{|\mathbf{p}|} \frac{1}{E\omega_{\mathbf{k}}} (-p \cdot k + |\mathbf{p}|^2 \sin^2\theta) d|\mathbf{k}|$$

Rate of energy loss:

$$\frac{dE}{dx} = -\frac{1}{v} \int d\Gamma(Q) w(Q)$$

$$\frac{dE}{dx} = -\frac{\alpha}{2} \frac{1}{p^2} \int k dk [-\frac{1}{2}(m_\gamma^2 + \eta k) + p^2(1 - \cos^2\theta)]$$

There are two relevant limits

$$E \ll \frac{m_p^2}{|\eta|} \longrightarrow \frac{dE}{dx} = -\frac{\alpha \eta^2 E^2}{4 m_p^2}.$$

$$E \gg \frac{m_p^2}{|\eta|} \longrightarrow \frac{dE}{dx} = -\frac{\alpha |\eta|}{3} E$$

The axion shield

There are two key scales in this problem

$$E_{th} \simeq 2m_\gamma m_p/\eta \qquad \text{and} \qquad m_p^2/\eta$$

If $E \gg m_p^2/|\eta|$

$$E(x) = \exp -\frac{\alpha|\eta|}{3} x$$

For the expected values of η this would give a mean free path $< \mathcal{O}(10)$ kpc.

This would imply that cold axions act as a powerful shield againts very energetic cosmic rays. This *would* in fact impose a rather stringent bound on the combination $\sqrt{\rho_a}/f_a$.

The axion (very weak) shield

However, this is not so because even for the most energetic cosmics, just below the GZK cut-off of 10^{20} eV, we are well below the cross-over scale $m_p^2/|\eta|$.

In this regime the expression for $E(x)$ is

$$E(x) = \frac{E(0)}{1 + \frac{\alpha\eta^2}{4m_p^2}E(0)x}$$

It is peculiar to see that for extremely large distances $E(x) \sim \frac{1}{x}$

From the obvious fact that we detect (likely) extragalactic rays of large energy we can set at present the largely irrelevant bound

$$\eta < 10^{-14}\ \mathrm{eV}$$

However this does not mean that the whole effect is irrelevant.
Consider the radioemission from the Bremsstrahlung.
For $m_\gamma = 10^{-18}$ eV and $\eta = 10^{-20}$ eV the emitted photon momenta fall in the range

$$10^{-16}\ \mathrm{eV}(0.024\ \mathrm{Hz}) < k < 100\ \mathrm{eV}(24\ \mathrm{PHz})$$

for primary protons and

$$10^{-16}\ \mathrm{eV} < k < 400\ \mathrm{MeV}$$

for primary electrons.
Spectrum of emission (per unit time)

$$\int_{E_{min}}^{E_{GZK}} dE\, n(E) \frac{d\Gamma}{dk} \qquad E_{min} = \sqrt{\frac{m^2 k}{\eta}} > E_{th} \qquad E_{th} = 2\frac{m_{p,e} m_\gamma}{|\eta|} \simeq 0$$

Broadcasters:

Proton primaries

$$n(E) = N \times \begin{cases} E^{-2.68} & 10^9 \le E \le 4 \cdot 10^{15} \\ 1.12 \cdot 10^{19} E^{-3.26} & 4 \cdot 10^{15} \le E \le 4 \cdot 10^{18} \\ 3.85 \cdot 10^{-4} E^{-2.59} & 4 \cdot 10^{18} \le E \le 2.9 \cdot 10^{19} \\ 7.34 \cdot 10^{29} E^{-4.3} & E \ge 2.9 \cdot 10^{19} \end{cases}$$

Electron (+positrons) primaries

$$n(E) = N \times \begin{cases} 0.01 E^{-2.68} & E \le 5 \cdot 10^{10} \\ 71.1 E^{-3.04} & E \ge 5 \cdot 10^{10} \end{cases}$$

Units: $\text{eV}^{-1}\ \text{m}^{-2}\ \text{s}^{-1}\ \text{sr}^{-1}$.

Need to assume a given function $t(E)$ and combine with isotropy hypothesis to find the photon yield.

The flux of photons is

$$\frac{d^3 N_\gamma}{dkdSdt} = \int_{E_{\min}(k)>E_{th}}^{\infty} dE\ t(E)J(E)\frac{d\Gamma(E,k)}{dk}\ , \quad E_{min}(k) = \sqrt{\frac{m^2 k}{\eta}}$$

$t(E)$ is approximately constant: $t(E) \approx T_p = 10^7$ yr for protons. $t(E) \approx T_e = 5 \cdot 10^5$ yr for electrons in average, but it is not constant:

$$t(E) \sim 1/E$$

The photon energy flux is obtained by multiplying the photon flux by the energy of a photon with momentum k:

$$I(k) = \omega(k) \int_{E_{min}(k)>E_{th}}^{\infty} dE\ t(E)J(E)\frac{d\Gamma}{dk}$$

$$\approx \frac{\alpha T}{8k} \int_{E_{min}(k)}^{\infty} dE\ N_i \left[A(k)E^{-\gamma_i} + B(k)E^{-(\gamma_i+1)} + C(k)E^{-(\gamma_i+2)} \right]$$

Only the first term is important; it is dominated by E_{min}.

Axion-induced radioemission

Radiation flux intensity

$$I_\gamma^p(k) \simeq \frac{\alpha\eta T}{2}\frac{J_p(E_{min}(k))E_{min}(k)}{\gamma_{min}-1}$$

and

$$I_\gamma^e(k) \simeq \frac{\alpha\eta T_0}{2}\frac{J_e(E_{min}(k))}{\gamma_{min}}$$

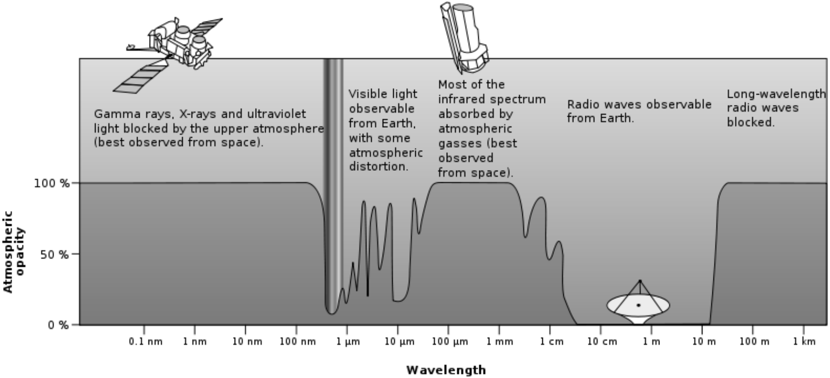
Energies are all expressed in eV. The value γ_{min} is determined by the cosmic ray flux in a given range of E.
The dominant contribution comes from electrons

$$I_\gamma^e(k) \simeq 3\times 10^2 \times \left(\frac{\eta}{10^{-20}\ \mathrm{eV}}\right)^{2.52}\left(\frac{k}{10^{-7}\ \mathrm{eV}}\right)^{-1.52}\ \mathrm{m}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}$$

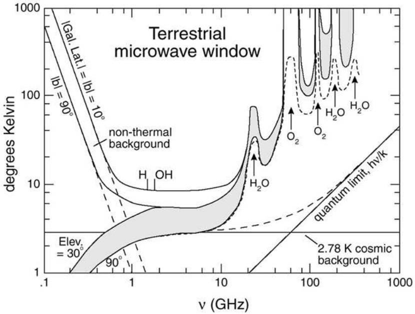
For protons

$$I_\gamma^p(k) \simeq 6\times\left(\frac{T}{10^7\ \mathrm{yr}}\right)\left(\frac{\eta}{10^{-20}\ \mathrm{eV}}\right)^{1.84}\left(\frac{k}{10^{-7}\ \mathrm{eV}}\right)^{-0.84}\ \mathrm{m}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}$$

Atmosphere opacity



Galactic noise



Unit in radio astronomy: 1 Jy $= 10^{-26}$ W Hz^{-1} m^{-2} sr^{-1}
$\simeq 1.5 \times 10^7$ eV eV^{-1} m^{-2} s^{-1} sr^{-1}

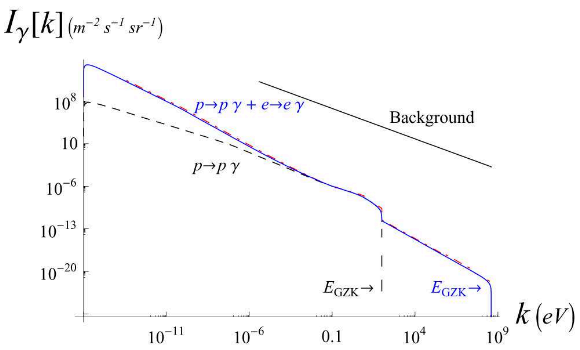


Figure : Energy radiated as a function of the wave vector.

Galactic noise

The window $\lambda = 10$ cm (3 GHz) to $\lambda = 100$ m (30 MHz) corresponds to 10^{-5} eV to 10^{-8} eV.
This region has a strong background from galactic noise from synchrotron radiation. In the 100 MHz region the signal is 9 orders of magnitude below the background.
However

- Sensitivity of planned antennas may be as low as $10^{-12}\times$ background
- Galactic magnetic field $\mathbf{H}$ ranges from $\sim \mu$G to $\sim$ mG and background $\sim \mathbf{H}^2$
- The power dependence of the electron yield is different from the SR in the galactic plane
- Regions of low magnetic field and high galactic latitude are to be explored
- Polarization is different

Antennas

LWA

- New Mexico (deployed). Sensitivity down to 30 MHz and 10^{-4} Jy

10^{-4} Jy $\simeq 10^{3}$ m^{-2} s^{-1} sr^{-1}

maybe even less depending on extension

SKA

- SA+Australia, under construction. Sensitivity down to 70 MHz and 650 nJy

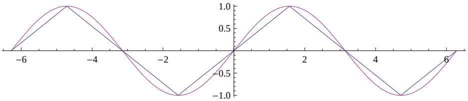
at the lowest frequency assuming an integration time of 50hrs

Far side of the Moon

- Not limited by atmosphere opacity
- Designs exist (ESA) reporting sensitivities down to 10^{-5} Jy

Sensitivity of antennas is not an issue, but the background is a tough enemy.

$a(t)$ changes sign with a period $2\pi/m_a$. Let us approximate the sinusoidal variation and solve exactly for the propagating modes



The equation for $\hat{A}_\nu(t, \vec{k})$ is

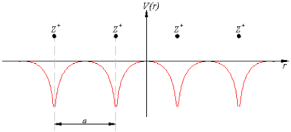
$$\left[g^{\mu\nu}(\partial_t^2 + \vec{k}^2) - i\epsilon^{\mu\nu\alpha\beta}\eta_\alpha k_\beta\right] \hat{A}_\nu(t, \vec{k}) = 0$$

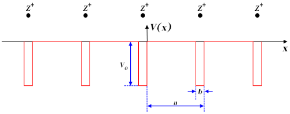
$$\hat{A}_\nu(t, \vec{k}) = \sum_{\lambda=+,-} f_\lambda(t)\varepsilon_\nu(\vec{k}, \lambda)$$

We write $f(t) = e^{-i\omega t}g(t)$ and demand that $g(t)$ have the same periodicity as $\eta(t)$.

Forbidden wavelengths

There is a similarity with the familiar 1D Kronig-Penney model exchanging space and time





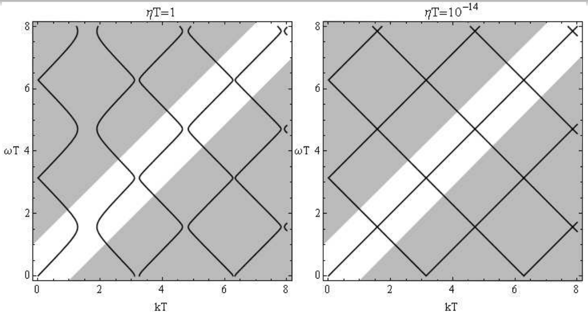
Defining

$$\alpha^2 = k^2 + \eta_0 k, \qquad \beta^2 = k^2 - \eta_0 k$$

Matching condition of the wave functions and their derivatives:

$$\cos(2\omega T) = \cos(\alpha T)\cos(\beta T) - \frac{\alpha^2 + \beta^2}{2\alpha\beta}\sin(\alpha T)\sin(\beta T) \qquad T = \frac{\pi}{M_a}$$

Forbidden wavelengths



Position and width of the gaps

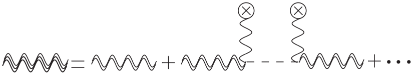
$$k_n = \frac{n m_a}{2}, \quad n \in \mathbb{N}; \qquad \Delta k \sim \begin{cases} \dfrac{\eta_0}{n\pi} & \text{for } n \text{ odd} \\ \\ \dfrac{\eta_0^2}{2 n m_a} & \text{for } n \text{ even} \end{cases}$$

Some photon wavelengths are forbidden in the universe if there is a cold axion background.

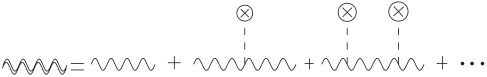
Could this be seen in table-top experiments?

Adding a magnetic field

If $\eta_0 = 0$ the theoretical technology is well known. Used to analyze the results of CAST, ADMX, ALPS ....



Interaction with the cold axion background implies that we need to take into account



Relevant parameters

$$b = 2 g_{a\gamma\gamma} \frac{\alpha}{\pi} \frac{B}{f_a} \qquad \eta_0 = 2 g_{a\gamma\gamma} \frac{\alpha}{\pi} \frac{a_0 m}{f_a}$$

assuming $f_a = 10^7$ GeV

$$B = 10 \text{ T} \Rightarrow b \leq 10^{-15}\,\text{eV} \qquad \eta_0 \leq 10^{-20}\,\text{eV}$$

Photon propagation in a CAB and a magnetic field

Photon propagator

$$D^{ij}(\omega,k) = -i\left(\frac{P_+^{ij}}{\omega^2-k^2-\eta_0 k}+\frac{P_-^{ij}}{\omega^2-k^2+\eta_0 k}\right) -i\omega^2\frac{b^i b^j}{(\omega^2-k^2)[(\omega^2-k^2)(\omega^2-k^2-m_a^2)-\omega^2 b^2]}.$$

$P_\pm^{ij}$: helicity projectors

Ellipticity and rotation

With the propagator, we can compute the evolution of a photon wave

- Initially, linear polarisation at an angle β with respect to $\vec{B}$.
- After a distance x the angle of the polarisation plane is

$$\alpha(x) \approx \beta - \frac{\eta_0 x}{2} - \frac{\epsilon}{2} \sin 2\beta$$

and an ellipticity appears: $e = \frac{1}{2} \left| \varphi \sin 2\beta \right|$,

$$\epsilon \approx -\frac{\omega^2 b^2}{m_a^4} \left(1 - \cos \frac{m_a^2 x}{2\omega} \right), \quad \varphi \approx \frac{\omega^2 b^2}{m_a^4} \left(\frac{m_a^2 x}{2\omega} - \sin \frac{m_a^2 x}{2\omega} \right).$$

To achieve long distances in small volumes: bouncing between mirrors.

Polarimetric experiments

Effects of the magnetic field $\vec{B}$ (well known, e.g. PVLAS experiment)

$$\Delta\beta = -\frac{\epsilon}{2}\sin 2\beta$$

- Proportional to $\sin 2\beta$.
- $\epsilon < 0$: the rotation always increases the angle.
- It can be accumulated when bouncing.

Polarimetric experiments

Effects of the CAB (new!)

$$\Delta\beta = -\frac{\eta_0 x}{2}$$

- A net rotation independent of the initial angle. It tends to cancel.
- If the distance betweeen mirrors is tuned to $L = \pi m_a^{-1}$, η changes sign when the light bounces.
- Tuning to the axion mass is a common feature of axion experiments.

Polarimetric experiments

For a distance $x = \mathcal{N} L$,

$$\frac{|\eta_0| x}{2} = g_{a\gamma\gamma} \frac{2\alpha}{\pi} \frac{\sqrt{2\rho}}{f_a m_a} \mathcal{N} \simeq 10^{-18}\ \mathrm{eV}^{-2} \times g_{a\gamma\gamma} \times \sqrt{\rho} \times \mathcal{N}.$$

Depends on

- The coupling $g_{a\gamma\gamma} \sim \mathcal{O}(1)$
- The local axion density $\rho \simeq 10^{-4} \mathrm{eV}^4$
- The combination $f_a m_a = 6 \times 10^{15}\ \mathrm{eV}^2$

Each bounce: increment of 10^{-20}. Finesse of $\mathcal{N} = 10^6$ is feasible. Observation of this rotation would reveal the existence of the CAB.

Recall the modification to QED brought about by an axion-like background

$$\Delta\mathcal{L} = \frac{1}{2}\eta_\alpha A_\beta \tilde{F}^{\alpha\beta}$$

This piece changes slightly the dispersion relation of photons. We will now explore different possible axion backgrounds (other than the cold background oscillating in time with period $\sim 1/m_a$)

$$-\tfrac{1}{4}\, F^{\mu\nu}(x)\widetilde{F}_{\mu\nu}(x)\,\zeta_\lambda x^\lambda\,\theta(-\zeta\cdot x) \leftrightarrow \tfrac{1}{2}\,\zeta_\mu A_\nu(x)\widetilde{F}^{\mu\nu}(x)\,\theta(-\zeta\cdot x),$$

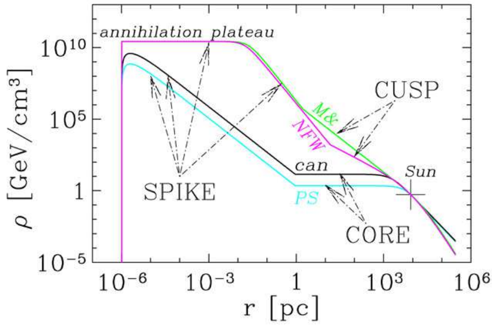
This associates a space-like boundary with a space-like CS vector

$$\zeta_\mu = \zeta \times (0, \vec{a}) \quad |\vec{a}| = 1$$

(LIV vector renamed from η_μ to ζ_μ to avoid confusion with CAB)

Crossing the boundary

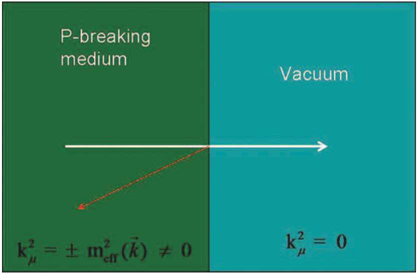
Compact dense stars filled by axions with density degrading to their surface?
Galactic dark matter profiles?



Crossing the boundary

Even if not totally realistic let us use a linearly varying background (it can be solved easily)

For simplicity let us place the boundary "wall" in the $\hat{X}$ direction



Matching on the boundary $\zeta \cdot x = 0$

$$\delta(\zeta \cdot x)\left[\, A^\mu_{\rm vacuum}(x) - A^\mu_{\rm CS}(x) \,\right] = 0$$

Crossing the boundary

Abnormal dispersion laws for different polarizations in the parity broken phase

$$\begin{cases} k_{1L} = k_{10} = \sqrt{\omega^2 - m^2 - k_\perp^2} \\ k_{1+} = \sqrt{\omega^2 - m^2 - k_\perp^2 + \zeta\sqrt{\omega^2 - k_\perp^2}} \\ k_{1-} = \sqrt{\omega^2 - m^2 - k_\perp^2 - \zeta\sqrt{\omega^2 - k_\perp^2}} \end{cases}$$

Usual dispersion law in the normal phase

$$k_1 = \sqrt{\omega^2 - m^2 - k_\perp^2}$$

Different dispersion relations lead to non-trivial reflection and transmision coefficients

Crossing the boundary

$$M^2 \equiv k_\mu k^\mu = m^2 - \zeta\sqrt{\omega^2 - k_\perp^2}$$

$$k_{1L} = \sqrt{\frac{(M^2-m^2)^2}{\zeta^2} - m^2} \qquad k_{1\pm} = \sqrt{\frac{(M^2-m^2)^2}{\zeta^2} - M^2}$$

In this notation the $\pm$ dispersion relations apparently coincide but M^2 has different domains of definition

$$M_+^2 < \left(\sqrt{m^2 + \frac{\zeta^2}{4}} - \frac{\zeta}{2}\right)^2 \quad M_-^2 < \left(\sqrt{m^2 + \frac{\zeta^2}{4}} + \frac{\zeta}{2}\right)^2$$

Then

$$\kappa_{ref}(M^2) = \frac{\left|\sqrt{\frac{(M^2-m^2)^2}{\zeta^2} - M^2} - \sqrt{\frac{(M^2-m^2)^2}{\zeta^2} - m^2}\right|}{\left|\sqrt{\frac{(M^2-m^2)^2}{\zeta^2} - M^2} + \sqrt{\frac{(M^2-m^2)^2}{\zeta^2} - m^2}\right|}$$

Photon escaping from the axion sphere.

Recall $M^2/\zeta = m^2/\zeta^2 - \sqrt{\omega^2 - k_\perp^2}$

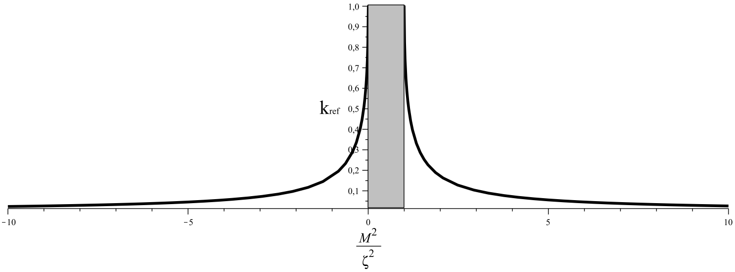


Figure : Reflection coefficient for photons ($m = 0$) escaping. The kinematically forbidden region is shaded

Crossing the boundary

In the context of axion physics the effect seems to depend crucially on a ratio of two numbers that are both small: m_V and ζ

It is also quite interesting to study to photons attempting to enter the axion-sphere.

The astrophysical consequences of the above results yet to be worked out...

Summary

Propagation of photons, electrons, protons,... in a pseudoscalar background is well described by a LIV version of QED. There are no hidden assumptions or model dependences of any kind in the predictions.

- Properties are rather unfamiliar
- The dispersion relation is modified and this makes possible processes such as $\gamma \to e^+ e^-$ or $p \to p\gamma$
- A background of cold axions has unexpected consequences on cosmic ray propagation
- CR emit circularly polarized light due to this effect. Backgrounds are a serious problem
- Photons in the universe have not well defined frequencies and some wave lengths are forbidden
- There is a small rotation in the polarization plane of photons with peculiar properties