Quantum Chromodynamics and the Renormalization Theory

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Outline

- Introduction/Motivation
- Objectives
- Method
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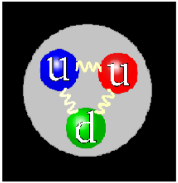

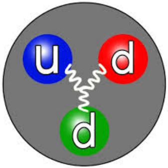

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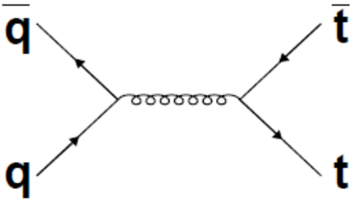
- There are 6 flavors of quarks: u, d, c, s, t, b together with their antiparticles
- The top quark t is the heaviest elementary particle observed so far.

Motivation

The top quark is produced in high energy scattering processes between protons (quarks) at Tevatron, LHC.



It decays into W bosons and b quarks, further generating jets of gluons and quarks. Due to its enormous mass it cannot form composite hadrons, but only individual quarks, which is a rare opportunity of studying these elementary particles.

Objectives

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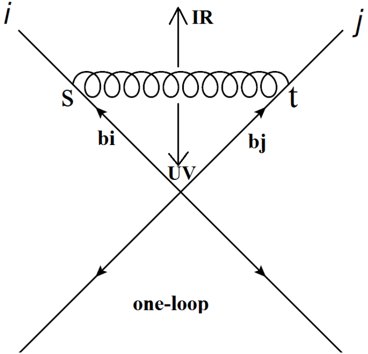
- Theoretical treatment of scattering processes, involving exchange of gluons between external massive partons
- Analytically computing the scattering amplitude (probability), which can be compared with the experimental data
- As a theoretical simplification we consider soft gluons, which is the case of top quark experiment
- Main problem: encounter of infinite terms (singularities), removed by renormalization: main goal

Method

- External particles are replaced by semi-infite Wilson lines, pointing in direction $\beta_i = p_i/s$, where p is the 4-momentum.
- Massive particles imply non-lightlike Wilson lines, i.e. $\beta_i^2 < 0$.

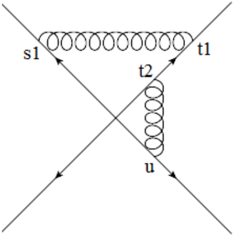
$$1_{ij} = \mathcal{F}_{ij}(\gamma_{ij})\; T_i^a\; T_j^a$$

$$\gamma_{ij} = 2\,\frac{\beta_i \cdot \beta_j}{\sqrt{\beta_i^2 \beta_j^2}}$$



Method

A web W is a set of diagrams D, obtained by permuting the way the gluons attach to the Wilson lines:



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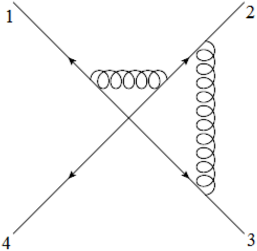

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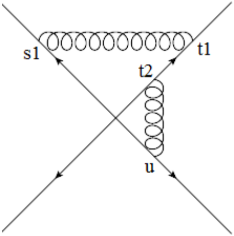

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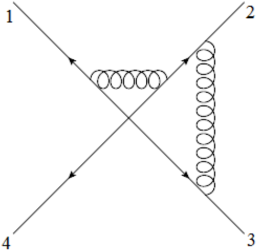

(2b)

$$W^{(n)} = \sum_{D} \mathcal{F}(D)\, R\, C(D); \qquad W^{(n)} = \sum_{k=0}^{n} W^{(n,-k)}\, \epsilon^{-k}$$

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Wilson line and exponential regulator gives the Feynman rule:

$$(ig_s)\beta_i^{\mu} \int_0^{\infty} ds\, e^{-m\,s\sqrt{-\beta_i^2}}$$

$$D_{\mu\nu} = -\mathcal{N} g_{\mu\nu}\,(-x^2)^{\epsilon-1}$$

To renormalize S we have to determine the factor Z (infinite), expressed as:

$$\Gamma = -\frac{d \ln Z(\mu, \epsilon)}{d \ln \mu} = -\frac{d \ln S_{ren.}(\mu, m)}{d \ln \mu}$$

where Γ is called the soft anomalous dimension and it is actually finite. Recall that μ is the renormalization scale.

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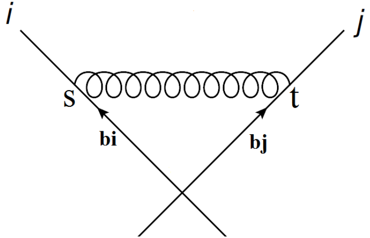
At one-loop and two-loop orders, Γ is given by:

$$\Gamma^{(1)} = -2\, W^{(1,-1)}$$
$$\Gamma^{(2)} = -4 W^{(2,-1)} - 2[W^{(1,-1)}, W^{(1,0)}]$$
$$\Gamma^{(3)} = -6 W^{(3,-1)} + 3\,[W^{(2,0)}, W^{(1,-1)}] + 3\,[W^{(1,0)}, W^{(2,-1)}] + [...]$$

Results

At one-loop order, the kinematic factor is:

$$\mathcal{F}_{ij}^{(1)}(\gamma_{ij}) = \mu^{2\epsilon} g_s^2 \mathcal{N} \beta_i \cdot \beta_j \int_0^\infty ds \int_0^\infty dt \, e^{-m(s\sqrt{-\beta_i^2}+t\sqrt{-\beta_j^2})} \left(-(s\beta_i - t\beta_j)^2\right)^{\epsilon-1}$$



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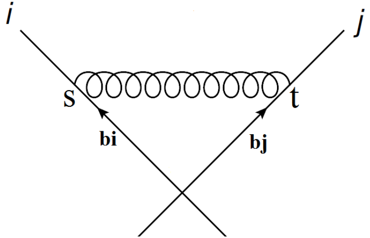


$$\mathcal{F}^{(1,-1)}(\gamma_{ij}) = \frac{g_s^2}{16\pi^2\epsilon} \frac{1+y_{ij}^2}{1-y_{ij}^2} \ln(y_{ij}); \qquad -\gamma_{ij} = y_{ij} + \frac{1}{y_{ij}}$$

Results

At one-loop order, we can compute $\Gamma^{(1)}$, using:

$$W^{(1,-1)} = \mathcal{F}^{(1,-1)}(\gamma_{ij})\, T_i^a T_j^a$$

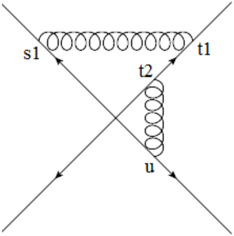
obtaining the result:

$$\Gamma^{(1)} = -\frac{g_s^2}{8\pi^2}\,\frac{1+y_{ij}^2}{1-y_{ij}^2}\,\ln(y_{ij})\, T_i^a T_j^a$$

Adrian Bodnarescu, "Renormalization of multiparton scattering amplitudes", Buletinul Institutului Politehnic Iasi. These results can be compared with: A. Mitov, G. F. Sterman, I. Sung, Phys. Rev. D 82, 034020 (2010), working in coordinate space.

Results

At two-loop order, we consider the two diagrams 2a, 2b, composing the web W_{121}, related by the symmetry $\gamma_{12} \leftrightarrow \gamma_{23}$:



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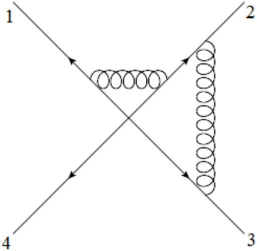

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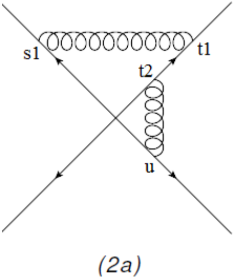
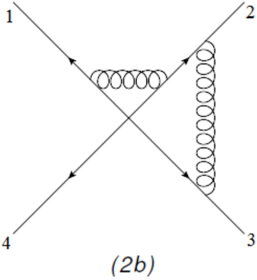


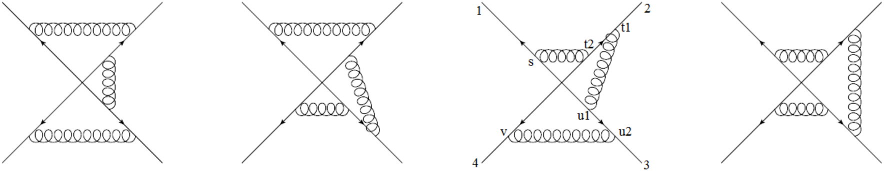


This web is given by:

$$W_{121}^{(2)} = \frac{1}{2}\left(\mathcal{F}^{(2)}(2a) - \mathcal{F}^{(2)}(2b)\right)\left(C(2a) - C(2b)\right)$$

Results

At three loop-order consider the web W_{1221}:

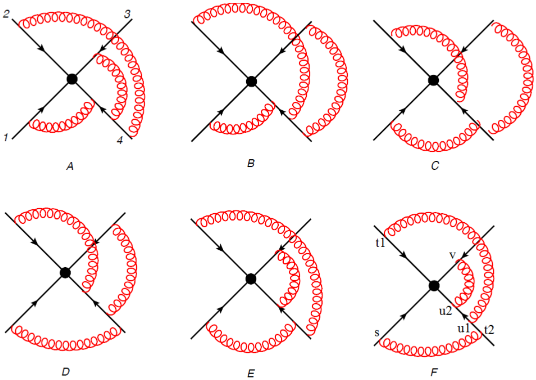


(3a) (3b) (3c) (3d)

$$W_{1221}^{(3)} = \frac{1}{6}\Big(\mathcal{F}(3a) - 2\,\mathcal{F}(3b) - 2\,\mathcal{F}(3c) + \mathcal{F}(3d)\Big)\, C$$

Results

At three-loop order, we consider also the web W_{1113}:



which are all related by symmetry relations: e.g. for E, F: $\gamma_{14} \leftrightarrow \gamma_{24}$; for D, F: $\gamma_{24} \leftrightarrow \gamma_{34}$; etc.

We compute the *kinematic factor* for the diagram 3F, using the previous Feynman rule:

$$\mathcal{F}(3F) = g_s^6 \mu^{6\epsilon} \mathcal{N}^3 (\beta_1 \cdot \beta_4)(\beta_2 \cdot \beta_4)(\beta_3 \cdot \beta_4) \int_0^\infty ds \int_0^\infty dt_1 \int_0^\infty dt_2 \int_0^\infty du_1 \int_0^\infty du_2 \int_0^\infty dv \, e^{-m(\sqrt{-\beta_1^2}s + \sqrt{-\beta_2^2}t_1 + \sqrt{-\beta_4^2}(u_1+u_2+t_2) + \sqrt{-\beta_3^2}v)} [(s\beta_1 - t_2\beta_4)^2 (t_1\beta_2 - u_1\beta_4)^2 (u_2\beta_4 - v\beta_3)^2]^{\epsilon-1} \, \Theta(t_2 - u_1) \, \Theta(u_1 - u_2).$$

The web W_{1113} is obtained by applying the mixing matrices method:

$$W_{1113} = \frac{1}{6}\Big(- \mathcal{F}(3A) + 2\,\mathcal{F}(3B) - \mathcal{F}(3C) - \mathcal{F}(3D) - \mathcal{F}(3E) + 2\,\mathcal{F}(3F) \Big)$$

The contribution to the soft anomalous dimension of third order $\Gamma^{(3)}$ is:

$$W_{1113}^{(3,-1)} = g_s^6 \frac{\mathcal{N}^3}{48\epsilon} \gamma_{14}\gamma_{24}\gamma_{34} \int_0^1 dxdydz P_0(x,\gamma_{14}) P_0(y,\gamma_{24}) P_0(z,\gamma_{34})$$
$$\left[4 \ln\left(\frac{y}{x}\right) \ln\left(\frac{z}{y}\right) + \ln^2\left(\frac{z}{x}\right) - \frac{7}{4} \ln^2\left(\frac{y}{x}\right) + \frac{11}{4} \ln^2\left(\frac{z}{y}\right) + \right.$$
$$\left. + 9\,\mathrm{Li}_2\left(-\frac{z}{y}\right) - 9\,\mathrm{Li}_2\left(-\frac{y}{x}\right)\right] \frac{1}{6} C_1,$$

where the propagator function $P_0(x,\gamma_{ij}) = [1 - 4x(1-x]\alpha_{ij}]^{-1}$, for $\alpha_{ij} = 1/2 + \gamma_{ij}/4$.

A. Bodnarescu, Rom. J. Phys. 60, 1052–1067 (2015). This method is described in: E. Gardi, J. M. Smillie, C. D. White, JHEP 1109, 114 (2011), where different sets of diagrams were considered.

Conclusions

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- All these higher order corrections are necessary to obtain more accurate theoretical predictions, which are compared with the experimental data, in order to test the proposed theories.

Bibliography

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