Review of High Frequency Impedance

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Introduction

In low emittance rings, in colliders, in linac-based X-ray free electron lasers (FELs), the impedance/wakes of the vacuum chamber objects can lead to performance degradation. They can cause *e.g.* emittance growth, instabilities, heating, …

With a typical beam pipe radius of a~ 10-20 mm, high frequency can mean:

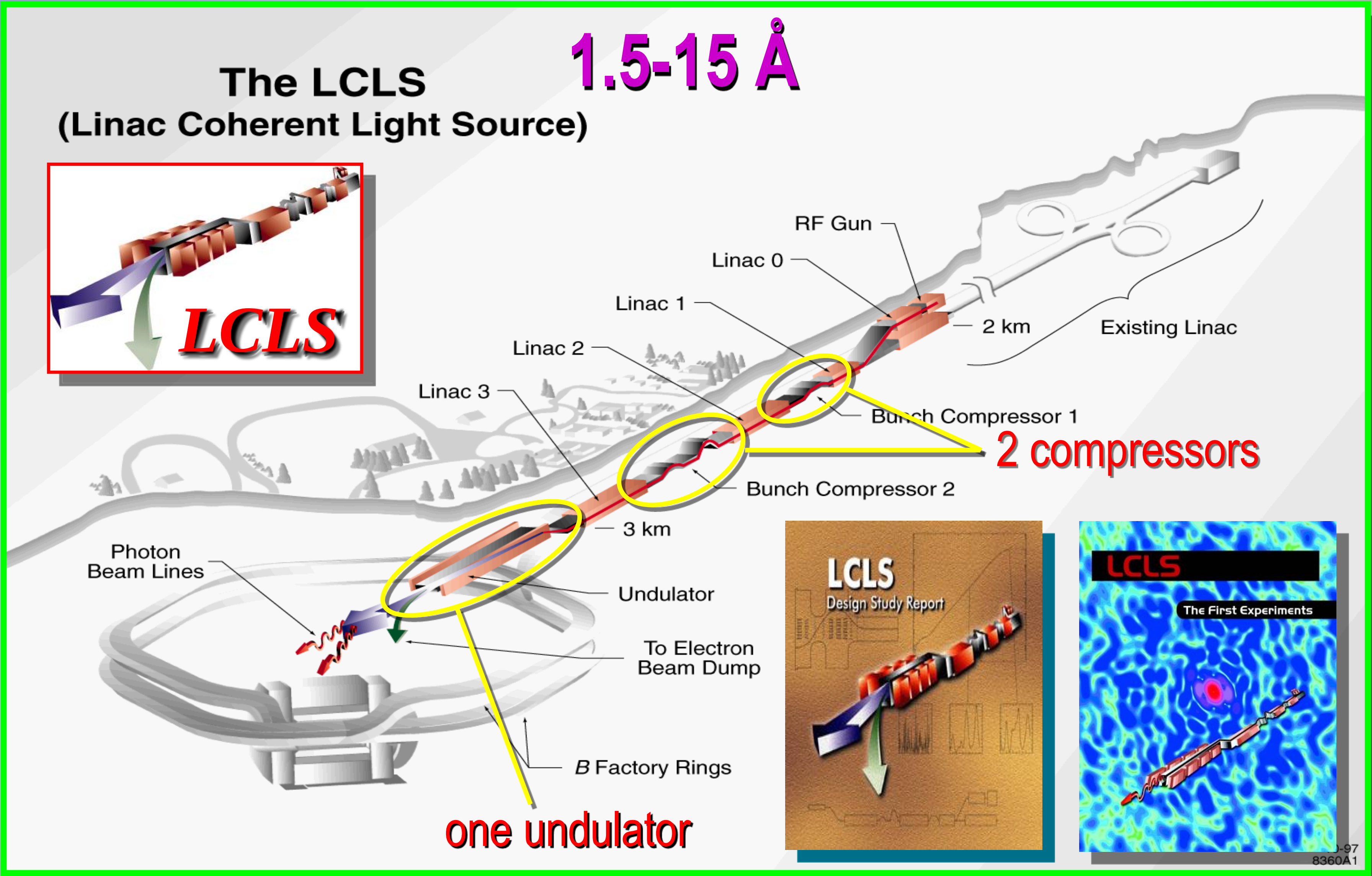
(1) ka>~ 1, or equivalently σ_z/a<~ 1, ($k = \omega/c$); normally in electron storage ring, σ_z~ 5-10 mm; f~ 10 GHz
(2) For a ring in low α mode, σ_z~ 0.5-1 mm; f~ 100 GHz
(3) Near the end of an X-ray FEL, σ_z~ 1-20 μm; f~ 5 THz

Numerically obtaining high frequency impedance/short-range wake is typically difficult; often, however, analytic approximations exist

Much understanding in the subject of wakes of short and extremely short bunches has been gained in the last ~15 years. Will describe wakes that are important for short bunches; focus on longitudinal plane, analytical expressions

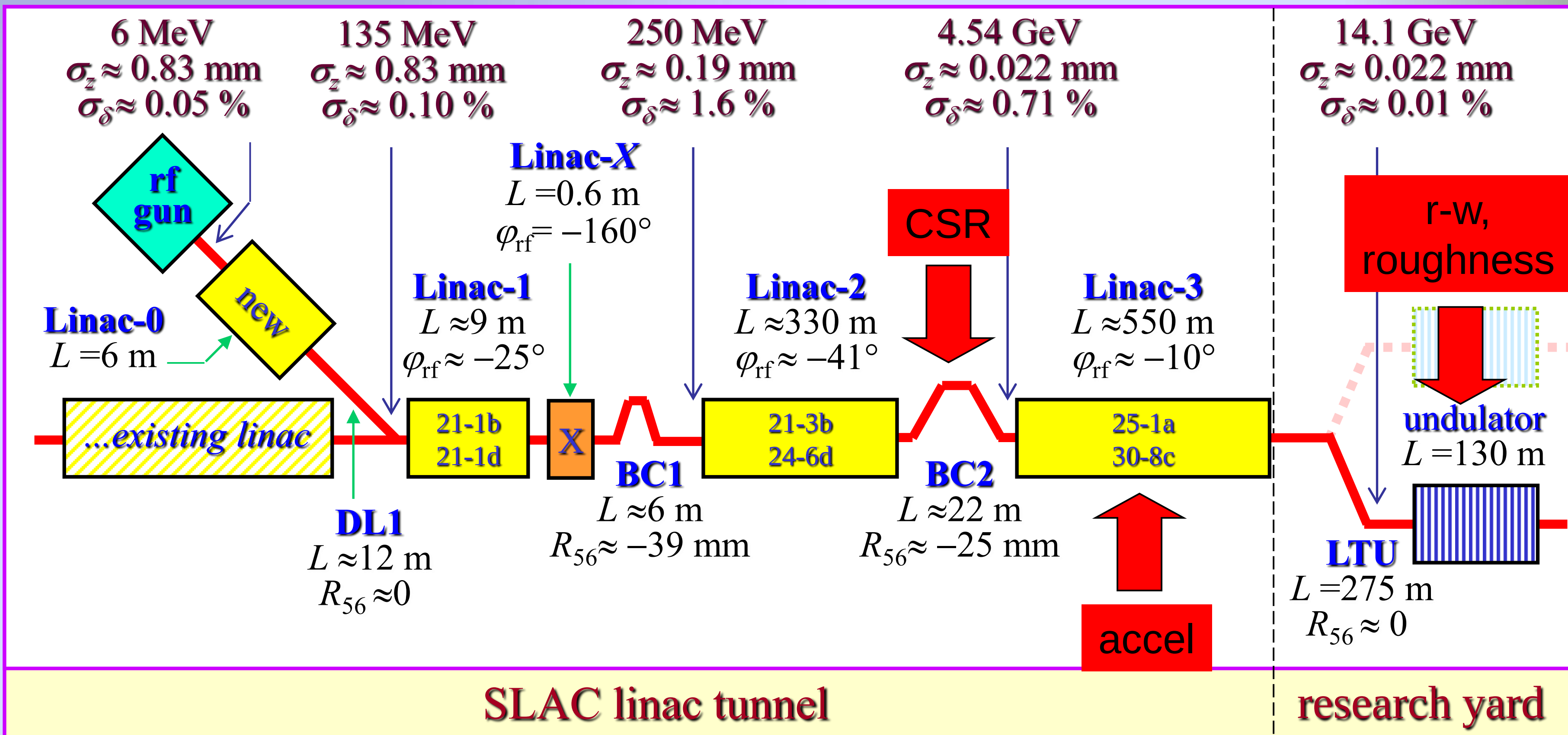
Motivator for understanding extremely high frequency impedances:

LCLS at *SLAC*



X-FEL based on last 1-km of existing *SLAC* linac

LCLS Accelerator and Compressor Schematic



Outline

- Principles
- Impedance of single objects in beam pipe
  - Diffraction model, optical model
- Periodic structures
  - Wake at origin
  - Examples: accelerating structures, resistive wall (rw) wake, roughness, coherent synchrotron radiation (CSR)
- Conclusions

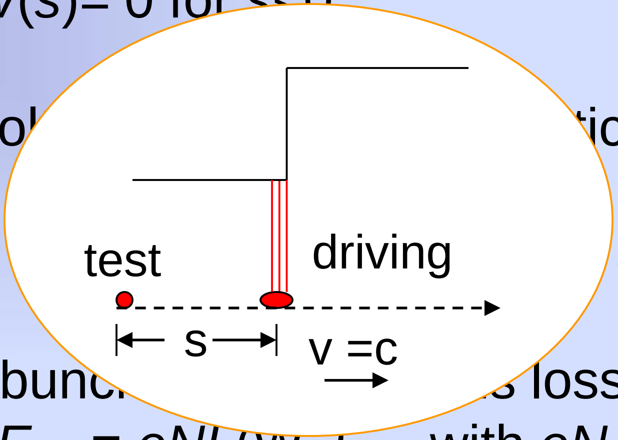
Some comparisons with measurements will also be shown

Note: the (longitudinal) rw, roughness, and CSR wakes only tend to be strong for short bunches

G. Stupakov will give a more detailed talk about CSR tomorrow

Wakes and Impedances

- consider a particle, moving at speed c at offset y through a structure, that is followed by a test particle at distance s; Wake $W(s)$ is voltage loss (per structure or per period) experienced by the test particle; $W(s) = 0$ for $s<0$



- bunch wake is vol ... ticle in a distribution

$$W_\lambda(s) \quad \ldots \quad s')\, ds'$$

average of minus bunc ... s loss factor; energy spread increase $\delta E_{rms} = eNL(W_\lambda)_{rms}$, with eN charge, L length of structure (in periodic case).

- impedance

$$Z(k) = \int_0^\infty W(s) e^{iks} ds \ ,$$

- similar for transverse: W_x, Z_x

- loss factor, $\kappa_{\|}$-- average of bunch wake; kick factor, $\kappa_{\perp}$ average of transverse bunch wake

- wakes can be used as driving terms in long. or trans. tracking; assume beta function $\beta >> L$, structure or period length

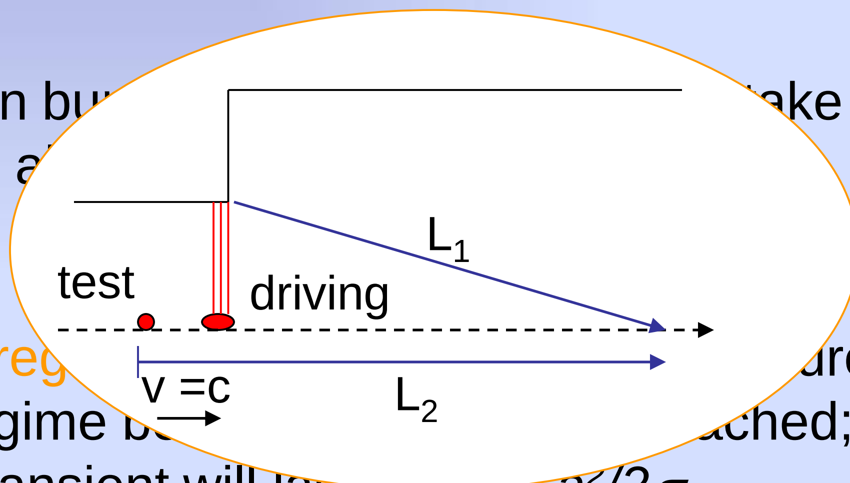
- assumed to act instantaneously

- at high energy space charge or PIC simulations not necessary; space charge ($\sim\gamma^{-2}$) can be ignored

finite energy: impedance drops sharply to 0 when $k>\gamma/a$ (γ Lorentz energy factor); for $\sigma_z< a/\gamma$, replace σ_z by a/γ in wake formulas; if a= 1 cm, energy E= 14 GeV, this occurs when σ_z = 0.4 µm.

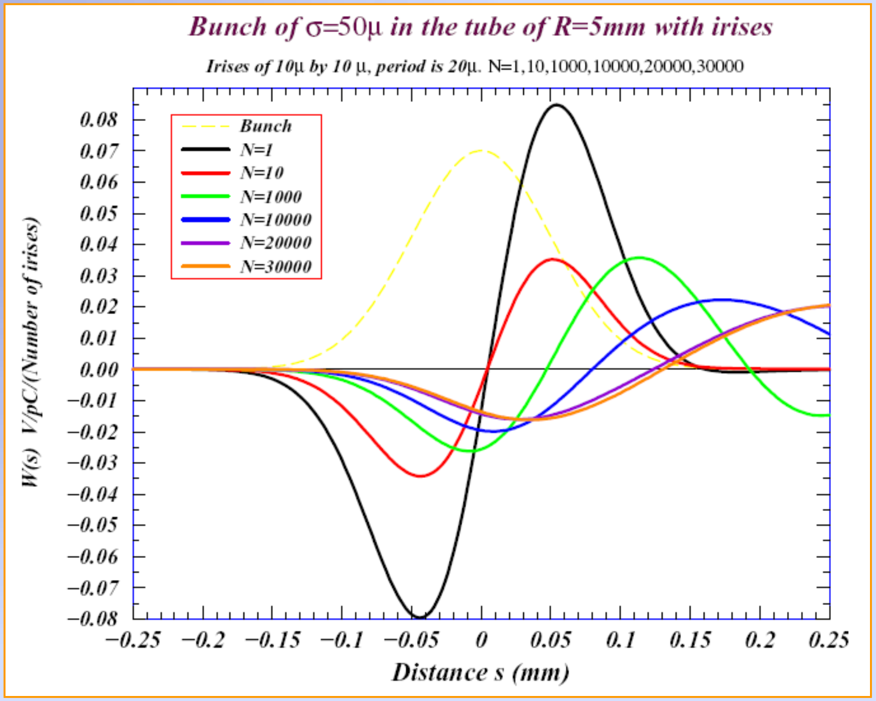
- catch-up distance: if head particle passes e.g. the beginning of a cavity, tail particle doesn't know it until $z = a^2/2s$ (a beam pipe radius, s separation of particles) later. If a= 1cm and s= 20 µm, then z= 2.5 m.

for Gaussian bu… take several times this distance for a…



- transient reg… ures, there will be a transient regime be… ached; for Gaussian with length σ_z, transient will last until $z \sim a^2/2\sigma_z$

if in transient regime, need to know initial conditions accurately



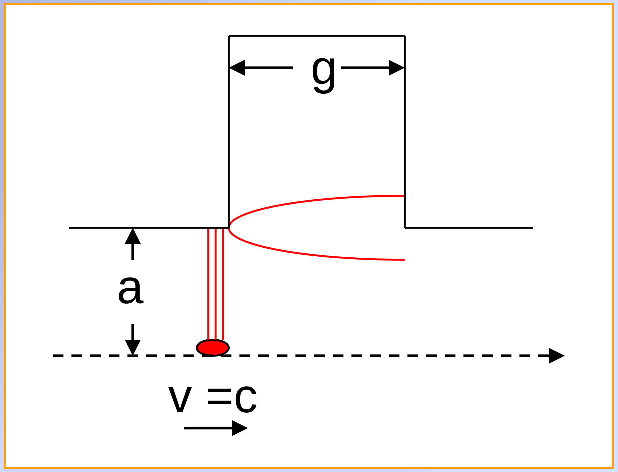
z_{cu}= 25 cm =>
N= 12000

Simulation of wake per period generated by a bunch in a tube with *N* small corrugations (A. Novokhatski).

Wakes of single structure in beam pipe

Single Cavity (Fresnel) diffraction model

(J.D. Lawson)



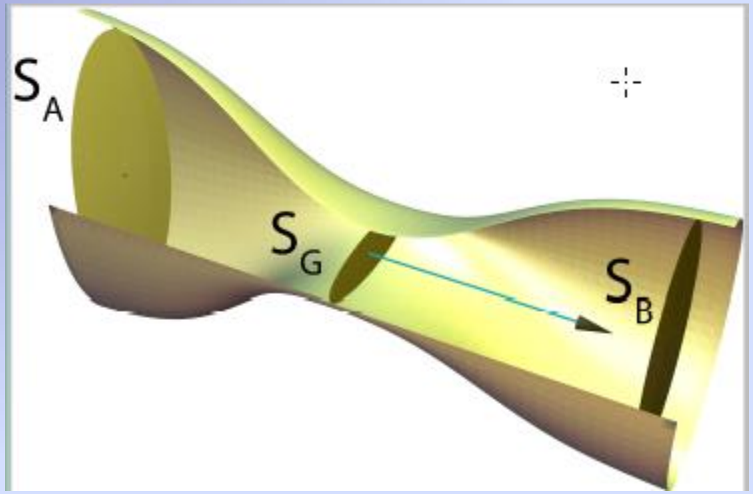
- integrate the total effect from $z = -\infty$ to $+\infty$
- high frequency impedance: $$Re(Z) = \frac{Z_0}{2\pi^{3/2} a}\sqrt{\frac{g}{k}}$$

=> short-range wake: $$W = \frac{Z_0 c}{\sqrt{2}\pi^2 a}\sqrt{\frac{g}{s}}$$

- dipole wake: $$W_x = \frac{2}{a^2}\int W(s)\, ds$$
- bunch wake: longitudinal $\sim\sigma_z^{-1/2}$; transverse (dipole) $\sim\sigma_z^{+1/2}$

Impedance calculation using the optical approximation

(K. Bane, G. Stupakov, I. Zagorodnov, PRST-AB 10: 054401, 074401, 2007)



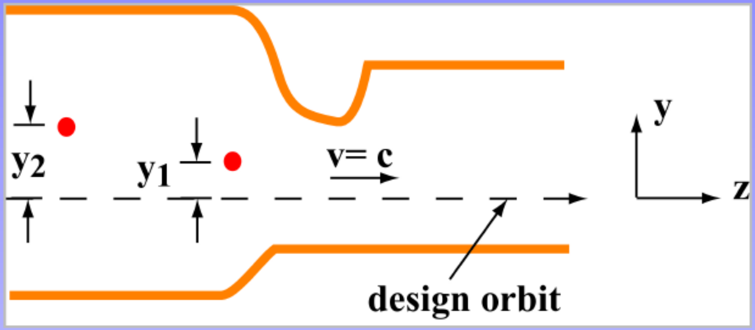
Transition from beam pipe A to beam pipe B

- Valid if:
  (i) frequency is high (ω>> *c*/*g*, with *g* aperture of S_G)
  (ii) transition is short compared to catch-up distance, $l \sim g^2\omega/c$

- Applies to transitions, irises, short collimators, long collimators (the sum of two transitions). Can obtain impedance of 3D transitions analytically or by performing 1D integral

- *Z* is a real constant (bunch wake $W_\lambda \sim -\lambda_z$) resistive; $Z_x \sim \omega^{-1}$ (bunch wake $W_{x\lambda} \sim -\int \lambda_z(s)\, ds$) capacitive

Impedance calculation



Sketch of a generalized 3D transition.

Driving and test particles are assumed to have small offsets with respect to a design orbit (assumed, for simplicity to lie in a vertical symmetry plane). The vertical impedance is given by a sum of (transverse) monopole, dipole, and quadrupole terms

$$Z_{\perp,tot} = Z_{\perp,m} + y_1 Z_{\perp,d} + y_2 Z_{\perp,q} \ .$$

If the design orbit lies also in a horizontal symmetry plane, and if $y_1 = y_2 = y_0$, then let

$$Z_{\perp} = \frac{Z_{\perp,tot}}{y_0} = Z_{\perp,d} + Z_{\perp,q}$$

From the Panofsky-Wenzel theorem

$$Z_{\perp,m} = \frac{c}{\omega} Z_{\parallel,m} \ , \qquad Z_{\perp,d} = \frac{c}{\omega} Z_{\parallel,d} \ , \qquad Z_{\perp,q} = \frac{2c}{\omega} Z_{\parallel,q} \ .$$

Solutions for the three terms are given as surface integrals of potentials φ_m, φ_d, φ_q, in regions A and B. In the dipole case

$$Z_{\parallel,d} = \frac{1}{2\pi c} \left[\iint_{S_B} (\nabla \varphi_{d,B})^2 \, dS - \int_{S_G} \nabla \varphi_{d,A} \cdot \nabla \varphi_{d,B} \, dS \right]$$

with $\nabla^2 \varphi_{d,A} = 4\pi\delta'(y)\delta(x)$. The potentials, in turn, are obtained from the Green function

$$\nabla^2 G(x,y,y_0) = -4\pi\delta(x)\delta(y-y_0) \ ,$$

with *e.g.*

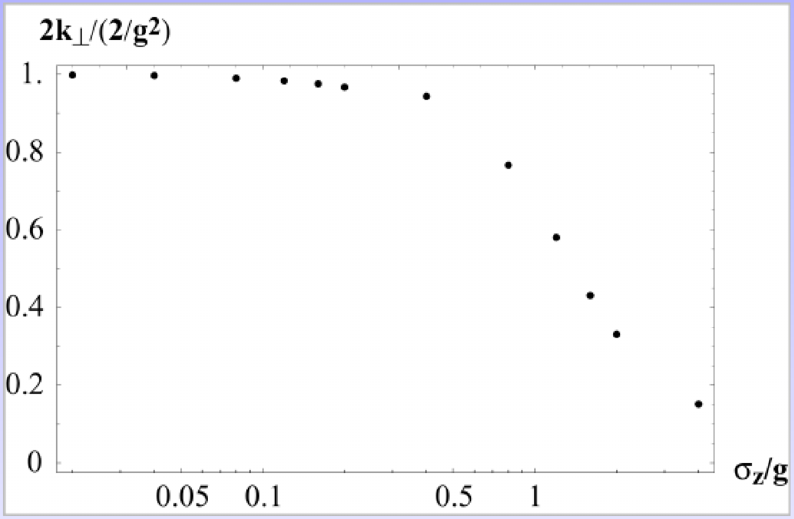
$$\varphi_d(x,y) = \left[\frac{\partial}{\partial y_0} G(x,y,y_0) \right]_{y_0=0} \ .$$

The 2D integrals can be converted to 1D integrals using Green's first identity. The longitudinal impedance $Z_{\parallel,long}$ is obtained similarly.

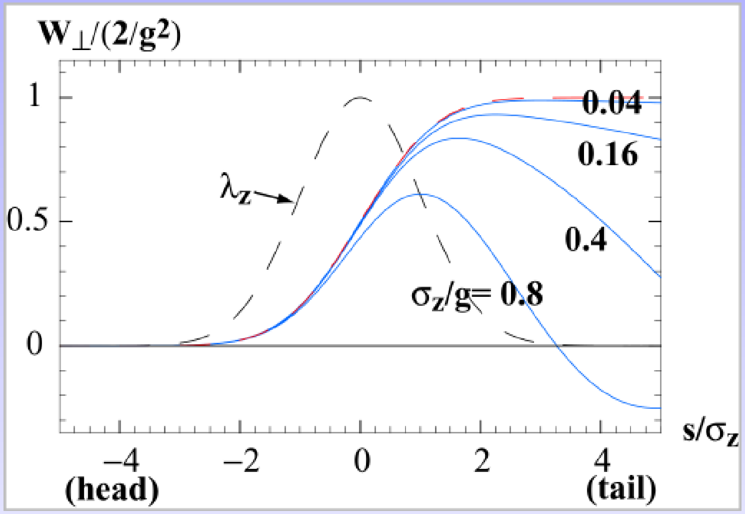
Numerical comparison

We verify our results using ECHO, a 3D, time-domain finite difference program that calculates wakefields of an ultra-relativistic bunch (author I. Zagorodnov).

As test example, consider a thin, round iris of radius g in a large beam pipe (Note $\kappa_\perp = \omega Z_\perp/2$):

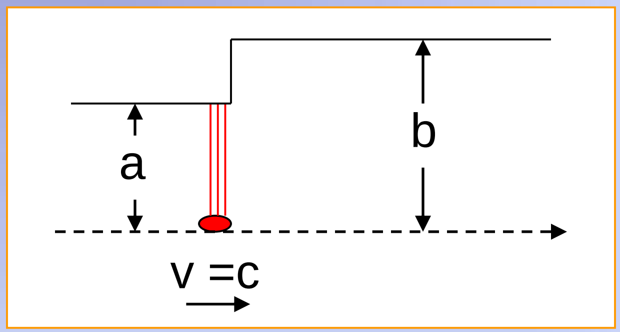


Kick factor vs bunch length



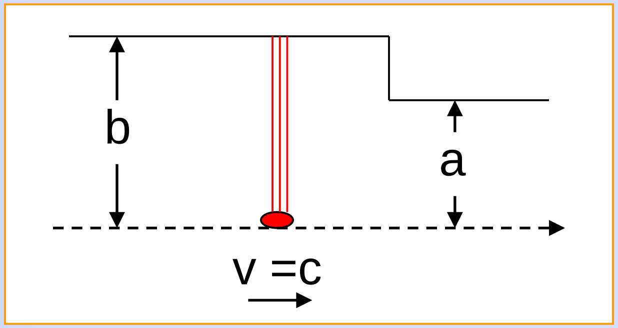
Transverse wake within bunch

Some well-known 2D examples



"out transition"

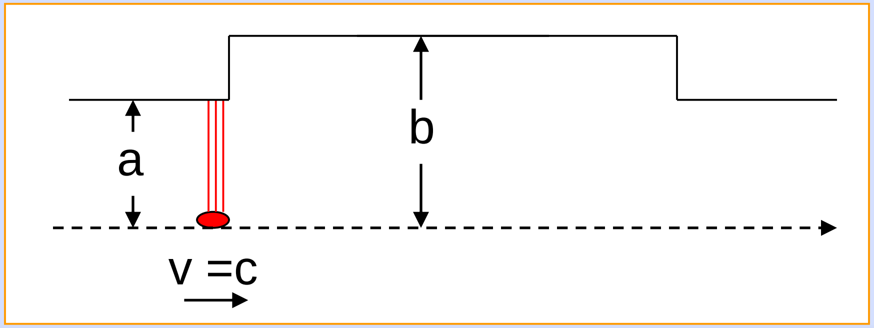
$$Z_{hi} \approx \frac{Z_0}{\pi} \ln(b/a)$$



"in transition"

$$Z_{hi} \approx 0$$

(Heifets & Kheifets)



$$Z_{hi} \approx \frac{Z_0}{\pi} \ln(b/a)$$, a constant

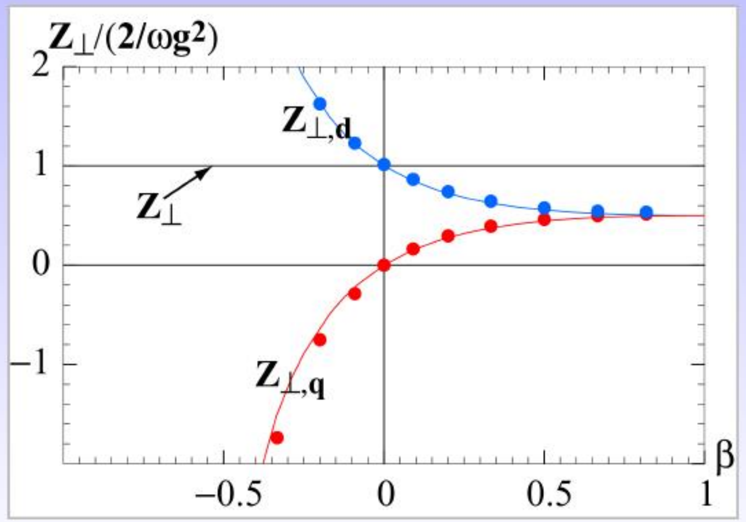
Example: Small Elliptical Iris in Beam Pipe

Consider a small iris of horizontal and vertical semi-axes w and g. To obtain the potentials of the beam pipe we can begin with the free-space Green function

$$G(x, y, y_0) = -\ln[x^2 + (y - y_0)^2] \ .$$

Performing the line integrals over the elliptical aperture we obtain

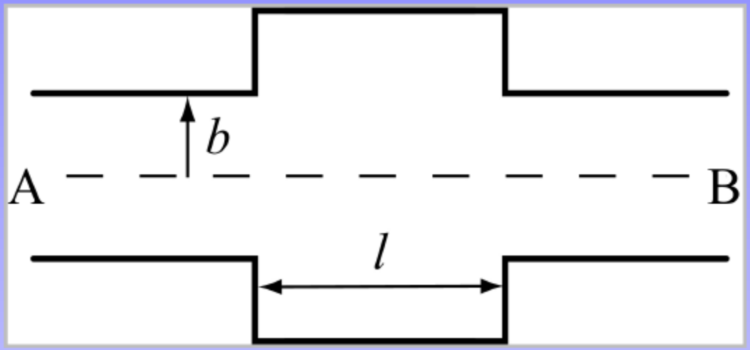
$$Z_{\perp,d} = \frac{1}{\omega g^2}\left(1 + \frac{g^2}{w^2}\right) , \quad Z_{\perp,q} = \frac{1}{\omega g^2}\left(1 - \frac{g^2}{w^2}\right) , \quad Z_{\perp} = \frac{2}{\omega g^2} \ .$$



For a small elliptical iris the transverse impedances as functions of $\beta = (w - g)/(w + g)$. Plotting symbols give ECHO numerical results.

Relation to diffraction model

Let us consider now the pillbox cavity.



Diffraction theory gives the longitudinal impedance for a cavity

$$Z_{\parallel,\mathrm{diffraction}} = \frac{2(1+i)}{\pi^{1/2}}\sqrt{\frac{l}{cb^2\omega}},$$

where b is the pipe radius and l is the length of the cavity.

The optical theory predicts zero impedance for the pillbox cavity.

The result of diffraction theory is of the next order in the small parameter $\sqrt{\sigma_z l/b^2}$, which also indicates the accuracy of the optical approximation.

Periodic Structures: Wake at Origin

For periodic, cylindrically symmetric structures whose closest approach to axis is *a*, the steady-state wakes seem to obey

$$W(0^+) = \frac{Z_0 c}{\pi a^2} \quad \text{and} \quad W_x'(0^+) = \frac{2Z_0 c}{\pi a^4} \ ,$$

with $W_x(0^+) = 0$, where $Z_0 = 377\ \Omega$.

● This is true for (1) the rw wake, (2) a disk-loaded accelerator structure, (3) a pipe with small periodic corrugations, and (4) a dielectric-lined metallic pipe; it is independent of the boundary material

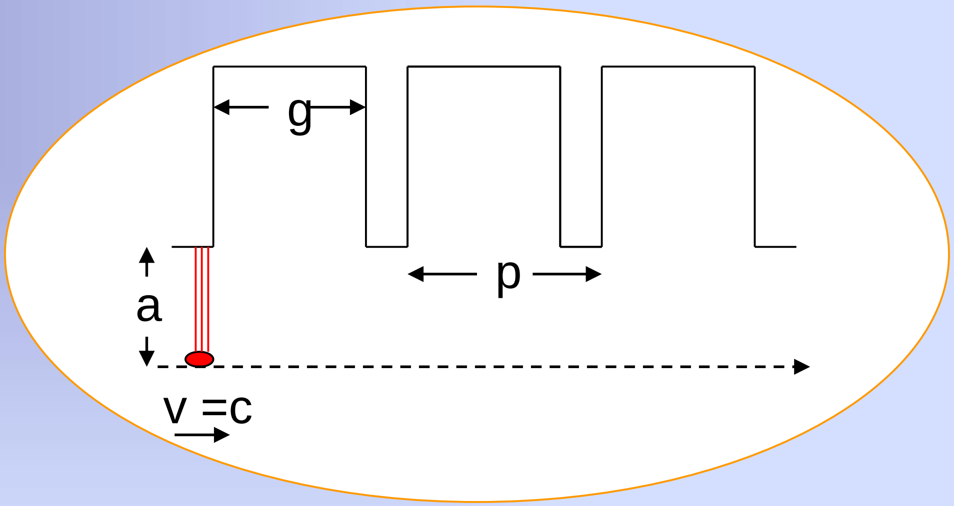
● It depends only on cross-section of beam pipe; *e.g*. for flat geometry with aperture 2*a*, $W(0^+) = (\pi^2/16)(Z_0 c/\pi a^2)$

It appears to be a general property (see S.S. Baturin and A.D. Kanareykin, PRL 113, 214801 (2014), who claim to have a general proof)

● The maximum wake loss is for short bunches: $V_w \approx W(0^+)/2$; for *e.g*. a round structure with $a = 1$ cm, $Q = 1$ nC, $V_w = 180$ kV/m

● Implies that wake/impedance contributions at short distance/high frequency do not simply add (*e.g*. for rw + roughness)

Periodic accelerating structures



- When a short bunch enters a periodic structure, the effect of the first cell is like the diffraction model. After a distance ~$a^2/2\sigma_z$, a steady state field pattern is reached

- High frequency (steady-state) impedance:

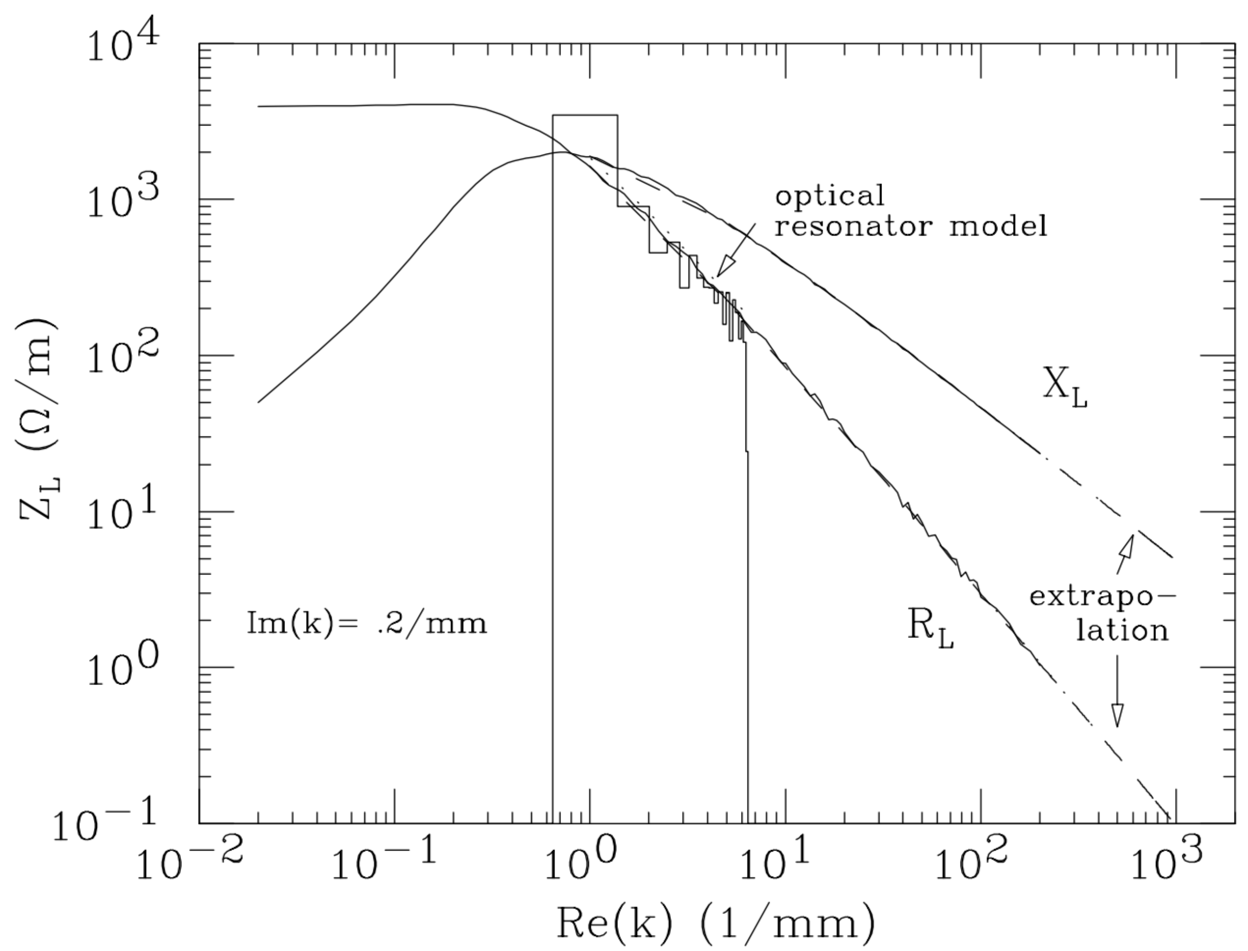
[Gluckstern; Yokoya and Bane]

$$Z(k) \approx \frac{iZ_0}{\pi k a^2}\left[1 + (1+i)\frac{\alpha(g/p)\,p}{a}\left(\frac{\pi}{kg}\right)^{1/2}\right]^{-1} ,$$

$$\alpha(x) \approx 1 - 0.465\sqrt{x} - 0.070x$$

- Re(Z)~ $k^{-3/2}$; $W(0^+)= Z_0 c/(\pi a^2)$

Next Linear Collider (NLC) example calculation



Real (R_L) and imaginary (X_L) parts of impedance for the NLC accelerating cavities showing results of numerical calculations (bins and solid curves) and the analytical, high frequency formula (dashes). (From K. Bane et al, SLAC-PUB-7862, Revised, 1998)

- wake obtained by detailed frequency domain calculation

- results can be fit to (over useful parameter range)

$$W(s) = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{s/s_1}\right) \quad \text{with} \quad s_1 = 0.41 \frac{a^{1.8} g^{1.6}}{p^{2.4}} \; .$$

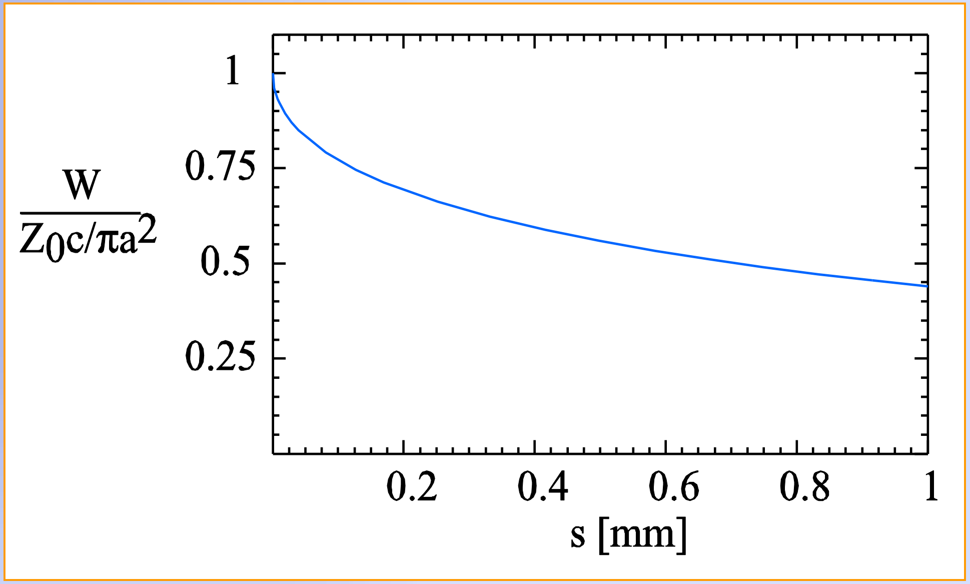
in SLAC linac, s_1=1.5 mm

- same has been done for transverse wake

[K. Bane, et al]

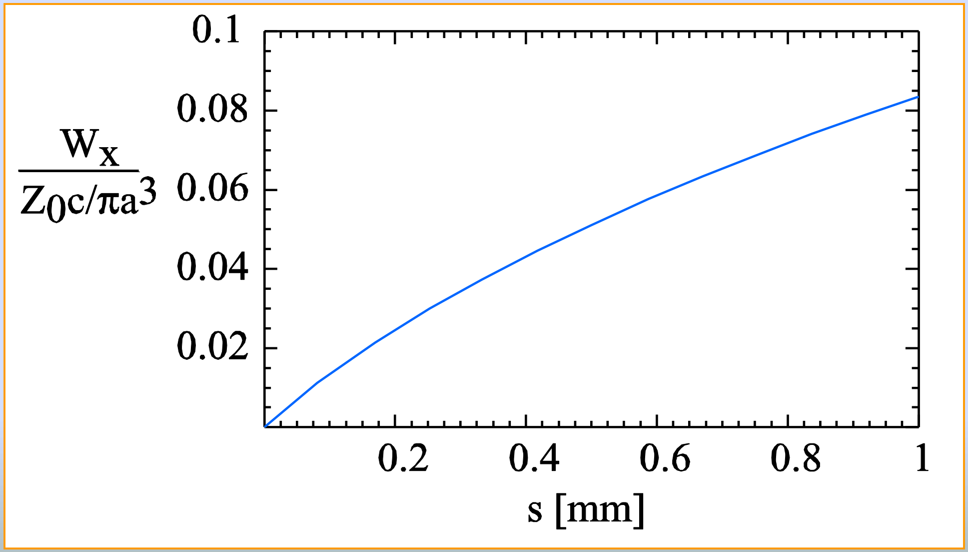
- Short-range SLAC linac wakes:

$$\frac{Z_0 c}{\pi a^2} = 0.27 \ \frac{\text{GV}}{\text{nC} \cdot \text{km}}$$



longitudinal

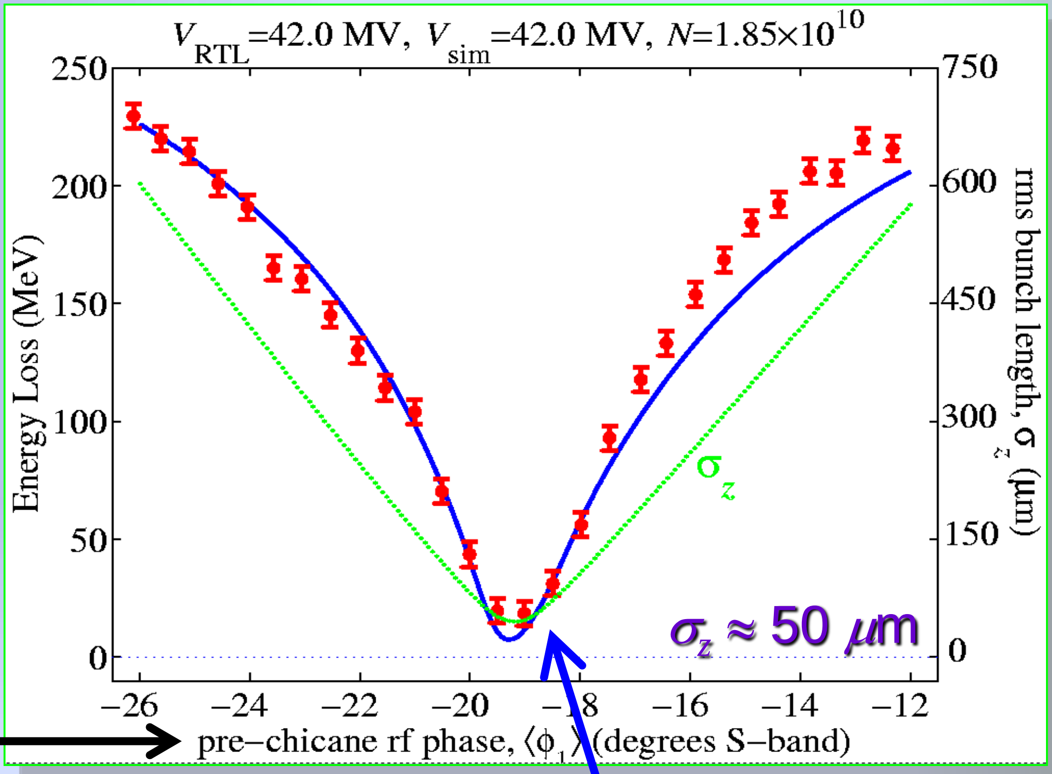
$$\frac{Z_0 c}{\pi a^3} = 23 \frac{\text{MV}}{\text{nC} \cdot \text{km} \cdot \text{mm}}$$



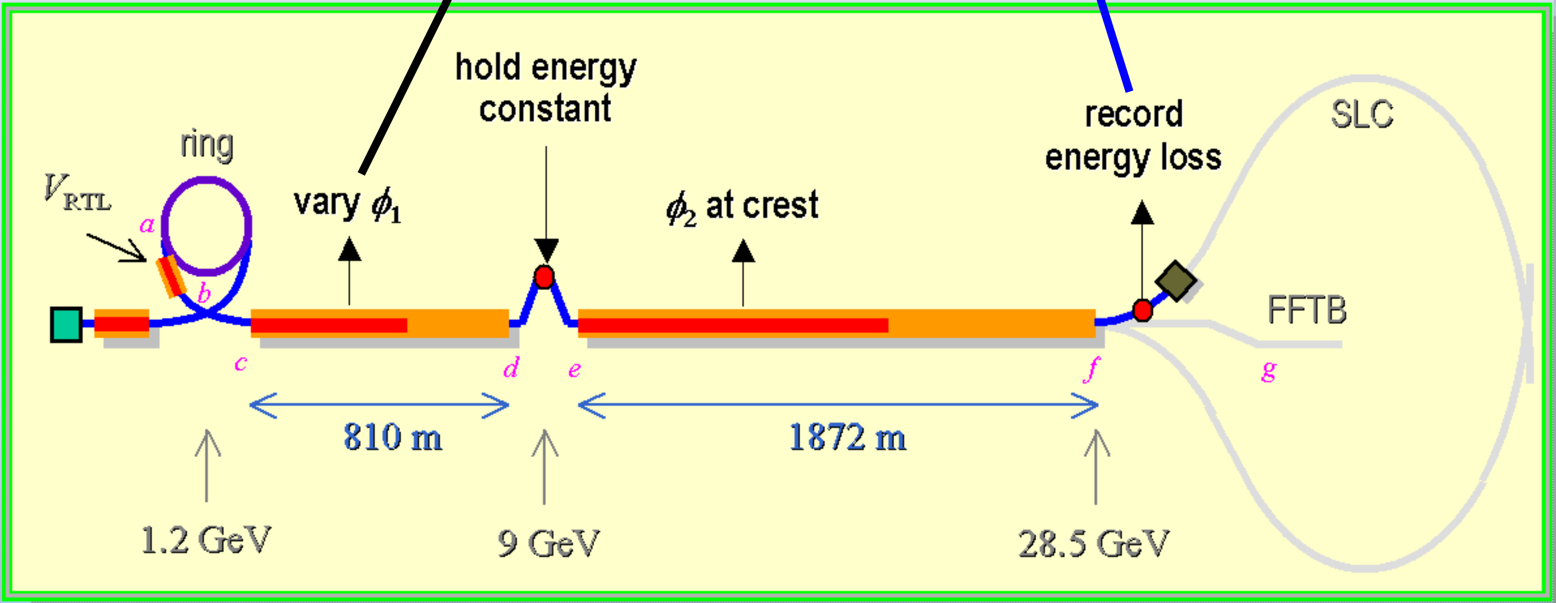
transverse

Wakefield energy-loss used to set and confirm minimum bunch length

- In the past we have confirmed through measurement the size and shape of the wake in the SLAC linac, but that was for σ_z~ 1 mm



K. Bane *et al.*, PAC'03



(P. Emma)

Resistive Wall Wake

- impedance (see A. Chao's book): $Z = \left(\frac{Z_0}{2\pi a}\right)\frac{1}{\frac{\lambda}{k} - \frac{ika}{2}}$

with $\lambda = \sqrt{\frac{2\pi\sigma|k|}{c}}\,[i + sgn(k)]$

- low frequency $Z \sim k^{1/2}$ => familiar long-range wake:

$$W(s) = -\frac{c}{4\pi^{3/2}a}\sqrt{\frac{Z_0}{\sigma}}\frac{1}{s^{3/2}}\ ,$$

bunch wake ~ $\sigma_z^{-3/2}$

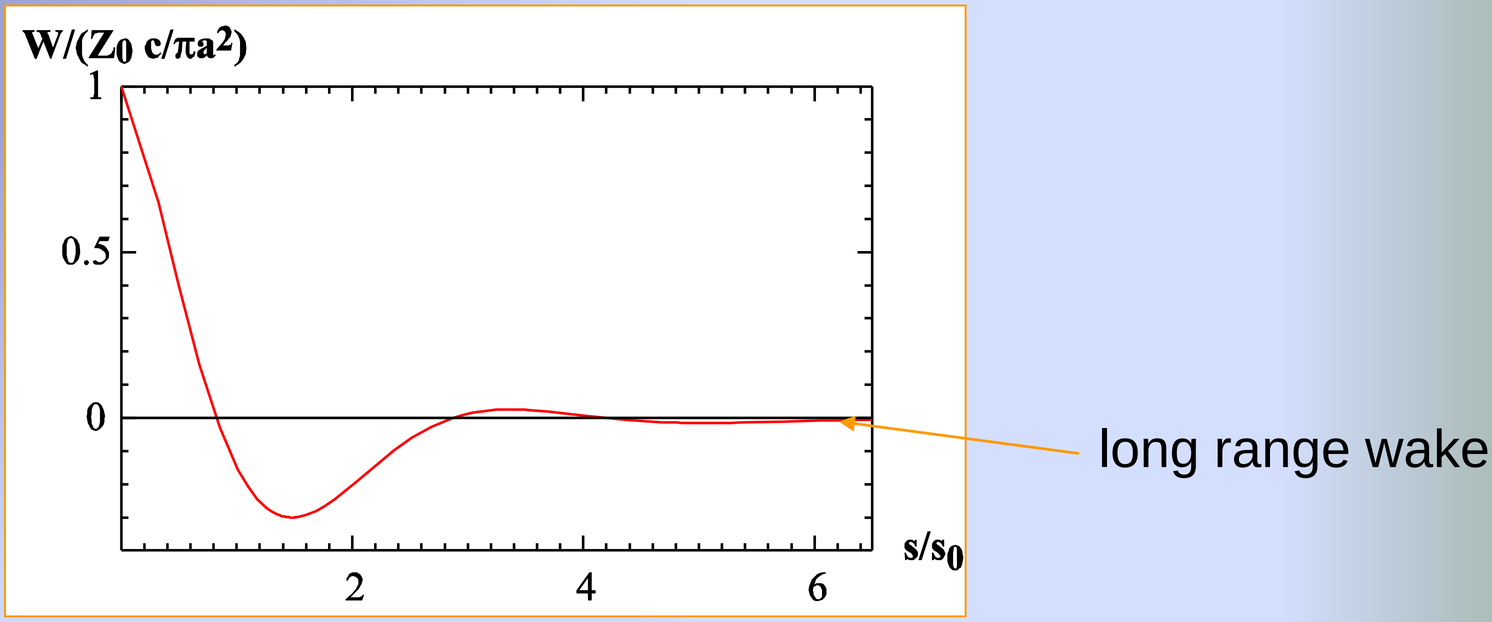
- general solution:

$$W = \frac{4Z_0c}{\pi a^2}\left(\frac{e^{-s/s_0}}{3}\cos\frac{\sqrt{3}s}{s_0} - \frac{\sqrt{2}}{\pi}\int_0^\infty \frac{dx\,x^2 e^{-x^2 s/s_0}}{x^6+8}\right)$$

- characteristic distance over which wake drops to zero:

$$s_0 = \left(\frac{2a^2}{Z_0\sigma}\right)^{\frac{1}{3}}$$

(for Al with a= 2.5 mm, s_0= 9.3 μm)



Resistive wall wake

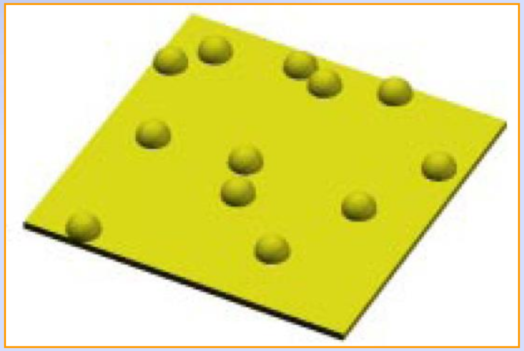
- Can include ac conductivity of metal and anomalous skin effect in calculation. But if $\sigma_z \sim s_0$, results are not sensitive to such details

- Normally (for $\sigma_z >> s_0$) longitudinal rw wake is a weak effect. But in undulator region of LCLS, where $\sigma_z \sim s_0 = 9\ \mu m$, it dominates wake effects

Roughness Impedance

A metallic beam pipe with a rough surface has an impedance that is enhanced at high frequencies. Two approaches to modeling are (i) random collection of bumps, (ii) small periodic corrugations

(i) Random bumps



Impedance of one hemispherical bump (of radius *h*) for $k \ll 1/h$

$$Z(k) = ikc\mathcal{L}_1 = ik\frac{Z_0 h^3}{4\pi a^2} \ ,$$

[S. Kurennoy]

- for many bumps (α filling factor, f form factor)

$$\mathcal{L}/L = \frac{2\alpha f a \mathcal{L}_1}{h^2} = \frac{\alpha f Z_0 h}{2\pi a c} \ ,$$

Gaussian $(W_\lambda)_{rms} \approx 0.06 c^2 \mathcal{L}/L \sigma_z^2$

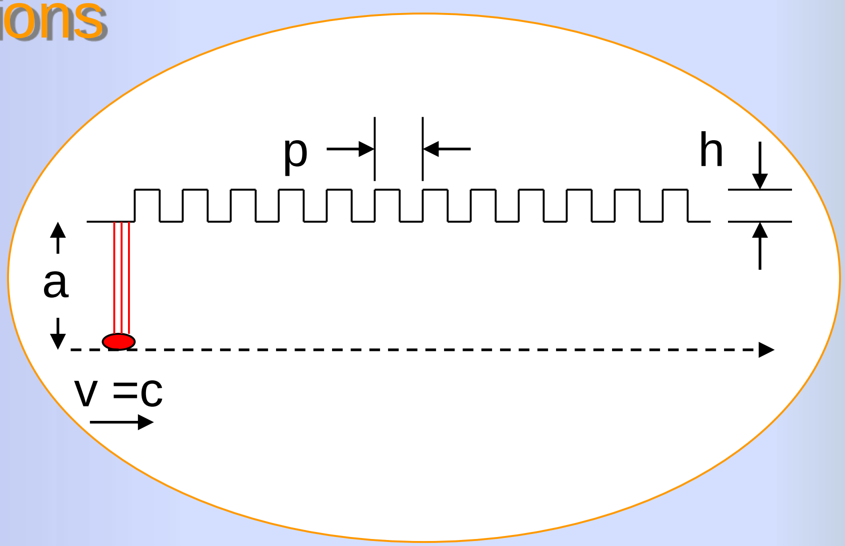
- idea has been systematized so that, from surface measurement, can find impedance:

$$\mathcal{L}/L = \frac{Z_0}{2\pi c a} \int_{-\infty}^{\infty} \frac{k_z^2}{\sqrt{k_\theta^2 + k_z^2}} S(k_z, k_\theta)\, dk_z dk_\theta \ ,$$

with S spectrum of surface, k_z, k_θ, longitudinal, azimuthal wave numbers

[K. Bane, et al; G. Stupakov]

(ii) Small periodic corrugations



motivation: numerical simulations of many randomly placed, small cavities on a pipe found that, in steady state, the short range wake is very similar to truly periodic case

● consider a beam pipe with small corrugations of height *h*, period *p*, and gap *p*/2. If $h/p \gtrsim 1$, wake

$$W(s) \approx \frac{Z_0 c}{\pi a^2} \cos k_0 s \quad \text{with} \quad k_0 = \frac{2}{\sqrt{ah}} \; .$$

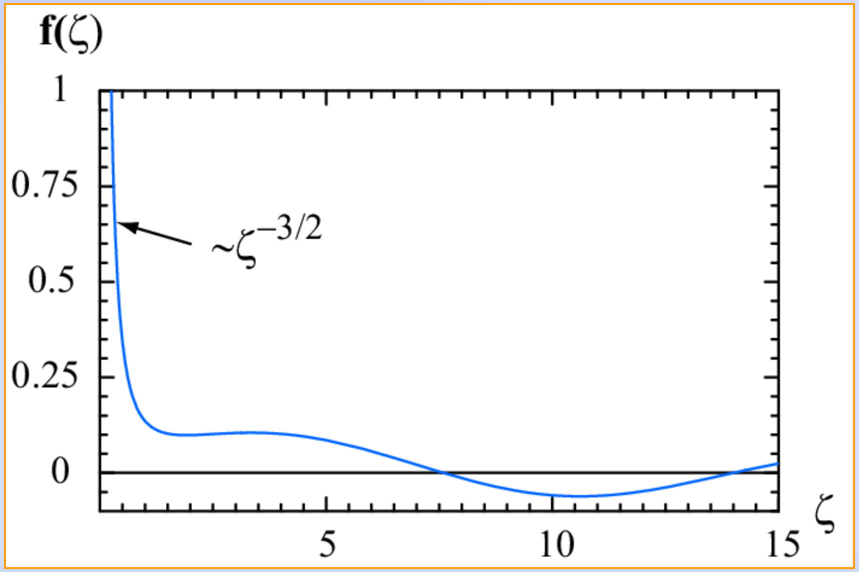
● for Gaussian, with $k_0\sigma_z \gg 1$, becomes inductive with $\mathcal{L}/L = Z_0 h/(4ac)$, similar to earlier model

[A. Novokhatski, et al; K. Bane and A. Novokhatski]

- however, when $h/p \ll 1$

$$W(s) = \frac{Z_0 c h^2 k_1^3}{4\pi a} f(k_1 s)\ , \quad f(\zeta) = -\frac{1}{2\sqrt{\pi}} \frac{\partial}{\partial \zeta} \frac{\cos(\zeta/2) + \sin(\zeta/2)}{\sqrt{\zeta}}$$

with $k_1 = 2\pi/p$



[G. Stupakov]

- For $k_1 s \lesssim 1$ (but not too small):
  - $W \sim s^{-3/2}$; for bunch $W_\lambda \sim \sigma_z^{-3/2}$ (vs. σ_z^{-2} for earlier model)
  - bunch wake weaker by $\sim h/p$ than single mode model

- Measurements of roughness of well-polished samples of Al show that $h/p \ll 1$; thus the second periodic model applies

- For LCLS, if we assume a surface profile with $h \sim 0.5$ µm, $p \sim 100$ µm, and $\sigma_z = 20$ µm => roughness wake 0.15 as strong as resistive wall wake in LCLS undulator

Combining rw + roughness impedance

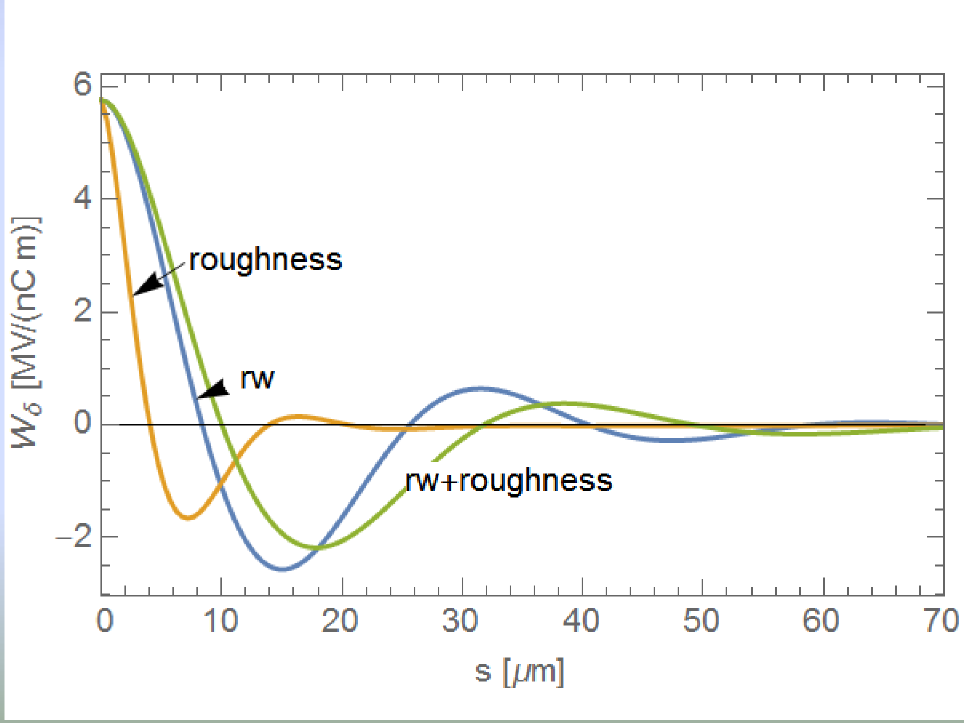
- LCLS undulator beam pipe has flat cross-section, with half-height *a*= 2.5 mm and length *L*= 130 m. With the short bunch σ_z<~ 20 μm, rw wake dominates, generating energy chirp

- Can write impedance in terms of surface impedances ζ_{rw} and ζ_{ro} (roughness) as (in the round case, as an example)

$$Z(k) = \frac{Z_0}{2\pi a}\left(\frac{1}{\zeta_{rw}(k)+\zeta_{ro}(k)} - \frac{ika}{2}\right)^{-1}$$

Note that for low frequencies the two components just add

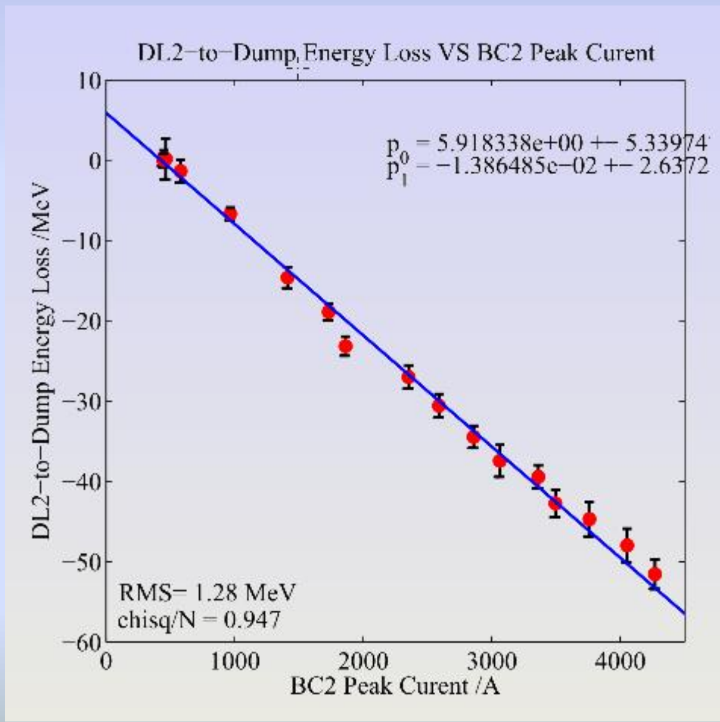
Wake for round pipe with radius a= 2.5 mm made of Al, with roughness model $(r')_{rms}$= 30 mr and λ_{ro}= 300 μm

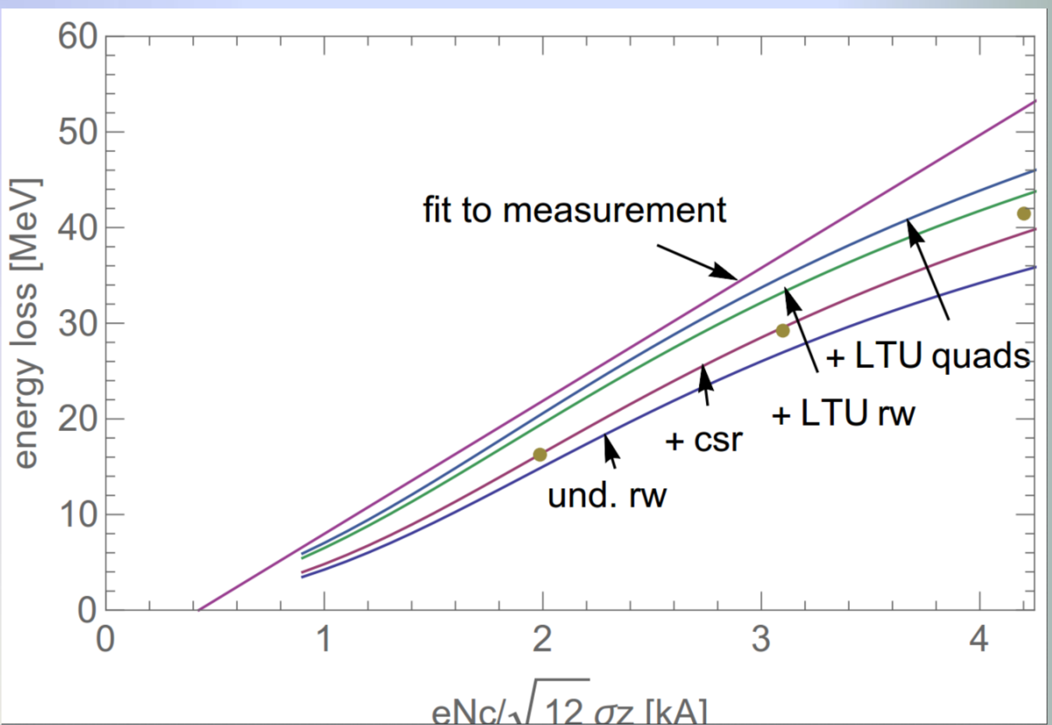


Measurement of wake of undulator beam pipe at the LCLS

● Energy loss in undulator beam pipe as function of peak current was measured at LCLS. However, the effect of the upstream, 350-m-long linac-to-undulator (LTU) transfer line was, unavoidably, included. (For another analysis, see A. Novokhatski FEL2009.)

● Undulator beam pipe has vertical aperture 5 mm and is 130 m long. Its calculated wake gives 70% of the measured loss. Apertures in LTU are much larger and wakes should be weaker. Also included in calculations are: (i) csr of dogleg, (ii) rw of 190 m Cu and 30 m SS pipes in LTU, (ii) small aperture quads in LTU

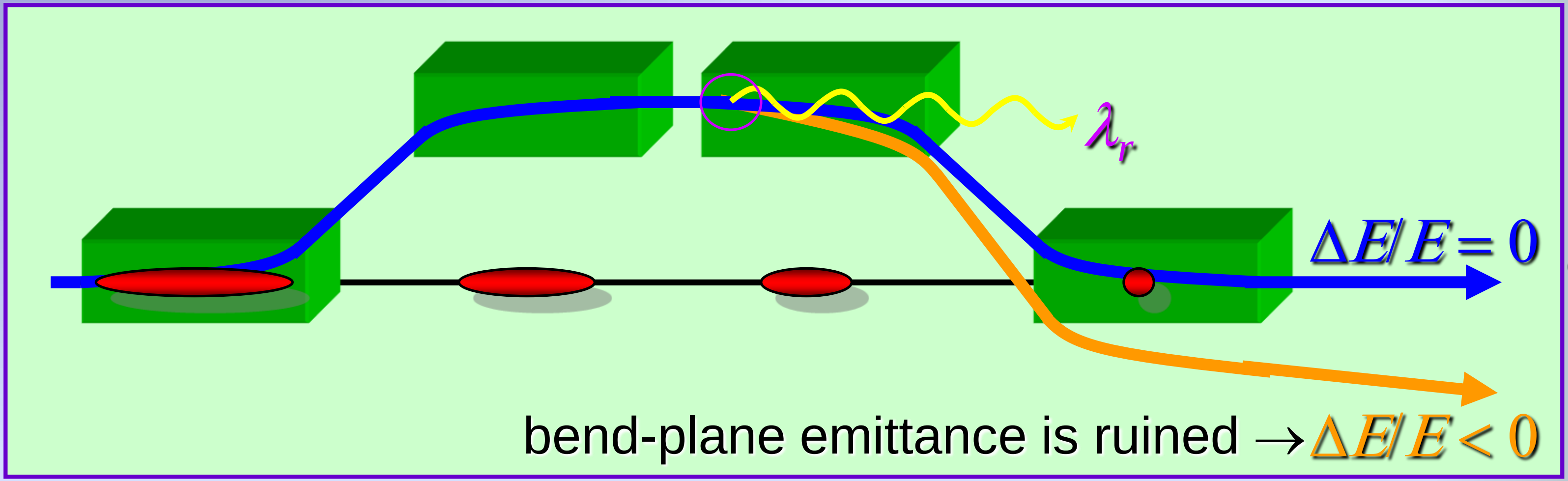




(Left) Raw data (minus energy loss is plotted); (Right) Calculations compared to measurement

CSR Wake

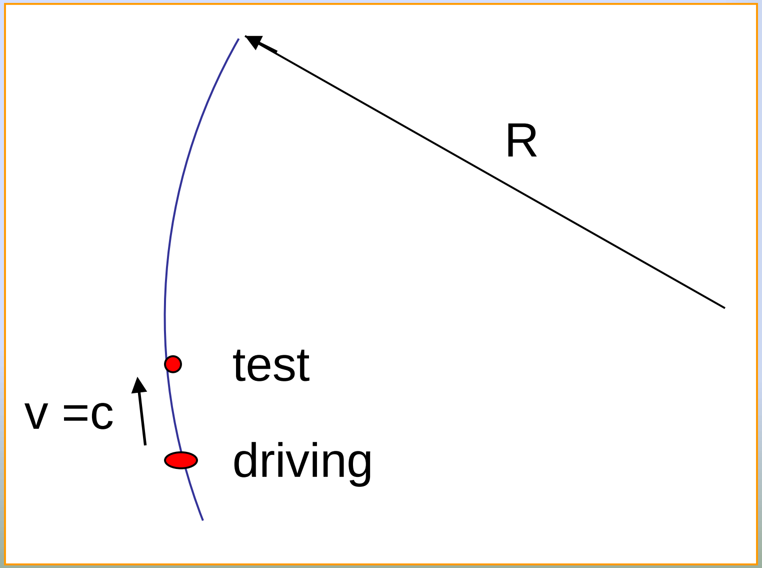
- CSR effect on bunch can be described as a wakefield effect



(P. Emma)

- Consider ultra-relativistic particle (and test particle) moving on circle of radius *R* in free space

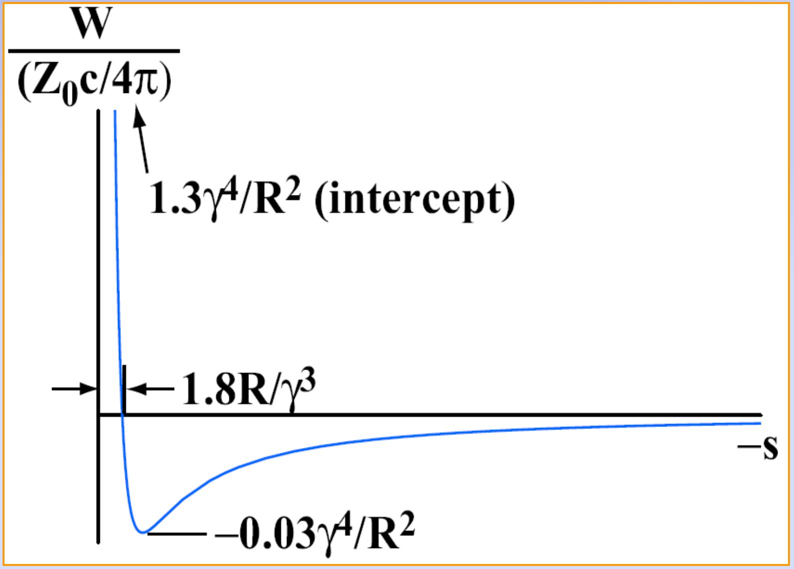
- for effect test charge needs to be *ahead* of driving charge



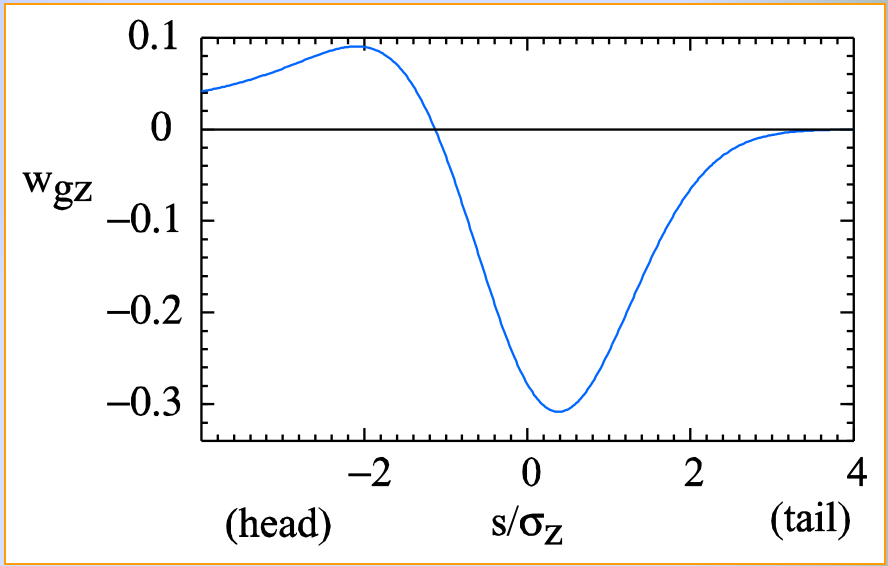
- (Steady-state) wake, for $(-s) \gg R/\gamma^3$

$$W(s) = -\frac{Z_0 c}{2 \cdot 3^{4/3} \pi R^{2/3} (-s)^{4/3}} \qquad s < 0 \ ,$$

while $W(0^-) = Z_0 c \gamma^4 / (3\pi R^2)$



point charge wake



Gaussian wake:

$$W_\lambda(s) = \frac{Z_0 c}{2\pi} w_{gz}(s) R^{-2/3} \sigma_z^{-4/3}$$

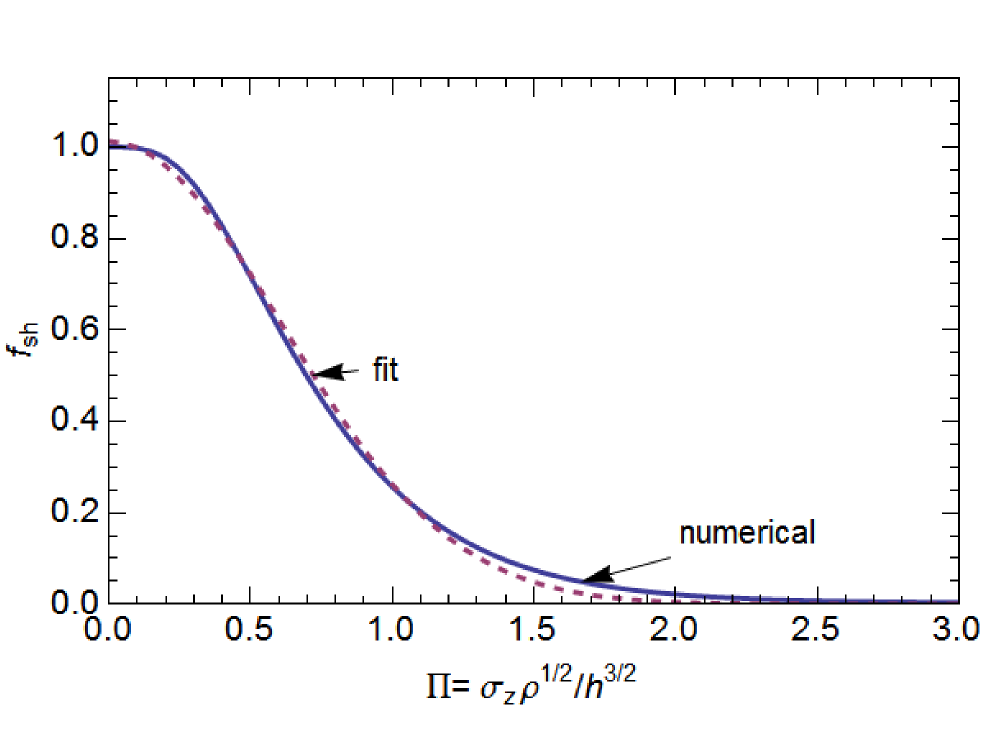
- $Z \sim k^{1/3}$
- formation length $z_f \approx (24 R^2 \sigma_z)^{1/3}$

[J. Murphy, et al; Y. Derbenev, et al]

Shielded CSR

- Murphy et al, derived the wake for particles moving in a circle between two perfectly conducting, parallel plates

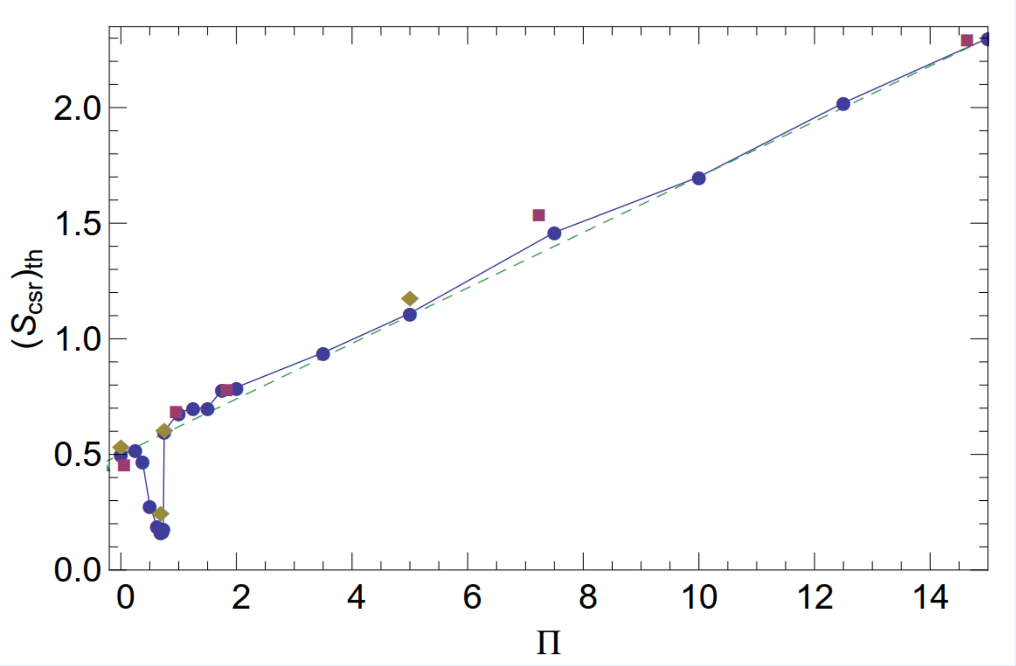
- A normalized shielding parameter can be given as $\Pi = \sigma_z \rho^{1/2}/h^{3/2}$, where the plate separation is $2h$, the bending radius ρ, bunch length σ_z



The energy loss of a Gaussian beam moving in a circle between parallel plates, normalized to the loss in free space, as function of Π. The fit $\exp[-(\Pi/\sigma)^2/2]$ with $\sigma = 0.609$, is given by the dashes.

Microwave Instability Threshold Study

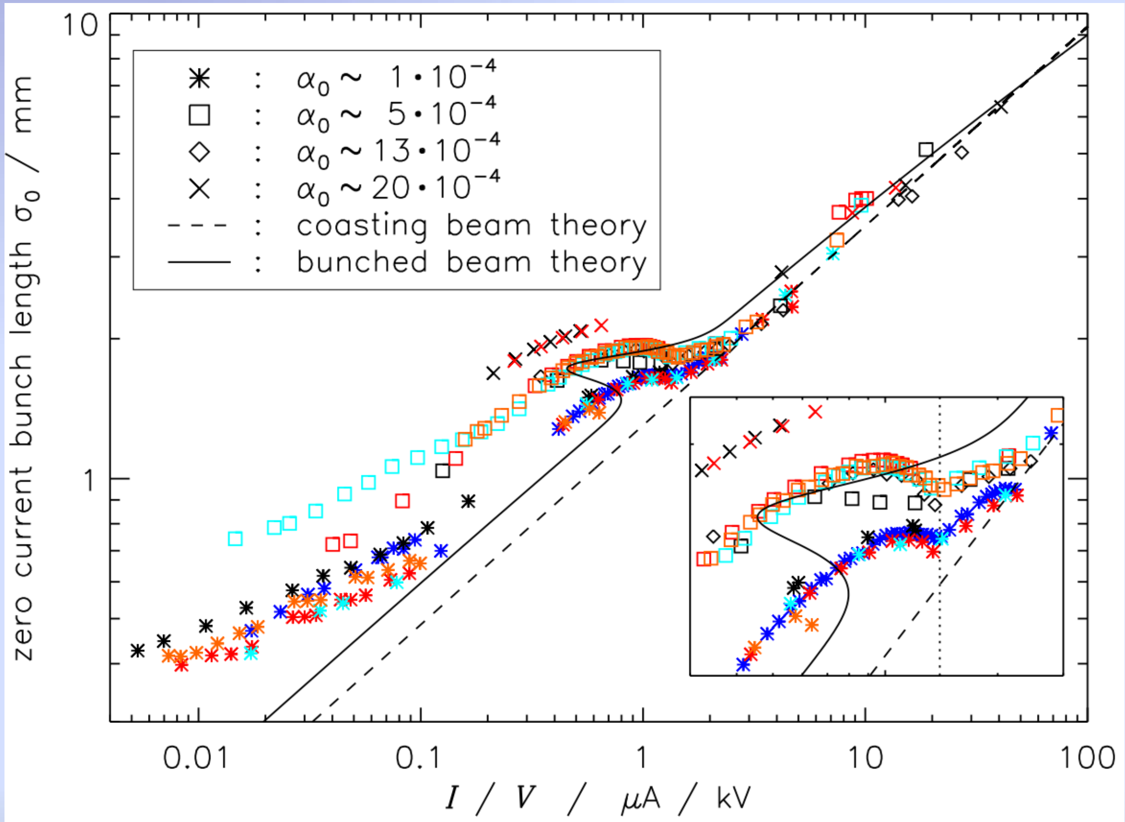
- Bane, Cai, Stupakov (2010) performed a series of Vlasov-Fokker-Planck simulations to study the microwave threshold $(S_{csr})_{th}$ as function shielding parameter, $\Pi = \sigma_{z0}\rho^{1/2}/h^{3/2}$, where σ_{z0} is nominal bunch length
- $(S_{csr})_{th} = I\rho^{1/3}/\sigma_{z0}^{4/3}$, where normalized current $I = r_e N_b/(2\pi\nu_{s0}\gamma\sigma_{\delta 0})$



Microwave threshold study assuming the ring impedance is given by the shielded CSR model. Except in the dip region near $\Pi \sim 0.8$, the results fit well to $(S_{csr})_{th} = 0.5 + 0.12\Pi$ (the dashes)

Comparison with measurements

- Several storage rings running with short bunches (MLS, BESSY-II, ANKA) have reported striking agreement with these simulation results, in spite of the fact that real vacuum chambers are much more complicated than parallel plates



Bursting thresholds measured at Metrology Light Source (MLS). Colors indicate measurement series within one α_0 set (from M. Ries et al, IPAC2012).

Conclusion

- Have discussed high frequency impedance/short range wakes of vacuum chamber transitions; the optical model
- Have given examples of steady-state wakes of periodic structures that become strong for short bunches, those of: accelerating structures, resistive walls (rw), surface roughness,CSR
- We find reasonably good agreement with measurements for very short bunches (at the LCLS) for: accelerating structure wake losses, rw wake losses in the undulator, and CSR in the bunch compressors (not discussed here)
- The wakes of very short bunches may be relatively easy to calculate approximately, in that there are often analytical expressions and *e.g*. there are limiting values for periodic wakes

However, when the catch-up distance becomes large or the structures become complicated, large numerical calculations may still be necessary