CP Violation

The asymmetry between Matter and AntiMatter

Gerhard Raven, Nikhef and Vrije Universiteit Amsterdam
1. Antimatter & Big Bang
2. Symmetries and the weak interactions
3. Discovery of CP violation
4. Describing CP violation and the weak interactions
5. CP violation and the Standard Model
6. Testing the Standard Model predictions of CP violation
7. Outlook for future measurements
8. Summary
Antimatter

matter and antimatter distinction in different from + versus - charge in electrodynamics

- In Maxwell’s theory, if we change all “+” into “-” and vice-versa, nothing happens...

matter & antimatter can be distinguished: the “stuff” in the universe is the “matter”

- There must be some fundamental difference in the laws of physics...
matter and antimatter distinction in different from + versus - charge in electrodynamics

• In Maxwell’s theory, if we change all “+” into “-” and vice-versa, *nothing* happens...

matter & antimatter *can* be distinguished: the “stuff” in the universe is the “matter”

• There must be *some* fundamental difference in the laws of physics...
Antiparticles: Dirac’s prediction

• Combining quantum mechanics with special relativity, and the wish to *linearize* $\partial/\partial t$, leads Dirac to the equation

$$ (i\gamma^\mu \partial_\mu - m) \psi(\vec{x}, t) = 0 $$

• Solutions describe particles with spin = 1/2

• But half of the solutions have *negative energy*

$$ E = \pm \sqrt{\vec{p}^2 + m^2} $$

• Vacuum represents a “sea” of such negative-energy particles (fully filled according to Pauli’s principle)

• Dirac identified holes in this sea as “antiparticles” with opposite charge to particles … (however, he conjectured that these holes were protons, despite their large difference in mass, because he thought “positrons” would have been discovered already)

• An electron with energy $E$ can fill this hole, emitting an energy $2E$ and leaving the vacuum (hence, the hole has effectively the charge $+e$ and positive energy).
Antiparticles: Stueckelberg/Feynman

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Stueckelberg/Feynman interpretation:

- consider the negative energy solution as *running backwards in time*
Antiparticles: Stueckelberg/Feynman

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Stueckelberg/Feynman interpretation:

- consider the negative energy solution as running backwards in time
- and re-label it as antiparticle, with positive energy, going forward in time
- emission of $E>0$ antiparticle = absorption of particle $E<0$
- Naturally describes creation and annihilation...
- ... and that particles and antiparticles must have the same mass, spin, ... and opposite charges
Discovery of Antiparticles

Back to experiment: does antimatter exists, and, if so, where is it?

Carl Anderson studies at cosmic rays on Pikes peak, using a Cloud chamber

Particles will show (temporarily) as condensation trail in gas volume (just like condensation trails of airplanes)
Antiparticles: Anderson’s discovery

- Result: discovery of a positively charged, electron-like particle dubbed the ‘positron’

![Image showing a 6 mm Pb plate with a 63 MeV positive track and a 23 MeV positive track significantly longer than expected for a proton.](image-url)
Antiparticles: Anderson’s discovery

- Confirmed with $\gamma \rightarrow e^+e^-$

Carl D. Anderson

The production and properties of positrons

Nobel Lecture, December 12, 1936
equal amounts of matter & antimatter produced (?)

Where is the antimatter?
Cosmic Antimatter...

- Antiparticles appear in cosmic ray showers
- But what about the original incoming (anti?)particle
- Must measure before the shower starts, eg. above the atmosphere..
AntiMatter Searches: AMS

Photo taken from Mir (1998)
AntiMatter Searches: AMS

AMS-2 currently scheduled for STS-134 (either the last or last but one shuttle flight!) for delivery to the ISS.

Look for anti-Helium: very unlikely to have been created as secondary product in collisions...

\[
\frac{N_{\overline{\text{He}}}}{N_{\text{He}}} < 1.1 \times 10^{-6} \text{ @ } 95\% CL
\]
No evidence for the original, “primordial” cosmic antimatter:

- Absence of anti-nuclei amongst cosmic rays in our galaxy
- Absence of intense γ-ray emission due to annihilation of distant galaxies in collision with antimatter
Antimatter & the Big Bang

Big Bang:

- Create equal amounts of matter & antimatter

Early universe
Big Bang:

- Create equal amounts of matter & antimatter
- Somewhere along the way, one (matter) is favored
- Final result: a bit of matter and lots of photons
  - $N_{\text{baryons}}/N_{\text{photons}} \approx 6 \times 10^{-10}$
VIOLATION OF CP INVARIANCE, C ASYMMETRY, AND BARYON ASYMMETRY OF THE UNIVERSE

A. D. Sakharov
Submitted 23 September 1966
ZhETF Pis'ma 5, No. 1, 32-35, 1 January 1967

The theory of the expanding Universe, which presupposes a superdense initial state of matter, apparently excludes the possibility of macroscopic separation of matter from antimatter; it must therefore be assumed that there are no antimatter bodies in nature, i.e., the Universe is asymmetrical with respect to the number of particles and antiparticles (C asymmetry). In particular, the absence of antibaryons and the proposed absence of baryonic neutrinos implies a non-zero baryon charge (baryonic asymmetry). We wish to point out a possible explanation of C asymmetry in the hot model of the expanding Universe (see [1]) by making use of effects of CP invariance violation (see [2]). To explain baryon asymmetry, we propose in addition an approximate character for the baryon conservation law.
Sakharov’s conditions on the Big Bang

• In 1967, Sakharov formulated three necessary conditions to generate universe with a baryon asymmetry:

1. a process that violates baryon number

2. C and CP violation, i.e. breaking of the C and CP symmetries

3. 1 & 2 should occur during a phase which is NOT in thermal equilibrium

• These lectures will focus on 2.

Andrei Sakharov
“Father” of Soviet hydrogen bomb & Nobel Peace Prize Winner
Existence of antimatter is a consequence of the combination of special relativity and quantum mechanics

No ‘primordial’ antimatter observed

Need something called ‘CP’ symmetry breaking to explain the absence of antimatter
Instructions by the VOC (Dutch East India Company) in Aug 1642:

“Since many rich mines and other treasures have been found in countries north of the equator between 15° and 40° latitude, there is no doubt that countries alike exist south of the equator. The provinces in Peru and Chili rich of gold and silver, all positioned south of the equator, are revealing proofs hereof.”

Abel Tasman discovered Tasmania (Nov. 1642), New Zealand (Dec. 1642), Fiji (Jan 1643), ...

From the point of view of the VOC, this was a disappointment.
Symmetries & “Hidden Observables”

“The root to all symmetry principles lies in the assumption that it is impossible to observe certain basic quantities; the non-observables”

1. Space translation symmetry:
   Hidden observable: **Absolute position**
   Conserved quantity: momentum

2. Time shift symmetry:
   Hidden observable: **Absolute time**
   Conserved quantity: Energy

3. Rotation symmetry:
   Hidden observable: **Absolute orientation**
   Conserved quantity: Angular momentum

See Noether’s theorem for more details
Symmetry & “Hidden Observable”

- Example: Potential energy between two charged particles:

\[ V = V (\vec{r}_1 - \vec{r}_2) \]

- translate origin by \( \vec{d} \):

\[
\begin{align*}
\vec{r}_1 & \rightarrow \vec{r}_1 - \vec{d} \\
\vec{r}_2 & \rightarrow \vec{r}_2 - \vec{d}
\end{align*}
\]

See Noether’s theorem for more details.
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  \[
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  \vec{r}_2 \rightarrow \vec{r}_2 - \vec{d}
  \]

- V is invariant under translations
  \[
  V (\vec{r}_1 - \vec{r}_2) \rightarrow V (\vec{r}_1' - \vec{r}_2')
  \]

- System is symmetric under translations

See Noether’s theorem for more details
Symmetry & “Hidden Observable”

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\[ V (\vec{r}_1 - \vec{r}_2) \rightarrow V (\vec{r}_1 - \vec{r}_2) \]

- System is symmetric under translations.

- Absolute position is a non-observable: the interaction is independent of the choice of origin.

- Result: total momentum is conserved

\[
\frac{d}{dt} (\vec{p}_1 + \vec{p}_2) = - \left( \vec{\nabla}_1 + \vec{\nabla}_2 \right) V (\vec{r}_1 - \vec{r}_2) = 0
\]

See Noether’s theorem for more details.
Symmetry & “Hidden Observable”

- Example: Potential energy between two charged particles:

\[ V = V (r_1^r - r_2^r) \]

- Translate particles by \( \vec{d} \):

\[ \begin{align*}
  \vec{r}_1 & \rightarrow \vec{r}_1 + \vec{d} \\
  \vec{r}_2 & \rightarrow \vec{r}_2 + \vec{d}
\end{align*} \]

- \( V \) is invariant under translations:

\[ V (r_1^r - r_2^r) \rightarrow V (\vec{r}_1^r - \vec{r}_2^r) \]

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\[ \frac{d}{dt} (\vec{p}_1 + \vec{p}_2) = - \left( \vec{\nabla}_1 + \vec{\nabla}_2 \right) V (\vec{r}_1^r - \vec{r}_2^r) = 0 \]

See Noether’s theorem for more details.
Discrete Symmetries

- Space, time translation & orientation symmetries are all continuous symmetries
  - Each symmetry operation associated with one or more continuous parameter
- There are also discrete symmetries
  - Spatial sign flip (x,y,z → -x,-y,-z) : P
  - Charge sign flip (Q → -Q) : C
  - Time sign flip (t → -t) : T

- Are these discrete symmetries exact symmetries that are observed in nature?
  - Key issue of these lectures

<table>
<thead>
<tr>
<th>Quantity</th>
<th>P</th>
<th>C</th>
<th>T</th>
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</thead>
<tbody>
<tr>
<td>Space vector</td>
<td>x</td>
<td>-x</td>
<td>x</td>
</tr>
<tr>
<td>Time</td>
<td>t</td>
<td>t</td>
<td>t</td>
</tr>
<tr>
<td>Momentum</td>
<td>p</td>
<td>-p</td>
<td>p</td>
</tr>
<tr>
<td>Spin</td>
<td>s</td>
<td>s</td>
<td>s</td>
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<td>Electrical field</td>
<td>E</td>
<td>-E</td>
<td>-E</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>B</td>
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Discrete Symmetries

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• Are these discrete symmetries exact symmetries that are observed in nature?
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In particle physics:

\[
P | e_L^- \rangle = | e_R^- \rangle \\
P | \pi^0 \rangle = -| \pi^0 \rangle \\
P | n \rangle = +| n \rangle \\
C | e_L^- \rangle = | e_L^+ \rangle \\
C | u \rangle = | \bar{u} \rangle \\
C | d \rangle = | \bar{d} \rangle \\
C | \pi^0 \rangle = +| \pi^0 \rangle
\]

\text{note: the definition of a 'left handed' particle will follow in 'a few slides' time}
Discrete Symmetries

- No evidence that electromagnetic & strong forces break C, P or T
- Example: $\pi^0$ decay into photons

\[ \pi^0 = \frac{1}{\sqrt{2}} \left[ u\bar{u} - d\bar{d} \right]_{L=0,S=0} \Rightarrow C\left|\pi^0\right\rangle = +\left|\pi^0\right\rangle \]

\[ C \cdot \vec{B} = -\vec{B}; C \cdot \vec{E} = -\vec{E} \Rightarrow C\left|\gamma\right\rangle = -\left|\gamma\right\rangle \]

- $\pi^0$ decays to two photons, but not three!
- Initial and final states are C even, thus C is conserved!
- Experimental test of P and C conservation in EM interaction:
  - C invariance: $\text{Br}(\pi^0 \rightarrow \gamma\gamma\gamma) < 3.1 \times 10^{-5}$
  - P invariance: $\text{Br}(\eta \rightarrow \pi^0 \pi^0 \pi^0 \pi^0) < 6.9 \times 10^{-7}$
- Experimental test of C invariance in strong interaction:
  - compare rates of positive and negative particles in eg. $p\bar{p} \rightarrow \pi^+ \pi^- X, K^+ K^- X, \ldots$
CPT theorem

“Any Lorentz-invariant local quantum field theory is invariant under the successive application of C, P and T”

G. Lüders, W. Pauli (1954); J. Schwinger (1951)

Assumptions:
1. Lorentz invariance
2. “principle of locality”
3. Causality
4. Vacuum lowest energy
5. Flat space-time
6. Point-like particles

Consequences:
1. Relation between spin and statistics: fields with integer spin commute and fields with half-numbered spin anticommute; Pauli exclusion principle
2. Particles and antiparticles have equal mass and lifetime, equal magnetic moments with opposite sign, and opposite quantum numbers

\[
\frac{M(K^0) - M(\bar{K}^0)}{\left(M(K^0) + M(\bar{K}^0)\right)/2} < 10^{-17} (95\% CL)
\]
Before 1956 physicists were *convinced* that the laws of nature were left-right symmetric. Strange?

A “gedanken” experiment:
Consider two perfectly mirror symmetric cars:

What would happen if the ignition mechanism uses, say, $^{60}\text{Co} \beta$ decay?
The $\theta$-$\tau$ puzzle

Observation of decays to two pions and three pions, but whatever decays (now known as $K^+$), has, in both decays, the same lifetime, mass, spin=0...

In 1953, Dalitz argued that since the pion has parity of $-1$,

- two pions (*) would combine to produce a net parity of $(-1)(-1) = +1$,
- and three pions (*) would combine to have total parity of $(-1)(-1)(-1) = -1$.

Hence, if conservation of parity holds, there are two distinct particles with parity +1 (the ‘$\theta$’) and parity -1 (the ‘$\tau$’)(**).

But how to explain the fact that the mass and lifetime are the same?

K$^+$ DECAY MODES

$K^-$ modes are charge conjugates of the modes below.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Fraction ($\Gamma_i/\Gamma$)</th>
<th>Scale factor/Confidence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_0$</td>
<td>$\pi^+\pi^0$</td>
<td>(21.13 ±0.14 ) %</td>
</tr>
<tr>
<td>$\Gamma_10$</td>
<td>$\pi^+\pi^0\pi^0$</td>
<td>( 1.73 ±0.04 ) %</td>
</tr>
<tr>
<td>$\Gamma_11$</td>
<td>$\pi^+\pi^+\pi^-$</td>
<td>( 5.576±0.031) %</td>
</tr>
</tbody>
</table>

Citation: S. Eidelman et al. (Particle Data Group), Phys. Lett. B 592, 1 (2004) (URL: http://pdg.lbl.gov)

(*) produced in the decay of a spin=0 mother

(**) Warning: do not confuse this ‘$\tau$’ with what is now known as the $\tau$ lepton...
Question of Parity Conservation in Weak Interactions*

T. D. Lee, Columbia University, New York, New York

AND

C. N. Yang,† Brookhaven National Laboratory, Upton, New York
(Received June 22, 1956)

Recent experimental data indicate closely identical masses\(^1\) and lifetimes\(^2\) of the \(\theta^+ (\equiv K_{\pi 1}^+)\) and the \(\tau^+ (\equiv K_{\pi 3}^+)\) mesons. On the other hand, analyses\(^3\) of the decay products of \(\tau^+\) strongly suggest on the grounds of angular momentum and parity conservation that the \(\tau^+\) and \(\theta^+\) are not the same particle. This poses a rather puzzling situation that has been extensively discussed.\(^4\)

One way out of the difficulty is to assume that parity is not strictly conserved, so that \(\theta^+\) and \(\tau^+\) are two different decay modes of the same particle, which necessarily has a single mass value and a single lifetime.

We wish to analyze this possibility in the present paper against the background of the existing experimental evidence of parity conservation. It will become clear that existing experiments do indicate parity conservation in strong and electromagnetic interactions to a high degree of accuracy, but that for the weak interactions (i.e., decay interactions for the mesons and hyperons, and various Fermi interactions) parity conservation is so far only an extrapolated hypothesis unsupported by experimental evidence. (One might even say that the present \(\theta - \tau\) puzzle may be taken as an indication that parity conservation is violated in weak interactions. This argument is, however, not to be taken seriously because of the paucity of our present knowledge concerning the nature of the strange particles. It supplies rather an incentive for an examination of the question of parity conservation.) To decide

---

The Nobel Prize in Physics 1957

"for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles"
Experimental Test of Parity Conservation in Beta Decay*

C. S. Wu, Columbia University, New York, New York

AND

E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson,
National Bureau of Standards, Washington, D. C.

(Received January 15, 1957)

Idea for experiment in collaboration with Lee and Yang: Look at spin of decay products of polarized radioactive nucleus

- Production mechanism involves exclusively weak interaction
Parity & Spin

How does the decay of a particle with spin tell you something about parity?

Gedanken-experiment: decay of a spin-1 particle to two spin-½ particles

- Spin: $|1, 1\rangle \rightarrow |\frac{1}{2}, \frac{1}{2}\rangle + |\frac{1}{2}, -\frac{1}{2}\rangle$

- It is important that initial state is maximally polarized: only then there is a single solution for the spin of the decay products. If not, e.g.
  - $|1, 0\rangle \rightarrow |\frac{1}{2}, +\frac{1}{2}\rangle + |\frac{1}{2}, -\frac{1}{2}\rangle$
  - $|1, 0\rangle \rightarrow |\frac{1}{2}, -\frac{1}{2}\rangle + |\frac{1}{2}, +\frac{1}{2}\rangle$
Parity & Spin

- A possible orientation

...
Parity & Spin

- A possible orientation
- And another...
Parity & Spin

- A possible orientation
- And another...
- And another...

Under parity transform $H \rightarrow -H$

If parity conserved, no reason to favour one value of $H$ over another
Parity & Spin: Helicity

- A possible orientation
- And another...
- And another...
- Introduce projection of spin on momentum, the helicity, to distinguish:
  \[ H = \frac{\vec{S} \cdot \vec{P}}{|\vec{S} \cdot \vec{P}|} \]
- Under parity transform \( H \rightarrow -H \)
- If parity conserved, no reason to favour one value of \( H \) over another

\[ H = +1 \quad \text{”Right Handed”} \]

\[ H = -1 \quad \text{”Left Handed”} \]

Warning: helicity assignment is not Lorentz invariant for massive particles: an observer can boost 'past' such that \( p \) changes direction.

For more details, please check on the difference between 'chirality' and 'helicity'.
How do you obtain a sample of $^{60}\text{Co}$ with spins aligned in one direction, and compare to non-aligned case?

Adiabatic demagnetization of $^{60}\text{Co}$ in a magnetic field at very low temperatures ($\sim 0.01 \text{ K}$!). Extremely challenging in 1956!
Mme Wu’s Experiment: setup

- How do you obtain a sample of $^{60}$Co with spins aligned in one direction, and compare to non-aligned case?

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- Adiabatic demagnetization of $^{60}$Co in a magnetic field at very low temperatures (~0.01 K!). Extremely challenging in 1956!
• The counting rate in the polarized case is different from the unpolarized case.

• Changing the direction of the B-field changes the counting rate!

• Electrons are preferentially emitted in the direction opposite the $^{60}$Co spin!

$^{60}$Co polarization decreases as a function of time as the temperature increases.
Mme Wu’s Experiment: conclusion

- The counting rate in the polarized case is different from the unpolarized case.
- Changing the direction of the B-field changes the counting rate!
- Electrons are preferentially emitted in the direction opposite the $^{60}$Co spin!
- Analysis of the results shows that data consistent with the emission of only left-handed (i.e. $H = -1$) electrons....
- ... and thus only right-handed anti-neutrinos.
Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon*

Richard L. Garwin,† Leon M. Lederman, and Marcel Weinrich

Physics Department, Nevis Cyclotron Laboratories, Columbia University, Irvington-on-Hudson, New York, New York

(Received January 15, 1957)
From P to C, P and CP

- Lederman et al.: Look at decay $\pi^+ \rightarrow \mu^+ \nu_\mu$

- Pion has spin 0; $\mu, \nu_\mu$ both have spin $\frac{1}{2}$
  - spin of decay products must be *oppositely* aligned
  - Helicity of muon is the *same* as that of neutrino.

---

\[ \begin{align*}
\mu^+ & \quad \pi^+ & \quad \nu_\mu \\
\bullet & \quad \bullet & \quad \bullet
\end{align*} \]
From P to C,P and CP

- Lederman et al.: Look at decay $\pi^+ \rightarrow \mu^+ \nu_\mu$

- Pion has spin 0; $\mu, \nu_\mu$ both have spin $\frac{1}{2}$
  $\rightarrow$ spin of decay products must be oppositely aligned
  $\rightarrow$ Helicity of muon is the same as that of neutrino.

- Nice bonus: can also measure polarization of both neutrino ($\pi^+$ decay) and anti-neutrino ($\pi^-$ decay)

- Result: All neutrinos produced are left-handed and all anti-neutrinos are right-handed
C, P and CP

\[
\begin{align*}
\pi^- + \mu^- (L) &\rightarrow \pi^+ + \mu^+ (R) \\
\pi^- + \mu^- (R) &\rightarrow \pi^+ + \mu^+ (L)
\end{align*}
\]
C, P and CP

C broken, P broken, but CP appears to be preserved in weak interaction!
Summary

- Existence of antimatter is a consequence of the combination of special relativity and quantum mechanics
- No ‘primordial’ antimatter observed
- Need something called ‘CP’ symmetry breaking to explain the absence of antimatter
- CPT is a very good symmetry
- C,P and CP are conserved in strong & EM interactions
- C,P completely broken by weak interactions, CP looks healthy...
Kaons...

$m_K \sim 494$ MeV/c$^2$

No strange particles lighter than kaons exist

⇒ Decay must violate “strangeness”

Strong force conserves “strangeness”

⇒ Decay is a pure weak interaction
**Kaons...**

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\[ K^+ \rightarrow \pi^+\pi^0, \pi^+\pi^-\pi^+, \pi^+\pi^0\pi^0 \]

\[ K^- \rightarrow \pi^-\pi^0, \pi^-\pi^+\pi^-, \pi^-\pi^0\pi^0 \]

\[ K^0 \rightarrow \pi^0\pi^0, \pi^0\pi^0\pi^0, \pi^+\pi^-, \pi^+\pi^-\pi^0 \]

\[ \overline{K}^0 \rightarrow \pi^0\pi^0, \pi^0\pi^0\pi^0, \pi^+\pi^-, \pi^+\pi^-\pi^0 \]
Kaons...

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<table>
<thead>
<tr>
<th>Isospin</th>
<th>( \bar{K}^0 )</th>
<th>( K^+ )</th>
<th>( K^- )</th>
<th>( K^0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1</td>
<td>((s\bar{d}))</td>
<td>((\bar{s}u))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>((s\bar{u}))</td>
<td>((\bar{s}d))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

-1 \( \Rightarrow \) “Strangeness”

**hadronic decays:**

\[ K^+ \rightarrow \pi^+\pi^0, \pi^+\pi^-\pi^+, \pi^+\pi^0\pi^0 \]

\[ K^- \rightarrow \pi^-\pi^0, \pi^-\pi^+\pi^-, \pi^-\pi^0\pi^0 \]

\[ K^0 \rightarrow \pi^0\pi^0, \pi^0\pi^0\pi^0, \pi^+\pi^-, \pi^+\pi^-\pi^0 \]

\[ \bar{K}^0 \rightarrow \pi^0\pi^0, \pi^0\pi^0\pi^0, \pi^+\pi^-, \pi^+\pi^-\pi^0 \]

**semi-leptonic decays:**

\[ K^+ \rightarrow \pi^0\mu^+\nu_\mu, \pi^0e^+\nu_e \]

\[ K^- \rightarrow \pi^0\mu^-\bar{\nu}_\mu, \pi^0e^-\bar{\nu}_e \]

\[ K^0 \rightarrow \pi^-\mu^+\nu_\mu, \pi^-e^+\nu_e \]

\[ \bar{K}^0 \rightarrow \pi^+\mu^-\bar{\nu}_\mu, \pi^+e^-\bar{\nu}_e \]
Kaons...

\( m_K \sim 494 \text{ MeV/c}^2 \)

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\[
\begin{array}{c|c}
\text{Isospin} & +1 \\
\bar{K}^0 (s\bar{d}) & K^+ (\bar{s}u) \\
-1 & \bar{K}^- (s\bar{u}) K^0 (\bar{s}d) \\
-1 & +1 \quad \text{“Strangeness”}
\end{array}
\]

hadronic decays:
\[
\begin{align*}
K^+ & \rightarrow \pi^+\pi^0, \pi^0\pi^-\pi^+, \pi^+\pi^0\pi^0 \\
K^- & \rightarrow \pi^-\pi^0, \pi^-\pi^+\pi^-, \pi^-\pi^0\pi^0 \\
K^0 & \rightarrow \pi^0\pi^0, \pi^0\pi^0\pi^0, \pi^+\pi^-, \pi^+\pi^-\pi^0 \\
\bar{K}^0 & \rightarrow \pi^0\pi^0, \pi^0\pi^0\pi^0, \pi^+\pi^-, \pi^+\pi^-\pi^0
\end{align*}
\]

semi-leptonic decays:
\[
\begin{align*}
K^+ & \rightarrow \pi^0\mu^+\nu_\mu, \pi^0 e^+\nu_e \\
K^- & \rightarrow \pi^0\mu^-\bar{\nu}_\mu, \pi^0 e^-\bar{\nu}_e \\
K^0 & \rightarrow \pi^-\mu^+\nu_\mu, \pi^- e^+\nu_e \\
\bar{K}^0 & \rightarrow \pi^+\mu^-\bar{\nu}_\mu, \pi^+ e^-\bar{\nu}_e
\end{align*}
\]

leptonic decays:
\[
\begin{align*}
K^+ & \rightarrow \mu^+\nu_\mu, \mu^+ e^+\nu_e \\
K^- & \rightarrow \mu^-\bar{\nu}_\mu, \mu^- e^-\bar{\nu}_e \\
K^0 & \rightarrow \mu^-\mu^+, \mu^- e^- e^+ \\
\bar{K}^0 & \rightarrow \mu^+\mu^-, \mu^+ e^- e^-
\end{align*}
\]
m_K \sim 494 \text{ MeV/c}^2

No strange particles lighter than kaons exist

⇒ Decay must violate “strangeness”

Strong force conserves “strangeness”

⇒ Decay is a pure weak interaction

<table>
<thead>
<tr>
<th>Isospin</th>
<th>K^0 \ (s\bar{d})</th>
<th>K^+ \ (\bar{s}u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1</td>
<td>\bar{K}^0</td>
<td>K^+</td>
</tr>
<tr>
<td>-1</td>
<td>K^-</td>
<td>K^0</td>
</tr>
</tbody>
</table>

-1 \quad +1 \quad “Strangeness”

Hadronic and leptonic decays: particle and anti-particle behave the same
Kaons...

\( m_K \sim 494 \text{ MeV/c}^2 \)

No strange particles lighter than kaons exist

\( \Rightarrow \) Decay must violate “strangeness”

Strong force conserves “strangeness”

\( \Rightarrow \) Decay is a pure weak interaction

\[ \begin{array}{c|c|c|c}
\text{Isospin} & \bar{K}^0 & K^+ & \bar{K}^0 \\
\hline
+1 & (s\bar{d}) & (\bar{s}u) & (s\bar{d}) \\
-1 & (s\bar{u}) & (\bar{s}u) & (s\bar{u}) \\
-1 & (\bar{s}d) & (s\bar{d}) & (\bar{s}d) \\
+1 & (\bar{s}d) & (s\bar{d}) & (\bar{s}d) \\
\end{array} \]

**Hadronic and leptonic decays:**

- Particle and anti-particle behave the same

**Semi-leptonic decays:**

- Particle and anti-particle are distinct!

“\( \Delta Q=\Delta S \) rule”
Behavior of Neutral Particles under Charge Conjugation

M. Gell-Mann,* Department of Physics, Columbia University, New York, New York

AND

A. Pais, Institute for Advanced Study, Princeton, New Jersey

(Received November 1, 1954)

Some properties are discussed of the $\theta^0$, a heavy boson that is known to decay by the process $\theta^0 \rightarrow \pi^+ + \pi^-$. According to certain schemes proposed for the interpretation of hyperons and $K$ particles, the $\theta^0$ possesses an antiparticle $\bar{\theta}^0$ distinct from itself. Some theoretical implications of this situation are discussed with special reference to charge conjugation invariance. The application of such invariance in familiar instances is surveyed in Sec. I. It is then shown in Sec. II that, within the framework of the tentative schemes under consideration, the $\theta^0$ must be considered as a “particle mixture” exhibiting two distinct lifetimes, that each lifetime is associated with a different set of decay modes, and that no more than half of all $\theta^0$’s undergo the familiar decay into two pions. Some experimental consequences of this picture are mentioned.
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Known:

$-K^0 \rightarrow \pi^+ \pi^-$
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Known:

$-K^0 \rightarrow \pi^+ \pi^-$

Hypothesis:

$-K^0$ is not equal to $K^0$
Some properties are discussed of the $K^0$, a heavy boson that is known to decay by the process $K^0 \rightarrow \pi^+ + \pi^-$. According to certain schemes proposed for the interpretation of hyperons and $K$ particles, the $K^0$ possesses an antiparticle $\overline{K^0}$ distinct from itself. Some theoretical implications of this situation are discussed with special reference to charge conjugation invariance. The application of such invariance in familiar instances is surveyed in Sec. I. It is then shown in Sec. II that, within the framework of the tentative schemes under consideration, the $K^0$ must be considered as a “particle mixture” exhibiting two distinct lifetimes, that each lifetime is associated with a different set of decay modes, and that no more than half of all $K^0$'s undergo the familiar decay into two pions. Some experimental consequences of this picture are mentioned.
Neutral Meson Mixing

\[ \Psi(t) = a(t) |K^0\rangle + b(t) |\bar{K}^0\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \]

\[ i \frac{\partial}{\partial t} \Psi = \hat{H} \Psi \]

\[ \hat{H} = \begin{pmatrix} M_K & 0 \\ 0 & M_K \end{pmatrix} \]
Neutral Meson Mixing

\[ \Psi(t) = a(t) \ket{K^0} + b(t) \ket{\bar{K}^0} \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \]

\[ i \frac{\partial}{\partial t} \Psi = \hat{H} \Psi \]

\[ \hat{H} = \begin{pmatrix} M_K & 0 \\ 0 & M_K \end{pmatrix} \]

As (eventually) \( K^0 \) and \( \bar{K}^0 \) decay, add an antihermitic part to the Hamiltonian

\[ \hat{H} = \begin{pmatrix} M_K - \frac{i}{2} \Gamma_K & 0 \\ 0 & M_K - \frac{i}{2} \Gamma_K \end{pmatrix} \]

\[ \frac{d}{dt} (|a|^2 + |b|^2) = - \begin{pmatrix} a^* \\ b^* \end{pmatrix} \begin{pmatrix} \Gamma_K & 0 \\ 0 & \Gamma_K \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \]

Can identify \( \Gamma_K \) as the decay width \((=1/\tau_K)\).
Neutral Meson Mixing

\[ \Psi(t) = a(t) |K^0\rangle + b(t) |\bar{K}^0\rangle \equiv \left( \begin{array}{c} a(t) \\ b(t) \end{array} \right) \]

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\[ \hat{H} = \begin{pmatrix} M_K - \frac{i}{2} \Gamma_K & 0 \\ 0 & M_K - \frac{i}{2} \Gamma_K \end{pmatrix} \]

Now consider the effect of CP symmetry:

\[ \text{CP} \quad \begin{array}{c} K^0 \\ \bar{K}^0 \end{array} \leftrightarrow \begin{array}{c} \pi^+ \pi^- \\ \pi^+ \pi^- \end{array} \]

\[ \hat{H} = \begin{pmatrix} M_K - \frac{i}{2} \Gamma_K & \Delta \\ \Delta & M_K - \frac{i}{2} \Gamma_K \end{pmatrix} \]
Neutral Meson Mixing

\[ \Psi(t) = a(t) |K^0\rangle + b(t) |\bar{K}^0\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \]

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\[ \hat{H} = \begin{pmatrix} M_K - \frac{i}{2} \Gamma_K & \Delta \\ \Delta & M_K - \frac{i}{2} \Gamma_K \end{pmatrix} \]

\[ K^0 \text{ and } \bar{K}^0 \text{ are no longer eigenstates of } H \]

their sum (K₁) & difference (K₂) are eigenstates...

and K₁ and K₂ have different masses and lifetimes.
Neutral Kaon Mixing

- $K_1$ and $K_2$ are their own antiparticle, but one is CP even, the other CP odd
- Only the CP even state can decay into 2 pions
  - $|K_1\rangle \ (CP=+1) \to \pi\pi \ (CP=-1 \times -1 = +1)$
- The CP odd state will decay into 3 pions instead
  - $|K_2\rangle \ (CP=-1) \to \pi\pi \pi \ (CP = -1 \times -1 \times -1 = -1)$
- There is a huge difference in available phasespace between the two ($\sim 600\times$) → the CP even state will decay much faster
  - Difference due to $M(K^0) \approx 3M(\pi)$
  - $\Delta$ has a large imaginary component!

\[
|K_1\rangle = \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}}
\]

\[
|K_2\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}
\]
Observation of Long-Lived Neutral $V$ Particles*

K. Lande, E. T. Booth, J. Impeduglia, and L. M. Lederman,
*Columbia University, New York, New York

AND

W. Chinowsky, Brookhaven National Laboratory,
*Upton, New York
(Received July 30, 1956)

At the present stage of the investigation one may only conclude that Table I, Fig. 2, and $Q^*$ plots are consistent with a $K^0$-type particle undergoing three-body decay. In this case the mode $\pi e\nu$ is probably prominent, the mode $\pi\mu\nu$ and perhaps other combinations may exist but are more difficult to establish, and $\pi^+\pi^-\pi^0$ is relatively rare. Although the Gell-Mann-Pais predictions (I) and (II) have been confirmed, long lifetime and “anomalous” decay mode are not sufficient to identify the observed particle with $\theta_2^0$. In particular,
Existence of antimatter is a consequence of the combination of special relativity and quantum mechanics.

No 'primordial' antimatter observed.

Need something called ‘CP’ symmetry breaking to explain the absence of antimatter.

CPT is a very good symmetry.

C,P and CP are conserved in strong & EM interactions.

C,P completely broken by weak interactions, CP looks healthy...

Neutral kaons can ‘mix’ (oscillate) into their antiparticles.

And this can causes lifetime & mass differences of the CP (!) eigenstates of the Hamiltonian.
Designing a CP violation experiment

- How do you obtain a pure ‘beam’ of $K_2$ particles?

- Exploit that decay of $K_1$ into two pions is much faster than decay of $K_2$ into three pions
  
  \[ \tau_1 = 0.89 \times 10^{-10} \text{ sec} \]
  
  \[ \tau_2 = 5.2 \times 10^{-8} \text{ sec} \ (\sim 600 \text{ times larger!}) \]

- Beam of neutral Kaons automatically becomes beam of $|K_2>$ as all $|K_1>$ decay very early on…

\[ K_1 \text{ decay early (into } \pi \pi \pi \pi \text{)} \quad \text{Pure } K_2 \text{ beam after a while!} \]
The Cronin & Fitch Experiment

Essential idea: Look for (CP violating) \( K_2 \rightarrow \pi^+\pi^- \) decays 20 meters away from \( K^0 \) production point

\[ K_2 \rightarrow \pi^+\pi^- \] decays 20 meters away from \( K^0 \) production point

If you detect two out of the three pions of a \( K_2 \rightarrow \pi\pi\pi \) decay their combined momentum will generally not point along the beam line
The Cronin & Fitch Experiment

Essential idea: Look for (CP violating) $K_2 \to \pi\pi$ decays 20 meters away from $K^0$ production point

**Decay of $K_2$ into 2 pions**

Plot the angle between the momentum direction of two pions and the beamline

Incoming $K_2$ beam
The Cronin & Fitch Experiment

Essential idea: Look for (CP violating) $K_2 \rightarrow \pi \pi$ decays 20 meters away from $K^0$ production point

**Incoming $K_2$ beam**

**Plot the angle between the momentum direction of two pions and the beamline**
Nobel Prize 1980

"for the discovery of violations of fundamental symmetry principles in the decay of neutral K-mesons"

“The discovery emphasizes, once again, that even almost self evident principles in science cannot be regarded fully valid until they have been critically examined in precise experiments.”
CP is (a bit) broken by weak decays...

Conclusion: weak decay violates CP (as well as C and P)
- But effect is tiny! (~0.2%)
- Maximal (100%) violation of P symmetry “easily” interpretable as absence of right-handed neutrino,

how to construct a physics law that violates a symmetry just a tiny bit?
Summary

- Existence of antimatter is a consequence of the combination of special relativity and quantum mechanics
- No ‘primordial’ antimatter observed
- Need something called ‘CP’ symmetry breaking to explain the absence of antimatter
- CPT is a very good symmetry
- C,P and CP are conserved in strong & EM interactions
- C,P completely broken by weak interactions, CP looks healthy...
- neutral kaons can ‘mix’ (oscillate) into their antiparticles
- and this can causes lifetime & mass differences of the CP eigenstates of the Hamiltonian
- CP is (a bit) broken in the neutral kaon system!
How to describe this?

EVIDENCE FOR THE $2\pi$ DECAY OF THE $K_2^0$ MESON

Princeton University, Princeton, New Jersey
(Received 10 July 1964)

three-body decays of the $K_2^0$. The presence of a
two-pion decay mode implies that the $K_2^0$ meson
is not a pure eigenstate of CP. Expressed as

$$K_2^0 = 2^{-1/2}[(K_0 - \bar{K}_0) + \epsilon (K_0 + \bar{K}_0)]$$

where $\tau_1$ and $\tau_2$ are the $K_1^0$ and $K_2^0$ mean lives
and $R_T$ is the branching ratio including decay to
two $\pi^0$. Using $R_T = \frac{3}{2} R$ and the branching ratio
quoted above, $|\epsilon| \approx 2.3 \times 10^{-3}$.

$$|K_L\rangle = |K_2\rangle + \epsilon |K_1\rangle$$

$$|K_S\rangle = |K_1\rangle + \epsilon |K_2\rangle$$

with $|\epsilon| \ll 1$
How to describe this?

Have a choice when ‘parameterizing’ $K_S$ and $K_L$:
1. in terms of $K^0$ and $\overline{K}^0$
2. in terms of $K_1$ and $K_2$

Historically, ‘kaon physics’ has chosen 2, but in ‘B physics’ (next lectures!), the equivalent of 1 is very much dominant...

This tends to be very confusing...

---

Evidence for the $2\pi$ Decay of the $K^0_L$ Meson

Princeton University, Princeton, New Jersey
(Received 10 July 1964)

Three-body decays of the $K^0_L$. The presence of a two-pion decay mode implies that the $K^0_L$ meson is not a pure eigenstate of $CP$. Expressed as $K^0_L = 2^{-1/2}[(K_0 - \overline{K}_0) + \epsilon (K_0 + \overline{K}_0)]$ then $|\epsilon|^2 \approx R_T \tau_1 \tau_2$ where $\tau_1$ and $\tau_2$ are the $K^0_L$ and $K^0_S$ mean lives and $R_T$ is the branching ratio including decay to two $\pi^0$. Using $R_T = 3/2 R$ and the branching ratio quoted above, $|\epsilon| \approx 2.3 \times 10^{-3}$.

$$|K_L\rangle = |K_2\rangle + \epsilon |K_1\rangle$$

$$|K_S\rangle = |K_1\rangle + \epsilon |K_2\rangle$$

with $|\epsilon| << 1$
How to describe this?

Have a choice when ‘parameterizing’ $K_S$ and $K_L$:
1. in terms of $K^0$ and $\bar{K}^0$
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Historically, ‘kaon physics’ has chosen 2, but in ‘B physics’ (next lectures!), the equivalent of 1 is very much dominant...

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EVIDENCE FOR THE 2$\pi$ DECAY OF THE $K_2^0$ MESON*†
J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

three-body decays of the $K_2^0$. The presence of a two-pion decay mode implies that the $K_2^0$ meson is not a pure eigenstate of $CP$. Expressed as $K_2^0 = 2^{-1/\pi}[(K_0 - \bar{K}_0) + \epsilon (K_0 + \bar{K}_0)]$ then $|\epsilon|^2 \approx R_T \tau_1 \tau_2$ where $\tau_1$ and $\tau_2$ are the $K_1^0$ and $K_2^0$ mean lives and $R_T$ is the branching ratio including decay to two $\pi^0$. Using $R_T = \frac{3}{2}R$ and the branching ratio quoted above, $|\epsilon| \approx 2.3 \times 10^{-3}$.

\[
\langle K_L | K_L \rangle \equiv 1 \Rightarrow |q|^2 + |p|^2 = 1
\]

\[
|K_L\rangle = p |K^0\rangle - q |\bar{K}^0\rangle
\]

\[
|K_S\rangle = p |K^0\rangle + q |\bar{K}^0\rangle
\]

\[
|q|^2 + |p|^2 = 1
\]

eg. $p = 1 + \epsilon$ with $|\epsilon| << 1$
Time Evolution of $K^0$ and $K^0$...

\[
\begin{pmatrix}
K_S(0) \\
K_L(0)
\end{pmatrix}
= 
\begin{pmatrix}
+q & +p \\
+q & -p
\end{pmatrix}
\begin{pmatrix}
K^0(0) \\
K^0(0)
\end{pmatrix}
\]

\[
\begin{pmatrix}
K_S(t) \\
K_T(t)
\end{pmatrix}
= 
\begin{pmatrix}
e^{-i\omega_S t} & 0 \\
0 & e^{-i\omega_L t}
\end{pmatrix}
\begin{pmatrix}
K_S(0) \\
K_L(0)
\end{pmatrix}
\]

\[
\begin{pmatrix}
K^0(t) \\
\bar{K}^0(t)
\end{pmatrix}
= 
\begin{pmatrix}
+1/2q & +1/2q \\
+1/2p & -1/2p
\end{pmatrix}
\begin{pmatrix}
K_S(t) \\
K_L(t)
\end{pmatrix}
\]
Time Evolution of $K^0$ and $\bar{K}^0$...

\[
\left( \frac{K^0(t)}{K^0(0)} \right) = \left( \begin{array}{cc} g_+(t) & pg_-(t) \\ \frac{q}{p} g_-(t) & g_+(t) \end{array} \right) \left( \frac{K^0(0)}{K^0(0)} \right) \quad g_\pm(t) = \frac{e^{-i\omega S t} \pm e^{-i\omega L t}}{2}
\]

\[t = 0\quad t\]

\[K^0\]  
\[\bar{K}^0\]  
\[\bar{K}^0\]  
\[K^0\]  
\[\bar{K}^0\]  
\[K^0\]  
\[\bar{K}^0\]  
\[K^0\]
Time Evolution of $K^0$ and $\bar{K}^0$...

\[
\left( \frac{K^0(t)}{\bar{K}^0(t)} \right) = \left( \begin{array}{cc}
g_+(t) & \frac{p}{q} g_-(t) \\
\frac{q}{p} g_-(t) & g_+(t) \end{array} \right) \left( \frac{K^0(0)}{\bar{K}^0(0)} \right)
\]

\[g_{\pm}(t) = \frac{e^{-i\omega_{St}} \pm e^{-i\omegaLt}}{2}\]

$t = 0$

\[\begin{align*}
K^0 & \rightarrow g_+(t) \\
\bar{K}^0 & \rightarrow \frac{p}{q} g_-(t)
\end{align*}\]

\[\begin{align*}
\pi^+ e^- \nu_e & \uparrow \frac{K^0}{\bar{K}^0} \\
\pi^- e^+ \nu_e & \uparrow K^0
\end{align*}\]
Time Evolution of $K^0$ and $\bar{K}^0$...

\[ A_T(t) = \frac{\bar{I}_{\pi-e+\nu}(t) - I_{\pi+e-\bar{\nu}}(t)}{\bar{I}_{\pi-e+\nu}(t) + I_{\pi+e-\bar{\nu}}(t)} = \frac{1 - |q/p|^4}{1 + |q/p|^4} = 4\mathcal{R}\epsilon \]

\[ A_T \approx (6.6 \pm 1.6) \times 10^{-3} \]

\[ \Rightarrow |q/p| = 0.9967 \pm 0.0008 \neq 1 \]
Time Evolution of $K^0$ and $\bar{K}^0$...

This measurement allows one to make an ABSOLUTE distinction between matter and anti-matter

- Positive charge is the charged carried by the lepton preferentially produced in the decay of the neutral $K$ meson

\[
A_T(t) = \frac{\bar{I}_{\pi^- e^+ \nu}(t) - I_{\pi^+ e^- \bar{\nu}}(t)}{\bar{I}_{\pi^- e^+ \nu}(t) + I_{\pi^+ e^- \bar{\nu}}(t)} = \frac{1 - |q/p|^4}{1 + |q/p|^4} = 4\Re\epsilon
\]

$A_T(t) = (6.6 \pm 1.6) \times 10^{-3}$

$\Rightarrow |q/p| = 0.9967 \pm 0.0008 \neq 1$
Summary

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- No ‘primordial’ antimatter observed
- Need something called ‘CP’ symmetry breaking to explain the absence of antimatter
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- C,P completely broken by weak interactions, CP looks healthy...
- Neutral kaons can ‘mix’ (oscillate) into their antiparticles
- And this can causes lifetime & mass differences of the CP eigenstates of the Hamiltonian
- CP is (a bit) broken in the neutral kaon system!
- And we can use this to unambiguously distinguish matter and antimatter
At $t=0$, events with a
- $K^+$ ‘tag’ are a pure $\bar{K}^0$ sample
- $K^-$ ‘tag’ are a pure $K^0$ sample
CPLEAR Detector @ CERN
At $t=0$, events with a $-K^+$ ‘tag’ are a pure $\bar{K}^0$ sample
- $K^-$ ‘tag’ are a pure $K^0$ sample

\[
p\bar{p} \rightarrow \left\{ \begin{array}{l}
\pi^- K^+ \bar{K}^0 \\
\pi^+ K^- \bar{K}^0
\end{array} \right. \]
Interference!

\[ g_{\pm}(t) = \frac{e^{-i\omega_st} \pm e^{-i\omega_L t}}{2} \]

\[ A_{+-} \equiv \langle \pi^+ \pi^- | K^0 \rangle \]

\[ \bar{A}_{+-} \equiv \langle \pi^+ \pi^- | \bar{K}^0 \rangle \]

<table>
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<td>( g_+(t) )</td>
<td>( A_{+-} \downarrow K^0 )</td>
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<td>( g_+(t) )</td>
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Interference!

\[ g_\pm(t) = \frac{e^{-i\omega_S t} \pm e^{-i\omega_L t}}{2} \]

\[ A_{+-} \equiv \langle \pi^+ \pi^- | K^0 \rangle \]
\[ \overline{A}_{+-} \equiv \langle \pi^+ \pi^- | \overline{K}^0 \rangle \]
\[ \lambda_{+-} \equiv \frac{q}{p} \overline{A}_{+-} \]

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<td>( A_{+-} \downarrow ) ( K^0 )</td>
</tr>
<tr>
<td>( \frac{q}{p} g_-(t) )</td>
<td>( \overline{A}_{+-} \uparrow )</td>
<td>( \overline{K}^0 )</td>
</tr>
<tr>
<td>( \overline{K}^0 )</td>
<td>( g_+(t) )</td>
<td>( A_{+-} \downarrow ) ( K^0 )</td>
</tr>
<tr>
<td>( \frac{p}{q} g_-(t) )</td>
<td>( A_{+-} \uparrow )</td>
<td>( \overline{K}^0 )</td>
</tr>
</tbody>
</table>
Interference!

\[ g_{\pm}(t) = \frac{e^{-i\omega_s t} \pm e^{-i\omega_L t}}{2} \]

\[ A_{+-} \equiv \langle \pi^+ \pi^- | K^0 \rangle \]
\[ \bar{A}_{+-} \equiv \langle \pi^+ \pi^- | \bar{K}^0 \rangle \]
\[ \lambda_{+-} \equiv \frac{q}{p} \frac{\bar{A}_{+-}}{A_{+-}} \]

<table>
<thead>
<tr>
<th>( t = 0 )</th>
<th>( t )</th>
<th>Rate</th>
</tr>
</thead>
</table>

\[ K^0 \]
\[ g_+(t) \]
\[ \frac{q}{p} g_-(t) \]
\[ A_{+-} \downarrow \]
\[ \pi^+ \pi^- \]
\[ \bar{A}_{+-} \uparrow \]
\[ \frac{K^0}{\bar{K}^0} \]
\[ \propto | A_{+-} [ g_+(t) + \lambda_{+-} g_-(t) ] |^2 \]

\[ \bar{K}^0 \]
\[ g_+(t) \]
\[ \frac{p}{q} g_-(t) \]
\[ A_{+-} \downarrow \]
\[ \pi^+ \pi^- \]
\[ A_{+-} \uparrow \]
\[ \frac{K^0}{\bar{K}^0} \]
\[ \propto | \bar{A}_{+-} [ g_+(t) + \frac{1}{\lambda_{+-}} g_-(t) ] |^2 \]
Three ways to break CP...

\[ g_{\pm}(t) = \frac{e^{-i\omega_st} \pm e^{-i\omega_Lt}}{2} \quad A_{+-} \equiv \langle \pi^+\pi^-|K^0 \rangle \quad \lambda_{+-} \equiv \frac{q}{p} \frac{\bar{A}_{+-}}{A_{+-}} \]

\[ \Gamma(K^0 \to \pi^+\pi^-) \propto |A_{+-}|^2 \left[ |g_+(t)|^2 + |\lambda_{+-}|^2 |g_-(t)|^2 + 2\Re(\lambda_{+-}g_+^*(t)g_-(t)) \right] \]

\[ \Gamma(\overline{K^0} \to \pi^+\pi^-) \propto |\bar{A}_{+-}|^2 \left[ |g_+(t)|^2 + \frac{1}{|\lambda_{+-}|^2} |g_-(t)|^2 + \frac{2}{|\lambda_{+-}|^2} \Re(\lambda_{+-}^*g_+^*(t)g_-(t)) \right] \]

1. CP violation in decay  \[ \left| \frac{\bar{A}_f}{A_f} \right| \neq 1 \]

2. CP violation in mixing:  \[ \left| \frac{q}{p} \right| \neq 1 \]

3. CP violation in interference mixing/decay:  \[ \mathcal{I}(\lambda_f) = \mathcal{I}\left(\frac{q}{p} \frac{\bar{A}_f}{A_f} \right) \neq 0 \]
Write in terms of observables...

\[ \eta_{+-} = \frac{1 - \lambda}{1 + \lambda} = \frac{pA - qA}{pA + qA} = \frac{\langle \pi^+\pi^- | K_L \rangle}{\langle \pi^+\pi^- | K_S \rangle} \]

\[ \eta_{-+} = |\eta_{+-}| e^{i\phi_{+-}} \]

\[ \lambda_{+-} \equiv \frac{q\bar{A}_{+-}}{pA_{+-}} \]

\[ \Gamma (K^0 \rightarrow \pi^+\pi^-) = N \left[ e^{-\Gamma_{st}} + |\eta_{+-}|^2 e^{-\Gamma_{Lt}} + 2e^{-\Gamma t} |\eta_{+-}| \cos (\Delta m t - \phi_{+-}) \right] \]

\[ \Gamma (\bar{K}^0 \rightarrow \pi^+\pi^-) = \bar{N} \left[ e^{-\Gamma_{st}} + |\eta_{+-}|^2 e^{-\Gamma_{Lt}} - 2e^{-\Gamma t} |\eta_{+-}| \cos (\Delta m t - \phi_{+-}) \right] \]

KS \quad KL \quad KS-KL \text{ interference}
Write in terms of observables...

\[ \eta_{+-} = \frac{1 - \lambda}{1 + \lambda} = \frac{pA - q\bar{A}}{pA + q\bar{A}} = \frac{\pi^+\pi^- |K_L\rangle}{\pi^+\pi^- |K_S\rangle} \]

\[ \eta_{+-} = |\eta_{+-}| e^{i\phi_{+-}} \]

\[ \lambda_{+-} \equiv \frac{q \bar{A}_{+-}}{p A_{+-}} \]

\[ \Gamma (K^0 \rightarrow \pi^+\pi^-) = N \left[ e^{-\Gamma_{st}} + |\eta_{+-}|^2 e^{-\Gamma_{Lt}} + 2e^{-\Gamma_t} |\eta_{+-}| \cos (\Delta mt - \phi_{+-}) \right] \]

\[ \Gamma (\bar{K}^0 \rightarrow \pi^+\pi^-) = \bar{N} \left[ e^{-\Gamma_{st}} + |\eta_{+-}|^2 e^{-\Gamma_{Lt}} - 2e^{-\Gamma_t} |\eta_{+-}| \cos (\Delta mt - \phi_{+-}) \right] \]

Interference term has a sign difference because:

\[ |K^0\rangle = \frac{1}{2p} (|K_L\rangle + |K_S\rangle) \]

\[ |\bar{K}^0\rangle = \frac{1}{2q} (|K_L\rangle - |K_S\rangle) \]
A determination of the CP violation parameter $\eta_{+-}$ from the decay of strangeness-tagged neutral kaons

CPLEAR Collaboration


\[
\mathcal{A} = \frac{\Gamma (K^0 \rightarrow \pi^+ \pi^-) - \Gamma (\overline{K}^0 \rightarrow \pi^+ \pi^-)}{\Gamma (K^0 \rightarrow \pi^+ \pi^-) + \Gamma (\overline{K}^0 \rightarrow \pi^+ \pi^-)}
\]

\[\bigcirc \quad K^0(t = 0) \rightarrow \pi^+ \pi^- \]

\[\bullet \quad \overline{K}^0(t = 0) \rightarrow \pi^+ \pi^- \]
Existence of antimatter is a consequence of the combination of special relativity and quantum mechanics

No ‘primordial’ antimatter observed

Need something called ‘CP’ symmetry breaking to explain the absence of antimatter

CPT is a very good symmetry

C,P and CP are conserved in strong & EM interactions

C,P completely broken by weak interactions, CP looks healthy...

neutral kaons can ‘mix’ (oscillate) into their antiparticles

and this can causes lifetime & mass differences of the CP eigenstates of the Hamiltonian

CP is (a bit) broken in the neutral kaon system!

And we can use this to unambiguously distinguish matter and antimatter

There are actually three ways in which CP can be broken!
CP and the Standard Model

Sofar:
- seen that Weak Interaction breaks both C and P ‘completely’ and CP ‘a bit’
- described what happens in very generic terms...

Next:
1. towards the Standard Model description of the Weak Interaction
2. how CP violation is integrated into the Standard Model
3. how can we test the Standard Model description of CP violation?
In the sixties, it seemed that there were

- 4 types of lepton: $e$, $ν_e$, $μ$, $ν_μ$
- 3 types of quark: $u$, $d$, $s$
- but many (most!) considered quarks a mathematical trick to explain the zoo of observed particles...

Let's sort them by their electrical charge:

0: $ν_e$, $ν_μ$  
$+\frac{2}{3}$: $u$

-1: $e$, $μ$  
$-\frac{1}{3}$: $d$, $s$
In the sixties, it seemed that there were

- 4 types of lepton: \( e, \nu_e, \mu, \nu_\mu \)
- 3 types of quark: \( u, d, s \)
- but many (most!) considered quarks a mathematical trick to explain the zoo of observed particles...

Let's sort them by their electrical charge:

\[
\begin{array}{ccc}
W^- & 0: & \nu_e, \nu_\mu \\
-1: & e, \mu & -\frac{1}{3}: d, s \\
\end{array}
\]

\[
\begin{array}{c}
W^+ \\
\end{array}
\]
Weak Interaction: Leptons vs Quarks

- Problem: using the measured muon lifetime, the *predicted* neutron lifetime is a bit too short -- and the *predicted* lifetime of strange particles way too short...

- Conclusion: measured strength (coupling constant) of weak interaction is systematically (!) different when measured in different types of processes???

- Or maybe we just overlooked something?
UNITARY SYMMETRY AND LEPTONIC DECAYS

Nicola Cabibbo
CERN, Geneva, Switzerland
(Received 29 April 1963)

\[
\ell^- \rightarrow W^- \nu_{\ell} \quad g \quad g \cos \theta_C \quad g \sin \theta_C
\]
UNITARY SYMMETRY AND LEPTONIC DECAYS

Nicola Cabibbo
CERN, Geneva, Switzerland
(Received 29 April 1963)

To determine $\theta$, let us compare the rates for $K^+ \to \mu^+ + \nu$ and $\pi^+ \to \mu^+ + \nu$; we find

$$\Gamma(K^+ \to \mu\nu)/\Gamma(\pi^+ \to \mu\nu) = \frac{\tan^2 \theta}{M_K^2(M_K^2 - M_{\mu\nu}^2)/M_{\pi}^2(1 - M_{\mu\nu}^2/M_{\pi}^2)^2}. \quad (3)$$

From the experimental data, we then get\(^5,^6\)

$$\theta = 0.257. \quad (4)$$
Weak Interaction: Universality

\[ \ell^- \rightarrow W^- \rightarrow \nu_\ell \quad d \cos \theta_C \quad s \sin \theta_C \]

\[ g = g = g \]
Weak Interaction: Universality

\[ \ell^- \rightarrow W^- \nu_\ell \]

\[ g = d \cos \theta_C + s \sin \theta_C \rightarrow W^- u \]

\[ g \]

\[ g \]
The $d$ quark as ‘seen’ by the $W$, the weak eigenstate $d'$, is not same as the mass eigenstate (the $d$)...

\[
\begin{pmatrix}
\nu_e \\
\mu
\end{pmatrix}_L, \quad \begin{pmatrix}
\nu_\mu \\
\mu
\end{pmatrix}_L, \quad \begin{pmatrix}
u \\\n\mu
\end{pmatrix}_L = \begin{pmatrix}
\nu \\
d' \\
\mu
\end{pmatrix}_L
\]
The $d'$ seen by the $W$ is a \textit{superposition} of the $d$ and $s$...
Weak Interaction: Universality

The $d'$ seen by the $W$ is a superposition of the $d$ and $s$...

- If $d'$ is a superposition of the $d$ and $s$, shouldn't there be an $s'$ as well? (*)

- If so, we can write $d'$ and $s'$ as rotated versions of $d$ and $s$

\[
\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}
\]

(*) yes: coupling of $Z$ to $d'$ without matching $s'$ causes a tree-level flavour changing neutral current, which is incompatible with eg. observed $\text{Br}(K_L \to \mu\mu)$
The d’ seen by the W is a superposition of the d and s...

- If d’ is a superposition of the d and s, shouldn’t there be an s’ as well? (*)

- If so, we can write d’ and s’ as rotated versions of d and s

- And if there is an s’, why no u-like partner for it?

\[
\begin{pmatrix}
    d' \\
    s'
\end{pmatrix}
= \begin{pmatrix}
    \cos \theta_C & \sin \theta_C \\
    -\sin \theta_C & \cos \theta_C
\end{pmatrix}
\begin{pmatrix}
    d \\
    s
\end{pmatrix}
\]

\[
\begin{pmatrix}
    u \\
    d'
\end{pmatrix}_L,
\begin{pmatrix}
    c \\
    s'
\end{pmatrix}_L
\]

(*) yes: coupling of Z to d’ without matching s’ causes a tree-level flavour changing neutral current, which is incompatible with eg. observed Br(K_L → μμ)
Cabibbo and the charm quark

- There was however one major exception which Cabibbo could not describe: $K^0 \rightarrow \mu^+ \mu^-$

- Observed rate much lower than expected from Cabibbos rate correlations (expected rate $\propto g^8 \sin^2 \theta_c \cos^2 \theta_c$)

\[ d \rightarrow \cos \theta_C \quad \mu^+ \]
\[ u \quad \nu_\mu \]
\[ \bar{s} \rightarrow \sin \theta_C \quad \mu^- \]
GIM and the charm quark

- How does it solve the $K^0 \rightarrow \mu^+\mu^-$ problem?

- Second decay amplitude added that is almost identical to original one, but has relative minus sign $\Rightarrow$ (Almost) fully destructive interference

- Cancellation not perfect because $u, c$ mass not quite the same...
Weak Interactions with Lepton-Hadron Symmetry*

S. L. Glashow, J. Iliopoulos, and L. Maiani†
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02139
(Received 5 March 1970)

We propose a model of weak interactions in which the currents are constructed out of four basic quark fields and interact with a charged massive vector boson. We show, to all orders in perturbation theory, that the leading divergences do not violate any strong-interaction symmetry and the next to the leading divergences respect all observed weak-interaction selection rules. The model features a remarkable symmetry between leptons and quarks. The extension of our model to a complete Yang-Mills theory is discussed.

\[
\begin{pmatrix}
\nu_e \\
e
\end{pmatrix}_L,
\begin{pmatrix}
\nu_\mu \\
\mu
\end{pmatrix}_L,
\begin{pmatrix}
u_e \\
e
\end{pmatrix}_L,
\begin{pmatrix}
u_\mu \\
\mu
\end{pmatrix}_L
\begin{pmatrix}
u_e \\
e
\end{pmatrix}_L,
\begin{pmatrix}
u_\mu \\
\mu
\end{pmatrix}_L
\]

One ‘tiny’ problem: no experimental evidence for a fourth quark...

...until 1974: Ting, Richter (Nobel prize 1976)
Cartoon shown by N. Cabibbo in 1966...

since then, there was tremendous progress in the understanding (better: describing) \( CP \) violation

⇒ next topic!
Existence of antimatter is a consequence of the combination of special relativity and quantum mechanics.

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And we can use this to unambiguously distinguish matter and antimatter.

There are actually three ways in which CP can be broken!

The weak and mass eigenstates of quarks are not the same...
CP-Violation in the Renormalizable Theory of Weak Interaction

Makoto KOBAYASHI and Toshihide MASKAWA

Department of Physics, Kyoto University, Kyoto

(Received September 1, 1972)

In a framework of the renormalizable theory of weak interaction, problems of CP-violation are studied. It is concluded that no realistic models of CP-violation exist in the quartet scheme without introducing any other new fields. Some possible models of CP-violation are also discussed.
The Nobel Prize winning part

Next we consider a 6-plet model, another interesting model of CP-violation. Suppose that 6-plet with charges \((Q, Q, Q, Q-1, Q-1, Q-1)\) is decomposed into \(SU_{weak}(2)\) multiplets as \(2+2+2\) and \(1+1+1+1+1+1\) for left and right components, respectively. Just as the case of \((A, C)\), we have a similar expression for the charged weak current with a \(3\times3\) instead of \(2\times2\) unitary matrix in Eq. (5). As was pointed out, in this case we cannot absorb all phases of matrix elements into the phase convention and can take, for example, the following expression:

\[
\begin{pmatrix}
\cos \theta_1 & -\sin \theta_1 \cos \theta_3 & -\sin \theta_1 \sin \theta_3 \\
\sin \theta_1 \cos \theta_3 & \cos \theta_1 \cos \theta_2 \cos \theta_3 - \sin \theta_2 \sin \theta_3 e^{i\phi_3} & \cos \theta_1 \cos \theta_2 \sin \theta_3 + \sin \theta_2 \cos \theta_3 e^{i\phi_3} \\
\sin \theta_1 \sin \theta_3 & \cos \theta_1 \sin \theta_2 \cos \theta_3 + \cos \theta_2 \sin \theta_3 e^{i\phi_3} & \cos \theta_1 \sin \theta_2 \sin \theta_3 - \cos \theta_2 \sin \theta_3 e^{i\phi_3}
\end{pmatrix}
\]

(13)

Then, we have CP-violating effects through the interference among these different current components. An interesting feature of this model is that the CP-violating effects of lowest order appear only in \(\Delta S \neq 0\) non-leptonic processes and in the semi-leptonic decay of neutral strange mesons (we are not concerned with higher states with the new quantum number) and not in the other semi-leptonic, \(\Delta S = 0\) non-leptonic and pure-leptonic processes.
How many ‘physical’ parameters in $V_{\text{CKM}}$?

• complex $N \times N$ matrix: $2N^2$ parameters

• must be unitary:
  • eg. $t$ must decay to either $b$, $s$ or $d$, so $|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$
  • in general: $V^* V = I \rightarrow N^2$ constraints

• freedom to change phase of quark fields $|q_j\rangle \rightarrow e^{i\phi_j} |q_j\rangle$

• $2N-1$ phases are irrelevant:
  \[
  \langle q_i | V_{ij} | q_j \rangle \rightarrow \langle q_i | e^{-i\phi_i} V_{ij} e^{i\phi_j} | q_j \rangle
  \]
  \[
  V_{ij} \rightarrow e^{i(\phi_j-\phi_i)} V_{ij}
  \]

• number of ‘physical’ parameters = $N^2-2N+1$
How many ‘physical’ parameters in $V_{\text{CKM}}$?

- complex $N \times N$ matrix: $2N^2$ parameters
- must be unitary:
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- in general: $V^*^T V = I \rightarrow N^2$ constraints
- freedom to change phase of quark fields $|q_j\rangle \rightarrow e^{i\phi_j} |q_j\rangle$
- $2N-1$ phases are irrelevant:

$$
\langle q_i \, | V_{ij} \, | q_j \rangle \rightarrow \langle q_i \, | e^{-i\phi_i} V_{ij} e^{i\phi_j} \, | q_j \rangle
$$

$$
V_{ij} \rightarrow e^{i(\phi_j-\phi_i)} V_{ij}
$$

- number of ‘physical’ parameters = $N^2-2N+1$
- how many can be rotation angles? $N(N-1)/2$
- For $N=2$: 1 parameter, with 1 rotation angle (Cabbibo!)
- For $N=3$: 4 parameters = 3 rotations + 1 irreducible complex phase!
What does CP (or, equivalently $T$) conjugation do with the Hamiltonian $H$?

\[
\begin{align*}
[\hat{x}, \hat{p}] &= i\hbar \\
T [\hat{x}, \hat{p}] T^{-1} &= TiT^{-1} \hbar \\
T \hat{x} &= \hat{x} \\
T \hat{p} &= -\hat{p}
\end{align*}
\]
Complex phases & CP

What does CP (or, equivalently T) conjugation do with the Hamiltonian $H$?

$$[\hat{x}, \hat{p}] = i\hbar$$

$$T[\hat{x}, \hat{p}]T^{-1} = TiT^{-1}\hbar$$

$T\hat{x} = \hat{x}$

$T\hat{p} = -\hat{p}$

$TiT^{-1} = -i$

The $T$ (and CP) operations must be **anti-unitary**, which implies **complex conjugation**!

With 3 (or more) generations $V_{CKM}$ can be complex $\rightarrow$ CP violation possible
Are there really 3 generations?

- Discovery of 5\textsuperscript{th} quark in 1977
  - Named ‘b’ for beauty/bottom
  - Mass around 4.5 GeV
  - Start of the 3\textsuperscript{rd} generation of quarks!

Observation of a Dimuon Resonance at 9.5 GeV in 400-GeV Proton-Nucleus Collisions

S. W. Herb, D. C. Hom, L. M. Lederman, J. C. Sens,\textsuperscript{(a)} H. D. Snyder, and J. K. Yoh
Columbia University, New York, New York 10027

and

Fermi National Accelerator Laboratory, Batavia, Illinois 60510

and

A. S. Ito, H. Jöstlein, D. M. Kaplan, and R. D. Kephart
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(Received 1 July 1977)

Accepted without review at the request of Edwin L. Goldwasser under policy announced 26 April 1976

Dimuon production is studied in 400-GeV proton-nucleus collisions. A strong enhancement is observed at 9.5 GeV mass in a sample of 8000 dimuon events with a mass $m_{\mu^+\mu^-} > 5$ GeV.
Discovery of the 6\textsuperscript{th} quark

- Discovery of top quark
  - complete 3-generation picture

- Took a long time (1994)
  - because $t$ quark is very heavy:
    $\sim 175 \text{ GeV/c}^2$!

Evidence for Top Quark Production in $\bar{p}p$ Collisions at $\sqrt{s} = 1.8 \text{ TeV}$

We summarize a search for the top quark with the Collider Detector at Fermilab (CDF) in a sample of $\bar{p}p$ collisions at $\sqrt{s} = 1.8 \text{ TeV}$ with an integrated luminosity of 19.3 pb$^{-1}$. We find 12 events consistent with either two $W$ bosons, or a $W$ boson and at least one $b$ jet. The probability that the measured yield is consistent with the background is 0.26%. Though the statistics are too limited to establish firmly the existence of the top quark, a natural interpretation of the excess is that it is due to $t\bar{t}$ production. Under this assumption, constrained fits to individual events yield a top quark mass of $174 \pm 10^{+2}_{-1}$ GeV/c$^2$. The $t\bar{t}$ production cross section is measured to be $13.9^{+4.1}_{-4.0}$ pb.

PACS numbers: 14.65.Ha, 13.85.Ni, 13.85.Qk
Are there \textit{more} than three generations?

- Surprisingly, you can actually say something about that…
  
  - Measure decay rate of $Z$ boson into all quarks, compare to total $Z$ boson decay rate
  
  - Because $Z$ can decay into $\nu \bar{\nu}$ each additional generation with a light neutrino increases the \textit{fraction} of $Z$ decaying to $\nu \bar{\nu}$, and thus decreases the \textit{fraction} of hadronic decays....
  
  - Shows conclusively that there are only 3 generations (of neutrinos, of the type we know, with mass $< \frac{M_Z}{2}$)
Existence of antimatter is a consequence of the combination of special relativity and quantum mechanics.

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And we can use this to unambiguously distinguish matter and antimatter.

There are actually three ways in which CP can be broken!

The weak and mass eigenstates of quarks are not the same... related by $V_{CKM}$.

With 3 or more families, one can have a complex phase(s) in $V_{CKM}$ and thus CP violation is possible!
Three generations, four parameters...

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}
= V_{CKM}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
= 
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\]

\[
V_{CKM} = 
\begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
\]

with \( s_{ij} = \sin \theta_{ij}, c_{ij} = \cos \theta_{ij} \)
so with four parameters \( \theta_{12}, \theta_{23}, \theta_{13}, \delta \)

... and many more observables!
How do you measure those numbers?

- **Magnitudes** are typically determined from ratio of decay rates

- Example 1 – Measurement of $|V_{ud}|$
  
  - Compare decay rates of neutron decay and muon decay
  
  - Ratio proportional to $|V_{ud}|^2$
  
  - $|V_{ud}| = 0.9735 \pm 0.0008$
  
  - $V_{ud}$ of order 1
How do you measure those numbers?

- Example 2 – Measurement of $|V_{us}|$
  - Compare decay rates of semileptonic $K^{-}$ decay and muon decay
  - Ratio proportional to $|V_{us}|^2$
  - $|V_{us}| = 0.2196 \pm 0.0023$

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]
How do you measure those numbers?

- Example 3 – Measurement of $V_{cb}$
  - Compare decay rates of $B^0 \rightarrow D^* \ell^+\nu$ and muon decay
  - Ratio proportional to $V_{cb}^2$
  - $|V_{cb}| = 0.0402 \pm 0.0019$
  - $|V_{cb}|$ is almost (but not quite) equal to $\cos(\theta_c)^2 = 0.0484$
How do you measure those numbers?

- Example 4 – Measurement of $V_{ub}$
  - Compare decay rates of $B^0 \rightarrow D^{*-}l^+\nu$ and $B^0 \rightarrow \pi^-l^+\nu$
  - Ratio proportional to $(V_{ub}/V_{cb})^2$
  - $|V_{ub}/V_{cb}| = 0.090 \pm 0.025$
Hierarchy...

\[
\begin{pmatrix}
|V_{ud}| & |V_{us}| & |V_{ub}| \\
|V_{cd}| & |V_{cs}| & |V_{cb}| \\
|V_{td}| & |V_{ts}| & |V_{tb}|
\end{pmatrix}
= \begin{pmatrix}
0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\
0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\
0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.99913^{+0.000044}_{-0.000043}
\end{pmatrix}
\]

Parametrization of the Kobayashi-Maskawa Matrix

Lincoln Wolfenstein

*Department of Physics, Carnegie–Mellon University, Pittsburgh, Pennsylvania 15213*  
(Received 22 August 1983)

The quark mixing matrix (Kobayashi–Maskawa matrix) is expanded in powers of a small parameter $\lambda$ equal to $\sin\theta_v = 0.22$. The term of order $\lambda^2$ is determined from the recently measured $B$ lifetime. Two remaining parameters, including the $CP$-nonconservation effects, enter only the term of order $\lambda^3$ and are poorly constrained. A significant reduction in the limit on $\epsilon'/\epsilon$ possible in an ongoing experiment would tightly constrain the $CP$-nonconservation parameter and could rule out the hypothesis that the only source of $CP$ nonconservation is the Kobayashi–Maskawa mechanism.

PACS numbers: 11.30.Er, 12.10.Ck, 13.25.+m
Hierarchy...

$$\begin{pmatrix}
| V_{ud} | & | V_{us} | & | V_{ub} | \\
| V_{cd} | & | V_{cs} | & | V_{cb} | \\
| V_{td} | & | V_{ts} | & | V_{tb} |
\end{pmatrix} =
\begin{pmatrix}
0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\
0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\
0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043}
\end{pmatrix}$$

---

Parametrization of the Kobayashi-Maskawa Matrix

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$$\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} \equiv
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} + \mathcal{O}(\lambda)$$
Hierarchy...

\[
\begin{pmatrix}
|V_{ud}| & |V_{us}| & |V_{ub}| \\
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\begin{pmatrix}
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V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
= 
\begin{pmatrix}
1 & \lambda & 0 \\
-\lambda & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
+ \mathcal{O}(\lambda^2)
\]
Hierarchy...

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
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\end{pmatrix}
\equiv 
\begin{pmatrix}
1 - \lambda^2/2 & \lambda & 0 \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
0 & -A\lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^3)
\]
Hierarchy...

\[
\begin{pmatrix}
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\end{pmatrix} =
\begin{pmatrix}
1 - \lambda^2/2 & \lambda & A\lambda^3 (\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & A\lambda^2 \\
A\lambda^3 (1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4)
\]
Hierarchy...

Flavour-changing transition by charged weak current (boldness indicates transition probability $\propto |V_{ij}|$)

Smallest couplings are complex $\rightarrow$ CP violation

What is the explanation for this structure? We don’t know!

- Transition within generation favored
- Transition from 1st to 2nd generation suppressed by $\lambda = \sin(\theta_c)$
- Transition from 2nd to 3rd generation suppressed by $\lambda^2 = \sin^2(\theta_c)$
- Transition from 1st to 3rd generation suppressed by $\lambda^3 = \sin^3(\theta_c)$
Existence of antimatter is a consequence of the combination of special relativity and quantum mechanics.

No 'primordial' antimatter observed.

Need something called 'CP' symmetry breaking to explain the absence of antimatter.

CPT is a very good symmetry.

C,P and CP are conserved in strong & EM interactions.

C,P completely broken by weak interactions, CP looks healthy...

Neutral kaons can 'mix' (oscillate) into their antiparticles.

And this can causes lifetime & mass differences of the CP eigenstates of the Hamiltonian.

CP is (a bit) broken in the neutral kaon system!

And we can use this to unambiguously distinguish matter and antimatter.

There are actually three ways in which CP can be broken!

The weak and mass eigenstates of quarks are not the same...

With 3 (or more) families, one can have a complex phase in the CKM matrix that defines the weak eigenstates, and this allows for CP violation!

There is a clear (and unexplained!) hierarchy in the CKM.
How to measure $|V_{td}|$ and $|V_{ts}|$?

$$
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
= \begin{pmatrix}
1 - \lambda^2/2 & \frac{\lambda}{1 - \lambda^2/2} & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 & -A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4)
$$
**Intermezzo: Neutral Meson Mixing**

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<td>$D^0$</td>
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<tr>
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<td>$\bar{b}$</td>
<td>$B^0$</td>
<td>$\bar{B}_s$</td>
<td>$\times$</td>
</tr>
</tbody>
</table>

- Need to be neutral and have distinct anti-particle ($\times$)
- Needs to have a non-zero lifetime
  - top is so heavy, it decays long before it can even form a meson ($\diamond$)
- That leaves four distinct cases...

Note: for (much!) more detail, see eg. arXiv:hep-ex/0103016v1
Intermezzo: Describing Mixing...

Time evolution of $B^0$ and $\overline{B^0}$ can be described by an effective Hamiltonian:

$$i \frac{\partial}{\partial t} \Psi = H \Psi$$

$$\Psi(t) = a(t) \left| B^0 \right\rangle + b(t) \left| \overline{B^0} \right\rangle \equiv \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$H = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix}_{\text{hermitian}} - \frac{i}{2} \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}_{\text{hermitian}}$$

what is the difference between $M_{12}$ and $\Gamma_{12}$?
Intermezzo: Describing Mixing…

Time evolution of $B^0$ and $\bar{B}^0$ can be described by an effective Hamiltonian:

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$$H = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

What is the difference between $M_{12}$ and $\Gamma_{12}$?

Remember: anti-hermitian part describes the ‘leaking’ out of the (sub)space spanned by $B^0$ and $\bar{B}^0$.

$$\frac{d}{dt} (|a|^2 + |b|^2) = -(a^* b^*) \begin{pmatrix} \Gamma \\ \Gamma_{12}^* \\ \Gamma \\ \Gamma_{12} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

$M_{12}$ describes $B^0 \leftrightarrow \bar{B}^0$ via virtual states.

$\Gamma_{12}$ describes $B^0 \leftrightarrow \bar{B}^0$ via real states, eg $\pi\pi$.

For details, look up “Wigner-Weisskopf” approximation…
Solving the Schrödinger Equation

\[ i \frac{\partial}{\partial t} \psi(t) = \begin{pmatrix} M - \frac{i}{2} \Gamma & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M - \frac{i}{2} \Gamma \end{pmatrix} \psi(t) \]

Solution (in terms of eigenvectors):

\[ \psi(t) = a \left| B_H(t) \right\rangle + b \left| B_L(t) \right\rangle \]

(a and b determined by initial conditions)

Eigenvectors:

\[ \left| B_H \right\rangle = p \left| B \right\rangle + q \left| \bar{B} \right\rangle \]
\[ \left| B_L \right\rangle = p \left| B \right\rangle - q \left| \bar{B} \right\rangle \]

From the eigenvector calculation:

\[ \frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}} \]

Evolution of eigenvectors:

\[ \left| B_H(t) \right\rangle = \left| B_H \right\rangle e^{-i \left( M + \frac{1}{2} \Delta m - \frac{i}{2} (\Gamma - \Delta \Gamma) \right) t} \]
\[ \left| B_L(t) \right\rangle = \left| B_L \right\rangle e^{-i \left( M - \frac{1}{2} \Delta m + \frac{i}{2} (\Gamma + \Delta \Gamma) \right) t} \]

\[ \Delta m \text{ and } \Delta \Gamma \text{ follow from the eigenvalues:} \]

\[ \Delta m + \frac{i}{2} \Delta \Gamma = 2 \sqrt{\left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right)} \]
Solving the Schrödinger Equation

\[ i \frac{\partial}{\partial t} \psi(t) = \begin{pmatrix} M - \frac{i}{2} \Gamma & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M - \frac{i}{2} \Gamma \end{pmatrix} \psi(t) \]

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\[ \left| B_L(t) \right> = \left| B_L \right> e^{-i \left( M - \frac{1}{2} \Delta m + \frac{i}{2} (\Gamma + \Delta \Gamma) \right) t} \]

\[ \Delta m \text{ and } \Delta \Gamma \text{ follow from the eigenvalues:} \]

\[ \Delta m + \frac{i}{2} \Delta \Gamma = 2 \sqrt{\left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right)} \]

if: \( \Gamma_{12} = 0 \Rightarrow \Delta \Gamma = 0, \left| \frac{q}{p} \right| = 1 \)
Mixing: Kaons vs. B mesons

- The difference between K mixing and ‘the rest’: \( \Gamma_{12} \)

- A large fraction of Kaon decays produce CP eigenstates:
  - all decays without leptons are CP eigenstates..
  - the CP even ones have more phase-space
  - Hence the lifetime difference (large \( \Gamma_{12} \!\!\) !)

- For \( B^0 \), (and, to a somewhat lesser extent \( B_s \)), the dominant decays are not CP eigenstates
  - hence \( \Delta \Gamma = 0 \) (smallish), and \( \Gamma_{12} \) does not contribute to \( B^0 \) mixing
  - note: as a result labeling eigenstates as ‘S’hort and ‘L’ong doesn’t make sense -- hence the ‘H’eavy and ‘L’ight

- so do \( B^0 \) (\( B_s \)) mesons actually mix?

Dominant decay amplitudes
Mixing: Box Diagrams

\[ \bar{b} \rightarrow V_{qb} \rightarrow V_{qb} \rightarrow \bar{d} \]

\[ W \]

\[ q = \bar{u}, \bar{c}, \bar{t} \]

\[ q = u, c, t \]

\[ d \rightarrow V_{qd} \rightarrow V_{qd} \rightarrow b \]

\[ q = u, c, t \]

\[ W^+ \]

\[ W^- \]
Mixing: Box Diagrams

\[ \bar{b} \rightarrow V_{\bar{q}b} V_{q\bar{b}} \rightarrow \bar{d} \]

\[ d \rightarrow V_{qd} V_{q^*d} \rightarrow b \]

\[ q = u, c, t \]

\[ q = \bar{u}, \bar{c}, \bar{t} \]

GIM(V_{CKM} unitarity):
if \( u, c, t \) same mass, everything cancels by construction!
Mixing: Box Diagrams

$W - W + q = u, c, t$

$W - W^+ q = u, c, t$

$t - \bar{t}: \propto m_t^2 |V_{td} V_{tb}^*|^2 \propto m_t^2 \lambda^6$

$c - \bar{c}: \propto m_c^2 |V_{cd} V_{cb}^*|^2 \propto m_c^2 \lambda^6$

$c - \bar{t}, \bar{c} - t: \propto m_c m_t V_{td} V_{tb}^* V_{cd} V_{cb}^* \propto m_c m_t \lambda^6$

$\Delta m_d = \frac{G_F^2}{6\pi^2} m_w^2 \eta_B S_0 (m_t^2 / m_W^2) m_{B_d} |V_{td}|^2 B_{B_d} f_{B_d}^2$

GIM($V_{CKM}$ unitarity): if $u, c, t$ same mass, everything cancels by construction!

Dominated by top quark mass: $\Delta m_B \approx 0.00002 \cdot \left( \frac{m_t}{\text{GeV}/c^2} \right)^2 \text{ps}^{-1}$

reference: $\tau_B \sim 1.5 \text{ ps}$
Before you decay, you’ve gotta ask yourself one question:

“do I feel like oscillating?”

well, do ya?

Dominated by top quark mass:

$$\Delta m_B \approx 0.00002 \cdot \left( \frac{m_t}{\text{GeV}/c^2} \right)^2 \text{ ps}^{-1}$$

reference: $$\tau_B \sim 1.5 \text{ ps}$$
**B⁰ Mixing: ARGUS, 1987**

- Produce an $b\bar{b}$ bound state, $\Upsilon(4S)$, in $e^+e^-$ collisions:
  - $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^0\overline{B}^0$

- and then observe:
  - $B^0_1 \rightarrow D^-_1 \mu^+_1 \nu_1$
  - $D^-_1 \rightarrow \overline{D}^0 \pi^-_1$
  - $D^0 \rightarrow K^+ \pi^-$
  - $B^0_2 \rightarrow D^-_2 \mu^+_2 \nu_2$
  - $D^-_2 \rightarrow D^- \pi^0$
  - $D^- \rightarrow K^+ \pi^-\pi^- \pi^0 \rightarrow \gamma \gamma$

- measure that $\sim17\%$ of $B^0$ and $\overline{B}^0$ mesons oscillate before they decay
  - $\tau_B \sim 1.5 \text{ ps} \Rightarrow \Delta m_d \sim 0.5/\text{ps}$,

First evidence of a really large top mass!
B<sub>s</sub> mixing:

\[ q = \bar{u}, \bar{c}, \bar{t} \]
\[ q = u, c, t \]

most important difference with B<sup>0</sup>: replace \( V_{td} \rightarrow V_{ts} \)

\[
\begin{align*}
\frac{\Delta m_d}{\Delta m_s} & \approx \frac{|V_{td}|^2}{|V_{ts}|^2} \approx \frac{\lambda^6}{\lambda^4} = \lambda^2 \approx 0.04 \\
& \Rightarrow \Delta m_s \approx 12 \text{ ps}^{-1} \\
\Delta m_d & = 0.502 \pm 0.006 \text{ ps}^{-1}
\end{align*}
\]

A more complete calculation leads to the SM expectation of \( \sim 18/\text{ps} \)
We report the observation of $B_s^0$-$\bar{B}_s^0$ oscillations from a time-dependent measurement of the $B_s^0$-$\bar{B}_s^0$ oscillation frequency $\Delta m_s$. Using a data sample of 1 fb$^{-1}$ of $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV collected with the CDF II detector at the Fermilab Tevatron, we find signals of 5600 fully reconstructed hadronic $B_s$ decays, 3100 partially reconstructed hadronic $B_s$ decays, and 61 500 partially reconstructed semileptonic $B_s$ decays. We measure the probability as a function of proper decay time that the $B_s$ decays with the same, or opposite, flavor as the flavor at production, and we find a signal for $B_s^0$-$\bar{B}_s^0$ oscillations. The probability that random fluctuations could produce a comparable signal is $8 \times 10^{-8}$, which exceeds 5σ significance. We measure $\Delta m_s = 17.77 \pm 0.10(\text{stat}) \pm 0.07(\text{syst})$ ps$^{-1}$ and extract $|V_{td}/V_{ts}| = 0.2060 \pm 0.0007(\Delta m_s) + 0.0001(\Delta m_d + \text{theor})$.

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PACS numbers: 14.40.Nd, 12.15.Ff, 12.15.Hh, 13.20.He
$D^0$ mixing

Look for ‘wrong sign’ $D^0$ decays

\[ D^0 \xrightarrow{} K^+ \pi^- \]
$D^0$ mixing

$D^0 \xrightarrow{\text{wrong sign}} \overline{D^0} \rightarrow K^+ \pi^-$

$|V_{cd}V_{us}^*| = \mathcal{O}(\lambda^2) \approx 0.04$

$d, s, b \text{ ‘in the loop’ instead of } u, c, t$
$\Rightarrow \text{GIM (almost) kills this amplitude...}$
Evidence for $D^0 - \overline{D^0}$ Mixing

We present evidence for $D^0 - \overline{D^0}$ mixing in $D^0 \rightarrow K^+ \pi^-$ decays from 384 fb$^{-1}$ of $e^+ e^-$ colliding-beam data recorded near $\sqrt{s} = 10.6$ GeV with the BABAR detector at the PEP-II storage rings at the Stanford Linear Accelerator Center. We find the mixing parameters $\Delta^2 = [-0.22 \pm 0.30 \text{(stat)} \pm 0.21 \text{(syst)}] \times 10^{-3}$ and $\gamma' = [9.7 \pm 4.4 \text{(stat)} \pm 3.1 \text{(syst)}] \times 10^{-3}$ and a correlation between them of $-0.95$. This result is inconsistent with the no-mixing hypothesis with a significance of 3.9 standard deviations. We measure $R_D$, the ratio of doubly Cabibbo-suppressed to Cabibbo-favored decay rates, to be $[0.303 \pm 0.016 \text{(stat)} \pm 0.010 \text{(syst)}]/%$. We find no evidence for $CP$ violation.

FIG. 2. (a) Projections of the proper-time distribution of combined $D^0$ and $\overline{D^0}$ WS candidates and fit result integrated over the signal region $1.843 < m_{K\pi} < 1.883$ GeV/$c^2$ and $0.1445 < \Delta m < 0.1465$ GeV/$c^2$. The result of the fit allowing (not allowing) mixing but not $CP$ violation is overlaid as a solid (dashed) curve. (b) The points represent the difference between the data and the no-mixing fit. The solid curve shows the difference between fits with and without mixing.
Summary of Neutral Meson Mixing

Blue:
given a $P^0$, at $t=0$, the probability of finding a $P^0$ at $t$.

Red:
given a $P^0$, at $t=0$, the probability of finding a $P^0\bar{b}$ at $t$.
Existence of antimatter is a consequence of the combination of special relativity and quantum mechanics
No 'primordial' antimatter observed
Need something called ‘CP’ symmetry breaking to explain the absence of antimatter
CPT is a very good symmetry
C,P and CP are conserved in strong & EM interactions
C,P completely broken by weak interactions, CP looks healthy...
normal kaons can 'mix' (oscillate) into their antiparticles
and this can causes lifetime & mass differences of the CP eigenstates of the Hamiltonian
CP is (a bit) broken in the neutral kaon system!
And we can use this to unambiguously distinguish matter and antimatter
There are actually three ways in which CP can be broken!
the weak and mass eigenstates of quarks are not the same...
with 3 (or more) families, one can have a complex phase in the CKM matrix that defines the weak eigenstates, and this allows for CP violation!
There is a clear (and unexplained!) hierarchy in the CKM
All four neutral mesons can mix -- and do, but some faster(slower) than others...
Heavy top quark needed for B mixing
How to put these measurements together?

- Many measurements, but in the end $V_{CKM}$ has only four parameters
- ...and only one of them is actually responsible for CP violation
- How to make a coherent/powerful/... test of the model?
- How to integrate CP measurements in this?

- $V_{CKM}$ has many relations amongst its elements....
Use the unitarity constraint(s)!

The 9 unitarity conditions of the 3×3 generations CKM matrix:

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]
Use the unitarity constraint(s)!

The 9 unitarity conditions of the 3×3 generations CKM matrix:

\[ |V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1 \]
Use the unitarity constraint(s)!

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\end{align*}
\]

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]
Use the unitarity constraint(s)!

The 9 unitarity conditions of the 3×3 generations CKM matrix:

\[ |V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1 \]

\[ |V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1 \]

\[ |V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1 \]

The 6 complex “Unitarity Triangles” involve different physics processes
Use the unitarity constraint(s)!

The 9 unitarity conditions of the $3 \times 3$ generations CKM matrix:

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1$$

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1$$

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$$

The 6 complex “Unitarity Triangles” involve different physics processes

$$V^{\ast}_{us}V_{ud} + V^{\ast}_{cs}V_{cd} + V^{\ast}_{ts}V_{td} = 0$$
Use the unitarity constraint(s)!

The 9 unitarity conditions of the 3×3 generations CKM matrix:

\[
|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1 \\
|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1 \\
|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1
\]

The 6 complex “Unitarity Triangles” involve different physics processes

\[
V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0 \\
V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0
\]
Use the unitarity constraint(s)!

The 9 unitarity conditions of the 3×3 generations CKM matrix:

\[
\begin{align*}
|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 &= 1 \\
|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 &= 1 \\
|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 &= 1
\end{align*}
\]

The 6 complex “Unitarity Triangles” involve different physics processes

\[
\begin{align*}
V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} &= 0 \\
V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} &= 0 \\
V_{us}^* V_{us} + V_{cs}^* V_{cs} + V_{ts}^* V_{ts} &= 0
\end{align*}
\]
Use the unitarity constraint(s)!

The 9 unitarity conditions of the 3×3 generations CKM matrix:

\[
\begin{align*}
|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 &= 1 \\
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\end{align*}
\]

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

The 6 complex “**Unitarity Triangles**” involve different physics processes

\[
\begin{align*}
V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} &= 0 \\
V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} &= 0 \\
V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{tb}^* V_{ts} &= 0
\end{align*}
\]
Use the unitarity constraint(s)!

The 9 unitarity conditions of the 3×3 generations CKM matrix:

\[ |V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1 \]
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\[ |V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1 \]

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\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
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The 6 complex “Unitarity Triangles” involve different physics processes

\[ V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0 \]
\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \]
\[ V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{tb}^* V_{ts} = 0 \]
\[ V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0 \]
\[ V_{ud} V_{td}^* + V_{us} V_{ts}^* + V_{ub} V_{tb}^* = 0 \]
\[ V_{cd} V_{td}^* + V_{cs} V_{ts}^* + V_{cb} V_{tb}^* = 0 \]
Use the unitarity constraint(s)!

The 9 unitarity conditions of the 3x3 generations CKM matrix:

\[
\begin{align*}
|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 &= 1 \\
|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 &= 1 \\
|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 &= 1
\end{align*}
\]

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
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\end{pmatrix}
\]

The 6 complex “Unitarity Triangles” involve different physics processes

\[
\begin{align*}
V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} &= 0 \\
V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} &= 0 \\
V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{tb}^* V_{ts} &= 0 \\
V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* &= 0 \\
V_{ud} V_{td}^* + V_{us} V_{ts}^* + V_{ub} V_{tb}^* &= 0 \\
V_{cd} V_{td}^* + V_{cs} V_{ts}^* + V_{cb} V_{tb}^* &= 0
\end{align*}
\]

‘sd’ triangle: $K^0$
Use the unitarity constraint(s)!

The 9 unitarity conditions of the 3×3 generations CKM matrix:

\[
\begin{align*}
|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 &= 1 \\
|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 &= 1 \\
|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 &= 1
\end{align*}
\]

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

The 6 complex “Unitarity Triangles” involve different physics processes

- \(V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0\)
- \(O(\lambda) + O(\lambda) + O(\lambda^5) = 0\) \(\text{‘sd’ triangle: } K^0\)

- \(V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0\)
- \(O(\lambda^3) + O(\lambda^3) + O(\lambda^3) = 0\) \(\text{‘bd’ triangle: } B^0\)

- \(V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{tb}^* V_{ts} = 0\)

- \(V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0\)

- \(V_{ud} V_{td}^* + V_{us} V_{ts}^* + V_{ub} V_{tb}^* = 0\)

- \(V_{cd} V_{td}^* + V_{cs} V_{ts}^* + V_{cb} V_{tb}^* = 0\)
Use the unitarity constraint(s)!

The 9 unitarity conditions of the 3×3 generations CKM matrix:

\[
\begin{align*}
|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 &= 1 \\
|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 &= 1 \\
|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 &= 1
\end{align*}
\]

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

The 6 complex “Unitarity Triangles” involve different physics processes

- **‘sd’ triangle:** \(K^0\)
  \[
  V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0
  \]
  \(\mathcal{O}(\lambda) + \mathcal{O}(\lambda) + \mathcal{O}(\lambda^5) = 0\)

- **‘bd’ triangle:** \(B^0\)
  \[
  V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0
  \]
  \(\mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) = 0\)

- **‘bs’ triangle:** \(B_s\)
  \[
  V_{ub} V_{us} + V_{cb} V_{cs} + V_{tb} V_{ts} = 0
  \]
  \(\mathcal{O}(\lambda^4) + \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^2) = 0\)
Use the unitarity constraint(s)!

The 9 unitarity conditions of the 3×3 generations CKM matrix:

\[ |V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1 \]
\[ |V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1 \]
\[ |V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1 \]

\[ \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} \]

The 6 complex “Unitarity Triangles” involve different physics processes:

\[ V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0 \]
\[ O(\lambda) + O(\lambda) + O(\lambda^5) = 0 \]
‘sd’ triangle: \( K^0 \)

\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \]
\[ O(\lambda^3) + O(\lambda^3) + O(\lambda^3) = 0 \]
‘bd’ triangle: \( B^0 \)

\[ V_{ub} V_{us}^* + V_{cb} V_{cs}^* + V_{tb} V_{ts}^* = 0 \]
\[ O(\lambda^4) + O(\lambda^2) + O(\lambda^2) = 0 \]
‘bs’ triangle: \( B_s \)

\[ V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0 \]
\[ \text{relative size of } CP\text{-violating effects} \]

\[ V_{ud} V_{td}^* + V_{us} V_{ts}^* + V_{ub} V_{tb}^* = 0 \]
\[ V_{cd} V_{td}^* + V_{cs} V_{ts}^* + V_{cb} V_{tb}^* = 0 \]
"The" Unitarity Triangle...

\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \]
“The” Unitarity Triangle...

\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \]

- The internal angles are quark rephasing independent and observable

\[
\begin{align*}
\alpha &= \arg \left( -\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}} \right) \\
\gamma &= \arg \left( -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right) \\
\beta &= \arg \left( -\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}} \right)
\end{align*}
\]
“The” Unitarity Triangle...

\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \]

- pick a quark phase convention such that \( V_{cb}^* V_{cd} \) is real

\begin{align*}
\alpha &= \arg \left( -\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}} \right) \\
\gamma &= \arg \left( -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right) \\
\beta &= \arg \left( -\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}} \right)
\end{align*}
"The" Unitarity Triangle...

\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \]

- Normalize all sides by \(-V_{cb}^* V_{cd}\)

\[ \alpha = \text{arg} \left( -\frac{V_{ub}^* V_{ud}}{V_{ub} V_{ud}} \right) \]

\[ \gamma = \text{arg} \left( -\frac{V_{ub} V_{ud}}{V_{cb}^* V_{cd}} \right) \]

\[ \beta = \text{arg} \left( -\frac{V_{cb} V_{cd}}{V_{tb}^* V_{td}} \right) \]
“The” Unitarity Triangle...

\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \]

- Put in the Wolfenstein parameterization of the \( V_{CKM} \) elements

\[ \alpha = \arg \left( -\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}} \right) \]
\[ \gamma = \arg \left( -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right) \]
\[ \beta = \arg \left( -\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}} \right) \]
“The” Unitarity Triangle...

\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \]

- And simplify...

\[ \bar{\rho} + i\eta \equiv (\rho + i\eta) \left( 1 - \frac{\chi^2}{2} \right) \]

\[ \alpha = \arg \left( -\frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}} \right) \]

\[ \gamma = \arg \left( -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right) \]

\[ \beta = \arg \left( -\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}} \right) \]
$(\bar{\rho}, \bar{\eta})$: the magnitudes only...
$(\bar{\rho}, \bar{\eta})$: the magnitudes and $\varepsilon_K$...
Existence of antimatter is a consequence of the combination of special relativity and quantum mechanics

No ‘primordial’ antimatter observed

Need something called ‘CP’ symmetry breaking to explain the absence of antimatter

CPT is a very good symmetry

C,P and CP are conserved in strong & EM interactions

C,P completely broken by weak interactions, CP looks healthy...

neutral kaons can ‘mix’ (oscillate) into their antiparticles

and this can causes lifetime & mass differences of the CP eigenstates of the Hamiltonian

CP is (a bit) broken in the neutral kaon system!

And we can use this to unambiguously distinguish matter and antimatter

There are actually three ways in which CP can be broken!

the weak and mass eigenstates of quarks are not the same...

with 3 (or more) families, one can have a complex phase in the CKM matrix that defines the weak eigenstates, and this allows for CP violation!

There is a clear (and unexplained!) hierarchy in the CKM

All four neutral mesons can mix -- and do, but some faster(slower) than others...

Heavy top quark needed for B mixing

Using the measured magnitudes of V_{CKM} elements, we can predict the weak phases!
Measuring the angles (phases!)..

- We’ve measured the sides, and have predictions for the angles,
- But how to measure the angles, i.e. phases?
- Interference!

\[
\begin{align*}
V_{ub}^*V_{ud} & \quad \alpha \\
V_{tb}^*V_{td} & \quad \beta \\
V_{cb}^*V_{cd} & \quad \gamma
\end{align*}
\]
Amplitudes and Observables

\[ A_j = \langle \text{final} | H_j | \text{initial} \rangle \]
\[ = |A_j| e^{+i \phi_j} \]

\[ P(i \rightarrow f) = |A_1 + A_2|^2 \]
Amplitudes and Observables

\[ A_j = \langle \text{final} | H_j | \text{initial} \rangle = |A_j| e^{+i\phi^{\text{weak}}_j} \]

\[ \bar{A}_j = A_j^* = |A_j| e^{-i\phi^{\text{weak}}_j} \]

\[ P(i \rightarrow f) = |A_1 + A_2|^2 \]

\[ P(\bar{i} \rightarrow \bar{f}) = |\bar{A}_1 + \bar{A}_2|^2 \]
Amplitudes and Observables

\[ A_j = \langle \text{final} | H_j | \text{initial} \rangle \]
\[ = |A_j| e^{i \phi_j^{\text{weak}}} \]

\[ \bar{A}_j = A_j^* \]
\[ = |A_j| e^{-i \phi_j^{\text{weak}}} \]

\[ P(i \rightarrow f) = |A_1 + A_2|^2 \]
\[ P(\bar{i} \rightarrow \bar{f}) = |\bar{A}_1 + \bar{A}_2|^2 \]
\[ = |A_1|^2 + 2 |A_1||A_2| \cos \phi_2 + |A_2|^2 \]
Amplitudes and Observables

\[ A_j = |A_j| e^{i(\phi_j + \text{weak}_j + \kappa_j)} \]

\[ P(i \rightarrow f) = |A_1 + A_2|^2 \]
Amplitudes and Observables

\[ A_j = |A_j|e^{i(\phi_j^{\text{weak}} + \kappa_j)} \]

\[ \bar{A}_j = |A_j|e^{i(-\phi_j^{\text{weak}} + \kappa_j)} \]

\[ P(i \to f) = |A_1 + A_2|^2 \]

\[ P(\bar{i} \to \bar{f}) = |\bar{A}_1 + \bar{A}_2|^2 \]
Amplitudes and Observables

\[ A_j = |A_j| e^{i(\phi_j^{\text{weak}} + \kappa_j)} \]

\[ \bar{A}_j = |A_j| e^{i(-\phi_j^{\text{weak}} + \kappa_j)} \]

\[ P(i \to f) = |A_1 + A_2|^2 \]

\[ = |A_1|^2 + 2|A_1||A_2| \cos (\phi_2 \pm \kappa_2) + |A_2|^2 \]

\[ P(i \to f) - \bar{P}(\bar{i} \to \bar{f}) = -4|A_1||A_2| \sin (\phi_2) \sin (\kappa_2) \]

(large) weak phases necessary but not sufficient for (large) CP violation...
Interference!

\[ g_\pm(t) = \frac{e^{-i\omega_s t} \pm e^{-i\omega_L t}}{2} \]

<table>
<thead>
<tr>
<th>( t = 0 )</th>
<th>( t )</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B^0 )</td>
<td>( f_{CP} )</td>
<td>( g_+(t) A_{f_{CP}} + \frac{q}{p} g_-(t) \overline{A}<em>{f</em>{CP}} )</td>
</tr>
<tr>
<td>( B^0 )</td>
<td>( \overline{A}_f )</td>
<td>( \frac{q}{p} g_-(t) \overline{A}<em>{f</em>{CP}} + \frac{p}{q} g_+(t) A_{f_{CP}} )</td>
</tr>
<tr>
<td>( \overline{B}^0 )</td>
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<td>( \overline{B}^0 )</td>
<td>( \overline{A}_f )</td>
<td>( \frac{q}{p} g_-(t) A_{f_{CP}} + \frac{p}{q} g_+(t) \overline{A}<em>{f</em>{CP}} )</td>
</tr>
</tbody>
</table>
Interference!

\[ g_\pm(t) = \frac{e^{-i\omega St} \pm e^{-i\omega Lt}}{2} \]

For neutral B mesons, \( g_- \) has a 90° phase difference wrt. \( g_+ \)

\[ g_+(t) = e^{-imt} e^{-\Gamma t/2} \cos \frac{\Delta mt}{2} \quad \lambda_{\text{fCP}} = \frac{q}{p} \frac{\bar{A}_{fCP}}{\bar{A}_{fCP}} \]

\[ g_-(t) = e^{-imt} e^{-\Gamma t/2} i \sin \frac{\Delta mt}{2} \]

\[ B^0 \quad g_+(t) \quad \downarrow A_f \quad B^0 \]

\[ f_{\text{CP}} \quad q \quad \frac{g_-(t)}{p} \quad \uparrow \bar{A}_f \quad \bar{B}^0 \]

\[ B^0 \quad \downarrow \bar{A}_f \quad \bar{B}^0 \]

\[ g_+(t) \quad \uparrow \bar{A}_f \]

\[ \bar{B}^0 \quad \downarrow A_f \quad B^0 \]

\[ f_{\text{CP}} \quad p \quad \frac{g_-(t)}{q} \quad \uparrow A_f \]

Amplitude

\[ A_{fCP} e^{-imt} e^{-\Gamma t/2} \left( \cos \frac{\Delta mt}{2} + \lambda_{fCP} i \sin \frac{\Delta mt}{2} \right) \]

\[ \bar{A}_{fCP} e^{-imt} e^{-\Gamma t/2} \left( \cos \frac{\Delta mt}{2} + \frac{1}{\lambda_{fCP}} i \sin \frac{\Delta mt}{2} \right) \]
Interference!

\[ t = 0 \quad \text{to} \quad t \]

<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>( B^0 \rightarrow f_{CP} )</td>
<td>( A_{f_{CP}} e^{-imt} e^{-\Gamma t/2} \left( \cos \frac{\Delta mt}{2} + \lambda_{f_{CP}} i \sin \frac{\Delta mt}{2} \right) )</td>
</tr>
<tr>
<td>( \overline{B^0} \rightarrow f_{CP} )</td>
<td>( \overline{A}<em>{f</em>{CP}} e^{-imt} e^{-\Gamma t/2} \left( \cos \frac{\Delta mt}{2} + \frac{1}{\lambda_{f_{CP}}} i \sin \frac{\Delta mt}{2} \right) )</td>
</tr>
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</table>

\( \Delta mt/2 = 0 \)
\( \Delta mt/2 = \pi/4 \)
\( \Delta mt/2 = \pi/2 \)
\( \Delta mt/2 = 3\pi/4 \)
Interference!

\[
\begin{align*}
t & = 0 \\
B^0 & \rightarrow f_{CP} \\
\overline{B}^0 & \rightarrow f_{CP}
\end{align*}
\]

\[
\begin{align*}
\lambda_{f_{CP}} & = \frac{q A_{f_{CP}}}{p A_{f_{CP}}}
\end{align*}
\]

\[
\begin{align*}
A_{f_{CP}} & = e^{-imt} e^{-\Gamma t/2} \left( \cos \frac{\Delta mt}{2} + \lambda_{f_{CP}} i \sin \frac{\Delta mt}{2} \right) \\
\overline{A}_{f_{CP}} & = e^{-imt} e^{-\Gamma t/2} \left( \cos \frac{\Delta mt}{2} + \frac{1}{\lambda_{f_{CP}}} i \sin \frac{\Delta mt}{2} \right)
\end{align*}
\]

\[
\begin{align*}
\Delta mt/2 & = 0 \\
\Delta mt/2 & = \pi/4 \\
\Delta mt/2 & = \pi/2 \\
\Delta mt/2 & = 3\pi/4
\end{align*}
\]

\[\Rightarrow \text{Time Dependent CP Asymmetry!!!}\]
Interference!

\[ \lambda_{fCP} = \frac{q \overline{A}_{fCP}}{p A_{fCP}} = \eta_{CP} \frac{q \overline{A}_{fCP}}{p A_{fCP}} \]

\[ B^0 \rightarrow f_{CP} \quad \frac{1}{2} e^{-\Gamma t} \left[ 1 + \left( \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \right) \cos(\Delta m t) - \left( \frac{2I(\lambda)}{1 + |\lambda|^2} \right) \sin(\Delta m t) \right] \]

\[ \overline{B^0} \rightarrow f_{CP} \quad \frac{1}{2} e^{-\Gamma t} \left[ 1 - \left( \frac{1 - |\lambda|^2}{1 + |\lambda|^2} \right) \cos(\Delta m t) + \left( \frac{2I(\lambda)}{1 + |\lambda|^2} \right) \sin(\Delta m t) \right] \]

\[ A_{CP} = \frac{\Gamma(\overline{B^0} \rightarrow f_{CP}) - \Gamma(B^0 \rightarrow f_{CP})}{\Gamma(\overline{B^0} \rightarrow f_{CP}) + \Gamma(B^0 \rightarrow f_{CP})} \]

\[ = -C_{fCP} \cos(\Delta m t) + S_{fCP} \sin(\Delta m t) \]

Next: find the right \( f_{CP} \)...
$B \rightarrow J/\psi K_S$

$$\lambda_{J/\psi K_S} = -\frac{q \overline{A_{J/\psi K_S}}}{p A_{J/\psi K_S}}$$
\[ \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \]

\[ \lambda_{J/\psi K_S} = -\frac{q}{p} \frac{A_{J/\psi K_S}}{A_{J/\psi K_S}} \]
$B \rightarrow J/\psi K_S$

$$\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}$$

$$\lambda_{J/\psi K_S} = -\frac{q}{p} \frac{A_{J/\psi K_S}}{A_{J/\psi K_S}}$$

$$|K_s\rangle = p_K |K^0\rangle + q_K |\bar{K}^0\rangle$$
$B \rightarrow J/\psi K_S$

\[
\frac{V^*_{tb} V_{td}}{V_{tb} V^*_{td}}
\]

$\lambda_{J/\psi K_S} = -\frac{q}{p} \frac{A_{J/\psi K_S}}{A_{J/\psi K_S}}$

\[
V^*_{cb} V_{cs} q K
\]

\[
\overline{B^0} \rightarrow \overline{d} \rightarrow c \ J/\psi
\]

$\overline{B^0} \rightarrow \overline{d} \rightarrow s \ K^0 \times \langle K_s | \overline{K^0} \rangle$

\[
V^*_{cb} V_{cs} p K
\]

$B^0 \rightarrow d \rightarrow \overline{c} \ J/\psi
\]

$B^0 \rightarrow d \rightarrow \overline{s} \ K^0 \times \langle K_s | K^0 \rangle$

$|K_s\rangle = p_K |K^0\rangle + q_K |\overline{K^0}\rangle$
\[ \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \]

\[ \lambda_{J/\psi K_S} = -\frac{q}{p} \frac{A_{J/\psi K_S}}{A_{J/\psi K_S}} \]

\[ \frac{q_K}{p_K} \approx \frac{V_{cs} V_{cd}}{V_{cs} V_{cd}^*} \]

\[ V_{cb} V_{cs} q_K \approx V_{cb} V_{cs} p_K \approx \left\langle K_s | K^0 \right\rangle \]

\[ \left\langle K_s | K^0 \right\rangle \approx \left\langle K_s | K^0 \right\rangle \]
\[
\lambda_{J/\psi K_S} = -\frac{q}{p} \frac{A_{J/\psi K_S}}{A_{J/\psi K_S}}
\]
B → J/ψK_S

\[ \lambda_{J/\psi K_S} = -\frac{q}{p} \frac{A_{J/\psi K_S}}{A_{J/\psi K_S}} \]

\[
\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \]

\[
\frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}} \]

\[ \overline{B^0} \rightarrow \bar{b} d \]

\[ \overline{B^0} \rightarrow b \bar{d} \]

\[ \lambda_{J/\psi K_S} = -\frac{q}{p} \frac{A_{J/\psi K_S}}{A_{J/\psi K_S}} \]

\[ \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \]

\[ \overline{B^0} \rightarrow \bar{b} d \]

\[ \overline{B^0} \rightarrow b \bar{d} \]
$B \rightarrow J/\psi K_S$

\[
\lambda_{J/\psi K_S} = -\frac{q}{p} \frac{\overline{A}_{J/\psi K_S}}{\overline{A}_{J/\psi K_S}} = - \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cb} V_{cd}}{V_{cb}^* V_{cd}}
\]
$\lambda_{J/\psi K_S} = -q \frac{\overline{A}_{J/\psi K_S}}{p A_{J/\psi K_S}}$

$= -\frac{V_{tb}V_{td} V_{cb}V_{cd}}{V_{tb}V_{td}^* V_{cb}V_{cd}^*}$

$= -e^{-2i\beta}$
$\mathbf{B \to J/\psi K_S}$

$$\lambda_{J/\psi K_S} = -\frac{q}{p} \frac{A_{J/\psi K_S}}{A_{J/\psi K_S}}$$

$$= -\frac{V_{tb}^* V_{td} V_{cb} V_{cd}}{V_{tb} V_{td}^* V_{cb}^* V_{cd}}$$

$$= -e^{-2i\beta}$$

$$\mathcal{A}_{CP} = \frac{\Gamma(B^0 \to J/\psi K_S) - \Gamma(B^0 \to J/\psi K_S)}{\Gamma(B^0 \to J/\psi K_S) + \Gamma(B^0 \to J/\psi K_S)} = \sin (2\beta) \sin (\Delta mt)$$
Currently running or recently ended programs

CKM Physics and CP Violation
Worldwide Experimental Facilities
Approved experiments (mostly in construction)

Currently running or recently ended programs

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Planned experiments that were not ratified

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New proposals

CKM Physics and CP Violation

Worldwide Experimental Facilities
B factories: $e^+e^- \rightarrow Y(4S) \rightarrow B\bar{B}$

Very clean environment
Measuring $A_{CP}(t)$ in $B^0 \to J/\psi \, K_S$

- **Times evolution of $Y(4s)$ decay**
  - $t=0$: Decay of $Y(4s)$ into 2 B mesons

  Neither B is in a specific eigenstate, but $B_1B_2$ system evolves *coherently*, i.e. flavor anti-correlation preserved in evolution

  - $t=t_1$ One of the two mesons ($B_1$) decays.

    If it decays into a flavor eigenstate, flavor conservation in the coherent $B_1B_2$ requires that also $B_2$ goes into a flavor eigenstate, even though it has not decayed yet!

  - $t=t_2$ The other B meson decays

    This meson can decay into any kind of state, a $B^0\bar{B}$ or $B^0$ flavor eigenstate. The latter means that mixing took place between $t_1$ and $t_2$. It can also decay into a CP eigenstate (either directly or after a mixing)
Measuring $A_{\text{CP}}(t)$ in $B^0 \rightarrow J/\psi \ K_S$

- We can use this process to measure $A_{\text{CP}}(t)$

$$A_{\text{CP}}(t) = \frac{N(B^0(t) \rightarrow J/\psi K_s) - N(B^0(t) \rightarrow J/\psi K_s)}{N(B^0(t) \rightarrow J/\psi K_s) + N(B^0(t) \rightarrow J/\psi K_s)} = \sin 2\beta \sin \Delta mt$$

- More precise reading of $A_{\text{CP}}$:
  - We don’t necessarily need to produce $B^0$ mesons in a flavor eigenstate, we just need to measure the decay into CP eigenstate after a known time $t$ since it was (through whatever means) in a flavor eigenstate

- Bottom line
  - Look for $\Upsilon(4s) \rightarrow B^0\bar{B}^0$ where ‘1st’ $B^0$ decays into flavor eigenstate and ‘2nd’ $B^0$ decays into CP eigenstate and interpret $t_2-t_1$ as the correct time for the $A_{\text{CP}}(t)$ formula
  - Note: formalism also works when $\Delta t<0$!
Measuring $A_{CP}(t)$ in $B^0 \rightarrow J/\psi \ K_S$

- The last little catch: How do you measure a decay time difference?
  - Naïve solution: measure both decay times
  - Impossible in practice because you measure decay times from flight distances, but nothing marks the decay point of the $Y(4s)$
  - And even if you knew the decay point, the produced $B$ are almost at rest in the $Y(4S)$ frame...

- $m(Y(4s)) = 10.58 \text{ GeV}$
- $m(B^0) = 5.28 \text{ GeV} \rightarrow p^*_B = 340 \text{ MeV/c} \rightarrow (\beta\gamma)^* = 0.064 \rightarrow 30 \text{ um for } \tau = 1.5 \text{ ps}$
Measuring $A_{\text{CP}}(t)$ in $B^0 \rightarrow J/\psi \ K_S$

- Solution: Make the $Y(4s)$ fly!

- Both $B^0$ mesons practically at rest in $Y(4s)$ rest frame
  
  - If $Y(4s)$ moves in lab frame at modest speed no $B^0$'s will be emitted ‘backwards’ as speed of $Y(4s)$ in lab is always larger than maximum backward speed of $B^0$ w.r.t the $Y(4s)$

- Result: spacing between vertices $\propto$ difference in decay time!
The B Factories: PEP–2 (SLAC, USA) and KEK–B (KEK, Japan)
The B Factories: PEP–2 (SLAC, USA) and KEK–B (KEK, Japan)

KEKB delivered $\int L = 950$ fb$^{-1}$
Peak record: $2.1 \times 10^{34}$ cm$^{-2}$s$^{-1}$

PEP-II delivered $\int L = 557$ fb$^{-1}$
Peak record: $1.2 \times 10^{34}$ cm$^{-2}$s$^{-1}$
switched off: April 7th, 2008
The BABAR Detector

1.5 T solenoid

EMC
6580 CsI(Tl) crystals

e^+ (3.1 GeV)

DiRC (PID)
144 quartz bars
11000 PMTs

e^- (9 GeV)

Instrumented Flux Return
iron / RPCs (muon / neutral hadrons)

Drift Chamber
40 stereo layers

Silicon Vertex Tracker
5 layers, double-sided strips
The BABAR Detector
The BABAR Detector
The BABAR Detector
Ingredients of the measurements
Ingredients of the measurements

PEP-2 (SLAC)

\[ E_{e^-} = 9 \text{ GeV} \quad E_{e^+} = 3.1 \text{ GeV} \]
\[ \sqrt{s} = 10.58 \text{ GeV} \]
\[ \langle \beta \gamma \rangle_{\tau(4S)} = 0.56 \]
Ingredients of the measurements

PEP-2 (SLAC)

\[ E_{e^-} = 9 \text{ GeV} \quad E_{e^+} = 3.1 \text{ GeV} \]
\[ \sqrt{s} = 10.58 \text{ GeV} \]
\[ \langle \beta \gamma \rangle_{\Upsilon(4S)} = 0.56 \]

\[ \Upsilon(4S) \]

\[ e^- \rightarrow B_{\text{rec}} \]
\[ B_{\text{rec}} \rightarrow D^{*-} \pi^+, \ldots \]
\[ f_{\text{flav}} = D^{*-} \pi^+, \ldots \]
\[ f_{\text{CP}} = J/\psi K_S^0, J/\psi K_L^0, \ldots \]

Exclusive B Meson Reconstruction
Ingredients of the measurements

**PEP-2 (SLAC)**

\[ E_{e^-} = 9 \text{ GeV} \quad E_{e^+} = 3.1 \text{ GeV} \]
\[ \sqrt{s} = 10.58 \text{ GeV} \]
\[ \langle \beta \gamma \rangle_{\Upsilon(4S)} = 0.56 \]

\[ e^- \rightarrow \Upsilon(4S) \rightarrow e^+ e^- B \rightarrow e^- e^+ B_{\text{rec}} \]

\[ \text{rec} = \text{flav, } \overline{\text{flav}}, \text{ } CP \]
\[ f_{\text{flav}} = D^{*-} \pi^+, \ldots \]
\[ f_{CP} = J/\psi K_S^0, J/\psi K_L^0, \ldots \]
Ingredients of the measurements

PEP-2 (SLAC)

\[ E_{e^-} = 9 \text{ GeV} \quad E_{e^+} = 3.1 \text{ GeV} \]
\[ \sqrt{s} = 10.58 \text{ GeV} \]
\[ \langle \beta \gamma \rangle_{T(4S)} = 0.56 \]

**rec = flav, \overline{\text{flav}}, CP**

\[ f_{\text{flav}} = D^{*-} \pi^+, \ldots \]
\[ f_{\text{CP}} = J/\psi K^0_S, J/\psi K^0_L, \ldots \]
Ingredients of the measurements

PEP-2 (SLAC)

\[ E_{e^-} = 9 \text{ GeV} \quad E_{e^+} = 3.1 \text{ GeV} \]
\[ \sqrt{s} = 10.58 \text{ GeV} \]
\[ \langle \beta \gamma \rangle_{T(4S)} = 0.56 \]

\[ \gamma(4S) \]

\[ e^- \rightarrow e^+ \]

\[ B_{tag} \rightarrow t_{tag} \]

\[ B_{rec} \rightarrow t_{rec} \]

\[ J/\psi \rightarrow \mu^+ \]

\[ K^0 \rightarrow \pi^- \]

\[ \ell^+ \rightarrow K^+ \]

\[ \pi^- \rightarrow \pi^+ \]

B-Flavor Tagging

Exclusive B Meson Reconstruction

rec = flav, flav, CP

\[ f_{\text{flav}} = D^{*-} \pi^+ , ... \]

\[ f_{CP} = J/\psi K^0_S, J/\psi K^0_L, ... \]

tag = \( B^0, \overline{B}^0 \)

\[ f_{B^0} = X \ell^+ \nu, X K^+, X \pi^- , ... \]
Ingredients of the measurements

PEP-2 (SLAC)

$E_{e^-} = 9 \text{ GeV}$, $E_{e^+} = 3.1 \text{ GeV}$

$\sqrt{s} = 10.58 \text{ GeV}$

$\langle \beta \gamma \rangle_{\Upsilon(4S)} = 0.56$

B-Flavor Tagging

Exclusive $B$ Meson Reconstruction

$\gamma(4S)$

$e^-$

$e^+$

$B^0_{\text{rec}}$

$B^0_{\text{tag}}$

$t_{\text{tag}}$

$t_{\text{rec}}$

$\Delta t \equiv t_{\text{rec}} - t_{\text{tag}}$

$\ell^+$

$K^+$

$\pi^-$

$J/\psi$

$\mu^+$

$K_S^0$

$\pi^−$

$\mu^−$

$\pi^+$

rec = flav, $\overline{\text{flav}}, \text{CP}$

$f_{\text{flav}} = D^{*-} \pi^+, ...$

$f_{\text{CP}} = J/\psi K_S^0, J/\psi K_L^0, ...$

tag = $B^0, B^0$

$f_{B^0} = X \ell^+ \nu, X K^+, X \pi^−, ...$
**Ingredients of the measurements**

**PEP-2 (SLAC)**

\[ E_{e^-} = 9 \text{ GeV} \quad E_{e^+} = 3.1 \text{ GeV} \]
\[ \sqrt{s} = 10.58 \text{ GeV} \]
\[ \langle \beta \gamma \rangle_{\Upsilon(4S)} = 0.56 \]

\[ \Upsilon(4S) \rightarrow e^+e^- \]

**B-Flavor Tagging**

**Exclusive B Meson Reconstruction**

**Vertexing & Time Difference Determination**

\[ \Delta t \equiv t_{\text{rec}} - t_{\text{tag}} \]

\[ \Delta t \approx \Delta z/c \langle \beta \gamma \rangle_{\Upsilon(4S)} \]

\[ \langle \Delta z \rangle_{B \bar{B}} \approx 260 \mu m \]
Example of fully reco’d event

At $\Delta t=0$ (i.e. when the $D^*\pi$ decay happened), the ‘CP’ $B$ was/would have been a $B^0$
Putting it all together:

\[ \mathcal{B}_0 (\Delta t) \quad \overline{\mathcal{B}_0 (\Delta t)} \]

\[ A_{\text{CP}} (\Delta t) = D \cdot \sin (\Delta m_d \Delta t) \]

Imperfect flavor tagging

Finite \( \Delta t \) resolution

\[ \sin 2\beta \]

\[ D \cdot \sin 2\beta \]
**CP violation in B system!**

\[ e^+ e^- \rightarrow \Upsilon(4S) \rightarrow B_{\text{rec}} B_{\text{tag}} \]

\[ B_{\text{rec}} \rightarrow J/\psi K_S \]

\[ B_{\text{tag}} \rightarrow \ell^\pm X, K^\pm X, \pi_{\text{soft}}^\mp, \pi_{\text{fast}}^\pm X, \ldots \]
CP violation in B system!

\[ e^+ e^- \rightarrow \Upsilon(4S) \rightarrow B_{\text{rec}} B_{\text{tag}} \]

- \( B_{\text{rec}} \rightarrow J/\psi K_S \)
- \( B_{\text{tag}} \rightarrow \ell^\pm X, K^\pm X, \pi^\mp_{\text{soft}}, \pi^\pm_{\text{fast}} X, \ldots \)
CP violation in B system!

\[ e^+ e^- \rightarrow \Upsilon(4S) \rightarrow B_{\text{rec}} B_{\text{tag}} \]

- \( B_{\text{rec}} \rightarrow J/\psi K_S \)
- \( B_{\text{tag}} \rightarrow \ell^\pm X \)

220 events

- 98% signal purity!
- 3.3% mistag rate!
- 20% better \( \Delta t \) resolution!

Caveat: these plots made on a dataset of 100 million \( \Upsilon(4S) \) decays…
Putting it all together...
Putting it all together...


excluded area has CL > 0.95

sin 2β

sol. w/ cos 2γ < 0 (excl. at CL > 0.95)
Putting it all together...
Precise measurement of sin(2\beta) agrees perfectly with other measurements and CKM assumptions.

There is a solution of \( \rho, \eta \) consistent with all measurements.
Putting it all together...

The CKM model of CP violation has successfully been confirmed!

At the scale of electroweak interaction, CKM is *dominates* CP violation

No need (at current level of precision!) for physics beyond the Standard Model to explain observed CP violation

Precise measurement of $\sin(2\beta)$ agrees perfectly with other measurements and CKM assumptions

There is a solution of $\rho, \eta$ consistent with all measurements
• Existence of antimatter is a consequence of the combination of special relativity and quantum mechanics
• No ‘primordial’ antimatter observed
• Need something called ‘CP’ symmetry breaking to explain the absence of antimatter
• CPT is a very good symmetry
• C,P and CP are conserved in strong & EM interactions
• C,P completely broken by weak interactions, CP looks healthy...
• neutral kaons can ‘mix’ (oscillate) into their antiparticles
• and this can causes lifetime & mass differences of the CP eigenstates of the Hamiltonian
• CP is (a bit) broken in the neutral kaon system!
• And we can use this to unambiguously distinguish matter and antimatter
• There are actually three ways in which CP can be broken!
• the weak and mass eigenstates of quarks are not the same...
• with 3 (or more) families, one can have a complex phase in the CKM matrix that defines the weak eigenstates, and
  this allows for CP violation!
• There is a clear (and unexplained!) hierarchy in the CKM
• All four neutral mesons can mix -- and do, but some faster(slower) than others...
• Heavy top quark needed for B mixing
• Using the measured magnitudes of $V_{\text{CKM}}$ elements, we can predict the weak phases!
• And the measurements agree with the predictions...
Penguins on the horizon...
We now turn to the “penguin” diagrams of figs. 2e and 2f.

We now turn to the “penguin” diagrams of figs. 2e and 2f.


In the spring of 1977, Mike Chanowitz, Mary K. and I wrote a paper on GUTs [Grand Unified Theories] predicting the b quark mass before it was found. When it was found a few weeks later, Mary K., Dimitri, Serge Rudaz and I immediately started working on its phenomenology.

That summer, there was a student at CERN, Melissa Franklin, who is now an experimentalist at Harvard. One evening, she, I, and Serge went to a pub, and she and I started a game of darts. We made a bet that if I lost, I had to put the word penguin into my next paper. She actually left the darts game before the end, and was replaced by Serge, who beat me. Nevertheless, I felt obligated to carry out the conditions of the bet.

For some time, it was not clear to me how to get the word into this b quark paper that we were writing at the time…. Later… I had a sudden flash that the famous diagrams look like penguins. So we put the name into our paper, and the rest, as they say, is history.”

John Ellis in Mikhail Shifman’s “ITEP Lectures in Particle Physics and Field Theory”, hep-ph/9510397
Are we sure that $A_{CP}(J/\psi K_S) = \sin(2\beta)$?

$$A_{B^0 \rightarrow J/\psi K^0} = V_{cb}V_{cs}^* T + V_{tb}V_{ts}^* P_t + V_{cb}V_{cs}^* P_c + V_{ub}V_{us}^* P_u$$

Use the ‘bs’ unitarity triangle relation: $V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0$

$$A_{B^0 \rightarrow J/\psi K^0} = V_{cb}V_{cs}^* (T + P_c - P_t) + V_{ub}V_{us}^* (P_u - P_t) = O(\lambda^2)$$

Relative phase: $\gamma$

$$= O(\lambda^4)$$

Penguin contribution with different weak phase suppressed by $\lambda^2$:

$\Rightarrow$ Extraction of $\sin(2\beta)$ from $J/\psi K_S$ is “theoretically clean”
Direct CP violation: $\Gamma( B^0 \to f) \neq \Gamma(\bar{B}^0 \to \bar{f})$

CP violation if $\Gamma( B^0 \to f) \neq \Gamma(\bar{B}^0 \to \bar{f})$

But: need 2 amplitudes $\to$ interference

$$A_{B^0 \to K^-\pi^+} = V_{ub} V_{us}^* (T + P_u - P_t) + V_{cb} V_{cs}^* (P_c - P_t)$$
**Direct CP violation:** $\Gamma(B^0 \to f) \neq \Gamma(B^0 \to \bar{f})$

CP violation if $\Gamma(B^0 \to f) \neq \Gamma(B^0 \to \bar{f})$

But: need 2 amplitudes $\to$ interference

\[ A_{B^0 \to K^- \pi^+} = V_{ub}V_{us}^*(T + P_u - P_t) + V_{cb}V_{cs}^*(P_c - P_t) \]

Only different if both $\delta$ and $\gamma$ are $\neq 0$ !

$\Rightarrow \Gamma(B^0 \to f) \neq \Gamma(B^0 \to \bar{f})$
Direct CP violation: $\Gamma(B^0 \rightarrow f) \neq \Gamma(\bar{B}^0 \rightarrow \bar{f})$

First observation of Direct CPV in B decays (2004):

$$A_{CP} = \frac{\Gamma(\bar{B}^0 \rightarrow K^-\pi^+) - \Gamma(B^0 \rightarrow K^+\pi^-)}{\Gamma(\bar{B}^0 \rightarrow K^-\pi^+) + \Gamma(B^0 \rightarrow K^+\pi^-)}$$

$B^0 \rightarrow K^-\pi^+$

$B^0 \rightarrow K^+\pi^-$

$A_{CP} = -0.133 \pm 0.030 \pm 0.009$

$4.2\sigma$

$A_{CP} = -0.114 \pm 0.020$

BaBar + Belle:
• Why are loop dominated decay processes very perceptible to ‘new’ particles?

• You can simply replace an ‘internal quark line’ (the circle) with ‘new’ particles without affecting the initial and final state of the decay

• Momentum flowing through loop should be integrated to “infinity” → Potential high masses of virtual particles don’t kill contribution…

• No tree-level diagrams: less ‘competition’ from boring Standard Model amplitudes..
Existence of antimatter is a consequence of the combination of special relativity and quantum mechanics

No ‘primordial’ antimatter observed

Need something called ‘CP’ symmetry breaking to explain the absence of antimatter

CPT is a very good symmetry

C,P and CP are conserved in strong & EM interactions

C,P completely broken by weak interactions, CP looks healthy...

neutral kaons can 'mix' (oscillate) into their antiparticles

and this can causes lifetime & mass differences of the CP eigenstates of the Hamiltonian

CP is (a bit) broken in the neutral kaon system!

And we can use this to unambiguously distinguish matter and antimatter

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There is a clear (and unexplained!) hierarchy in the CKM

All four neutral mesons can mix -- and do, but some faster(slower) than others...

Heavy top quark needed for B mixing

Using the measured magnitudes of $V_{CKM}$ elements, we can predict the weak phases!

And the measurements agree with the predictions...

Penguins and rare decays could provide hints of physics beyond the Standard Model
A. Höcker: The Violation of Symmetry between Matter and Antimatter (3 & 4)
ALTAS and CMS concentrate on "high-\(p_T\)" discovery physics.

Their \(B\)-physics potential relies on the low-\(p_T\) performance of the Trigger systems.

LHCb is not a fixed-target experiment (looks like one). It concentrates on low-\(p_T\) \(B\) physics.

Virtues over ATLAS & CMS:
Low-\(p_T\) track trigger, particle ID & better mass resolution
**B Physics at Tevatron and LHC**

*B* physics at hadron colliders is complementary to the $e^+e^-$ *B* factories.

**Strengths:**  High statistics: LHC will produce $10^{12}$ bb/year at $2 \times 10^{32}$ cm$^{-2}$s$^{-1}$; accesses the $B_s$; sensitive to very rare modes, if clean signature; production of *b* baryons and $B_c$ mesons

**Weaknesses:**  Worse tagging (no quantum coherence) and background; no rare modes with neutrinos can be reconstructed; less efficient for $\pi^0$;
B physics at the Tevatron & LHC

**Prime Measurements:** (many, many more interesting measurements to be done!)

- $B_s$ mixing phase: very small in SM, excellent probe for new physics: $2\beta_s \approx -2 \arg V_{ts} V_{tb}^*$

- $B_s \rightarrow \mu^+\mu^-$: FCNC (box & EW-penguin-mediated) rare decay

- SM BR $\sim 3 \cdot 10^{-9}$; current limit (CDF) < $5.8 \cdot 10^{-8}$ at 95% CL

- in MSSM, BR enhanced by $\tan(\beta)^6$ (note: $\beta =$ ratio of VEVs, not $\sin(2\beta)$)

**New Result:**

- Combined Tevatron result (NEW)
- Full inclusion of systematics and non-Gaussian effects
- No constraints
- Make available to combination groups

- SM p-value = 0.034 (2.1σ) (2.0σ at nearest point)

- Compared to HFAG 2008:
  - Larger CDF sample + Better accounting for tails
  - "same level of SM agreement.

- Both CDF and D0 currently working on 2x samples.
- Expect improved precision by simultaneous fit of CDF and D0 samples.

**Graphical Diagram:**

- $B_s^0 \rightarrow \bar{B}_{s0}^0$
- $B_s \rightarrow J/\Psi$
- $\Delta \Gamma_s$ vs $\beta_s^{J/\Psi\phi}$
- 68% CL, 95% CL, 99% CL

- SM p-value = 0.034 (2.1σ)
  - (2.0σ at nearest point)

- $\beta_s^{J/\Psi\phi}$ range:
  - $[0.27, 0.59]$ at 68%
  - $[0.10, 1.42]$ at 95%
CKM phase and the universe

• could the CKM phase generate the observed baryon asymmetry?

• KM $CP$-violating asymmetries, $d_{CP}$, must be proportional to the Jarlskog invariant $J$

\[ d_{CP} = J \times \tilde{F}_U \times \tilde{F}_D \]

where:

\[ J = \text{Im}(V_{ud}V_{cs}V_{us}^*V_{cd}^*) \approx A^2 \lambda^6 \eta \quad \text{and:} \]

\[ \tilde{F}_U = (m_t^2 - m_c^2) \cdot (m_t^2 - m_u^2) \cdot (m_c^2 - m_u^2) \]

\[ \tilde{F}_D = (m_b^2 - m_s^2) \cdot (m_b^2 - m_d^2) \cdot (m_s^2 - m_d^2) \]

Area of every unitarity triangle!

• If any two up- or down-type masses equal, can redefine mass eigenstates, 'effectively' reducing the CKM from 3x3 to 2x2

• Since non-zero quark masses are required, $CP$ symmetry can only be broken where the Higgs field has acquired a vacuum expectation value $\rightarrow T_{EW}$

• (But with $M(\text{Higgs}) > 70$ GeV, insufficient deviation from thermal equilibrium...)

• To make $d_{CP}$ dimensionless, we divide by dimensioned parameter $D = T_c$ at the EW scale ($T_c = T_{EW} \sim 100$ GeV), with $[D] = \text{GeV}^{12}$

\[ \hat{d}_{CP} = \frac{d_{CP}}{D^{12}} \approx 10^{-19} \ll \eta = O\left(10^{-10}\right) \]

KM $CP$ violation seems irrelevant for baryogenesis!
What about neutrinos?

- However, we now know that neutrinos also have flavour oscillations
- thus they must have a (very small) mass...
- ... and thus there is the equivalent of a CKM matrix for them:
  - the Pontecorvo-Maki-Nakagawa-Sakata matrix
  - which has a completely different hierarchy!
- and, because neutrinos have no electric charge, you can do things you cannot do with quarks...
- there are scenarios (leptogenesis) where CP violation in the neutrino sector would generate (eventually) baryogenesis...
Existence of antimatter is a consequence of the combination of special relativity and quantum mechanics

No ‘primordial’ antimatter observed, need CP violation

CP broken by the charged weak interaction

The weak and mass eigenstates of quarks are different, and this difference is described by the CKM matrix

There is a clear (and unexplained!) hierarchical structure to the CKM matrix...

With 3 (or more) families, one can have a complex phase(s) in the CKM matrix, and this allows for CP violation!

Measurements show that CKM describes the dominant (only?) source of CP violation (at the EW scale).

But it doesn’t explain the matter -- antimatter asymmetry of the universe.
Instructions by the VOC (Dutch East India Company) in Aug 1642:

“Since many rich mines and other treasures have been found in countries north of the equator between 15° and 40° latitude, there is no doubt that countries alike exist south of the equator. The provinces in Peru and Chili rich of gold and silver, all positioned south of the equator, are revealing proofs hereof.”

Abel Tasman discovered Tasmania (Nov. 1642), New Zealand (Dec. 1642), Fiji (Jan 1643), ...

From the point of view of the VOC, this was a disappointment.
I’d be happy to discover Fiji instead!