II

SYMMETRIES IN FIELD THEORY

NATURAL LANGUAGE FOR PARTICLE PHYSICS:

QUANTUM FIELD THEORY

- FUNDAMENTAL EXCITATION $\rightarrow$ QUANTUM FIELD

SYMMETRIES: INCORPORATED IN THE LAGRANGIAN

$$\mathcal{L} = \mathcal{L}(\phi, \partial_\mu \phi)$$

* CONTINUOUS SYMMETRIES

(TRANSFORMATIONS OF FIELDS UNDER A GROUP $\underline{G}$)

$\chi_\alpha(x)$: QUANTUM FIELD

α : INTERNAL INDEX (COMPONENTS)

MEMBER OF AN IRREDUCIBLE MULTIPLET

GROUP TRANSFORMATION $(a \in G)$ $\rightarrow$

- $\chi_\alpha(x) \rightarrow \chi'_\alpha(x) = R_{\alpha\beta}(a)\,\chi_\beta(x)$

$R_{\alpha\beta}(a)$: MATRIX REPRESENTATION OF $\underline{G}$

IF $\psi_\alpha(x) \xrightarrow[a]{} \psi'_\alpha(x) \xrightarrow[a']{} \psi''_\alpha(x) \underset{\text{eq.}}{\sim} \psi_\alpha(x) \xrightarrow[a'']{} \psi''_\alpha(x)$

EQUIVALENT

$$\rightarrow \quad R_{\alpha\beta}(a')\, R_{\beta\gamma}(a) = R_{\alpha\gamma}(a'')$$

* IN HILBERT SPACE:

* TRANSFORMATION $\rightarrow$ UNITARY OPERATOR $U(a)$

- $U^{-1}(a)\, \psi_\alpha(x)\, U(a) = \psi'_\alpha(x) = R_{\alpha\beta}(a)\, \psi_\beta(x)$

$$U(a)\, U(a') = U(a'')$$

CONTINUOUS SYMMETRIES

→ TO STUDY INFINITESIMAL

↓

GENERATORS

G_i

$* \quad U(\delta a) = 1 + i\, \delta a_i\, G_i$

δa_i : REAL

G_i : HERMITIAN

COMPOSITION OF $U(a)$ →

$$[G_i, G_j] = i\, C_{ijk} G_k$$

LIE ALGEBRA

- C_{ijk}: STRUCTURE CONSTANTS OF G

↓

- $R_{\alpha\beta}(\delta a) = \delta_{\alpha\beta} + i\, \delta a_i\, (g_i)_{\alpha\beta}$

— g_i : REPRESENTATIONS OF GENERATORS. OBEY THE ALGEBRA

USING $(U^{-1}\psi U = R\psi)$

$$* \quad [G_i, \psi_\alpha(x)] = -(g_i)_{\alpha\beta}\psi_\beta(x)$$

(HOW THE FIELDS TRANSFORM UNDER G)

INVARIANCE OF A THEORY UNDER G $\Rightarrow$

QUANTUM ACTION INVARIANT $\Rightarrow$

THE CURRENTS $\ast$ $J_i^\mu(x) = \dfrac{\partial \mathcal{L}}{\partial \partial_\mu \psi_\alpha(x)} \dfrac{1}{i} (g_i)_{\alpha\beta} \psi_\beta(x)$

ARE CONSERVED:

NOETHER THEOREM!

$$\partial_\mu J_i^\mu = 0$$

IDENTIFICATION :

$$G_i = \int d^3x \, J_i^0(x)$$

$\longrightarrow$

CONSERVATION

$$[H, G_i] = 0$$

HAMILTONIAN

STATES

$|p;\alpha\rangle$: ONE PARTICLE STATE WITH $p^2=-m^2$

PARTICLE STATES OF $\chi_\alpha(x)$ TRANSFORM AS $\chi_\alpha(x)$

— IF AND ONLY IF THE VACUUM IS G INVARIANT —

- $\mathcal{U}(a)|0\rangle = |0\rangle$

$$\Downarrow$$

- $G_j|0\rangle = 0$

LSZ:

$$|p;\alpha\rangle = \lim_{x^0\to\pm\infty}\int d^3x\, e^{ipx}\,\frac{1}{i}\overleftrightarrow{\partial_0}\,\chi_\alpha(x)\,|0\rangle$$

$$\Downarrow$$

$$U^{-1}(a)\,|p;\alpha\rangle = \lim_{x^0\to\pm\infty}\int d^3x\, e^{ipx}\,\frac{1}{i}\overleftrightarrow{\partial_0}\;U^{-1}(a)\,\chi_\alpha(x)\,|0\rangle$$

$$U^{-1}(a)\,\chi_\alpha(x)\,\underbrace{U(a)\,|0\rangle}_{|0\rangle}$$

$$R_{\alpha\beta}(a)\,\chi_\beta(x)\,|0\rangle$$

using $(U^{-1}\chi\, U = R\chi)$

$$\downarrow$$

$$* \; U^{-1}(a)\,|p;\alpha\rangle = R_{\alpha\beta}(a)\,|p;\beta\rangle \; *$$

* WIGNER-WEYL REALIZATION OF SYMMETRY *

THE VACUUM IS G INVARIANT

$$* \; U^{-1}(a)\,|\phi;\alpha\rangle = R_{\alpha\beta}(a)\,|\phi;\beta\rangle \; *$$

$\Downarrow$

* ALL STATES OF THE MULTIPLETE $|\phi;\alpha\rangle$ *

* HAVE THE SAME MASS

$G_j|0\rangle = 0 \Rightarrow$ PARTICLES BELONGING TO A MULTIPLET IN AN IRREDUCIBLE REPRESENTATION HAVE THE SAME MASS

IF $p^\mu \equiv (m_\alpha, \vec{0}) \rightarrow$ THE REST STATE

$$H|p;\alpha\rangle_{REST} = m_\alpha |p;\alpha\rangle_{REST}$$

$$[H, G_i] = 0 \rightarrow [H, U^{-1}(a)] = 0 \rightarrow$$

$$0 = [H, U^{-1}(a)]|p;\alpha\rangle_{REST}$$

$$0 = R_{\alpha\beta}(a)\{m_\beta - m_\alpha\}|p;\alpha\rangle_{REST}$$

$$\Downarrow$$

$$m_\beta = m_\alpha$$

$$[G_j, \chi_\alpha(x)] = -(g_j)_{\alpha\beta}\, \chi_\beta(x) \longrightarrow$$

$$\langle 0 | [G_j, \chi_\alpha(x)] | 0 \rangle = -(g_j)_{\alpha\beta} \langle 0 | \chi_\beta(x) | 0 \rangle$$

$$\Downarrow$$

W - W $\Rightarrow$ $\underline{\langle 0 | \chi_\beta(x) | 0 \rangle = 0}$

* NAMBU-GOLDSTONE REALIZATION OF SYMMETRY *

* VACUUM IS NOT G INVARIANT

$$* \quad U(a)\,|0\rangle \neq |0\rangle$$

$$\Downarrow$$

$$U^{-1}(a)\,|p;\alpha\rangle \neq R_{\alpha\beta}(a)\,|p;\beta\rangle$$

$$\text{N-G} \Rightarrow \langle 0|X_\rho(x)|0\rangle \neq 0$$

GOLDSTONE THEOREM:

FOR EACH GENERATOR G_j THAT DOES NOT ANNIHILATE VACUUM $\longrightarrow$ ONE BOSON MASSLESS "GOLDSTONE BOSON"

$$\left\{ G_j^{NA} |0\rangle = |\phi ; j\rangle_G \quad (m^2=0) \right\}$$

GAUGE THEORY

- **HERMANN WEYL** (1919) : IDEA OF GAUGE INVARIANCE

(ONLY ELECTRON AND PROTON)

PHYSICAL INTERPRETATION **WRONG** !

↓

GAUGE INVARIANCE SURVIVED AS MAXWELL EQUATIONS' SYMMETRY

~ 1950 : **YANG AND MILLS**: EXTENSION OF GAUGE SYMMETRY (BEYOND e.m.)

✗ RELATIVITY (S. & G.) : → NO ABSOLUTE FRAME OF REFERENCE IN THE UNIVERSE

- S: LORENTZ SYMMETRY GROUP OF SPECIAL RELATIVITY IS A GLOBAL SYMMETRY

- G: A REFERENCE FRAME CAN ONLY BE DEFINED LOCALLY (AT A SINGLE POINT)

↓

IN G. TO RELATE FRAMES ONE NEEDS A CONNECTION

- GRAVITATIONAL FIELD → CONNECTION BETWEEN LOCAL FRAMES IN SPACE-TIME

- WEYL: ELECTROMAGNETISM CAN BE ASSOCIATED WITH A CONNECTION ?

TO RELATE THE LENGTHS OF VECTORS AT DIFFERENT POINTS

↓

SCALE (OR GAUGE) INVARIANCE

* LOCAL *

- WEYL: GAUGE CONNECTION ≡ ELECTROMAGNETIC A_μ

- $\partial_\mu S \to \partial_\mu S + \partial_\mu \Lambda$
- $A_\mu \to A_\mu + \partial_\mu \Lambda$

↓

MAGNITUD OF A VECTOR: CHANGES ALONG A TIME-LIKE WORLD-LINE

↓

<u>NO</u> WAY OF DEFINING A STANDARD CLOCK

(RATE OF CLOCK <u>DEPENDS ON THE HISTORY</u> OF CLOCK)

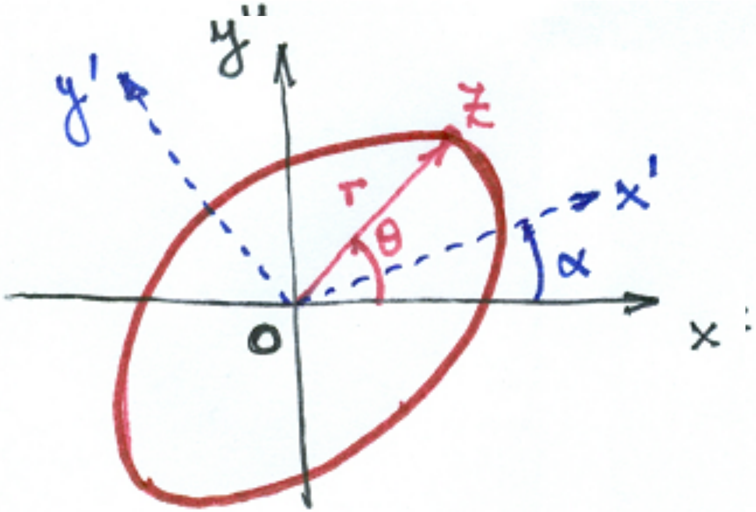
↓

ATOMS IN DIFFERENT POINTS → DIFFERENT SPECTRA !

<u>NOT OBSERVED</u>

BASIC IDEAS (CARICATURE...)

- HARMONIC OSCILLATOR (IN THE PLANE)



$$\begin{pmatrix} y = A \sin \omega t \\ x = B \cos \omega t \end{pmatrix}$$

$\Downarrow$

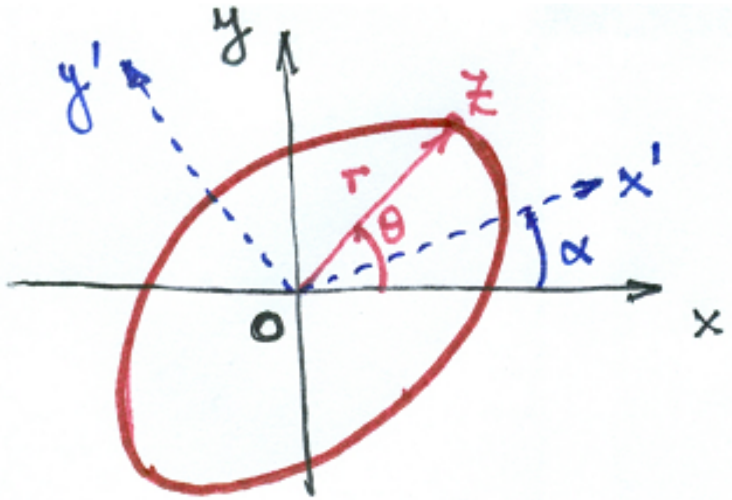
— COMPLEX VARIABLE

- $z = x + i y$

$\cdot\ (z = r e^{i\theta}) \cdot\ r = \sqrt{x^2 + y^2}\ ;\ \cdot\ \theta = \operatorname{arctg} \frac{y}{x}$

* OSCILLATOR EQUATION

- $$\frac{d^2 z}{d t^2} + \omega^2 z = 0 \qquad \left(T = \frac{2\pi}{\omega} \right)$$



BUT: TO MEASURE θ FROM X-AXIS ARBITRARY!

→ ROTATE α TO (x', y') → TO MEASURE FROM x'

↓

* $\theta \rightarrow \theta - \alpha$

[θ WAS "REGAUGED"]

- MULTIPLY THE EQUATION BY $e^{-i\alpha}$ (PHASE)
- REDEFINE $z' = z e^{-i\alpha}$

- $\dfrac{d^2 z}{dt^2} + \omega^2 z = 0$ $\longrightarrow$ - $\dfrac{d^2 z'}{dt^2} + \omega^2 z' = 0$

NOTHING CHANGED!

ABSOLUTE VALUE OF θ IRRELEVANT! $\equiv$ <u>GLOBAL</u> <u>GAUGE</u> INVARIANCE (θ INDEPENDENT OF t)

IF $\alpha = \alpha(t)$ LOCAL GAUGE TRANSFORMATION

$e^{-i\alpha(t)}$ CANNOT BE ABSORBED IN z'

- $$\frac{d^2 z'}{dt^2} = \frac{d^2}{dt^2}\left[z(t)\, e^{-i\alpha(t)}\right]$$

$$\frac{d\alpha(t)}{dt} \neq 0$$

HOW TO REGAIN INVARIANCE ?

(TO COMPENSATE $\frac{d\alpha}{dt}(t)$)

- REPLACE : • $\frac{d}{dt} \longrightarrow \frac{d}{dt} - i\,A(t)$

- WITH THE PROPERTY : IF : • $\theta \rightarrow \theta - \alpha(t)$

  THEN : • $A(t) \rightarrow A(t) - \frac{d\alpha}{dt}(t)$

- $A(t)$: COMPENSATOR $\equiv$ GAUGE FIELD

* AND ?

$\alpha = \alpha(t)$ ⇒ TO ROTATE THE REFERENCE FRAME

WITH $\omega = \dfrac{d\alpha(t)}{dt}$

ROTATING FRAME ⇒ "FICTITIOUS" FORCES

(CENTRIFUGE - CORIOLIS)

A(t) GENERATES THESE FORCES !

MAXWELL GAUGE INVARIANCE

- $\begin{cases} \vec{\nabla} \wedge \vec{A} = \vec{B} \\ \vec{\nabla} \cdot \vec{A} = ? \end{cases}$

- $\vec{E} = -\vec{\nabla}\phi - k'' \dfrac{\partial \vec{A}}{\partial t}$

* PANOFSKY - PHILLIPS (p.2)

POEMA "VECTORIAL"

Esto el Papa exclamó al firmar la bula
con que furioso excomulgó a Lutero:
La ***divergencia*** de un ***rotor*** es nula
y el ***rotor*** de un ***gradiente*** es siempre cero.
El gran fraile alemán invocó a dios
y exclamó con su habitual vehemencia:
El ***rotor*** de un ***rotor*** mas ***nabla dos***
da el ***gradiente*** de toda ***divergencia***.

Enrique Loedel Palumbo (1901-1962), ¿físico o poeta?

$$\Downarrow$$

$$* \begin{cases} \vec{A} \to \vec{A}' = \vec{A} + \vec{\nabla}\Lambda \\ \phi \to \phi' = \phi - k'' \dfrac{\partial \Lambda}{\partial t} \end{cases}$$

Λ: SCALAR FUNCTION

$\Rightarrow$ IDENTICAL $\vec{E}$ AND $\vec{B}$

GAUGE SYMMETRY OF e.m.

* ARBITRARINESS OF A_μ (IN CLASSICAL PHYSICS)

(e.m.) GAUGE INVARIANCE

$\longleftrightarrow$ CHARGE CONSERVATION (e.m.)

ENERGY $\int A_\mu J^\mu$

IS INVARIANT UNDER $A_\mu \to A_\mu + \partial_\mu \Lambda$

INTEGRATION BY PARTS $\to$ • $\partial_\mu J^\mu = 0$

$(\dot{Q} = 0)$

* AHARONOV - BOHM EFFECT

(PHASE SHIFT DUE TO A SOLENOID)

↓

POTENTIAL $\vec{A}$ IS A PHYSICAL FIELD DIRECTLY OBSERVED

$$\frac{e}{\hbar}(S_1 - S_2) = \frac{e}{\hbar}\left[\int_1 \vec{A}\cdot d\vec{x} - \int_2 \vec{A}\cdot d\vec{x}\right]$$

NO PROBLEM WITH ARBITRARINESS OF $\vec{A}$

[OBSERVABLE: RELATIVE CHANGE OF PHASE DUE TO $\vec{A}$]

ELECTROMAGNETIC INTERACTIONS

(MEDIATED BY PHOTONS)

FROM THE REQUIREMENT OF

LOCAL ABELIAN GAUGE INVARIANCE

⊕ YANG-MILLS (FOR NON ABELIAN)

↓

FUNDAMENTAL INTERACTIONS DESCRIBED BY

LOCAL GAUGE THEORIES

MEDIATED BY **GAUGE FIELDS**

$m_{\text{GAUGE FIELD}} \equiv 0$

$\mathcal{L}_m = \frac{1}{2} m A_\mu A^\mu$ NOT ALLOWED!

* "STANDARD" WAY OF CONSTRUCTING A GAUGE THEORY

(FOR MATTER FIELDS)

- CHOOSE AN APPROPRIATE UNITARY LIE GROUP G $\{g\}$
- PROPOSE AN ACTION WITH GLOBAL INVARIANCE UNDER G AS THE MATTER SYMMETRY GROUP

$\downarrow$

ACTION INVARIANT UNDER CONSTANT PHASE TRANSFORMATION

$\downarrow$

CONSERVATION OF A NOETHER CHARGE

- TO PROMOTE GLOBAL INVARIANCE TO LOCAL ONE

$$g \to g(x) \in \bar{G}$$

(PHASES SPACE-TIME DEPENDENT)

↓

TO INTRODUCE GAUGE FIELDS THAT COMPENSATE (VIA GAUGE TRANSFORM) CHANGES PRODUCED BY LOCAL PHASE CHANGES

↓

KINETIC PART OF THE MATTER LAGRANGIAN WRITTEN IN TERMS OF COVARIANT DERIVATIVES

- A KINETIC TERM FOR GAUGE FIELDS HAS TO BE INCLUDED

$g(x)$ WELL BEHAVED → CONSERVED NOETHER CHARGE UNCHANGED (FROM THE GLOBAL CASE)

* BUILDING A GAUGE THEORY *

"THE STANDARD WAY"

EX:

* ONE PARAMETER GROUP SYMMETRY

COMPLEX SCALAR FIELD:

- $\mathcal{L}(\phi, \partial_\mu \phi) = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$

$$\phi(x) \rightarrow \phi'(x) = e^{i\alpha} \phi(x)$$

- MULTIPLICATION BY A PHASE $(\phi \rightarrow e^{i\alpha} \phi)$
- IMPLEMENTS ROTATIONS OF A CIRCLE $(U(1))$

$\mathcal{L}$ INVARIANT UNDER PHASE (CONSTANT) $\Rightarrow$ $U(1)$

* α CONSTANT *

INFINITESIMAL $\Rightarrow$ $\begin{cases} \delta\phi = i\alpha\phi \\ \delta\phi^* = -i\alpha\phi^* \end{cases}$

NOETHER $\rightarrow$ $J_\mu = i\left[\partial_\mu\phi^*\phi - \phi^*\partial_\mu\phi\right]$

CONSERVED

$\Downarrow$

$Q = \int d^3x\, J_0$ = CONSTANT $\rightarrow$ $\dot{Q} = 0$

Q : "CHARGE" OF THE FIELD

INTRODUCING: $\begin{cases} \phi_1 = \frac{1}{\sqrt{2}}(\phi + \phi^*) \\ \phi_2 = \frac{1}{\sqrt{2}}(\phi - \phi^*) \end{cases}$

$\Downarrow$

- $\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2$
- $\mathcal{L}_k = \frac{1}{2}\partial_\mu \phi_k \partial^\mu \phi_k - m^2 \phi_k^2 \quad ; (k=1,2)$

$\Downarrow$

PHASE TRANSFORMATION $\Rightarrow$

$$\begin{cases} \phi_1 \to \phi_1' = \cos\alpha\, \phi_1 - \sin\alpha\, \phi_2 \\ \phi_2 \to \phi_2' = \cos\alpha\, \phi_2 + \sin\alpha\, \phi_1 \end{cases}$$

ROTATION IN (ϕ_1, ϕ_2)-PLANE

- <u>LOCAL</u> PHASE INVARIANCE

$$\alpha \longrightarrow \alpha(\underline{x})$$

YANG AND MILLS MOTIVATION:

"THE CONCEPT OF <u>FIELD</u> AND THE CONCEPT OF <u>LOCAL</u> INTERACTIONS IMPLY A SPREADING OF INFORMATION TO NEIGHBOURING POINTS AND ELIMINATE THE ACTION AT A DISTANCE. THEN <u>GLOBAL</u> PHASE INVARIANCE SEEMS TO CONTRADICT THE GENERALIZED IDEA OF <u>LOCALITY</u>"...

* $\phi(x) \to \phi'(x) = e^{i\alpha(x)} \phi(x)$ IN

- $\mathcal{L}(\phi, \partial_\mu \phi) = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$

SYMMETRY IS PRESERVED? **NO**

MASS TERM : O.K. ✓

KINETIC TERM :

- $\partial_\mu \phi(x) \to e^{i\alpha(x)} \left[\partial_\mu + i\, \partial_\mu \alpha(x) \right] \phi(x)$ **NO**

KINETIC TERM: **NO**

- $\partial_\mu\phi\,\partial^\mu\phi^* \rightarrow [\partial_\mu - i\partial_\mu\alpha(x)]\phi^*\,[\partial^\mu + i\partial^\mu\alpha(x)]\phi$

$\Downarrow$

$\mathcal{L}$ IS NOT INVARIANT UNDER LOCAL PHASE TRANSFORMATIONS

* HOW TO TURN $\mathcal{L}$ INVARIANT?

• FREEDOM OF A_μ (e.m. FIELD) •

REPLACE • $\partial_\mu \phi$

BY • $D_\mu \phi \equiv (\partial_\mu - iqA_\mu)\phi$

(GAUGE COVARIANT DERIVATIVE)

WHEN $\phi(x)$ CHANGES BY $\alpha(x)$ $\longrightarrow$

• $A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) - \frac{1}{q}\partial_\mu \alpha(x)$

THEN: • $D_\mu \phi(x) \rightarrow D'_\mu \phi'(x) = e^{i\alpha(x)} D_\mu \phi(x)$ •

TRANSFORMS AS THE FIELD

$$\mathcal{L} = \left[\partial_\mu + iqA_\mu(x)\right]\phi^*(x)\left[\partial^\mu - iqA^\mu(x)\right]\phi(x) - m^2\phi^*(x)\phi(x)$$

IS LOCAL PHASE TRANSFORMATION INVARIANT !

$\mathcal{L}$ NOW CONTAINS INTERACTIONS

(THE MINIMAL INTERACTION)

- $A_\mu(x)$: GAUGE FIELD

- COMPENSATING FIELD (RESTORE THE SYMMETRY)
- COMPARATIVE FIELD (DISTINGUISHES CHARGES)

$m_A \equiv 0$

FERMIONS

$$* \quad \mathcal{L}_{f-A} = \underbrace{i\bar{\psi}\gamma^\mu(\partial_\mu - ieA_\mu)\psi}_{i\bar{\psi}\gamma^\mu\partial_\mu\psi + e\bar{\psi}\gamma^\mu\psi A_\mu} - m\bar{\psi}\psi$$

PROPOSAL

(POSSIBLE AND NATURAL)

* TO INVERT THE PROCESS *

TO AVOID THE UKASE THAT PROMOTES GLOBAL TO LOCAL SYMMETRIES OF THE MATTER $\mathcal{L}$

"UKASE" (УКАЗ)

(HAVE THE POWER OF LAWS BUT MAY NOT ALTER THE REGULATIONS OF EXISTING LAWS)

Building Gauge Theories: The Natural Way
Fundamental Journal of Modern Physics 2 (2011) 15.
C. A. García Canal, F.A. Schaposnik

* SYMMETRIES : **OF FIELDS**

- IMPOSING SYMMETRIES **TO FIELDS**

{ WHY STARTING FROM PHASE INVARIANCE OF MATTER FIELDS ?

TO BUILD THE THEORY OF FUNDAMENTAL INTERACTIONS STARTING FROM THE INTERACTION MEDIATORS: THE GAUGE FIELDS

SOURCES ARE ADDED IMPOSING LOCAL GAUGE INVARIANCE AND LORENTZ INVARIANCE

(OF MAXWELL EQUATION OR NON-ABELIAN ONES)

- STARTING POINT:

ELECTRODYNAMICS

* MAXWELL EQUATIONS

NO SOURCES
\+
("PHYSICAL" PAIR)

- $$\partial_\mu F^{\mu\nu} = 0$$

(THE OTHER PAIR GIVEN BY BIANCHI IDENTITY)

(VIA $U(1)$ GAUGE FIELD $A_\mu(x)$)

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

— GAUGE TRANSFORMATION:

- $A_\mu(x) \to \hat{A}_\mu(x) = A_\mu(x) + \partial_\mu \Lambda(x)$

$F_{\mu\nu}(x)$ INVARIANT

- $\partial_\mu F^{\mu\nu} = 0$ (GAUGE INVARIANT)

- MAXWELL FROM THE LAGRANGIAN:

- $$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x)$$

(LORENTZ <u>AND</u> GAUGE INVARIANT)

PRESENCE OF EXTERNAL SOURCE j^{μ}_{ext}

$\Downarrow$

MAXWELL $\Rightarrow$ $\partial_{\mu} F^{\mu\nu} = e\, j^{\nu}_{ext}$

[$j^{\nu}_{ext}(x)$: NON-DYNAMICAL] ($e \in \mathbb{R}$: COUPLING OF SOURCE)

NATURAL AND SIMPLEST LORENTZ INVARIANT TERM IN THE LAGRANGIAN FORMULATION:

- $L_{INT} = e\, A_{\mu}(x)\, j^{\mu}_{ext}(x)$

IF $L = L_M + L_{INT}$ GAUGE INVARIANT

$\Downarrow$

$$J^\mu_{ext}(x) \xrightarrow[\Lambda]{} J^{\mu\,\Lambda}_{ext}(x) = J^\mu_{ext}(x)$$

AND

- $\partial_\mu J^\mu_{ext}(x) = 0$ (FORCED BY MAXWELL)

(L CHANGES AT MOST AS A TOTAL DERIVATIVE (UNDER GAUGE))

STRUCTURE OF MAXWELL PRESCRIBED BY LORENTZ SYMMETRY

↓

ONE CAN ANTICIPATE COUPLING OF MATTER TO GAUGE FIELDS

* CONSIDER A DYNAMICAL DIRAC FERMION $\underline{\psi(x)}$

AS THE ORIGIN OF THE CURRENT $\underset{\sim\sim\sim}{j^\mu(x)}$ $\left(+\, j^\mu_{ext}(x)\right)$

↓

MOST ECONOMIC FORM: BILINEAR OF FERMIONS

CONSTRUCTION GUIDED BY LORENTZ AND GAUGE

i) LORENTZ:

- $j^\mu(x) = \bar{\psi}(x)\,\gamma^\mu\,\psi(x)$

ii) GAUGE:

(INVARIANCE OF $j^\mu(x)$)

$$\psi(x) \to \psi^\Lambda(x) = e^{iq\Lambda(x)}\,\psi(x)$$

$(q \in \mathbb{R})$

FERMION DYNAMICAL $\Rightarrow$ $\underline{L_D = i\,\bar{\psi}(x)\,\gamma^\mu\,\partial_\mu\,\psi(x)}$

• $\underline{L} = L_M + L_D + L_{INT}$: GAUGE INVARIANT

IF $q \equiv e$

⊕ COVARIANT DERIVATIVE : $D_\mu = \partial_\mu - e A_\mu(x)$

USUAL

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \gamma^\mu D_\mu \psi$$

(MINIMAL e.m. COUPLING)

NOTICE:

1) IF $J^5_\mu = \bar{\psi}\gamma_\mu\gamma^5\psi$ (INSTEAD OF $J_\mu = \bar{\psi}\gamma_\mu\psi$)

PSEUDOVECTOR COUPLED TO A^μ $\Rightarrow$ PSEUDOSCALAR

ONE NEEDS $\mathcal{L}$ SCALAR

2) NON MINIMAL COUPLING : CORRECTS MAXWELL AT CLASSICAL LEVEL !

NON ABELIAN CASE

GAUGE FIELDS: $A_\mu(x) = A^a_\mu(x)\, t^a$

$$\downarrow$$

- $L = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{\psi}_i \left(i \not{\partial}\, \delta_{ij} - e \not{A}^a T^a_{ij} \right) \psi_j$

(GAUGE FIELD THEORY OF SU(N))

* SUMMARIZING

- USUAL WAY: TO PROMOTE THE GLOBAL UNITARY SYMMETRY OF THE MATTER LAGRANGIAN TO A LOCAL ONE $\rightarrow$ REQUIRING A GAUGE FIELD

- "NATURAL" WAY (UPSIDE DOWN): START FROM THE PURE GAUGE FIELD AND IMPOSING LORENTZ AND GAUGE INVARIANCE WHEN COUPLING MATTER $\Rightarrow$ SAME $\underline{L}$ AND CONSERVED $\underline{J_\mu}$

– PARALLEL TO THE GEOMETRIC APPROACH –

(STARTING POINT: DEFINE A CONNECTION IN A
PRINCIPAL FIBER BUNDLE AND
INTRODUCE MATTER FIELDS AS SECTIONS
IN THE ASSOCIATED VECTOR BUNDLE

UPSIDE DOWN

Upside Down Cake

You have some latitude with the fruits that you use. Just make sure that whatever you use overs the bottom in a substantial layer, around double-thickness, since the fruit will cook down while baking and settle nicely into place.
Berries and such as good nestled in the gaps between the slices of fruits.

For the fruit layer:
3 tablespoons butter (45g), salted or unsalted
3/4 cup packed (135g) light brown sugar
fruit: 8 quartered plums or apricots, 3-4 thickly-sliced pears or nectarines,
or 2 cups cranberries; add a handful of huckleberries, cherries, raspberries, or another bushberry

For the cake layer:
8 tablespoons (115g) unsalted butter
3/4 cup (150g) sugar
1 teaspoon vanilla extract
2 large eggs, at room temperature.
1 1/2 cups (210g) flour
1 1/2 teaspoon baking powder, preferably aluminum-free
1/4 teaspoon salt
1/2 cup (125ml) whole milk, at room temperature

1. Melt the 3 tablespoons (45g) of butter in a cast iron skillet, or cake pan.
Add the brown sugar and cook while stirring, until the sugar is melted and begins to bubble. Remove from heat and let cool.
2. Once cool, arrange the fruit in a pinwheel design, added berries if desired. Set aside.
3. To make the cake, preheat the oven to 350F. (190C)
4. Beat the 8 tablespoons (115g) of butter and sugar until fluffly. Add the vanilla, then the eggs, one at a time, until smooth.
5. Whisk or sift together the flour, baking powder, and salt.
6. Stir in half of the flour mixture, then the milk, then the remaining dry ingredients.
Do not overmix: stir just until the flour is barely incorporated into the batter.
7. Spread the batter over the fruit, then bake for 45 minutes to one hour (depending on the size of the pan, and the thickness of the batter).The cake is ready when it begins to pull away from the sides of the pan and the center feels just set.
8. Remove from oven, let cool about 20 minutes, then place a cake plate on top, and wearing oven mitts,
flip the cake out on to the plate.
Serving: Upside Down Cake is best served warm, perhaps with whipped cream or vanilla ice cream.

SEE YOU AT III

CONNECTION BETWEEN

E.M. FIELD STRENGTH → GAUGE INVARIANCE

AND COVARIANT DERIVATIVE

PARAMOUNT PROPERTY OF ELECTRODYNAMICS

ALREADY IN NON-RELATIVISTIC QUANTUM DYNAMICS

(GALILEO RELATIVISTIC)

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CHARGED PARTICLE IN E.M. FIELD:

GAUGE INVARIANCE IMPOSES A PHASE IN THE

PARTICLE WAVE FUNCTION

PHASE FACTOR IS DIRECTLY ASSOCIATED TO THE COVARIANT DERIVATIVE →

GAUGE COVARIANCE IS TRANSMITTED FROM THE FIELD STRENGTH TO THE WAVE FUNCTION

NON ABELIAN CASE

GAUGE FIELDS: $A_\mu(x) = A^a_\mu(x)\, t^a$

VALUES IN THE LIE ALGEBRA OF SU(N) WITH GENERATORS t^a

GAUGE TRANSFORMATION

- $A_\mu(x) \to A^\Lambda_\mu(x) = g^{-1}(x)\, A_\mu(x)\, g(x) - \frac{i}{e}\, g^{-1}(x)\, \partial_\mu g(x)$

$$\begin{cases} g(x) = e^{i\Lambda(x)} \\ \Lambda(x) = \Lambda^a(x)\, t^a \end{cases}$$

INFINITESIMALLY:

$$A_\mu(x) \to A^\Lambda_\mu(x) = A_\mu(x) + \frac{1}{e} D_\mu[A]\,\Lambda(x)$$

COVARIANT DERIVATIVE: $D_\mu[A] = \partial_\mu + ie\left[A_\mu(x), \;\right]$

TRANSFORMING AS:

- $D_\mu[A^\Lambda] = g^{-1}(x)\, D_\mu[A]\, g(x)$

- $F_{\mu\nu} = F^a_{\mu\nu} t^a = \partial_\mu A_\nu - \partial_\nu A_\mu + e\left[A_\mu, A_\nu\right]$

MAXWELL $\xrightarrow[\text{(NON-A)}]{}$ YANG - MILLS

$$\bullet\ \underline{D_\mu[A]\, F^{\mu\nu} = 0}$$

UNDER GAUGE $F_{\mu\nu}(x) \to F^{\Lambda}_{\mu\nu}(x) = g^{-1}(x)\, F_{\mu\nu}(x)\, g(x)$

(COVARIANTLY)

$*\ D_\mu[A]\, F^{\mu\nu} = 0$ ALSO

FROM $\bullet\ L_{YM} = -\frac{1}{2} T_r \left\{ F_{\mu\nu} F^{\mu\nu} \right\}$

INTRODUCE (AS BEFORE!): J^{μ}_{ext} $\Rightarrow$

- $D_{\mu} F^{\mu\nu} = e J^{\nu}_{ext}$

CONSISTENCY $\Rightarrow$ J^{ν}_{ext} TAKES VALUES IN THE LIE ALGEBRA OF SU(N)

- $$J^{\nu}_{ext}(x) = J^{\nu a}_{ext}(x)\, t^{a}$$

- $$L_{INT} = e\, \mathrm{Tr}\left\{ A_{\mu} J^{\mu}_{ext} \right\}$$

$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_{INT}$ GAUGE INVARIANT <u>IF</u>

- $J^{\mu}_{ext}(x) \to J^{\mu\,\Lambda}_{ext}(x) = g^{-1}(x)\, J^{\mu}_{ext}(x)\, g(x)$

AND SATISFIES : • $D_{\mu}[A]\, J^{\mu}_{ext}(x) = 0$

MAKING THE SOURCE DYNAMICAL: $\Rightarrow$ DIRAC FERMIONS

MAKING THE SOURCE DYNAMICAL: $\Rightarrow$ DIRAC FERMIONS

$$\Downarrow$$

DEFINE :

$$J^a_\mu(x) = \bar{\psi}^i \gamma_\mu T^a_{ij} \psi^j \qquad i,j = 1,2,\dots N$$

(FERMIONS IN THE FUNDAMENTAL OF SU(N) WITH GENERATORS T^a)

$$\psi(x) \to \psi^\Lambda(x) = e^{iq\Lambda(x)}\,\psi(x)$$

$$q \equiv e$$

$\oplus$ KINETIC TERM FOR FERMIONS

NOTICE: THE MATTER CURRENT IS **COVARIANTLY** $[D_\mu[A]\, j^\mu = 0]$ CONSERVED $\Rightarrow$ DOES NOT LEAD TO A CONSERVED CHARGE

ONLY WHEN THE GAUGE NOETHER CURRENT

$$\left(j^{\text{GAUGE}}\right)^a_\mu = f^{abc} F^b_{\mu\nu} A^{c\nu}$$

(ASSOCIATED TO THE YANG-MILLS LAGRANGIAN)

IS INCLUDED

$\rightarrow$ • $\underline{J_\mu = \left(j^{\text{GAUGE}}\right)_\mu + j_\mu}$ IS CONSERVED

$$\underline{\partial^\mu J_\mu = 0}$$