Flavour Physics & $\not{CP}$

1. Quark Mixing
2. P^0-$\bar{P}^0$ Mixing and CP Violation
3. Searching for New Physics

arXiv:1112.4094

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QUARK MIXING MATRIX

- **Unitary** $N_G \times N_G$ **Matrix:** N_G^2 **parameters**

$$\mathbf{V} \cdot \mathbf{V}^\dagger = \mathbf{V}^\dagger \cdot \mathbf{V} = \mathbf{1}$$

$\frac{1}{2} N_G (N_G - 1)$ moduli, $\frac{1}{2} N_G (N_G + 1)$ phases

- $2 N_G - 1$ **arbitrary phases:** $\bar{u}_i \mathbf{V}_{ij} d_j$

$$u_i \to e^{i\phi_i} u_i \quad ; \quad d_j \to e^{i\theta_j} d_j \qquad \Rightarrow \qquad \mathbf{V}_{ij} \to e^{i(\theta_j - \phi_i)} \mathbf{V}_{ij}$$

$\Downarrow$

$\mathbf{V}_{ij}$ **Physical Parameters:**

$\frac{1}{2} N_G (N_G - 1)$ **moduli** ; $\frac{1}{2} (N_G - 1) (N_G - 2)$ **phases**

- **$N_f = 2$: 1 angle, 0 phases (Cabibbo)**

$$\mathbf{V} = \begin{bmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{bmatrix} \Rightarrow \text{No } \not{CP}$$

- **$N_f = 3$: 3 angles, 1 phase (CKM)** $\quad c_{ij} \equiv \cos\theta_{ij} \;;\; s_{ij} \equiv \sin\theta_{ij}$

$$\mathbf{V} = \begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & c_{23}\, c_{13} \end{bmatrix}$$

$$\approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$\lambda \approx \sin\theta_C \approx 0.223 \;;\; A \approx 0.82 \;;\; \sqrt{\rho^2+\eta^2} \approx 0.4$ $\qquad \delta_{13} \neq 0 \;\; (\eta \neq 0) \Rightarrow \not{CP}$

PDG parametrization of the CKM matrix

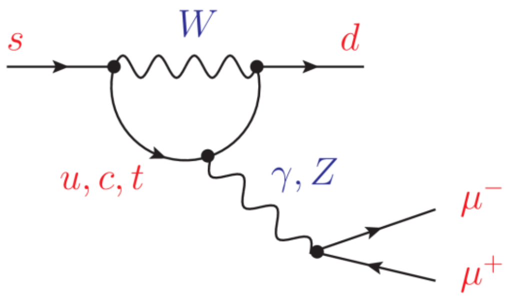
$$\mathbf{V} = \begin{bmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta_{13}} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta_{13}} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta_{13}} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta_{13}} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta_{13}} & c_{23}\,c_{13} \end{bmatrix}$$

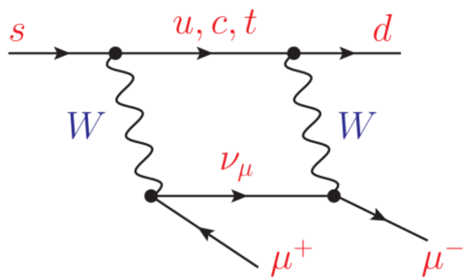
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Wolfenstein: $s_{12} \equiv \lambda \quad , \quad s_{23} \equiv A\lambda^2 \quad , \quad s_{13}\,e^{-i\delta_{13}} \equiv A\lambda^3(\rho - i\eta)$

$$\Rightarrow \quad \mathbf{V} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3\,(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\,\lambda^2 \\ A\lambda^3\,(1-\rho-i\eta) & -A\,\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

GIM Mechanism





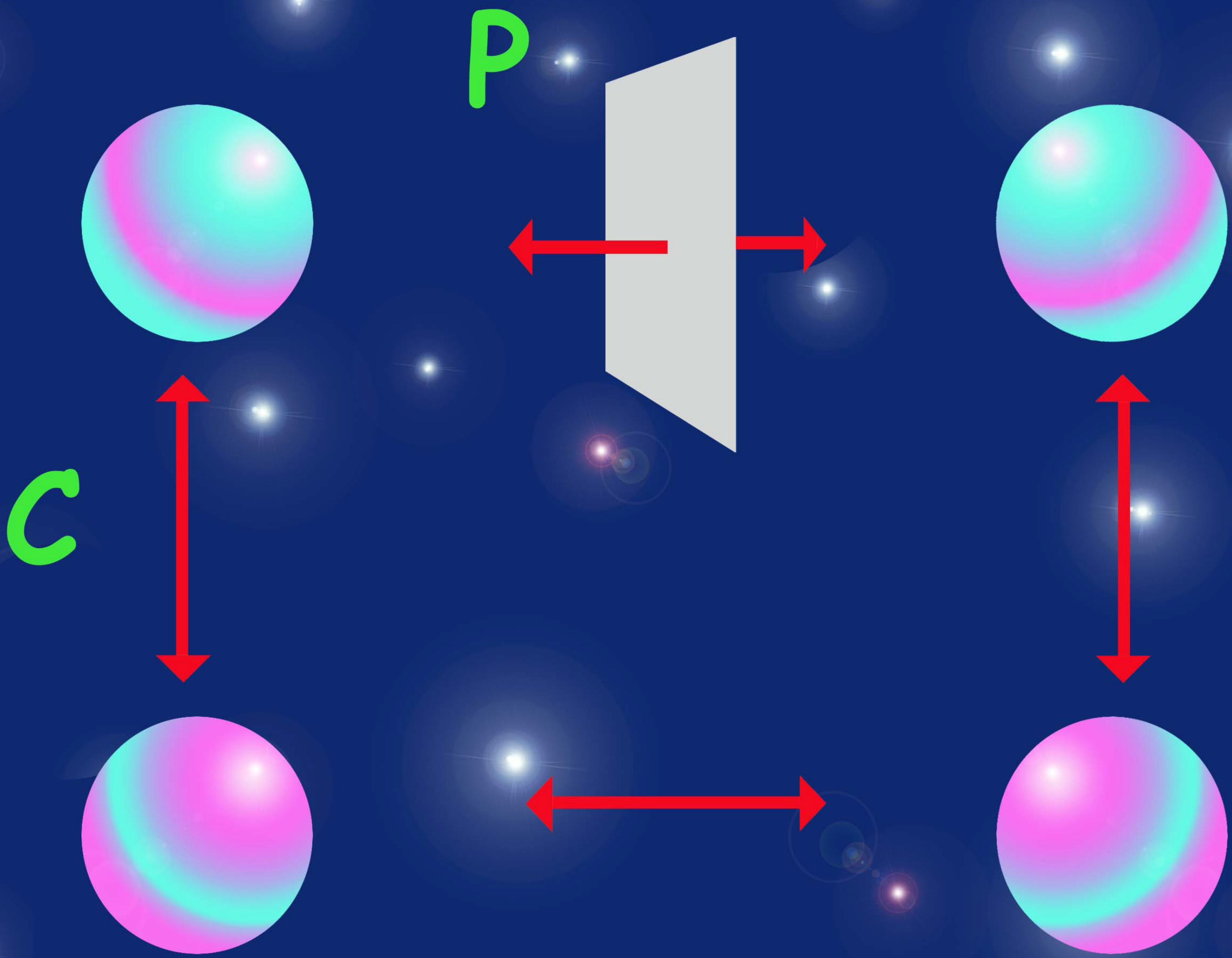
$$\mathcal{M} \propto \sum_{i=u,c,t} V_{is} V_{id}^* F(m_i^2/M_W^2)$$

$$\sum_{i=u,c,t} V_{is} V_{id}^* = 0 \quad \Rightarrow \quad \mathcal{M} = 0 \quad \text{if} \quad m_u = m_c = m_t$$

$\tilde{F}(x) \equiv F(x) - F(0)$ ⇒

$$\mathcal{M} \propto V_{cs} V_{cd}^* \tilde{F}(m_c^2/M_W^2) + V_{ts} V_{td}^* \tilde{F}(m_t^2/M_W^2)$$
$$\approx -\lambda \tilde{F}(m_c^2/M_W^2) - \lambda^5 A^2 (1-\rho+i\eta) \tilde{F}(m_t^2/M_W^2)$$

- **Charm contribution dominates. Strong suppression:** $\mathcal{M} \propto \lambda \frac{g^4}{16\pi^2} \frac{m_c^2}{M_W^2}$
- **CP effects governed by top contribution** $\left[\mathrm{Im}(V_{cs} V_{cd}^*) = -\mathrm{Im}(V_{ts} V_{td}^*)\right]$


P
C

- $\mathcal{C}\,,\,\mathcal{P}$: Violated maximally in weak interactions
- $\mathcal{CP}$: Symmetry of nearly all observed phenomena
- Slight (~ 0.2 %) $\not{\mathcal{CP}}$ in K^0 decays (1964)
- Sizeable $\not{\mathcal{CP}}$ in B^0 decays (2001)
- Huge Matter — Antimatter Asymmetry in our Universe → Baryogenesis

$\mathcal{CPT}$ Theorem: $\not{\mathcal{CP}} \longleftrightarrow \not{\mathcal{T}}$

Thus, $\not{\mathcal{CP}}$ requires:

- Complex Phases
- Interferences

Standard Model $\not{CP}$: 3 fermion families needed

$$\not{CP} \quad \Longleftrightarrow \quad \mathbf{H}(M_u^2) \cdot \mathbf{H}(M_d^2) \cdot \mathbf{J} \neq 0$$

$$\mathbf{H}(M_u^2) \equiv (m_t^2 - m_c^2)\,(m_c^2 - m_u^2)\,(m_t^2 - m_u^2)$$

$$\mathbf{H}(M_d^2) \equiv (m_b^2 - m_s^2)\,(m_s^2 - m_d^2)\,(m_b^2 - m_d^2)$$

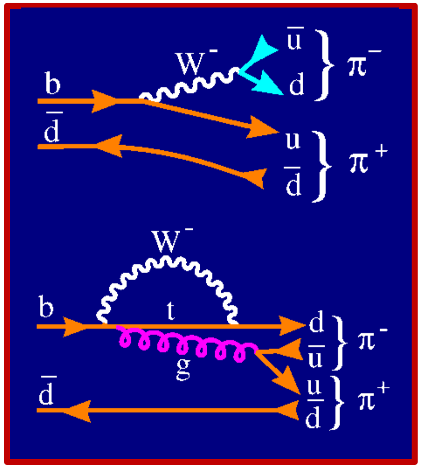
$$\mathbf{J} = c_{12}\, c_{13}^2\, c_{23}\, s_{12}\, s_{13}\, s_{23} \sin\delta_{13} = \left| A^2 \lambda^6 \eta \right| < 10^{-4}$$

- **Low-Energy Phenomena**
- **Small Effects ~ J**
- **Big Asymmetries ⟺ Suppressed Decays**
- **B Decays are an optimal place for $\not{CP}$ signals**

DIRECT $\not{\mathcal{CP}}$

$$\left|T\left(P \to f\right)\right| \neq \left|T\left(\bar{P} \to \bar{f}\right)\right|$$

$\bar{B}_d^0$



$$T\left(P \to f\right) = T_1\, e^{i\phi_1}\, e^{i\delta_1} + T_2\, e^{i\phi_2}\, e^{i\delta_2}$$

$\mathcal{CP}$

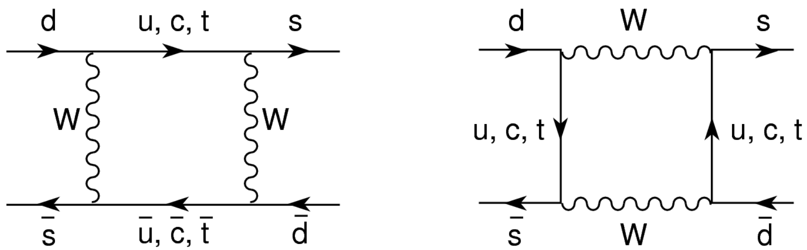
$$T\left(\bar{P} \to \bar{f}\right) = T_1\, e^{-i\phi_1}\, e^{i\delta_1} + T_2\, e^{-i\phi_2}\, e^{i\delta_2}$$

$$A^{CP}_{P\to f} \equiv \frac{\Gamma\left(P \to f\right) - \Gamma\left(\bar{P} \to \bar{f}\right)}{\Gamma\left(P \to f\right) + \Gamma\left(\bar{P} \to \bar{f}\right)} = \frac{-2\, T_1\, T_2\, \sin(\phi_2 - \phi_1)\, \sin(\delta_2 - \delta_1)}{T_1^{\,2} + T_2^{\,2} + 2\, T_1\, T_2\, \cos(\phi_2 - \phi_1)\, \cos(\delta_2 - \delta_1)}$$

One needs:

- **2 Interfering Amplitudes**
- **2 Different Weak Phases** $\left[\sin(\phi_2 - \phi_1) \neq 0\right]$
- **2 Different FSI Phases** $\left[\sin(\delta_2 - \delta_1) \neq 0\right]$

INDIRECT $\not{CP}$: $K^0 - \bar{K}^0$ MIXING



$$|K^0_{S,L}\rangle \sim p\,|K^0\rangle \mp q\,|\bar{K}^0\rangle$$

$$q/p \equiv (1-\bar{\varepsilon}_K)/(1+\bar{\varepsilon}_K)$$

$$\langle \bar{K}^0 | \mathbf{H} | K^0 \rangle \sim \sum_{ij} \lambda_i \lambda_j \, S(r_i, r_j)\, \eta_{ij} \quad \langle O_{\Delta S=2} \rangle$$

$$\langle O_{\Delta S=2} \rangle = \alpha_s(\mu)^{-2/9} \langle \bar{K}^0 | (\bar{s}_L \gamma^\alpha d_L)(\bar{s}_L \gamma_\alpha d_L) | K^0 \rangle \equiv \left(\frac{4}{3} M_K^2 f_K^2\right) \hat{B}_K$$

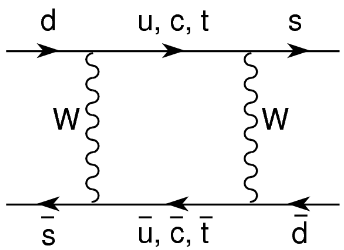
$$\lambda_i \equiv V_{id} V_{is}^* \quad ; \quad r_i \equiv m_i^2 / M_W^2 \quad (i = u, c, t)$$

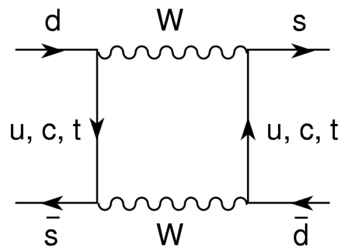
- **GIM Mechanism:** $\lambda_u + \lambda_c + \lambda_t = 0$

$$(M_{K_L} - M_{K_S})/M_{K^0} = (7.00 \pm 0.01) \times 10^{-15}$$

- $\mathcal{CP}$ **:** $\mathrm{Im}\,\lambda_t = -\mathrm{Im}\,\lambda_c \simeq \eta \lambda^5 A^2$
- **Hard GIM Breaking:** $S(r_i, r_i) \sim r_i$ ➡ **t quark**

INDIRECT $\not{CP}$: $K^0 - \bar{K}^0$ MIXING





$$\left|K^0_{S,L}\right\rangle \sim p\,\left|K^0\right\rangle \mp q\,\left|\bar{K}^0\right\rangle$$

$$q/p \equiv \left(1-\bar{\varepsilon}_K\right)/\left(1+\bar{\varepsilon}_K\right)$$

$$\left\langle \bar{K}^0 \left| \mathbf{H} \right| K^0 \right\rangle \sim \sum_{ij} \lambda_i\,\lambda_j\; S(r_i,r_j)\;\eta_{ij}\quad \left\langle O_{\Delta S=2}\right\rangle$$

$$\left\langle O_{\Delta S=2}\right\rangle = \alpha_s(\mu)^{-2/9}\left\langle \bar{K}^0 \left| \left(\bar{s}_L\,\gamma^\alpha d_L\right)\left(\bar{s}_L\,\gamma_\alpha d_L\right) \right| K^0 \right\rangle \equiv \left(\frac{4}{3} M_K^2\, f_K^2\right)\, \hat{B}_K$$

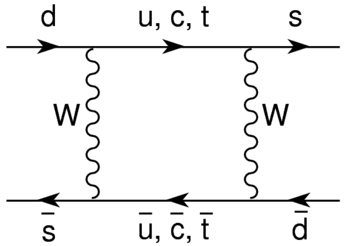
$$\lambda_i \equiv V_{id}\, V_{is}^* \qquad ; \qquad r_i \equiv m_i^2 / M_W^2 \qquad (i = u,c,t)$$

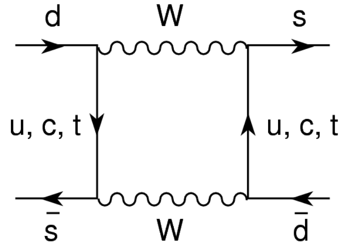
$$\mathcal{C}\,\left|K^0\right\rangle = \left|\bar{K}^0\right\rangle \quad , \quad \mathcal{P}\,\left|K^0\right\rangle = -\left|K^0\right\rangle \quad , \quad \mathcal{CP}\,\left|K^0\right\rangle = -\left|\bar{K}^0\right\rangle$$

$$\left|K^0_{1,2}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle \mp \left|\bar{K}^0\right\rangle\right) \qquad , \qquad \mathcal{CP}\,\left|K^0_{1,2}\right\rangle = \pm\,\left|K^0_{1,2}\right\rangle$$

$$\left|K^0_S\right\rangle \simeq \left|K^0_1\right\rangle + \bar{\varepsilon}_K\left|K^0_2\right\rangle \qquad , \qquad \left|K^0_L\right\rangle \simeq \left|K^0_2\right\rangle + \bar{\varepsilon}_K\left|K^0_1\right\rangle$$

INDIRECT $\not{CP}$: $K^0 - \bar{K}^0$ MIXING





$$\left|K^0_{S,L}\right\rangle \sim p\left|K^0\right\rangle \mp q\left|\bar{K}^0\right\rangle$$

$$q/p \equiv \left(1-\bar{\varepsilon}_K\right)/\left(1+\bar{\varepsilon}_K\right)$$

$$K^0 \to \pi^- l^+ \nu_l \quad (\bar{s} \to \bar{u}) \qquad ; \qquad \bar{K}^0 \to \pi^+ l^- \bar{\nu}_l \quad (s \to u)$$

$$\frac{\Gamma\left(K^0_L \to \pi^- l^+ \nu_l\right) - \Gamma\left(K^0_L \to \pi^+ l^- \bar{\nu}_l\right)}{\Gamma\left(K^0_L \to \pi^- l^+ \nu_l\right) + \Gamma\left(K^0_L \to \pi^+ l^- \bar{\nu}_l\right)} = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} = \frac{2\,\mathrm{Re}\left(\bar{\varepsilon}_K\right)}{1+|\bar{\varepsilon}_K|^2} = (0.332 \pm 0.006)\%$$

➡ $\mathbf{Re}(\bar{\varepsilon}_K) = (1.66 \pm 0.03) \cdot 10^{-3}$

$$\eta_{+-} \equiv \frac{T(K_L \to \pi^+\pi^-)}{T(K_S \to \pi^+\pi^-)} \approx \varepsilon_K \qquad \eta_{00} \equiv \frac{T(K_L \to \pi^0\pi^0)}{T(K_S \to \pi^0\pi^0)} \approx \varepsilon_K$$

$$\varepsilon_K = (2.228 \pm 0.011) \cdot 10^{-3}\, e^{i\phi_\varepsilon}$$

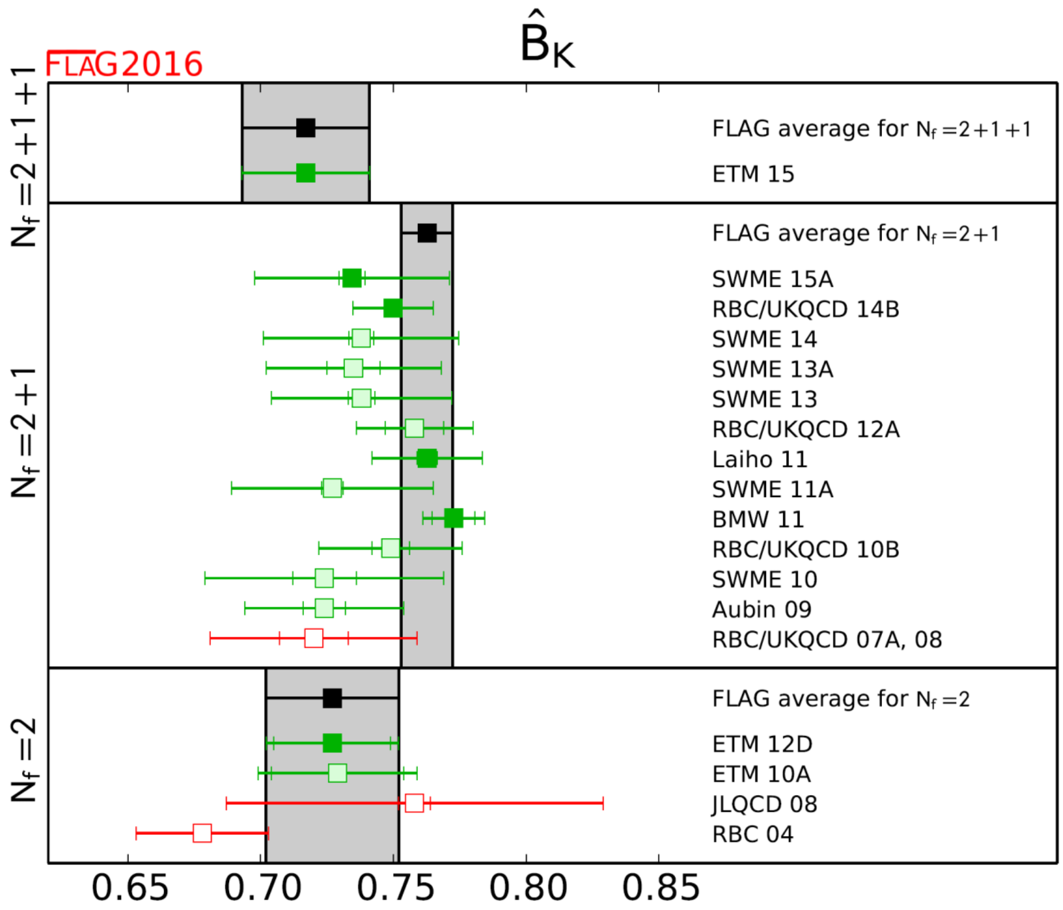
$$\phi_\varepsilon = (43.52 \pm 0.05)^\circ$$

➡ $$\eta\left[\left(1-\rho\right)A^2 + 0.22\right]A^2\,\hat{B}_K = 0.143$$

Buras et al

Lattice Results for $\hat{B}_K$

$$B_K^{\overline{MS}}(2\,\mathrm{GeV}) = 0.557 \pm 0.007 \quad , \quad \hat{B}_K = 0.763 \pm 0.010 \qquad (N_f = 2+1)$$



Flavianet Lattice Averaging Group

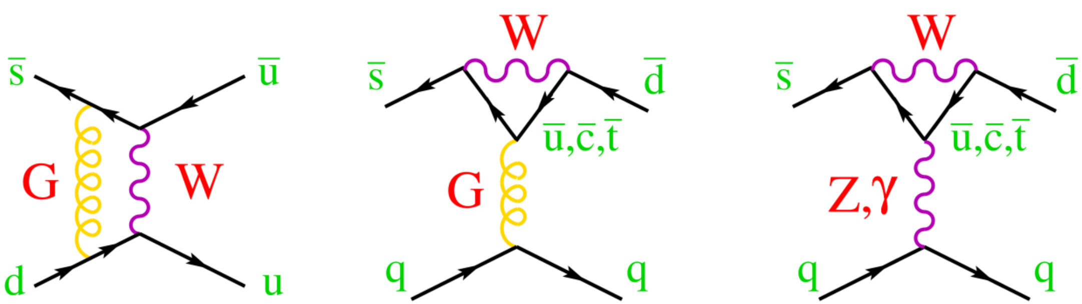
DIRECT $\not{CP}$ in $K \to \pi\,\pi$

$$\eta_{+-} \equiv \frac{T(K_L \to \pi^+\pi^-)}{T(K_S \to \pi^+\pi^-)} \approx \varepsilon_K + \varepsilon'_K \qquad\qquad \eta_{00} \equiv \frac{T(K_L \to \pi^0\pi^0)}{T(K_S \to \pi^0\pi^0)} \approx \varepsilon_K - 2\,\varepsilon'_K$$

$$\mathrm{Re}\left(\varepsilon'_K/\varepsilon_K\right) \approx \frac{1}{6}\left\{1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right\} = (16.8 \pm 1.4)\cdot 10^{-4}$$

NA48, NA31

KTeV, E731



$$\mathrm{Re}\left(\varepsilon'_K/\varepsilon_K\right)_{\mathrm{Th}} = \left(19\ ^{+11}_{-9}\right)\cdot 10^{-4}$$

- Short-distance OPE

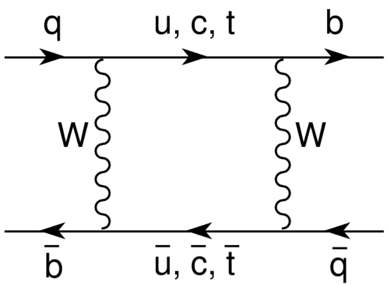
  Ciuchini et al, Buras et al

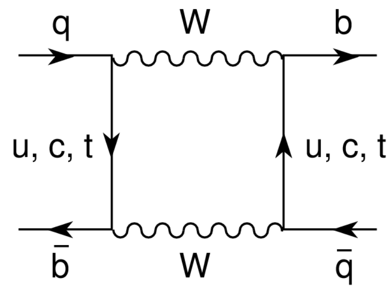
- Long-distance χPT

  Pallante-Pich-Scimemi

  Cirigliano-Ecker-Neufeld-Pich

$B^0 - \bar{B}^0$ MIXING





$$V_{ud}\, V_{ub}^{*} \sim V_{cd}\, V_{cb}^{*} \sim V_{td}\, V_{tb}^{*} \sim A\lambda^3$$

$$\langle \bar{B}^0 | \mathrm{H} | B^0 \rangle \sim |V_{td}|^2\, S(r_t, r_t) \left(\frac{4}{3} M_B^2 f_B^2 \right) \hat{B}_B$$

$$\Delta M_{B_d^0} = (0.5064 \pm 0.0019)\ \mathrm{ps}^{-1} \quad \rightarrow \quad |V_{td}|$$

- $\Delta M_{B_d^0} / \Gamma_{B_d^0} = 0.770 \pm 0.004$
- $\Delta M_{B_s^0} = (17.757 \pm 0.021)\ \mathrm{ps}^{-1}$
- $\Delta \Gamma_{B^0} / \Delta M_{B^0} \sim m_b^2 / m_t^2 \ll 1$
- $\mathrm{Re}\left(\bar{\varepsilon}_{B_d^0}\right) = -0.0010 \pm 0.0008$

$\Delta M_{B_s^0} / \Gamma_{B_s^0} = 26.72 \pm 0.09$

$|V_{ts}|^2 \gg |V_{td}|^2$

$\Delta \Gamma_{B_s^0} / \Gamma_{B_s^0} = -0.129 \pm 0.009$

$\mathrm{Re}\left(\bar{\varepsilon}_{B_s^0}\right) = -0.0003 \pm 0.0014$

$\not{CP}$ very small $\quad |q/p| - 1 \sim m_c^2 / m_t^2$

$P^0 - \bar{P}^0$ MIXING

Phase convention: $\mathcal{CP}\, |P^0\rangle = -|\bar{P}^0\rangle$

$$|\psi(t)\rangle = a(t)\,|P^0\rangle + b(t)\,|\bar{P}^0\rangle \qquad\qquad i\frac{d}{dt}|\psi(t)\rangle = \mathbf{M}\,|\psi(t)\rangle$$

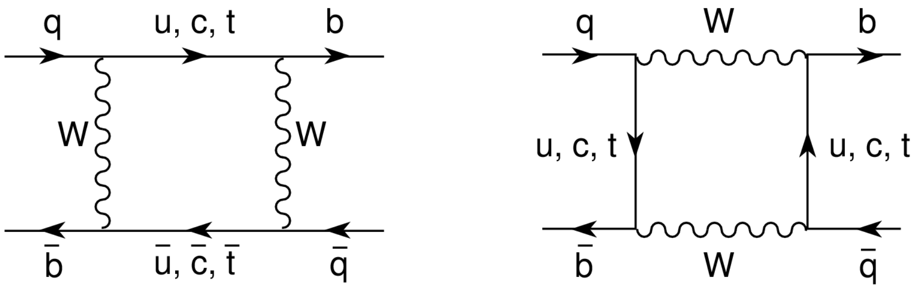
$\mathcal{CPT}$: $$\mathbf{M} = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

$\mathcal{CP}$: $$M_{12} = M_{12}^* \quad , \quad \Gamma_{12} = \Gamma_{12}^*$$

- **Dispersive:** $$M_{12} = \frac{1}{2M}\left\{\langle P^0 | H_{\Delta P=2} | \bar{P}^0\rangle + PP\int \frac{ds}{M^2 - s}\sum_X dQ_X\, \langle P^0 | H_{\Delta P=1} | X\rangle\langle X | H_{\Delta P=1} | \bar{P}^0\rangle\right\}$$
- **Absorptive:** $$\Gamma_{12} = \frac{\pi}{M}\int ds \sum_X dQ_X\, \delta(s - M^2)\,\langle P^0 | H_{\Delta P=1} | X\rangle\langle X | H_{\Delta P=1} | \bar{P}^0\rangle$$
- **Eigenvalues:** $$|P^0_\mp\rangle = \frac{p\,|P^0\rangle \mp q\,|\bar{P}^0\rangle}{\sqrt{|p|^2 + |q|^2}} \quad , \quad \frac{q}{p} \equiv \frac{1-\bar{\varepsilon}}{1+\bar{\varepsilon}} = \left(\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right)^{1/2}$$

$$\langle P_- | P_+\rangle = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} = \frac{2\,\mathrm{Re}(\bar{\varepsilon})}{1 + |\bar{\varepsilon}|^2}$$

$\mathcal{CP}$ ➡ $q/p = 1$ ➡ $$|P_{1,2}\rangle = \frac{1}{\sqrt{2}}\left(|P^0\rangle \mp |\bar{P}^0\rangle\right) \quad , \quad \mathcal{CP}\,|P_{1,2}\rangle = \pm|P_{1,2}\rangle$$



$$\mathbf{M} = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

$$|B^0_{\mp}\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \left(p\,|B^0\rangle \mp q\,|\bar{B}^0\rangle \right) \quad , \quad \frac{q}{p} \equiv \frac{1-\bar{\varepsilon}_B}{1+\bar{\varepsilon}_B} = \left(\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}} \right)^{1/2}$$

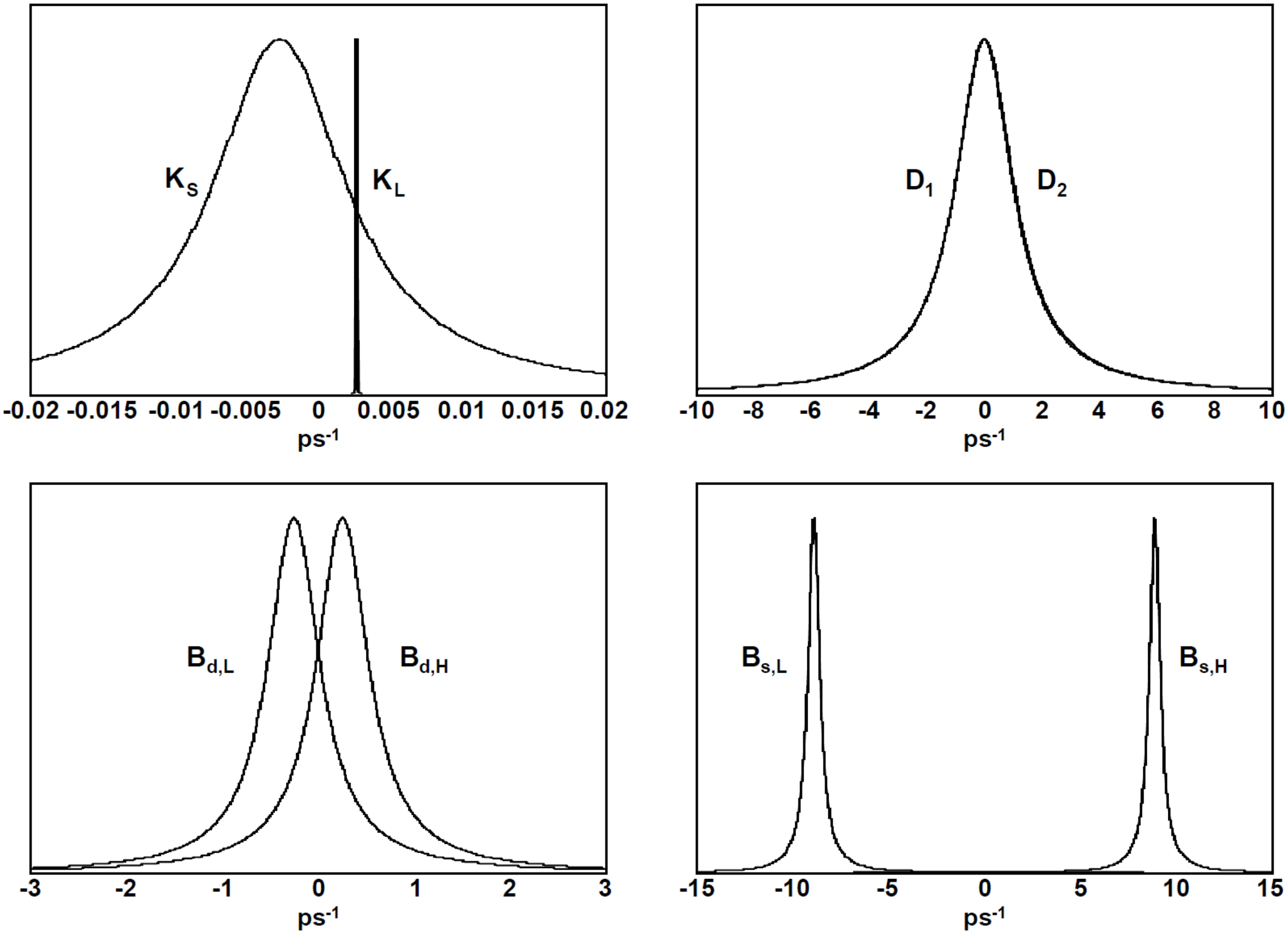
$$(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4\,|M_{12}|^2 - |\Gamma_{12}|^2 \quad , \quad \Delta M\; \Delta\Gamma = 4\,\mathrm{Re}(M_{12}\,\Gamma_{12}^*)$$

$$\Delta\Gamma/\Delta M \approx \Gamma_{12}/M_{12} \sim m_b^2/m_t^2 \ll 1 \quad \Rightarrow \quad \left|\frac{q}{p}\right| \approx 1 + \frac{1}{2}\left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_{\Delta B=2} \quad , \quad \phi_{\Delta B=2} \equiv \arg\left(M_{12}/\Gamma_{12}\right)$$

$$\Delta M \equiv M_{B_+} - M_{B_-} \quad , \quad \Delta\Gamma \equiv \Gamma_{B_+} - \Gamma_{B_-}$$

$$\begin{pmatrix} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{pmatrix} = \begin{pmatrix} g_1(t) & \frac{q}{p} g_2(t) \\ \frac{p}{q} g_2(t) & g_1(t) \end{pmatrix} \begin{pmatrix} |B^0\rangle \\ |\bar{B}^0\rangle \end{pmatrix} \quad , \quad \begin{pmatrix} g_1(t) \\ g_2(t) \end{pmatrix} = e^{-iMt}\, e^{-\Gamma t/2} \begin{pmatrix} \cos\left[\left(\Delta M - \frac{i}{2}\Delta\Gamma\right)\frac{t}{2}\right] \\ -i\sin\left[\left(\Delta M - \frac{i}{2}\Delta\Gamma\right)\frac{t}{2}\right] \end{pmatrix}$$

Widths & Mass Differences

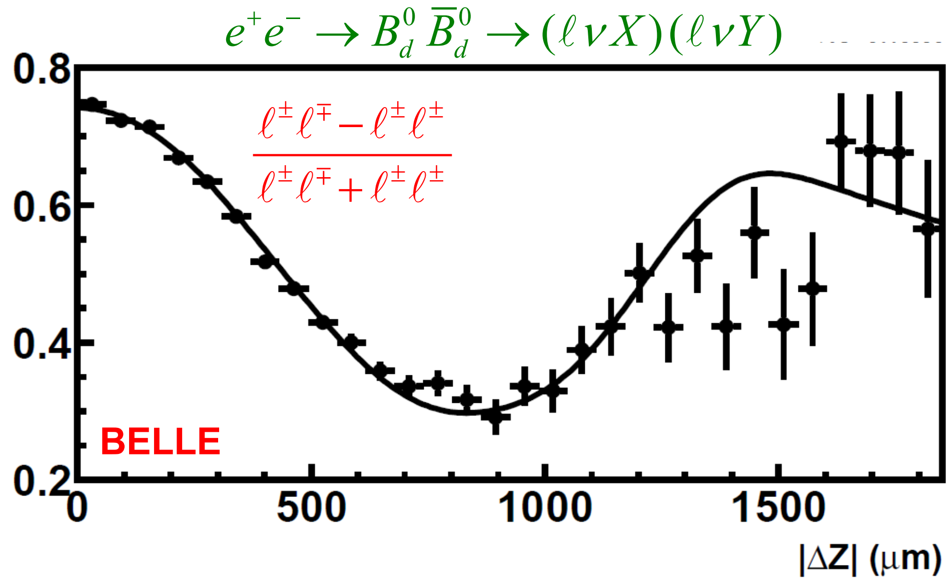


M. Gersabeck

Time Scales:

Oscillation ~ $\sin\left[(x-iy)\Gamma t/2\right]$

$$x \equiv \frac{\Delta M}{\Gamma} \quad , \quad y \equiv \frac{\Delta\Gamma}{2\Gamma}$$

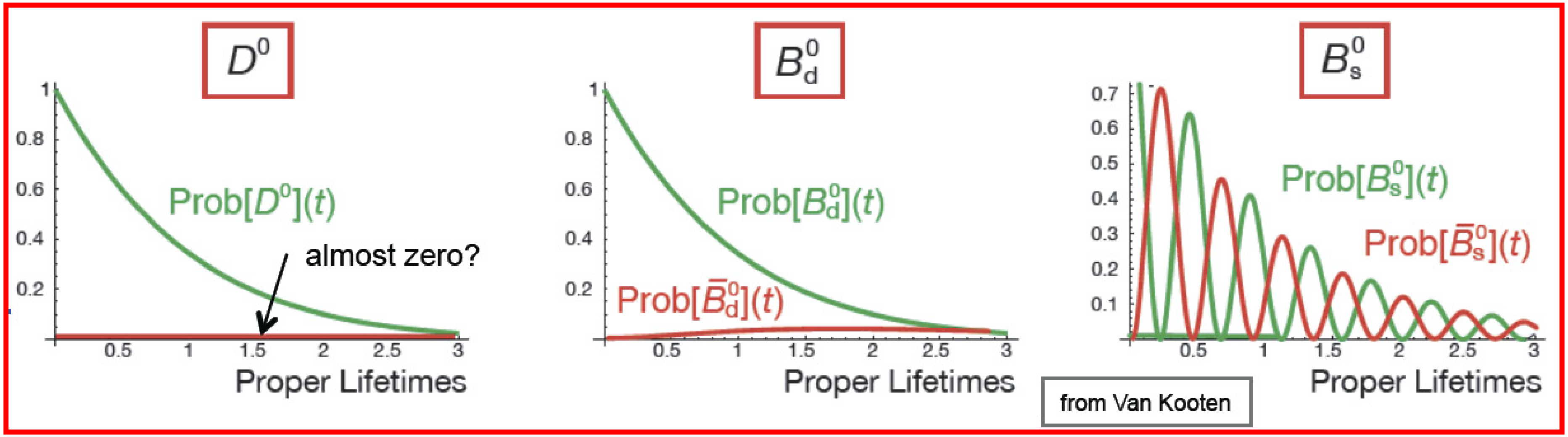


- **K⁰ :** x ~ y ~ 1
- **D⁰ :** x ~ y ~ 0.01 — Slow oscillation (decays faster)
- **B_d :** x ~ 1 , y ~ 0.01
- **B_s :** x ~ 25 , y ~ 0.05 — Fast oscillation (averages out to 0)

Time Scales:

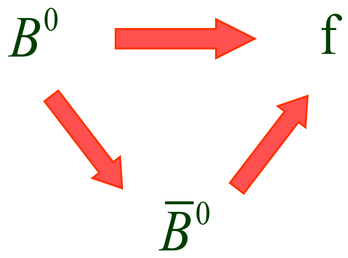
Oscillation ~ $\sin\left[(x-iy)\Gamma t/2\right]$

$$x \equiv \Delta M/\Gamma \quad , \quad y \equiv \Delta\Gamma/2\Gamma$$



- **K^0 :** x ~ y ~ 1
- **D^0 :** x ~ y ~ 0.01 Slow oscillation (decays faster)
- **B_d :** x ~ 1 , y ~ 0.01
- **B_s :** x ~ 25 , y ~ 0.05 Fast oscillation (averages out to 0)

$B^0 - \bar{B}^0$ MIXING AND DIRECT $\not{CP}$



$$T_f \equiv T[B^0 \to f] \quad ; \quad \bar{T}_f \equiv -T[\bar{B}^0 \to f] \quad ; \quad \bar{\rho}_f \equiv \bar{T}_f / T_f$$

$$T_{\bar{f}} \equiv T[B^0 \to \bar{f}] \quad ; \quad \bar{T}_{\bar{f}} \equiv -T[\bar{B}^0 \to \bar{f}] \quad ; \quad \rho_{\bar{f}} \equiv T_{\bar{f}} / \bar{T}_{\bar{f}}$$

$$\mathcal{CP}\; B^0 = -\bar{B}^0 \qquad ; \qquad \mathcal{CP}\; f = \bar{f}$$

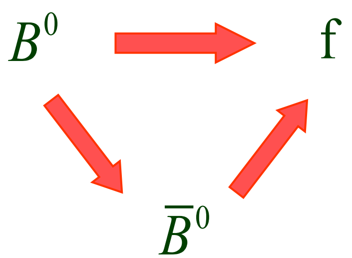
$$\Gamma[B^0(t) \to f] \sim \frac{1}{2}\, e^{-\Gamma t} \left(|T_f|^2 + |\bar{T}_f|^2\right) \left\{1 + \mathbf{C_f} \cos(\Delta M\, t) - \mathbf{S_f} \sin(\Delta M\, t)\right\}$$

$$\Gamma[\bar{B}^0(t) \to \bar{f}] \sim \frac{1}{2}\, e^{-\Gamma t} \left(|\bar{T}_{\bar{f}}|^2 + |T_{\bar{f}}|^2\right) \left\{1 - \mathbf{C_{\bar{f}}} \cos(\Delta M\, t) + \mathbf{S_{\bar{f}}} \sin(\Delta M\, t)\right\}$$

$$\mathbf{C_f} \equiv \frac{1 - |\bar{\rho}_f|^2}{1 + |\bar{\rho}_f|^2} \quad ; \quad \mathbf{S_f} \equiv \frac{2\,\mathrm{Im}\left(\frac{q}{p}\,\bar{\rho}_f\right)}{1 + |\bar{\rho}_f|^2} \quad ; \quad \mathbf{C_{\bar{f}}} \equiv -\frac{1 - |\rho_{\bar{f}}|^2}{1 + |\rho_{\bar{f}}|^2} \quad ; \quad \mathbf{S_{\bar{f}}} \equiv \frac{-\,2\,\mathrm{Im}\left(\frac{p}{q}\,\rho_{\bar{f}}\right)}{1 + |\rho_{\bar{f}}|^2}$$

$$\Delta\Gamma \ll \Delta M \quad \Longrightarrow \quad \frac{q}{p} \approx \sqrt{\frac{M_{12}^*}{M_{12}}} \approx \frac{\mathbf{V}_{tb}^* \,\mathbf{V}_{tq}}{\mathbf{V}_{tb}\, \mathbf{V}_{tq}^*} = e^{-2i\phi_M} \quad ; \quad \phi_M \approx \begin{cases} \beta & (B_d^0) \\ -\beta_s \approx -\lambda^2 \eta & (B_s^0) \end{cases}$$

$B^0 - \bar{B}^0$ MIXING AND DIRECT $\not{CP}$



$$T_f \to T[B^0 \to f] \quad ; \quad \bar{T}_f \to -T[\bar{B}^0 \to f] \quad ; \quad \bar{\rho}_f \equiv \bar{T}_f / T_f$$

$$T_{\bar{f}} \to T[B^0 \to \bar{f}] \quad ; \quad \bar{T}_{\bar{f}} \to -T[\bar{B}^0 \to \bar{f}] \quad ; \quad \rho_{\bar{f}} \equiv T_{\bar{f}} / \bar{T}_{\bar{f}}$$

$$\mathcal{CP}\; B^0 = -\bar{B}^0 \qquad ; \qquad \mathcal{CP}\; f = \bar{f}$$

$$\Gamma[B^0(t) \to f] \;\sim\; \frac{1}{2}\, e^{-\Gamma t} \left(|T_f|^2 + |\bar{T}_f|^2\right) \left\{1 + \mathbf{C_f}\, \cos(\Delta M\, t) - \mathbf{S_f}\, \sin(\Delta M\, t)\right\}$$

$$\Gamma[\bar{B}^0(t) \to \bar{f}] \;\sim\; \frac{1}{2}\, e^{-\Gamma t} \left(|\bar{T}_{\bar{f}}|^2 + |T_{\bar{f}}|^2\right) \left\{1 - \mathbf{C_{\bar{f}}}\, \cos(\Delta M\, t) + \mathbf{S_{\bar{f}}}\, \sin(\Delta M\, t)\right\}$$

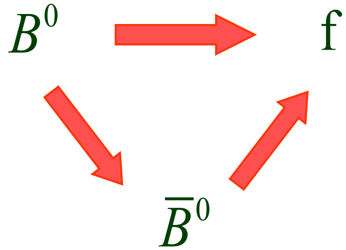
$$\mathbf{C_f} \equiv \frac{1 - |\bar{\rho}_f|^2}{1 + |\bar{\rho}_f|^2} \quad ; \quad \mathbf{S_f} \equiv \frac{2\,\mathrm{Im}\left(\frac{q}{p}\,\bar{\rho}_f\right)}{1 + |\bar{\rho}_f|^2} \quad ; \quad \mathbf{C_{\bar{f}}} \equiv -\frac{1 - |\rho_{\bar{f}}|^2}{1 + |\rho_{\bar{f}}|^2} \quad ; \quad \mathbf{S_{\bar{f}}} \equiv \frac{-\,2\,\mathrm{Im}\left(\frac{p}{q}\,\rho_{\bar{f}}\right)}{1 + |\rho_{\bar{f}}|^2}$$

CP self-conjugate: $\bar{f} = \eta_f\, f$ $\Rightarrow$

$$T_{\bar{f}} = \eta_f\, T_f \quad ; \quad \bar{T}_{\bar{f}} = \eta_f\, \bar{T}_f \quad ; \quad \rho_{\bar{f}} \equiv 1/\bar{\rho}_f$$

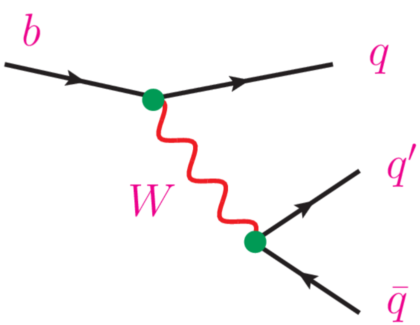
$$\mathbf{C_{\bar{f}}} = \mathbf{C_f} \quad ; \quad \mathbf{S_{\bar{f}}} = \mathbf{S_f}$$

$B^0 - \bar{B}^0$ MIXING AND DIRECT $\not{CP}$



CP self-conjugate: $\bar{\mathrm{f}} = \eta_{\mathrm{f}}\, \mathrm{f}$

$$\frac{q}{p} \approx \frac{\mathbf{V}_{tb}^* \mathbf{V}_{tq}}{\mathbf{V}_{tb} \mathbf{V}_{tq}^*} = e^{-2i\phi_M} \quad ; \qquad \phi_M \approx \begin{cases} \beta & (B_d^0) \\ -\beta_s \approx -\lambda^2 \eta & (B_s^0) \end{cases}$$

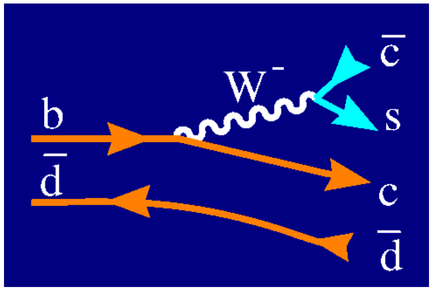
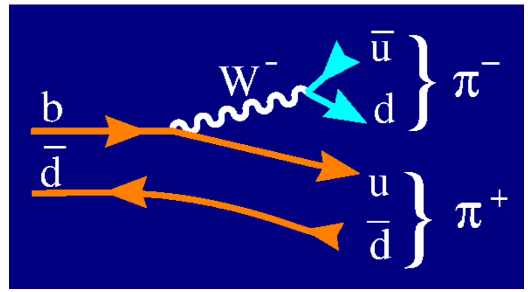
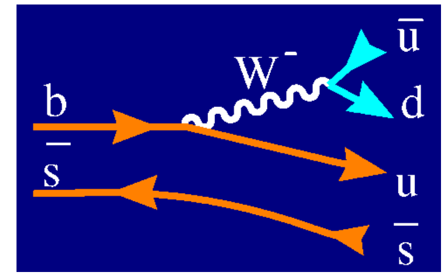
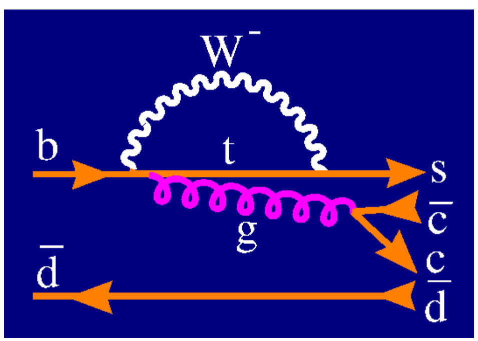
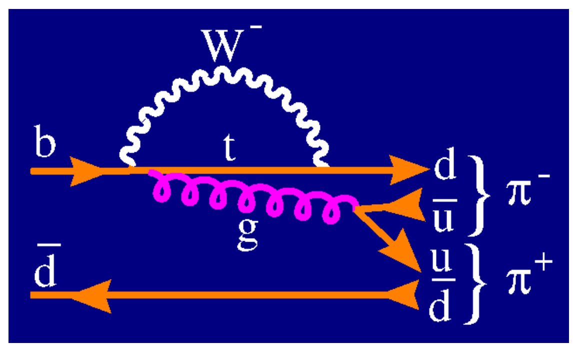
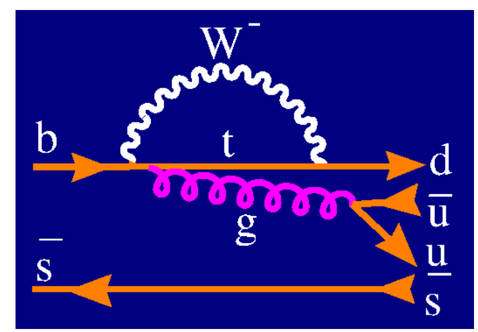


Assumption: **Only 1 decay amplitude**

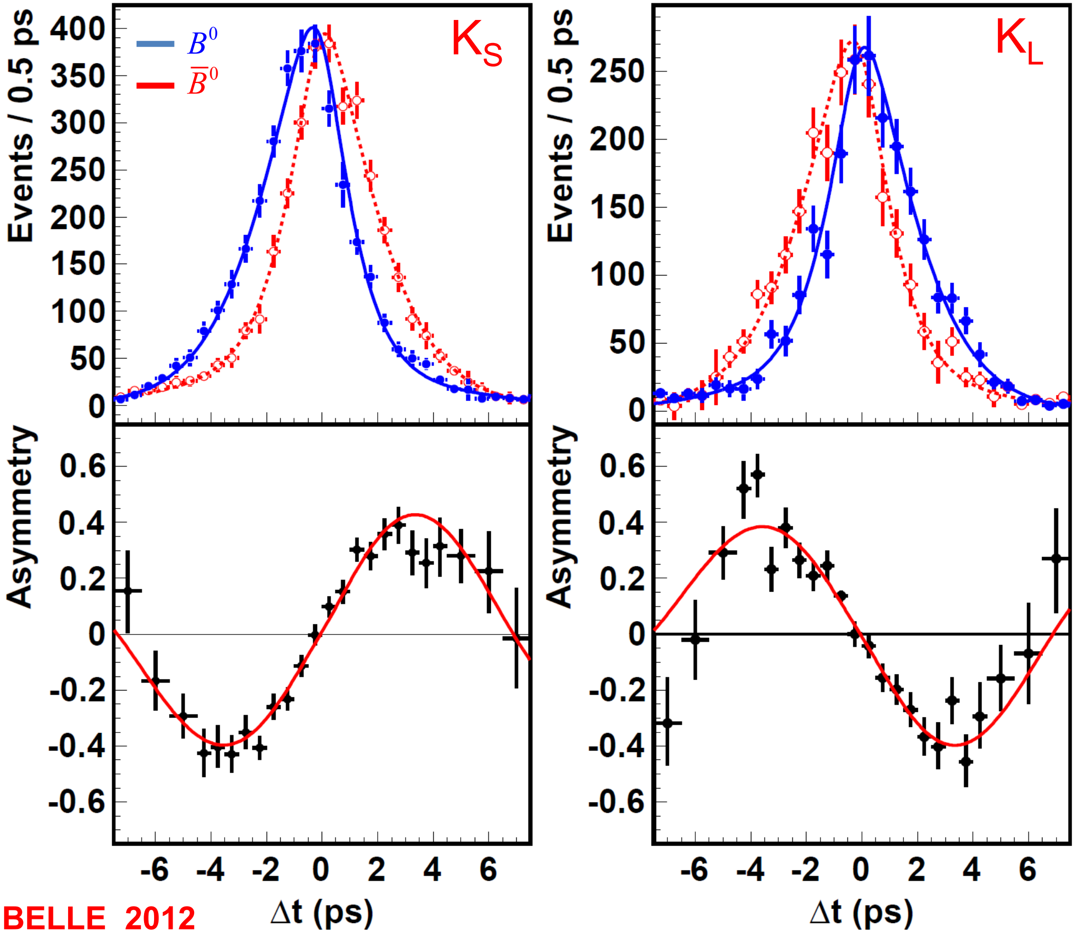
$$\frac{A_{b\to q\bar{q}q'}}{A_{\bar{b}\to \bar{q}q\bar{q}'}} = \frac{\mathbf{V}_{qb} \mathbf{V}_{qq'}^*}{\mathbf{V}_{qb}^* \mathbf{V}_{qq'}} = e^{-2i\phi_D} \quad \Rightarrow \quad \rho_{\bar{f}} = \bar{\rho}_f^* = \eta_f\, e^{2i\phi_D} \, , \quad C_f = 0$$

$$\Rightarrow \quad \frac{\Gamma\left(\bar{B}^0 \to \bar{\mathrm{f}}\right) - \Gamma\left(B^0 \to \mathrm{f}\right)}{\Gamma\left(\bar{B}^0 \to \bar{\mathrm{f}}\right) + \Gamma\left(B^0 \to \mathrm{f}\right)} = -\eta_{\mathrm{f}}\, \sin(2\phi)\, \sin(\Delta M\, t) \quad ; \qquad \phi = \phi_M + \phi_D$$

Direct information on the CKM matrix

<table>
<tr><th>$\bar{B}_d^0 \to J/\Psi\, K_S^0$</th><th>$\bar{B}_d^0 \to \pi^+\pi^-$</th><th>$\bar{B}_s^0 \to \rho^0 K_S^0$</th></tr>
<tr><td>$\phi \simeq \beta$</td><td>$\phi \simeq \beta + \gamma = \pi - \alpha$</td><td>$\phi \neq \gamma$</td></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td>$V_{cb}\, V_{cs}^* \sim A\lambda^2$</td><td>$V_{ub}\, V_{ud}^* \sim A\lambda^3(\rho - i\eta)$</td><td>$V_{ub}\, V_{ud}^* \sim A\lambda^3(\rho - i\eta)$</td></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td>$V_{tb}\, V_{ts}^* \sim -A\lambda^2$</td><td>$V_{tb}\, V_{td}^* \sim A\lambda^3(1 - \rho + i\eta)$</td><td>$V_{tb}\, V_{td}^* \sim A\lambda^3(1 - \rho + i\eta)$</td></tr>
<tr><td>***</td><td>**</td><td>BAD</td></tr>
</table>

$$\frac{\Gamma(\bar{B}^0 \to J/\psi\, K_S) - \Gamma(B^0 \to J/\psi\, K_S)}{\Gamma(\bar{B}^0 \to J/\psi\, K_S) + \Gamma(B^0 \to J/\psi\, K_S)} \;=\; -\eta_f\; \sin(2\beta)\; \sin(\Delta M\, t)$$

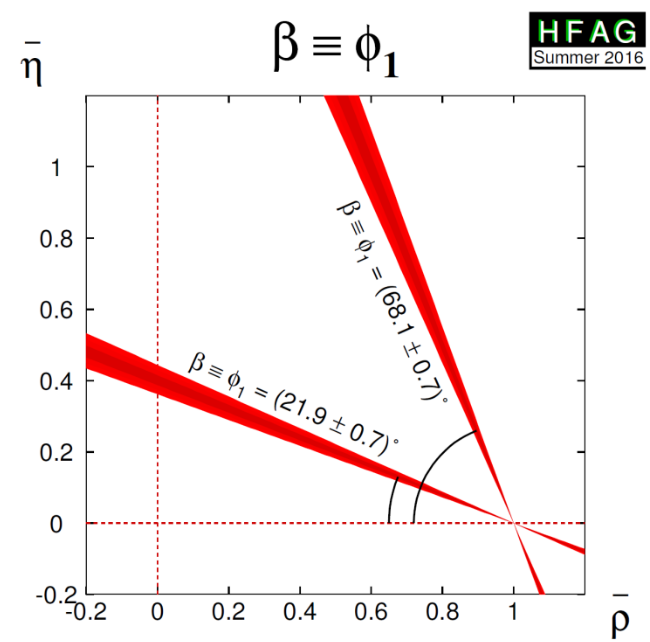


BELLE 2012

HFAG:

sin(2β) = 0. 69 ± 0.02

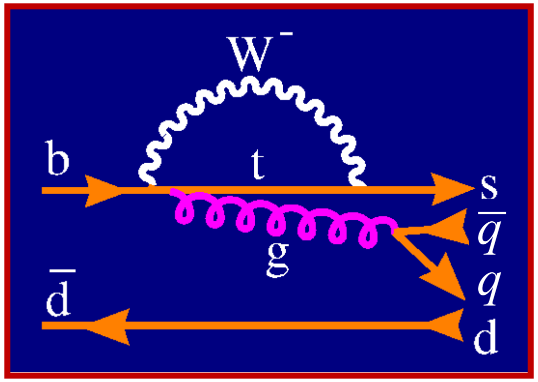
$B^0 \to J/\psi\, K_{S,L}\,,\, \psi(2S)\, K_S\,,\, \chi_c\, K_S\,,\, \eta_c\, K_S$



$\not{CP}$ Signal

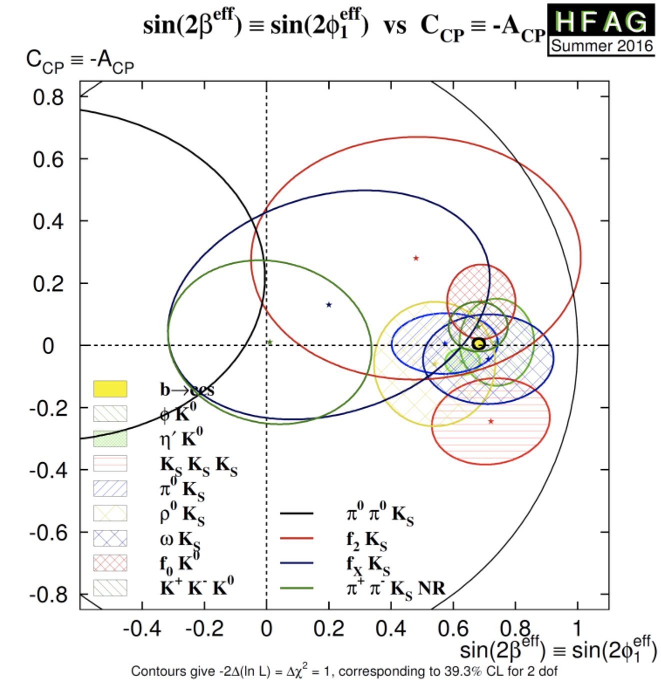
$b \to q\bar{q}s$

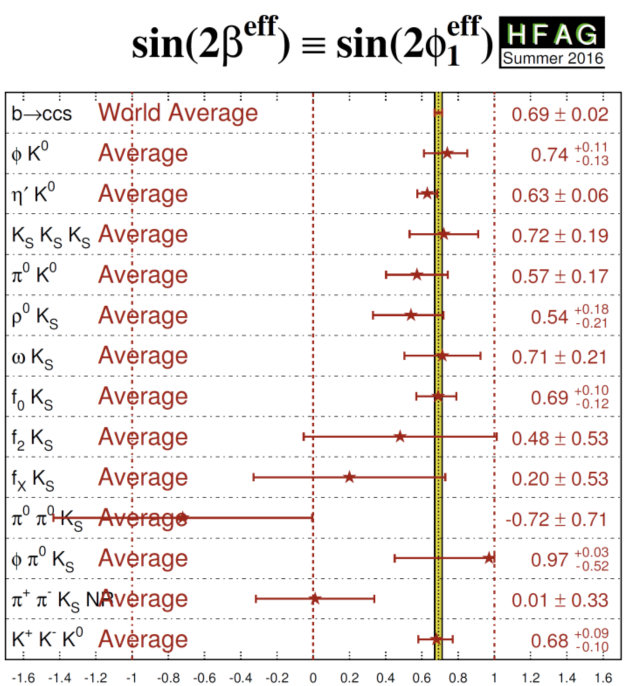
$q = d, s$

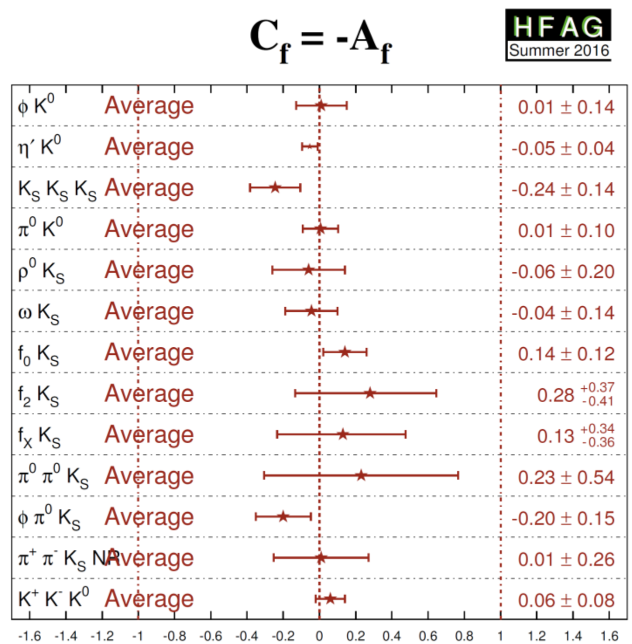


$V_{tb} V_{ts}^* \sim -A\lambda^2$

Sensitive to New Physics in Penguin diagram







Agreement with

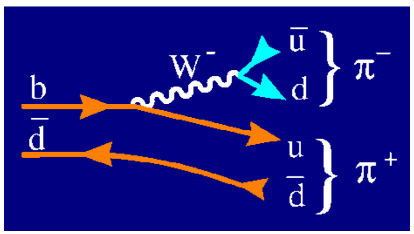
$B^0 \to J/\Psi\, K_S \quad (b \to c\bar{c}s)$

No signal of direct $\not{CP}$

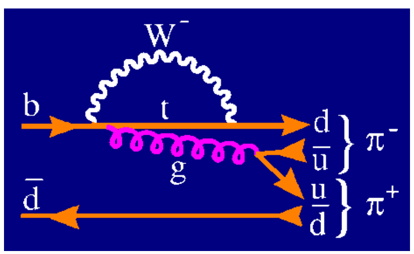
$B^0 \to \pi\pi$

$$\frac{\Gamma\left(\bar{B}^0 \to \bar{f}\right) - \Gamma\left(B^0 \to f\right)}{\Gamma\left(\bar{B}^0 \to \bar{f}\right) + \Gamma\left(B^0 \to f\right)} = -C_f \cos(\Delta M\, t) + S_f \sin(\Delta M\, t)$$

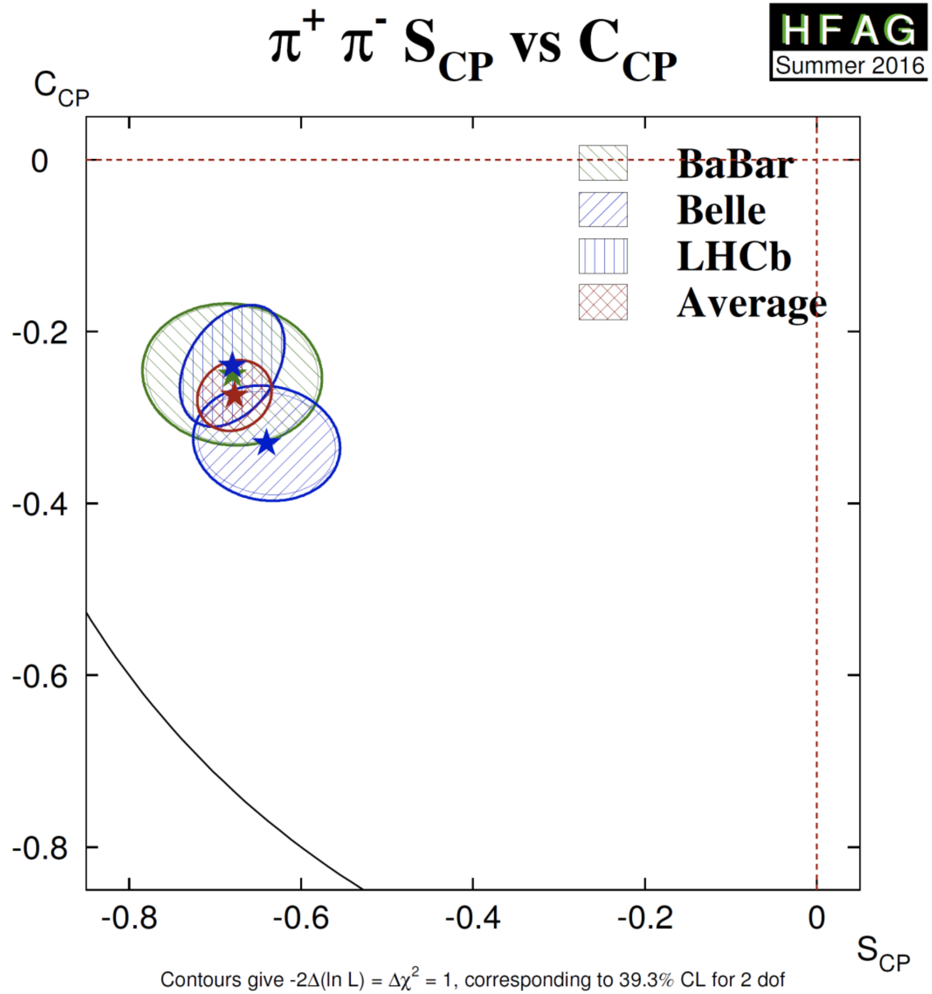
$$\alpha \equiv \arg\left[-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*}\right]$$



$$V_{ub} V_{ud}^* \sim A\lambda^3(\rho - i\eta)$$



$$V_{tb} V_{td}^* \sim A\lambda^3(1-\rho+i\eta)$$



$$C_f \equiv \frac{1-|\bar{\rho}_f|^2}{1+|\bar{\rho}_f|^2} \neq 0$$

Direct $\not{CP}$

Penguins

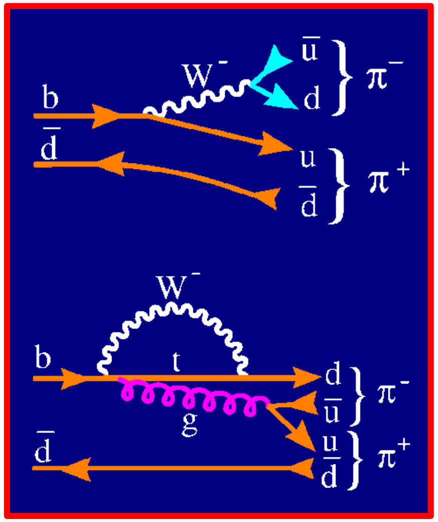
$$\Rightarrow \quad S_f \approx -\sin(2\alpha) \quad ?$$

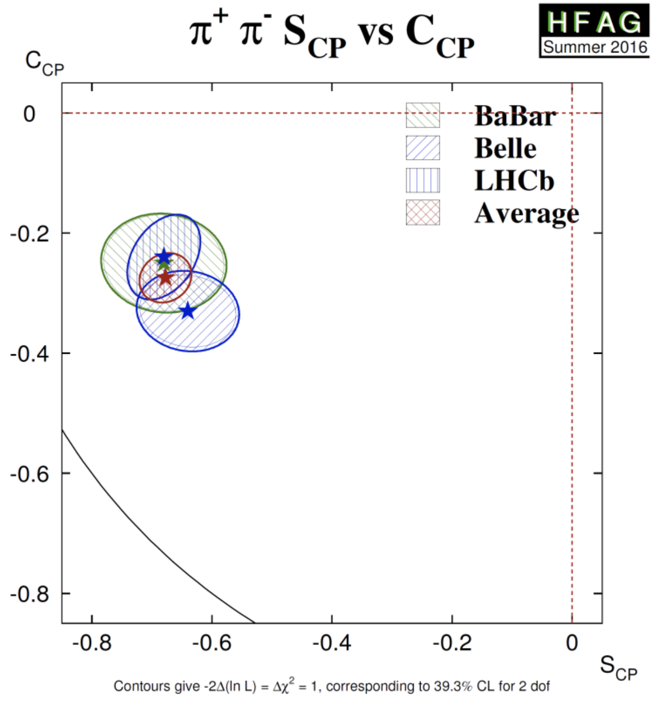
$B^0 \to \pi\pi, \rho\rho, \rho\pi$

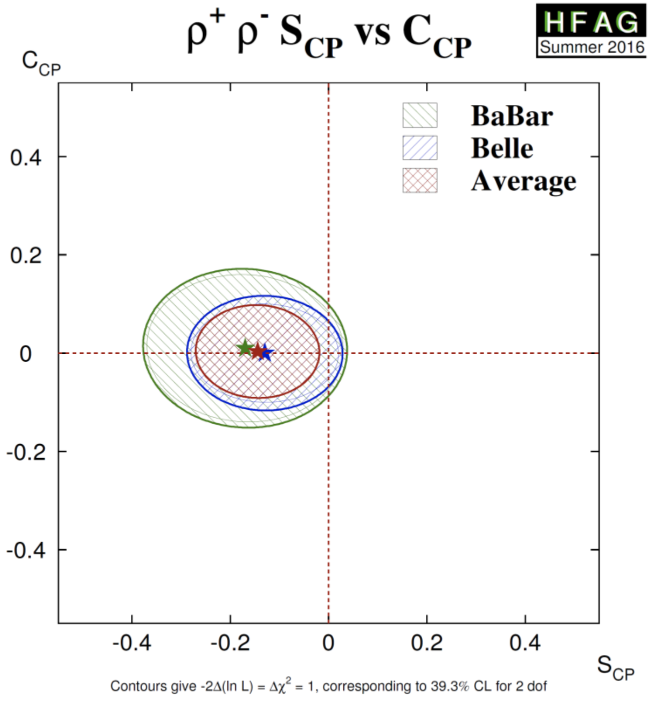
$$C_f \equiv \frac{1-|\bar{\rho}_f|^2}{1+|\bar{\rho}_f|^2} \neq 0$$

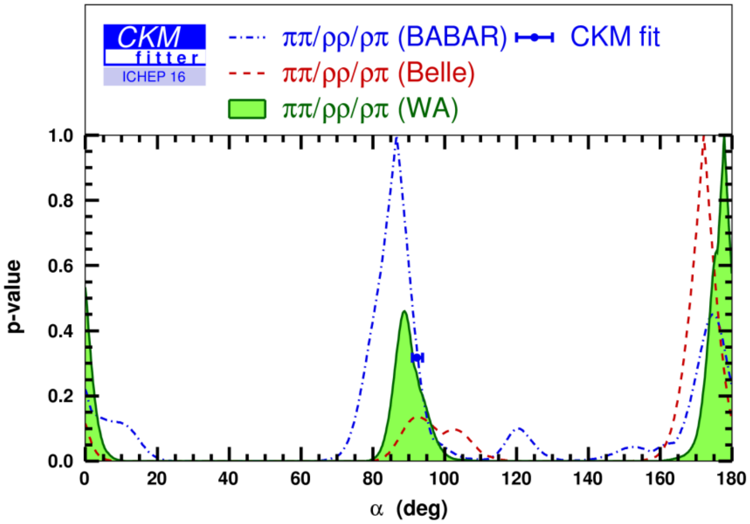
Direct $\mathcal{CP}$

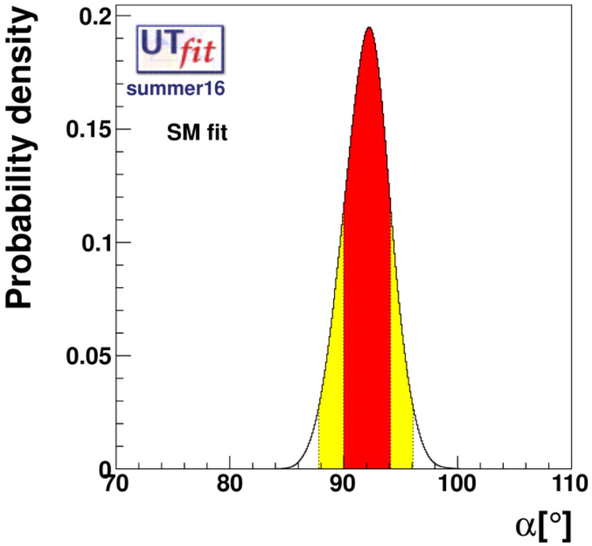
Penguins











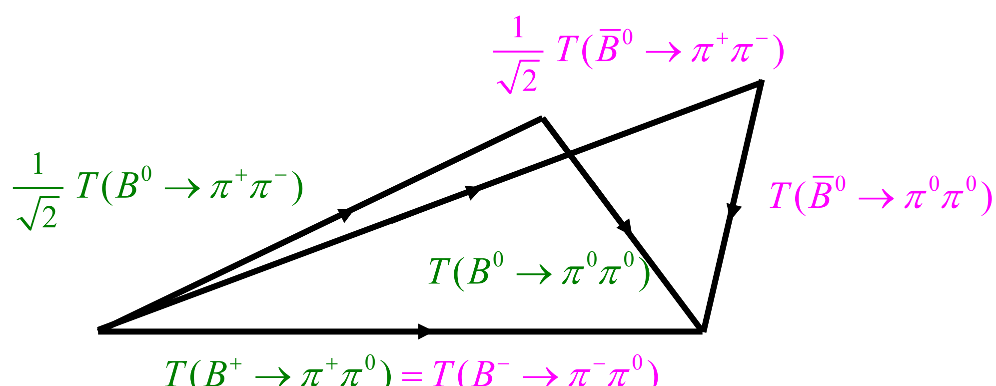
MEASURING HADRONIC CONTAMINATIONS

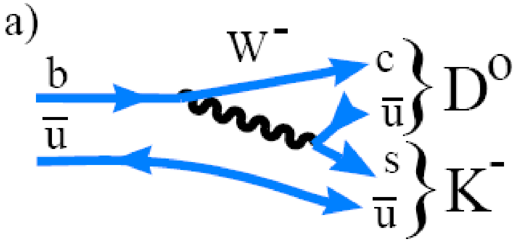
- **Time Evolution**
- **Transversity Analysis:** **B → V V**
- **Isospin Relations** (Gronau-London)
- **D^0-$\bar{D}^0$ Mixing** (Gronau-London-Wyler, Atwood-Dunietz-Soni)

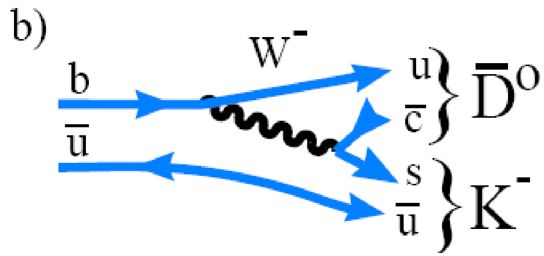
$$\sqrt{2}\,\mathrm{T}(B^+ \to D^0_+\, K^+) = \mathrm{T}(B^+ \to D^0\, K^+) + \mathrm{T}(B^+ \to \bar{D}^0\, K^+)$$

$$\sqrt{2}\,\mathrm{T}(B^0_d \to D^0_+\, K_S) = \mathrm{T}(B^+ \to D^0\, K_S) + \mathrm{T}(B^+ \to \bar{D}^0\, K_S)$$

- **Dalitz Analysis**
- **SU(3) Relations:** **B → π K , π π , ...**
- …







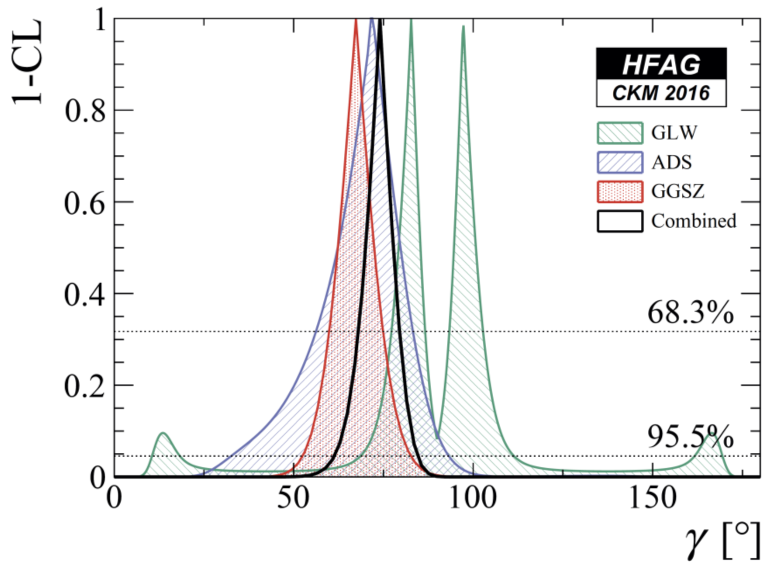
D^0-$\bar{D}^0$ Mixing

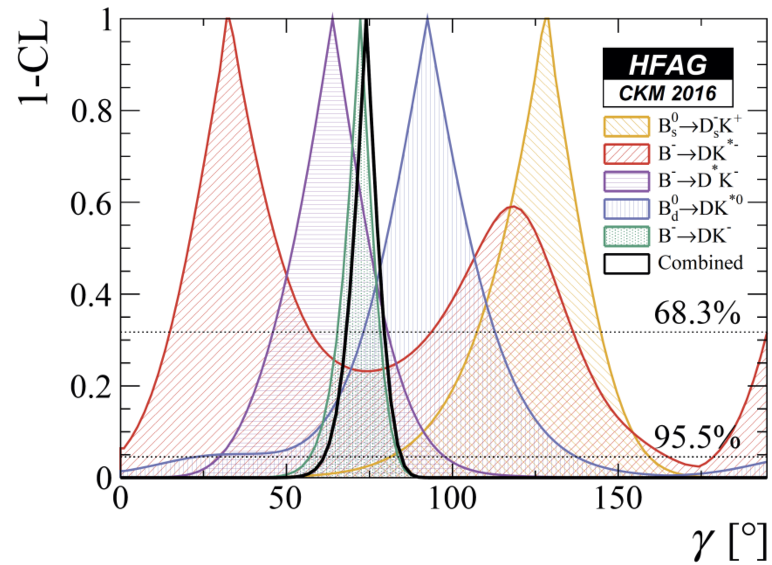
Gronau-London-Wyler

Atwood-Dunietz-Soni

$$\sqrt{2}\, \mathrm{T}(B^+ \to D^0_+ K^+) = \mathrm{T}(B^+ \to D^0 K^+) + \mathrm{T}(B^+ \to \bar{D}^0 K^+)$$

$$\sqrt{2}\, \mathrm{T}(B^0_d \to D^0_+ K_S) = \mathrm{T}(B^+ \to D^0 K_S) + \mathrm{T}(B^+ \to \bar{D}^0 K_S)$$

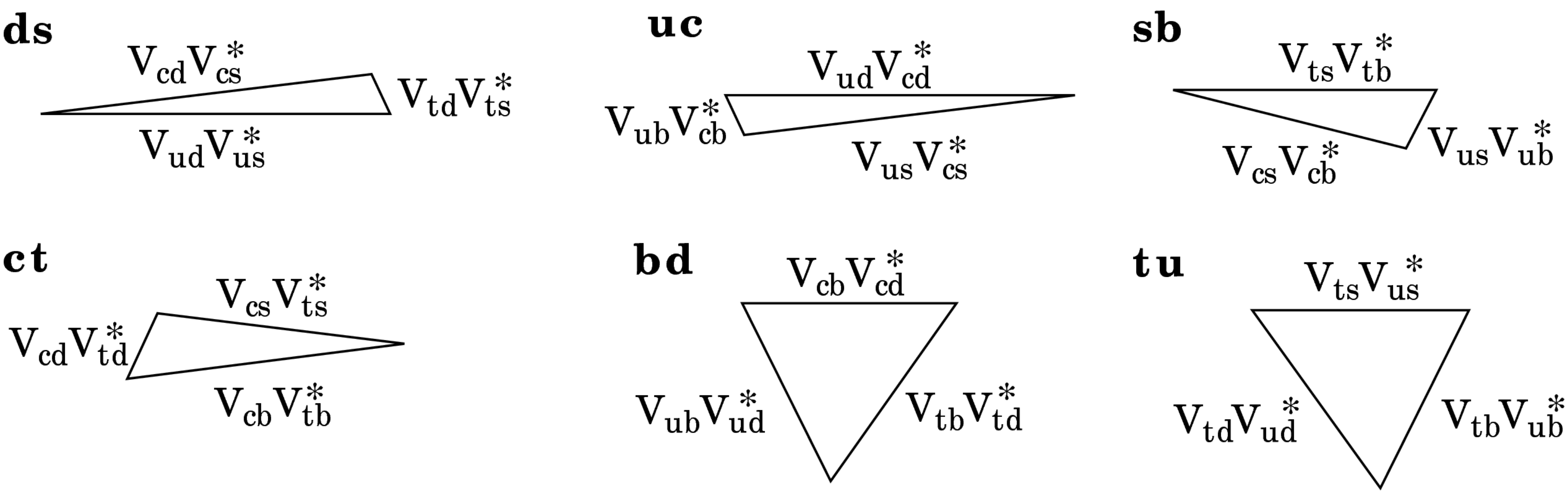




$$\gamma \equiv \arg\left[-\frac{\mathbf{V}_{ud}\,\mathbf{V}_{ub}^*}{\mathbf{V}_{cd}\,\mathbf{V}_{cb}^*}\right] = \left(74.0\ ^{+5.8}_{-6.4}\right)^\circ$$

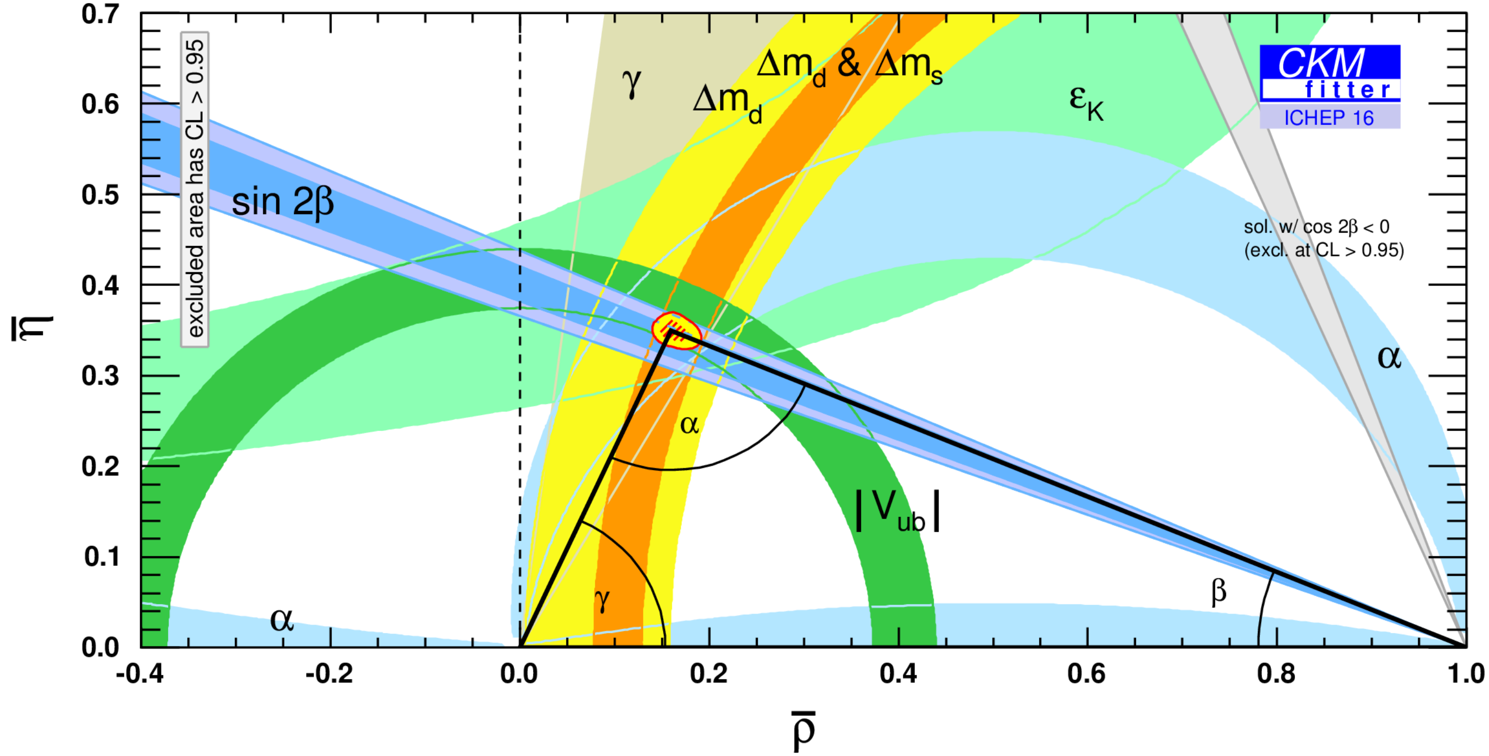
UNITARITY TRIANGLES

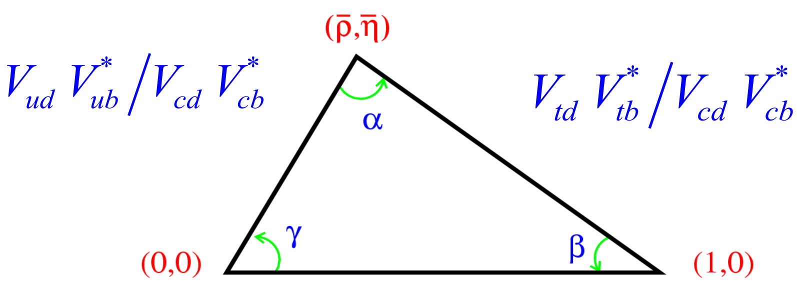
$$\mathbf{V_{ui}\, V^*_{uj} + V_{ci}\, V^*_{cj} + V_{ti}\, V^*_{tj} = 0} \qquad (i \neq j)$$



$$\mathbf{V} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$$V_{ud}\, V_{ub}^{*} + V_{cd}\, V_{cb}^{*} + V_{td}\, V_{tb}^{*} = 0$$





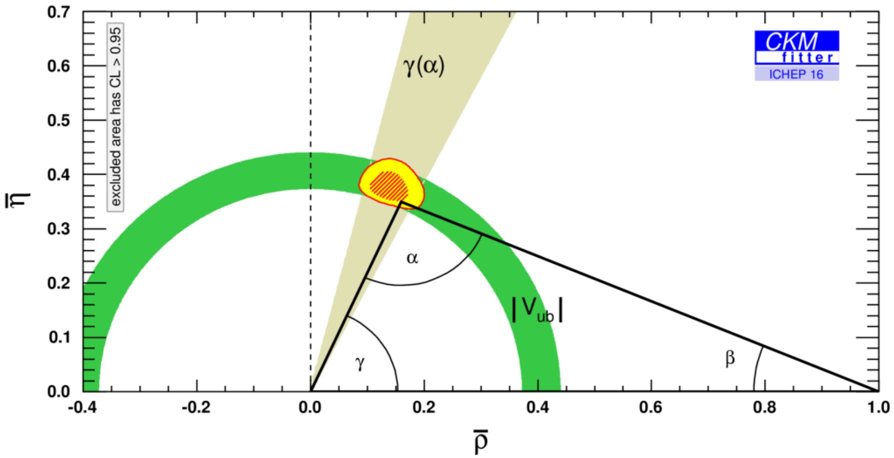
UT*fit*

$$\bar\eta \equiv \eta \left(1-\frac{1}{2}\lambda^2\right) = 0.343 \pm 0.011$$

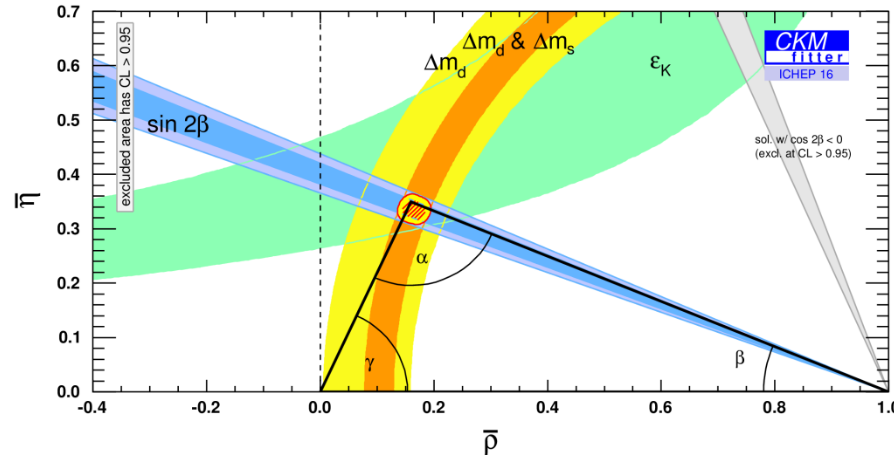
$$\bar\rho \equiv \rho \left(1-\frac{1}{2}\lambda^2\right) = 0.153 \pm 0.013$$

$\alpha = 91.0 \pm 2.5^\circ$; $\beta = 23.2 \pm 1.2^\circ$; $\gamma = 65.3 \pm 2.0^\circ$

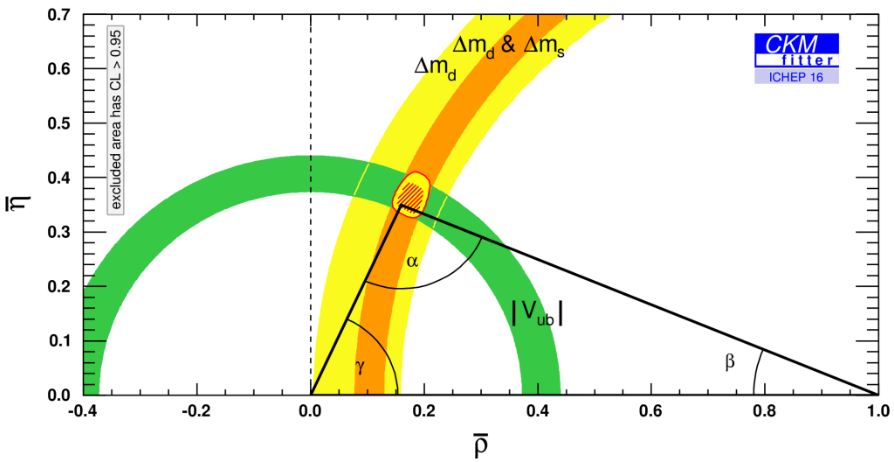
Tree-level determinations



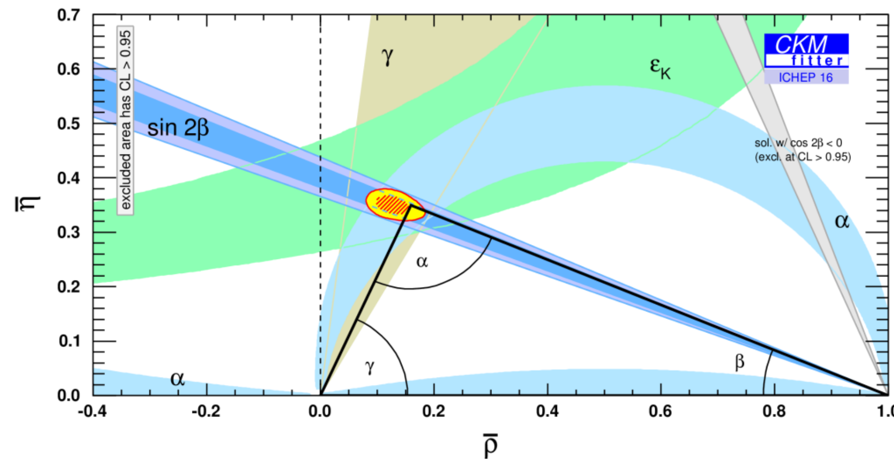
Loop processes



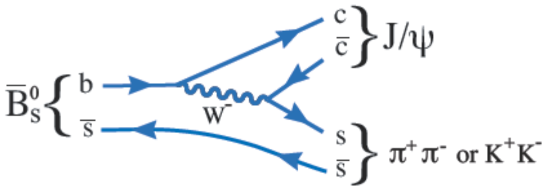
CP Conserving



CP Violating

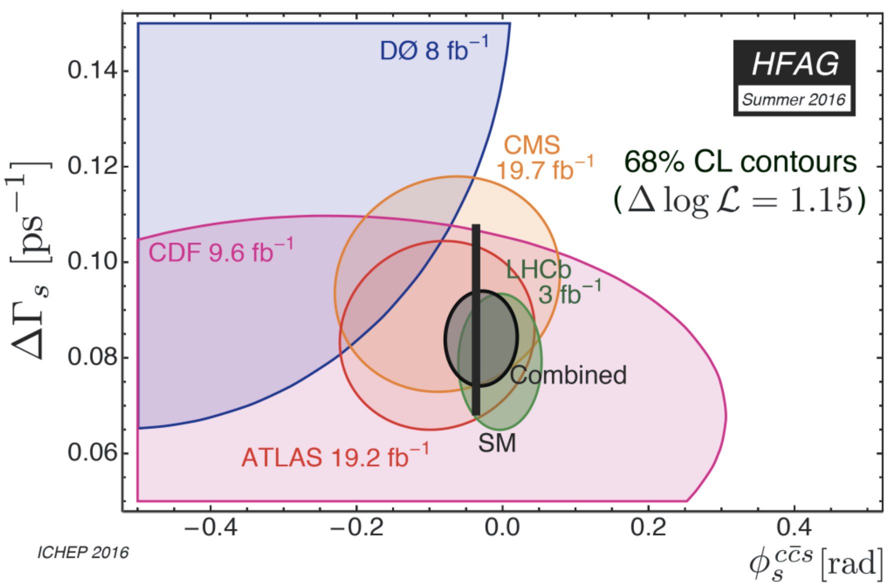


B_s Asymmetries



$$\phi_s^{c\bar{c}s} \equiv 2\left(\phi_s^M + \phi_s^D\right)$$

$$\phi_s^{c\bar{c}s}\Big|_{SM} \approx -2\beta_s \equiv -2\,\arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right)$$

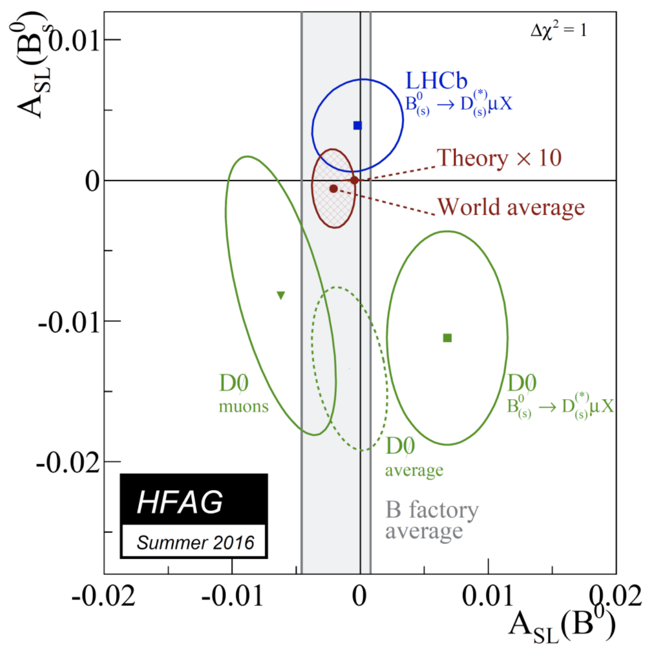


$$\Delta\Gamma_s = \left(0.084 \pm 0.007\right)\ \text{ps}^{-1} \quad , \quad \phi_s^{c\bar{c}s} = \left(-0.030 \pm 0.033\right)\ \text{rad}$$

$$A_{SL}(B_q^0) \equiv \frac{\Gamma(\bar{B}_q^0 \to \ell^+ X) - \Gamma(B_q^0 \to \ell^- X)}{\Gamma(\bar{B}_q^0 \to \ell^+ X) + \Gamma(B_q^0 \to \ell^- X)} = \frac{|p|^4 - |q|^4}{|p|^4 + |q|^4}$$

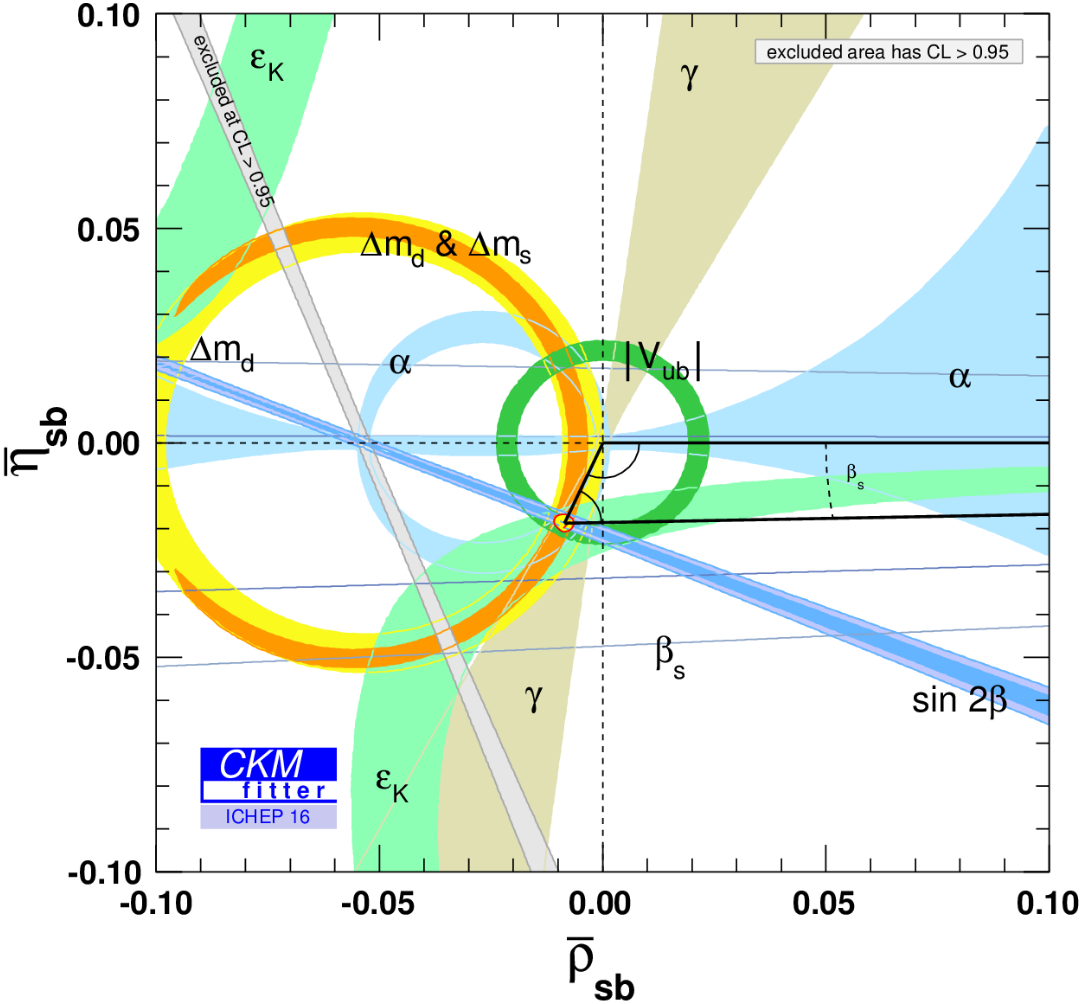
$$\approx 4\,\mathrm{Re}(\bar{\varepsilon}_{B_q^0}) \approx \frac{|\Gamma_{12}|}{|M_{12}|}\sin\phi_q \approx \frac{|\Delta\Gamma_{B_q^0}|}{|\Delta M_{B_q^0}|}\tan\phi_q$$

$$\phi_q \equiv \arg\left(-M_{12}^q/\Gamma_{12}^q\right) \sim \frac{m_c^2}{m_b^2}$$



$$A_{SL}(B_d^0) = -\,0.0021 \pm 0.0017 \quad , \quad A_{SL}(B_s^0) = -\,0.0006 \pm 0.0028$$

$$\underbrace{V_{us}V_{ub}^{*}}_{\lambda^4} + \underbrace{V_{cs}V_{cb}^{*}}_{\lambda^2} + \underbrace{V_{ts}V_{tb}^{*}}_{\lambda^2} = 0$$



$$\bar{\rho}_{sb} + i\,\bar{\eta}_{sb} = -V_{us}V_{ub}^{*}/V_{cs}V_{cb}^{*}$$

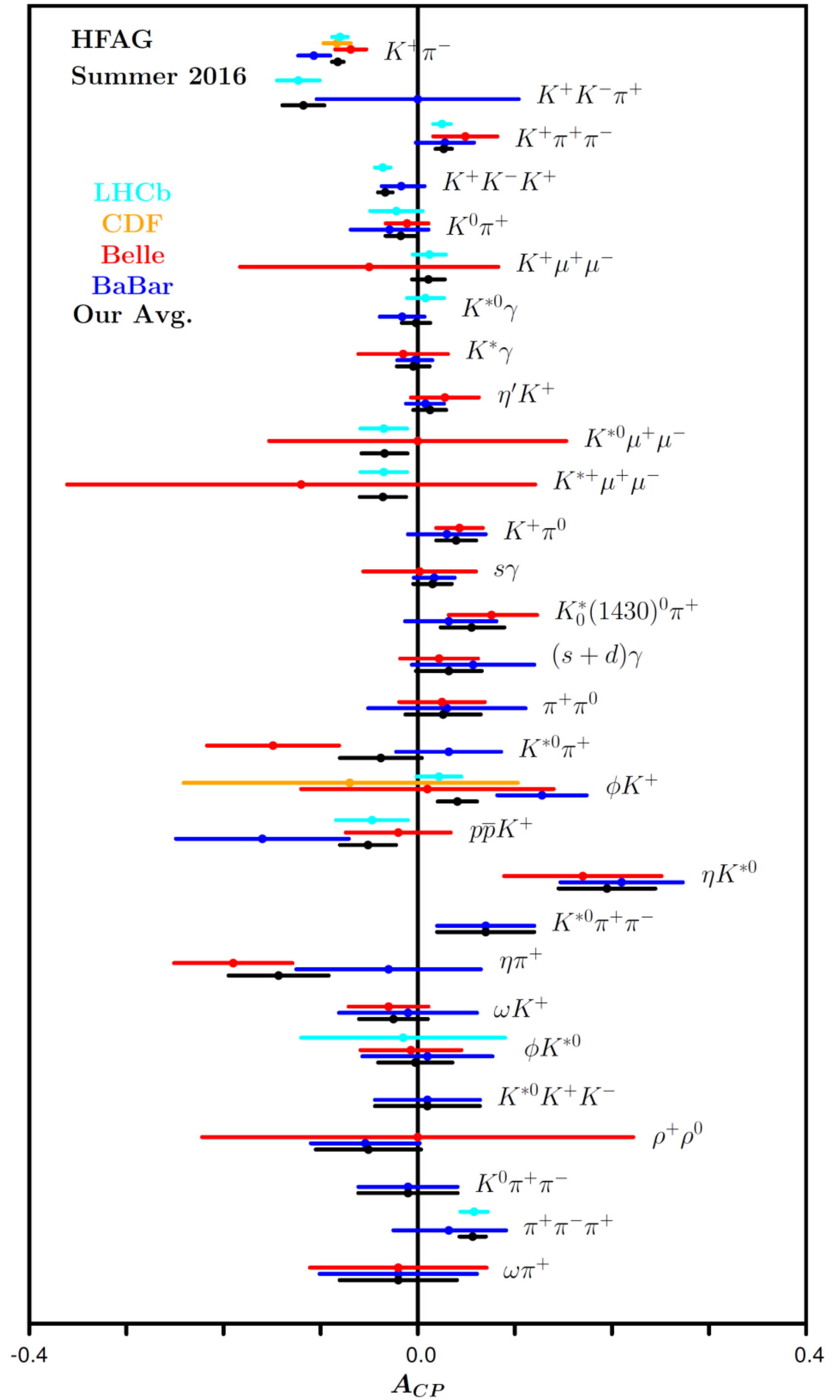
DIRECT $\not{CP}$

$$A_{CP}(B \to f) \equiv \frac{\mathrm{Br}(\bar{B} \to \bar{f}) - \mathrm{Br}(B \to f)}{\mathrm{Br}(\bar{B} \to \bar{f}) + \mathrm{Br}(B \to f)}$$

$$A_{CP}(B_d^0 \to \pi^- K^+) = -0.082 \pm 0.006 \qquad \textbf{(13.7 σ)}$$

$$A(B_s^0 \to \pi^- K^+) = -0.26 \pm 0.04 \qquad \textbf{(6.5 σ)}$$

$$A_{CP}(B^+ \to K^+ K^- \pi^+) = -0.118 \pm 0.022 \qquad \textbf{(5.4 σ)}$$



Large & Interesting Signals

Big challenge: Get reliable SM predictions

Severe hadronic uncertainties