Cosmology and Particle Physics

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IFT-UNESP & ICTP-SAIFR & LIneA

- Lecture I: The average Universe
- Lecture II: Origins
- Lecture III: The perturbed Universe

9th CLASHEP

San Juan del Rio, Mexico, 8–21 March 2017

Plan:

I.0 – Introduction

I.1 – Brief review of GR

I.2 – Dynamics of the Universe

I.3 – Thermal history of the Universe

"Our whole universe was in a hot dense state
Then nearly fourteen billion years ago expansion started"

Shameless advertising

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05. Mar 2017

UPCOMING SCHOOLS AND ACTIVITIES (COMPLETE LIST)

Jan. 16 - Jan. 27 VI Southern-Summer School on Mathematical Biology

Jan. 30 - Feb. 10 School on Interaction of Light with Cold Atoms

Mar. 08 Papos de Física
13 maneiras de morrer com um buraco negro

Mar. 13 - Mar. 24 Codello Minicourse on Effective Field Theory of Gravity

Mar. 13 - Mar. 24 School on Biological Soft Matter

Mar. 27 - Apr. 07 School on AdS/CMT Correspondence

Apr. 17 - Apr. 20 Workshop on Solitons: Integrability, Duality and Applications

May 10 - May 12 South American Dark Matter Workshop
Registration deadline: March 20, 2017

May 15 - Jun 16 Simons Program on the Non-perturbative Bootstrap

May 22 - May 29 Simons Non-perturbative Bootstrap School
Application deadline: March 17, 2017

Jul. 10 - Jul. 15 II IFT-Perimeter-SAIFR Journeys into Theoretical Physics
Application deadline: April 28, 2017

Jul. 15 Prêmio Jovens Físicos 2017
Application deadline: July 7, 2017

Jul. 17 - Jul. 28 School on Open Problems in Cosmology
Application deadline: May 19, 2017

Jul. 31 - Aug. 02 IV CosmoSul – Cosmology and Gravitation in the Southern Cone
Registration deadline: May 26, 2017

APPLICATIONS FOR...

- Simons-FAPESP Professor Positions
- Postdoctoral Positions
- Science Journalism Fellowship
- IT Internship
- Scientific Visits
- Proposals to Organize 2018 Activities

RECENT NEWS

On November 6, three renowned physicists presented seminars to the general public on "100 years of General Relativity: The enduring legacy of Albert Einstein" (David Gross - 2004 Nobel Prize in Physics), "Detecting gravitational waves from black holes" (Gabriela González - spokesperson for LIGO gravitational wave collaboration) and "Explorando as sutilezas do mundo quântico: De Einstein e Schrödinger à Informação Quântica" (Luiz Davidovich - president of Brazilian Academy of Science).

ICTP-Trieste/ICTP-SAIFR School on Open Problems in Cosmology

July 17 – 28, 2017

São Paulo, Brazil

ICTP-SAIFR/IFT-UNESP

FAPESP ICTP unesp IFT

Home Lecturers Registration

The primary temperature anisotropies of the cosmic microwave background (CMB) have now been mapped by the Planck satellite to near the limits set by cosmic variance and foregrounds. As a result, the future of the field will now shift toward large-scale structure (LSS) surveys and CMB polarization. This new territory presents both a challenge and an opportunity. While data from the Sloan Digital Sky Survey and smallscale CMB experiments like ACTpol and SPTpol have been available for a few years, these experiments have yet to compete with measurements of the CMB temperature in raw sensitivity. Ultimately, new experiments must surpass existing constraints if we are going to address unresolved questions we face today in regards to the physics of the early universe. Existing data supports the framework of inflation at early times, nevertheless, we know very little about the mechanism that leads to an accelerated expansion, either in the past or in our current universe. In addition, motivations from particle physics, and from the observation of late time acceleration, suggest our cosmological history may be more intricate than previously thought. Cosmological observations may be thus our only hope to address these issues.

The aim of the school is to familiarize students with both the important theoretical questions left in cosmology and the observations that may shed light on them in the future. The first week will develop the theoretical background, while in the second week future observational probes are discussed.

On Saturday, July 22, ICTP-SAIFR will host the workshop on open problems in cosmology. Presentations will provide a review of the newest developments in both theoretical tools, forecasts and observations. The discussion will be oriented towards the problems that are most relevant for further progress.

The application for the school automatically includes participation in the one0day workshop. *There is no registration fee and limited funds are available for travel and local expenses.*

The School will be followed by the "IVCosmoSul – Cosmology and Gravitation in the Southern Cone". If you would like to register for the workshop, please access http://www.ictp-saifr.org/IVCosmoSul

Organizers:

- Daniel Baumann (University of Amsterdam, The Netherlands)
- Paolo Creminelli (ICTP-Trieste, Italy)
- Daniel Green (UC Berkeley, USA)
- Rafael Porto (ICTP-SAIFR/IFT-UNESP, Brazil)
- Matias Zaldarriaga (IAS Princeton, USA)

I.0- Introduction

Why should a particle physicist learn cosmology?

• main evidences from BSM comes from cosmology: Dark matter, dark energy, inflation;

• particle physics affect cosmology: eg origin of matter-anti-matter asymmetry, Higgs as inflaton, neutrinos and the formation of structures, phase transitions;

• cosmology affects particle physics: eg evolution of the Universe may be responsible for electroweak symmetry breaking (relaxion idea).

- early Universe is a testbed for SM and BSM: stability or metastability of SM vacuum;

- gravity (geometry) may play an important role in particle physics: eg models with warped extra dimensions

- new particles from geometry: KK excitations, radion, etc

- Models with extra dimensions can change the evolution of the Universe (and hence be tested).

Standard Model of Particle Physics works fine but it is unsatisfactory (neutrino masses, dark matter, hierarchy problem, etc). Beyond SM!

Standard Model of Cosmology (ΛCDM) works fine but it is unsatisfatory (value and nature of Λ). Beyond ΛCDM!

Models abound! We have to see what Nature has chosen...

Cosmology has recently become a data driven science!

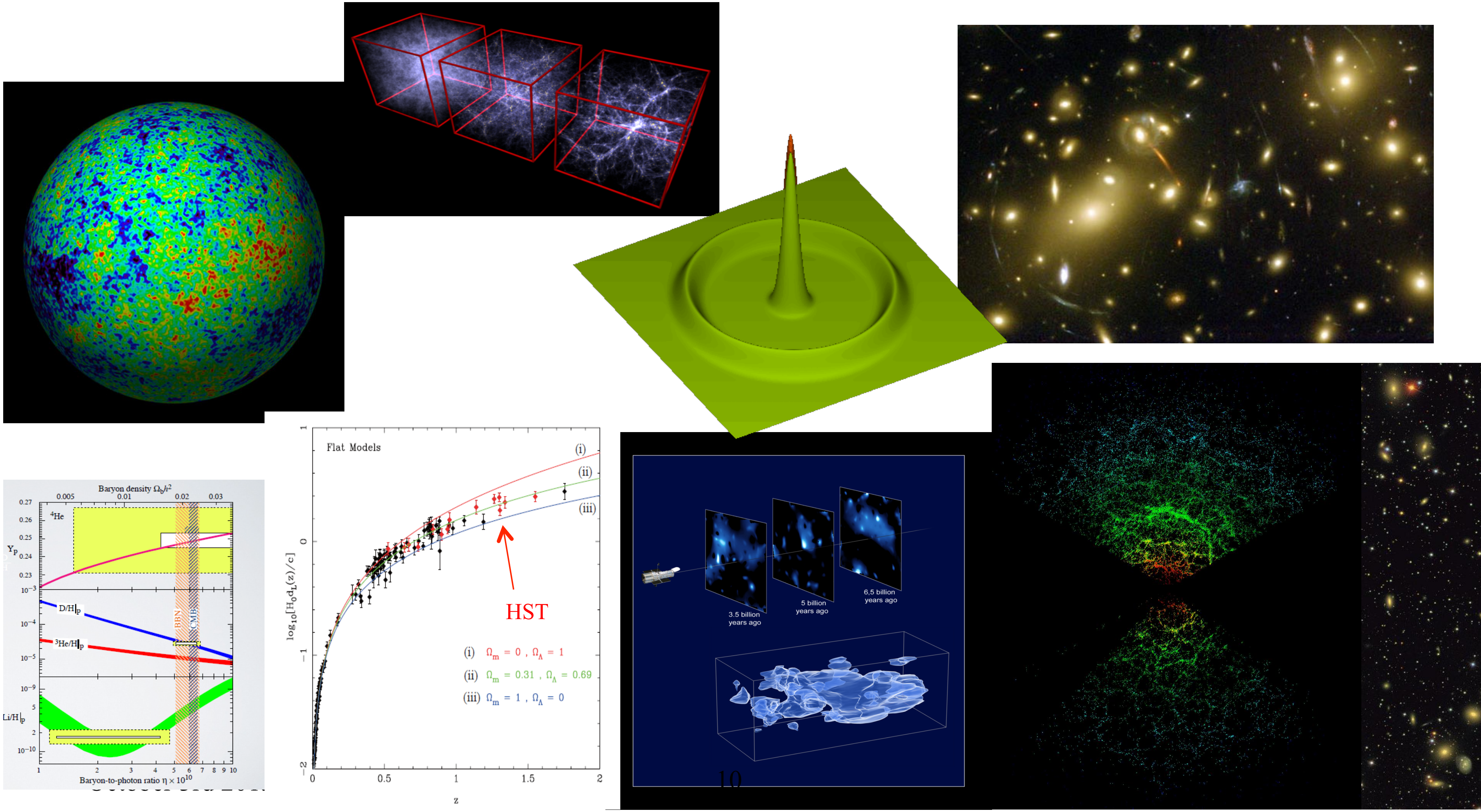
$t_U = (13.799 \pm 0.021) \times 10^9$ years [used to be $10^{9\pm1}$ years]

Many experiment are taking a huge amount of data that are being analyzed in order to find out which model best describes the universe.

Cosmological probes

- Cosmic Microwave Background (CMB)
- Big bang nucleosynthesis (BBN)
- Supernovae (type Ia)
- Baryon acoustic oscilation (BAO)
- Gravitational lensing
- Number count of clusters of galaxies

Cosmological probes



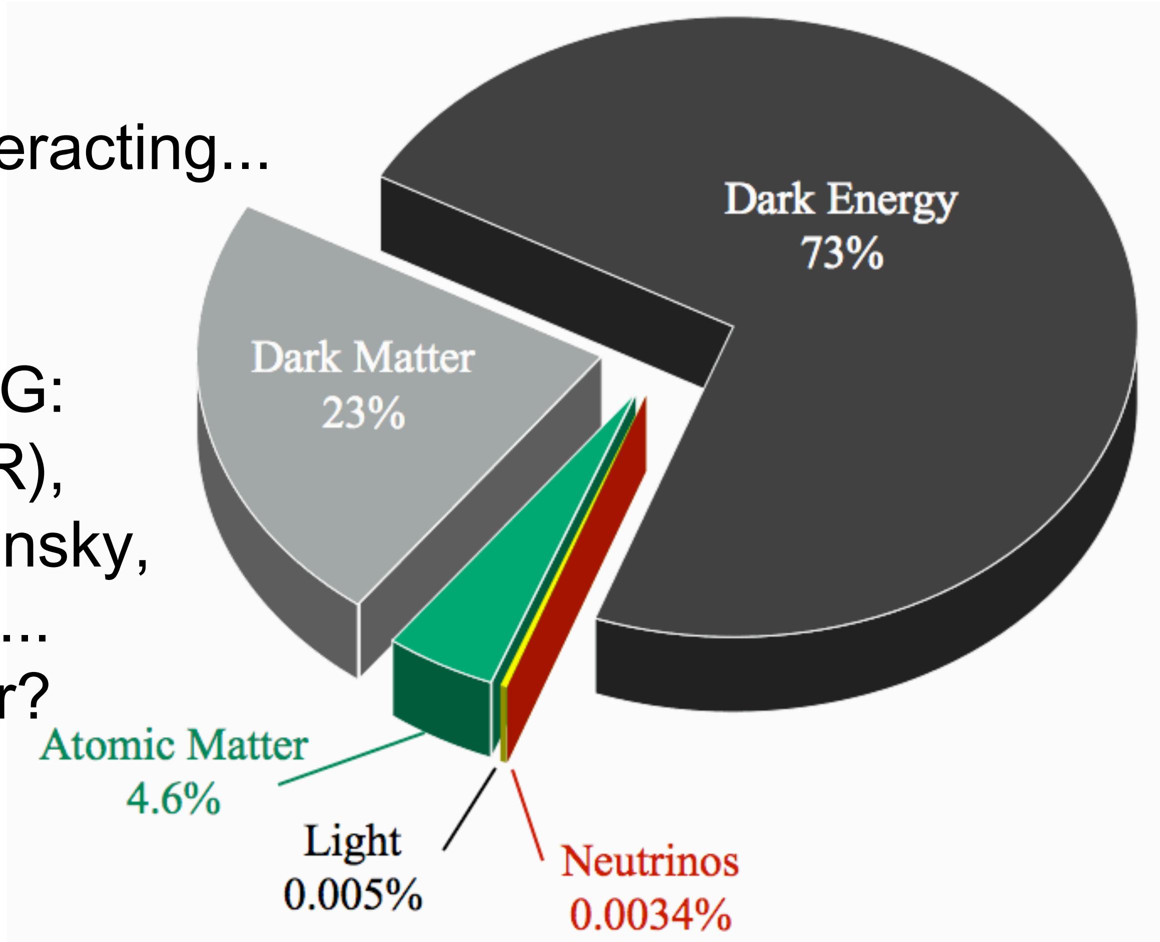
We know that we don't know what 95% of the Universe is made of:

What is dark matter?

Cold, warm, fuzzy, self-interacting...

What is dark energy?

New degree of freedom/MG:
Quintessence, galileon, f(R), Hordensky, beyond Hordensky, massive gravity, EFTofDE...
Does it interact with matter?
Does it cluster?



I.1- Brief Review of GR

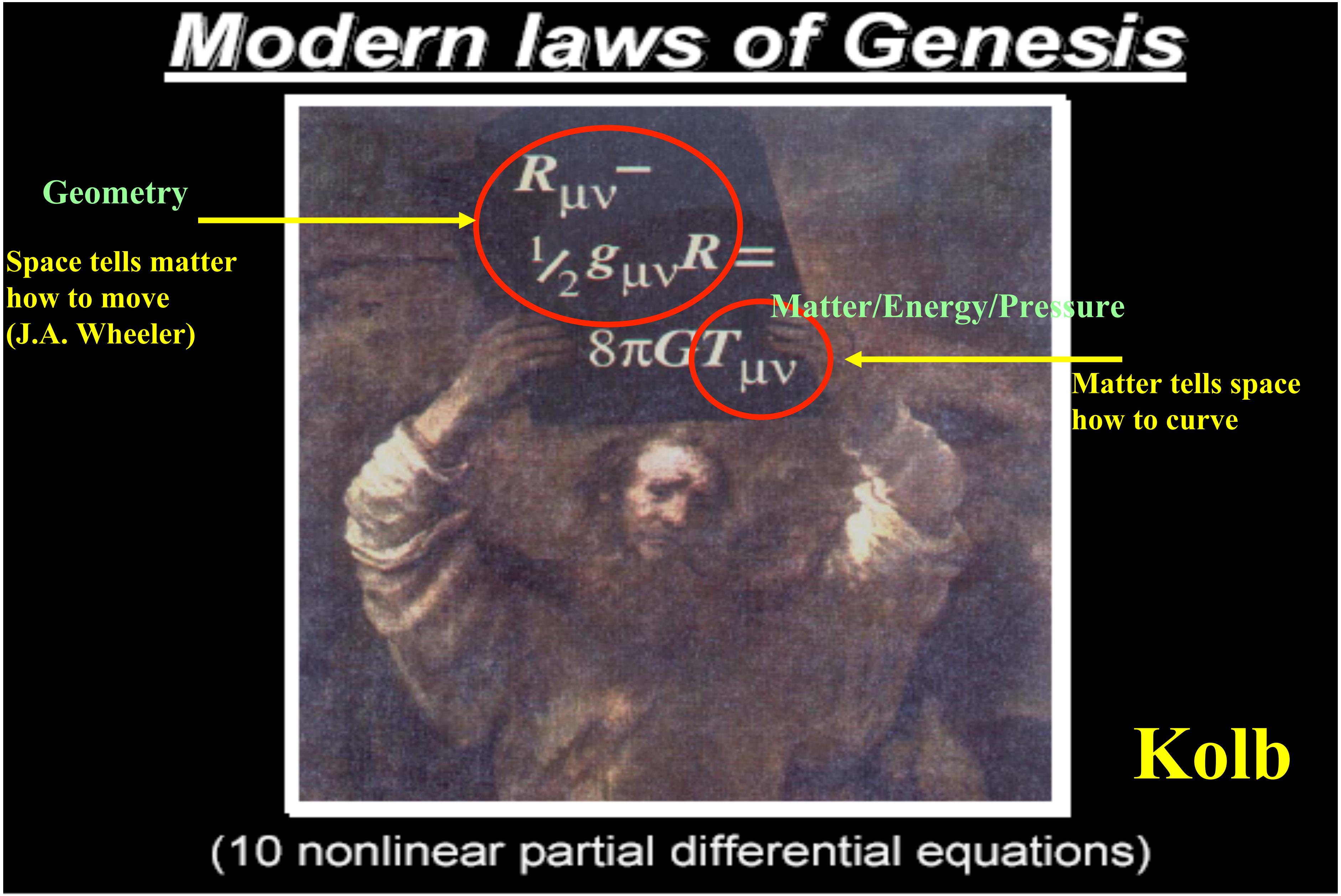
General Relativity rules the Universe at large scales

I.1.1 – Einstein's equation

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

10 nonlinear differential equations
Numerical GR, eg gravitational waves from coalescence of binary black holes

Standard Cosmological Model



Details of Einstein's equation:

- G: Newton's constant

$$G = \frac{1}{M_{\rm Pl}^2} \quad (\hbar = c = 1)$$

$$M_{\rm Pl} = 1.2 \times 10^{19}\ {\rm GeV}$$

Obs.: sometimes the *reduced* Planck mass is used:

$$\tilde{M}_{\rm Pl} = \frac{M_{\rm Pl}}{\sqrt{8\pi}} = 2.4 \times 10^{18}\ {\rm GeV}$$

- Metric:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \qquad g_{\mu\alpha} g^{\alpha\nu} = \delta_\mu^\nu$$

$$g_{\mu\nu} g^{\mu\nu} = 4$$

Details of Einstein's equation:

- Christoffel symbols (aka metric conection, affine connection) – first derivative of the metric :

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left\{ \frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} + \frac{\partial g_{\beta\nu}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}} \right\}$$

- Ricci tensor – second derivative of the metric:

$$R_{\mu\nu} = \frac{\partial}{\partial x^{\alpha}} \Gamma^{\alpha}_{\mu\nu} - \frac{\partial}{\partial x^{\nu}} \Gamma^{\alpha}_{\alpha\mu} + \Gamma^{\alpha}_{\mu\nu} \Gamma^{\beta}_{\alpha\beta} - \Gamma^{\beta}_{\alpha\mu} \Gamma^{\alpha}_{\beta\nu}$$

- Ricci scalar: $R = g^{\mu\nu} R_{\mu\nu}$

I.1.2 – Einstein-Hilbert action

$$S_{\mathrm{E-H}}[g_{\mu\nu}] = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, R[g_{\mu\nu}]$$

- Action is invariant under general coordinate transformations:

$$x^\mu \to x'^\mu(x^\mu)$$

- $g = \det(g_{\mu\nu})$

I.1.2 – Einstein-Hilbert action

- Dimensional analysis: $(\hbar = c = 1)$

$$[g] : \text{ dimensionless; } \quad [R] : \; E^2$$
$$[d^4x] : E^{-4}; \quad [S] : \text{ dimensionless}$$
$$\Rightarrow \boxed{[G] : E^{-2}}$$
$$G = \frac{1}{M_{\mathrm{Pl}}^2}$$

- Einstein equation in vacuum obtained from:

$$\frac{\delta S_{\mathrm{E-H}}}{\delta g_{\mu\nu}} = 0$$

I.1.3 – The cosmological constant

February 1917 (100 years ago): “Cosmological Cosniderations in the General Theory of Relativity” introduces the cosmological constant in the theory without violating symmetries:
a new constant of Nature!

It has an “anti-gravity” effect (repulsive force) and it was introduced to stabilize the Universe.

$$S_{\mathrm{E-H}} + S_{\Lambda} = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G} - \Lambda \right)$$

With the discovery of the expansion of the Universe (Hubble, 1929) it was no longer needed – “my biggest blunder”.

43. "Cosmological Considerations in the General Theory of Relativity"

[*Einstein 1917b*]

SUBMITTED 8 February 1917
PUBLISHED 15 February 1917

Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie.

Von A. Einstein.

Es ist wohlbekannt, daß die Poissonsche Differentialgleichung

$$\Delta\phi = 4\pi K\rho \qquad (1)$$

in Verbindung mit der Bewegungsgleichung des materiellen Punktes die Newtonsche Fernwirkungstheorie noch nicht vollständig ersetzt. Es muß noch die Bedingung hinzutreten, daß im räumlich Unend-

IN: *Königlich Preußische Akademie der Wissenschaften* (Berlin). *Sitzungsberichte* (1917): 142–152.

§ 4. On an Additional Term for the Field Equations of Gravitation

Poisson's equation given by equation (2). For on the left-hand side of field equation (13) we may add the fundamental tensor $g_{\mu\nu}$, multiplied by a universal constant, $-\lambda$, at present unknown, without destroying the general covariance. In place of field equation (13) we write

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa(T_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} T) \quad . \quad . \quad (13a)$$

George Gamow – My Worldline

correct, and changing it was a mistake. Much later, when I was discussing cosmological problems with Einstein, he remarked that the introduction of the cosmological term was the biggest blunder he ever made in his life. But this "blunder," rejected by Einstein, is still sometimes used by cosmologists even today, and the cosmological constant denoted by the Greek letter Λ rears its ugly head again and again and again.

I.1.4 – Modified gravity

Modified gravity is anything different from E-H (+ Λ) action (see 1601.06133)

Example: f(R) theories (see 1002.4928)

$$S[g_{\mu\nu}] = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\, f(R)$$

$$f(R) = a_0 + a_1 R + a_2 R^2 + \cdots + \frac{\alpha_1}{R} + \frac{\alpha_2}{R^2} + \cdots$$

cosmological constant

GR

Higher order derivatives

Issues with modified gravity:

• introduces new light degrees of freedom – new forces
Since there are stringent constraints one has to invoke “screening mechanisms” – chameleon, symmetron, Vainshtein, ...

• may have classical instabilities due to higher derivatives in equations of motion (Ostrogadski instabilities)

• may have quantum instabilities – “ghosts”

• may be brought in the form of GR with a suitable change of coordinates (Jordan frame -> Einstein frame) introducing non-standard couplings in the matter sector

• search for MG: use simple parametrizations (more later)

I.1.5 – Adding matter to the action

$$S_{\mathrm{matter}} = \int d^4x \sqrt{-g}\, \mathcal{L}_{\mathrm{matter}}$$

Examples:

Electromagnetism: $$S_{\mathrm{EM}} = -\frac{1}{4} \int d^4x \sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta}$$

Real scalar field: $$S_\phi = \int d^4x \sqrt{-g}\, \left[\frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - V(\phi) \right]$$

I.1.6 – Energy-momentum tensor

Definition:

$$\delta S_{\mathrm{matter}} = \frac{1}{2}\int d^4x\sqrt{-g}\, T^{\mu\nu}(x)\delta g_{\mu\nu}$$

which implies

$$T^{\mu\nu}(x) = \frac{2}{\sqrt{-g}}\frac{\delta S_{\mathrm{matter}}}{\delta g_{\mu\nu}}$$

Exercise 1: Show that for a real scalar field

$$T_\phi^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi - \mathcal{L}_\phi g^{\mu\nu}$$

Exercise 2: Show that for a cosmological constant

$$T_\Lambda^{\mu\nu} = \Lambda g^{\mu\nu}$$

Finally, Einstein equation for GR is obtained from the requirement:

$$\delta\left(S_{\mathrm{total}}\right) = \delta\left(S_{\mathrm{E-H}} + S_{\Lambda} + S_{\mathrm{matter}}\right) = 0$$

I.2- Dynamics of the Universe

I.2.1 – Friedmann-Lemaître-Robertson-Walker

Universe is spatially homogeneous and isotropic on average.

It is described by the FLRW metric (for a spatially flat universe):

$$ds^2 = dt^2 - a(t)^2 \left[dx^2 + dy^2 + dz^2\right]$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -a^2 & 0 & 0 \\ 0 & 0 & -a^2 & 0 \\ 0 & 0 & 0 & -a^2 \end{pmatrix}$$

FLRW metric is determined by one time-dependent function:
the so-called scale factor $a(t)$.
Distances in the universe are set by the scale factor.

Scale factor is the key function to study how the average universe evolves with time.

convention: $a=1$ today
physical distances: $d(t) = a(t)\, d_0$

Average evolution of the universe

- specified by <u>one function</u>: scale factor a(t)
- determines measurement of large scale distances, velocities and acceleration

$$a(t) \qquad \dot{a}(t) \qquad \ddot{a}(t)$$

- measured through standard candles (SNIa's) and standard rulers (position of CMB peak, BAO peak,...)

Redshift z: $$a(t) = \frac{1}{1+z}$$ z=0 today.

Expansion of the universe

Hubble parameter: $$H = \frac{\dot{a}(t)}{a(t)}$$

Expansion rate of the universe

Hubble constant: Hubble parameter today (H_0)

Analogy of the expansion of the universe with a balloon:

Space itself expands and galaxies get a free "ride".

Exercise 3: Show that for a spatially flat FLRW metric the Ricci tensor and the Ricci scalar are given by:

$$R_{00} = -3\frac{\ddot{a}}{a}; \; R_{ii} = a\ddot{a} + 2a^2;$$

$$R = -6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$$

I.2.2 – The right-hand side of Einstein equation: the energy-momentum tensor simplified

It is usually assumed that one can describe the components of the Universe as “perfect fluids”: at every point in the medium there is a locally inertial frame (rest frame) in which the fluid is homogeneous and isotropic (consistent with FLRW metric):

$$T^{00} = \rho(t); \;\; T^{ij} = \delta^{ij} P(t); \;\; T^{0i} = 0$$

density

pressure

isotropy

Homegeneity: density and pressure depend only on time.

Energy-momentum in the rest frame (indices are important):

$$T^{\mu}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}$$

In a frame with a given 4-velocity:

$$T^{\mu\nu} = -P g^{\mu\nu} + (\rho + P) u^{\mu} u^{\nu}$$

$$u^{\mu} = \gamma\,(1, \vec{v})$$

[Imperfect fluids: anisotropic stress, dissipation, etc.]

I.2.3 – Solving Einstein equation for the average Universe: Friedmann's equations

00 component:

$$R_{00} - \frac{1}{2} g_{00} = 8\pi G T_{00} \implies$$

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho}$$

1st Friedmann equation

ii component:

$$R_{ii} - \frac{1}{2} g_{ii} = 8\pi G T_{ii} \implies$$

$$\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\ddot{a}}{a} = -8\pi G P \implies$$

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)}$$

2nd Friedmann equation

I.2.4 – Evolution of different fluids

Exercise 4: Take a time derivative of 1st Friedmann equation to derive the “continuity”equation :

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P) = 0$$

Also follows from the conservation of the energy-momentum tensor and also from the 1st law of thermodynamics:

$$dU = -PdV,\; U = \rho V,\; V \propto a^{-3}$$

In order to study the evolution of a fluid we need to postulate a relation between density and pressure: the equation of state

Assume a simple relation:

$$P = \omega\rho$$

ω is called the equation of state parameter.

Examples:

- Non-relativistic matter (dust): $P \ll \rho \longrightarrow \omega = 0$
- Relativistic matter (radiation): $\omega = 1/3$

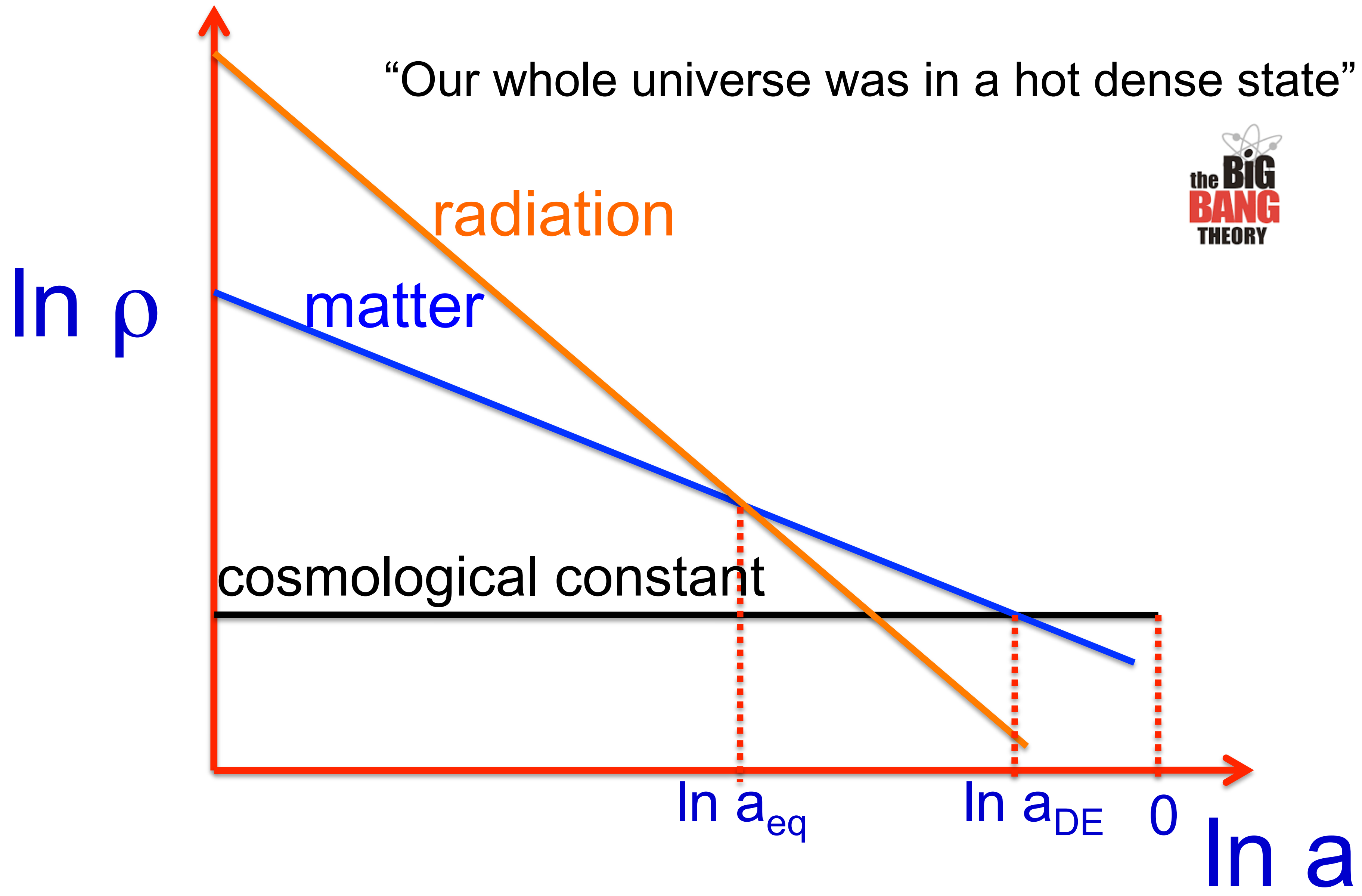
- Cosmological constant: $\omega = -1$

$$T^{\mu}_{\nu,\Lambda} = \begin{pmatrix} \Lambda & 0 & 0 & 0 \\ 0 & \Lambda & 0 & 0 \\ 0 & 0 & \Lambda & 0 \\ 0 & 0 & 0 & \Lambda \end{pmatrix} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}$$

Exercise 5: From the continuity equation show that the evolution of the density for a constant equation of state is:

$$\rho(t) = \rho(t_i) \left(\frac{a(t)}{a(t_i)} \right)^{-3(1+\omega)}$$

- Non-relativistic matter (dust): $\rho \propto a^{-3}$
- Relativistic matter (radiation): $\rho \propto a^{-4}$
- Cosmological constant: $\rho \propto a^{0}$


"Our whole universe was in a hot dense state"
the BIG BANG THEORY
radiation
ln ρ
matter
cosmological constant
ln a_eq
ln a_DE
0
ln a

I.2.5 – Evolution of the scale factor

Exercise 6: Using 1st Friedmann equation and the result from last section:

$$\frac{\dot{a}}{a} \propto \sqrt{\rho}, \;\; \rho \propto a^{-3(1+\omega)}$$

show that:

$$a(t) \propto t^{\frac{2}{3(1+\omega)}} = \begin{cases} t^{2/3} \; (\text{matter}) \\ t^{1/2} \; (\text{radiation}) \end{cases}$$

but for the cosmological constant one has an **exponential growth**:

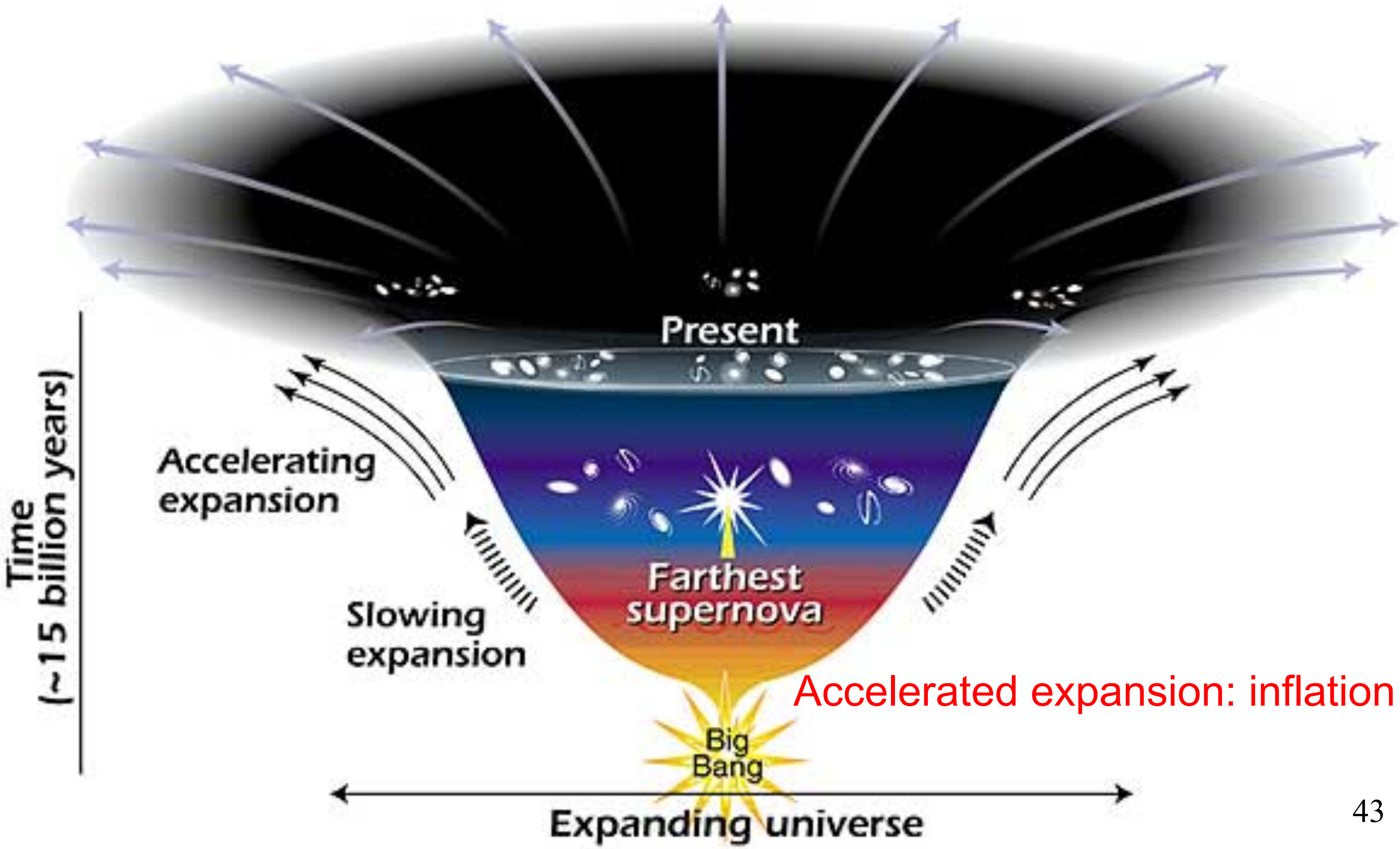
$$\frac{\dot{a}}{a} = \text{const.} = H \rightarrow a(t) \propto e^{Ht}$$

Exponential growth: universe is accelerating!

2nd Friedmann equation is:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) = \frac{8\pi G}{3}\rho > 0$$

The Universe started to accelerate a couple of billion years ago. Before that there was a period of normal decelerated expansion, essential for the formation of galaxies.



Accelerated expansion: inflation

Curiosity: what happens if w<-1 (“phantom” dark energy)?

$$\omega = -1 - \delta \to \rho_{\mathrm{phantom}} \propto a^{-3(1+\omega)} = a^{3\delta}$$

Density increases with time. It can be shown that there is a singularity where $a \to \infty$ at finite time: the “big rip”
[see astro-ph/0302506]

I.2.6 – Recipe of the Universe

Critical density: density at which the Universe is spatially flat.

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

Hubble constant today has been measured with some precision and there is a mild tension:

$$H_0 = (67.8 \pm 0.9)\ \mathrm{km/s/Mpc}\ (\mathrm{Planck})$$

$$H_0 = (72.0 \pm 3)\ \mathrm{km/s/Mpc}\ (\mathrm{HST})$$

Exercise 6: estimate the critical density in units of protons/m^3

Different contributions to the energy density budget of the Universe

$$\Omega_i = \frac{\rho_i}{\rho_c}$$

Spatially flat universe:

$$\sum_i \Omega_i = 1$$

1st Friedmann equation:

$$\frac{H(t)^2}{H_0^2} = \sum_i \Omega_i^{(0)} a^{-3(1+\omega_i)}$$

Exercise 7: given a Universe with

$$\Omega_\Lambda^{(0)} = 0.7,\ \Omega_{\rm matter}^{(0)} = 0.3,\ \Omega_{\rm rad}^{(0)} = 5 \times 10^{-5}$$

compute:

a. H_0^{-1} in units of years
b. the age of the Universe
c. $(\rho_c)^{1/4}$ in unites of eV – energy scale associated with the cosmological constant
d. H_0 in units of eV
e. the redshift z_{eq}
f. the redshift z_Λ

I.2.7 – Beyond Λ: dark energy

For a real homogeneous scalar field the energy-momentum tensor gives:

$$T^{00}_{\phi} = \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi);$$

$$T^{ii}_{\phi} = -g^{ii}P = -\left(\frac{1}{2}\dot{\phi}^2 - V(\phi)\right)g^{ii}$$

and therefore the time-dependent equation of state in this case is:

$$\omega(t) = \frac{P}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 + V(\phi)}{\frac{1}{2}\dot{\phi}^2 - V(\phi)} \Rightarrow -1 \leq \omega \leq 1$$

If potential energy dominates w~-1 and scalar field resembles a cosmological constant: quintessence field. Can be ultralight (~H_0)!

Some examples of dark energy models:

Cosmological Constant $$p_\Lambda = -\rho_\Lambda$$

Canonical Scalar Field:
Quintessence $$\mathcal{L}_Q = \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - V(\varphi)$$

Perfect Fluid $$p_0 = w\rho_0 \qquad \delta p = c_{\text{eff}}^2\delta\rho$$

Chaplygin Gas $$\rho_{Ch} = -A\rho_{Ch}^{-\alpha}$$

K-essence $$\mathcal{L} = F(X,\varphi) \qquad X = \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi$$

e.g. Tachyon,
Born-Infeld $$\mathcal{L}_T = V(\varphi)\sqrt{1-\partial^\mu\varphi\partial_\mu\varphi}$$

I.2.8 – Vacuum energy: the elephant in the room

Quantum mechanics – zero point energy:

$$E = \hbar\omega\left(n + 1/2\right)$$

In Quantum Field Theory, the energy density of the vacuum is (free scalar field of mass m):

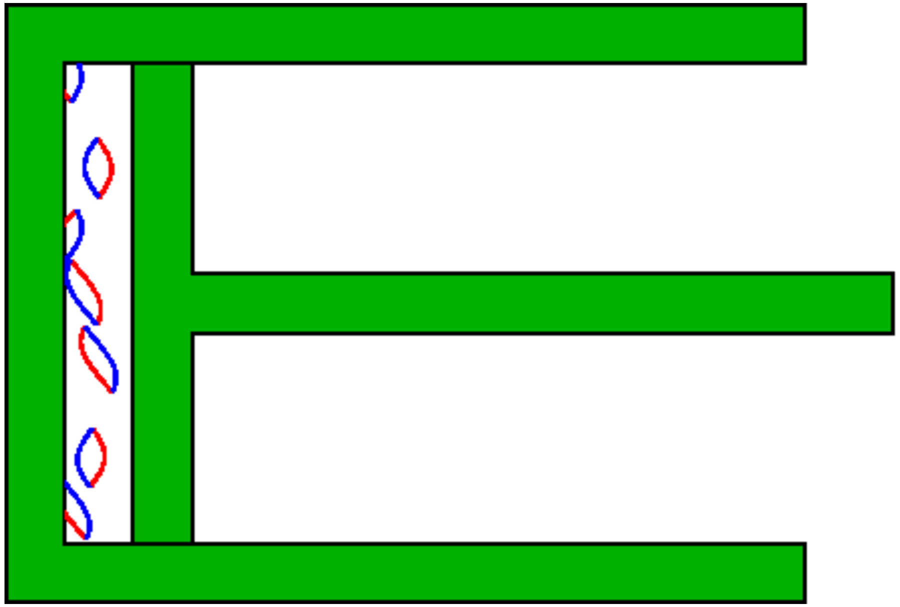
$$\rho_{vac} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2}$$

and is infinite! Integral must be cut-off at some physical energy scale. If integral is cutoff at the Planck scale, disagreement of ~ 10^{120} with data.

This is know as the cosmological constant problem.

Vacuum energy

$$\rho_\Lambda \propto \text{constant}$$



E. L. Wright

$$dE = -pdV \Rightarrow p_\Lambda < 0$$

I.2.9 – Distances in the Universe

There are 2 ways to measure large distances in the Universe:
from known luminosities (standard candles – eg SNIa) or
from known scales (standard rulers – eg BAO).

Let's recall that
physical distance = a(t) comoving distance
and discuss some other typical distances in the Universe.

a. Comoving distance between us (z=0) and an object at redshift z:

$$ds^2 = 0 \Rightarrow dt^2 = a(t)^2 d\chi^2 \Rightarrow$$

$$\boxed{\chi(z) = \int_0^z \frac{dz'}{H(z')}}$$

b. Comoving particle horizon: largest region in causal contact since the Big Bang:

$$\chi(z)_{\rm phys} = \int_0^t \frac{dt'}{a(t')}$$

c. Luminosity distance (d_L):

$$F = \frac{L}{4\pi d_L^2}$$

Flux of photons

Known luminosity of the source

In FLRW there are 2 extra source of dilution of the flux:

- redshift of photons (1/(1+z))
- rate of arrival decrease by (1/(1+z)) – time dilation

Therefore: $$\boxed{d_L = (1+z)\chi(z)}$$

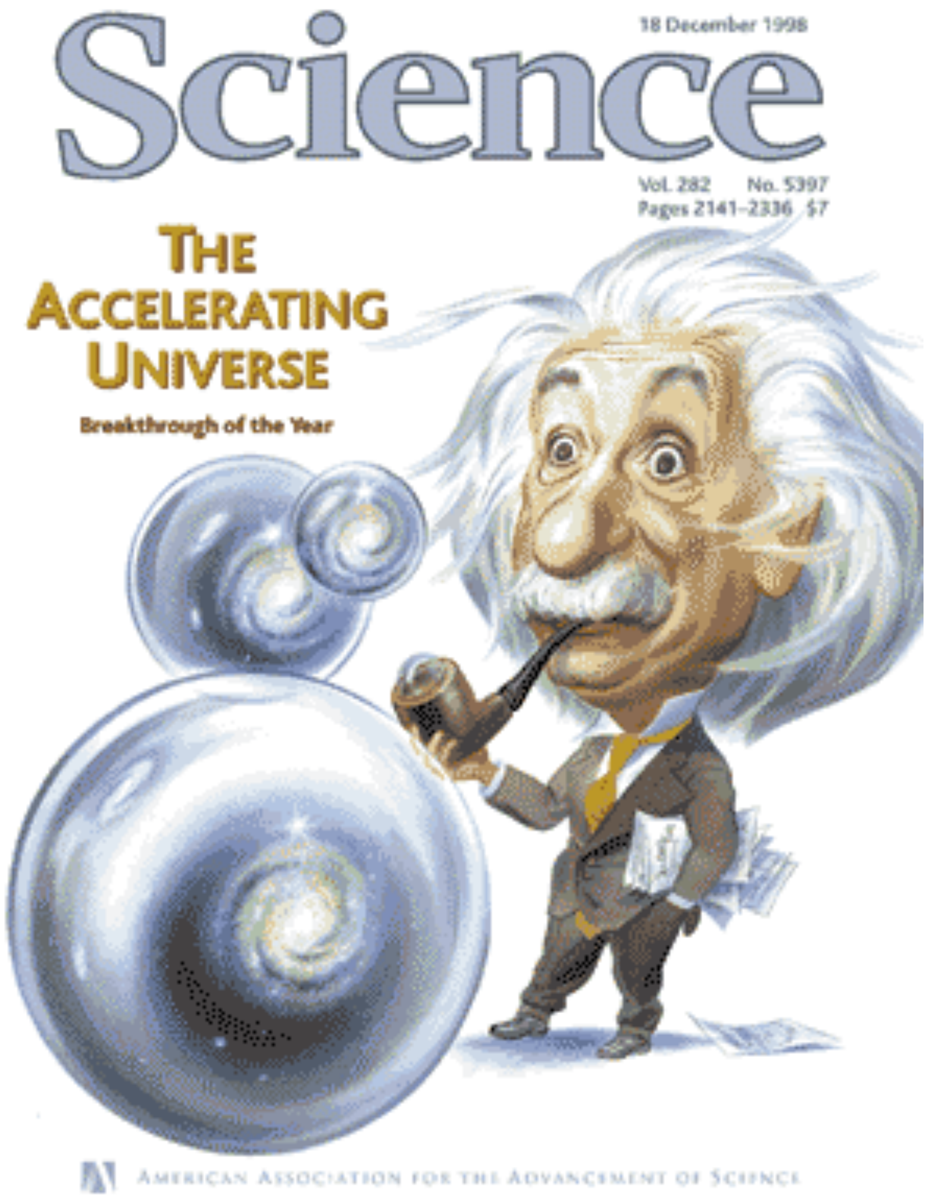
Exercise 8: plot $d_L(z)$ for $0<z<2$ for a flat Universe with

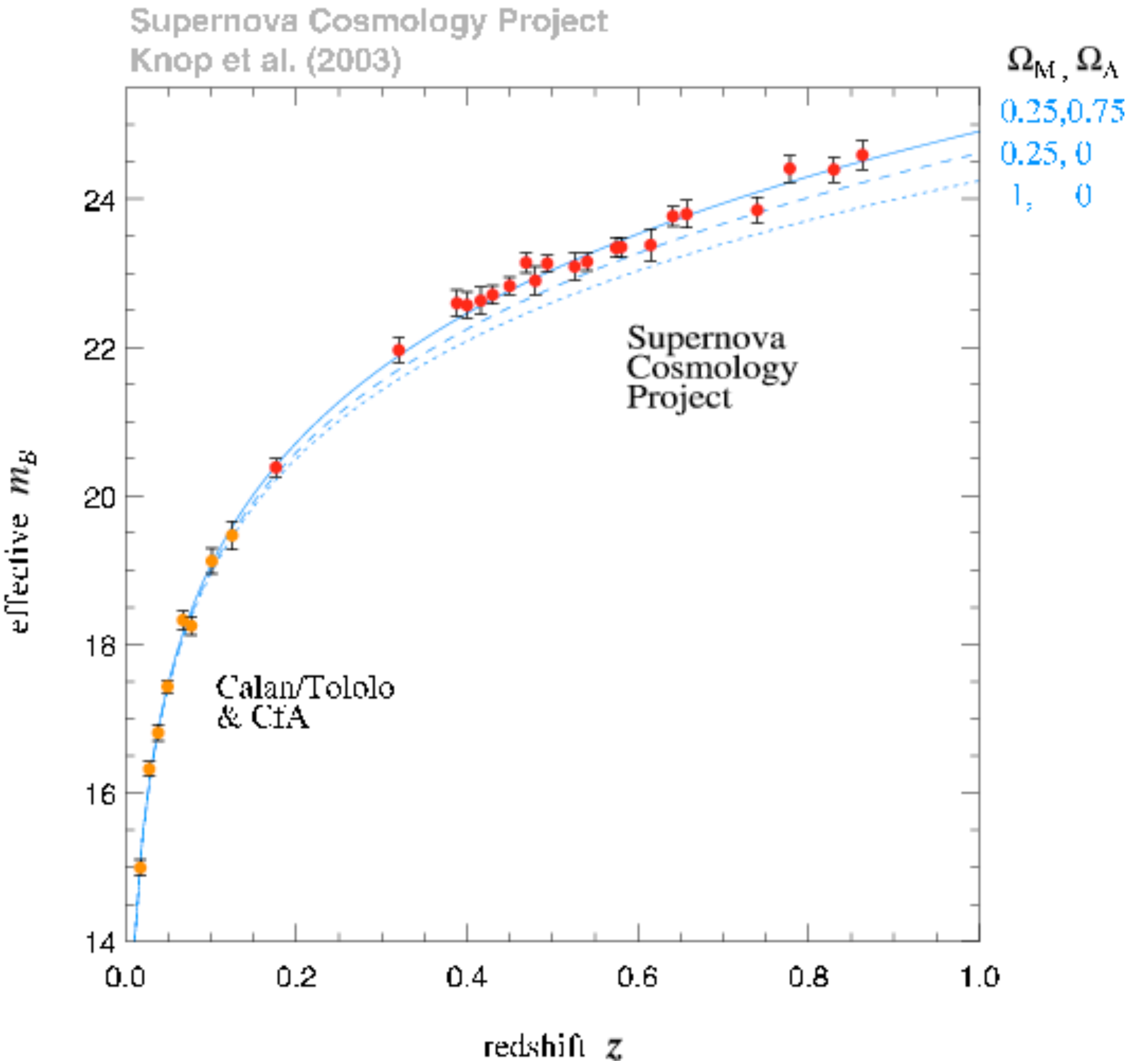
α. $\Omega_m = 1$ and $\Omega_\Lambda = 0$
β. $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$

d_L is larger for a Universe with Λ -> objects with same z look fainter

This is how the accelerated expansion of the Universe was discovered in 1998 using SNIa

The big surprise in 1998:





Supernova Cosmology Project
Knop et al. (2003)
Ω_M, Ω_Λ
0.25,0.75
0.25, 0
1, 0
Supernova Cosmology Project
Calan/Tololo & CfA
effective m_B
redshift z
14
16
18
20
22
24
0.0
0.2
0.4
0.6
0.8
1.0

The Nobel Prize in Physics 2011

Saul Perlmutter, Brian P. Schmidt, Adam G. Riess

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The Nobel Prize in Physics 2011

Photo: U. Montan

Saul Perlmutter

Prize share: 1/2

Photo: U. Montan

Brian P. Schmidt

Prize share: 1/4

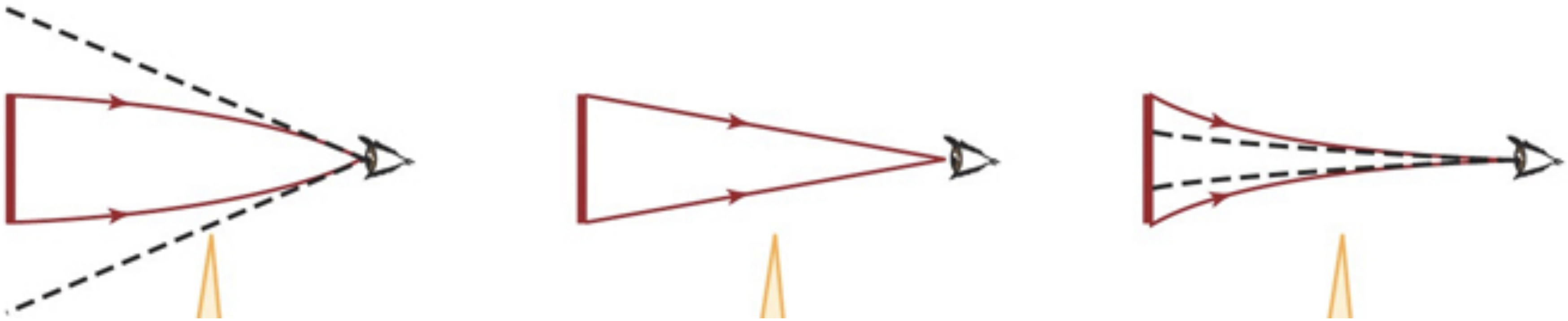
Photo: U. Montan

Adam G. Riess

Prize share: 1/4

The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae".*

d. Angular diameter distance (d_A):
related to the angle subentended by a physical lenght (l)



Closed Universe Flat Universe Open Universe

$$d_A = \frac{l}{\delta\theta}, \; l = a\chi\delta\theta \Rightarrow$$

$$\boxed{d_A = \frac{1}{1+z}\chi(z)}$$

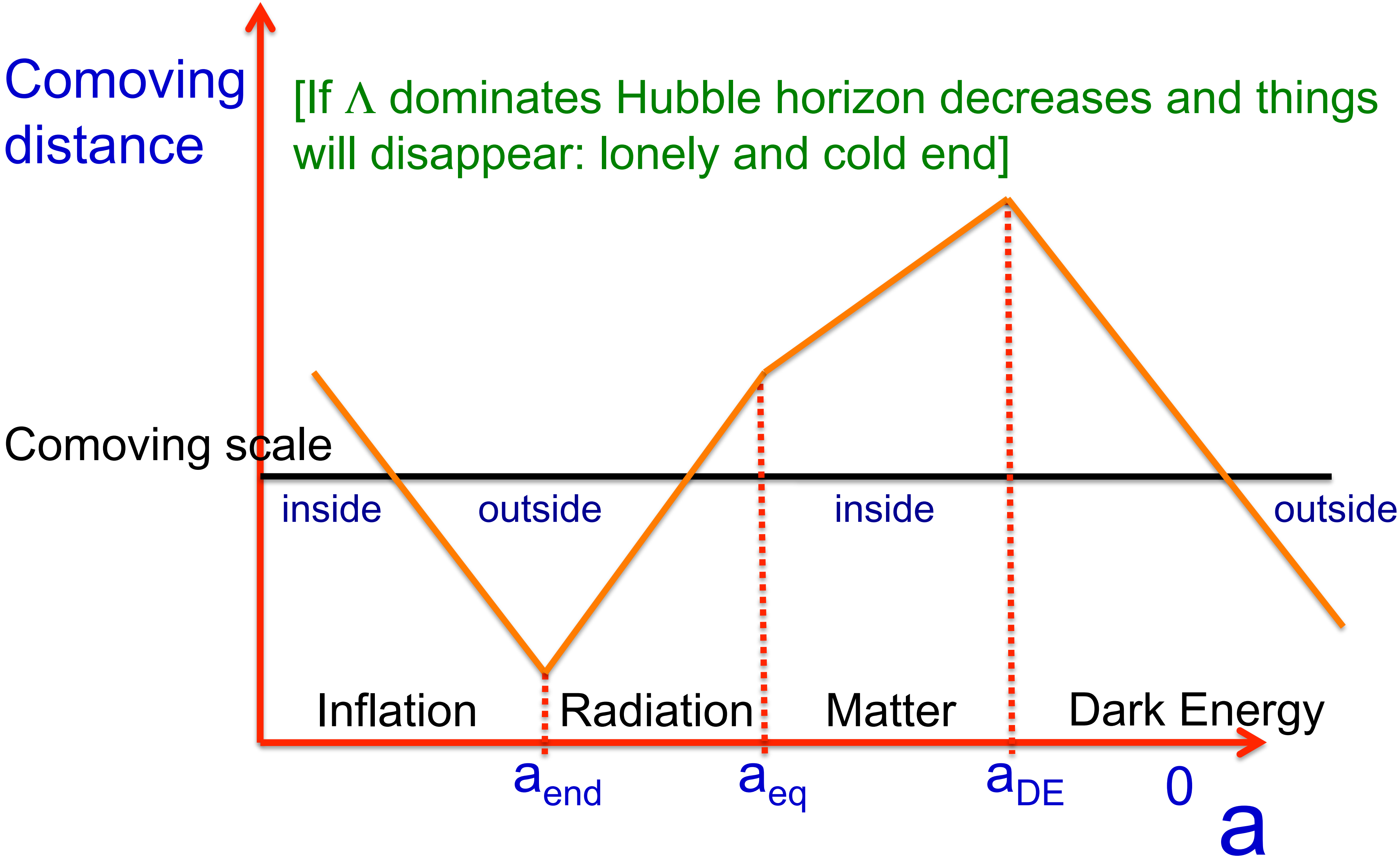
e. Hubble radius: distance particles can travel in a Hubble time

$$R_H = \frac{1}{H(t)}$$

Comoving Hubble radius: $$r_H = \frac{1}{aH} = \frac{1}{\dot{a}}$$

- Radiation dominated $\quad r_H \propto a$
- Matter dominated $\quad r_H \propto a^{1/2}$
- Λ dominated $\quad r_H \propto 1/a$

Comoving Hubble radius during the evolution of the Universe



I.3- Thermal history of the Universe

I.3.1 – Brief review of thermodynamics

Quick way to derive relation between temperature and scale factor:

$$\rho_r \propto T^4; \; \rho_r \propto a^{-4} \Rightarrow a \propto T^{-1}$$

Stefan-Boltzmann law

More formally the number density and energy density are:

$$n = \frac{g}{(2\pi)^3} \int d^3p \, f(\vec{p})$$

$$\rho = \frac{g}{(2\pi)^3} \int d^3p \, E(\vec{p}) f(\vec{p})$$

$$E = \sqrt{|\vec{p}|^2 + m^2}$$

E is the energy of a state, f(p) is the phase-space distribution and g is number of internal degrees of freedom (eg g=2 for photons, g=8 for gluons, g=12 for quarks, etc).

Phase-space distribution (+ for FD, - for BE), k_B=1,
μ chemical potential:

$$f(\vec{p}) = \frac{1}{e^{(E-\mu)/T} \pm 1}$$

Relativistic limit (T>>m) and T>>μ

$$\rho = \left(\frac{\pi^2}{30}\right) g T^4 \begin{cases} 1 \text{ (Bose - Einstein)} \\ \frac{7}{8} \text{ (Fermi - Dirac)} \end{cases}$$

$$n = \frac{\zeta(3)}{\pi^2} g T^3 \begin{cases} 1 \text{ (Bose - Einstein)} \\ \frac{3}{4} \text{ (Fermi - Dirac)} \end{cases} \qquad \zeta(3) = 1.202\cdots$$

Exercise 9: compute the number of CMB photons (T=2.73 K) in a cm^3

Non-relativistic limit (T<<m) and μ=0 [same for B-E and F-D]

$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

$$\rho = mn$$

Exponential Boltzmann suppression

Density of relativistic particles in the Universe is set by the effective number of relativistic degrees of freedom g_*:

$$\rho_r = \frac{\pi^2}{30} g_* T^4$$

$$g_* = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \sum_{\text{fermions}} \frac{7}{8} g_i \left(\frac{T_i}{T}\right)^4$$

$g_*(T)$ changes when mass thresholds are crossed as T decreases. At high T (>200 GeV) $g_*^{(SM)}$ ~100.

I.3.2 – Temperature-time relationship

From Friedmann's 1st equation for a radiation-dominated era:

$$H = \sqrt{\frac{\rho_r}{3\tilde{M}_{\rm PL}}} \propto T^2$$

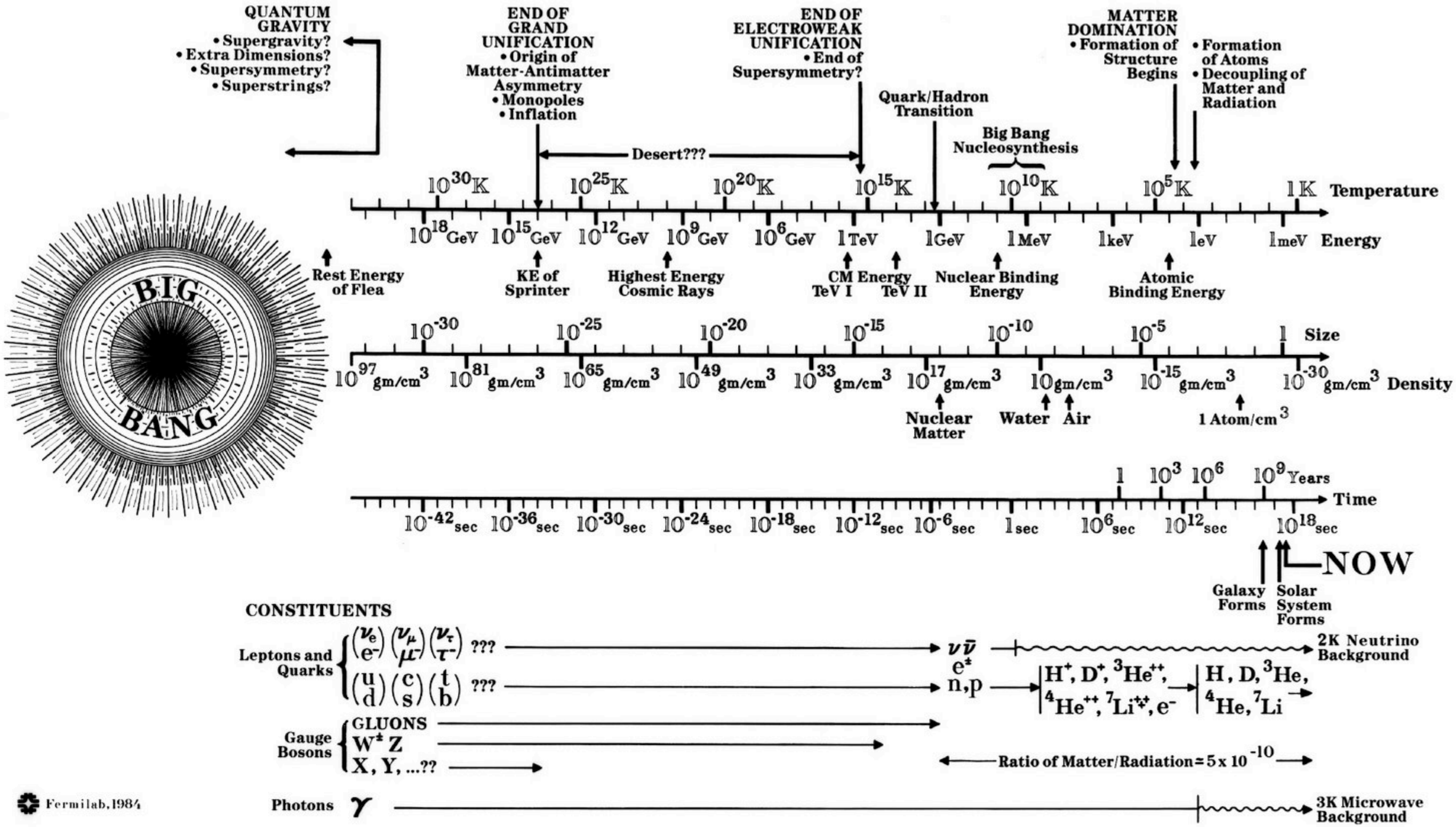
and $$H = \frac{\dot{a}}{a} \propto t^{-1}$$

one finds: $$T \propto t^{-1/2}$$

Putting numbers: $$T(\mathrm{MeV}) \simeq 1.5 g_*^{-1/4} t(\mathrm{s})^{-1/2}$$

Thermal history of the Universe

Kolb & Turner



I.3.3 – Decoupling of species

Different particles are in thermal equilibrium when they can interact efficiently. There are 2 typical rates that can be compared:

- rate of particle interactions:

$$\Gamma(T) = n\langle\sigma v\rangle$$

Number density

Thermal averaged cross section x velocity

- expansion rate of the Universe:

$$H(T)$$

When

$$\Gamma(T) \gg H(T)$$

particles are in thermal equilibrium.

As a first estimate particles decouple when $\Gamma(T) \sim H(T)$

More precise estimate requires solving a Boltzmann equation (more later)

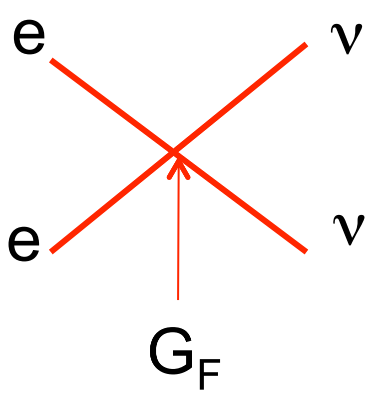
Example: decoupling of neutrinos from the thermal bath

Weak interactions:

$$\nu_e + \bar{\nu}_e \leftrightarrow e^+ + e^-$$

$$e^- + \bar{\nu}_e \leftrightarrow e^- + \bar{\nu}_e$$

Low energy cross section (4-fermi interaction):



$G_F = 10^{-5}$ GeV^{-2}: Fermi constant

$$\sigma \sim G_F^2 T^2;\ n_\nu \sim T^3 \Rightarrow \Gamma_\nu(T) \sim G_F^2 T^5$$

$$H(T) \sim T^2/M_{\rm Pl}$$

$$\boxed{T_{\nu,\rm dec} = \left(\frac{1}{G_F^2 M_{\rm Pl}}\right)^{1/3} \sim 1\,\rm MeV}$$

After decoupling neutrinos cool down as T α 1/a.

They would have the same temperature as photons except for the fact that photons get heated up by the annihilation of e^+e^- at around T~0.5 MeV. Hence neutrinos are a bit cooler (T_ν=1.95 K)

Obs.1: In SM ν's are massless and only ν_L exist. Now we know that this is incorrect. Some extensions of the SM postulate the existence of ν_R to explain ν masses. This is a new degree of freedom and is gauge singlet under SM interactions [eg 1303.6912].

Obs.2: Today there is a cosmic ν background which is very difficult to detect – experiment Ptolomy is being designed for this search.

Obs.3: Experiments such as Planck are sensitive to the number of relativistic degrees of freedom present at the time of CMB. This is characterized by the so-called N_{eff} parameter. In the SM, N_{eff} = 3.046 (some ν's are heated by e^+e^- annihilation). There were some measurements giving a larger N_{eff} which prompted many papers postulating new relativistic degrees of freedom dubbed **"dark radiation"**.

In 2015 Planck measured N_{eff} = 3.15 +/- 0.23 and most people are now happy.

Obs.4: If massless $\rho_\nu \sim \rho_\gamma$. If massive:

$$\rho_\nu \simeq \sum_i m_{\nu,i} n_{\nu,i} \Rightarrow \Omega_\nu^{(0)} = \frac{\sum_i m_{\nu,i}}{94\mathrm{eV}}$$

Obs.5: Most stringent bounds on ν masses comes from cosmology. More later.