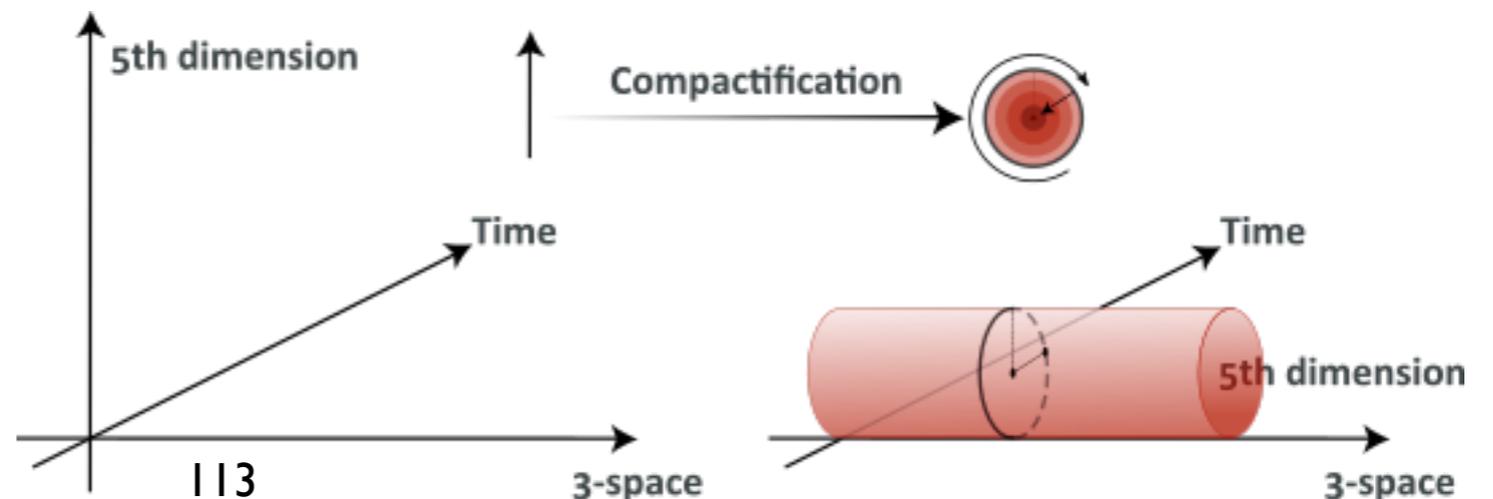


Kaluza-Klein theories

- Add extra space-time dimensions to unify gravity and electromagnetism (T. Kaluza 1921)
- The fifth dimension is compact, periodic, very small (O. Klein 1926)
- Start with a theory of Einstein gravity in 5-D. One of the dimensions is compactified in a very small circle
- Tower of massive scalars
- The 5-D general coordinate invariance broken in ground state \rightarrow ordinary gravity in 4-D plus an Abelian gauge field
- U(1) gauge symmetry appears associated with coordinate transformations on circle
- Parameters of the two theories connected, since they have same origin

$$M_{KK}^2 \propto n/R^2$$



Kaluza theory

- Consider a 5D theory only with gravity, the gravitons are

$$h_{MN} \quad M, N = \mu, 5$$

- Correspond to fluctuations around flat space

$$g_{MN} = \eta_{MN} + h_{MN}$$

which decompose into a spin 2 $h_{\mu\nu}$, identified with a graviton,

a spin 0, $h_{\mu 5}$ identified with a photon, and a scalar h_{55}

- The first proposal did not include massless charged matter particles

Kaluza-Klein theories

- The action in 5D, y is fifth-dimension. Only scalars

$$S_5 = - \int d^4x dy M_* \left[|\partial_\mu \phi|^2 + |\partial_y \phi|^2 + g_5^2 |\phi|^4 \right]$$

- y compactified in a circle $y = y + 2\pi R$

- Expand the complex scalar in a Fourier series

$$\phi(x, y) = \sum_{n=-\infty}^{\infty} e^{iny/R} \phi^{(n)}(x) = \phi^{(0)}(x) + \sum_{n \neq 0} e^{iny/R} \phi^{(n)}(x)$$

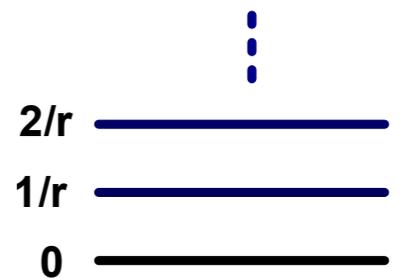
- Integrating over y gives $S_5 = S_4^{(0)} + S_4^{(n)}$

massless scalar

$$S_4^{(0)} = - \int d^4x 2\pi R M_* \left[|\partial_\mu \phi^{(0)}|^2 + g_5^2 |\phi^{(0)}|^4 \right],$$

$$S_4^{(n)} = - \int d^4x 2\pi R M_* \sum_{n \neq 0} \left[|\partial_\mu \phi^{(n)}|^2 + \left(\frac{n}{R}\right)^2 |\phi^{(n)}|^2 \right] + \text{quartic - couplings}$$

tower of massive modes



- We can describe 5D theories as 4D ones

$$S_4^{(0)} = - \int d^4x \left[|\partial_\mu \phi^{(0)}|^2 + g_4^2 |\phi^{(0)}|^4 \right] \quad g_4^2 = \frac{g_5^2}{2\pi R M_*}$$

- Same for gauge fields and graviton:

$$- \int d^4x \left[\frac{1}{16\pi G_N} \mathcal{R}^{(0)} + \frac{1}{4g_4^2} F^{(0)\mu\nu} F_{\mu\nu}^{(0)} \right] + \dots,$$

$$g_4^2 = \frac{g_5^2}{2\pi R M_*},$$

$$G_N = \frac{1}{16\pi^2 R M_*^3}$$

g_4 is 4D gauge coupling
 G_N is 4D Newton constant

strength of interaction suppressed by
radius of extra dimension

- If we lived in 5D, then g_5 should be perturbative, but from the 4D perspective g_4 should be strongly interacting \rightarrow

$$R \sim 1/M_*$$

$$M_P^2 = 2\pi R M_*^3 .$$

$$R \sim \frac{1}{M_P} = l_P \sim 10^{-32} \text{ cm}$$

- The compactification radius is of the order of the Planck length (reduced Planck mass)

- Gauge coupling in 5D has negative mass dimension
→ non-renormalizable
From 4D perspective this is due to the K-K modes accessible at the energy scale
- M_* is the cut-off of the theory, which we treat as an effective field theory below this mass scale

- Gauss' law for the electric field and potential of a point charge

$$\oint_{S^2} \vec{E} \cdot d\vec{S} = Q \Rightarrow \|\vec{E}\| \propto \frac{1}{R^2}, \Phi \propto \frac{1}{R} : 4D$$

$$\oint_{S^3} \vec{E} \cdot d\vec{S} = Q \Rightarrow \|\vec{E}\| \propto \frac{1}{R^3}, \Phi \propto \frac{1}{R^2} : 5D$$

$$\|\vec{E}\| \propto \begin{cases} \frac{1}{R^3} & : R < r \\ \frac{1}{R^2} & : R \gg r \end{cases}$$

- At small distances E falls as $1/R^3$
- When the extra dimension is small compared to the distances then it falls as $1/R^2$, as usual

Large extra dimensions

- Similarly for the gravitational fields G_{MN} in 5D becomes the graviton $g_{\mu\nu}$, gravivectors $G_{\mu n}$ and graviscalars G_{mn}
- Is it possible to solve the hierarchy problem with large extra dimensions?
- The Planck mass is now a derived quantity, in d dimensions, V is the volume

$$M_P^2 = V^d M_*^{2+d} .$$

$$M_P^2 = (RM_*)^d M_*^2 .$$

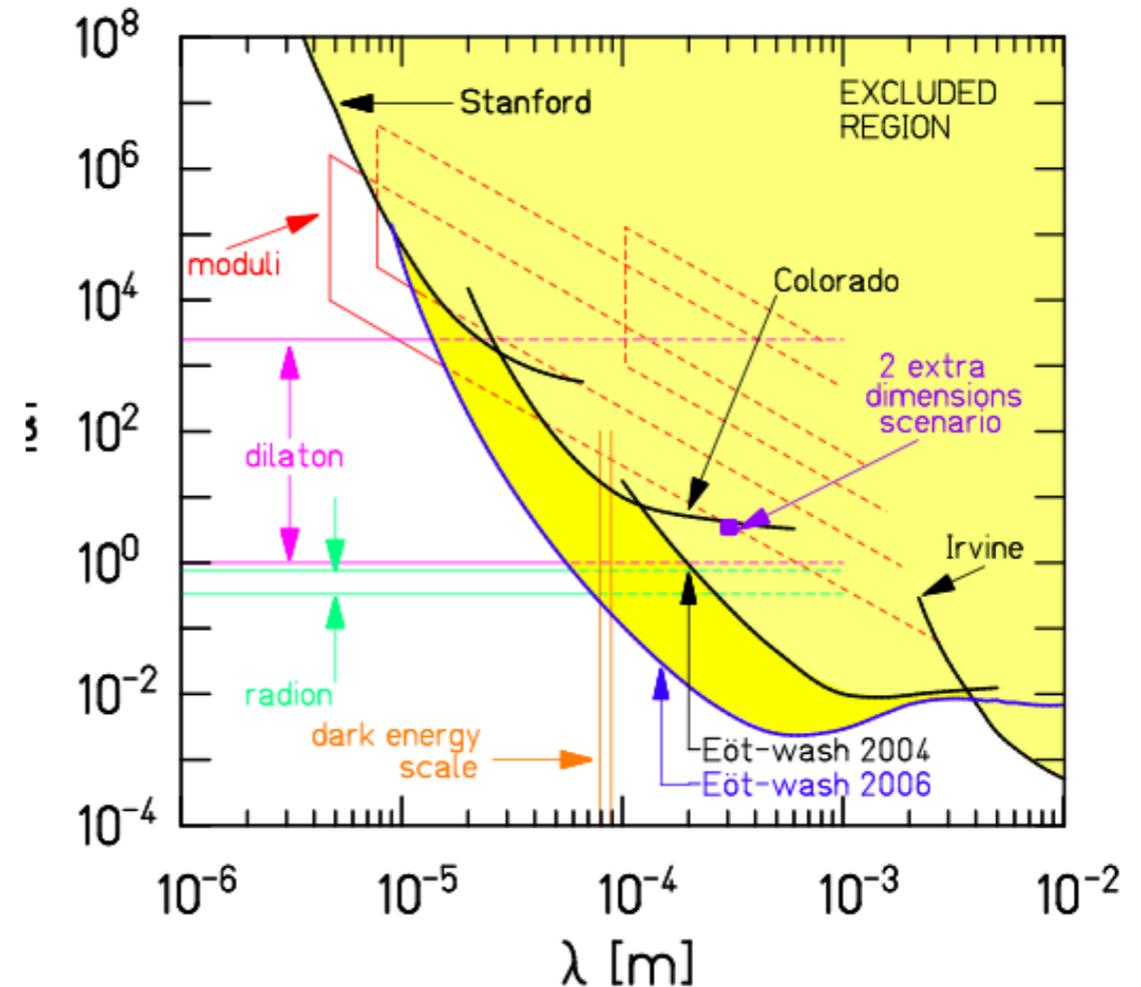
- Gravity lives in more dimensions than the gauge bosons, it propagates in the extra dimensions, thus explaining the gravitational couplings smallness compared to the other couplings

- KK tower of graviton \rightarrow new forces

$$F_{KK}(r) = -\alpha G_N \frac{m_1 m_2}{r} e^{-r/\lambda},$$

$\lambda = R$, α constant of compactification type

- If we assume M_* above 1 TeV:
- For $d=1 \rightarrow R \sim 10^9$ km
 $d=2 \rightarrow R \sim 0.5$ mm
- $d = 2$ is ruled out, unless $M_* > 3$ TeV

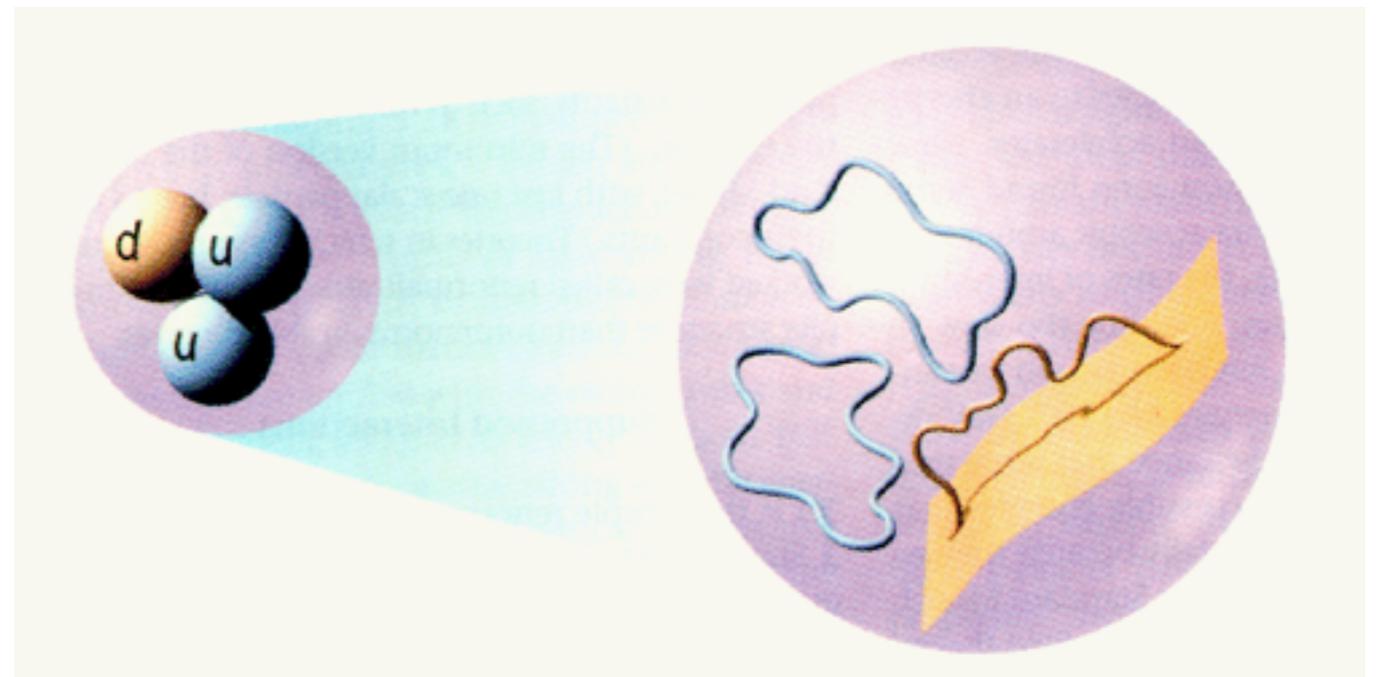


E. G. Adelberger, J. H. Gundlach, B. R. Heckel et al., Prog. Part. Nucl. Phys. 62 (2009) 102–134.

Superstrings and branes

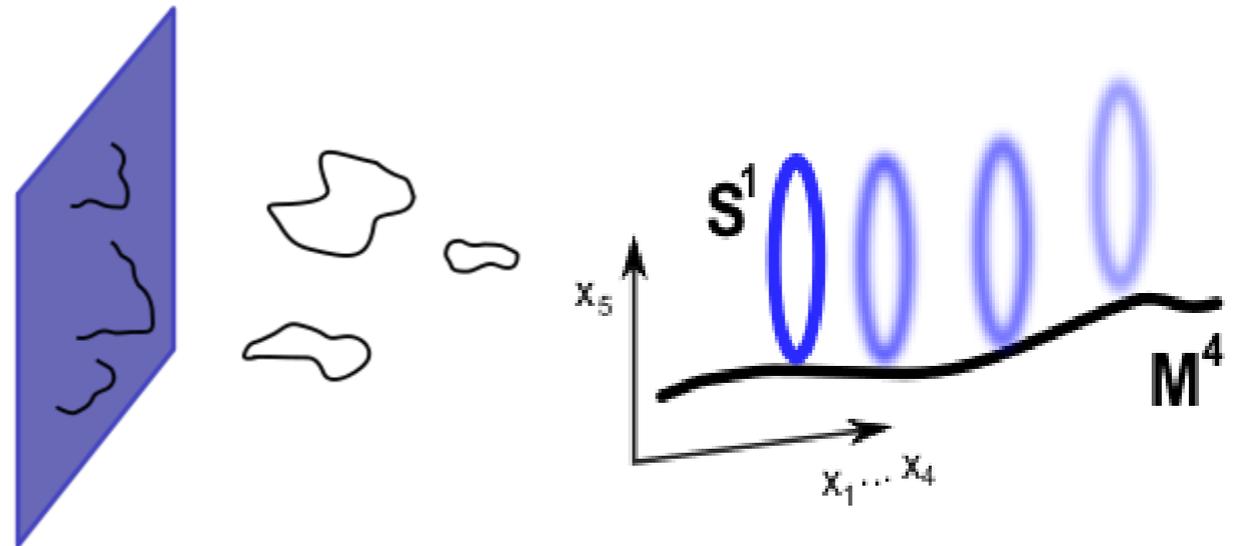
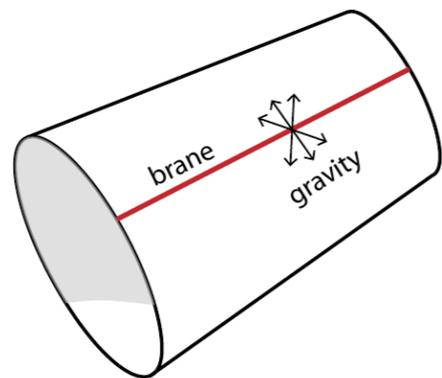
more symmetries, more unification, more particles, more dimensions

- Particles are excitations of vibrating strings
- Branes are p -dimensional surfaces in space time
- Many require SUSY
- Require extra space-time dimensions for consistency
- Similar idea as K-K compactification
- Many ways to compactify these extra dimensions
- Spectrum includes a graviton
- Duality relates strongly coupled to weakly coupled theories
- Could be ultimate unification

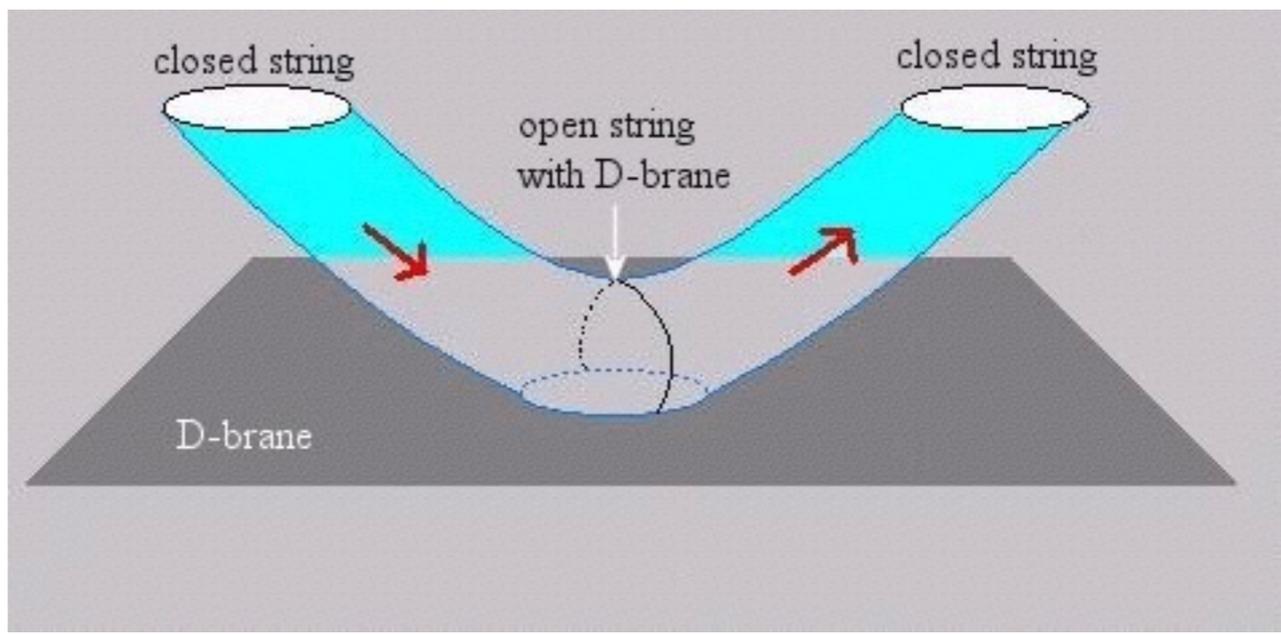
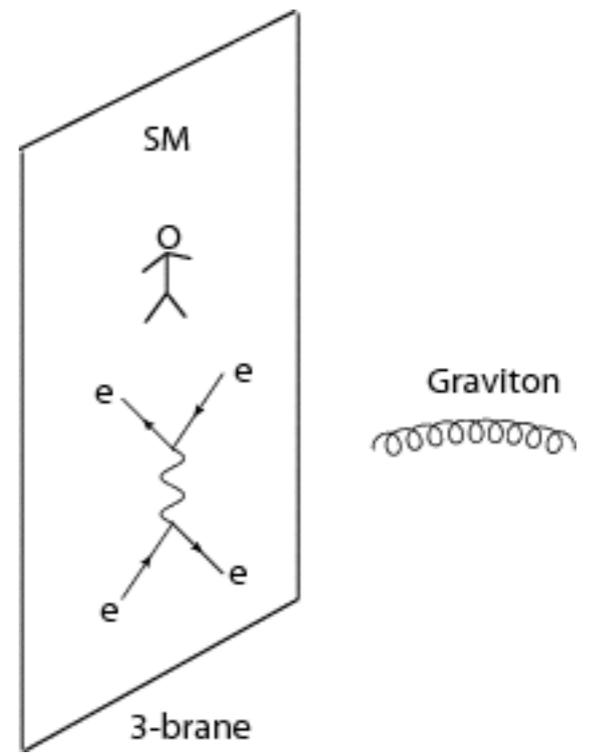
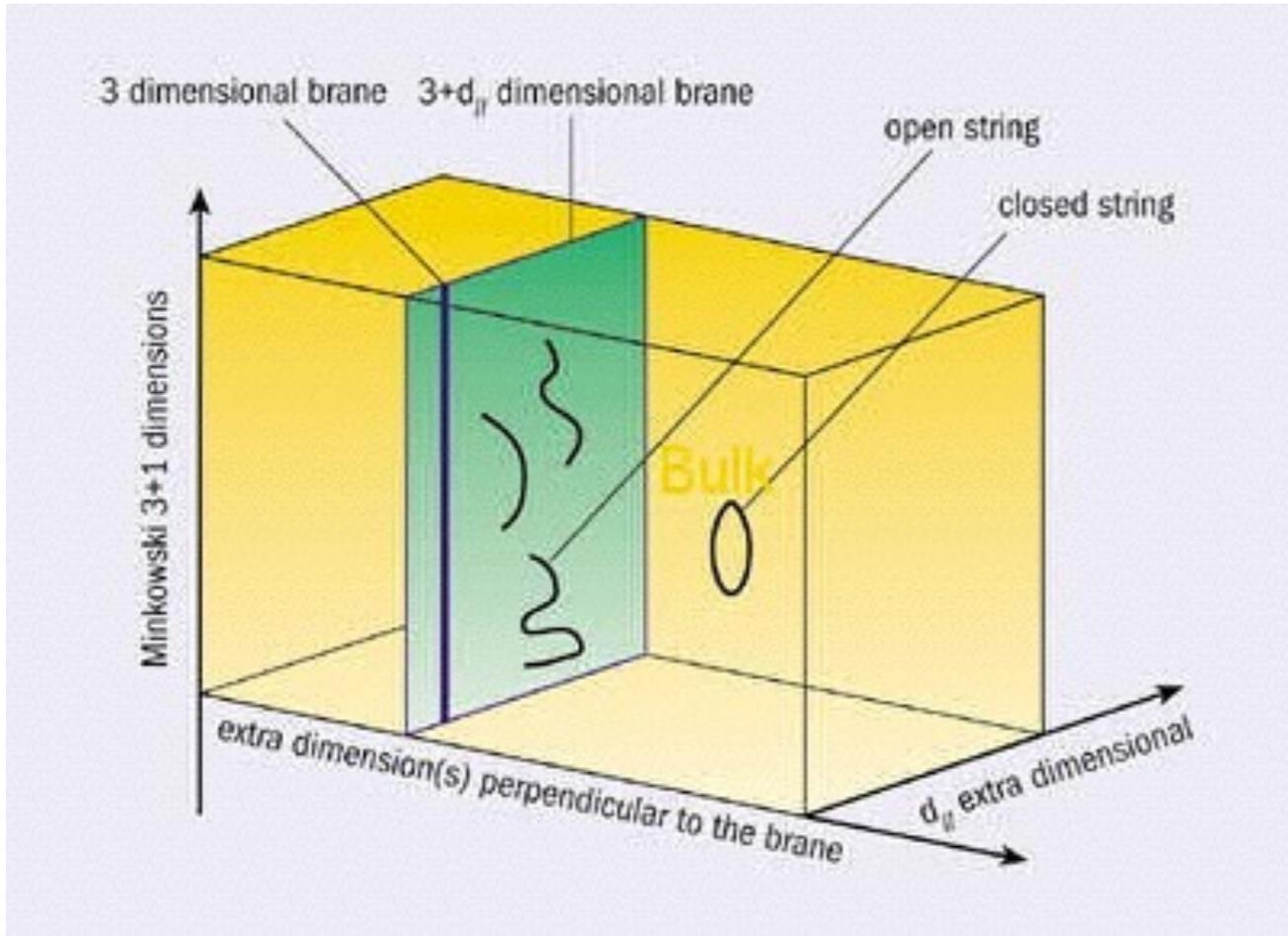


- Adding more dimensions we extend the Poincaré symmetries of SM and general coordinate transformations of general relativity, to each
- Superstring theory also requires extra dimensions 10 for heterotic string, 11 for supergravity and M theory
- Why don't we perceive them?

- They are compactified or



- We live on a brane — bulk of space time lives in $D+3+1$ dimensions, but SM (us) are fixed to a brane = $3+1$ dimensional hyper surface. Gravity moves through space-time, i.e. all dimensions. We don't.



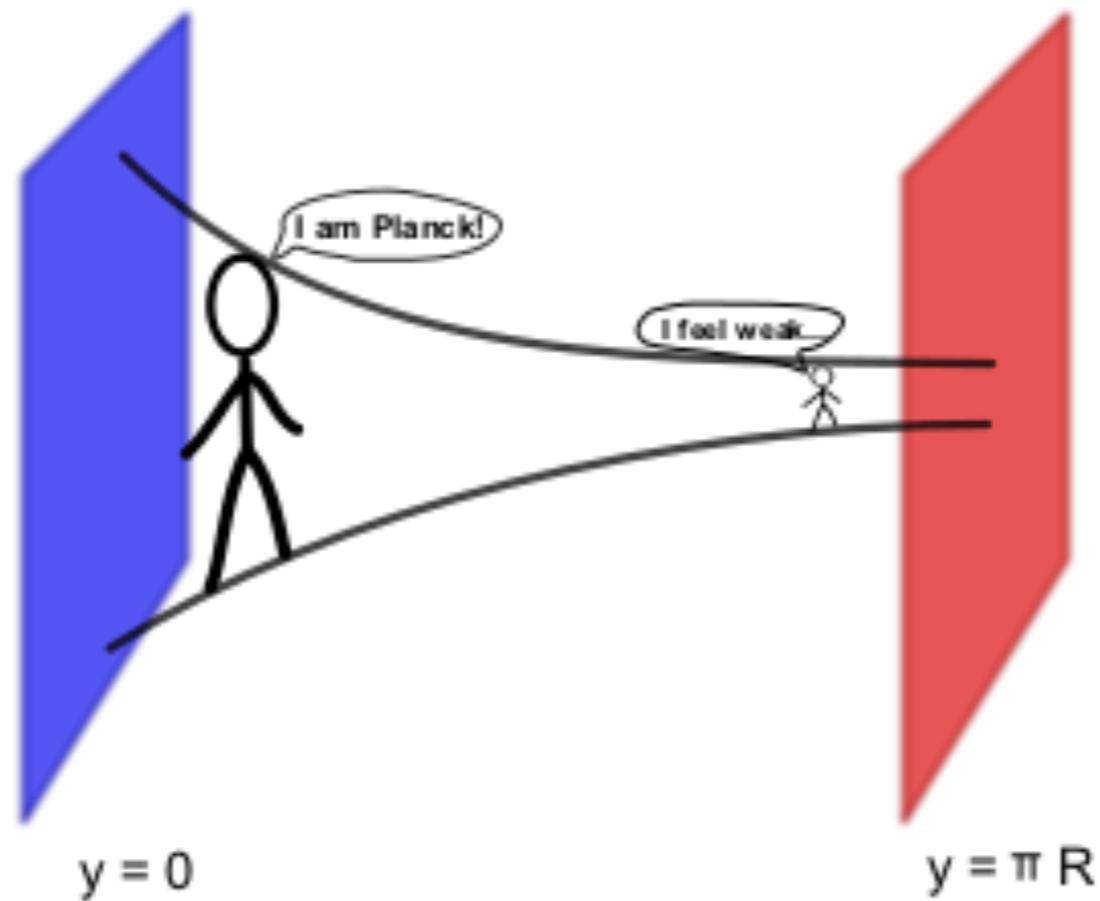
Warped extra dimensions

- 5th dimension bounded by two (3+1) branes
- Warped space-times, the metric warps exponentially along the extra dimension

$$ds^2 = e^{-|ky|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2.$$

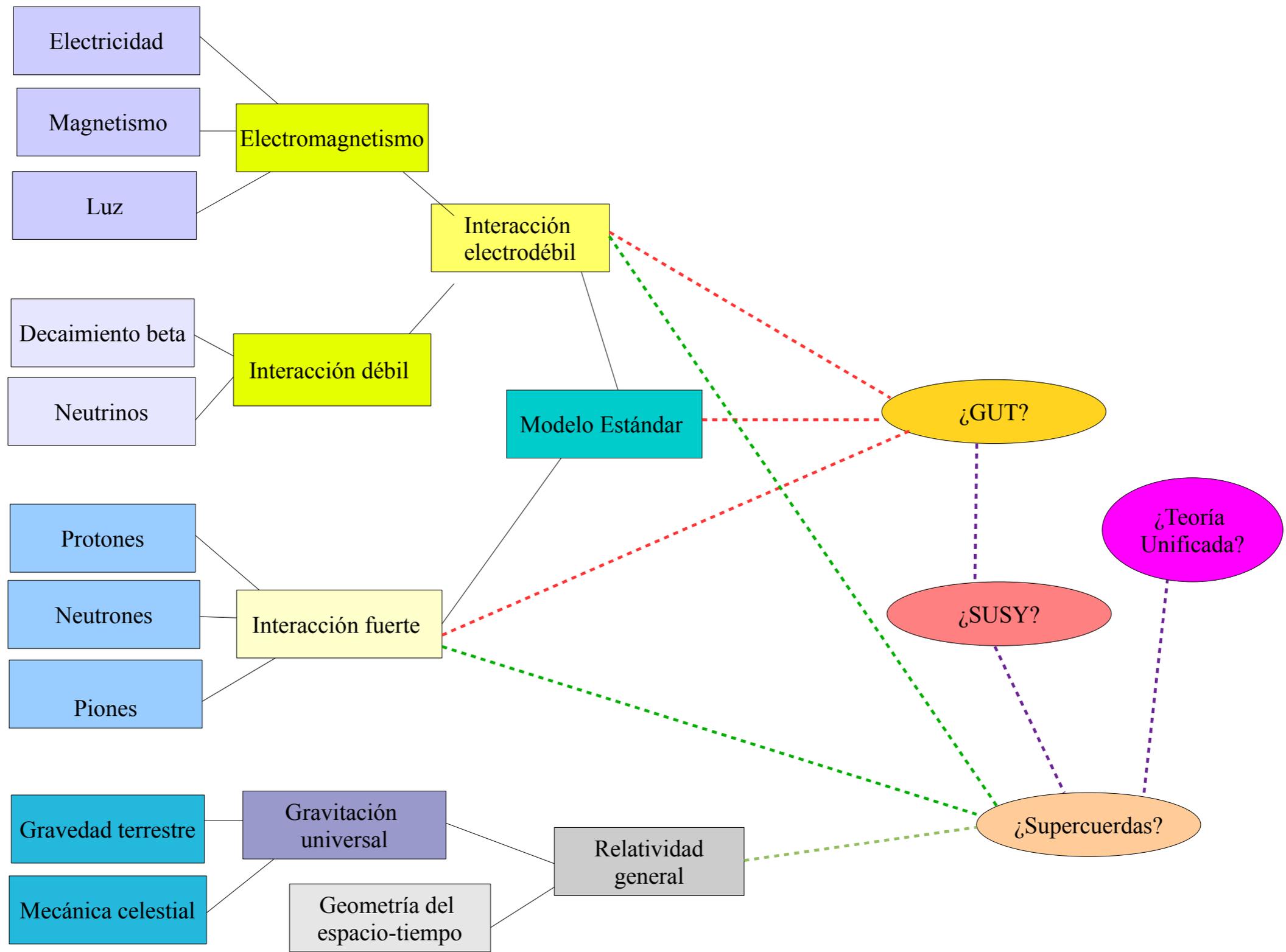
- MD = MPlanck, but it gets warped down to eV scale, $r \sim 10/k$, $k \sim$ Planck scale \implies small extra dimension

$$\Lambda_\pi \sim M_{Pl} e^{-k\pi r} \sim \mathcal{O}(\text{TeV})$$



Planck brane is warped down to weak brane. Higgs and other fields live on the weak brane

Gravity may propagate in the extra dimension \implies
might produce a TeV resonance that is detectable —
“RS graviton”



Multi-Higgs models more particles

- 2HDM without SUSY
 - widely studied
 - different versions depending on how they couple to the other SM particles
- 3 or more HDM also possible
- Extra singlet Higgs models NMSSM
- Complex MSSM CMSSM
 - More sources of CP violation
 - Candidates for dark matter, with some discrete symmetry
 - Might give explanation for the mass hierarchies and mixing of quarks and leptons

N-Higgs doublet models — NHDM

- Add more complex scalar electroweak doublets
All with same hyper charge $Y=1$

$$V(\phi) = Y_{ij}\phi_i^\dagger\phi_j + Z_{ijkl}(\phi_i^\dagger\phi_j)(\phi_k^\dagger\phi_l).$$

- $N^2 + N^2(N^2 + 1)/2$ real parameters:
12 for 2HDM, 54 for 3HDM...
- Potential must be bounded by below, no charge or colour breaking minima
- Must respect unitarity bounds
- Can have CP breaking minima \rightarrow baryogenesis (or disaster)

2HDM

- Most studied, experimental limits exist
- **4 parameters** in the quadratic part
10 parameters (6 real, 4 complex) in the quartic part
- Again: minimum must be bounded from below, neutral and satisfy perturbative unitarity bounds

$$\begin{aligned}
 V = & \underbrace{m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 - m_{12}^2 \phi_1^\dagger \phi_2 - (m_{12}^2)^* \phi_2^\dagger \phi_1}_{\text{quadratic}} + \underbrace{\frac{\lambda_1}{2} (\phi_1^\dagger \phi_1)^2 + \frac{\lambda_2}{2} (\phi_2^\dagger \phi_2)^2}_{\text{quartic}} \\
 & + \underbrace{\lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) + \frac{\lambda_5}{2} (\phi_1^\dagger \phi_2)^2 + \frac{\lambda_5^*}{2} (\phi_2^\dagger \phi_1)^2}_{\text{quartic}} \\
 & + \underbrace{\left[\lambda_6 (\phi_1^\dagger \phi_1) (\phi_1^\dagger \phi_2) + \lambda_7 (\phi_2^\dagger \phi_2) (\phi_1^\dagger \phi_2) + h.c. \right]}_{\text{quartic}} .
 \end{aligned}$$

- To analyse it: usually impose symmetries

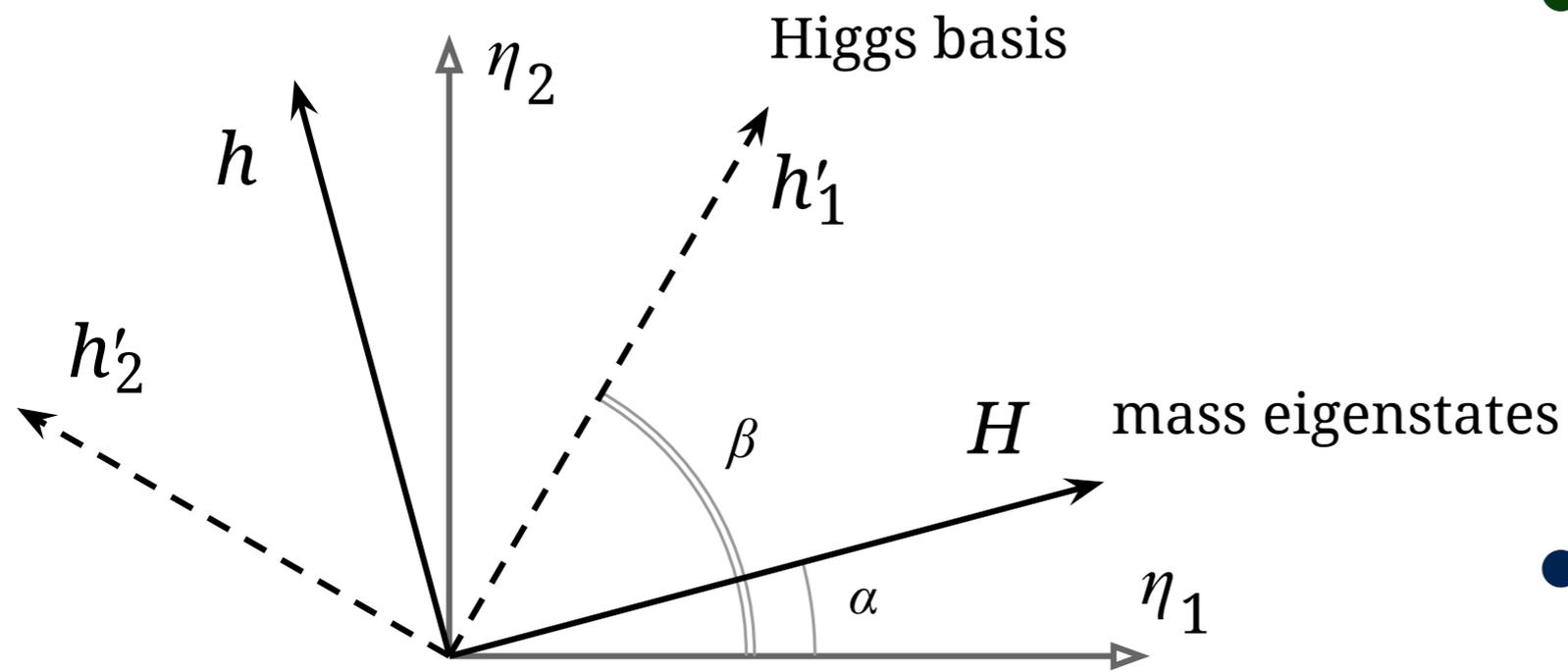
- Not all parameters are physical, only 11.
- Add more symmetries: No FCNC \rightarrow Z_2 symmetry, softly broken by m_{12}^2 term
- Mainly two types of 2HDM analysed, 8 parameters:

Type I: all fermions couple only to one doublet

Type II: up quarks couple to one doublet, down quarks and leptons couple to the second doublet

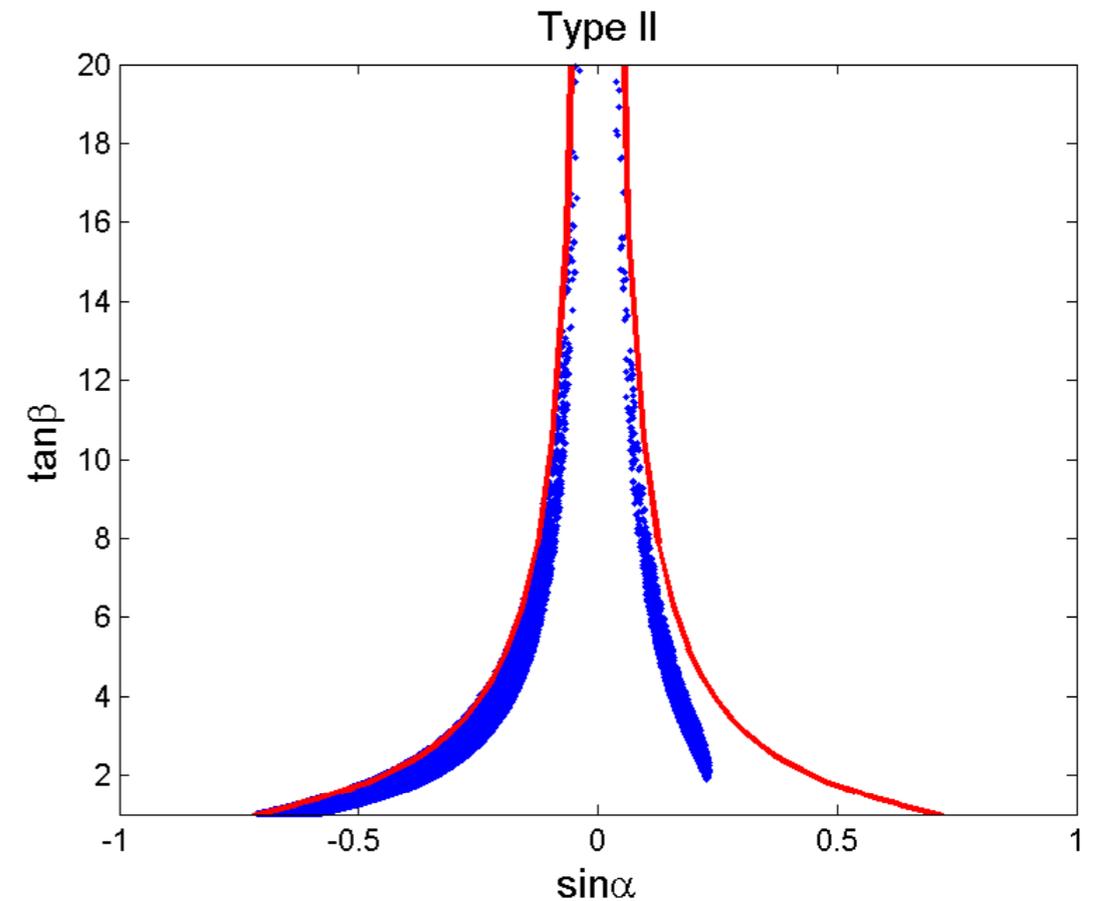
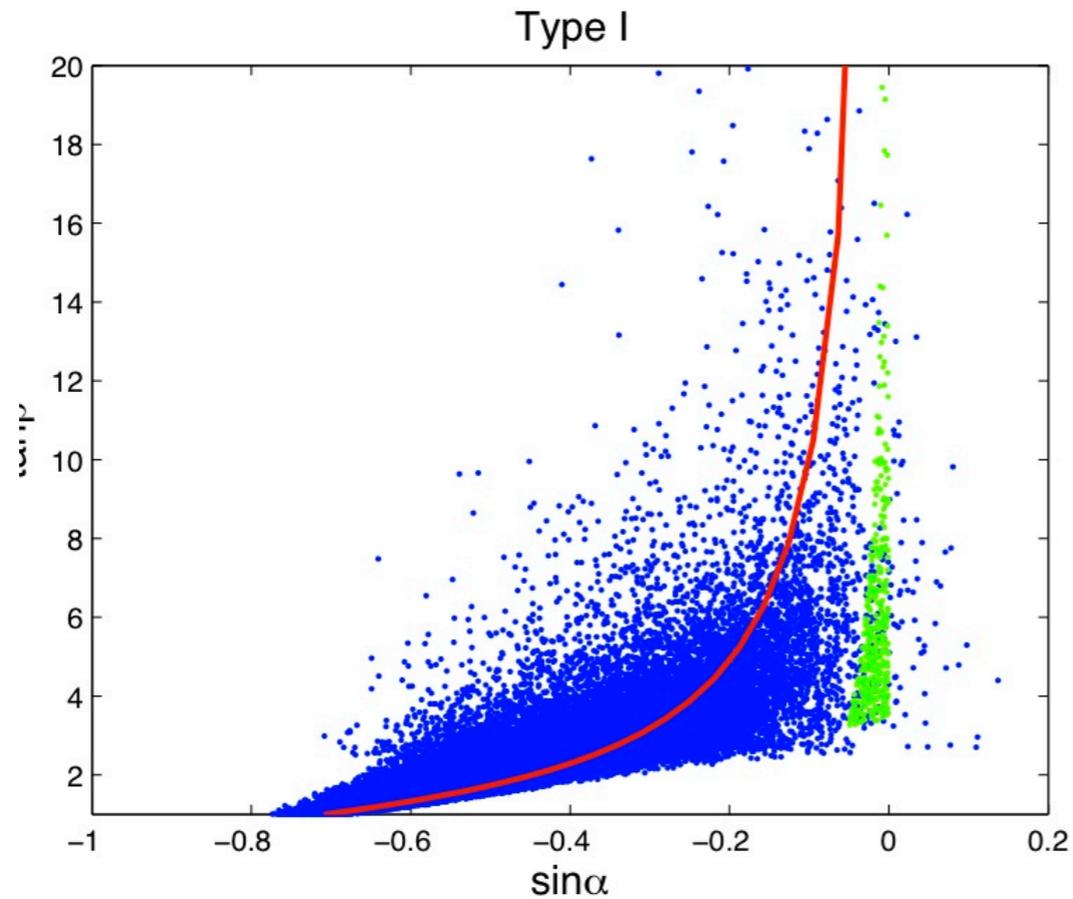
- After eW symmetry breaking there are five massive physical Higgs bosons $m_h, m_H, m_A, m_{H^\pm}, \tan\beta$ (v_1/v_2), m_{12}^2 , $\sin(\beta-\alpha)$

α mixing angle between scalars



- (η_1, η_2) original basis in which there is natural flavour conservation
- (h'_1, h'_2) Higgs basis, all the vev is in one of the Higgses, rotated by β from original
- (h, H) physical Higgs bases, rotated by α from original, and by $(\beta - \alpha)$ from Higgs one

- How to test them? Look for signals of new physics
- Usual test focus on 2HDM type I and II, and on MSSM
- Heavy extra Higgs usually decoupled, difficult to discard the 2HDM
- Other new scalars would give similar results, so searches for new Higgses and scalars are complementary
- LEP excluded charge scalars $M_H > 80$ GeV, and $b \rightarrow s\gamma$ places stringer constraints $M_H > 480$ GeV



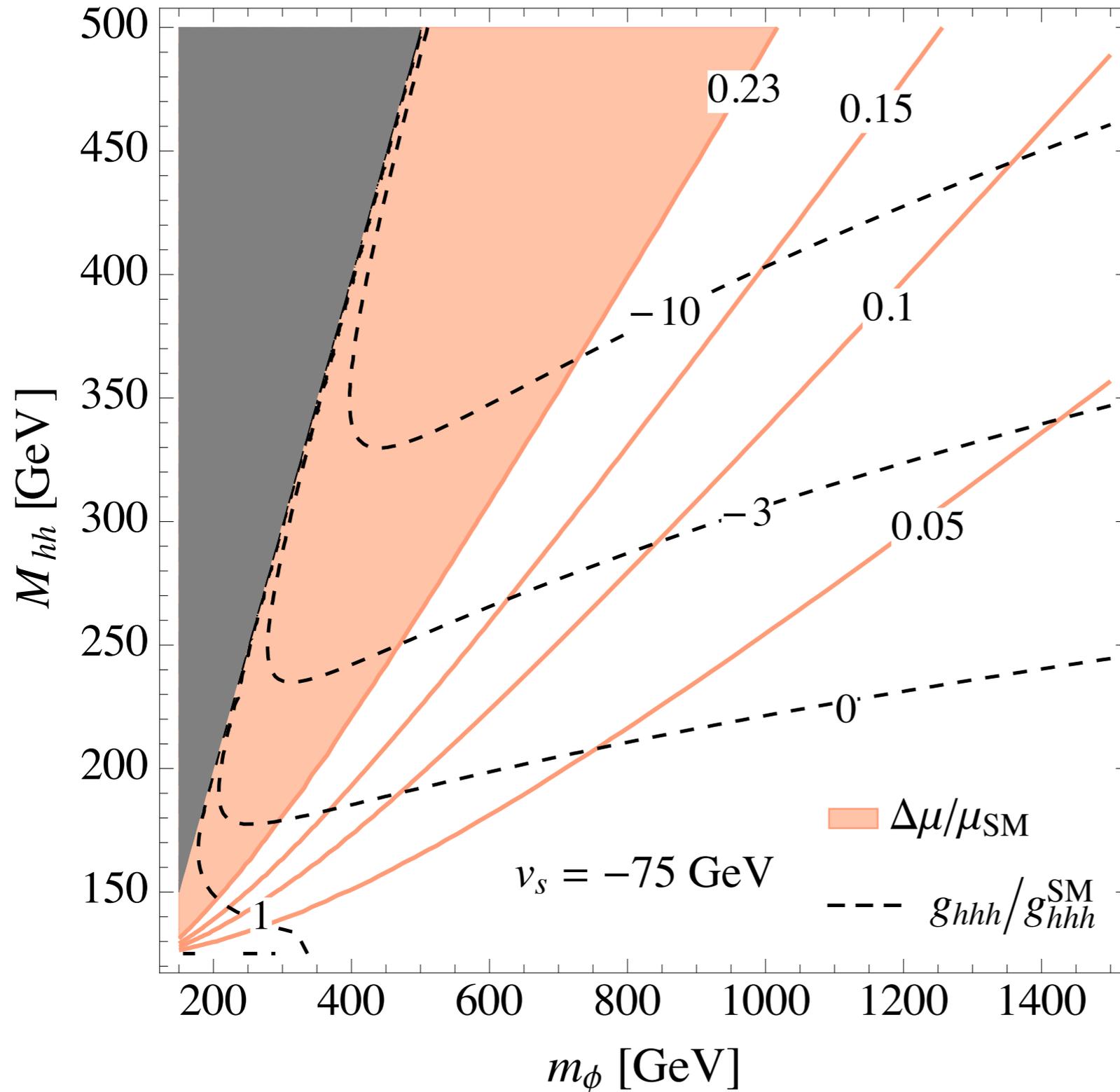
$$R_f = \frac{\sigma(pp \rightarrow h)_{2\text{HDM}} \text{BR}(h \rightarrow f)_{2\text{HDM}}}{\sigma(pp \rightarrow h_{\text{SM}}) \text{BR}(h_{\text{SM}} \rightarrow f)}$$

Points that pass all constraints: R_f is the ratio of events predicted by 2HDM to the ones from SM, f is the final state particle
 Green 1 sigma, blue 2 sigma, red SM limit

A. Barroso, P. M. Ferreira, R. Santos, M. Sher, J. P. Silva, 2HDM at the LHC - the story so far, in: Proceedings, 1st Toyama International Workshop on Higgs as a Probe of New Physics 2013 (HPNP2013): Toyama, Japan, February 13-16, 2013, 2013. arXiv:1304.5225.
 URL <http://inspirehep.net/record/1228915/files/arXiv:1304.5225.pdf>

Adding singlets

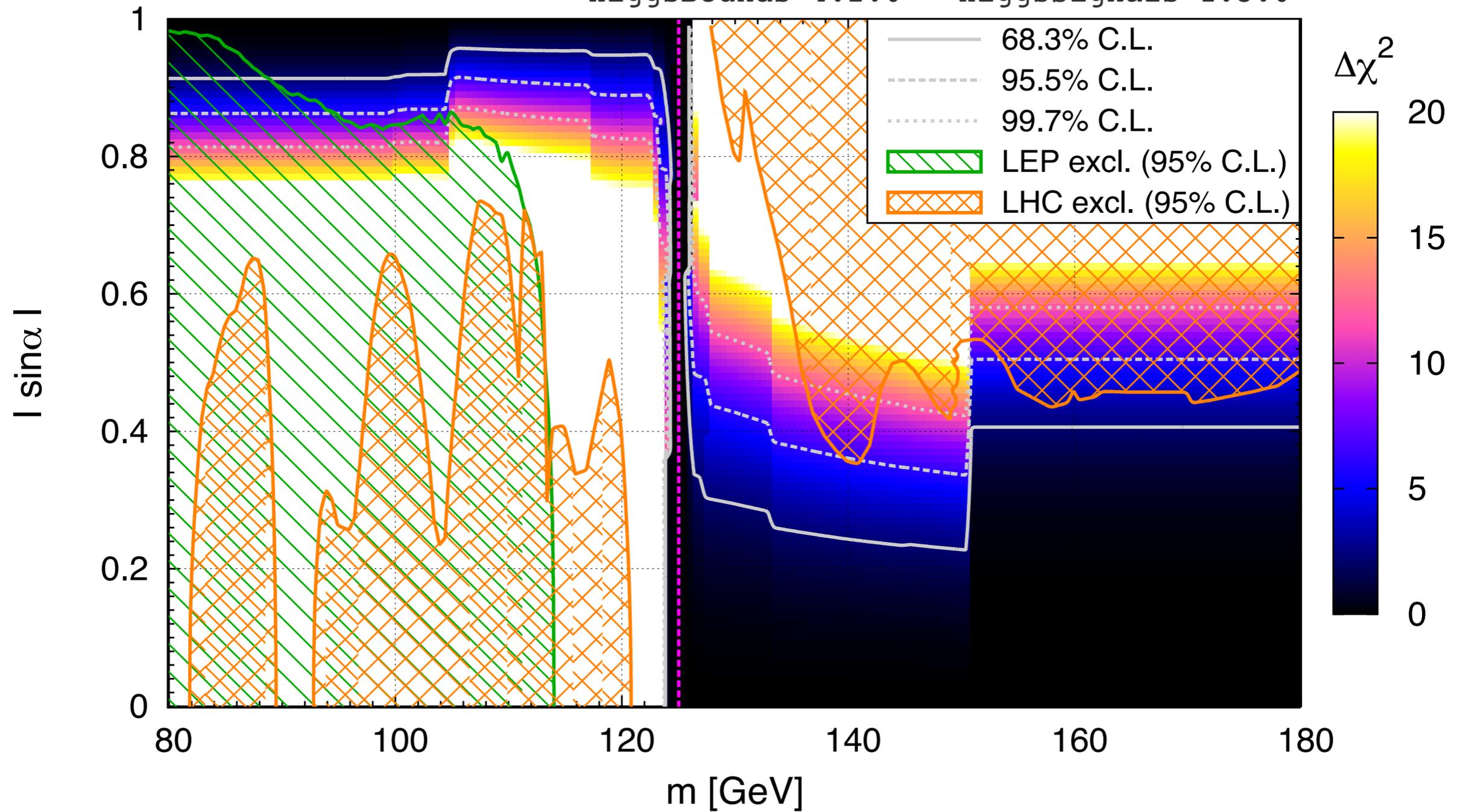
- Add only one scalar singlet, either real or complex
- Transforms trivially under SM, i.e. is a singlet
- Only couples to SM via the $\phi\phi$ operator
- Could be there is a whole hidden sector, SM blind to it
- $V_{int} = \lambda_{hs}\phi^\dagger\phi S^2$, for a real scalar
- In the quartic interaction, the self-interaction of S can get a non-zero vev, which is equal to the $\mu^2\phi^\dagger\phi$ term in SM. This mass scale is normally put by hand
- They can give a natural candidate for dark matter, complex ones can give the right DM relic abundance



Parameters of real singlet extension:
 v , $\tan\beta = v/v_s$, m_h , m_H ,
 α (mixing angle of h_s h_{SM})

Ratio of trilinear coupling to SM value, for $v_s = -75$ GeV
 Grey unphysical
 Pink, exclusion at 95% from Higgs signal
 Pink lines, different values of the mixing angle \sin^2
 $M_{hh} (I, I)$ of the mass matrix before diagonalization

D. Buttazzo, F. Sala, A. Tesi, Singlet-like Higgs bosons at present and future colliders, JHEP 11 (2015) 158. arXiv:1505.05488, doi:10.1007/JHEP11(2015)158.



Constraints on m_H , $|\sin\alpha|$ plane, and from Higgs signal $\Delta\chi^2$
 magenta line is the normal Higgs

NMSSM

- Add a singlet to the MSSM → Next to MSSM
- The super potential gets modified, where $f(S)$ is a polynomial of at most order three

$$\mathcal{W}_{\text{NMSSM}} = \mathcal{W}_{\text{MSSM}} + \lambda S H_u H_d + f(S)$$

- The superpotential is

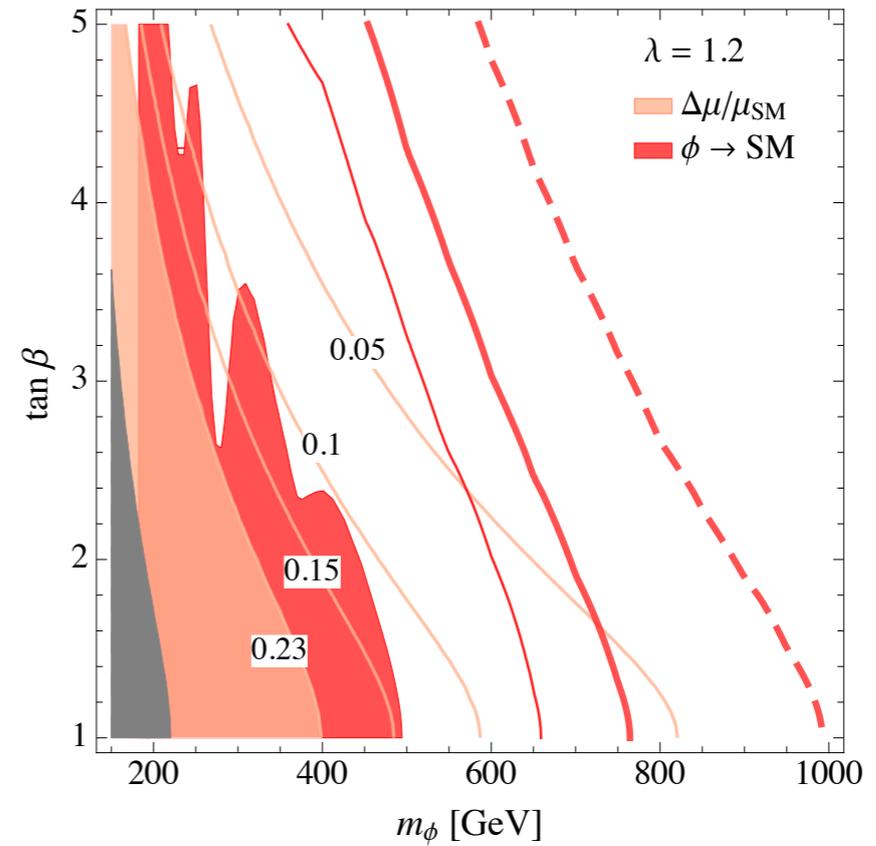
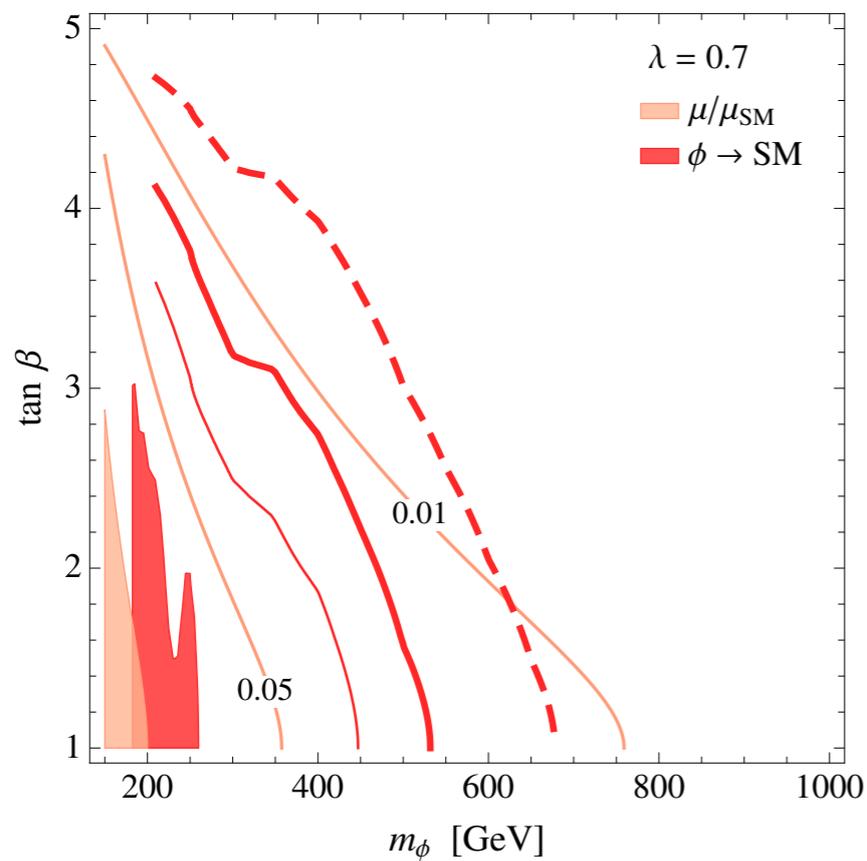
$$V_{\text{soft}}^{\text{NMSSM}} \supset m_S^2 |S|^2 + \left(\lambda A_\lambda H_u H_d S + \frac{\kappa}{3} A_\kappa S^3 + \text{h.c.} \right)$$

- The Higgs mass is now

$$m_h^2 \lesssim \frac{\lambda^2 v^2}{2} s_{2\beta}^2 + m_Z^2 c_{2\beta}^2 + \Delta^2$$

- 7 physical Higgs bosons
- Large SUSY contribution to the Higgs mass → possible to achieve $M_h = 125$ GeV at tree level
- Allows larger masses for stops and gluinos $O(\lambda/g)$
- The coupling λ is not asymptotically free, $\lambda > 0.7$ theory becomes strongly coupled, below the GUT scale
- $\lambda \sim 1.2$ strong coupling regime
 $\lambda \sim 0.7$ still perturbative at GUT scale
- If λ is sizeable → lightest new particles are the extra scalar bosons of the Higgs sector

Grey unphysical, pink and red excluded regions
Contours of fixed $s_2\gamma$



Projected: thin solid red LHC13
thick solid red LHC14
dashed red I4 HL-LHC

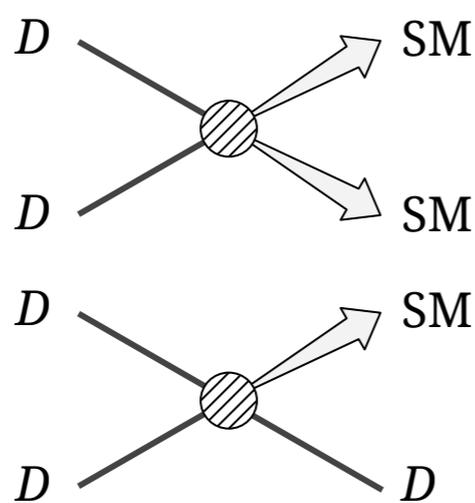
D. Buttazzo, F. Sala, A. Tesi, Singlet-like Higgs bosons at present and future colliders, JHEP 11 (2015) 158. arXiv:1505.05488, doi:10.1007/JHEP11(2015)158.

Dark Matter?

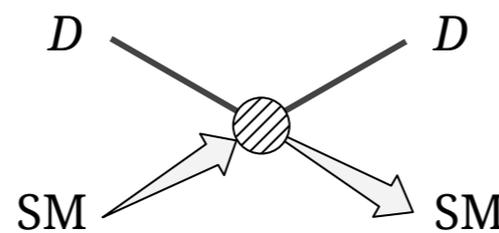
- Many models provide candidates for DM
 - sterile neutrinos, neutrinos
 - scalars, Higgses, K-K scalars, others
 - axion (solution to the strong CP problem), pseudoscalar
 - SUSY particles, neutralino, sneutrino, axino
 - Decaying DM
- DM-DM interactions
 - Most models assume WIMP
 - Usually require a discrete symmetry to make them stable
 - Constraints from cosmology: relic density, structure formation, BBN
 - DM could be more than one type of particle

Scalar dark matter popular now

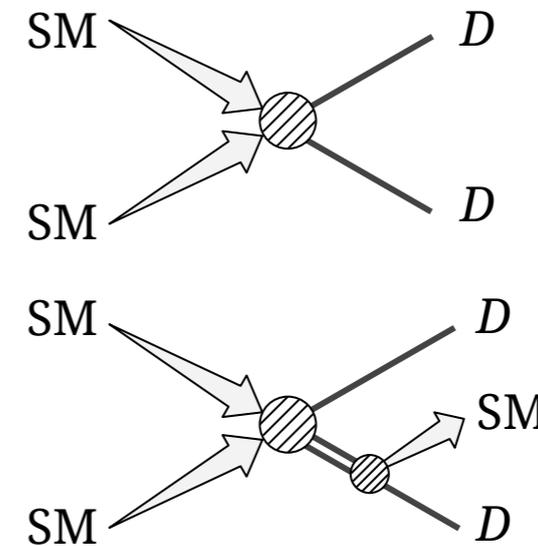
- Natural in models with extended Higgs sectors and global symmetries, like Z_2
- These symmetries may remain unbroken after eW symmetry breaking \implies DM candidate stable



(semi-)annihilation



direct detection

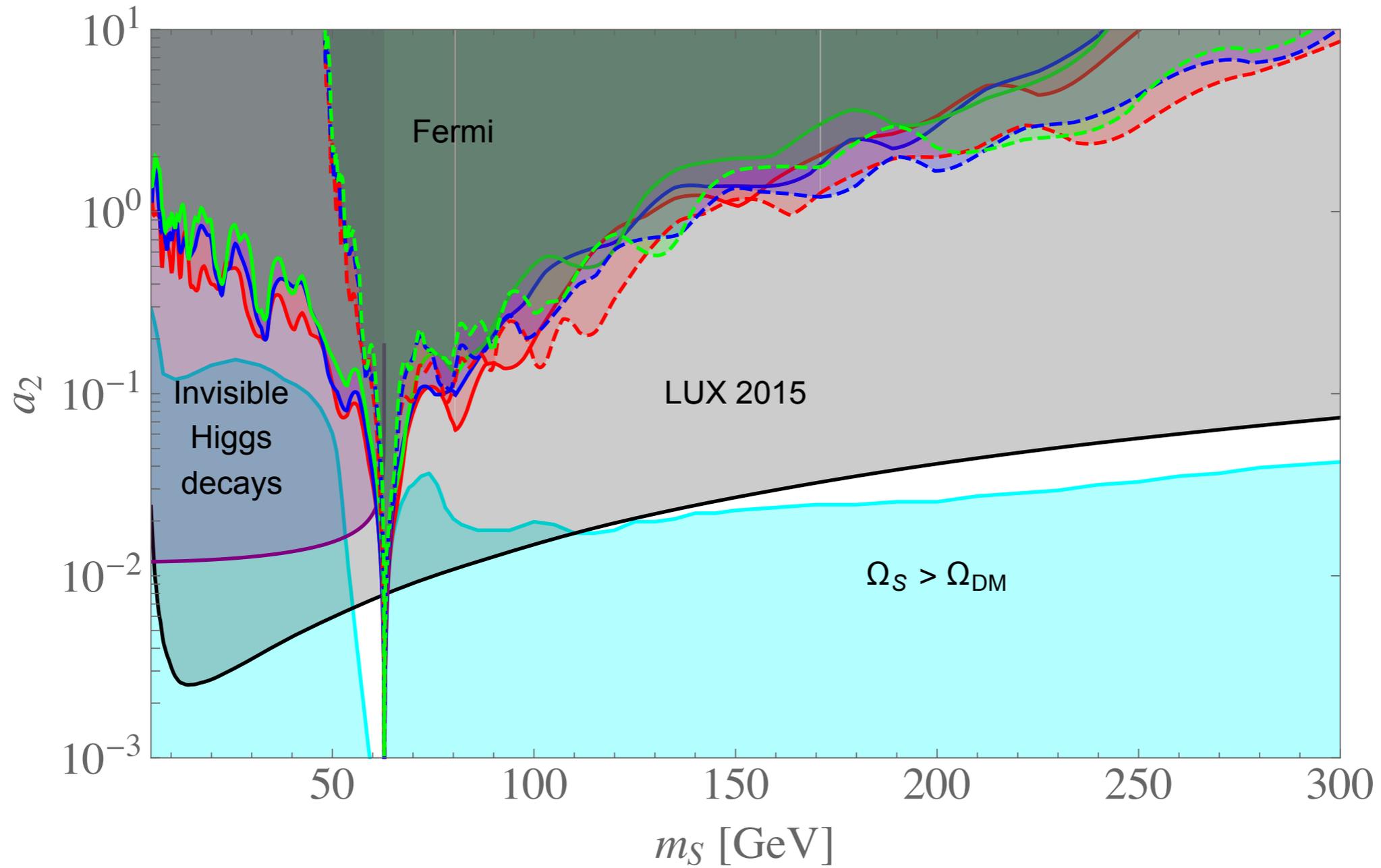


production

- annihilation, co-annihilation and semi-annihilation will determine DM abundance
- also missing energy signature in colliders
- Example: minimal dark matter. Add extra scalars in higher dimensional reps of $SU(2)$, only renormalizable interactions in Lagrangian.
- If lightest component neutral \implies DM candidate
- They are used also to explain small neutrino masses

Real singlet extension

- One real gauge singlet and a Z_2 symmetry
- Symmetry makes DM stable and prevents it from mixing with neutral part of the doublet
- Correct DM relic abundance for some values of the coupling λ between SM-DM
- Cannot be too heavy (above TeV), or runs into trouble with perturbative constraints

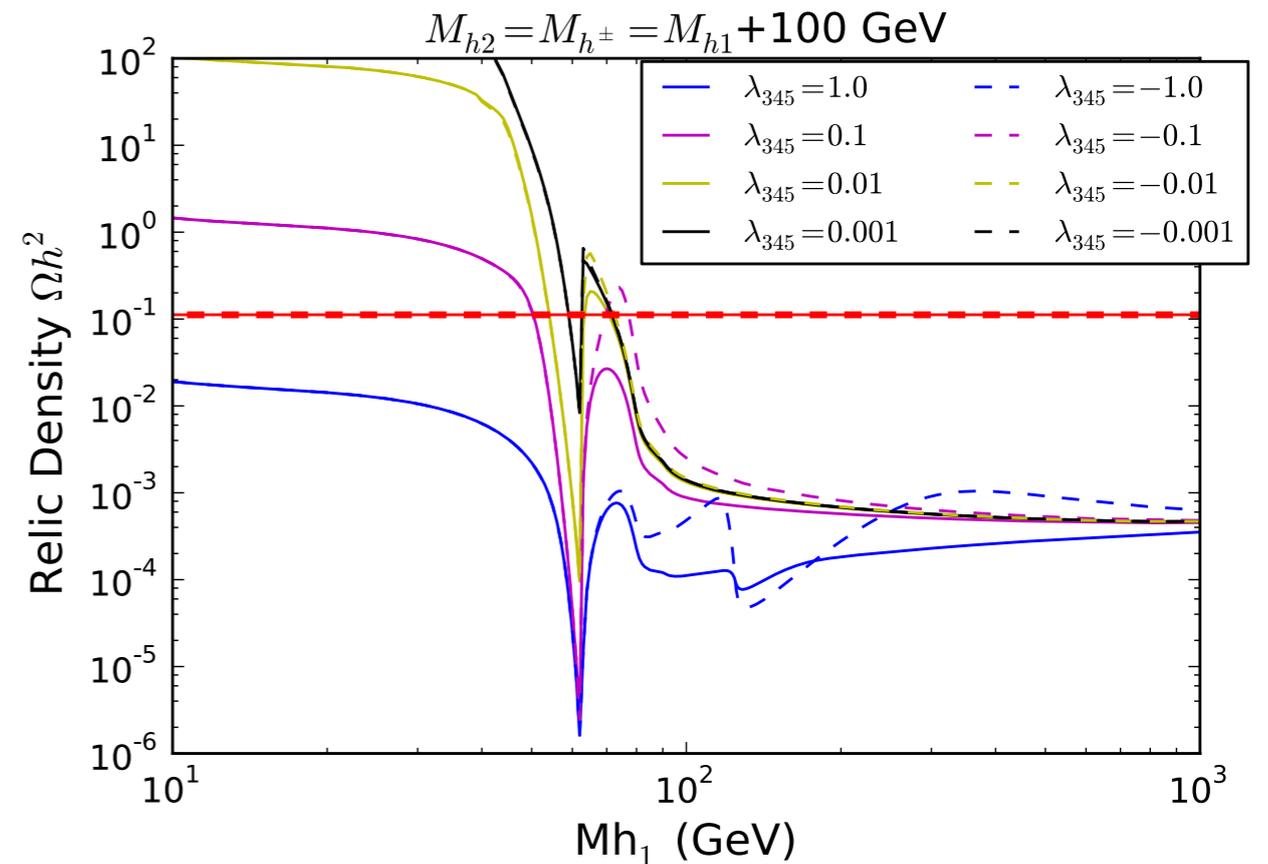
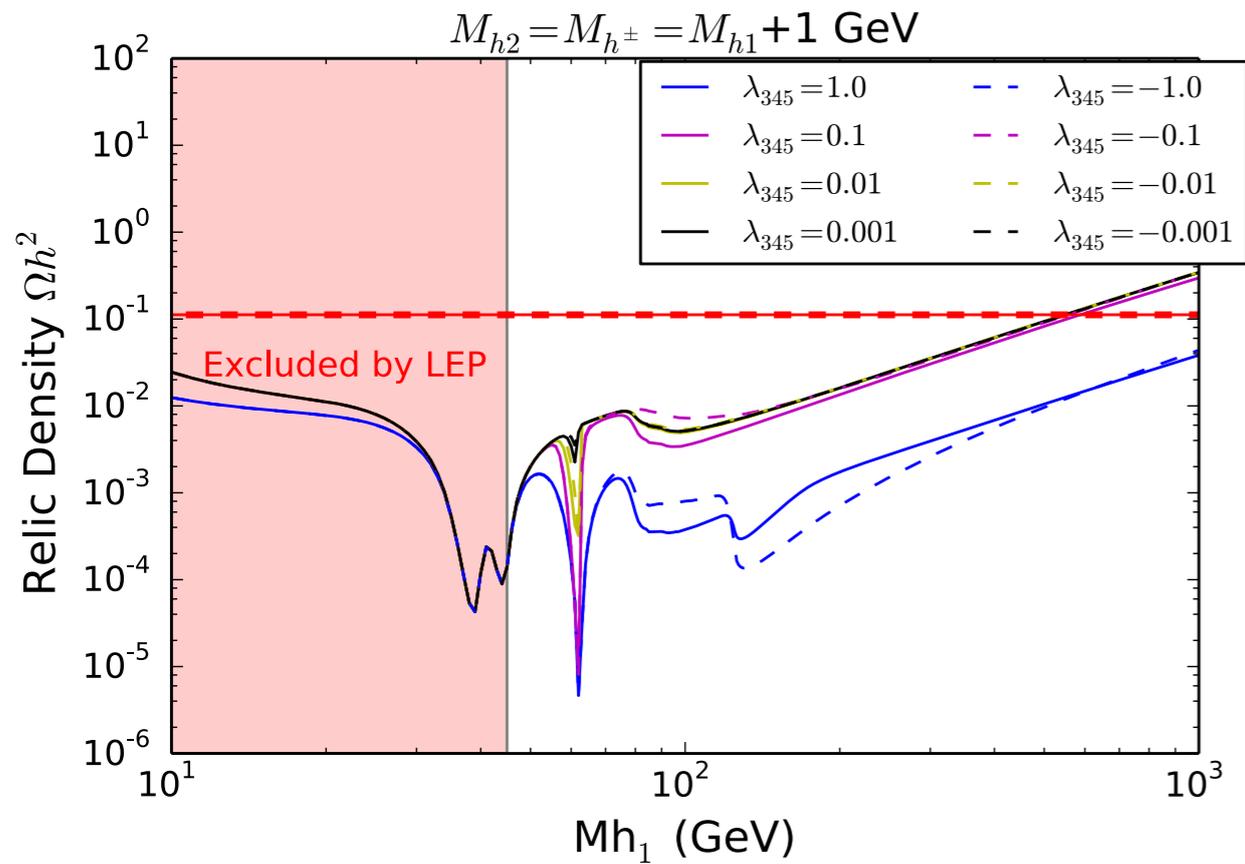


singlet DM: almost excluded by LHC, Planck, Fermi and LUX
white region still allowed

Inert doublet model

- 2HDM type I with Z_2 symmetry has natural DM candidate
- $\langle H_2 \rangle = 0$, i.e. inert, in large parts of parameter space, odd under Z_2
- Heavier scalars from this doublet decay into the lightest one
- DM annihilation, co-annihilation and collider signatures possible
- Masses are

$$M_{h^+}^2 = \frac{1}{2}\lambda_3 v^2 + m_{22}^2, \quad M_{h_1}^2, M_{h_2}^2 = \frac{1}{2}(\lambda_3 + \lambda_4 \mp |\lambda_5|)v^2 + m_{22}^2.$$



quasi degenerate and large mass splitting limits in IDM
 small mass region ruled out by LHC and Planck
 red line: relic density

Flavour/family symmetries more symmetries

- Add the flavour symmetry of your choice
- Continuous → upon breaking might generate massless Goldstone bosons $U(1)$, $SU(3)$...

- Discrete Z_N , A_4 , S_3 , S_4 , Q_6 , Δ_{27} ...
→ might also generate accidental continuous symmetries

- Explain masses and mixings of quarks and leptons
- Usually also add more particles → Higgs



Flavour symmetries

- Mass hierarchy of quarks and charged leptons very hierarchical
- Mass hierarchy also in neutrino sector, but whether normal or inverted hierarchy unknown
- Small mixing in quarks, but large mixing in leptons
- Perhaps there is an underlying flavour/family symmetry

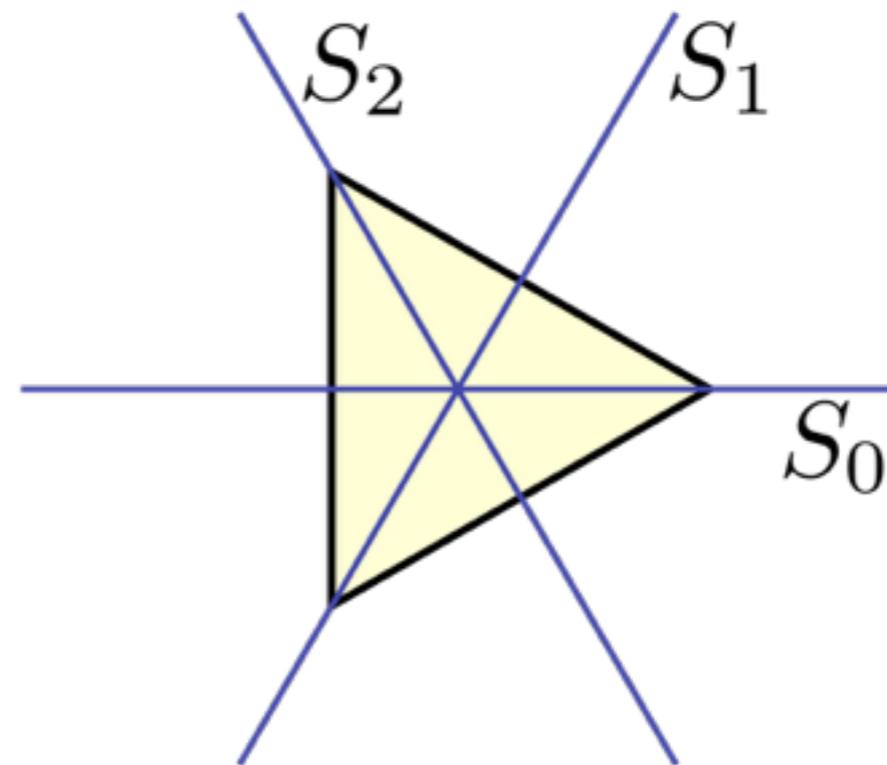
- Abelian or non-Abelian?
- Abelian symmetries have all uni-dimensional irreps
- Need non-Abelian symmetries, have irreps of more than one dimension \implies possible to explain the patterns of masses and mixings
- Discrete or continuous?
- Breaking of continuous symmetries leads to Goldstone bosons
- Discrete symmetries have several representations of small dimension \implies appropriate to describe three generations

- Non-Abelian, discrete groups widely studied recently
- Permutational symmetries: S_N , A_N
- Dihedral symmetries: D_N
- Double valued dihedral symmetries: D_N'
Other double valued groups: T' , O'
- Subgroups of $SU(3)$: $\Delta(3n^2)$, $\Delta(6n^2)$

3HDM with S_3

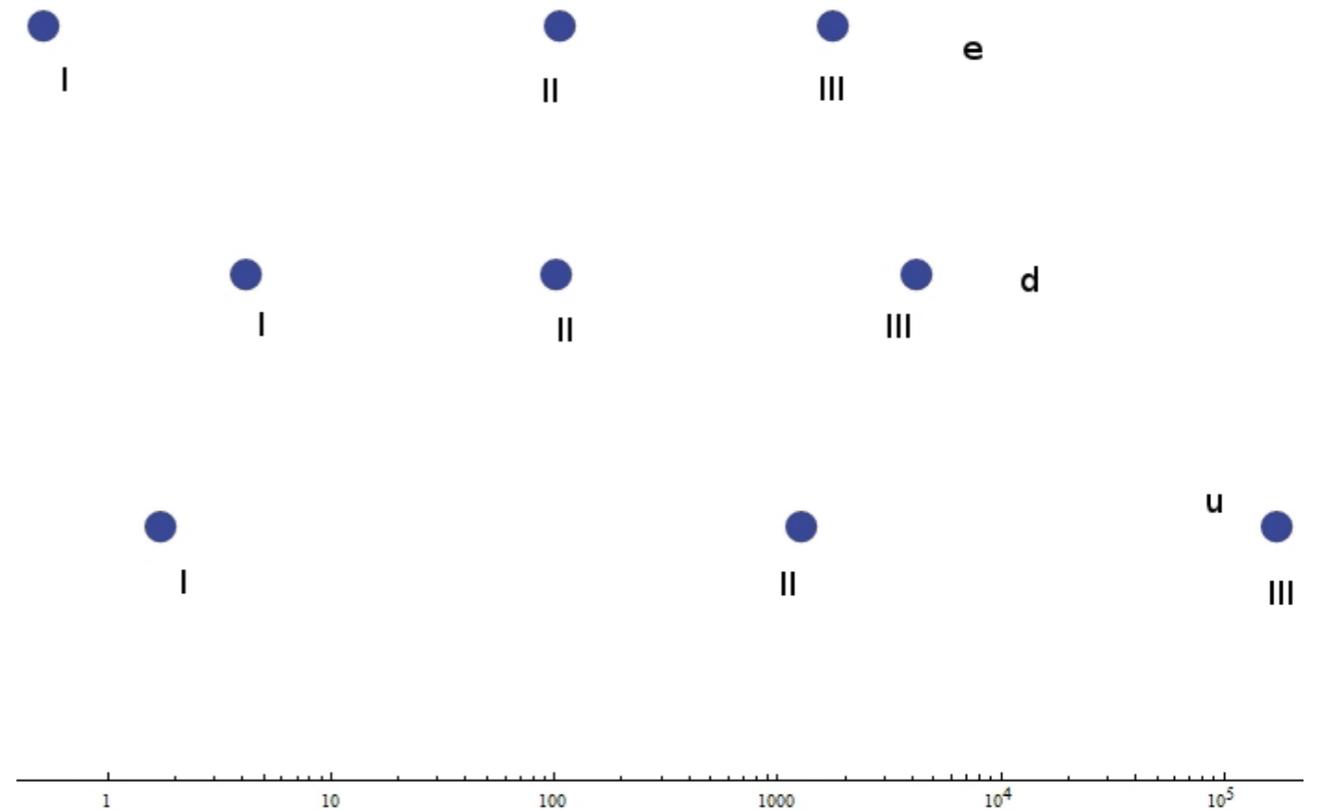
- Low-energy model
- Extend the concept of flavour to the Higgs sector by adding two more eW doublets
- Add symmetry: permutation symmetry of three objects, symmetry operations (reflections and rotations) that leave an equilateral triangle invariant

- 3HDM without symmetry: 57 couplings in the Higgs potential

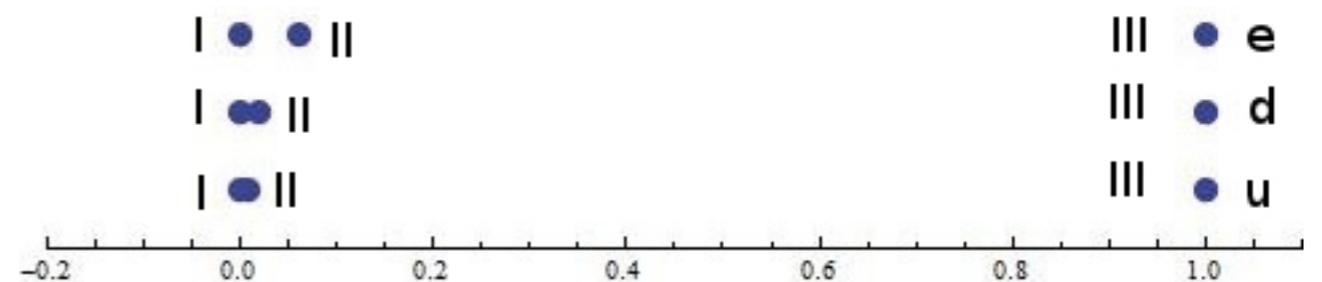


Why a particular symmetry?

- Prior to the eW symmetry breaking all families look the same \rightarrow permutation symmetry
- Smallest non-Abelian group discrete group S_3
- Has irreducible representations: $2, 1_S, 1_A$



Logarithmic plot of fermions



Mass ratios

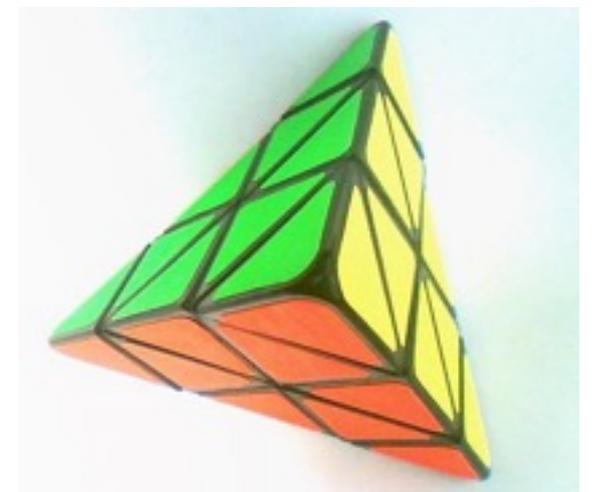
Predictions, advantages?

- Possible to reparametrize mixing matrices in terms of mass ratios, successfully
- Reproduces well CKM → one less parameter as SM
- PMNS → fix one mixing angle, predictions for the other two within experimental range
- Predicts reactor angle $\theta_{13} \neq 0$
- No extra flavons
- Higgs potential has 8 couplings
- Underlying symmetry in quark, leptons and Higgs → residual symmetry of a more fundamental one?
- Lots of Higgses:
3 neutral, 4 charged,
2 pseudoscalars
Natural decoupling limit
- Further predictions will come from Higgs sector:
decays, branching ratios

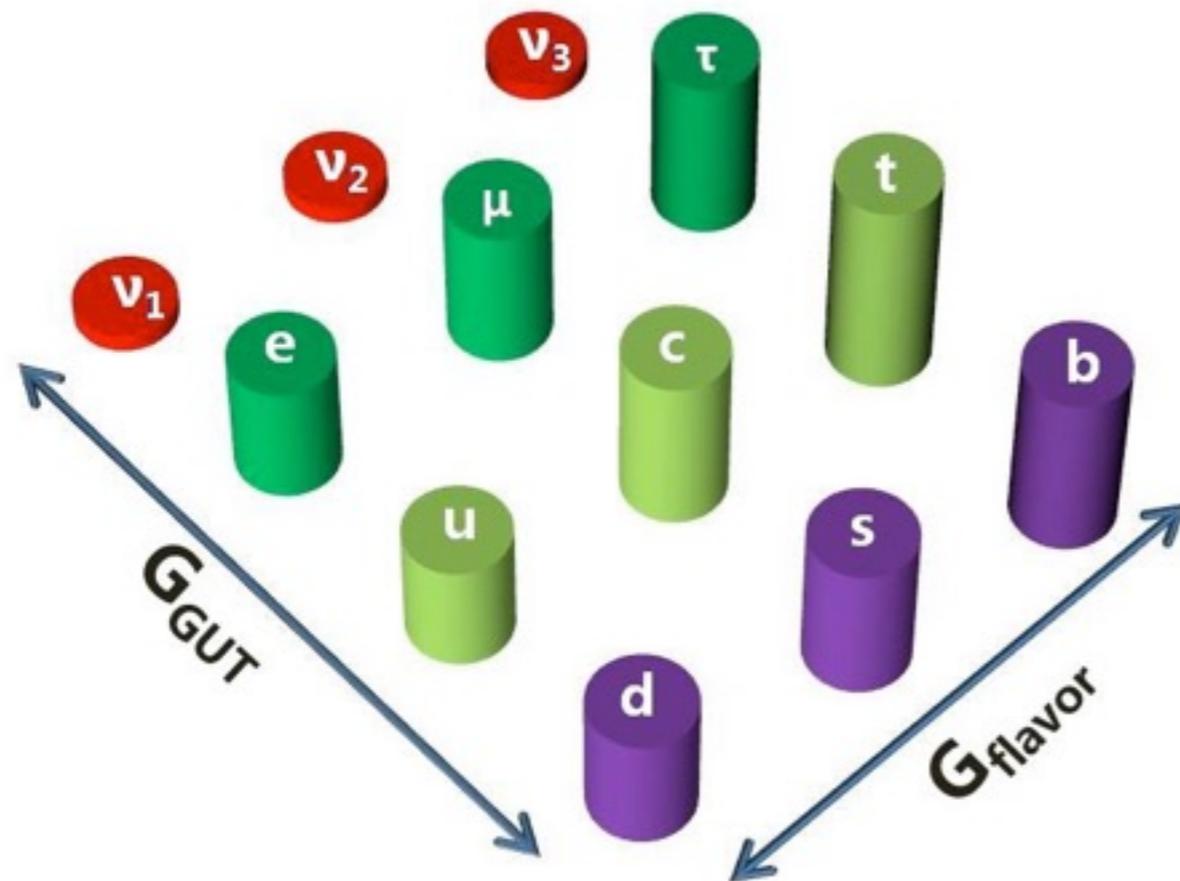
A4 symmetry

- Symmetry group of a tetrahedron, even permutations of four objects
- Four irreducible representations: 1, 1', 1'', 3
- Fermions can be nicely assigned to the 3
- It predicts the tri-bimaximal mixing, which is now ruled out by experiment
- Symmetry has to be broken softly to reproduce the mixing pattern
- Deviations from tri-bimaximal mixing may come from higher dimensional operators from neutrino sector

$$\sin^2(\theta_{12}^{TB}) = \frac{1}{3}, \quad \sin^2(\theta_{23}^{TB}) = \frac{1}{2}, \quad \sin^2(\theta_{13}^{TB}) = 0.$$



Add a discrete flavour symmetry to a SUSY GUT theory



Interplay between
the different symmetries
(and their breaking)
would lead to an explanation
of the fermion mass hierarchies

- $SU(5) \times A_4$, $SU(5) \times Q_6$, $SU(5) \times S_4 \times U(1)$,
 $SU(5) \times T_7 \dots$
- $SO(10) \times S_3$, $SO(10) \times S_4$, $SO(10) \times D_4$, $SO(10) \times A_4$,
 $SO(10) \times \Delta(54) \dots$
- Many of these models also have extra Abelian discrete symmetries Z_N , besides the GUT and the flavour symmetries

- The addition of discrete symmetries might lead to accidental continuous symmetries
- After eW symmetry breaking there might be surprises \implies unwanted Goldstone bosons from the continuous symmetries
- There might also be residual discrete symmetries \implies can spoil the pattern of masses and mixings

Extra $U(1)$'s

- Well motivated, they appear naturally when breaking large unification groups to SM
- Also appear in string compactifications
- In SUSY GUTs and string compactifications $U(1)$ ' and $SU(2) \times U(1)$ breaking tied to soft SUSY breaking
- At TeV, proposed to alleviate the quadratic divergences in the Higgs mass
- Dynamical symmetry breaking models often include $U(1)$'s
- In some models associated with almost-hidden sectors

How do we sieve through models/ideas

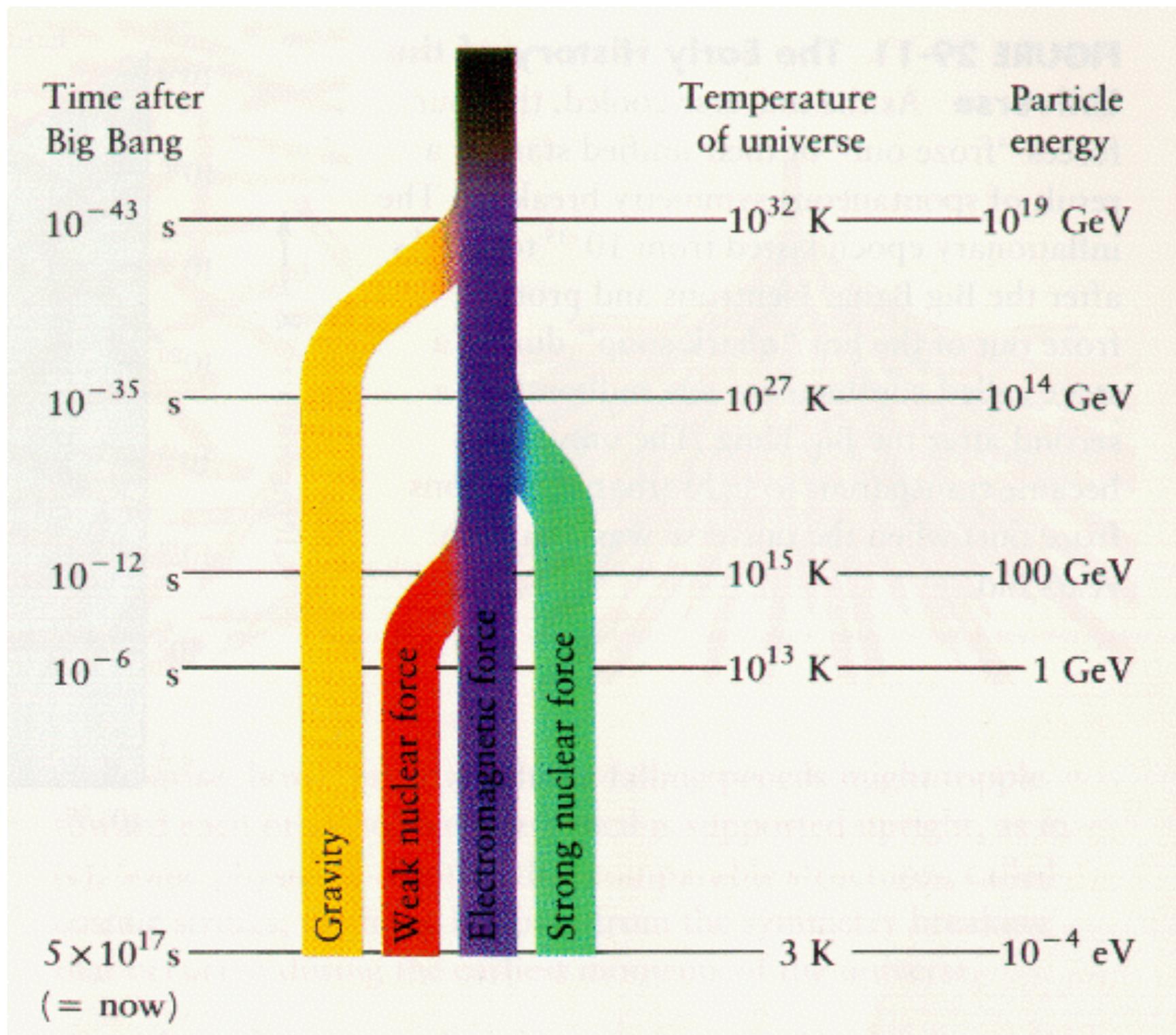
- Interplay between theory and experiment
- Models have to provide predictions and/or explanations to the SM open questions
- Each extension BSM has assumptions that have to be spelled out explicitly to be able to test it
- Applies to theory and experiment
- Theoretical uncertainties have to be taken into account (not only experimental)
- Most successful models combine more than one idea

Going BSM — assumptions

- Usually do not break Lorentz or gauge invariance
These symmetries are too fundamental to mess around with...
- Respect experimental results (not everybody likes to)
These are facts of life, even if for some of them origin not understood
- Guiding principles: mathematical consistency and experimental compatibility
- Many assumptions are a matter of “taste”, acknowledge it

Where (in energy) are these models realized?

- Depends...
 - some models are valid around few TeV and a “bit” above
 - some models assumed valid to TeV to the GUT or Planck scale
 - some models valid only at GUT or Planck scale
- If they are described by a QFT we can use Renormalization Group Equations (RGE) to test them at different scales



String theory

GUTS

ν_R

SUSY

Extra
Higgses

other
symmetries

Conclusions

- There is unknown physics to explore → BSM
- To discover it we need an interplay between mathematical consistency (theories), experiments and observations in particle physics, cosmology and astrophysics
- There is a lot of work to do...