

Constructing Stable Observables with Energy Correlation Functions



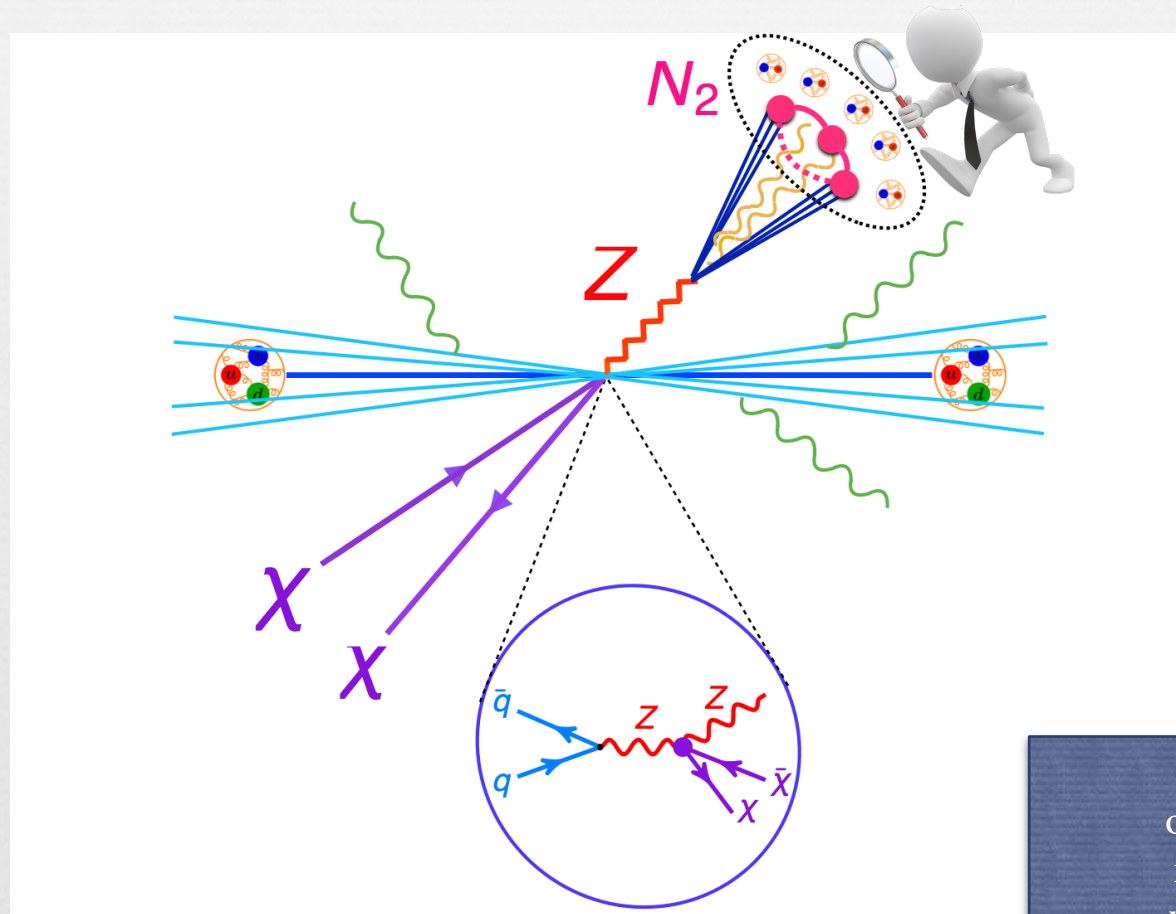
Lina Necib

MIT

Based on arXiv:1609.07483

In collaboration with Ian Moutl and Jesse Thaler

Jet Substructure



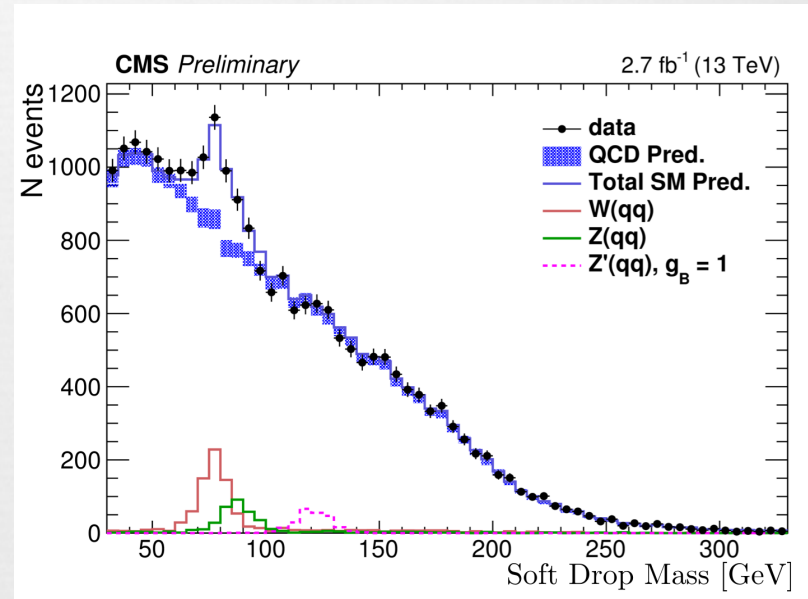
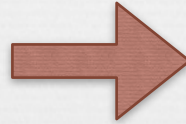
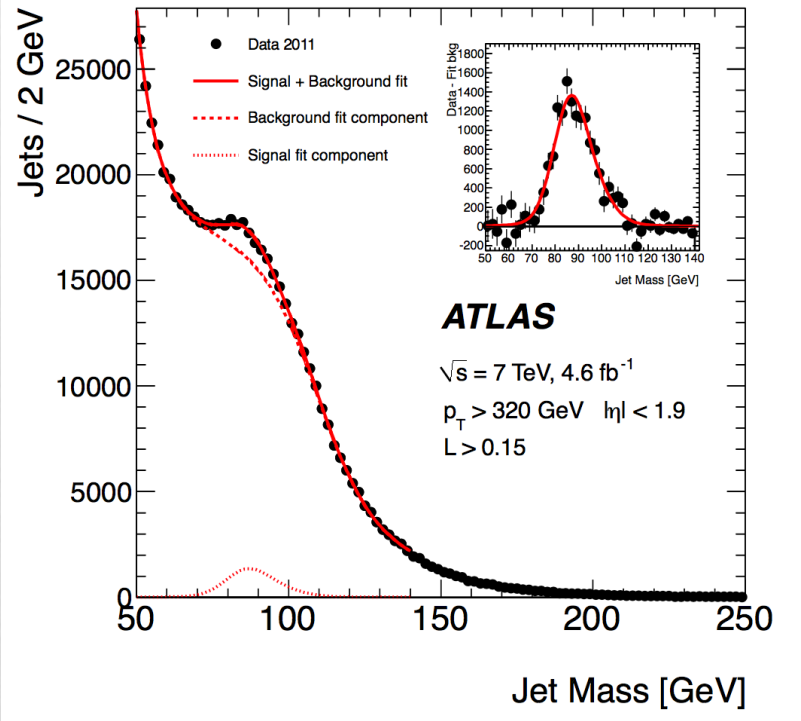
Jet substructure observables play an important role in a variety of searches, e.g. dark matter.



Absolute Performance

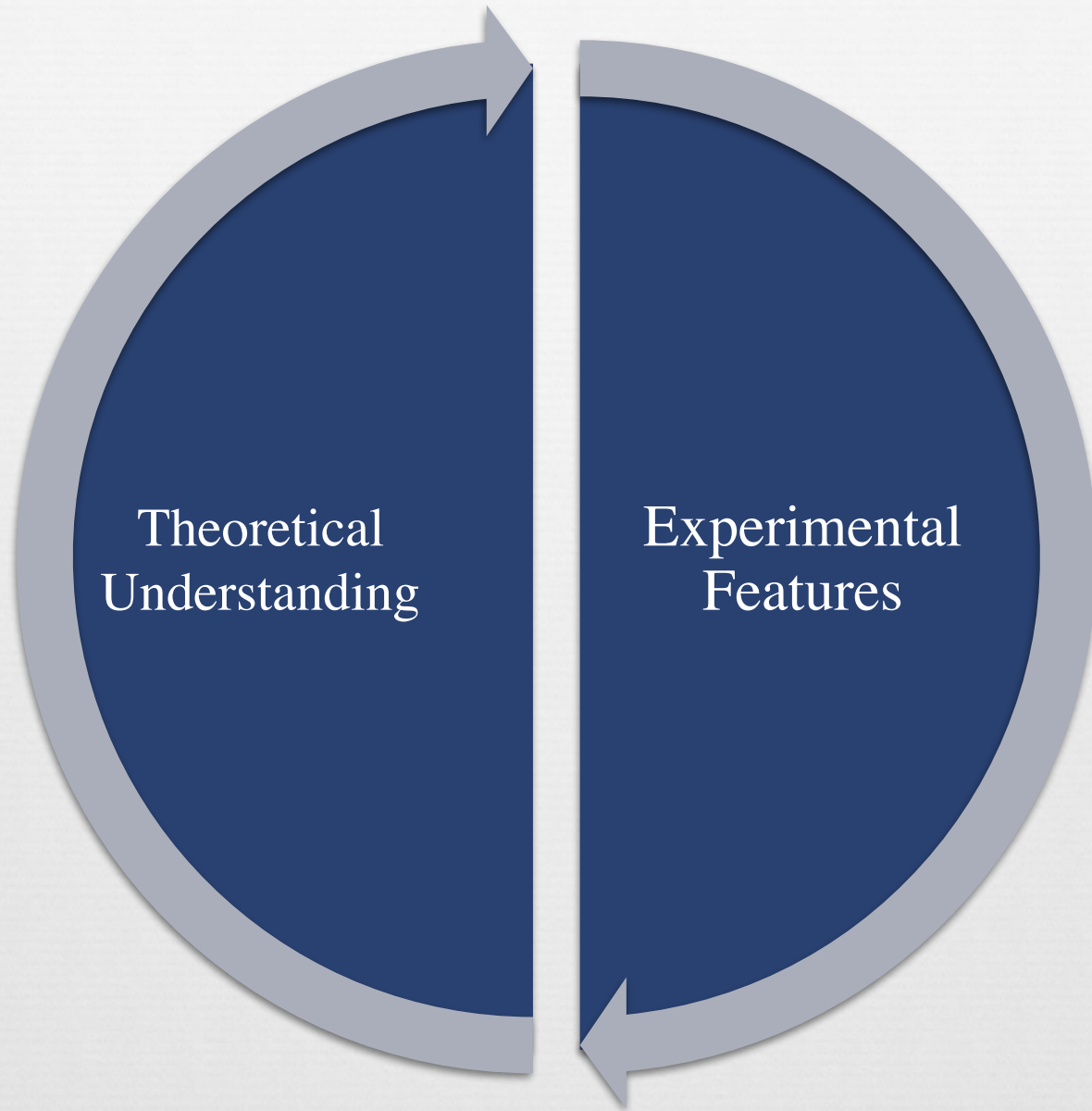
Stability

Stability in m_J and p_{TJ}

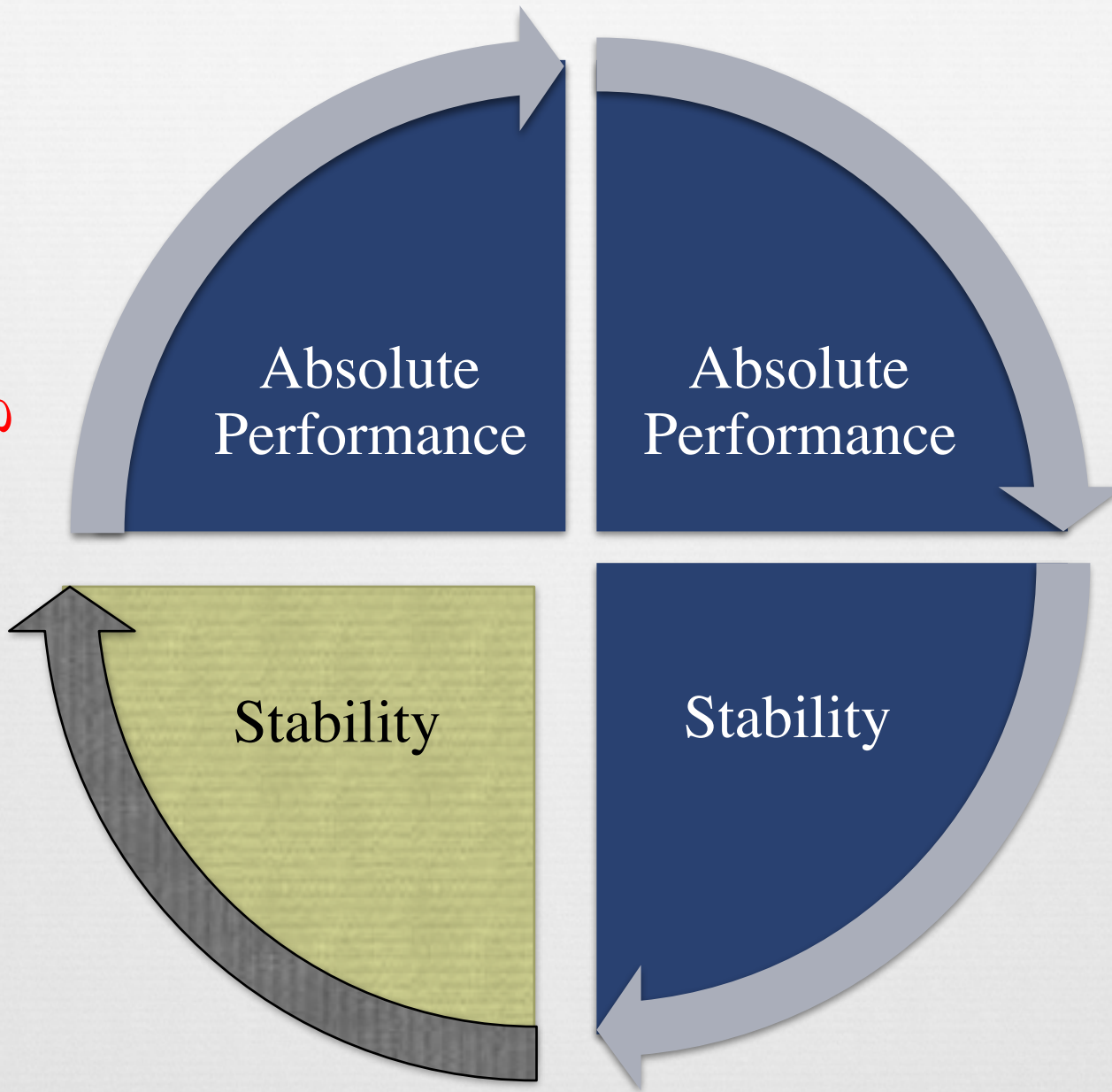


Use more stable substructure observables leads to improved performance. For example, DDT.

[Dolen, Harris, Marzani, Rappoccio, Tran 1603.00027]

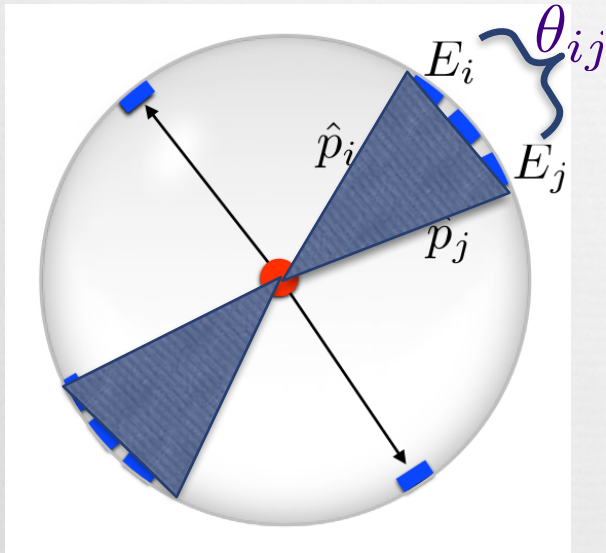


**Theoretical
Understanding**



**Experimental
Features**

Problem: Unstable Observables



$$D_2^{(\beta)} = \frac{e_3^{(\beta)}}{\left(e_2^{(\beta)}\right)^3} = \frac{\text{[Diagram of a fan of lines]} }{\left(\text{[Diagram of a fan of lines]}\right)^3}$$

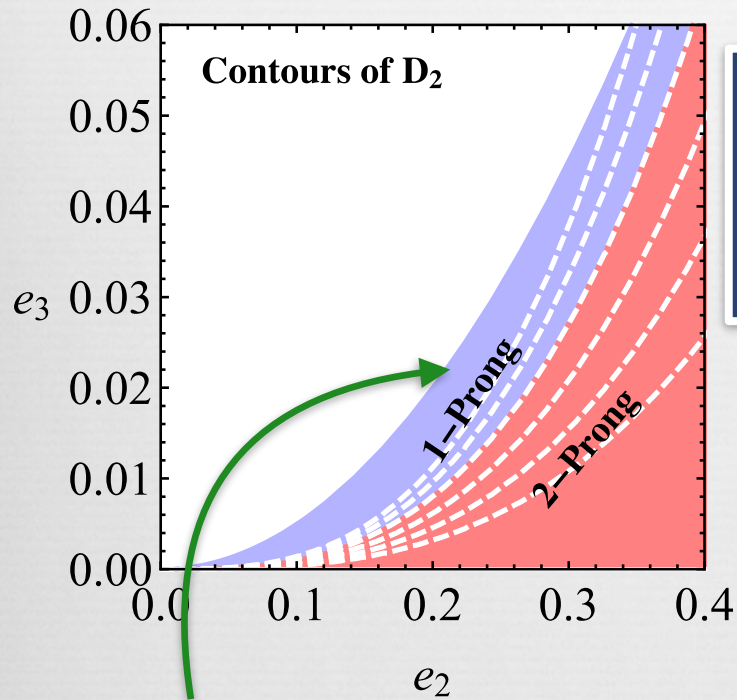
The equation shows the definition of $D_2^{(\beta)}$ as a ratio of $e_3^{(\beta)}$ to $(e_2^{(\beta)})^3$. The diagrams illustrate the variables: a fan of lines for $e_3^{(\beta)}$ and a cube of such fans for $(e_2^{(\beta)})^3$.



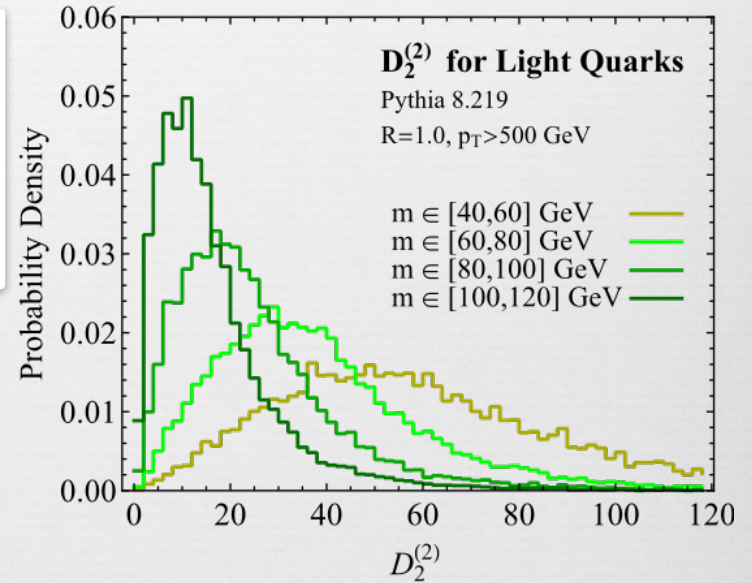
∞ D_2 is designed from power counting to optimize performance.

[Larkoski, Moul, Neill 1507.03018]

Problem: Unstable Observables



Can easily detect
power law
scaling with
power counting!



$$D_2^{(2),\max} \sim \frac{e_3^{(2)}}{(e_2^{(2)})^3} \sim \frac{(e_2^{(2)})^2}{(e_2^{(2)})^3} \sim \frac{1}{(e_2^{(2)})} \sim \frac{p_T^2}{m_J^2}$$

No power law scaling
means “parametric
stability”

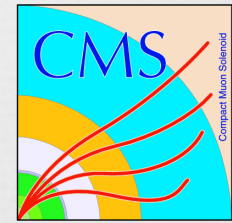
Stability



There are multiple ways we can make stable observables:

Groom away soft radiation

Construct an intrinsically stable observable

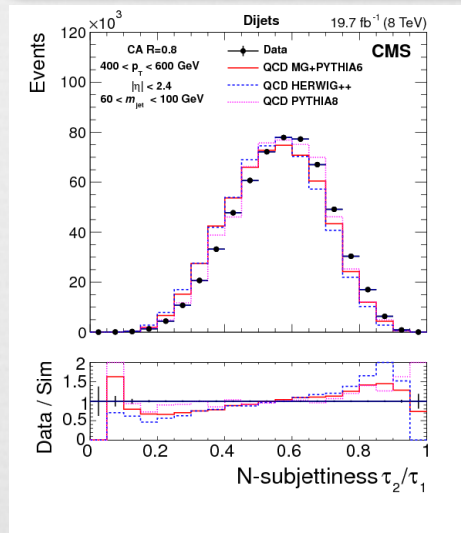
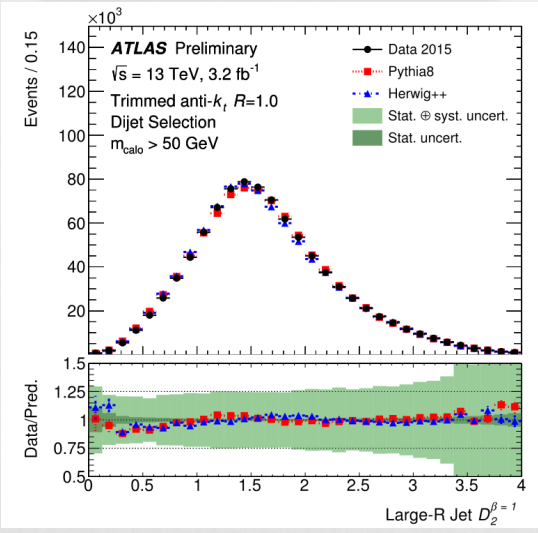


N-subjettiness

Further numerical instability can be removed by DDT.



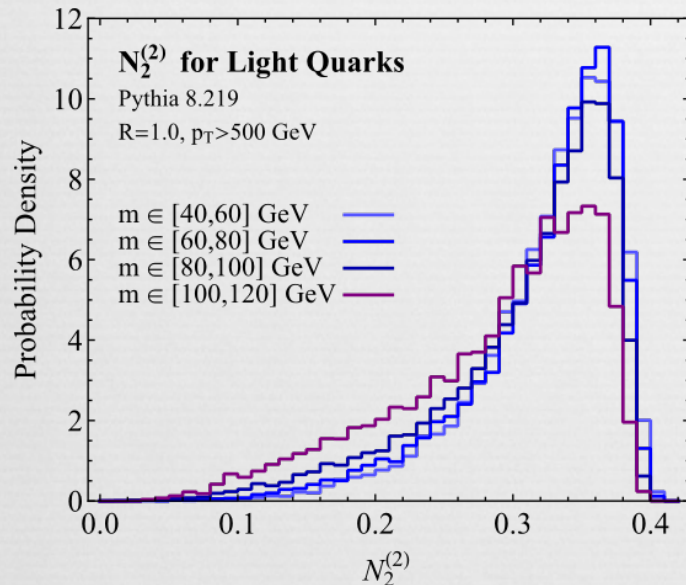
R2D2 tagger



Stability



☞ New energy correlation based observable N_2 **parametrically stable** with and without grooming!



$$N_2^{(\beta)} = \frac{2e_3^{(\beta)}}{\left(e_2^{(\beta)}\right)^2} = \frac{\text{Diagram 1}}{\left(\text{Diagram 2}\right)^2}$$

The diagram on the right shows two jet-like structures. The top one has three red dots at the ends of lines, and the bottom one has two red dots. Lines connect the dots, representing energy correlations.

Further numerical instability can be removed by DDT.

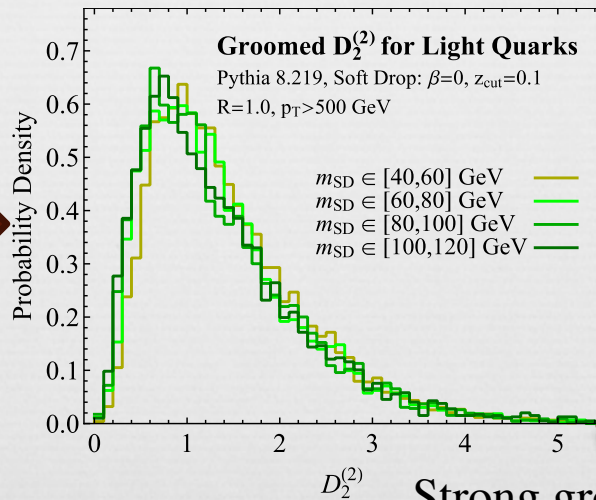
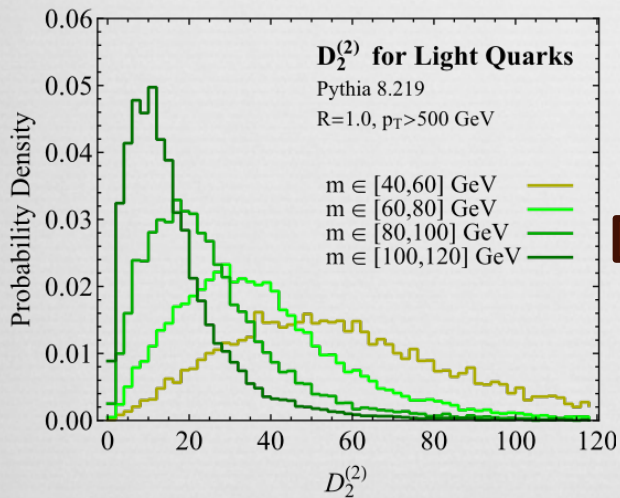
☞ N_2 power counts similarly as N-subjettiness, but avoids pathological issues with axes.

Strategy 1: Groom away soft radiation



ATLAS used trimming to improve stability of D_2

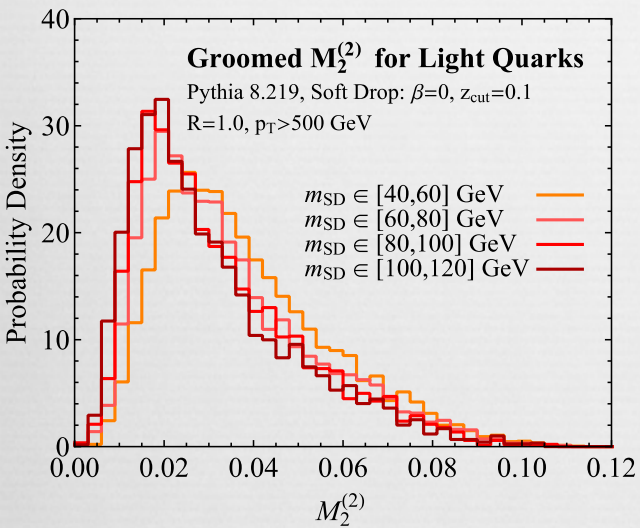
- Softdrop
- Pruning
- Trimming
- Mass Drop
- ...



[Ellis, Vermilion, Walsh 0903.5081]
 [Krohn,Thaler,Wang 0912.1342]
 [Dasgupta, Fregoso, Marzani, Salam 1307.0007]
 [Larkoski, Marzani, Soyez, Thaler 1402.2657]

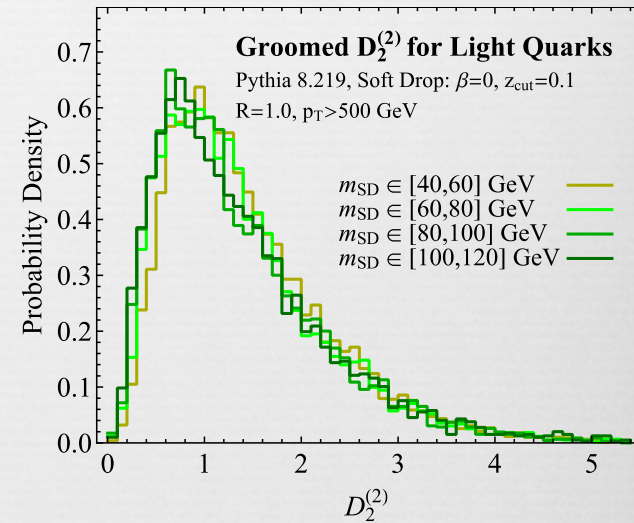
Strong grooming stabilizes the shape

Strategy 1: Groom away soft radiation



$$M_2^{(\beta)} = \frac{1 e_3^{(\beta)}}{(e_2^{(\beta)})}$$

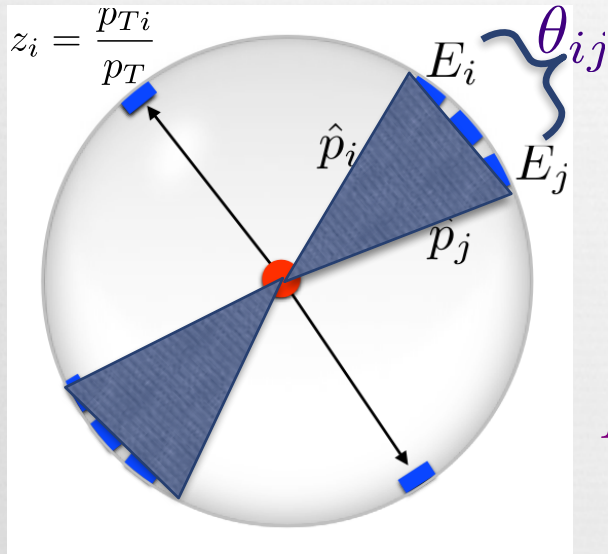
$$D_2^{(1,2)} = \frac{e_3^{(1)}}{(e_2^{(2)})^{3/2}}$$



We can build more observables which are stable after grooming.



Strategy 2: Build a stable observable

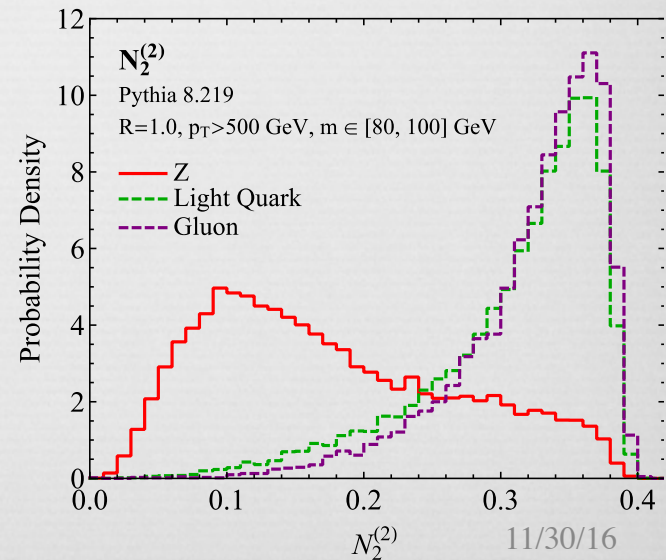


$$N_2^{(\beta)} = \frac{2e_3^{(\beta)}}{(e_2^{(\beta)})^2} = \frac{\text{[Diagram of Z decay]} }{\text{[Diagram of quark decay]}}$$

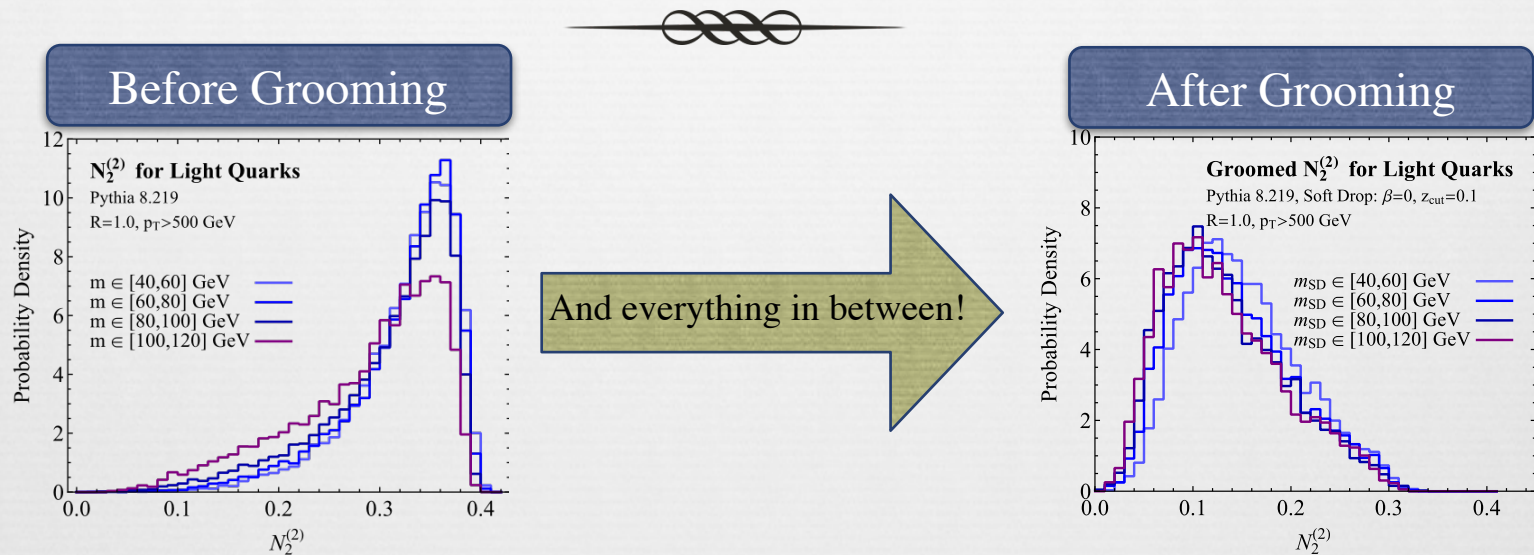
$$N_2^{(\beta)} = \frac{2e_3^{(\beta)}}{(e_2^{(\beta)})^2} = \frac{\sum_{i,j,k} z_i z_j z_k \min(\theta_{ij}^\beta \theta_{kj}^\beta, \theta_{ij}^\beta \theta_{ik}^\beta, \theta_{kj}^\beta \theta_{ik}^\beta)}{(\sum_{i,j} z_i z_j \theta_{ij}^\beta)^2}$$

- ☞ Theoretically motivated for a good discrimination between 1 and 2-prong jets.
- ☞ Stable under changes of p_T and mass cuts!

Further numerical instability can be removed by DDT.



Strategy 2: Build a stable observable

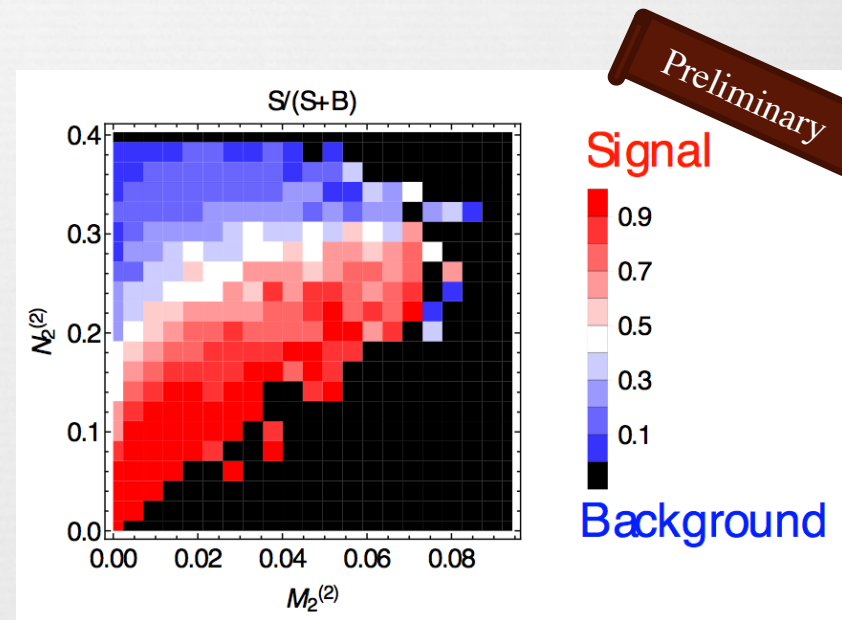
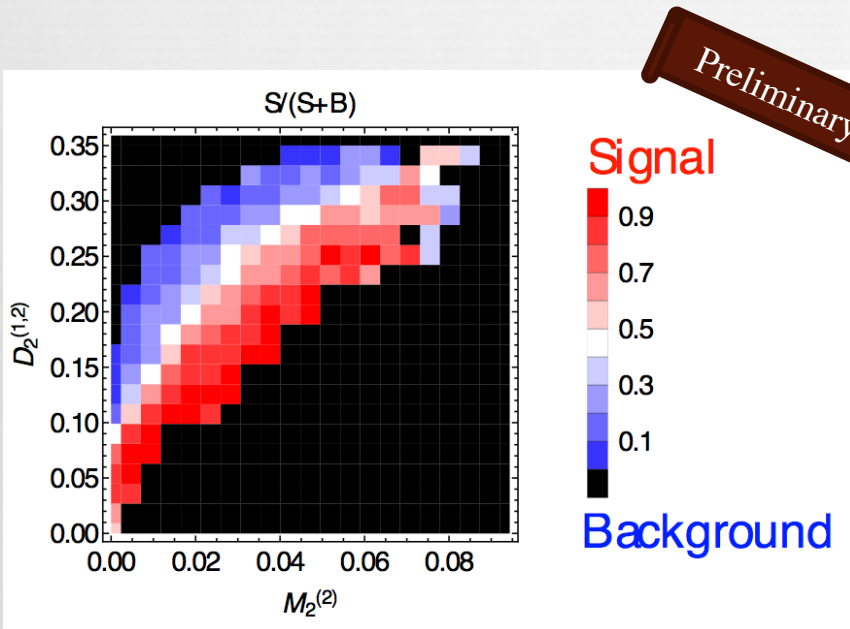


- ☞ Theoretically motivated for a good discrimination between 1 and 2-prong jets.
- ☞ Stable under changes of p_T and mass cuts!

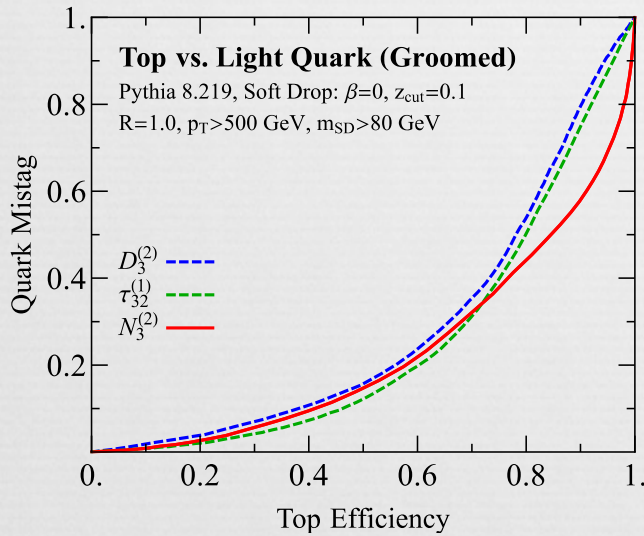
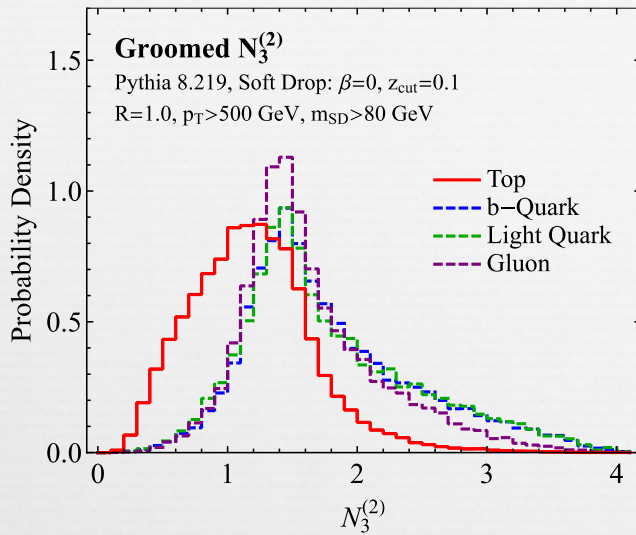
Further instability can be removed by DDT.



Looking ahead: Correlations



- Now we have 3 observables constructed from the same functions.
- Interesting to think about extending 1-D to 2-D observables.



Top Tagging



Extension of the same observables to
3 prong.

$$N_3^{(\beta)} = \frac{2e_4^{(\beta)}}{(e_3^{(\beta)})^2} = \frac{\text{Diagram 1}}{(\text{Diagram 2})^2}$$

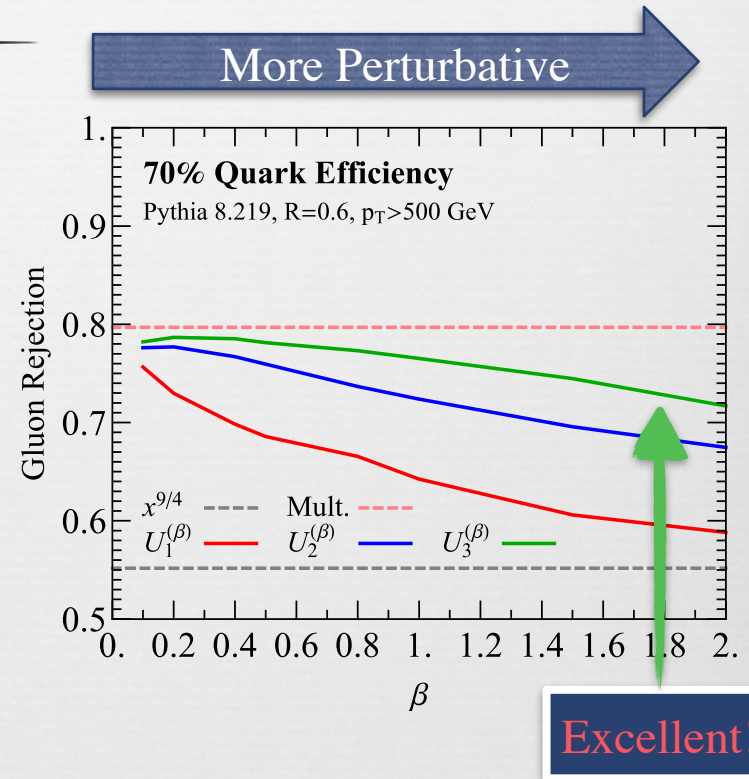
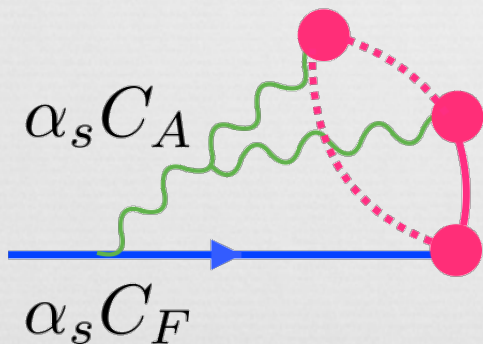
$$N_3^{(\beta)} = \frac{2e_4^{(\beta)}}{(e_3^{(\beta)})^2}$$

Quark/Gluon discrimination

Quarks and Gluons have different color factors, which allows for discrimination. Probing multiple emissions improves discrimination.

$$U_n^{(\beta)} = 1 e_n^{(\beta)}$$

This observable is a measure of the “number” of emissions!



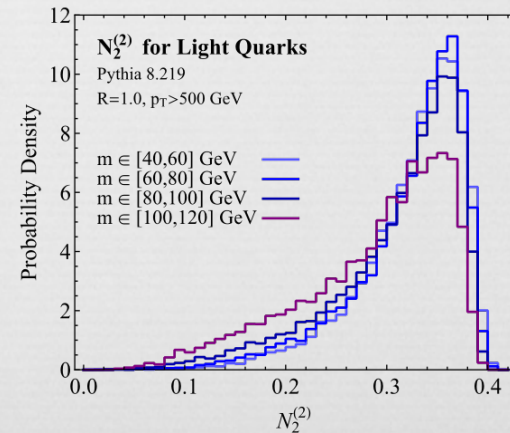
Stability

- ⌘ We have the ability to construct observables with particular parametric features.
- ⌘ Power counting is a useful diagnostic tool (see backup slides for CMS hybrid strategy).
- ⌘ We focused on stability because it is of experimental significance.
- ⌘ **Experimental input is appreciated!**

Groom away soft radiation

Construct an intrinsically stable observable

$$N_2^{(\beta)} = \frac{2e_3^{(\beta)}}{\left(e_2^{(\beta)}\right)^2} = \frac{\text{Diagram 1}}{\left(\text{Diagram 2}\right)^2}$$

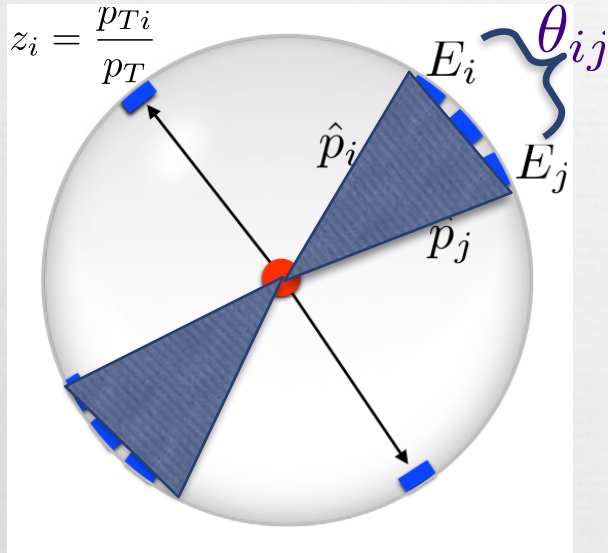


⌘ All observables are available in fastjet contrib under EnergyCorrelator 1.2.0

Backup Slides

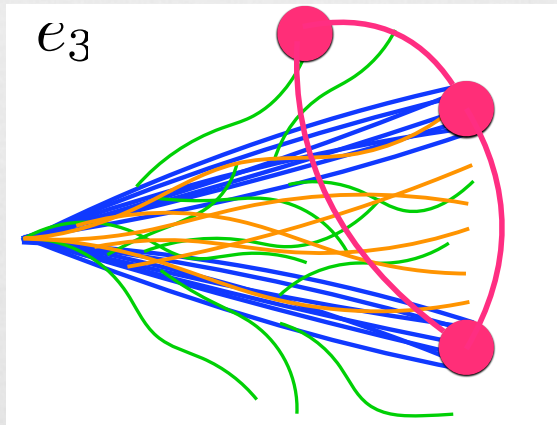


Exploring Old Angles



$$e_2^{(\beta)} = \sum_{i,j} z_i z_j \theta_{ij}^\beta$$

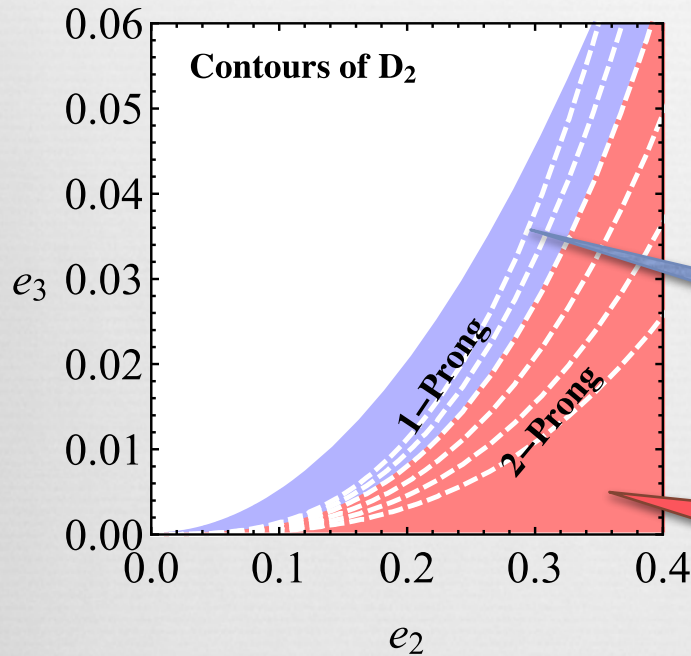
$$e_3^{(\beta)} = \sum_{i,j,k} z_i z_j z_k \theta_{ij}^\beta \theta_{kj}^\beta \theta_{ik}^\beta$$



D_2



$$D_2^{(\beta)} = \frac{e_3^{(\beta)}}{(e_2^{(\beta)})^3}$$



Background:

$$(e_2^{(\beta)})^3 < (e_3^{(\beta)}) < (e_2^{(\beta)})^2$$

Signal:

$$(e_3^{(\beta)}) < (e_2^{(\beta)})^3$$

From power counting, D_2 is the optimal observable obtained from the "old" energy correlation functions for 2-prong discrimination.

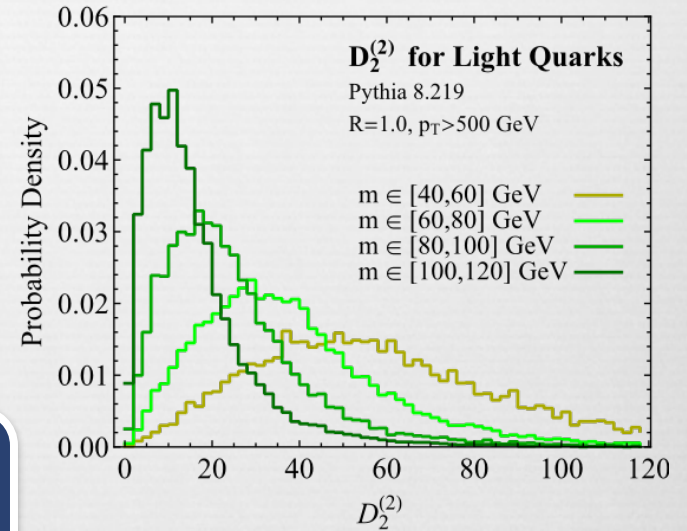
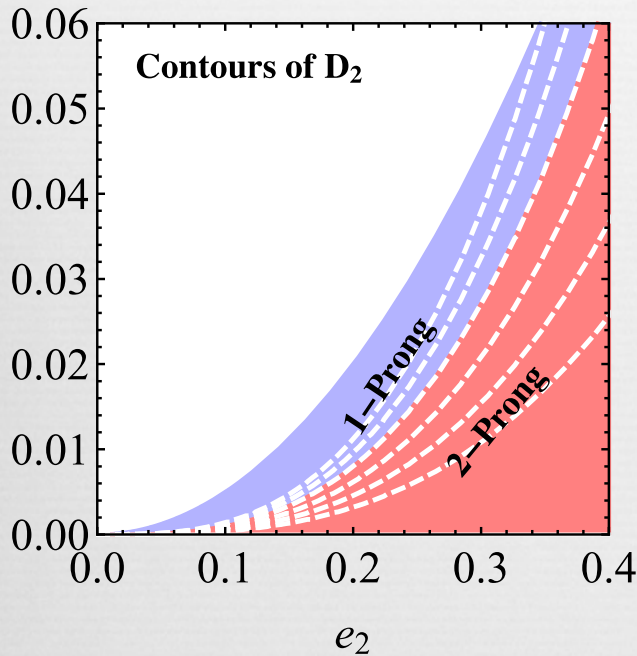
[Larkoski, Moult, Neill 1507.03018]

D_2



$$D_2^{(\beta)} = \frac{e_3^{(\beta)}}{(e_2^{(\beta)})^3}$$

BUT:
 D_2 is unstable!

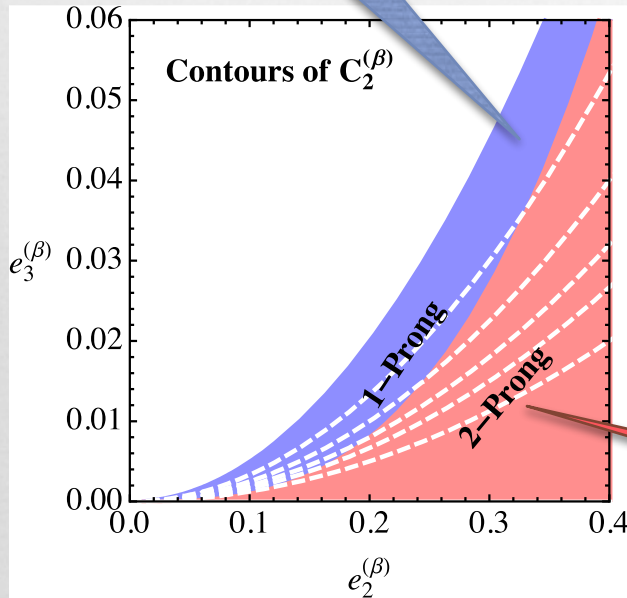


- From power counting, D_2 is the optimal observable obtained from the "old" energy correlation functions for 2-prong discrimination.
- Indeed it has been adopted at ATLAS.

[Larkoski, Moult, Neill 1507.03018]

Background

C_2



$$C_2^{(\beta)} = \frac{e_3^{(\beta)}}{(e_2^{(\beta)})^2}$$

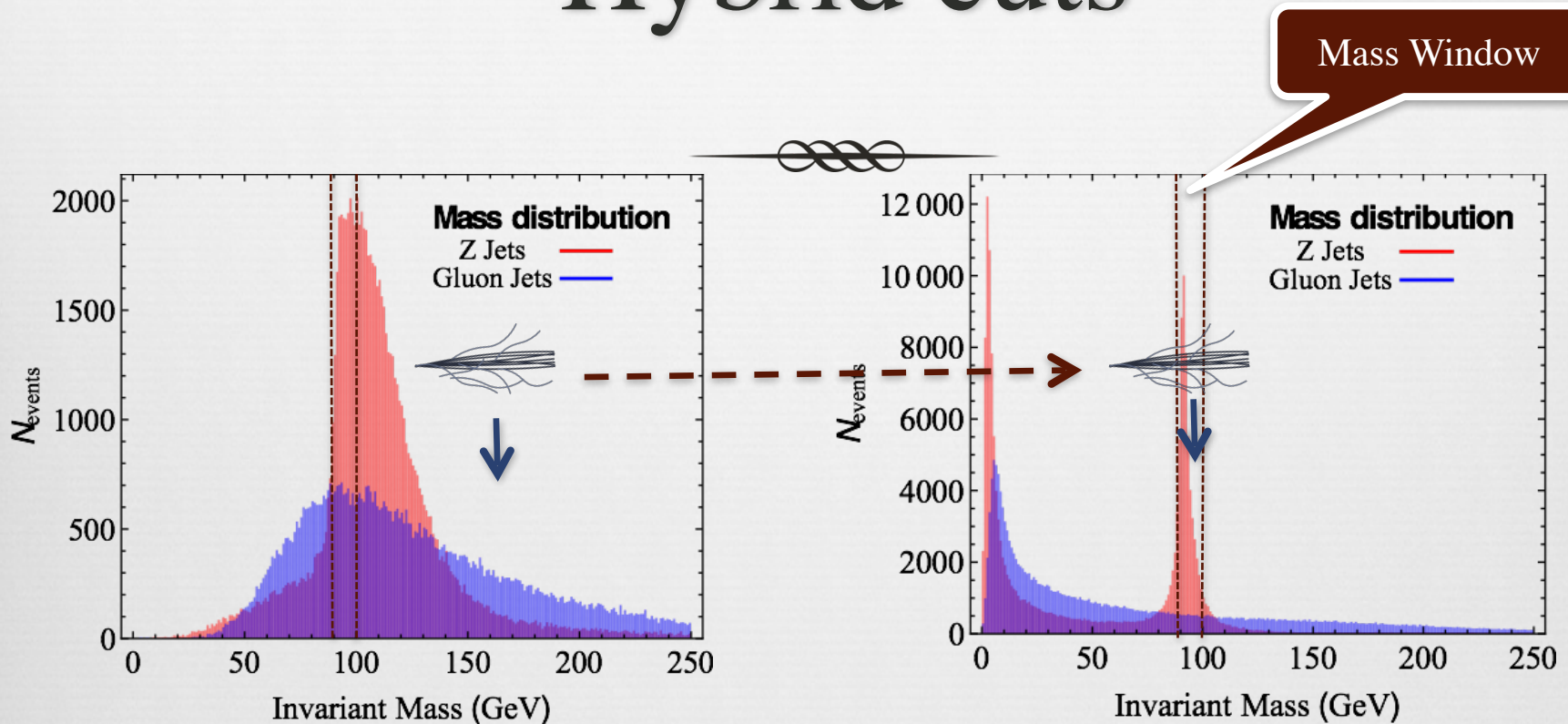
Signal

BUT:

It can be shown from power counting that C_2 is not an optimal choice.

It is used by CMS with a "hybrid" cut.

Hybrid cuts

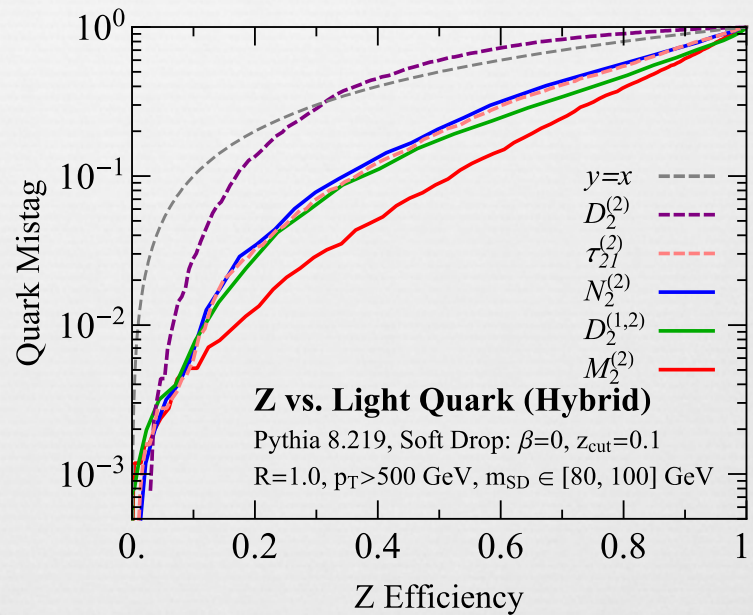


- ⌘ Strategy adopted by CMS, which makes C_2 perform well.
- ⌘ Mass cut on the groomed mass.
- ⌘ Effectively selects for higher mass.

Hybrid cuts



- ☞ Strategy adopted by CMS.
- ☞ Mass cut on the groomed mass.
- ☞ Effectively selects for higher mass.



$$D_2^{(2), \text{max}} \sim \frac{P_{TJ}^2}{m_J^2}$$

$$N_2^{(2), \text{max, peak}} \sim \text{const}$$

$$M_2^{(2), \text{peak}} \sim \frac{m_J^2}{P_{TJ}^2}$$



Hybrid cuts

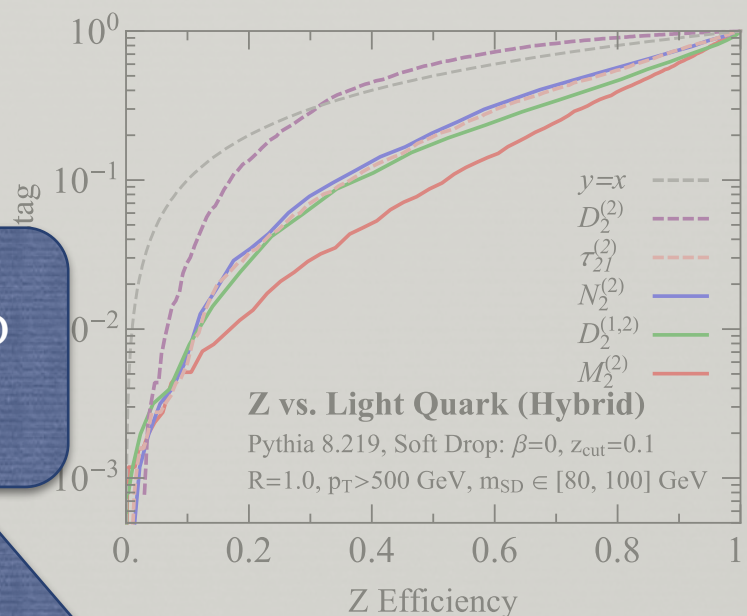


Strategy adopted by CMS.

Max
gro

Stable under standard AND hybrid cuts!

Eff
higher mass.



$$D_2^{(2),\text{max}} \sim \frac{P_{TJ}^2}{m_J^2}$$

$$N_2^{(2),\text{max,peak}} \sim \text{const}$$

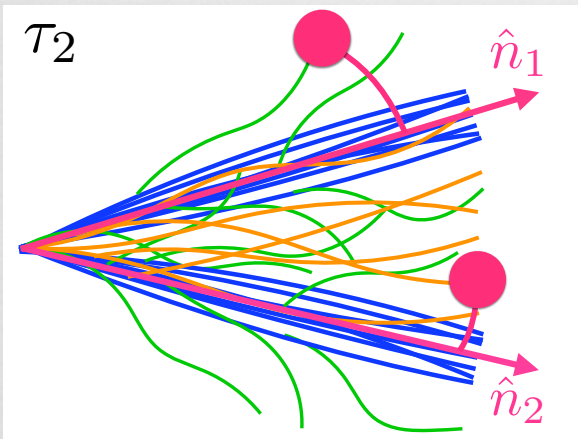
$$M_2^{(2),\text{peak}} \sim \frac{m_J^2}{P_{TJ}^2}$$

N-subjettiness



∞ N-subjettiness

$$\tau_{2,1} = \frac{\tau_2}{\tau_1}$$



Background:

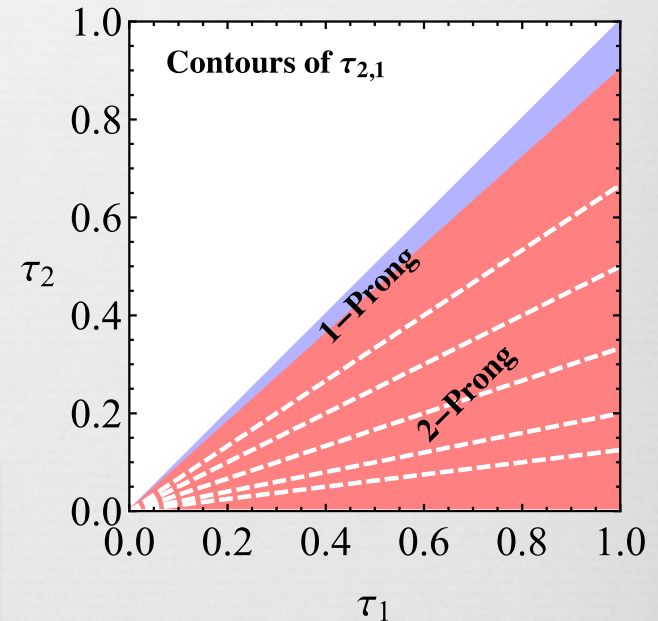
$$\tau_2 \sim \tau_1$$

Signal:

$$\tau_2 < \tau_1$$

BUT:

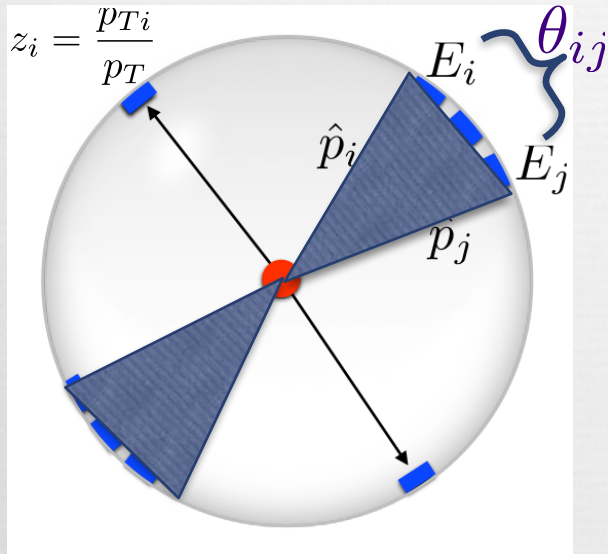
Axes are undefined in the unresolved limit.
Harder to calculate!



[Stewart, Tackmann, Waalewijn 1004.2489]

[Thaler, Van Tilburg 1011.2268]

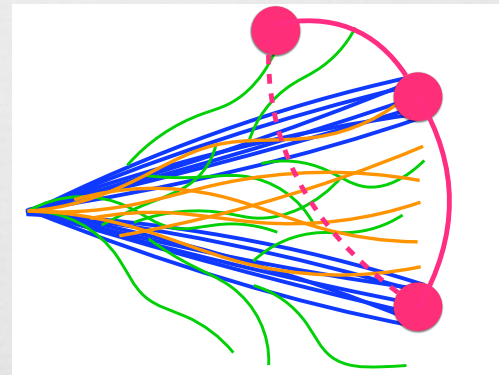
Exploring New Angles



$$e_2^{(\beta)} = \sum_{i,j} z_i z_j \theta_{ij}^\beta$$

$$1e_3^{(\beta)} = \sum_{i,j,k} z_i z_j z_k \min(\theta_{ij}^\beta, \theta_{kj}^\beta, \theta_{ik}^\beta)$$

$$2e_3^{(\beta)} = \sum_{i,j,k} z_i z_j z_k \min(\theta_{ij}^\beta \theta_{kj}^\beta, \theta_{ij}^\beta \theta_{ik}^\beta, \theta_{kj}^\beta \theta_{ik}^\beta)$$



Observables of the Day



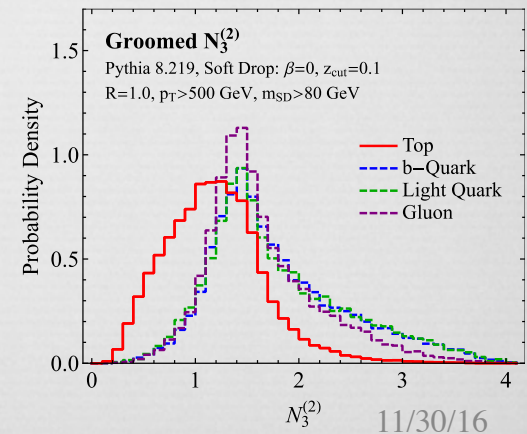
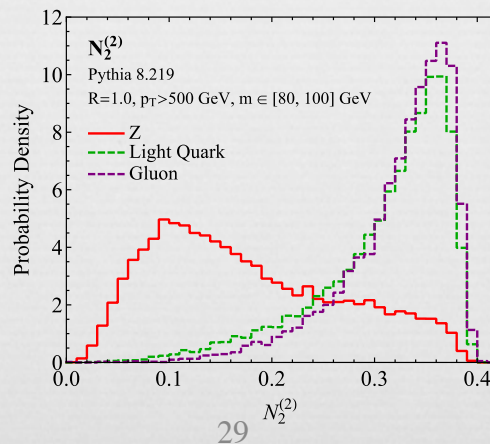
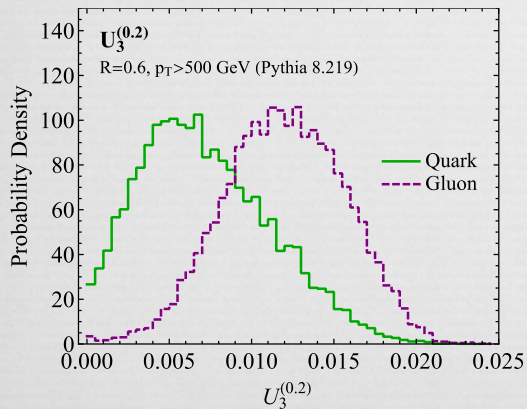
U_n



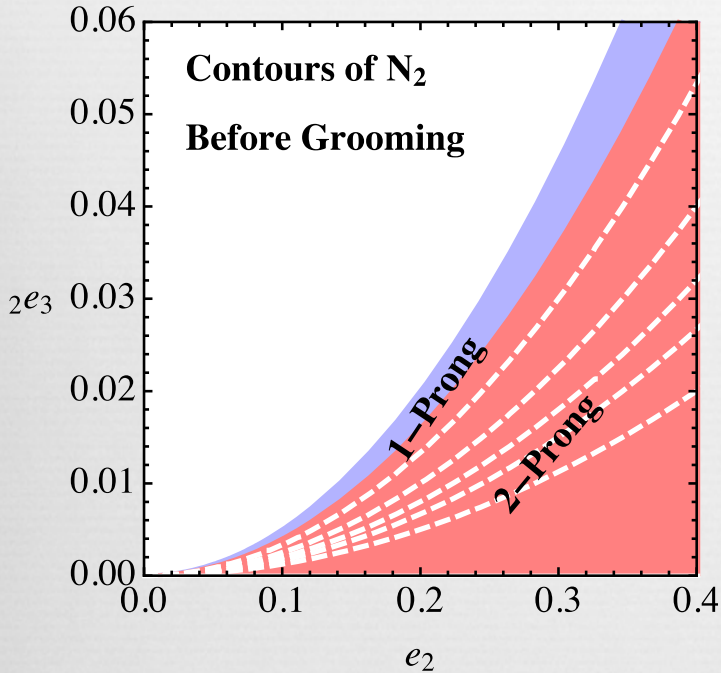
$N_2, M_2, D_2^{(1,2)}$



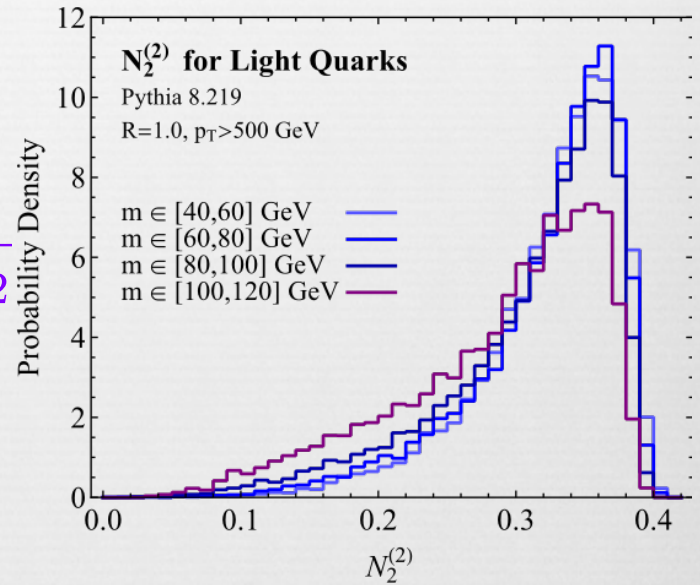
N_3



N_2



$$N_2^{(\beta)} = \frac{2e_3^{(\beta)}}{(e_2^{(\beta)})^2}$$



But most importantly, N_2 is **stable!**

$$N_2^{(2), \max} \sim \frac{(e_2^{(\beta)})^2}{(e_2^{(\beta)})^2} \sim \text{const}$$

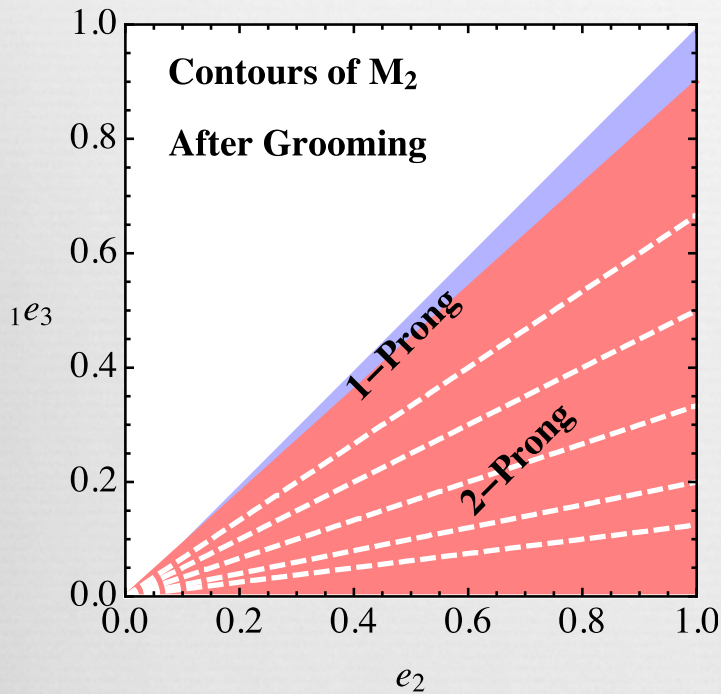
Background:

$$(2e_3^{(\beta)}) \sim (e_2^{(\beta)})^2$$

Signal:

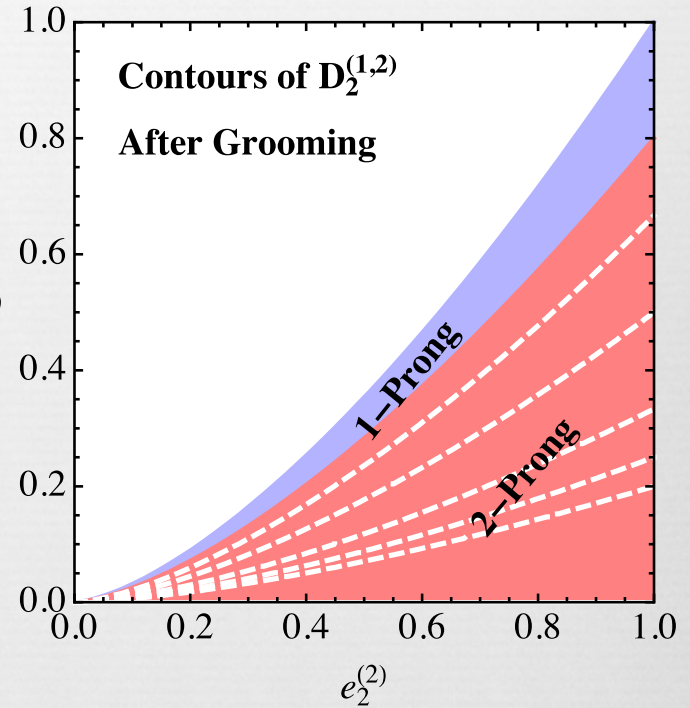
$$(2e_3^{(\beta)}) < (e_2^{(\beta)})^2$$

More Observables After Grooming



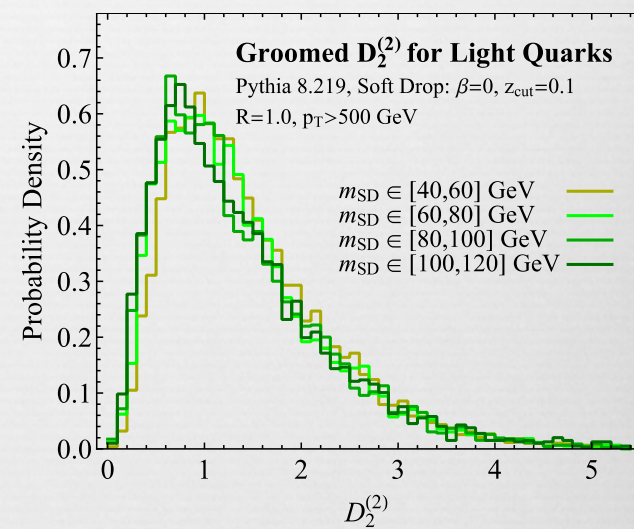
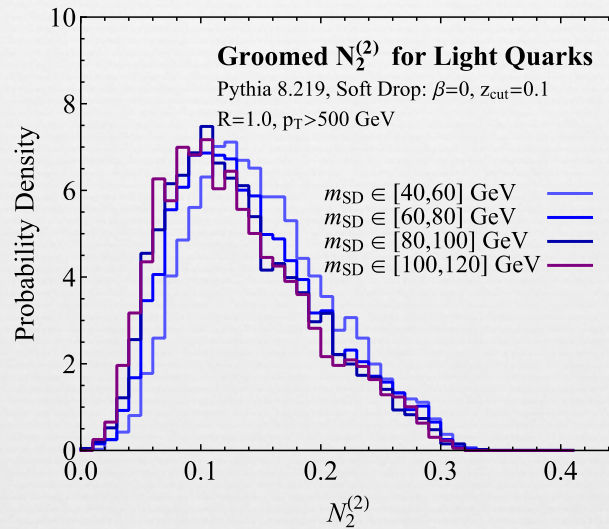
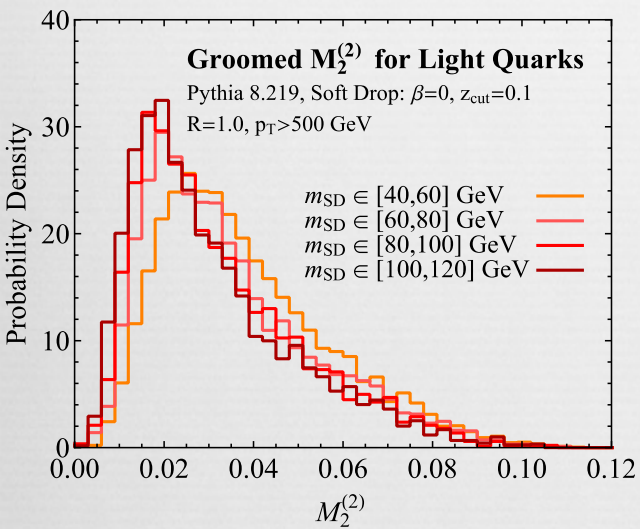
$$M_2^{(\beta)} = \frac{1 e_3^{(\beta)}}{(e_2^{(\beta)})}$$

$$D_2^{(1,2)} = \frac{e_3^{(1)}}{(e_2^{(2)})^{3/2}}$$



☞ New observables are stable after grooming!

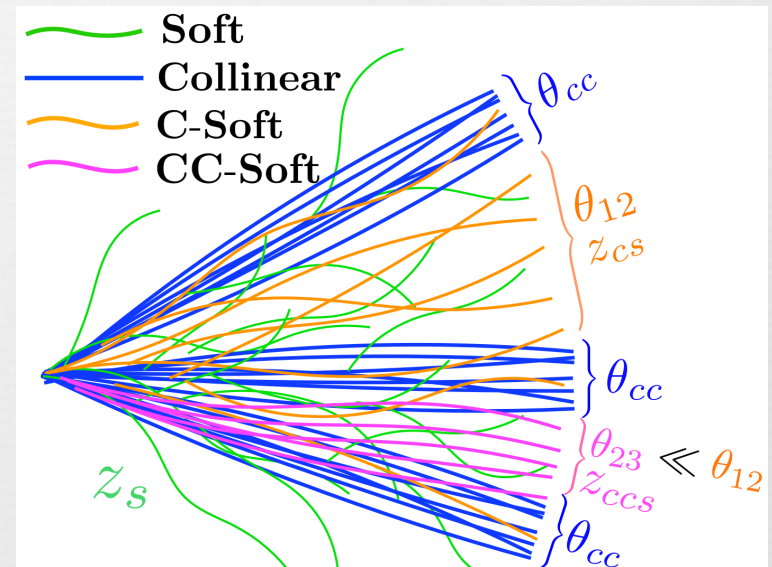
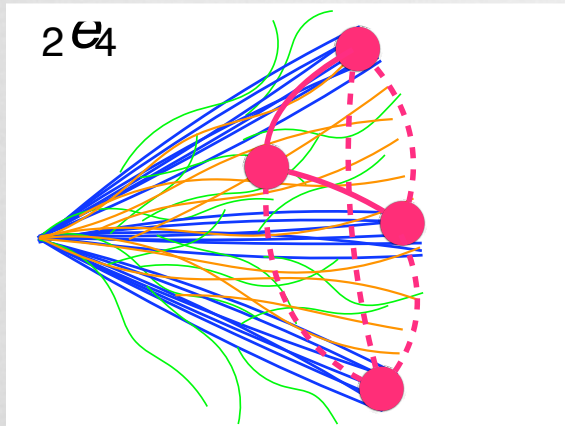
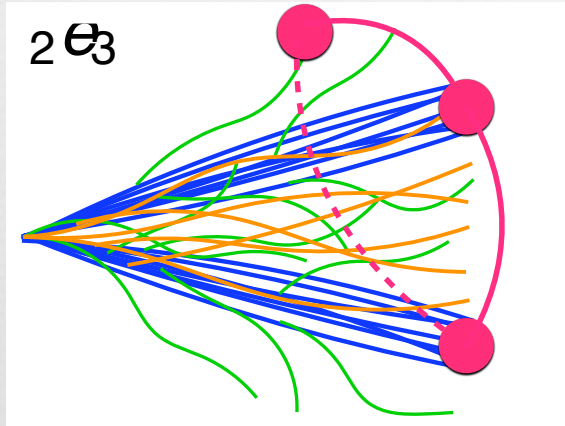
Stability after Grooming



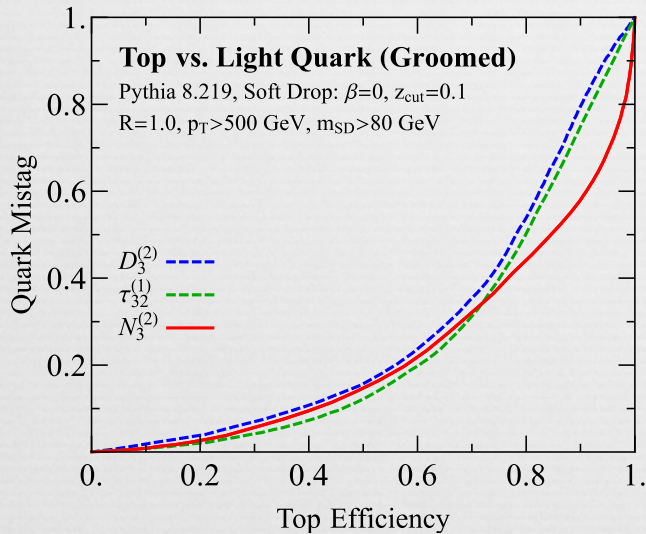
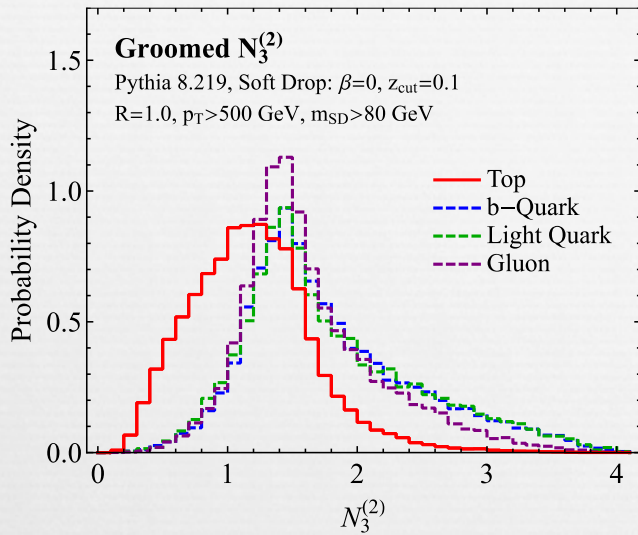
☞ Groomed observables are **stable!**

Build the same strategy for other searches:

searches:

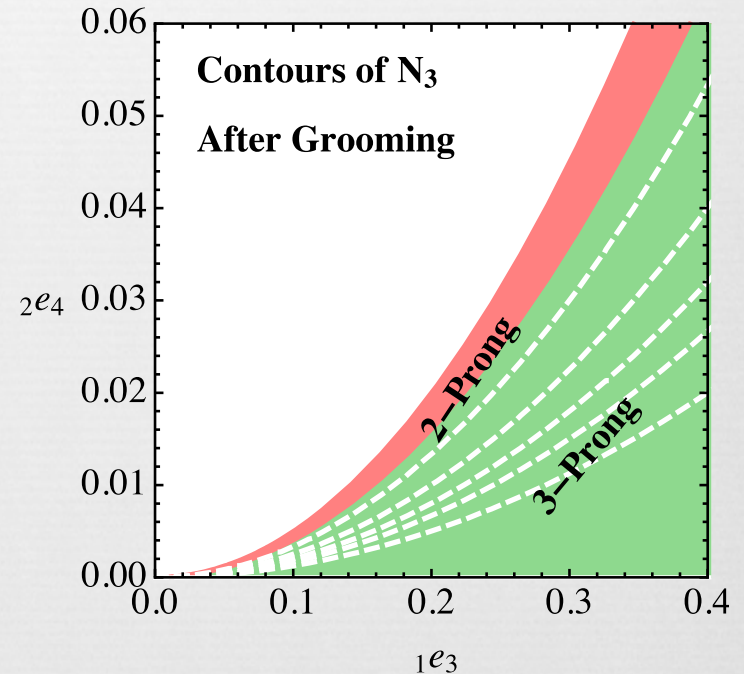


☞ Extending the generalized energy correlation functions from 2 prong, to 3 prong.



$$N_3^{(\beta)} = \frac{2e_4^{(\beta)}}{(e_3^{(\beta)})^2}$$

Top Searches

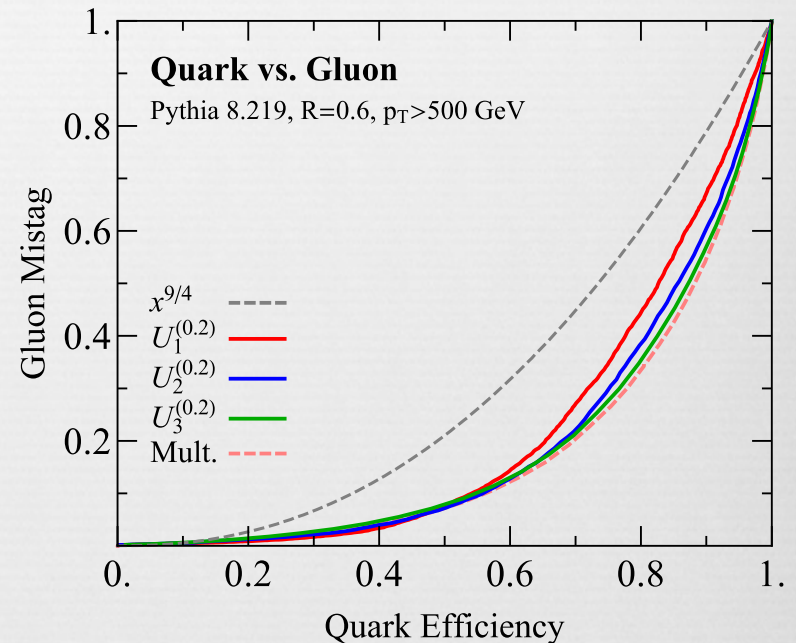


Quark/Gluon discrimination

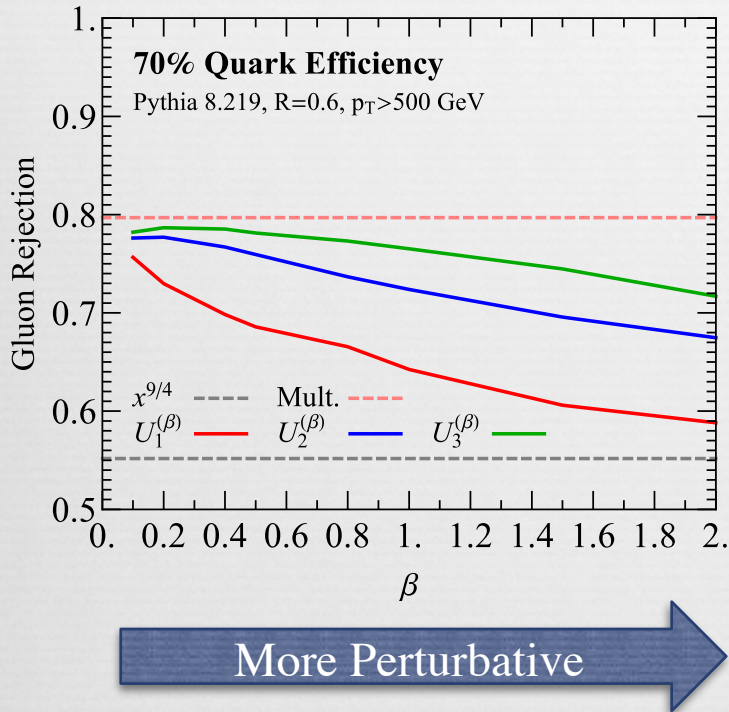


- Beyond Casimir scaling by probing multiple emissions.
- U_n asymptotes to a measurement of multiplicity.

$$U_n^{(\beta)} = 1 e_n^{(\beta)}$$



Quark/Gluon discrimination



$$U_n^{(\beta)} = 1 e_n^{(\beta)}$$

- ⌘ Small beta, small angles, non perturbative regime.
- ⌘ Stability of U_3 as a function of beta is a great feature. High beta, more control.