

# Fluctuating fluid dynamics for the QGP in the LHC and BES era

**Marcus Bluhm** Marlene Nahrgang Thomas Schäfer Steffen Bass

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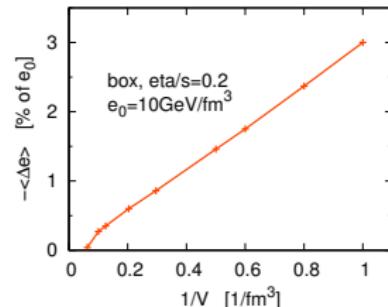
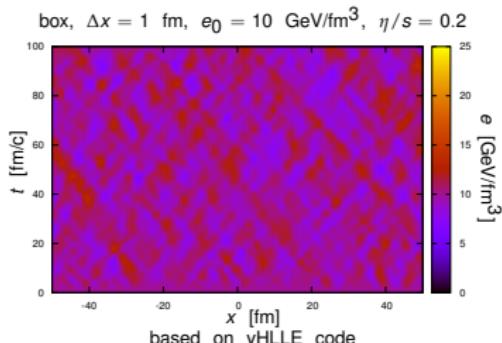
# Fluid dynamical fluctuations in heavy-ion collisions

The average particle spectra are successfully described using conventional fluid dynamics. → What about fluctuations?

$$\partial_\mu T^{\mu\nu} = \partial_\mu \left( T_{\text{eq}}^{\mu\nu} + \Delta T_{\text{visc}}^{\mu\nu} + \Xi^{\mu\nu} \right) = 0$$
$$\partial_\mu N^\mu = \partial_\mu \left( N_{\text{eq}}^\mu + \Delta N_{\text{visc}}^\mu + J^\mu \right) = 0$$

- important consequence: nonlinearities cause cutoff dependent corrections to EoS,  $\eta$  ... (e.g.  $\langle \Delta e \rangle \sim -\Lambda^3$ )

P. Kovtun et al., JHEP1407 (2014)

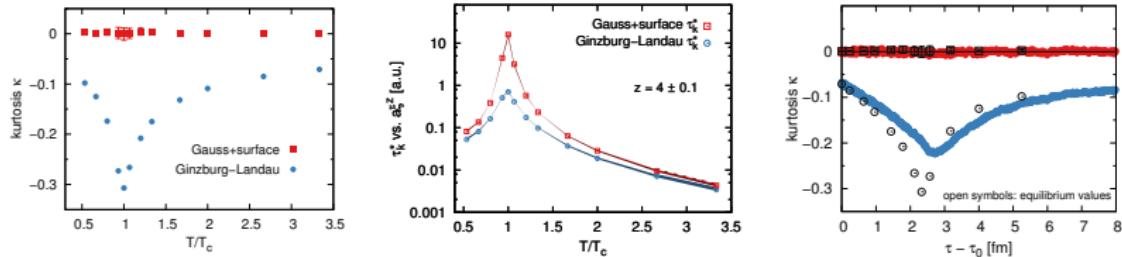


⇒ Implementing fluid dynamical fluctuations is important, but requires a sustained and systematic effort!

M. Nahrgang et al., CPOD2016  
I. Karpenko et al., Comput.Phys.Commun.185 (2014)

# Goal of this talk

Present the first numerical implementation of the diffusive dynamics of net-baryon density fluctuations near the QCD critical point:



- thoroughly tested against equilibrium expectations
- discuss effects of resolution, system-size and net-baryon charge conservation
- ⇒ generation of non-Gaussian fluctuations from Gaussian white noise!
- ⇒ dynamical critical scaling of model B! Hohenberg, Halperin, Rev.Mod.Phys.49 (1977)
- ⇒ nonequilibrium and memory effects in the dynamics!

## Diffusive dynamics of the net-baryon density

$$\partial_\mu N_B^\mu = 0 \quad \text{net-baryon charge conservation}$$

The diffusive dynamics follows the minimized free energy  $\mathcal{F}$ :

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To study intrinsic fluctuations include a stochastic current:

$$J(t, x) = \sqrt{2 T \kappa} \zeta(t, x)$$

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⇒ respects the fluctuation-dissipation theorem:

$$P_{\text{eq}}[n_B] = \frac{1}{Z} \exp(-\mathcal{F}[n_B]/T)$$

# Couplings motivated by 3-dimensional Ising model

$$\mathcal{F}[n_B] = T \int d^3r \left( \frac{m^2}{2n_c^2} \Delta n_B^2 + \frac{K}{2n_c^2} (\nabla \Delta n_B)^2 + \frac{\lambda_3}{3n_c^3} \Delta n_B^3 + \frac{\lambda_4}{4n_c^4} \Delta n_B^4 + \frac{\lambda_6}{6n_c^6} \Delta n_B^6 \right)$$

The couplings depend on temperature via the correlation length  $\xi(T)$ :

$$m^2 = 1/(\xi_0 \xi^2)$$

$$K = \tilde{K}/\xi_0$$

$$\lambda_3 = n_c \tilde{\lambda}_3 (\xi/\xi_0)^{-3/2}$$

$$\lambda_4 = n_c \tilde{\lambda}_4 (\xi/\xi_0)^{-1}$$

$$\lambda_6 = n_c \tilde{\lambda}_6$$

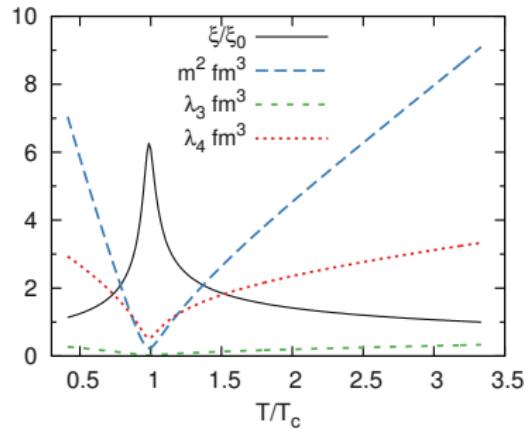
M. Tsypin PRL73 (1994); PRB55 (1997)

parameter choice:  $\Delta n_B = n_B - n_c$

$\xi_0 \sim 0.5 \text{ fm}$ ,  $T_c = 0.15 \text{ GeV}$ ,  $n_c = 1/3 \text{ fm}^{-3}$

$K = 1/\xi_0$  (surface tension)

$\tilde{\lambda}_3, \tilde{\lambda}_4, \tilde{\lambda}_6$  (universal, but mapping to QCD)



in this Fig.:  $\tilde{\lambda}_3 = 1$ ,  $\tilde{\lambda}_4 = 10$

## Different physical scenarios

The diffusion equation:

$$\begin{aligned}\partial_t n_B = & \frac{D}{n_c} \left( m^2 - K \nabla^2 \right) \nabla^2 n_B \\ & + D \nabla^2 \left( \frac{\lambda_3}{n_c^2} \Delta n_B^2 + \frac{\lambda_4}{n_c^3} \Delta n_B^3 + \frac{\lambda_6}{n_c^5} \Delta n_B^5 \right) + \sqrt{2Dn_c} \nabla \zeta\end{aligned}$$

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**Gauss+surface**

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**Ginzburg-Landau**

**noise**

- consider  $3 + 1$ d system with propagation in 1 spatial dimension
- diffusion coefficient  $D = \kappa T / n_c$ 
  - ▶ for equilibrium calculations  $D = 1$  fm
  - ▶ for dynamical calculations  $T$ -dependent
- equilibrium: let system equilibrate for long times
- dynamics: equilibrate first at high  $T$ , then evolve
- $\langle N_B \rangle$  perfectly conserved in the numerics!

# Different physical scenarios

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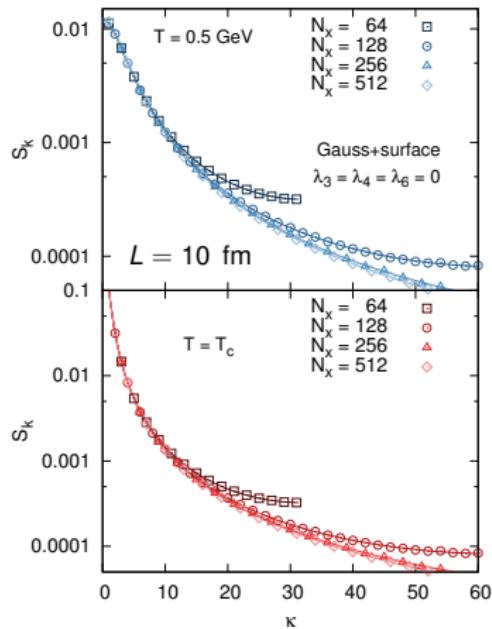
$$\partial_t n_B = \frac{D}{n_c} \left( m^2 - K \nabla^2 \right) \nabla^2 n_B \quad \text{Gauss+surface}$$

advection       $-\vec{v} \cdot \nabla n_B$

$$+ D \nabla^2 \left( \frac{\lambda_3}{n_c^2} \Delta n_B^2 + \frac{\lambda_4}{n_c^3} \Delta n_B^3 + \frac{\lambda_6}{n_c^5} \Delta n_B^5 \right) + \sqrt{2Dn_c} \nabla \zeta \quad \text{Ginzburg-Landau} \quad \text{noise}$$

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- equilibrium: let system equilibrate for long times
- dynamics: equilibrate first at high  $T$ , then evolve
- $\langle N_B \rangle$  perfectly conserved in the numerics!
- (including advection does not change results qualitatively)

# Gaussian model in equilibrium - structure factor



static structure factor in continuum  
 $(\xi^2 = K/m^2)$ :

$$S(k) = \frac{n_c^2}{m^2} \frac{1}{1 + \xi^2 k^2}$$

discretized form ( $k = 2\pi\kappa/L$ ):  
(semi-implicit predictor-corrector scheme)

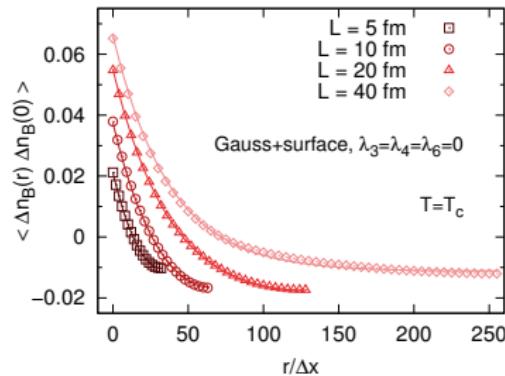
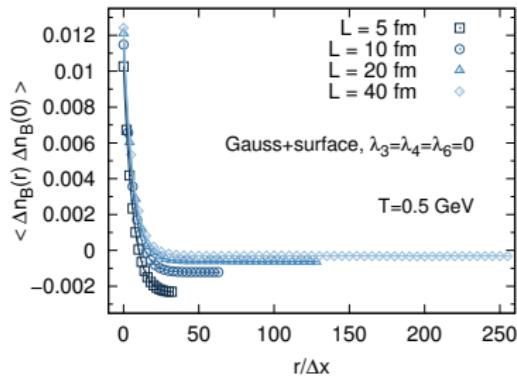
$$S_k = \frac{n_c^2}{m^2} \frac{1}{1 + \frac{2K}{m^2 \Delta x^2} (1 - \cos(k \Delta x))}$$

$\Rightarrow$  perfectly reproduced!  
 $\rightarrow S(k)$  for  $\Delta x = L/N_x \rightarrow 0!$

non-zero surface tension suppresses short-wavelength fluctuations!  
(in contrast to purely Gaussian model)

# Gaussian model in equilibrium - correlation function

- for  $K = 0$ : fluctuations are delta-correlated!
- for  $K \neq 0$ : numerical correlation length  $\xi > \Delta x$ , where spatial correlations broaden near  $T_c$ !

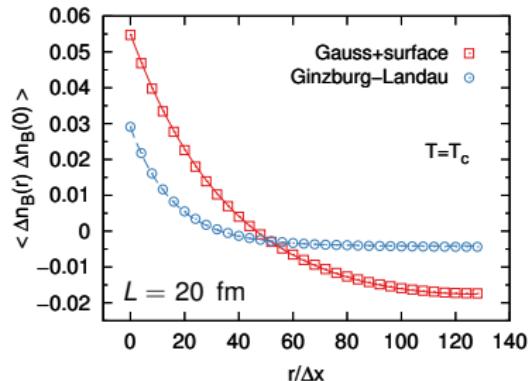
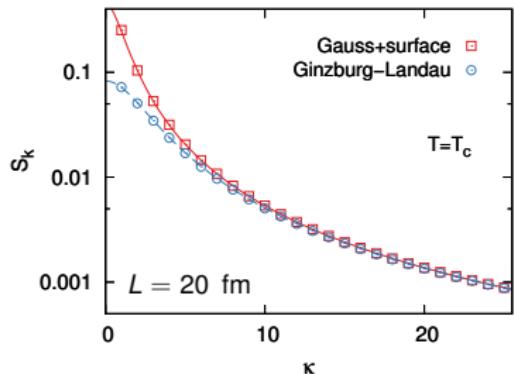


impact of exact  $\langle N_B \rangle$ -conservation: local  $n_B$ -fluctuations need to be balanced within  $L$

$\int_L dr \langle \Delta n_B(r) \Delta n_B(0) \rangle = 0 \rightarrow$  negative correction term expected  
 $\Rightarrow$  perfectly reproduced by the numerics!

Note: with increasing  $L$  larger equilibration times needed!

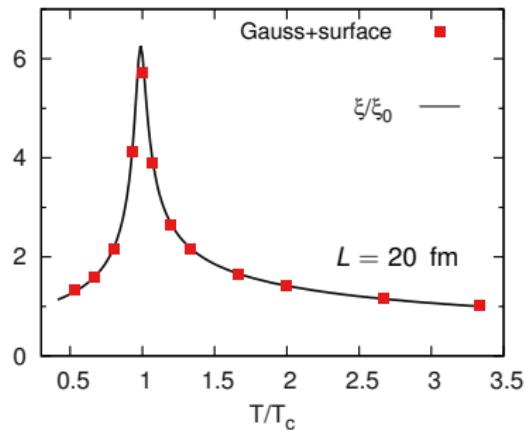
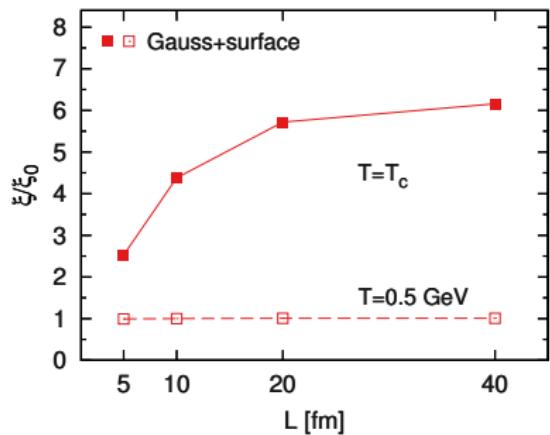
# Ginzburg-Landau model in equilibrium



- nonlinear interactions reduce  $S_k$  for long-wavelength fluctuations!
  - spatial correlations are significantly smaller!
- ⇒ at the level of 2-point correlations, results of Ginzburg-Landau model can be described by a **renormalized** Gauss+surface model (with  $m^2$  modified,  $K$  essentially unaffected)!

# Correlation length and finite-size effects

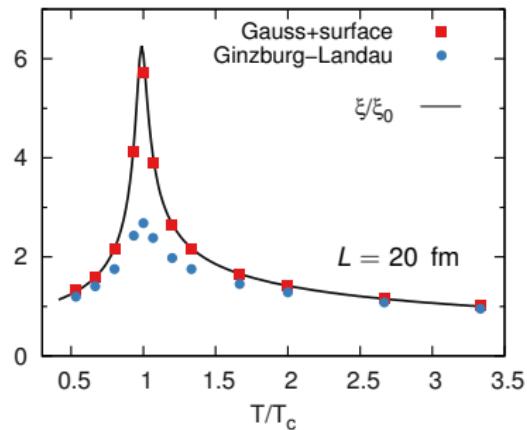
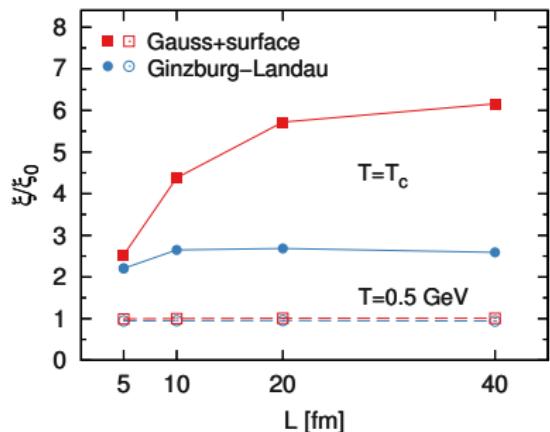
For finite size  $L$ , the numerical correlation length is strongly limited due to  $\langle N_B \rangle$ -conservation.



- very good realization of the input  $\xi(T)$ ! (only achievable with  $K \neq 0$ !)

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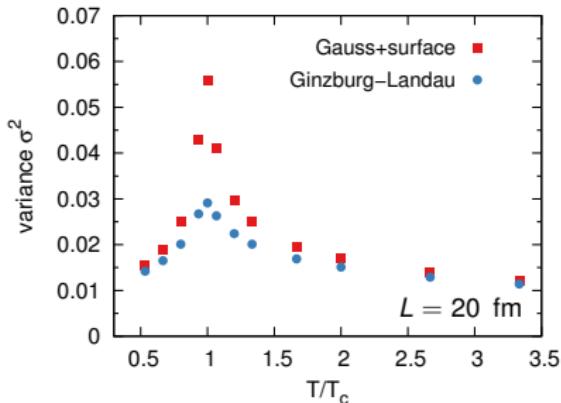
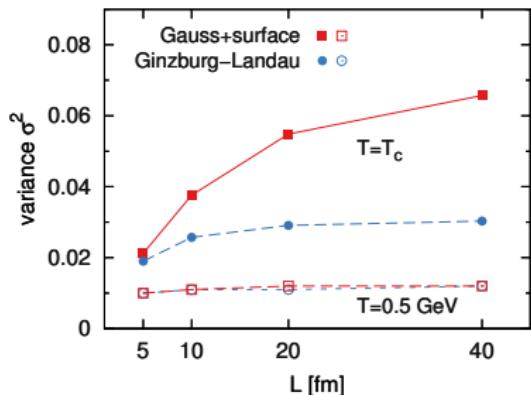
For finite size  $L$ , the numerical correlation length is strongly limited due to  $\langle N_B \rangle$ -conservation.



- very good realization of the input  $\xi(T)$ ! (only achievable with  $K \neq 0$ !)
- reduction of the numerically realized correlation length for Ginzburg-Landau model in accordance with the renormalized  $m^2$ !

# $T$ -dependence of Gaussian fluctuations

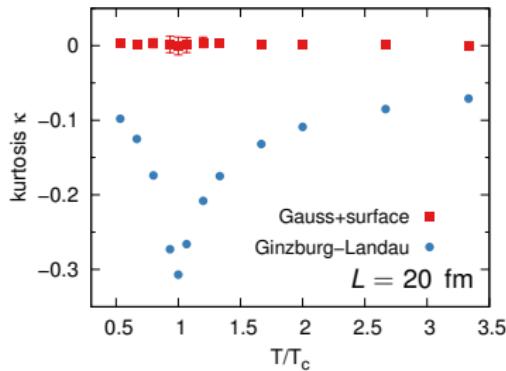
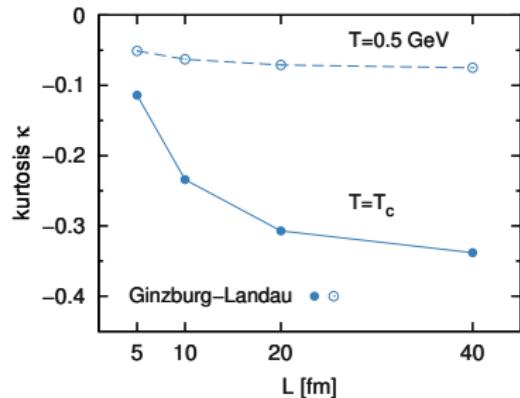
In finite systems: variance is reduced compared to TD expectations.



- nonlinear couplings in the Ginzburg-Landau model reduce the variance!
- ⇒ Could explain why no sign of criticality is observed in second-order moments at NA49 and RHIC BES phase I.

# $T$ -dependence of non-Gaussian fluctuations

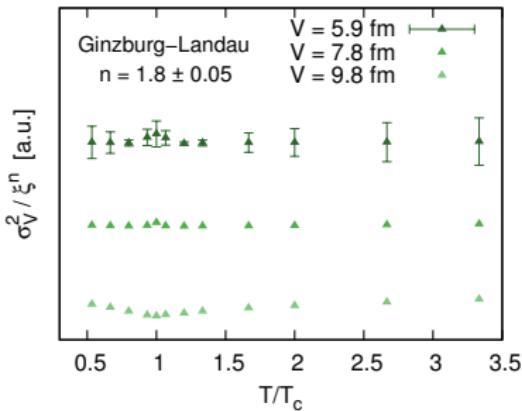
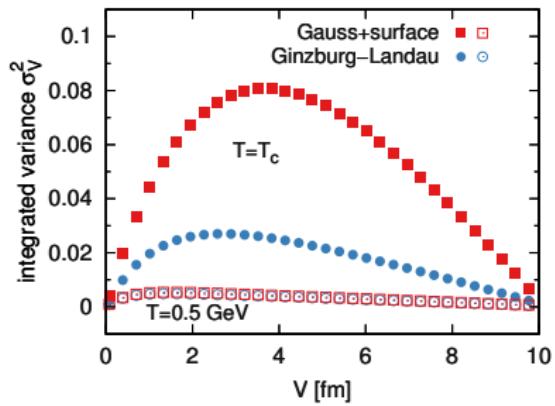
Kurtosis vanishes in the absence of nonlinear interactions!



- negative kurtosis observed for Ginzburg-Landau model with a pronounced signal near  $T_c$ !

Note: we choose  $L = 20 \text{ fm}$  ( $N_x = 256$ ) for the following studies!

# Integrated variance



- in this talk: study of local variance, taken over  $V = \Delta x$ , where  $\sigma^2 \propto \xi^!$
- integrated variance:  $\sigma_V^2 = \frac{1}{V^2} \int dx \int dy \langle \Delta n_B(x) \Delta n_B(y) \rangle \propto \xi^2$

M. Stephanov et al. PRL81 (1998); PRD60 (1999); PRL102 (2009)

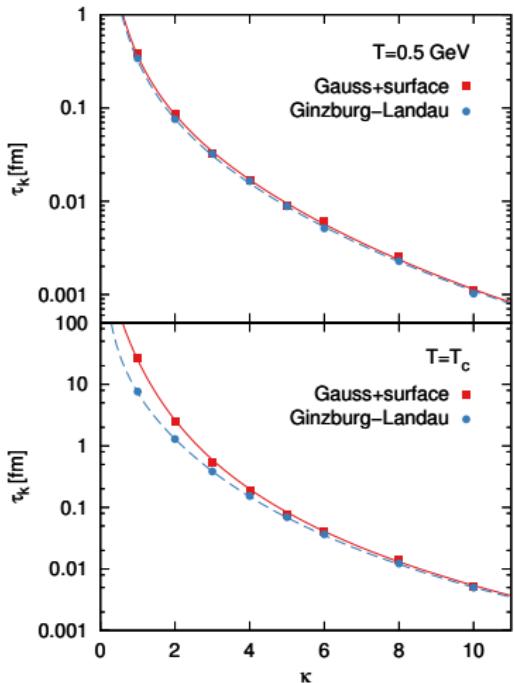
- including finite-size and  $\langle N_B \rangle$ -conservation we find:

$$\sigma_V^2 \propto \xi^n \text{ with } n \sim 1.80 \pm 0.05 !$$

# Dynamics: relaxation time

dynamic structure factor for Gaussian model in continuum:

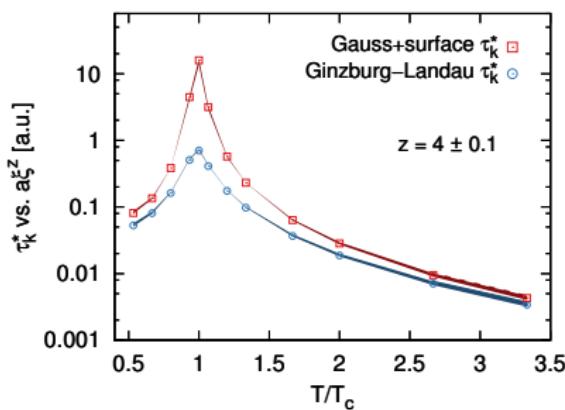
$$S(k, t) = S(k) \exp(-t/\tau_k) \text{ with } \tau_k^{-1} = \frac{Dm^2}{n_c} \left(1 + \frac{K}{m^2} k^2\right) k^2$$



- analytic results reproduced by numerics!
- long-wavelength modes relax slowly!
- $\tau_k$  significantly enhanced near  $T_c$ !
- nonlinear interactions reduce  $\tau_k$  for modes with small  $k$ !  
(described by renormalized Gauss+surface model)

# Dynamics: dynamical critical scaling

analyze  $\xi$ -dependence of relaxation time for modes with  $k^* = 1/\xi$ :



- for both models:  $\tau_k^* = a \xi^z$
- best fit gives:

$$z = 4 \pm 0.1$$

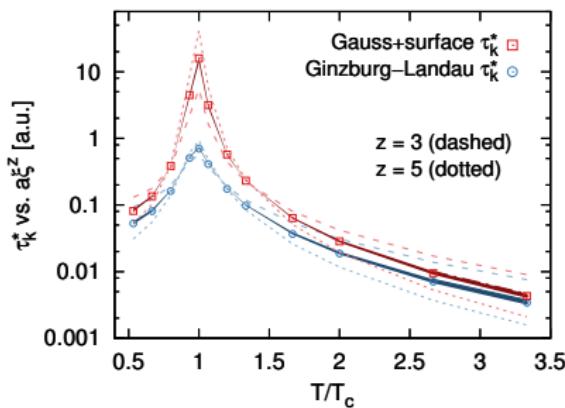
$$a = \frac{n_c \xi_0}{D(1 + \tilde{K})}$$

⇒ Simulations reproduce scaling of model B!

Hohenberg, Halperin, Rev.Mod.Phys.49 (1977)

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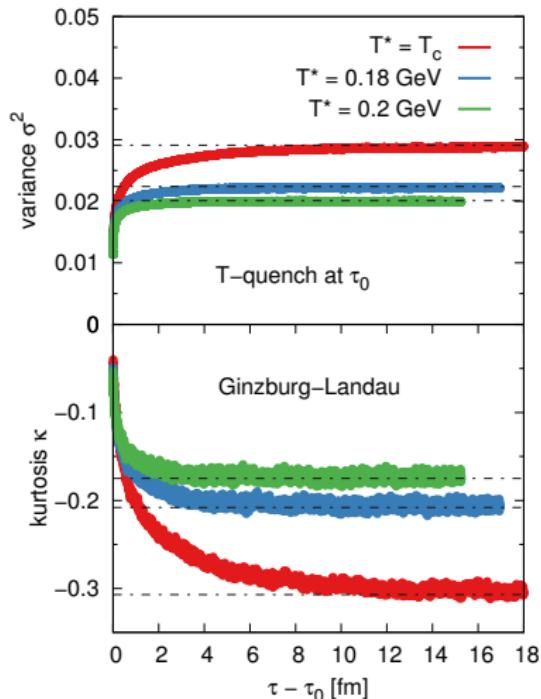
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- $z = 3, 5$  ruled out! (nice accuracy!)

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Hohenberg, Halperin, Rev.Mod.Phys.49 (1977)

# Dynamics: temperature quench and equilibration



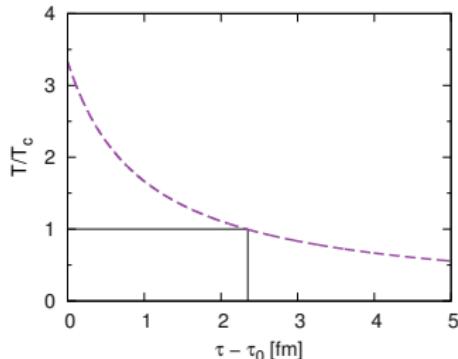
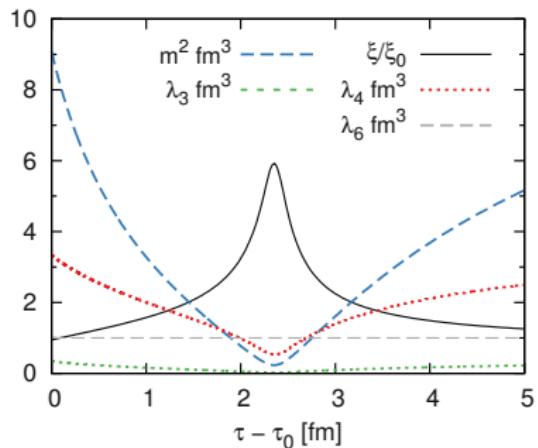
- temperature quench:  
at  $\tau_0$  temperature drops from  $T_0 = 0.5$  GeV to  $T^*$
- fast initial relaxation
- variance approaches equilibrium value faster than kurtosis
- long relaxation times near  $T_c$

B. Berdnikov, K. Rajagopal PRD61 (2000)

# Dynamics: time-dependent couplings

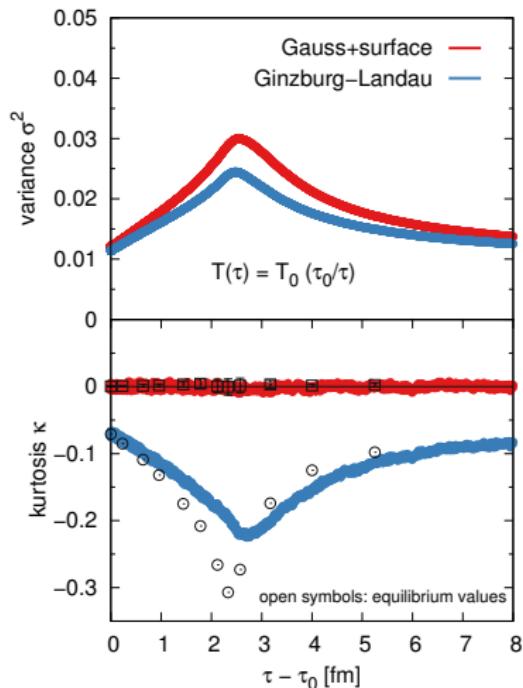
time-dependent temperature:

$$T(\tau) = T_0 \left( \frac{\tau_0}{\tau} \right)^{dc_s^2}$$



- choose  $c_s^2 = 1/3$   
(should be  $c_s^2 = c_s^2(T)$ )
- initialize system at  
 $T_0 = 0.5$  GeV,  $D(T_0) = 1$  fm
- $T_c$  reached at  $\tau - \tau_0 = 2.33$  fm

# Dynamics: critical (non-)Gaussian fluctuations



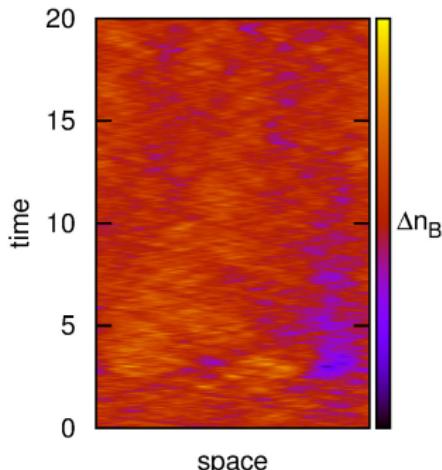
- shift of extrema for variance and kurtosis (memory effects)

B. Berdnikov, K. Rajagopal PRD61 (2000), S. Mukherjee et al., PRC92(3) (2015)

- |extremal values| in dynamical simulations below equilibrium values (nonequilibrium effects)
- no dynamical effects on non-Gaussianity in Gauss+surface model
- expected behavior with varying  $D$  and  $c_s^2$  (expansion rate)

# Conclusions

- successful test of numerical implementation vs. analytic expectations!
- significant effect of net-baryon charge conservation!
- in presence of nonlinear interactions:
  - variance is reduced (consistent with experimental data)!
  - generation of non-Gaussian fluctuations (from purely Gaussian white noise)!
- dynamics: critical scaling of model B, nonequilibrium and memory effects!



Thanks to:



**BEST**  
COLLABORATION