Theoretical perspective on strangeness production

Che-Ming Ko, Texas A&M University

- Introduction
- Nuclear equation of state
- Deeply subthreshold strangeness production
- Mean-field potentials
- Phi meson
- Statistical hadronization vs recombination
- Lambda polarization
- Exotic hadrons
- Summary

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First theoretical studies on strangeness production

KAON PRODUCTION IN RELATIVISTIC NUCLEAR COLLISIONS †

J. RANDRUP and C. M. KO NPA 343, 519 (1980)

Nuclear Science Division, Lawrence Berkeley Laboratory, Berkeley, CA 94720, USA

Received 19 February 1980

Abstract: Kaon production in relativistic nuclear collisions is studied on the basis of a conventional multiple-collision model. The input is the differential cross sections for kaon production in elementary baryon-baryon collisions, estimated in a simple model. Inclusive kaon spectra are calculated at 2.1 GeV/nucleon for a number of experimental cases. The calculated kaon yield is approximately isotropic in the mid-rapidity frame and extends considerably beyond the nucleon-nucleon kinematical limit.

Strangeness Production in the Quark-Gluon Plasma

Johann Rafelski and Berndt Müller PRL 48, 1066 (1982)

Institut für Theoretische Physik, Johann Wolfgang Goethe-Universität, D-6000 Frankfurt am Main, Germany

(Received 11 January 1982)

Rates are calculated for the processes $gg \to s\bar{s}$ and $u\bar{u}, d\bar{d} \to s\bar{s}$ in highly excited quark-gluon plasma. For temperature $T \geqslant 160$ MeV the strangeness abundance saturates during the lifetime ($\sim 10^{-23}$ sec) of the plasma created in high-energy nuclear collisions. The chemical equilibration time for gluons and light quarks is found to be less than 10^{-24} sec.

Statistical Thermodynamics in Relativistic Particle and Ion Physics: Canonical or Grand Canonical?

R. Hagedorn and K. Redlich[1]

Zeitschrift Fur PhysiK C 27, 541 (1985)

CERN, CH-1211 Geneva 23, Switzerland

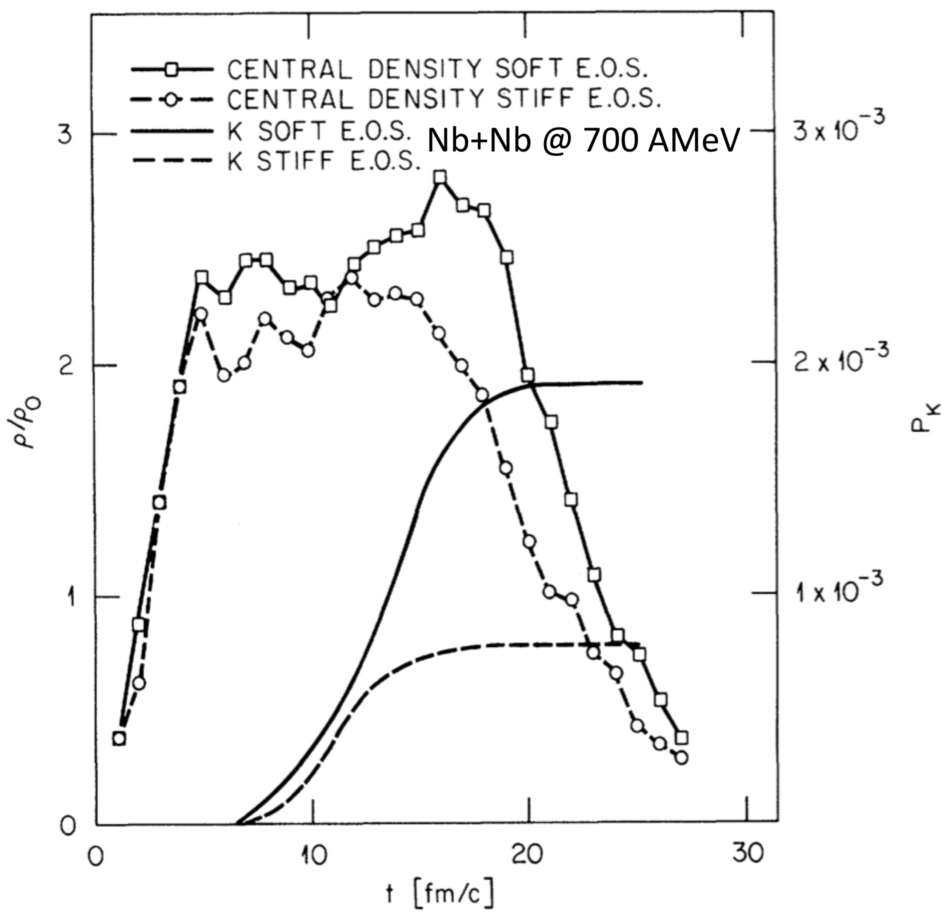
Received 10 September 1984

We consider relativistic statistical thermodynamics of an ideal Boltzmann gas consisting of the particles$K$, N, Λ, Σ and their antiparticles. Baryon number (B) and strangeness (S) are conserved. While any relativistic gas is necessarily grand canonical with respect to particle numbers, conservation laws can be treated canonically or grand canonically. We construct the partition function for canonical$B \times S$ conservation and compare it with the grand canonical one. It is found that the grand canonical partition function is equivalent to a large$B$ approximation of the canonical one. The relative difference between canonical and grand canonical quantities seems to decrease like const/B (two numerical examples) and from this a simple thumb rule for computing canonical quantities from grand canonical ones is guessed. For precise calculations, an integral representation is given.

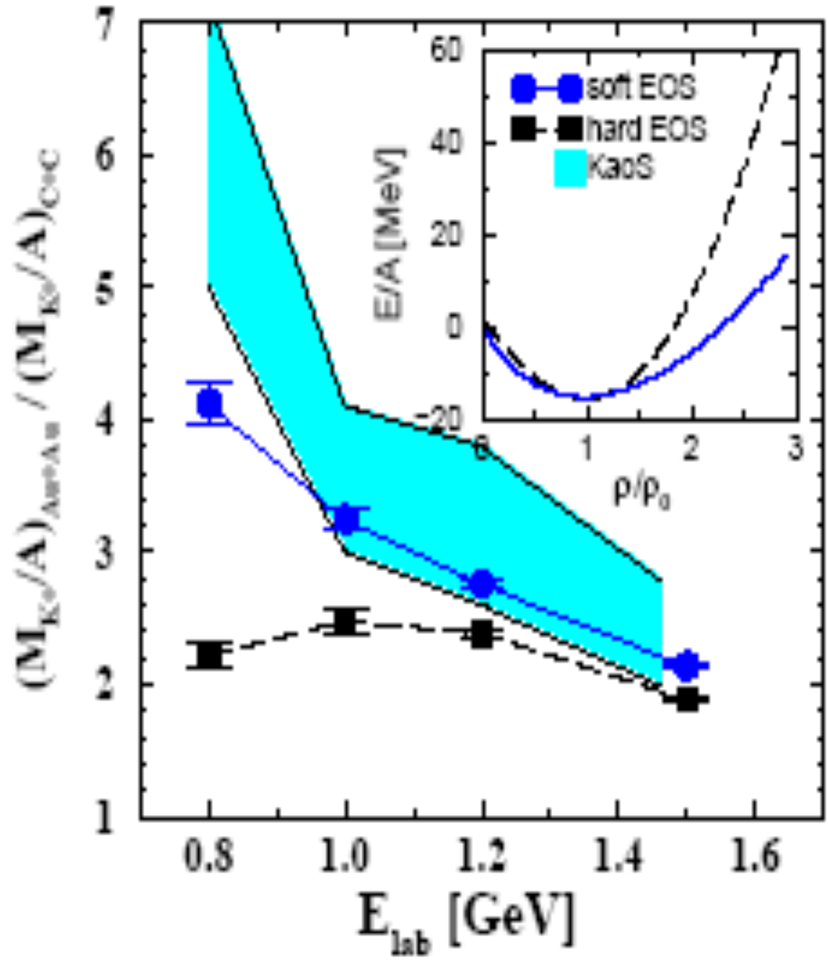
On leave from: Institute for Theoretical Physics, Wroclaw, Poland.

Subthreshold kaon production in high-energy HIC

Aichelin & Ko, PRL 55, 2661 (1985)



Fuchs, PRL 86, 1974 (2001)



- Kaon production at subthreshold energy in HI collisions is sensitive to nuclear EOS, and data are consistent with a soft one.

Nuclear symmetry energy

Li, Chen & Ko, Phys. Rep. 464, 113 (2008)

EOS of asymmetric nuclear matter

$$E(\rho,\delta) \approx E(\rho,\delta=0) + E_{sym}(\rho)\delta^2, \quad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

Symmetry energy

$$E_{sym}(\rho) = E_{sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho-\rho_0}{\rho_0}\right)^2$$

Symmetry energy coefficient $E_{sym}(\rho_0) \approx 30$ MeV from mass formula

Slope $L = 3\rho_0 \left.\frac{\partial E_{sym}(\rho)}{\partial \rho}\right|_{\rho=\rho_0}$ theoretical values -50 to 200 MeV

Curvature $K_{sym} = 9\rho_0^2 \left.\frac{\partial^2 E_{sym}(\rho)}{\partial^2 \rho}\right|_{\rho=\rho_0}$ theoretical values -700 to 466 MeV

Nuclear matter Incompressibility $K(\delta) = K_0 + K_{asy}\delta^2, \quad K_{asy} = K_{sym} - 6L$

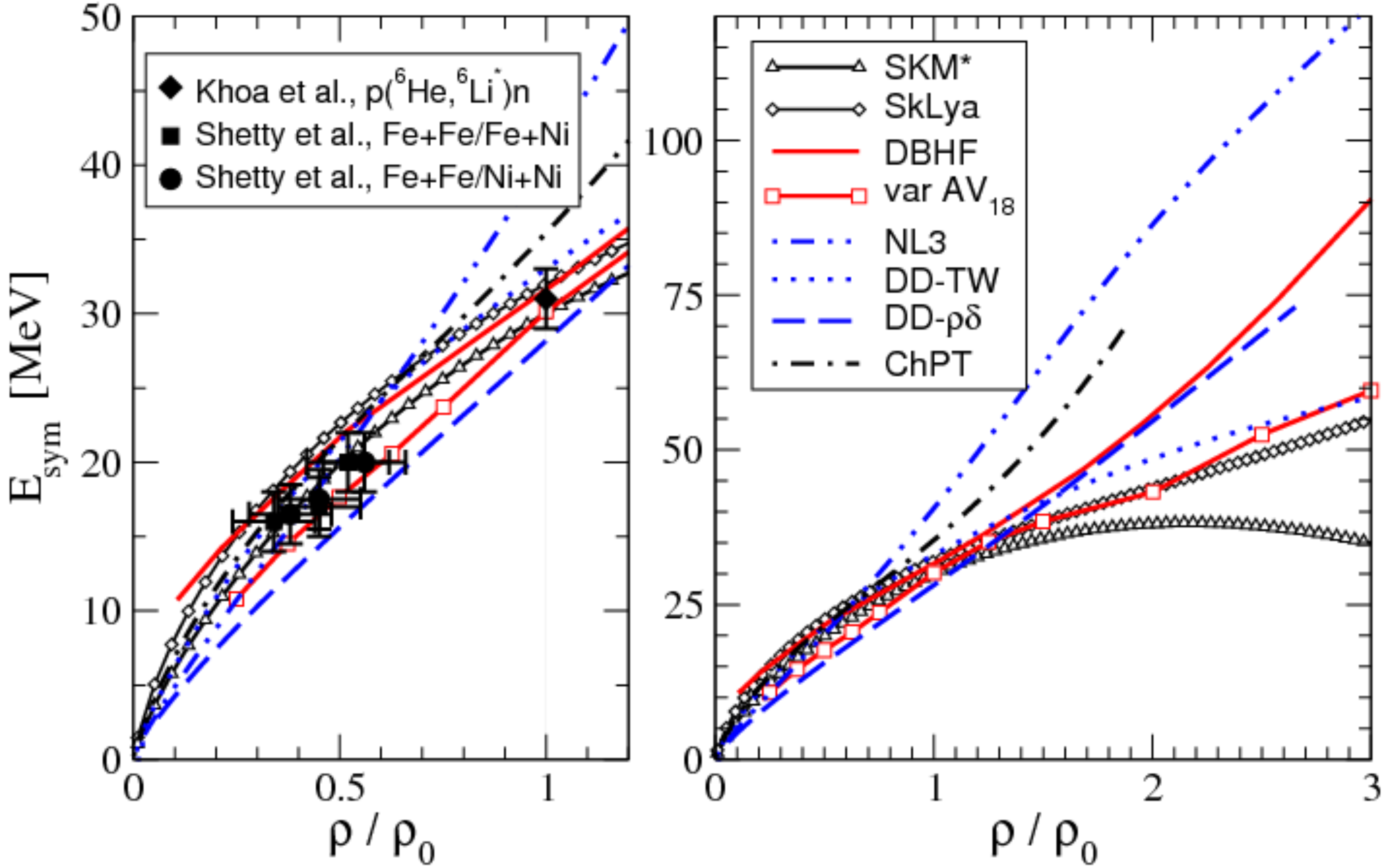
Empirically, $K_0 \sim 230\pm10$ MeV, $K_{asy} \sim -500 \pm 50$ MeV, $L \sim 88\pm50$ MeV

$E_{sym}(\rho) \sim 32\,(\rho/\rho_0)^\gamma$ with $0.7<\gamma<1.1$ for $\rho<1.2\rho_0$

- Symmetry energy at high densities is practically undetermined !

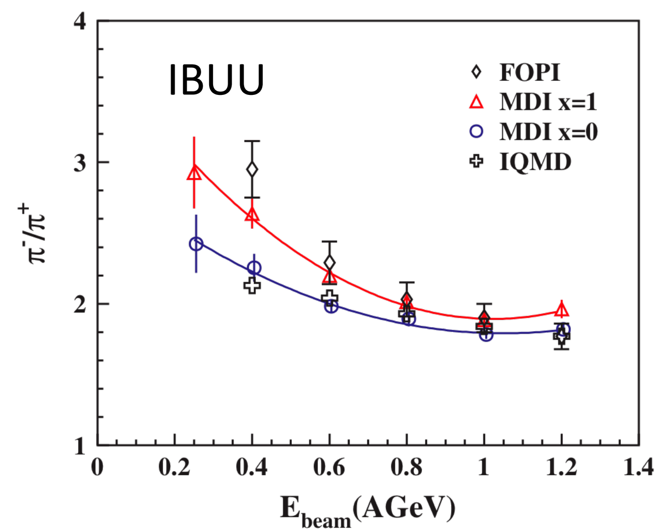
Theoretical predictions on nuclear symmetry energy

Fuchs, JPG 35, 014049 (2008)

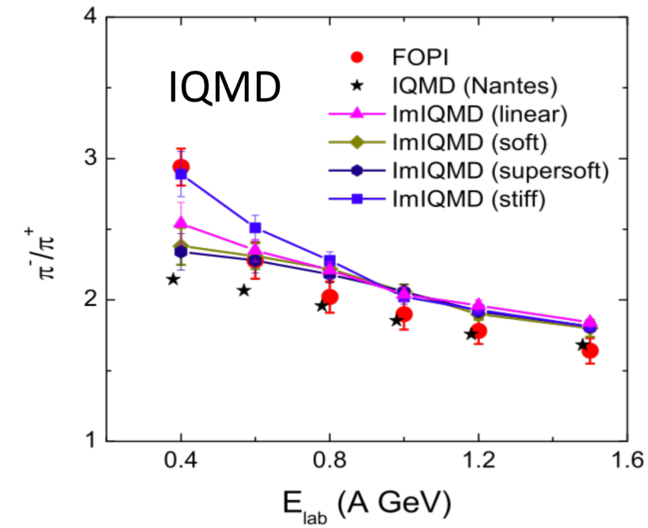


- Large uncertainties at both low and high densities

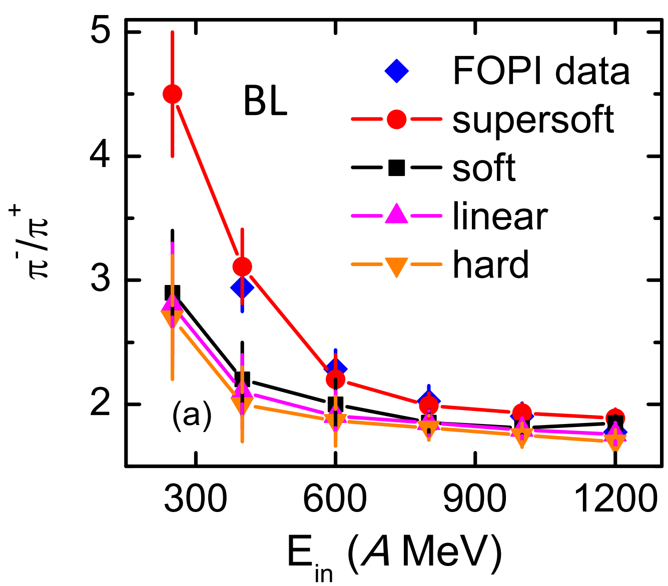
Conflicting results on symmetry energy from charged pion ratio



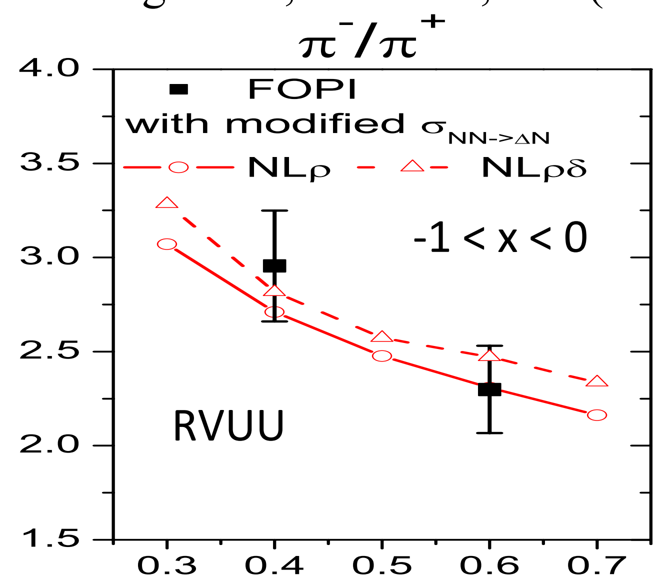
Xiao et al, PRL 102, 062502 (2009)



Feng & Jin, PLB 683, 140 (2010)



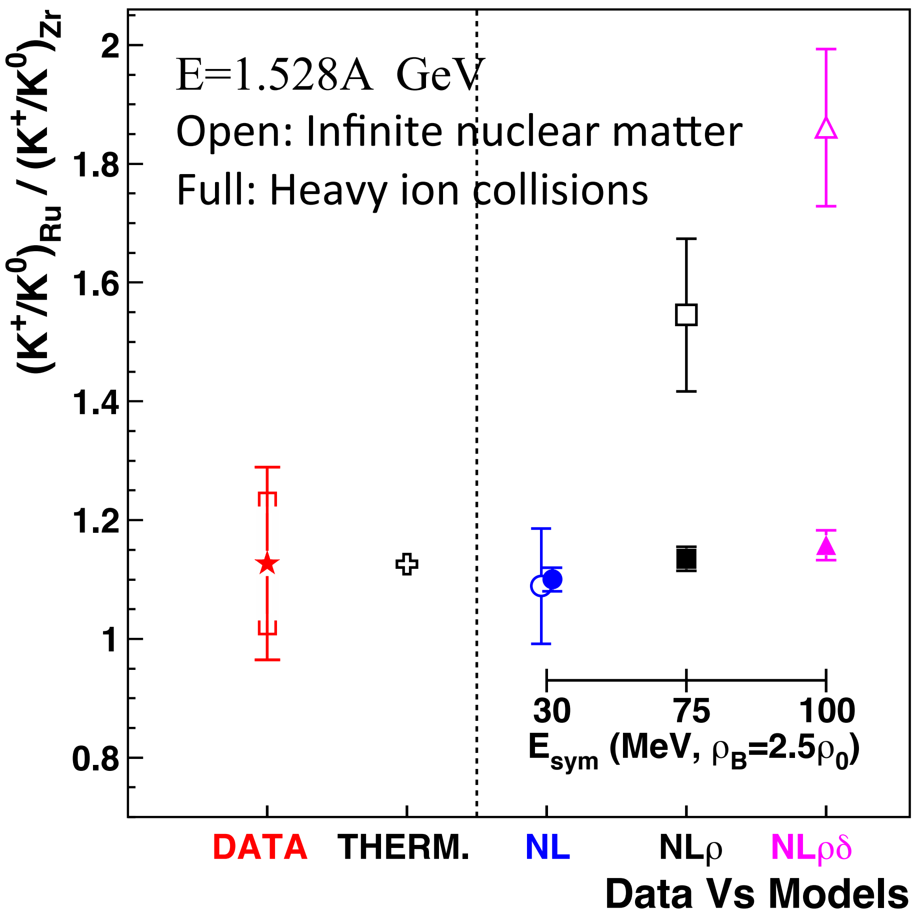
Shi et al., PLB 718, 1510 (2013)



Song & Ko, PRC 91, 014901 (2015)

Symmetry energy effect on K^+/K^0 ratio

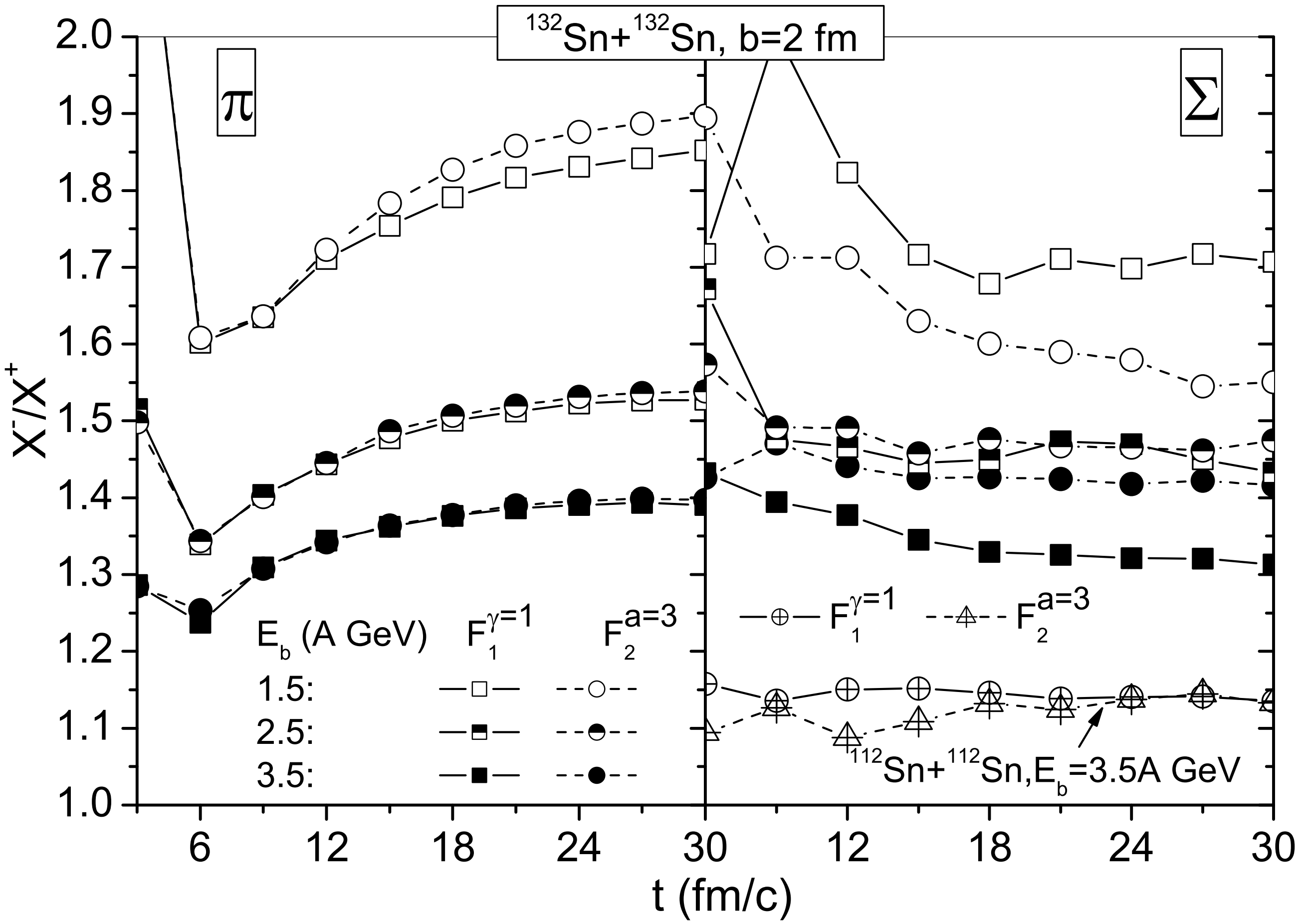
Lopez et al. (FOPI Collaboration) , Phys. Rev. C75, 011901(R) (2007)



- K^+/K^0 ratio only increases slightly with the stiffness of symmetry energy

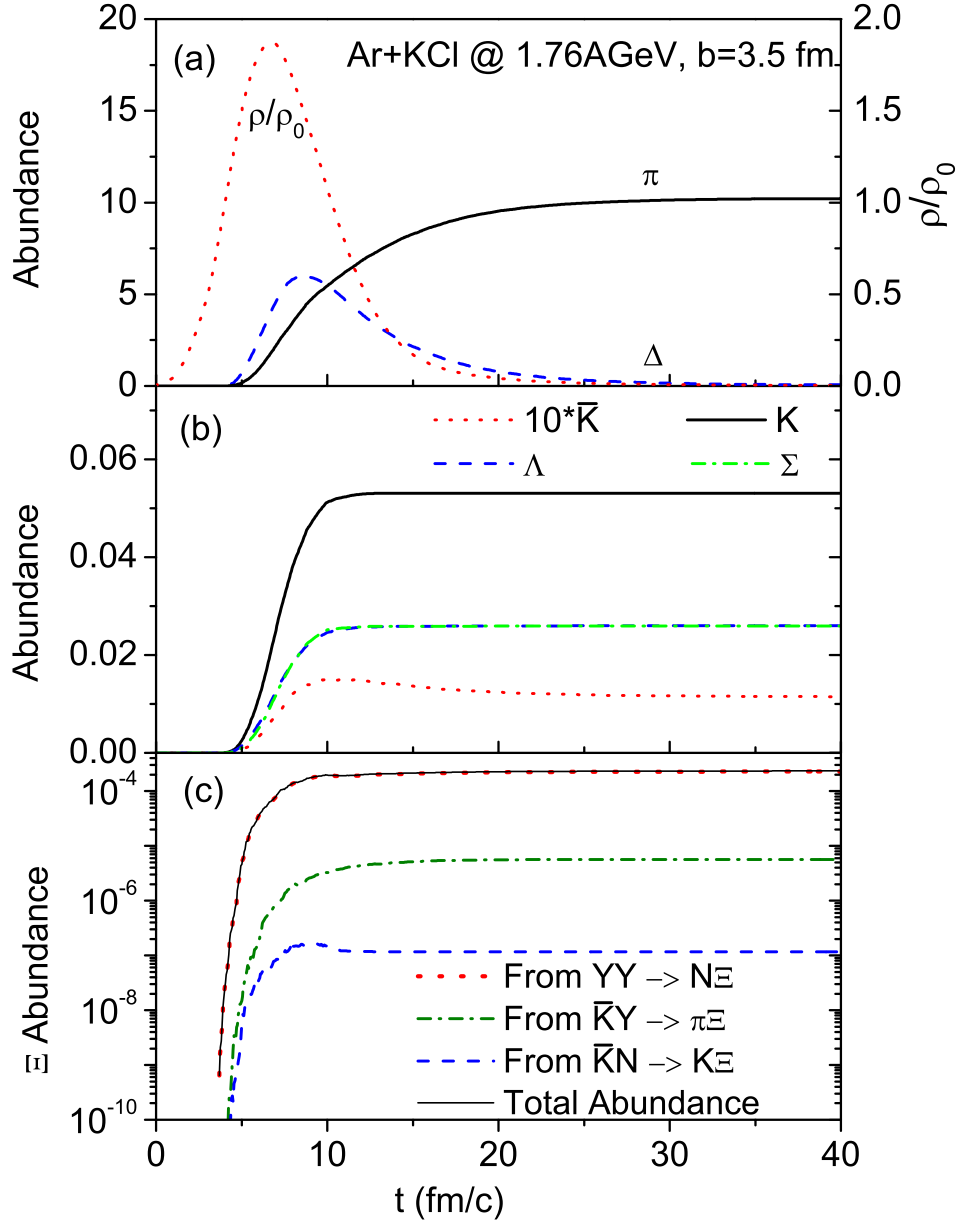
Symmetry energy effect on Σ^-/Σ^+ ratio

Li, Li, Zhao & Gupta, PRC 71, 054907 (2005)



- Soft symmetry energy (F_2) leads to a smaller Σ^-/Σ^+ ratio

Subthreshold production of Cascade (Ξ)



Li, Chen, Ko & Lee, PRC 85, 064902 (2012)

- Using cross sections calculated from meson-exchange model

  σ(Kbar Λ→πΞ) ~ 5-10 mb

  σ(Kbar Σ→πΞ) ~ 5-10 mb

  σ(ΛΛ→NΞ) ~ 40 mb

  σ(ΛΣ→NΞ) ~ 40-60 mb

  σ(ΣΣ→NΞ) ~ 15-30 mb

  σ(KN→Kbar Ξ) ~ 0.2 mb

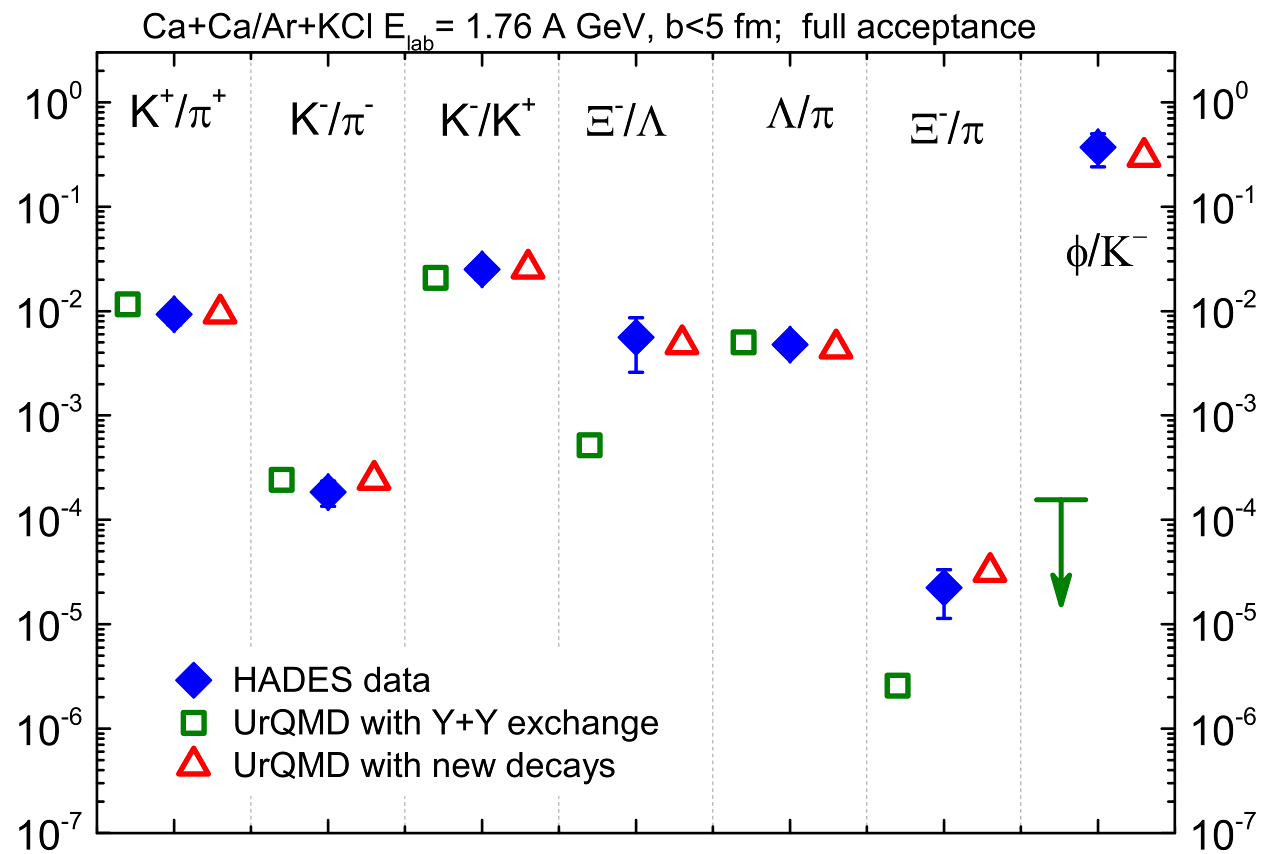
→ $\Xi^-/(\Lambda+\Sigma^0) \sim 3.4\text{x}10^{-3}$

compared with HADES data of $5.6\text{x}10^{-3}$ [Agakishiev et al., PRL 103, 132301 (2009)]

- Sensitivity of Ξ^-/Ξ^0 to nuclear symmetry energy?

Role of high mass baryon resonances

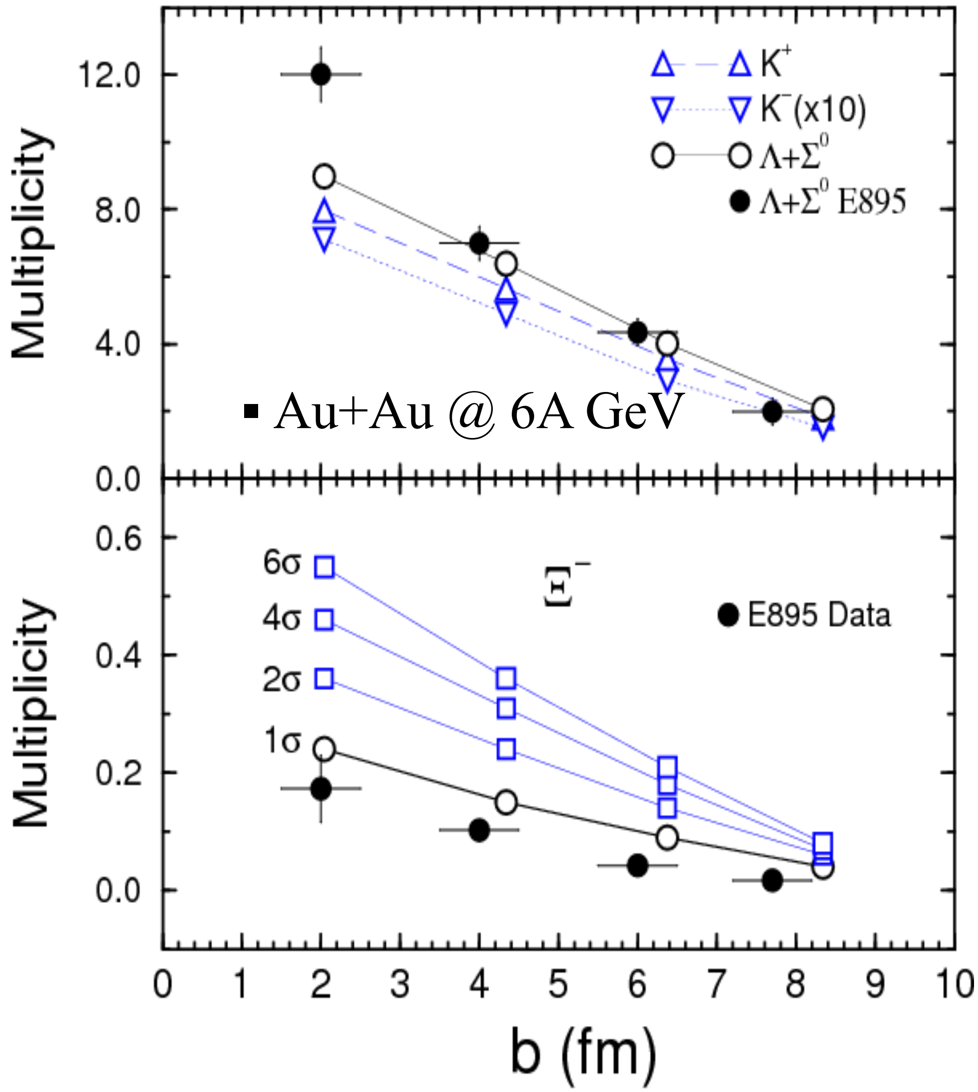
J. Steiheimer and M. Bleicher, J. Phys. G43, 015104 (2016)



- Good agreement with data after including in UrQMD decays of N*(1990), N*(2080), N*(2190), N*(2220), and N*(2250) to ΞKK with a branching ratio of 10% and to Nϕ with a BR of 0.2%.

Hyperon production at AGS

Pal, Ko, Alexander, Chung, and Lacey, PLB 595, 158 (2004)



- Based on hadronic transport model ART
- Strangeness-exchange reactions

$$\bar{K}N \rightarrow \Lambda(\Sigma)\pi$$
$$\bar{K}\Lambda(\Sigma) \rightarrow \Xi\pi$$

- Reproduce reasonable E895 data
- Ξ does not reach chemical equilibrium

Hadronic potentials in nuclear medium

Ko & Li, JPG 22, 1673 (1996); Ko, Koch & Li, ARNPS 47, 505 (1997)

- **Kaons and antikaons:** Chiral effective Lagrangian → repulsive potential for kaons and attractive potential for antikaons

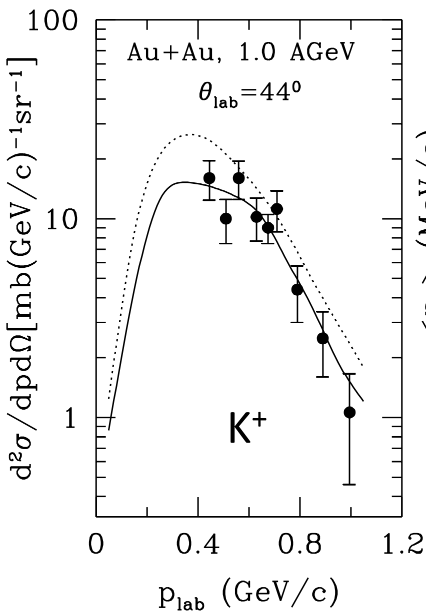
$$U_{K,\bar{K}} = \omega_{K,\bar{K}} - \omega_0, \quad \omega_0 = \sqrt{m_K^2 + p^2}$$

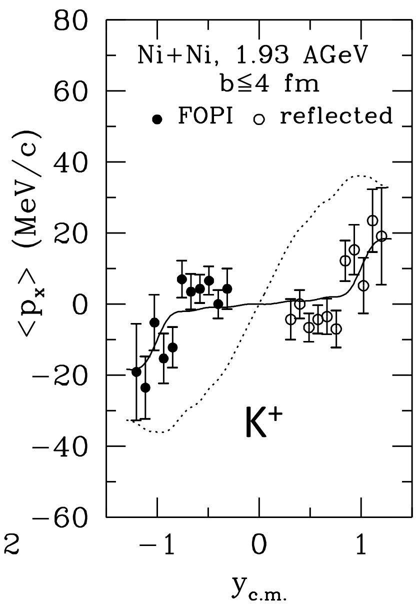
$$\omega_{K,\bar{K}} = \sqrt{m_K^2 + p^2 - a_{K,\bar{K}}\rho_s + (b_K\rho_B)^2} \pm b_K\rho_B$$

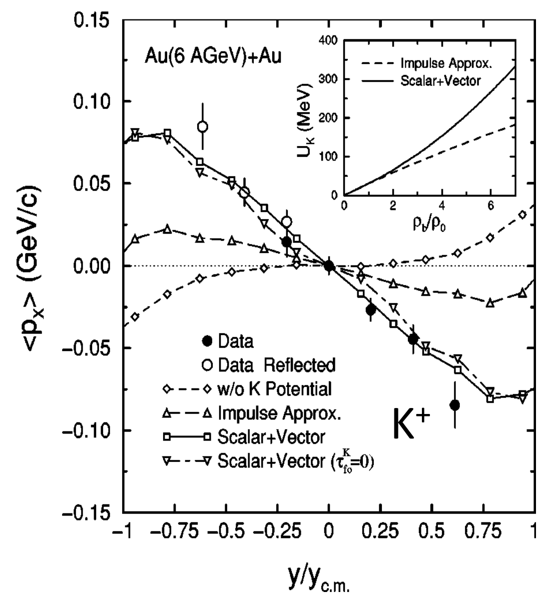
$$a_K = 0.22\ \mathrm{GeV^2fm^3}, \quad a_{\bar{K}} = 0.45\ \mathrm{GeV^2fm^3}$$

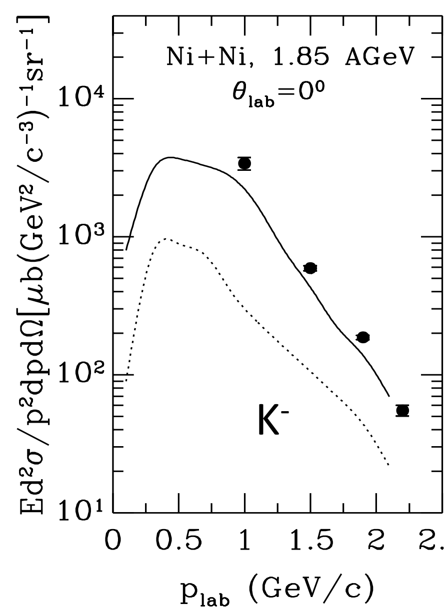
$$b_K = 0.33\ \mathrm{GeV^2fm^3}$$

$$\Rightarrow \quad U_K = 20\ \mathrm{MeV}, U_{\bar{K}} = -120\ \mathrm{MeV\ at\ } \rho_0 = 0.16\ \mathrm{fm^{-3}}$$





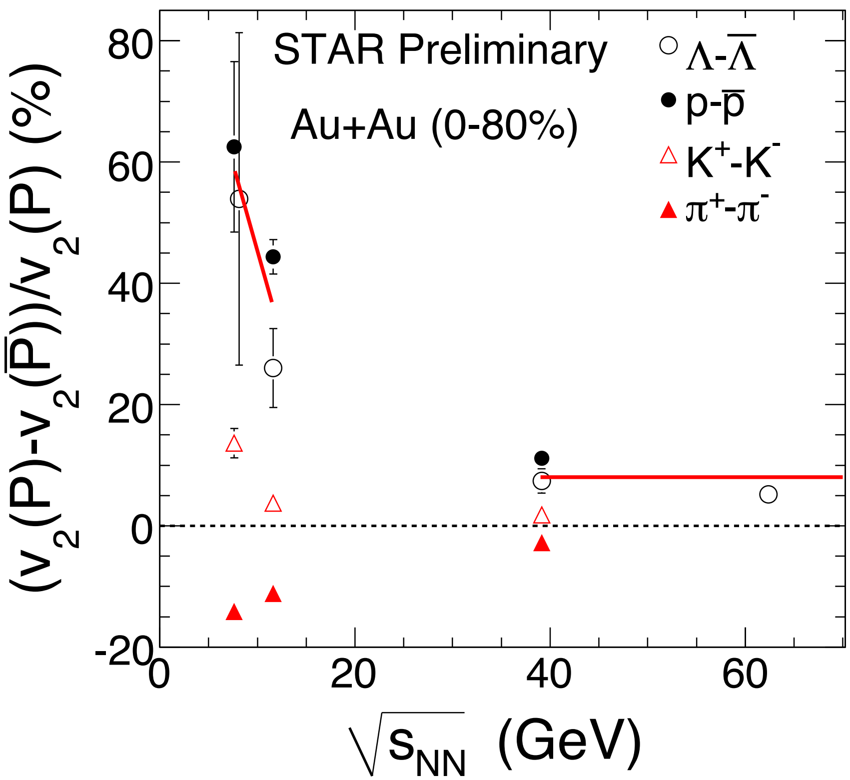


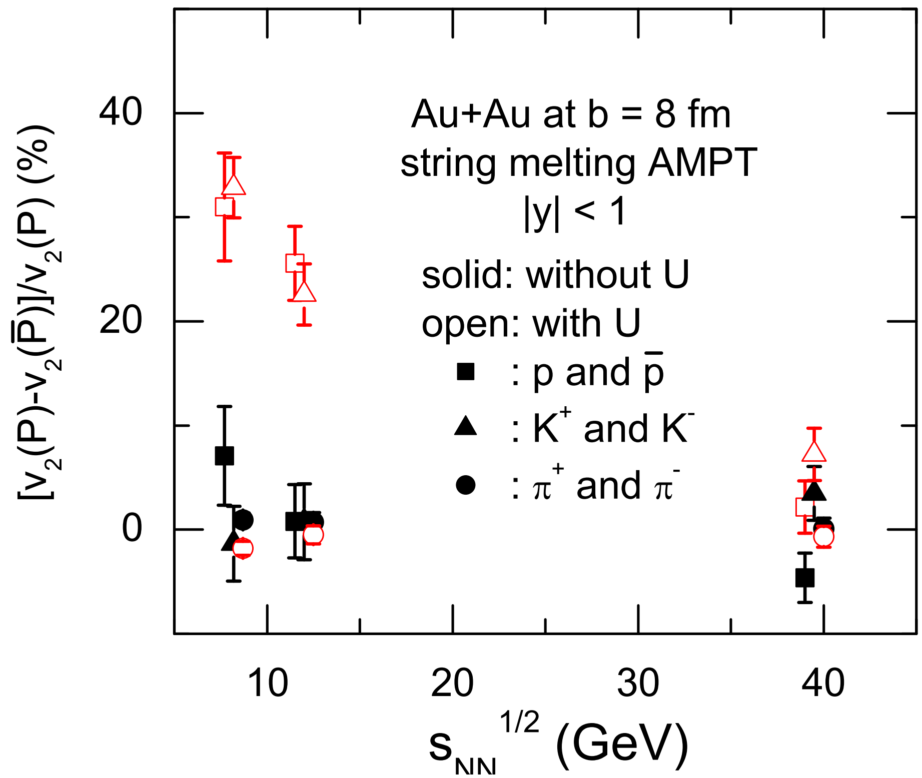


- Experimental data on spectrum and directed flow are consistent with repulsive kaon and attractive antikaon potentials.

Mean-field effects on particle and antiparticle elliptic flows

Xu, Chen, Ko & Lee, PRC 85, 045905 (2012)

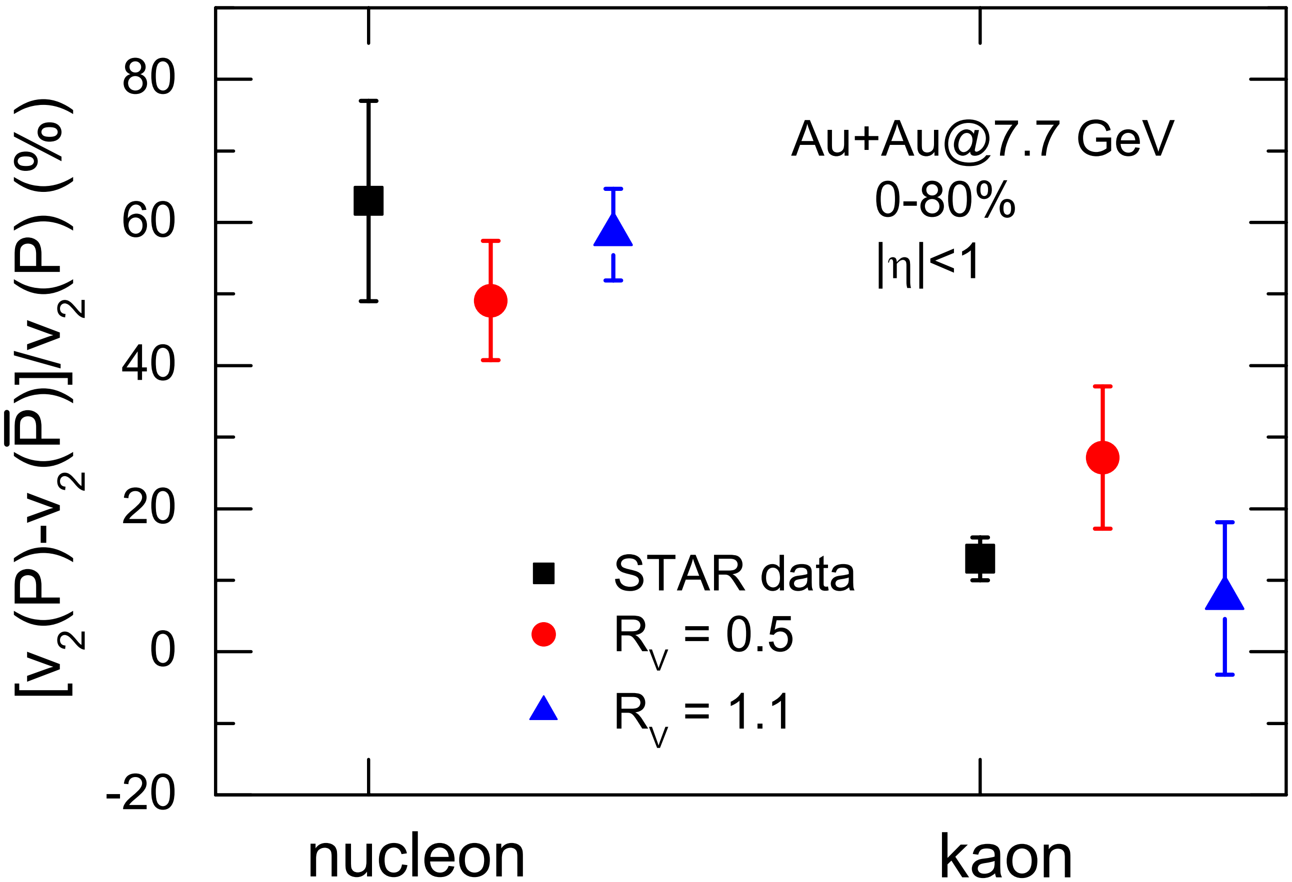




- Hadronic mean fields lead to splitting of particle and antiparticle elliptic flows in baryon-rich matter, diminish with increasing collision energies, similar to experimental observations by STAR
- Expect additional effects from partonic mean-fields

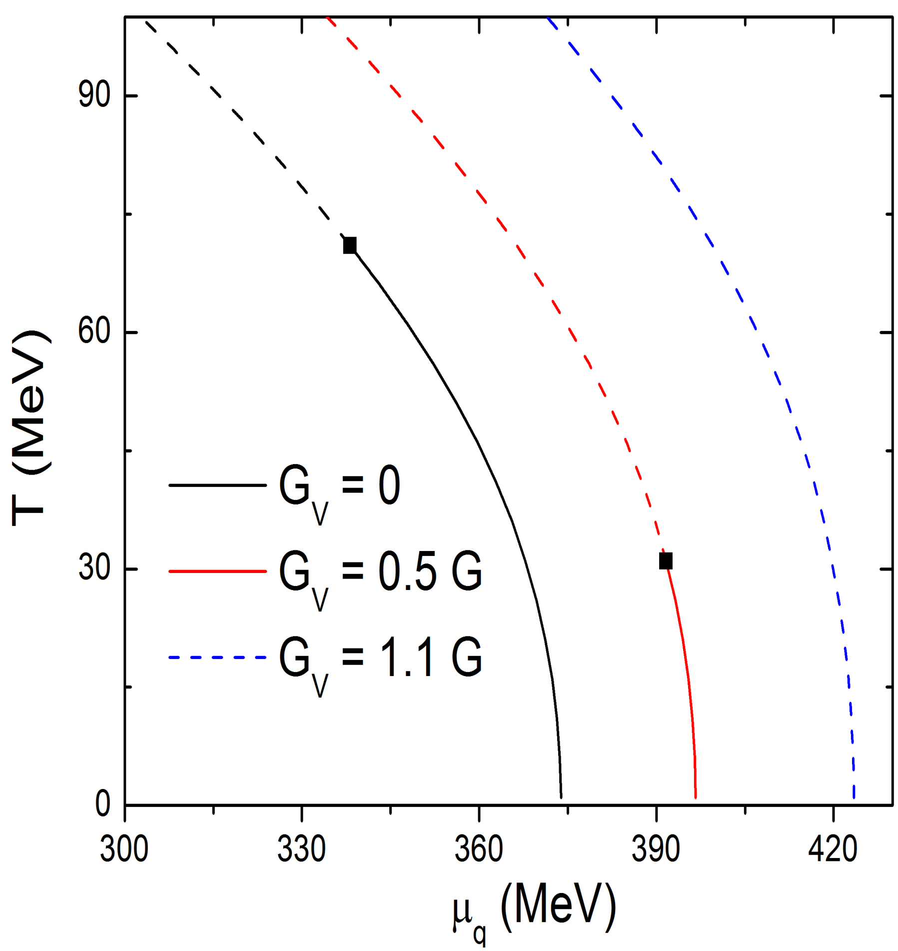
Relative v_2 difference including both partonic and hadronic potentials

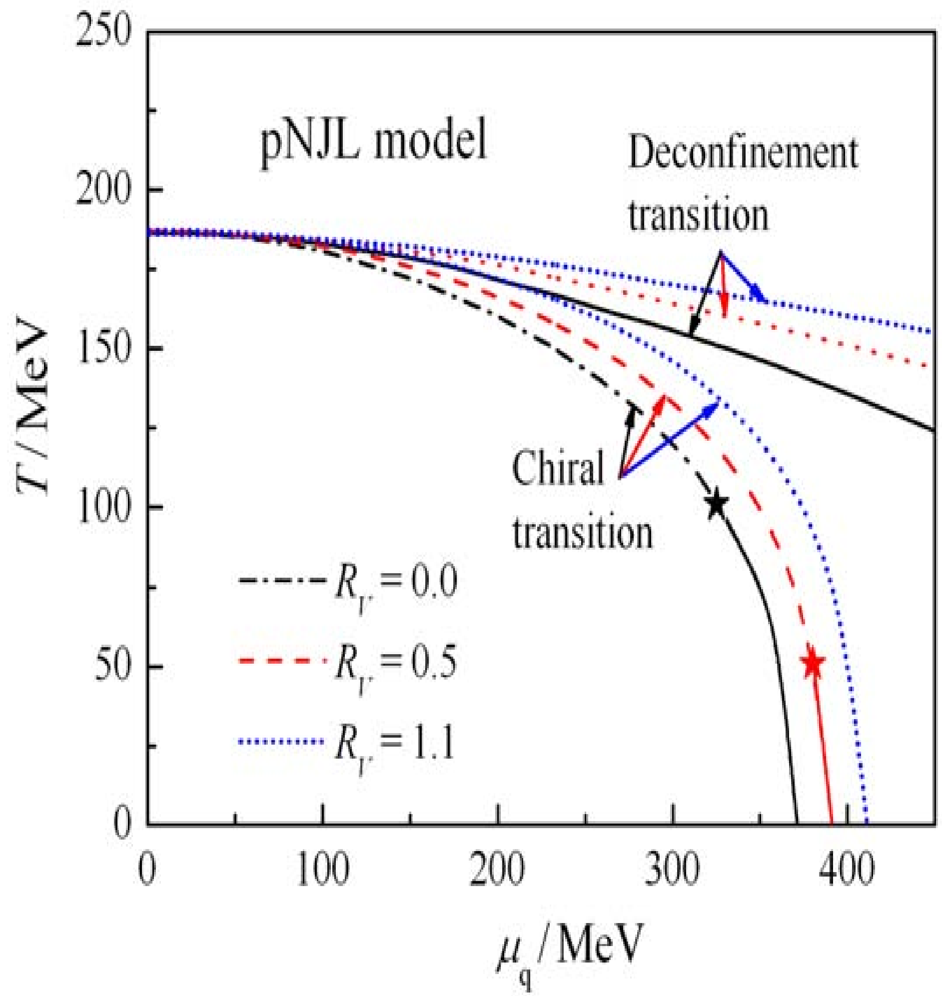
Xu, Song, Li & Ko, PRL 112, 012301 (2014)



- Finite partonic vector mean field with G_v/G=0.5 -1.1 is needed to describe STAR data.

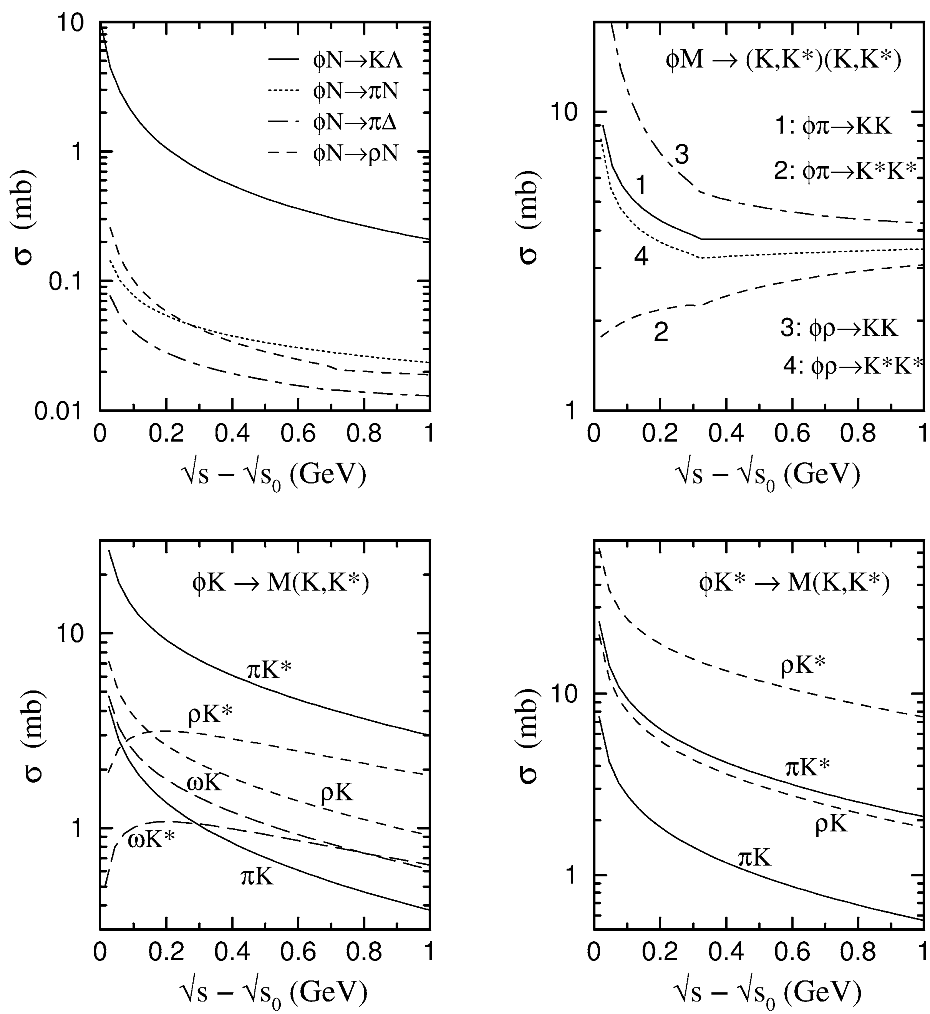
Effect of partonic vector interaction on QCD phase diagram





- Location of critical point depends strongly on G_V; moving to lower temperature and larger baryon chemical potential as G_V increases.
- Critical point disappears for $G_V > 0.6$ G in NJL and somewhat larger in PNJL.

Phi meson absorption and production cross sections



Pal, Ko & Lin, NPA 707, 525 (2002)

Besides $\phi \leftrightarrow K\overline{K}$, phi meson can be produced and absorbed via various hadronic reactions, calculable by

- Meson-exchange model

  Chung, Li, and Ko, NPA 625, 347 (97)

- Chiral Lagrangian with hidden local symmetry

  Averez-Ruso and Koch, PRC 65, 054901 (02)

Phi meson rapidity distribution at SPS

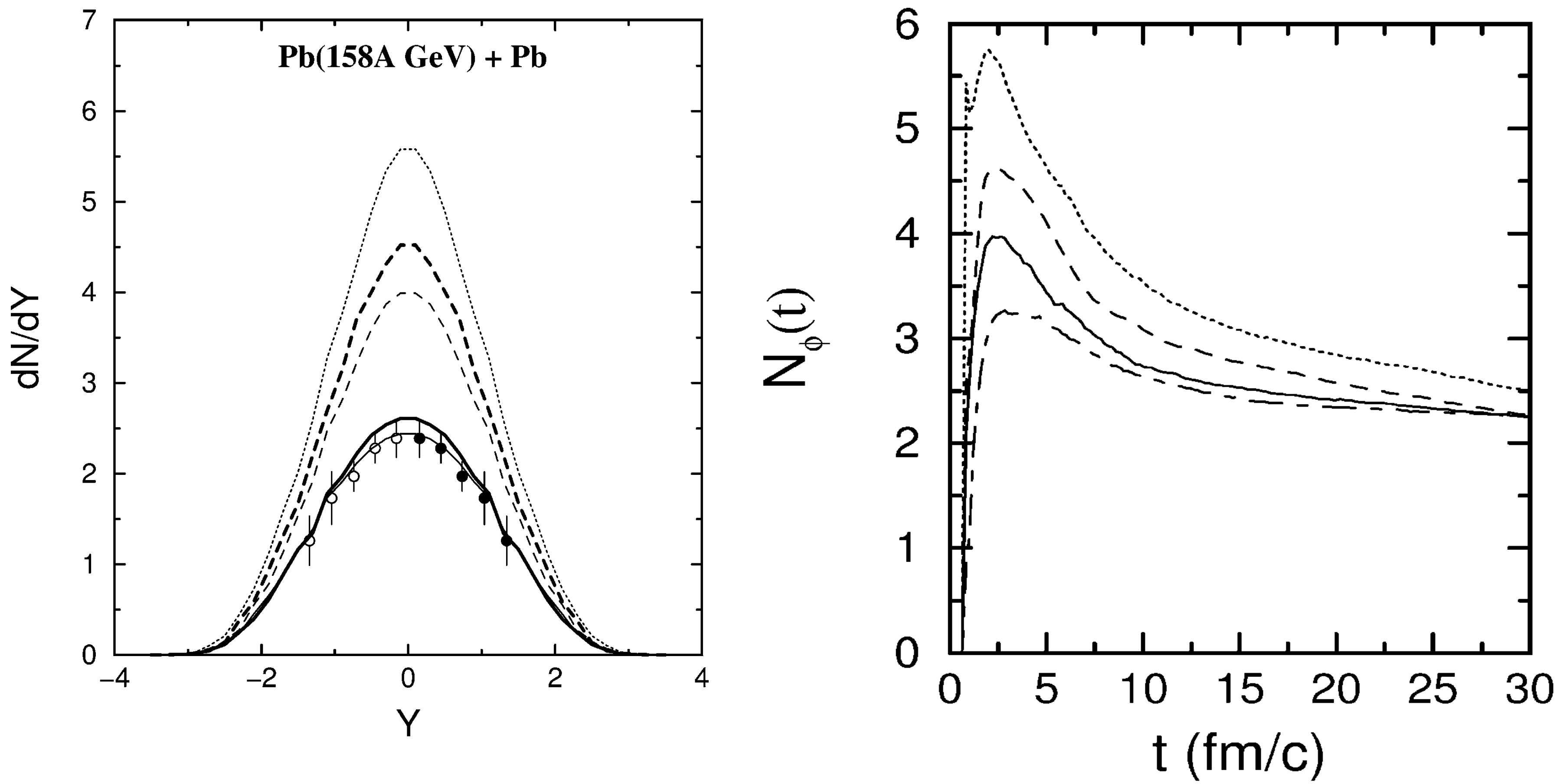


Fig. 3. Rapidity distribution of phi meson reconstructed from K^+K^- pairs (solid curves) and from $\mu^+\mu^-$ channel (dashed curves) for Pb+Pb collisions at 158 A GeV at an impact parameter of $b \leqslant 3.5$ fm in the AMPT model. The results are for without (thin curves) and with (thick curves) in-medium mass modifications. The dotted curve corresponds to phi mesons from the dimuon channel with in-medium masses and with the phi meson number from HIJING increased by a factor of two. The solid circles are the NA49 experimental data [7] from the K^+K^- channel.

Phi meson production at RHIC

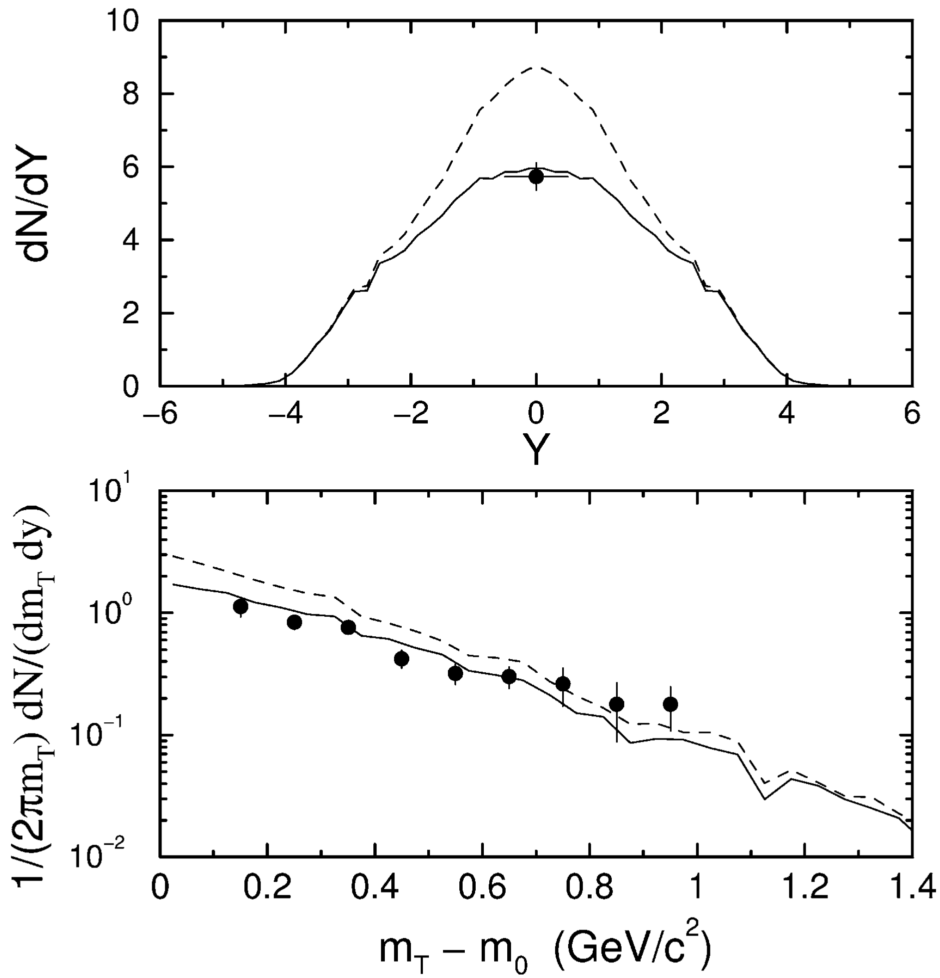
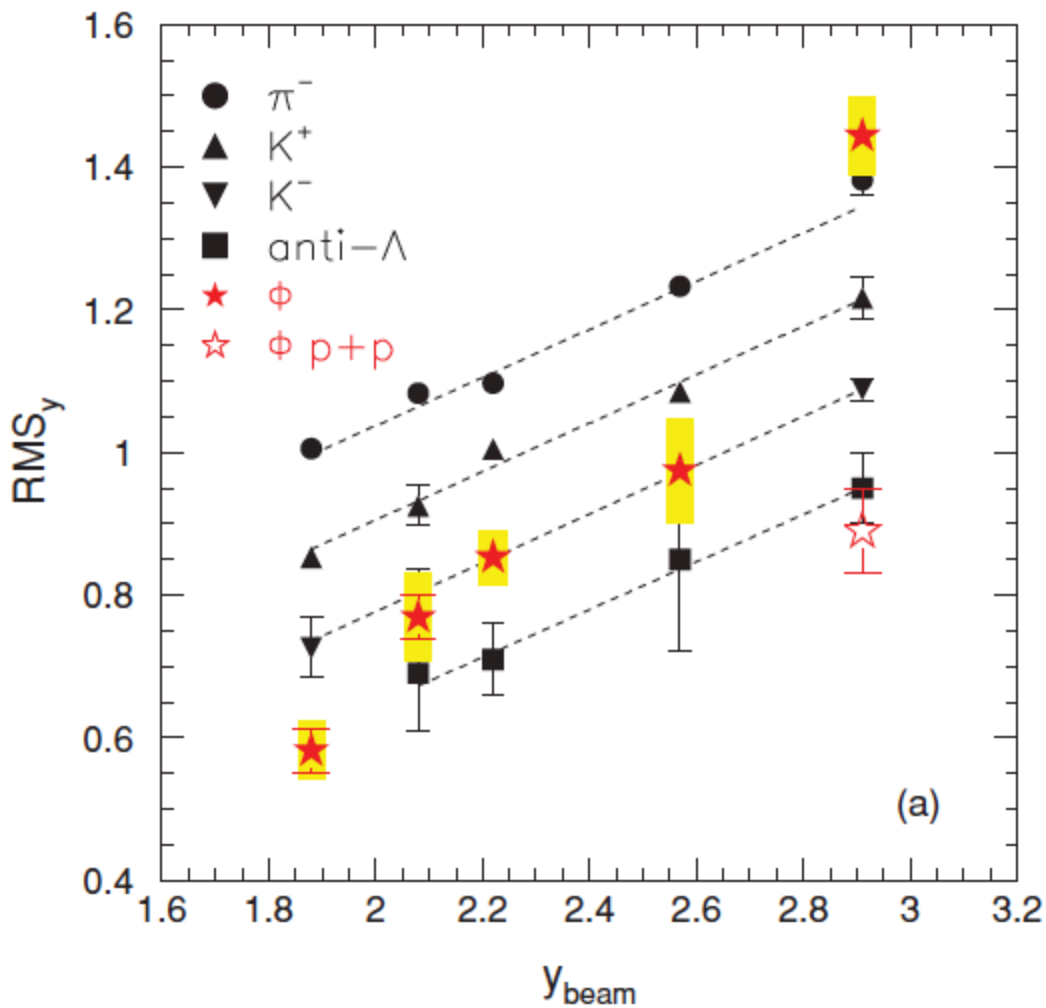


Fig. 5. The rapidity distribution (top panel) and the transverse mass spectra (bottom panel) for midrapidity ($|y| < 0.5$) phi mesons reconstructed from K^+K^- pairs (solid curves) and from $\mu^+\mu^-$ channel (dashed curves) for Au+Au collisions at RHIC energy of $\sqrt{s} = 130$ AGeV at an impact parameter of $b \leqslant 5.3$ fm in the AMPT model. The solid circles are the STAR experimental data [41] for 0–11% central collisions for ϕ reconstructed from K^+K^- decay.

Energy dependence of width of rapidity distribution

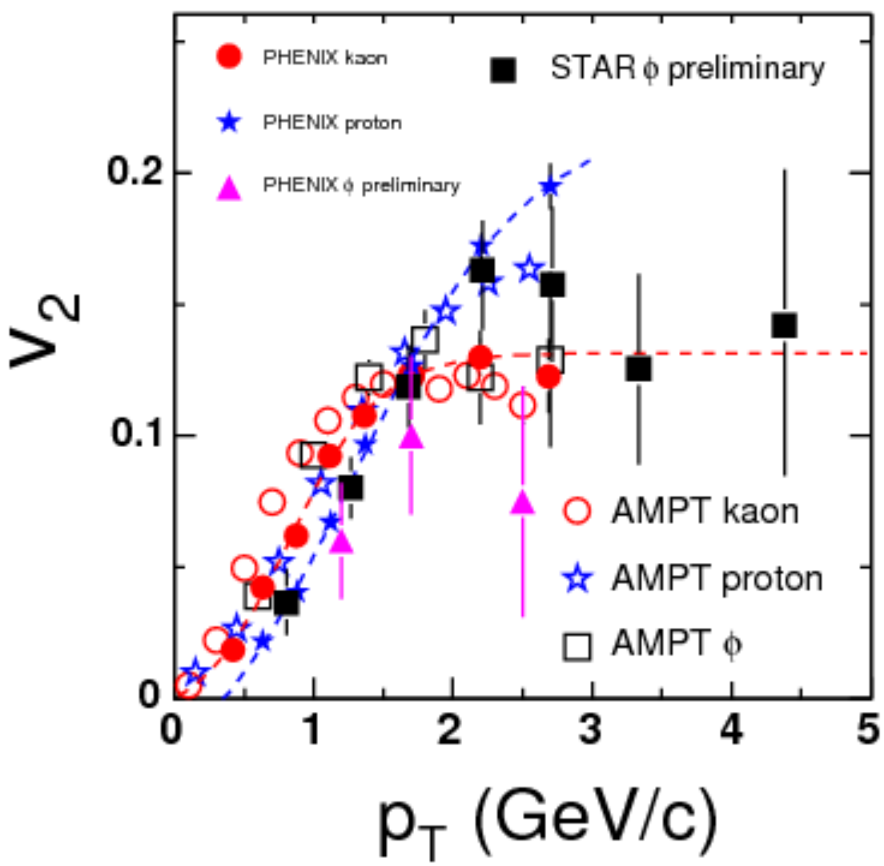
Pb+Pb @ SPS

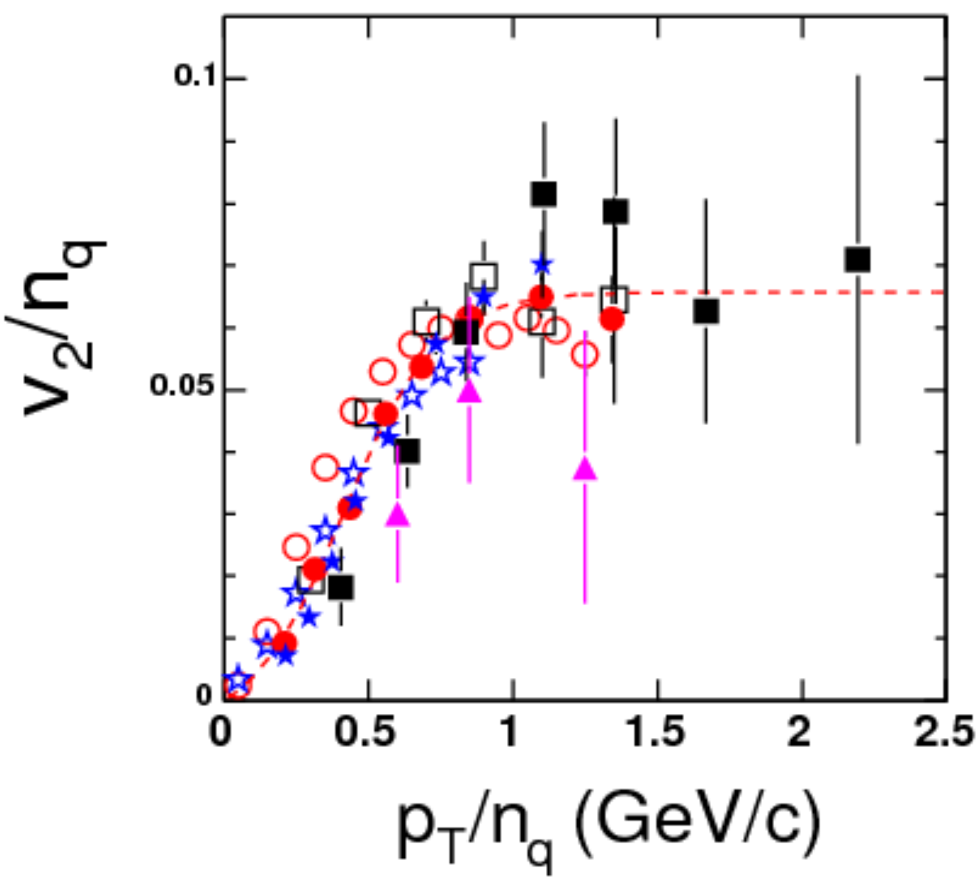


- Increases with energy
- Larger for light hadron
- Larger for K^+ than K^-?
- Phi behaves anomalously, like Λ at lower energy but like pion at high energy.

Phi flow from AMPT

Y. G. Ma et al., EPJ A29, 11 (2006)



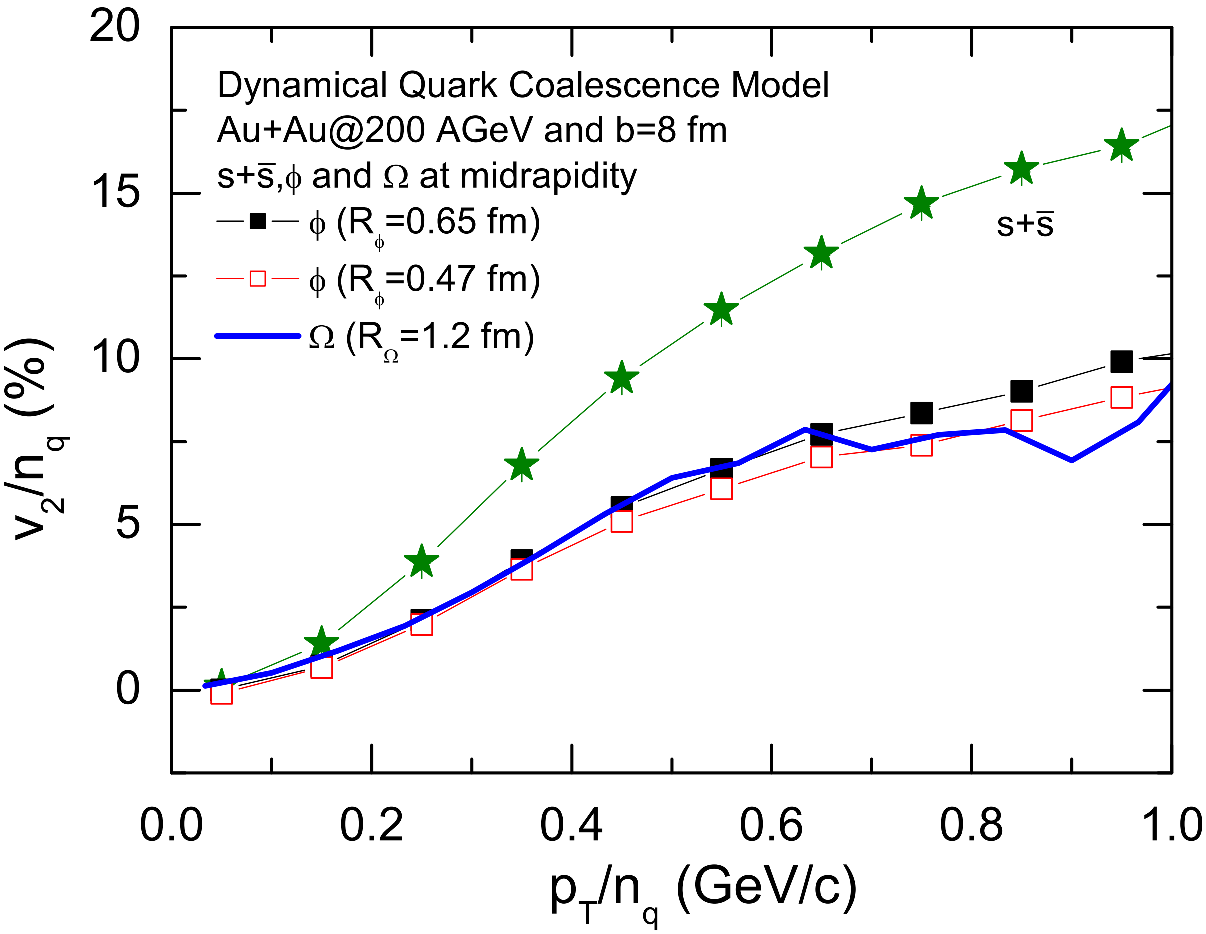


Phi meson v_2 is similar to that of kaon → quark number scaling

Dynamical quark coalescence model

Chen & Ko, PRC 73, 044903 (06)

Based on the phase-space distribution of strange quarks from AMPT and including quark spatial and momentum distributions in hadrons



Although scaled phi and Omega satisfy constituent quark number scaling, they are smaller than the strange quark elliptic flow.

Flavor dependence of escaping contribution to v_2

$\langle v_2 \rangle_{random\text{-}\phi} / \langle v_2 \rangle_{normal}$ ratio
~ fraction from pure escape:

Courtesy of Ziwei Lin

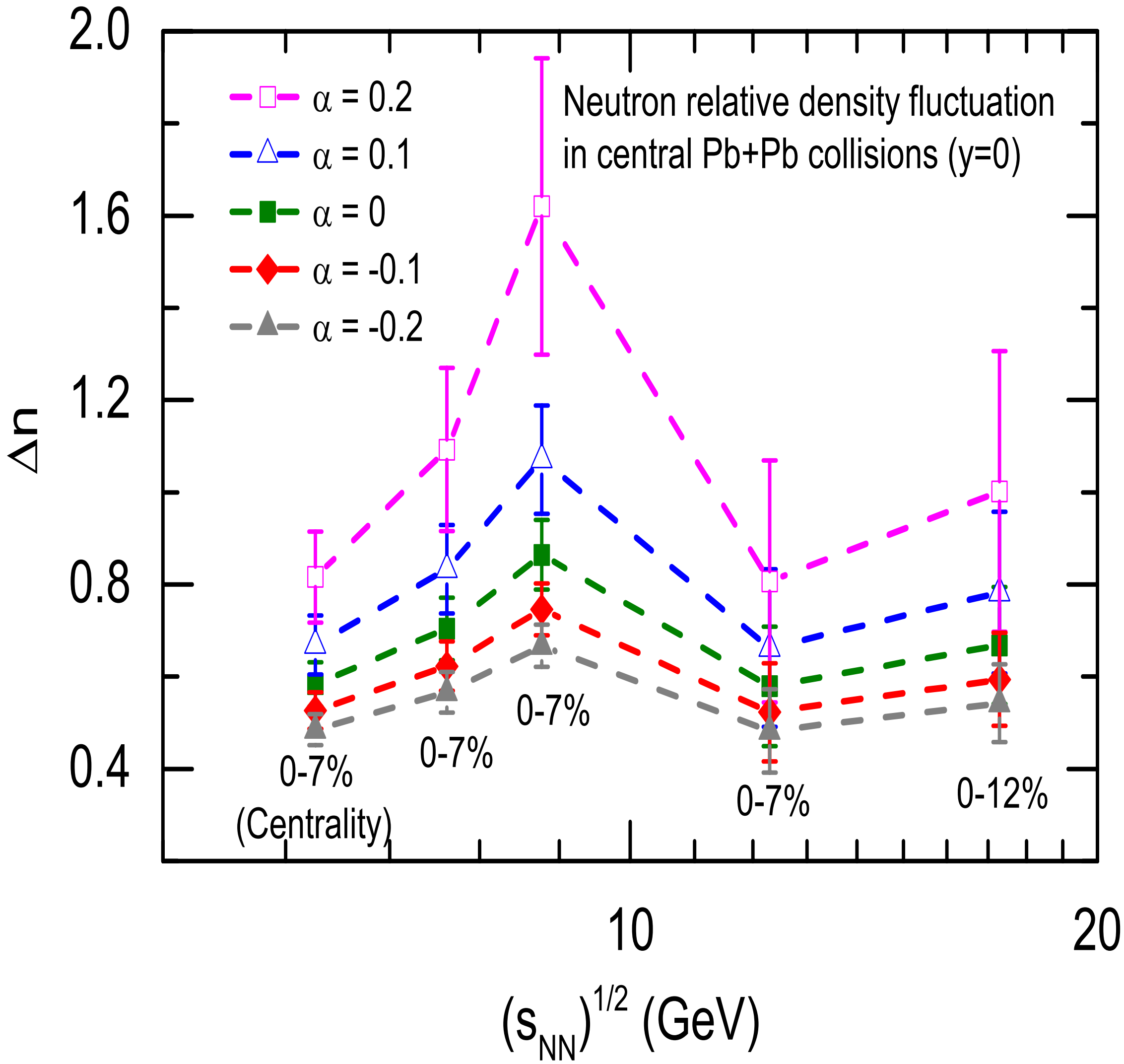
	dAu@200GeV b=0 fm	pPb@5TeV b=0 fm	AuAu@200GeV b=6.6-8.1 fm	PbPb@2.76TeV b=8 fm
u/d	93%(all quarks)	72.9%	65.6%	42.5%
s		59.1%	47.4%	26.5%
c		56.8%	21.8%	8.5%

v2 of charm quarks in AuAu@RHIC-200GeV & PbPb@LHC:
mostly comes from collective flow (not the escape mechanism).
➔heavy quarks are more sensitive probes of collective flow & the medium.
Esha, Md. Nasim & Huang, JPG44 (2017)

v2 of light quarks:
escape mechanism is more important for AuAu@RHIC, pPb@LHC
and smaller/lower-energy systems;
hydro-type collective flow is more important for PbPb@LHC
although with significant contribution from the escape mechanism.

Neutron relative density fluctuation from yield ratio of light nuclei

Sun, Chen, Ko & Xu, arXiv: 1702.07620 [nucl-th]



$$\mathcal{O}_{\text{p-d-t}} = \frac{N_{^3\text{H}} N_p}{N_\text{d}^2}$$

$$= g\frac{1 + (1+2\alpha)\Delta n}{(1+\alpha\Delta n)^2}$$

$$\Delta n = \frac{\langle(\delta n)^2\rangle}{\langle n\rangle^2}$$

$$\langle\delta n \delta n_p\rangle = \alpha\frac{\langle n_p\rangle}{\langle n\rangle}\langle(\delta n)^2\rangle$$

α: correlation factor

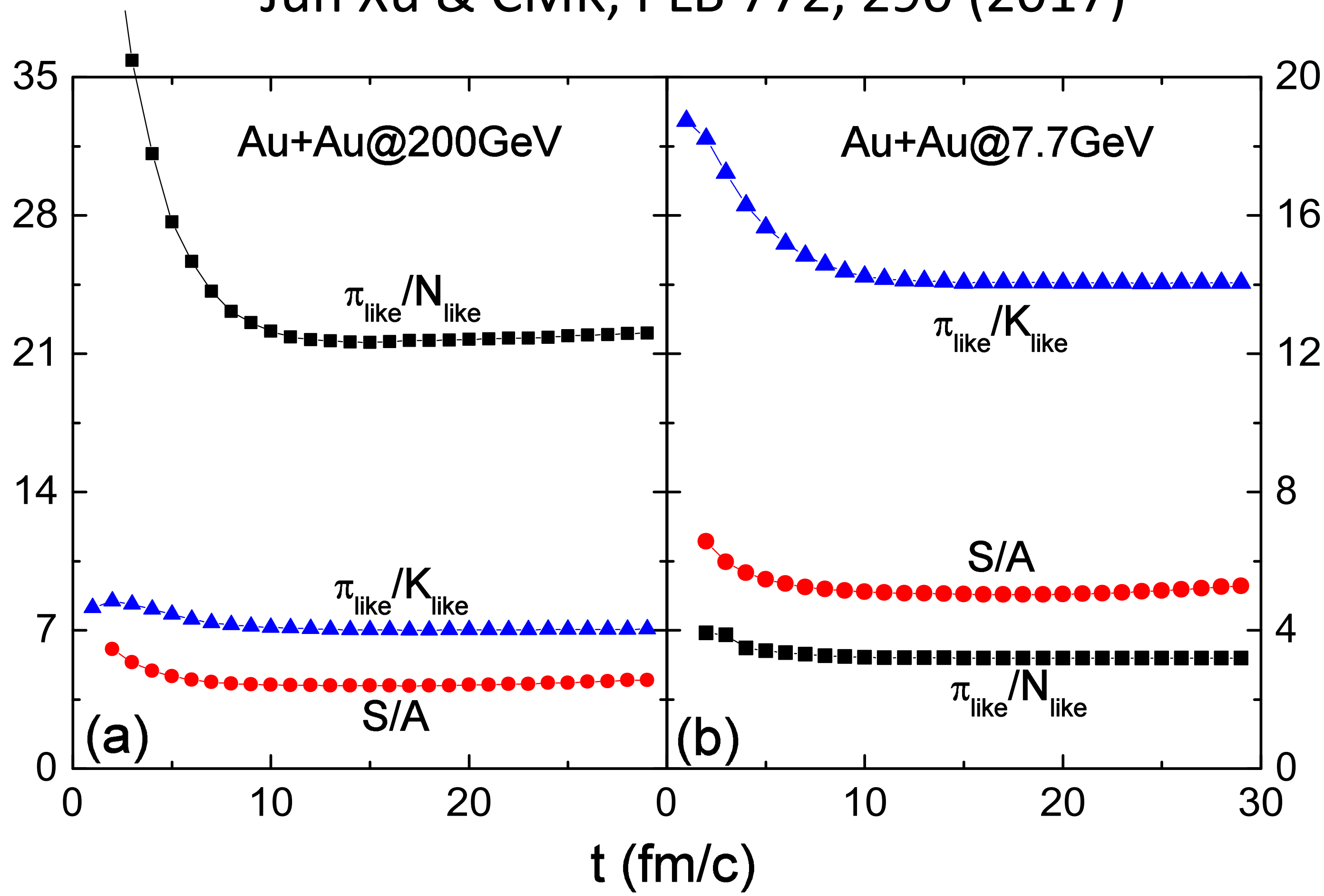
$T_C \approx 144$ MeV

$\mu_C \approx 385$ MeV

- Expect a similar behavior for $\frac{pK_0}{\pi^+\Lambda}$ from u-quark density fluctuation.

Chemical freeze-out in relativistic heavy ion collisions

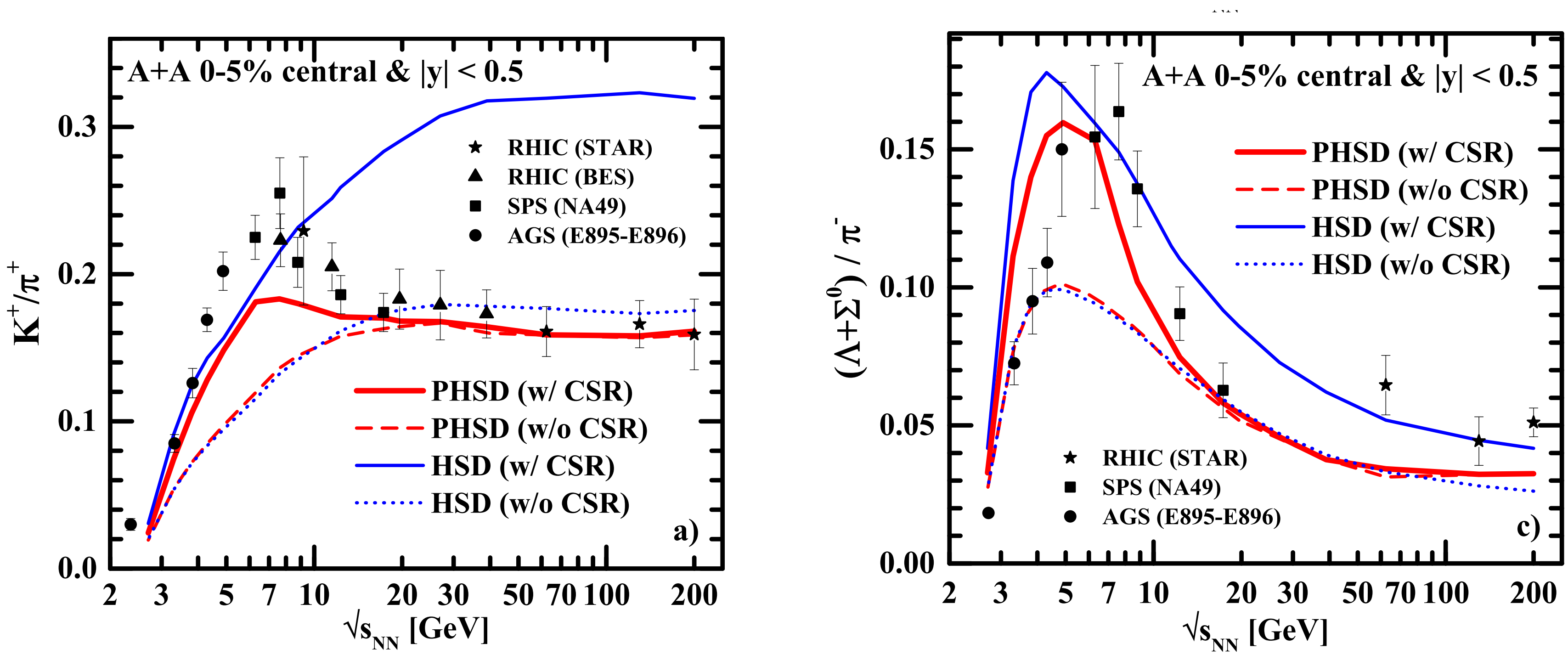
Jun Xu & CMK, PLB 772, 290 (2017)



- Both ratio of effective particle numbers and entropy per particle remain essentially constant from chemical to kinetic freeze-out.

Energy dependence of strange hadron to pion ratio (horn)

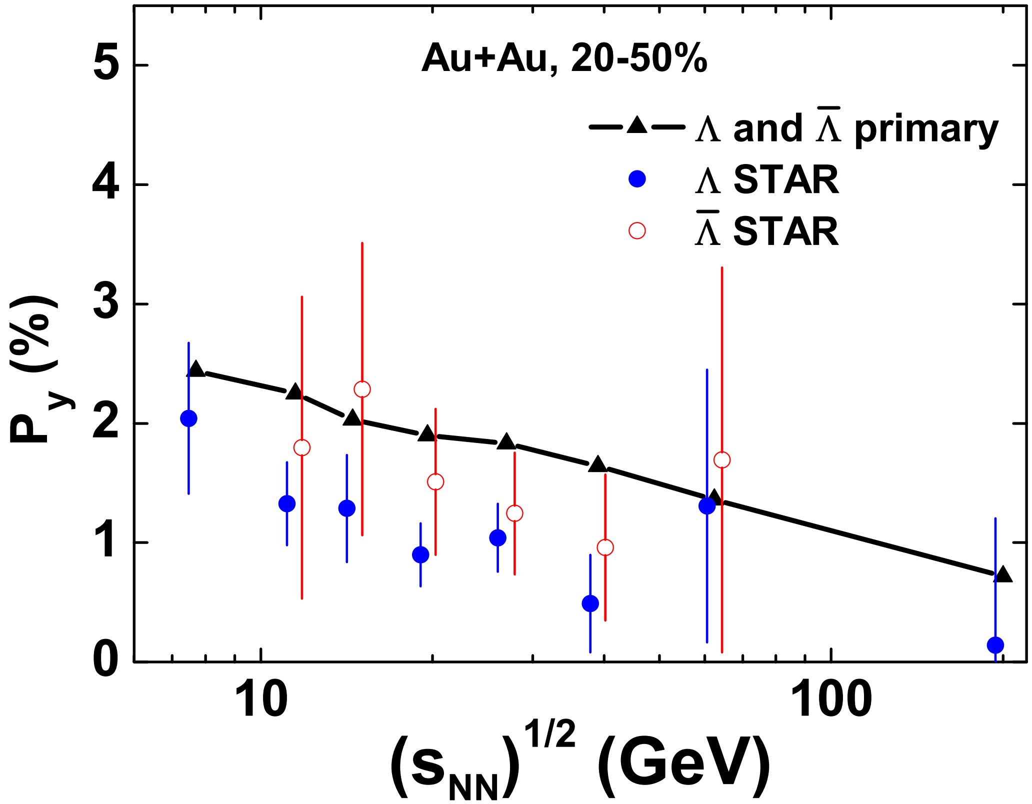
Gazdzicki and Gorentein, Acta Phys. Polom B30, 2750 (1999)



- Formation of QGP in statistical model: Cleymans et al., PLB 615, 50 (2005); Andronic, Braun-Munzinger & Stachel, NPA 772, 167 (2006)
- Chiral restoration in PHSD: A. Palmese et al., PRC 94, 044912 (2016).

Lambda polariztion in heavy ion collisions

Yifeng Sun and CMK, arXiV:1706 [nucl-th]



- Quarks follow chiral vortical equations of motion

$$\dot{\mathbf{x}} = \frac{1+\lambda\frac{\boldsymbol{\omega}}{p}}{1+3\lambda\frac{\boldsymbol{\omega}}{p}\cdot\hat{\mathbf{p}}}$$

$$\dot{\mathbf{p}} = 0$$

$$\boldsymbol{\omega} = \frac{1}{2}\nabla\times\mathbf{v}$$

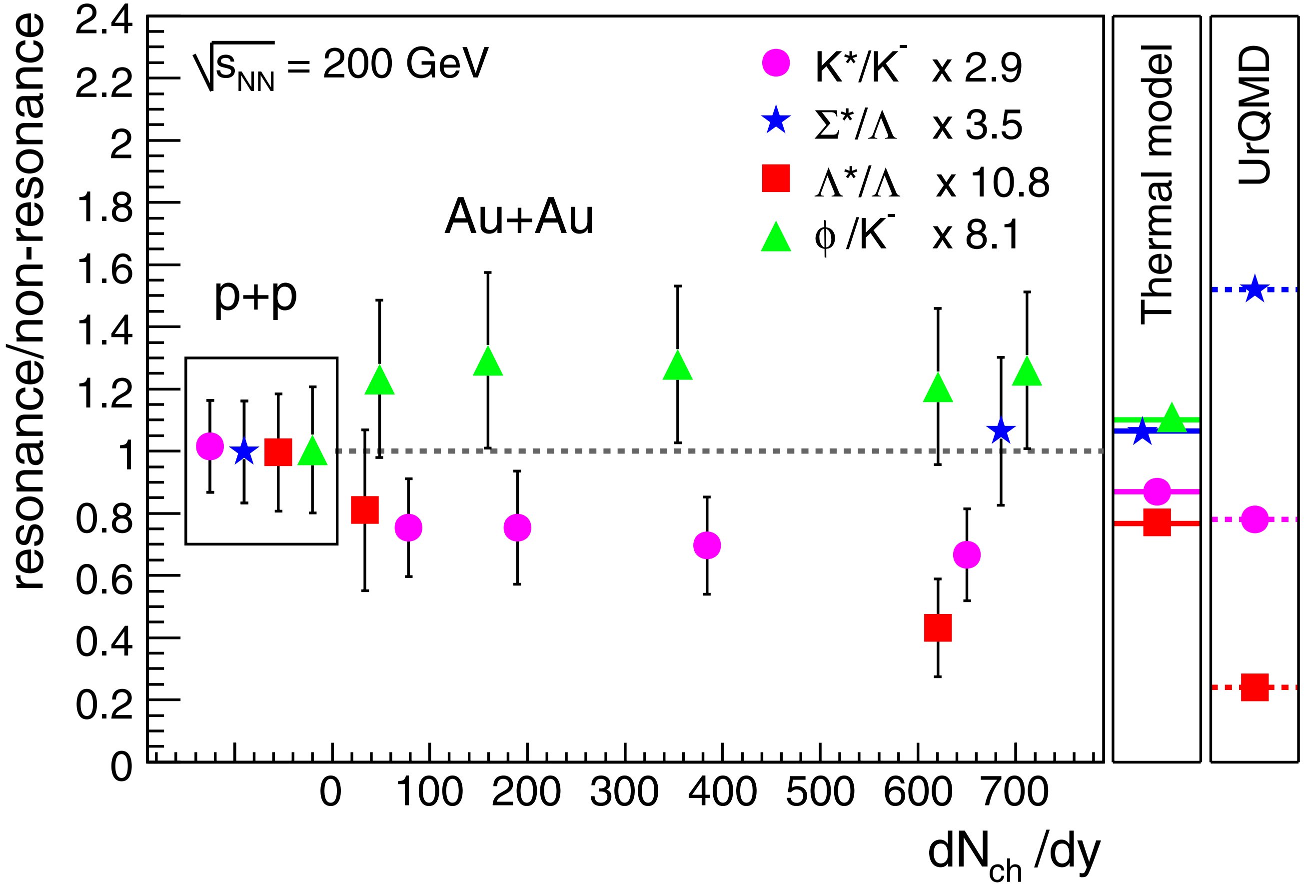
$$\lambda : \text{helicity} = \pm 1$$

$$\mathbf{v} : \text{velocity field}$$

- Quarks and antiquarks of opposite helicities undergo helicity flip scattering.
- Polarized Lambda formed from polarized quarks via coalescence.
- Lambda polarization decreases with energy due to decreasing vorticity.
- Similar to results based on statistical model based on final vorticity from hydro transport model.
- Seemingly larger Lambda than anti-Lambda polarizations in data is not understood.

Suppression of Λ(1520) in HIC

Abelev [Star Collaboration], PRL 97, 132301 (2006)



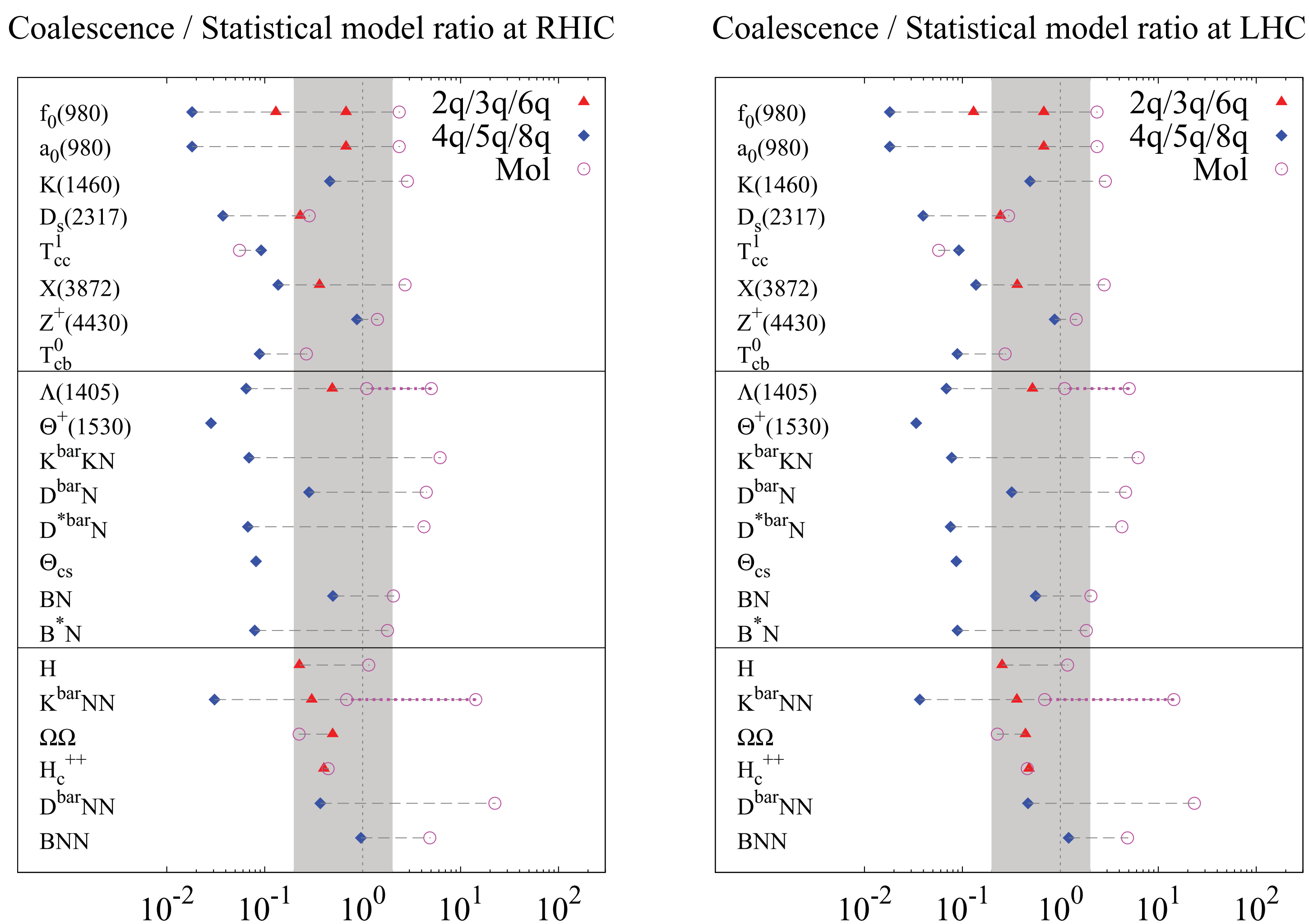
- Measured Λ(1520)/Λ(1115) ratio is significantly smaller than thermal model prediction, but can be explained by the coalescence model after taking into account the p-wave state of strang quark in Λ(1520) (Kanada-En' yo and Meuller, PRC 74, 061901(R) (2006).

Exotic mesons, baryons and dibaryons

Cho, Furumoto, Hyodo, Jido, Ko, Lee, Nielsen, Ohnishi, Sekihara, Yasui, and Yazaki [ExHIC Collaboration], PRL 106, 212001 (2011); PRC 84, 064910 (2011); PPNP 95, 279 (2017)

Particle	m (MeV)	g	I	J^P	$2q/3q/6q$	$4q/5q/8q$	Mol.	$\omega_{\text{Mol.}}$ (MeV)	decay mode
Mesons									
$f_0(980)$	980	1	0	0^+	$q\bar{q}, s\bar{s}$ $(L=1)$	$q\bar{q}s\bar{s}$	$\bar{K}K$	67.8(B)	$\pi\pi$ (strong decay)
$a_0(980)$	980	3	1	0^+	$q\bar{q}$ $(L=1)$	$q\bar{q}s\bar{s}$	$\bar{K}K$	67.8(B)	$\eta\pi$ (strong decay)
$K(1460)$	1460	2	1/2	0^-	$q\bar{s}$	$q\bar{q}q\bar{s}$	$\bar{K}KK$	69.0(R)	$K\pi\pi$ (strong decay)
$D_s(2317)$	2317	1	0	0^+	$c\bar{s}$ $(L=1)$	$q\bar{q}c\bar{s}$	DK	273(B)	$D_s\pi$ (strong decay)
T^1_{cc} †)	3797	3	0	1^+	—	$qq\bar{c}\bar{c}$	$\bar{D}\bar{D}^*$	476(B)	$K^+\pi^- + K^+\pi^- + \pi^-$
$X(3872)$	3872	3	0	1^+, 2^- *)	$c\bar{c}$ $(L=2)$	$q\bar{q}c\bar{c}$	$\bar{D}D^*$	3.6(B)	$J/\psi\pi\pi$ (strong decay)
$Z^+(4430)$ ‡)	4430	3	1	0^- *)	—	$q\bar{q}c\bar{c}$ $(L=1)$	$D_1\bar{D}^*$	13.5(B)	$J/\psi\pi$ (strong decay)
T^0_{cb} †)	7123	1	0	0^+	—	$qq\bar{c}\bar{b}$	$\bar{D}B$	128(B)	$K^+\pi^- + K^+\pi^-$
Baryons									
$\Lambda(1405)$	1405	2	0	$1/2^-$	qqs $(L=1)$	$qqqs\bar{q}$	$\bar{K}N$	20.5(R)-174(B)	$\pi\Sigma$ (strong decay)
$\Theta^+(1530)$ ‡)	1530	2	0	$1/2^+$ *)	—	$qqqq\bar{s}$ $(L=1)$	—	—	KN (strong decay)
$\bar{K}KN$ †)	1920	4	1/2	$1/2^+$	—	$qqqs\bar{s}$ $(L=1)$	$\bar{K}KN$	42(R)	$K\pi\Sigma$, $\pi\eta N$ (strong decay)
$\bar{D}N$ †)	2790	2	0	$1/2^-$	—	$qqqq\bar{c}$	$\bar{D}N$	6.48(R)	$K^+\pi^-\pi^- + p$
$\bar{D}^*N$ †)	2919	4	0	$3/2^-$	—	$qqqq\bar{c}$ $(L=2)$	$\bar{D}^*N$	6.48(R)	$\bar{D} + N$ (strong decay)
Θ_{cs} †)	2980	4	1/2	$1/2^+$	—	$qqqs\bar{c}$ $(L=1)$	—	—	$\Lambda + K^+\pi^-$
BN †)	6200	2	0	$1/2^-$	—	$qqqq\bar{b}$	BN	25.4(R)	$K^+\pi^-\pi^- + \pi^+ + p$
B^*N †)	6226	4	0	$3/2^-$	—	$qqqq\bar{b}$ $(L=2)$	B^*N	25.4(R)	$B + N$ (strong decay)
Dibaryons									
H †)	2245	1	0	0^+	$qqqqss$	—	ΞN	73.2(B)	$\Lambda\Lambda$ (strong decay)
$\bar{K}NN$ ‡)	2352	2	1/2	0^- *)	$qqqqqs$ $(L=1)$	$qqqqqq\,s\bar{q}$	$\bar{K}NN$	20.5(T)-174(T)	ΛN (strong decay)
$\Omega\Omega$ †)	3228	1	0	0^+	$ssssss$	—	$\Omega\Omega$	98.8(R)	$\Lambda K^- + \Lambda K^-$
H_c^{++} †)	3377	3	1	0^+	$qqqqsc$	—	$\Xi_c N$	187(B)	$\Lambda K^-\pi^+\pi^+ + p$
$\bar{D}NN$ †)	3734	2	1/2	0^-	—	$qqqqqq\,q\bar{c}$	$\bar{D}NN$	6.48(T)	$K^+\pi^- + d$, $K^+\pi^-\pi^- + p + p$
BNN †)	7147	2	1/2	0^-	—	$qqqqqq\,q\bar{b}$	BNN	25.4(T)	$K^+\pi^- + d$, $K^+\pi^- + p + p$

Ratio of exotic hadron yields from coalescence and statistical models

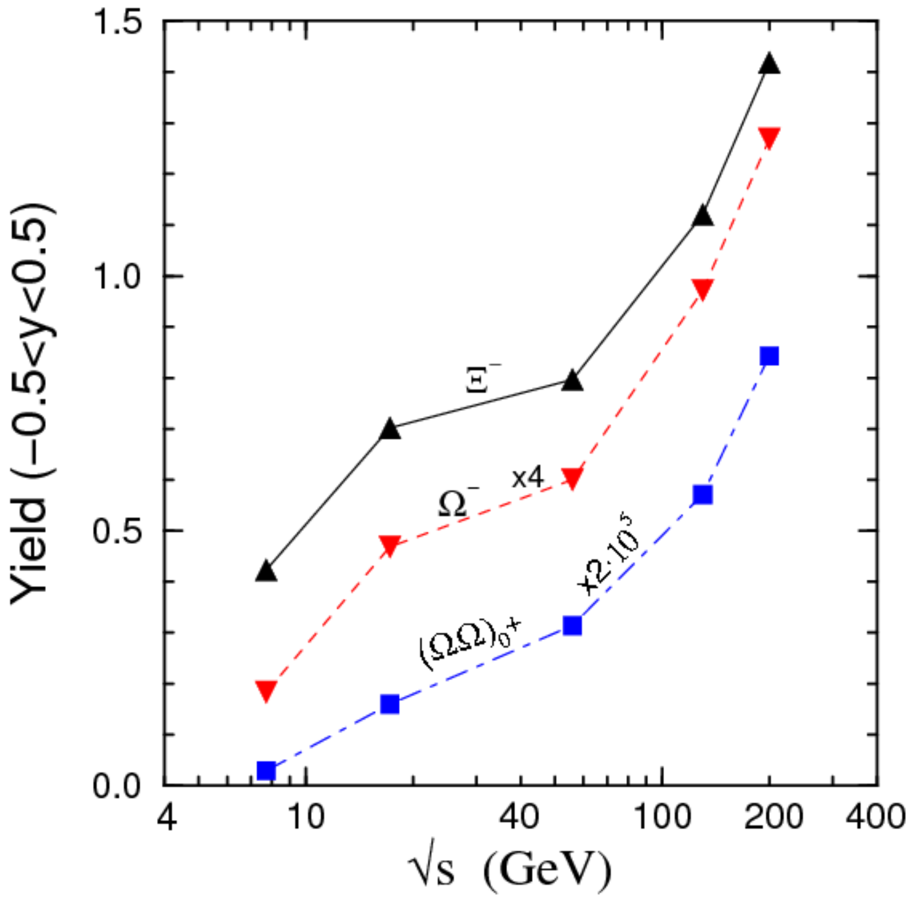


- Multiquark hadrons are suppressed while hadronic molecules are enhanced in coalescence model, compared to the statistical model predictions

Diomega prodcution in HIC

According to the chiral quark model of Zhang et al. (PRC 61, 065204 (2000)), diomega $(\Omega\Omega)_{0+}$ is bound by ~ 116 MeV with a root-mean-square radius ~ 0.84 fm and lifetime ~ 10^{-10} sec.

No.	Channel
I	$\Omega+\Omega\to(\Omega\Omega)_0+\gamma$
II	$\Omega+\Omega\to(\Omega\Omega)_0+\eta$
III	$\Omega+\Omega\to(\Omega\Omega)_0+\eta'$
IV	$\Omega+\Omega\to(\Omega\Omega)_0+\phi$
V	$\Omega+\Xi\to(\Omega\Omega)_0+K$
VI	$\Omega+\Xi\to(\Omega\Omega)_0+K^*$
VII	$\Omega+N\to(\Omega N)_2+\gamma$
VIII	$\Omega+N\to(\Omega N)_2+\pi$
IX	$\Omega+(\Omega N)_2\to(\Omega\Omega)_0+N$



Pal, Ko & Zhang, PLB 624, 210 (05)

- Cross sections for I-VI are ~ 2-25 μb and thus unimportant.
- Production is dominated by two-step processes through VII, VIII (~ 50-175 μb) and IX (~ 20-50 mb).
- Yield is order of magnitude smaller than in statistical or coalescence model (neglect of three-body process?)

Hidden-charm pentaquark hadrons: $P_c^+(4380)$, $P_c^+(4450)$

Wang, Song, Sun, Chen, Li, & Shao, PRC 94, 044913 (2016)

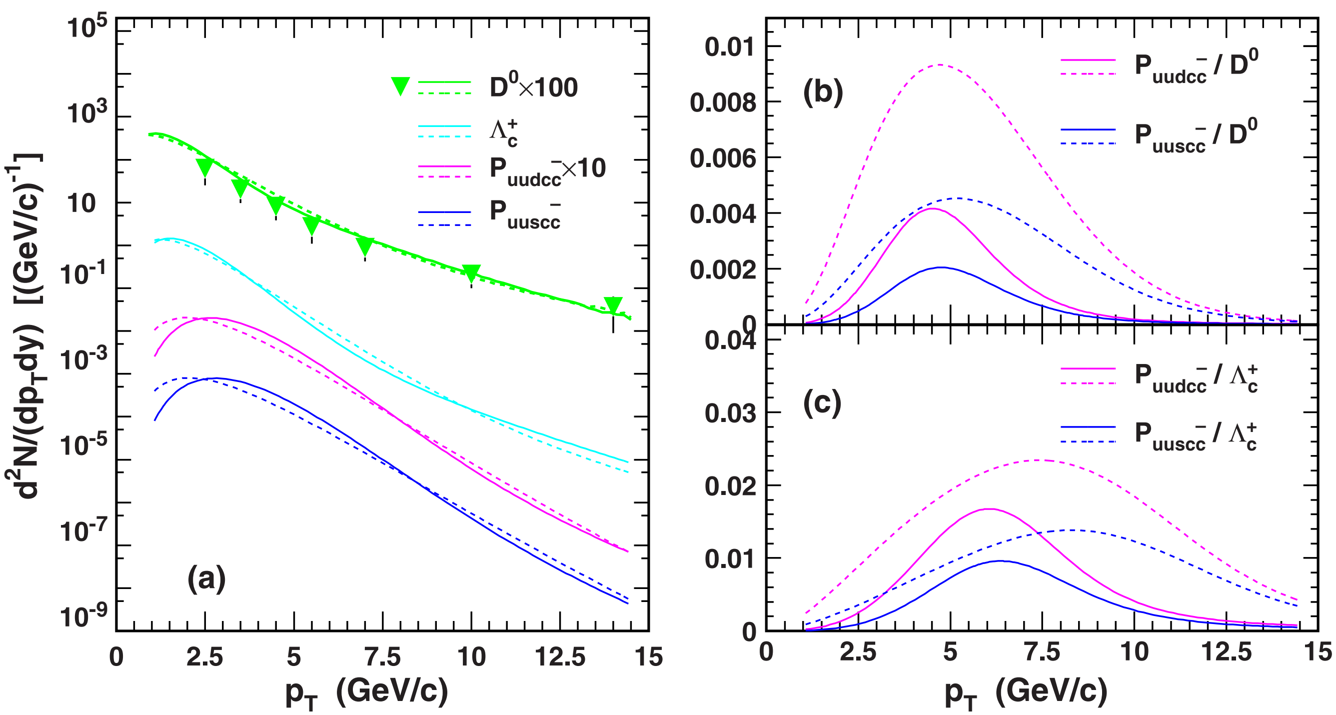


FIG. 1. (a) p_T spectra of D^0 mesons, Λ_c^+ baryons, and hidden-charm pentaquark states $P_{uudc\bar{c}}$ and $P_{uusc\bar{c}}$; (b) ratios $P_{uudc\bar{c}}/D^0$ and $P_{uusc\bar{c}}/D^0$; (c) ratios $P_{uudc\bar{c}}/\Lambda_c^+$ and $P_{uusc\bar{c}}/\Lambda_c^+$ at midrapidity in central Pb + Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. Filled symbols represent the experimental data from Ref. [45], and different lines show our results. Solid lines show results under the equal p_T combination scenario; dashed lines, results under the equal v_T combination scenario. In (a) p_T distributions of D^0 and $P_{uudc\bar{c}}$ are scaled by 100 and 10, respectively, for clarity.

Summary

- Are K^0/K^+ and Σ^-/Σ^+ good probes of symmetry energy at high densities?
- How important are mean-field potentials in BES and FAIR?
- Does phi meson scatter frequently in expanding hadronic matter?
- Where can one see quark recombination or coalescence for strange hadrons?
- Why does hadronic evolution not affect strange hadron abundances?
- What is the cause of "horn" in strange/pion ratio?
- Can the ratio $\frac{pK_0}{\pi^+\Lambda}$ provide information on u-quark density fluctuation?
- What is the chance to find exotic hadrons in HIC such as diomega as well as $P_c^+(4380)$ and $P_c^+(4450)$?