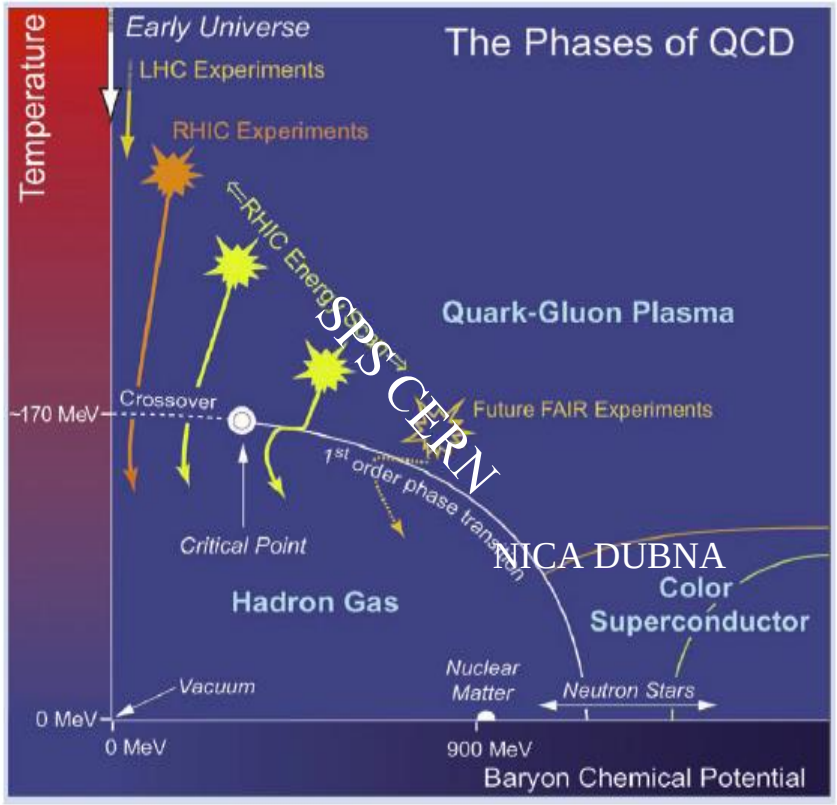
Overlap between Lattice and HIC data at the pseudo-critical temperature

Krzysztof Redlich, University of Wroclaw & EMMI/GSI

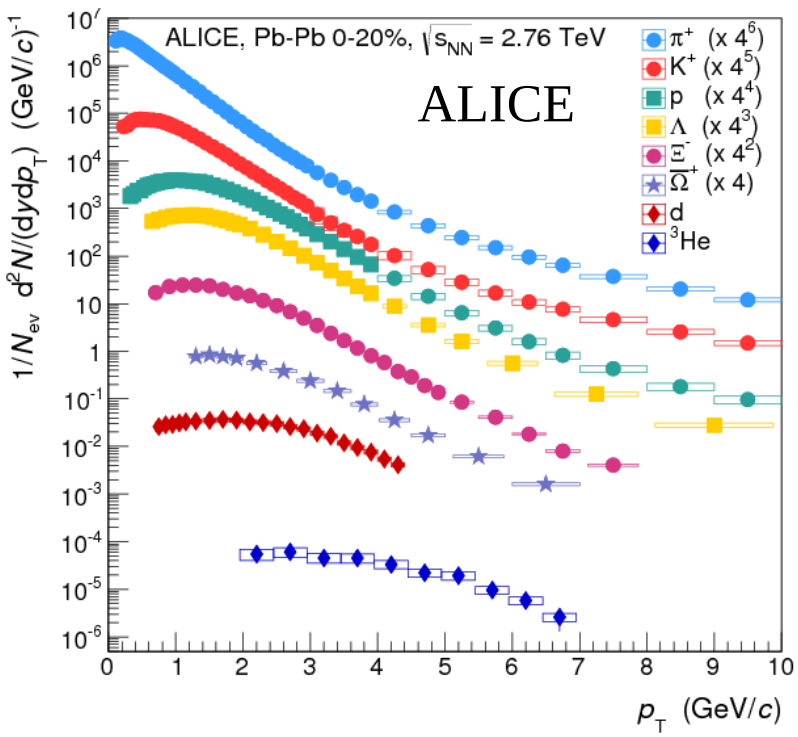
- Probing thermalization, particles composition and parameters of the collision fireball in HIC at the LHC
  - ⇨ linking LQCD results to HIC data of ALICE coll.
- Modelling QCD thermodynamic potential within HRG
  - ⇨ importance of dynamical widths of resonances: the S-matrix approach



Compare HIC data and Lattice QCD results

Can the thermal nature and composition of the collision fireball in HIC be verified ?

HIC → **Lattice QCD**



- The strategy:
  - Compare directly measured fluctuations and correlations with LGT

    F. Karsch and K. R, Phys. Lett. B 695, 136 (2011)

    F. Karsch, Central Eur. J. Phys. 10, 1234 (2012)

    A. Bazavov et al., Phys. Rev. Lett. 109, 192302 (2012):

    P. Alba, R. Bellwied, M. Bluhm, V. Mantovani Sarti, M.Nahrgang and C. Ratti, Phys. Rev. C 92, 064910 (2015)

    see also talk: Swagato Mukheriee , Claudia Ratti
  - Construct the 2nd order fluctuations and correlations from measured yields and compare with LGT

    P. Braun-Munzinger, A. Kalweit, J. Stachel, K.R. Phys. Lett. B 47, 292 (2015), Nucl.Phys. A956, 805 (2016)

Consider fluctuations and correlations of conserved charges to be compared with LQCD

Excellent probe of:

- QCD criticality

A. Asakawa at. al.

S. Ejiri et al.,…

M. Stephanov et al.,

K. Rajagopal et al.

B. Frimann et al.

- freezeout conditions in HIC

F. Karsch &

S. Mukherjee et al.,

C. Ratti et al.

P. Braun-Munzinger et al.

- They are quantified by susceptibilities:

If $P(T,\mu_B,\mu_Q,\mu_S)$ denotes pressure, then

$$\frac{\chi_N}{T^2} = \frac{\partial^2(P)}{\partial(\mu_N)^2} \qquad \frac{\chi_{NM}}{T^2} = \frac{\partial^2(P)}{\partial\mu_N\partial\mu_M}$$

$N = N_q - N_{-q}$, $N,M = (B,S,Q)$, $\mu = \mu/T$, $P = P/T^4$

- Susceptibility is connected with variance

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(<N^2> - <N>^2)$$

- If $\mathrm{P}(N)$ probability distribution of N then

$$<N^n> = \sum_N N^n \mathrm{P}(N)$$

Consider special case:

$< N_q > \equiv N_q$ =>

Charge carrying by particles $q = \pm 1$

- Charge and anti-charge uncorrelated and Poisson distributed, then
- $\mathrm{P}(N)$ the Skellam distribution

$$\mathrm{P}(N) = \left(\frac{N_q}{N_{-q}} \right)^{N/2} I_N(2\sqrt{N_q N_{-q}}) \exp[-(N_q + N_{-q})]$$

- Then the susceptibility

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(< N_q > + < N_{-q} >)$$

Consider special case: particles carrying $q = \pm 1, \pm 2, \pm 3$

- The probability distribution

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

$$< S_{-q} > \equiv S_{-q}$$

$$q = \pm 1, \pm 2, \pm 3$$

$$P(S) = \left(\frac{S_1}{S_{\bar{1}}}\right)^{\frac{S}{2}} \exp\left[\sum_{n=1}^{3}\left(S_n + S_{\bar{n}}\right)\right]$$

$$\sum_{i=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \left(\frac{S_3}{S_{\bar{3}}}\right)^{\frac{k}{2}} I_k\left(2\sqrt{S_3 S_{\bar{3}}}\right) \left(\frac{S_2}{S_{\bar{2}}}\right)^{\frac{i}{2}} I_i\left(2\sqrt{S_2 S_{\bar{2}}}\right)$$

$$\left(\frac{S_1}{S_{\bar{1}}}\right)^{-i-\frac{3k}{2}} I_{2i+3k-S}\left(2\sqrt{S_1 S_{\bar{1}}}\right)$$

Fluctuations

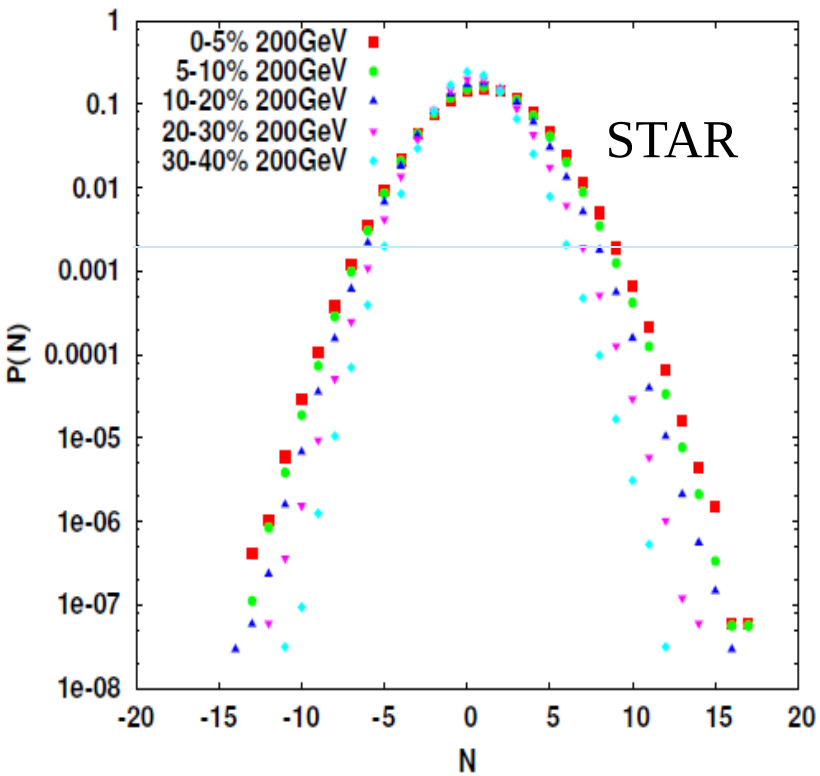
$$\frac{\chi_S}{T^2} = \frac{1}{VT^3} \sum_{n=1}^{|q|} n^2 \left(\langle S_n \rangle + \langle S_{-n} \rangle\right)$$

Correlations

$$\frac{\chi_{NM}}{T^2} = \frac{1}{VT^3} \sum_{m=-q_M}^{q_M} \sum_{n=-q_N}^{q_N} nm \langle S_{n,m} \rangle$$

$\langle S_{n,m} \rangle$ is the mean number of particles carrying charge $N = n$ and $M = m$

Variance at 200 GeV AA central coll. at RHIC

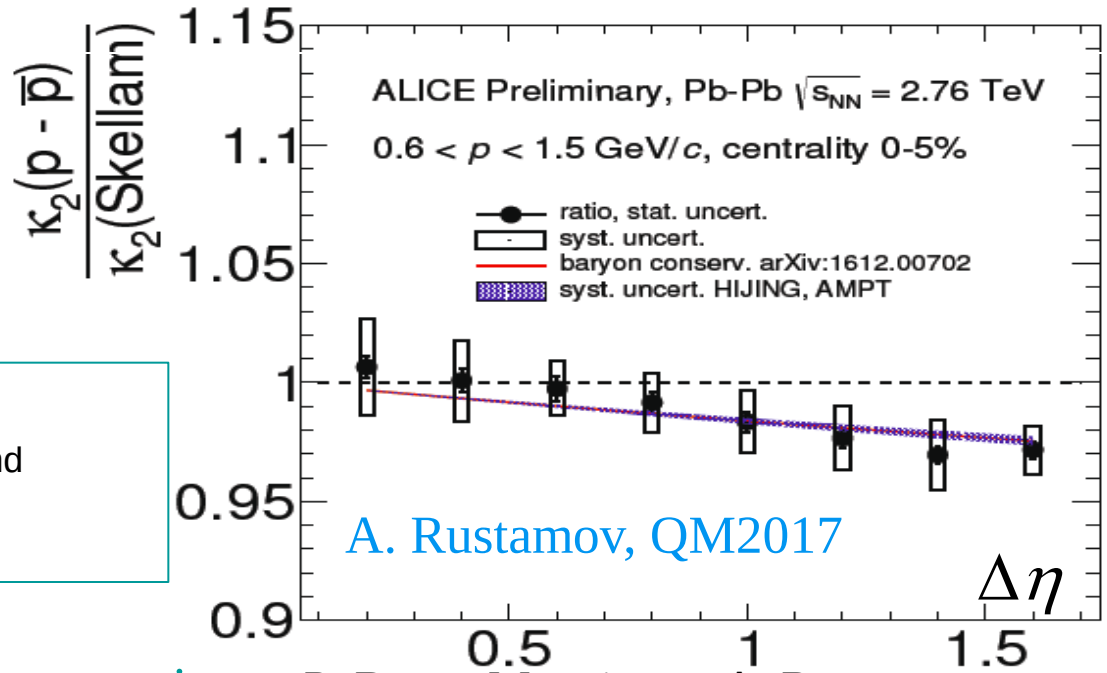


STAR Collaboration data in central coll. 200 GeV

- Consistent with Skellam distribution

$$\frac{<p>+<\bar{p}>}{\sigma^2} = 1.022 \pm 0.016 \qquad \frac{\chi_1}{\chi_3} = 1.076 \pm 0.035$$

- ALICE data consistent with Skellam $\Delta\eta < 1$



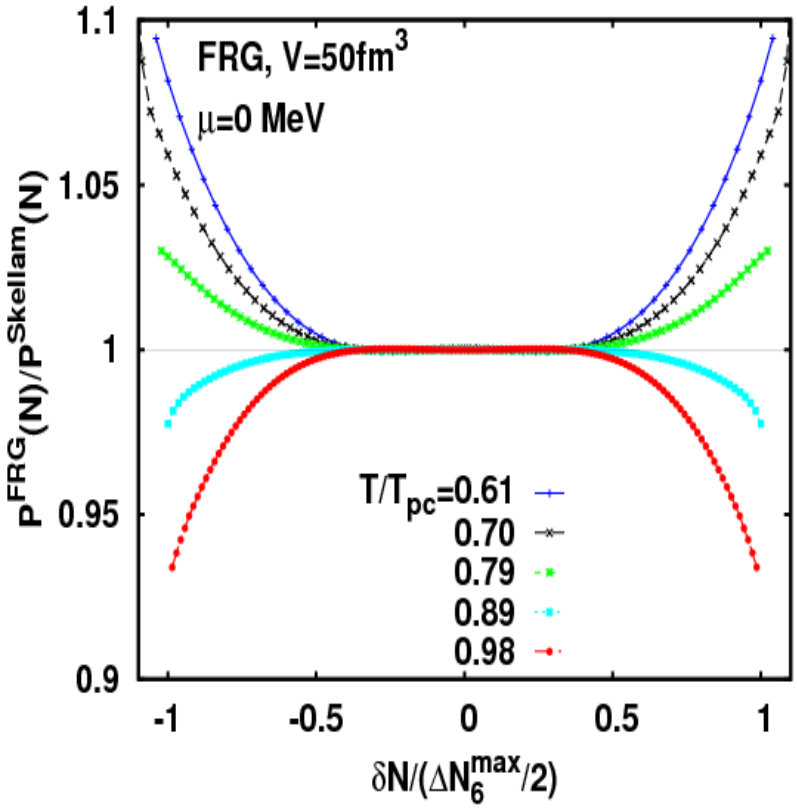
A. Rustamov, QM2017

- Skellam distribution is a good approximation to calculate the 2nd order charge fluctuations in HIC

The influence of baryon number conservation: P. Braun-Munzinger, A. Rustamov, J. Stachel. Nucl Phys. A960 (2017) 114

Variance at 200 GeV AA central coll. at RHIC

K. Morita, B. Friman and K.R.
Phys.Lett. B741 (2015) 178



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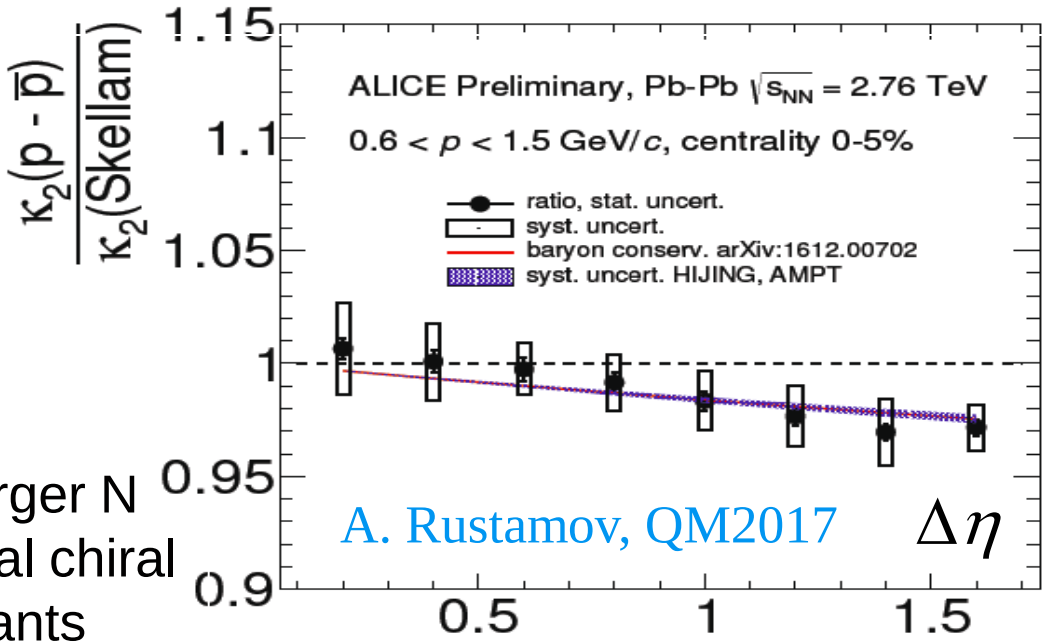


A. Rustamov, QM2017

- Shrinking of Skellam distr. at larger N needed to capture the O(4) critical chiral properties of higher order cumulants

Constructing net charge fluctuations and correlation from ALICE data

- Net baryon number susceptibility

$$\frac{\chi_B}{T^2} = \frac{1}{VT^3}(\langle p\rangle + \langle N\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + \langle \Xi^-\rangle + \langle \Xi^0\rangle + \langle \Omega^-\rangle + \overline{par}\ )$$

- Net strangeness

$$\frac{\chi_S}{T^2} \approx \frac{1}{VT^3}(\langle K^+\rangle + \langle K_S^0\rangle + \langle \Lambda + \Sigma_0\rangle + \langle \Sigma^+\rangle + \langle \Sigma^-\rangle + 4\langle \Xi^-\rangle + 4\langle \Xi^0\rangle + 9\langle \Omega^-\rangle + \overline{par}$$
$$-(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-} + \Gamma_{\varphi\to K_S^0} + \Gamma_{\varphi\to K_L^0})\langle \varphi\rangle\ )$$

- Charge-strangeness correlation

$$\frac{\chi_{QS}}{T^2} \approx \frac{1}{VT^3}(\langle K^+\rangle + 2\langle \Xi^-\rangle + 3\langle \Omega^-\rangle + \overline{par}$$
$$-(\Gamma_{\varphi\to K^+} + \Gamma_{\varphi\to K^-})\langle \varphi\rangle - (\Gamma_{K_0^*\to K^+} + \Gamma_{K_0^*\to K^-})\langle K_0^*\rangle\ )$$

Direct comparisons of Heavy ion data at LHC with LQCD

STAR and ALICE results => the 2nd order cumulants are consistent with Skellam distribution, thus χ_N and χ_{NM} with $N,M=\{B,Q,S\}$ are expressed by particle yields.

$$\frac{\chi_B}{T^2}=\frac{1}{VT^3}(203.7\pm11.4)$$

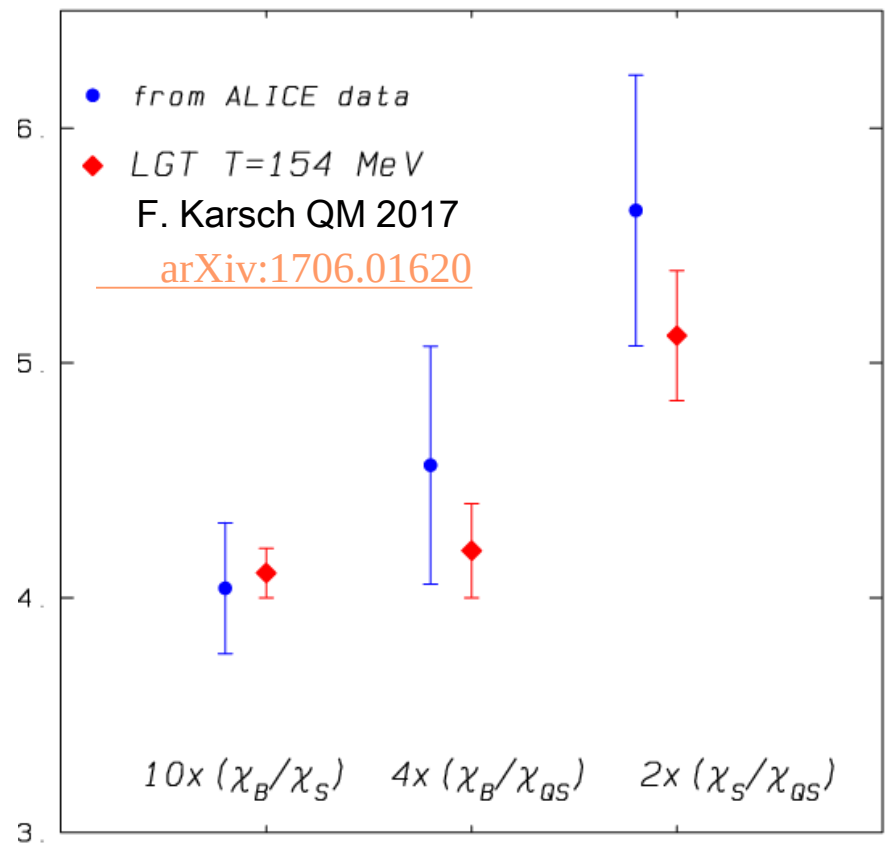
$$\frac{\chi_S}{T^2}=\frac{1}{VT^3}(504.2\pm16.8)$$

$$\frac{\chi_{QS}}{T^2}=\frac{1}{VT^3}(191.1\pm12)$$

- The Volume at T_c

$$V_{T_c}=3800\pm500\ \ fm^3$$

LGT results from: A. Bazavov et al., Phys. Rev. D 95}, 054504 (2017)



The cumulant ratios extracted from ALICE data are consistent with LQCD at the chiral crossover:
Evidence for thermalization at the phase boundary

The ratio of cumulants in LGT and ALICE data

- The ratio

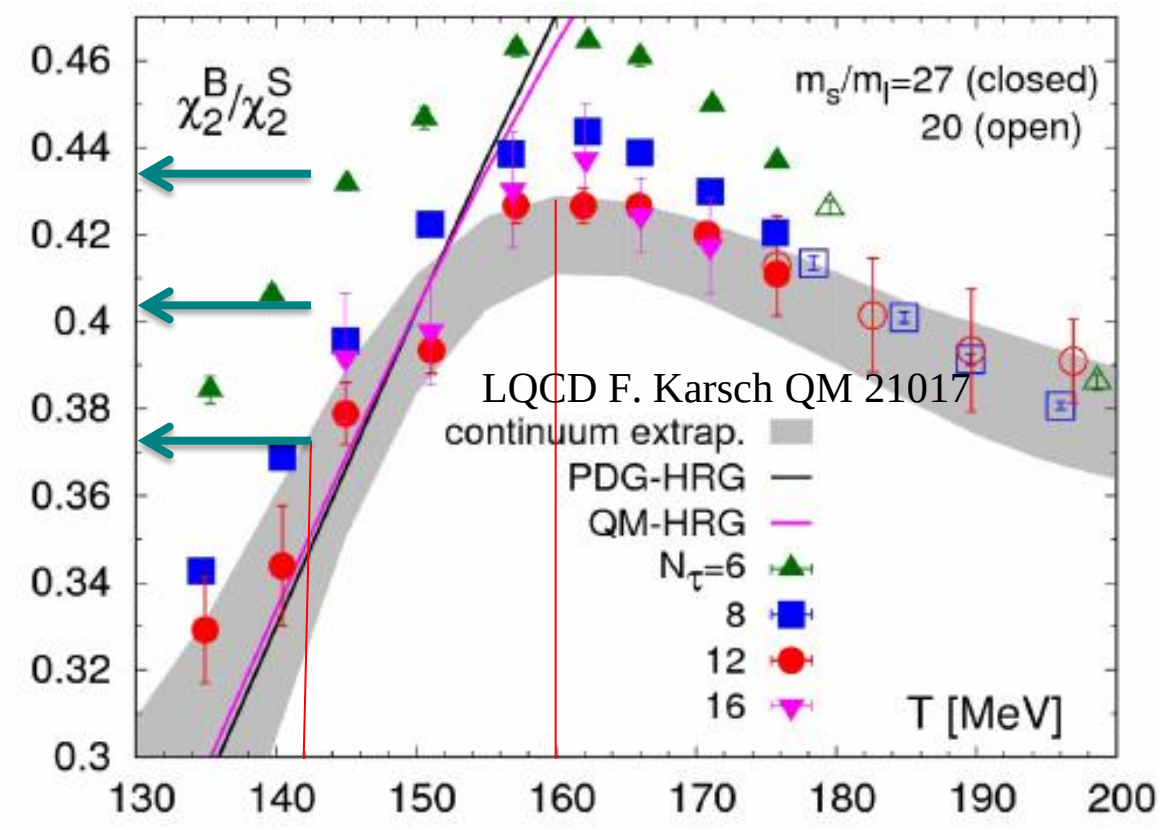
$$0.376 \le \frac{\chi_2^B}{\chi_2^S} \le 0.432$$

extracted from ALICE data is consistent with LQCD for

$$142 < T_f \le 160 \text{ MeV}$$

thus excellently overlaps with chiral crossover

$$145 < T_c \le 163 \text{ MeV}$$



Charge - Strangeness correlations

- The ratio

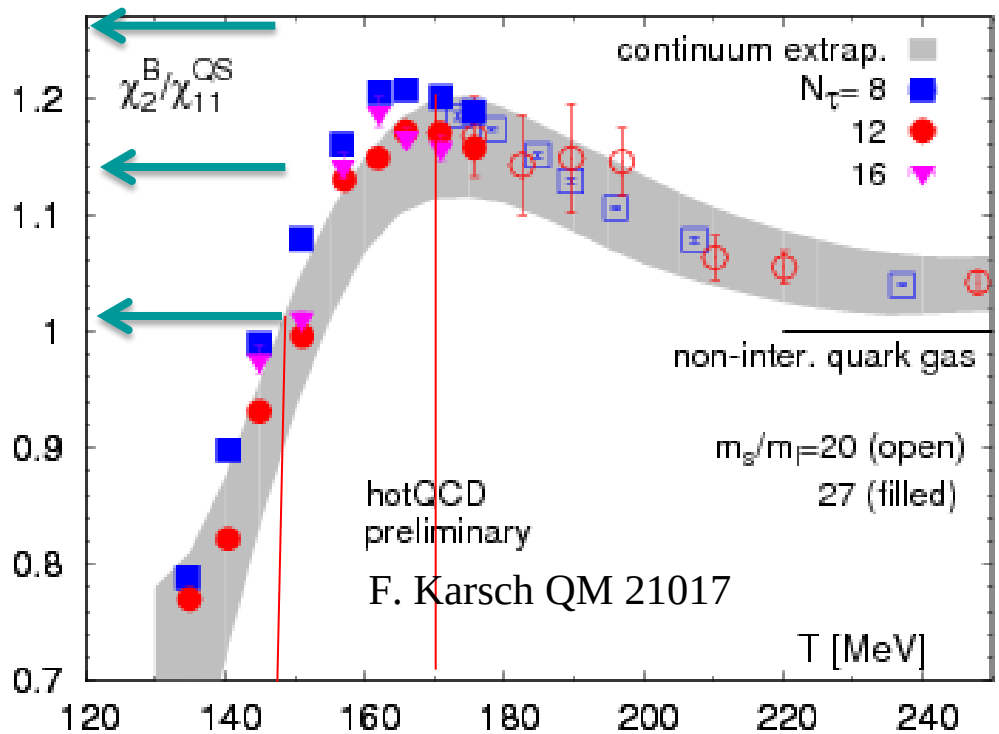
$$1.014 \le \frac{\chi_2^B}{\chi_2^{QS}} \le 1.267$$

extracted from ALICE data is consistent with LQCD for

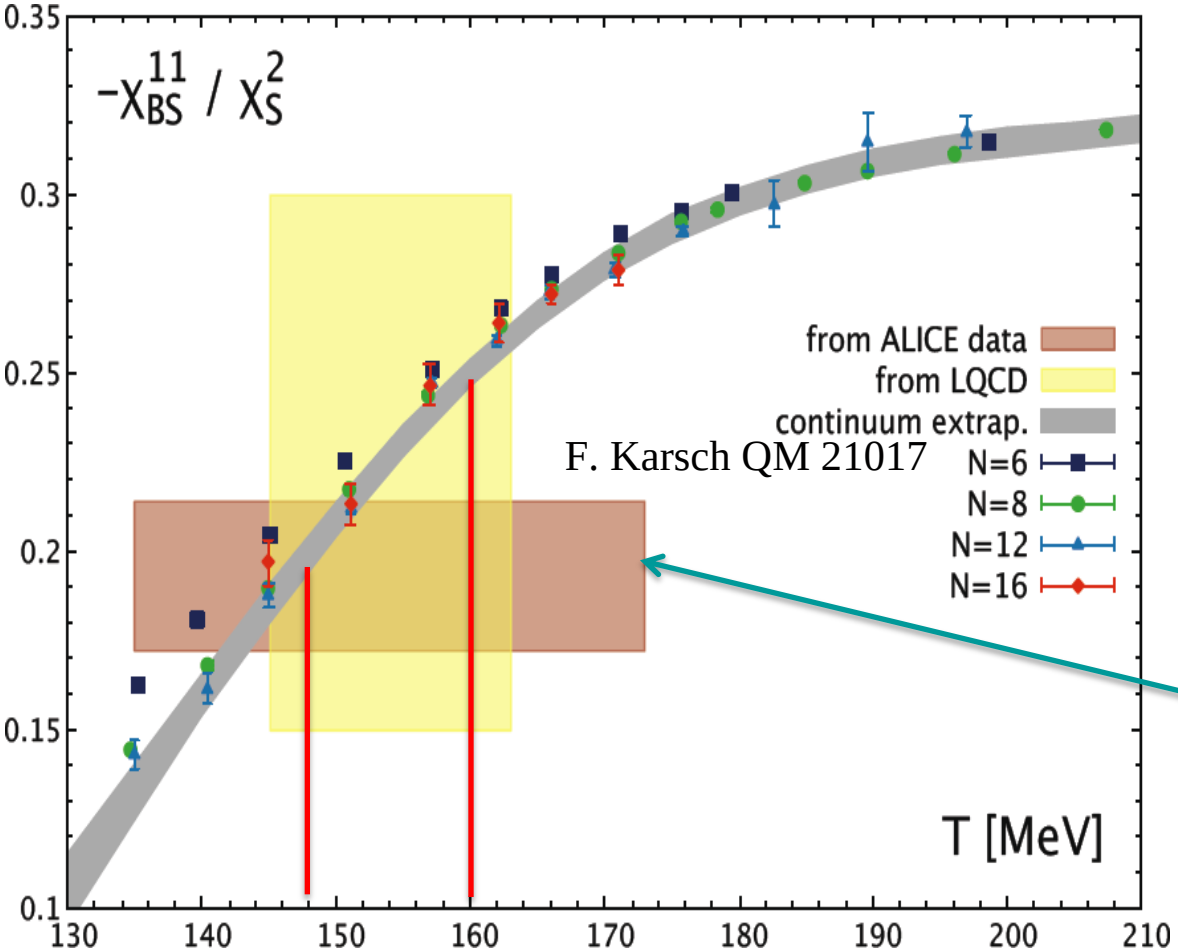
$$148 < T_f \le 170 \quad \text{MeV}$$

when combined with T_f obtained from χ_2^B / χ_2^S one concludes that, data consistent with LGT for

$$148 \le T_f < 160$$



Constraining chemical freezeout temperature at the LHC



$$C_{BS} = -\frac{<(\delta B)(\delta S)>}{<(\delta S)^2>} = -\frac{\chi_{BS}}{\chi_S}$$

- Excellent observable to fix the temperature

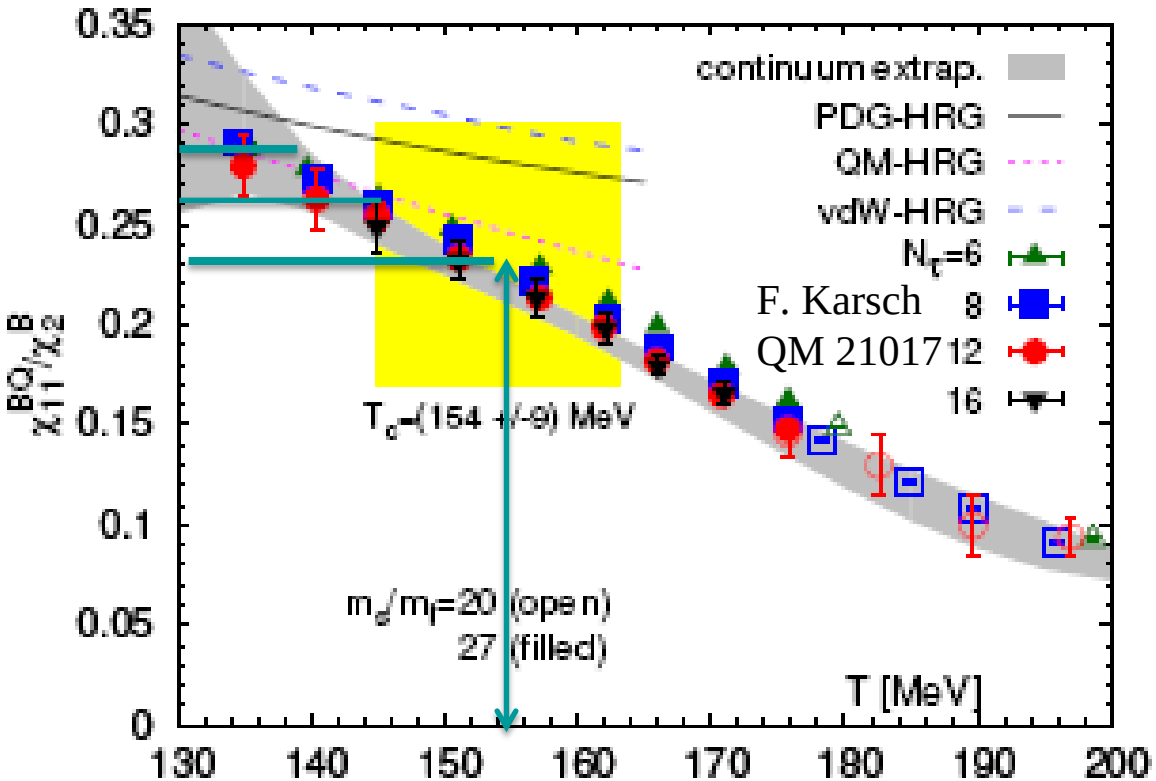
$$-\frac{\chi_{BS}}{T^2} \approx \frac{1}{VT^3}[2<\Lambda+\Sigma^0>+4<\Sigma^+> +8<\Xi>+6<\Omega^->] = (97.4\pm5.8)/VT^3$$

However, this is the lower limit since e.g. $\Sigma^*(\geq 1660) \to N\bar{K}$ $\Lambda^*(\geq 1520) \to N\bar{K}$ are not included

- Data on χ_B / χ_S and χ_B / χ_{QS} consistent with LQCD results for $0.148 \leq T_f < 160\ MeV$

At the LHC energy the fireball created in HIC is a QCD medium at the chiral cross over temperature.

Constraining the upper value of the chemical freeze-out temperature at the LHC



- Considering the ratio

$$\frac{<(\delta B)(\delta Q)>}{<(\delta B)^2>} = \frac{\chi_{BQ}}{\chi_B} = 0.26 \pm 0.03$$

one gets $T < 156\ MeV$

- From the comparison of 2nd order fluctuations and correlations observables constructed from ALICE data and LQCD, one gets agreement at

$$148 \leq T_f < 156\ MeV$$

Particle yields data at the LHC consistent with LQCD at the phase boundary

Thermal origin of particle yields with respect to HRG

Rolf Hagedorn => the Hadron Resonace Gas (HRG):

"uncorrelated" gas of hadrons and resonances

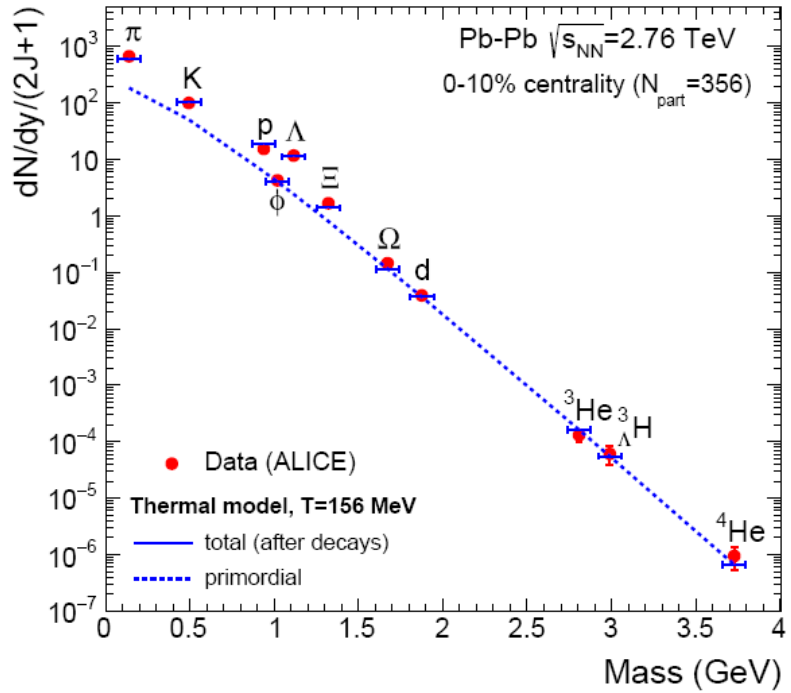
$$< N_i > = V\,[n_i^{th}(T,\vec{\mu}) + \sum_K \Gamma_{K\to i}\, n_i^{th-\mathrm{Res.}}(T,\vec{\mu})]$$

A. Andronic, Peter Braun-Munzinger, Johanna Stachel & K.R.

Particle yields with no resonance decay contributions:

$$\frac{1}{2j+1}\frac{dN}{dy} = V(m/T)^2 K_2(m/T)$$

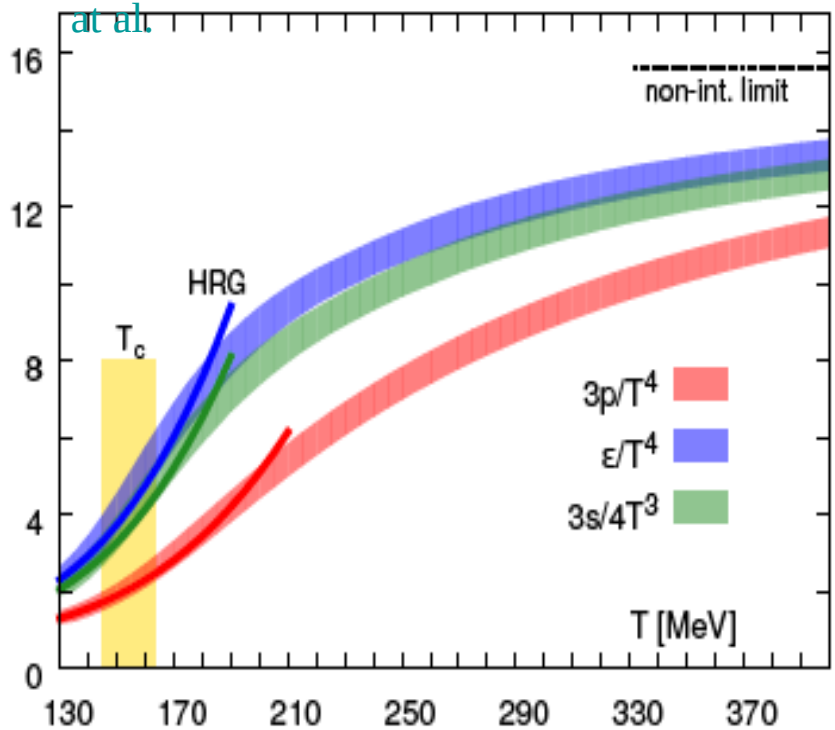
$$\sim m^{3/2} e^{-m/T}$$



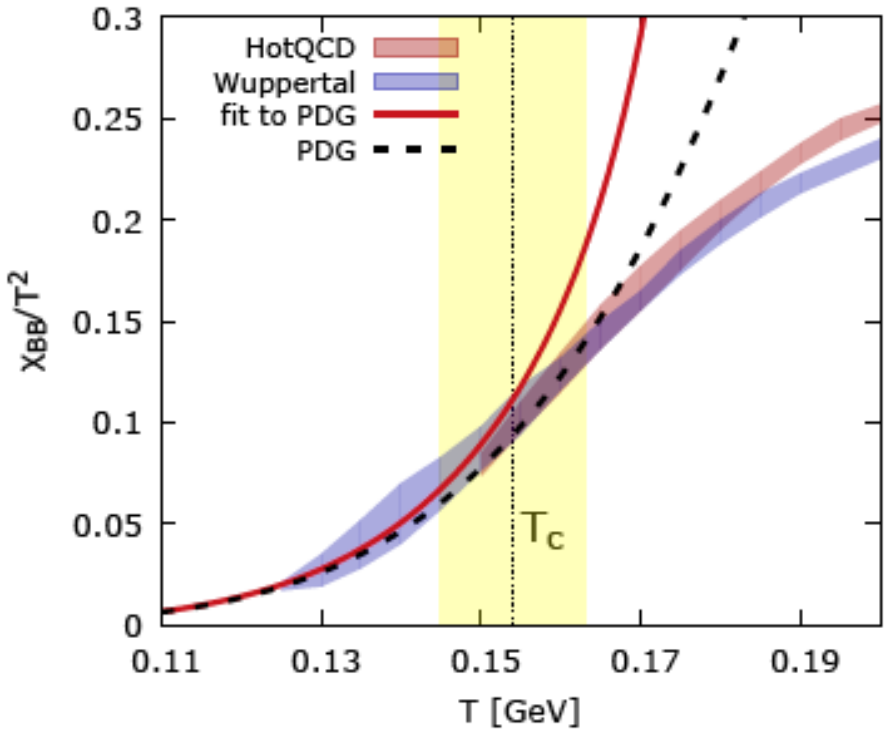
- Measured yields are well reproduced within HRG with $T = 156 \pm 1.5\ MeV$ that coincides with the chiral crossover

Good description of the QCD Equation of States by Hadron Resonance Gas

A. Bazavov et al. HotQCD Coll. July 2014 et al.

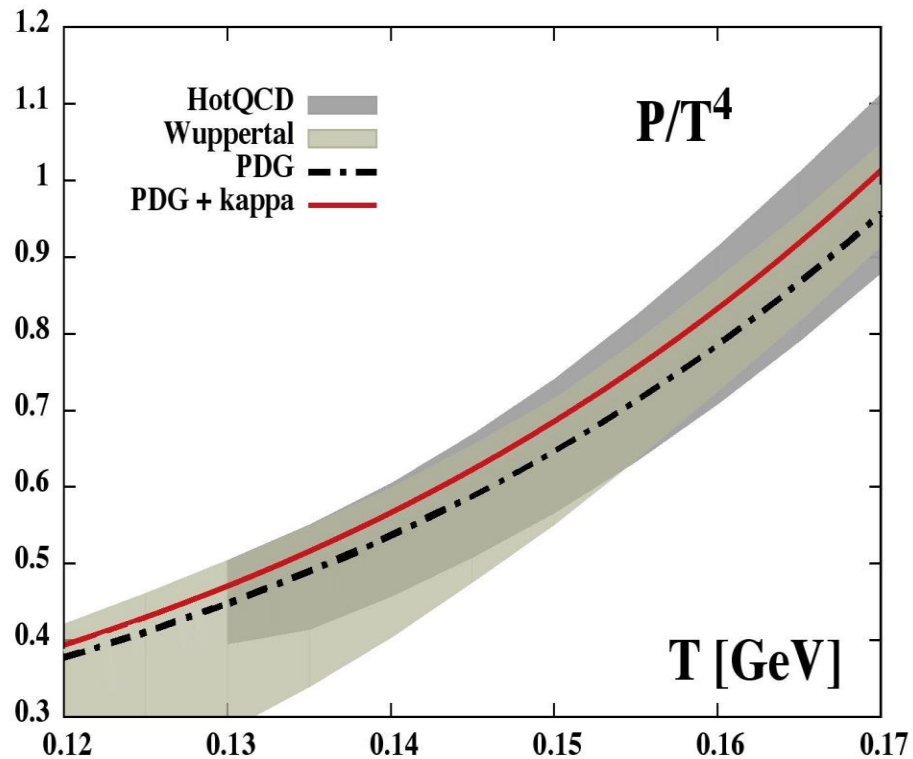


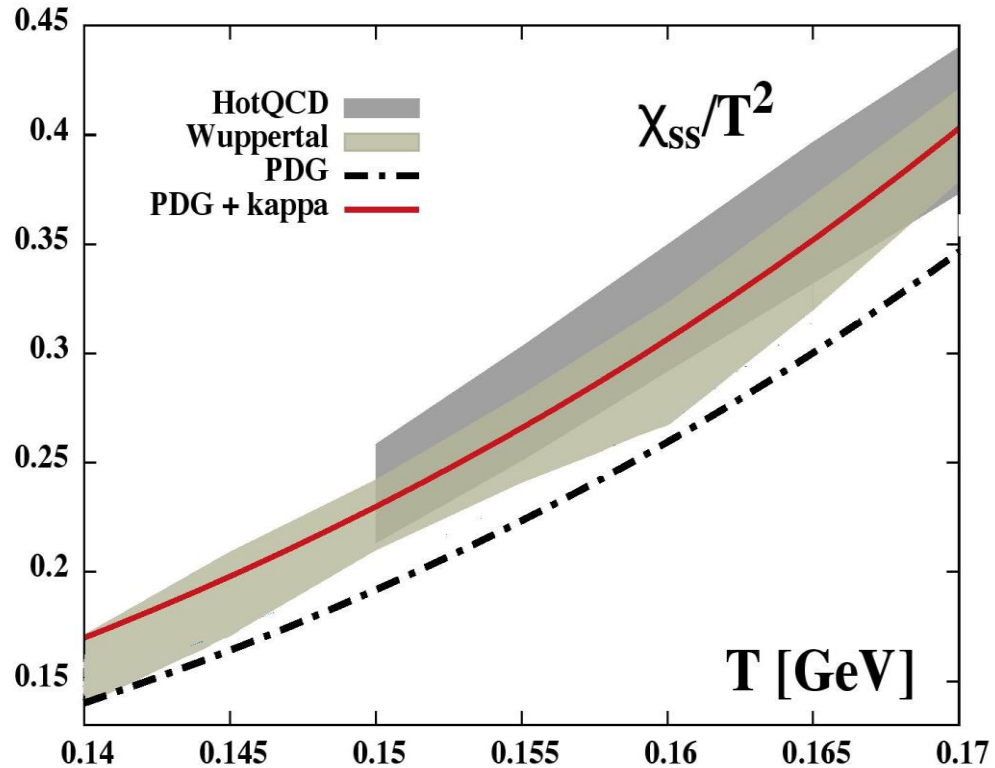
P.M. Lo, M Marczenko et al. Eur. Phys.J. A52 (2016)



- Hadron Gas thermodynamic potential provides an excellent approximation of the QCD equation of states in confined phase
- As well as, good description of the net-baryon number fluctuations which can be improved by adding baryonic resonances expected in the Hagedorn exponential mass spectrum

Leading missing resonance contribution to strangeness fluctuations





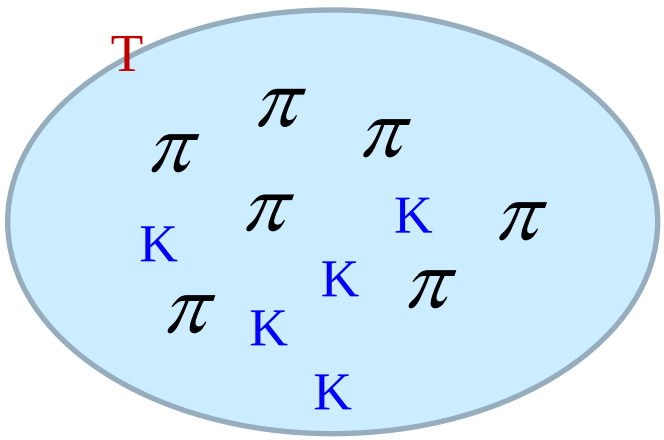
The strange scalar meson channel, with the unconfirmed kappa, $\kappa^{0^*}(800)$

$I(J^P) = \frac{1}{2}(0^+)$, provides large contribution to χ_{SS} when added to the PDG HRG

S-MATRIX APPROACH

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187, 345 (1969)

W. Weinhold, & B. Friman
Phys. Lett. B 433, 236 (1998).



- Consider interacting pions and kaons gas in thermal equilibrium at temperature T
- Due to *K*π scattering resonances are formed

  I =1/2, s -wave : κ(800), K0*(1430) [*JP* = 0+]

  I =1/2, p -wave : *K**(892), *K**(1410), *K**(1680) [*JP* =1−]
- In the S-matrix approach the thermodynamic pressure in the low density approximation

$$P(T) \approx P_{\pi}^{id} + P_{K}^{id} + P_{\pi K}^{\text{int}}$$

Thermodynamic pressure of an ideal gas:

$$P = P^{id} / T^4 = -\int \frac{d^3 p}{(2\pi)^3} \left\{ \ln\left[1 - e^{-\sqrt{p^2+M^2}-\mu}\right] + \ln\left[1 - e^{-\sqrt{p^2+M^2}+\mu}\right] \right\}$$

S-MATRIX APPROACH: INTERACTING PART

The leading order corrections , determined by the two-body scattering phase shift, which is equivalent to the second virial coefficient

$$P_{\text{int}} = \int_{m_{th}}^{\infty} \frac{dM}{2\pi} B(M) P_T(M)$$

$$B(M) = 2\frac{d}{dM}\delta(M)$$

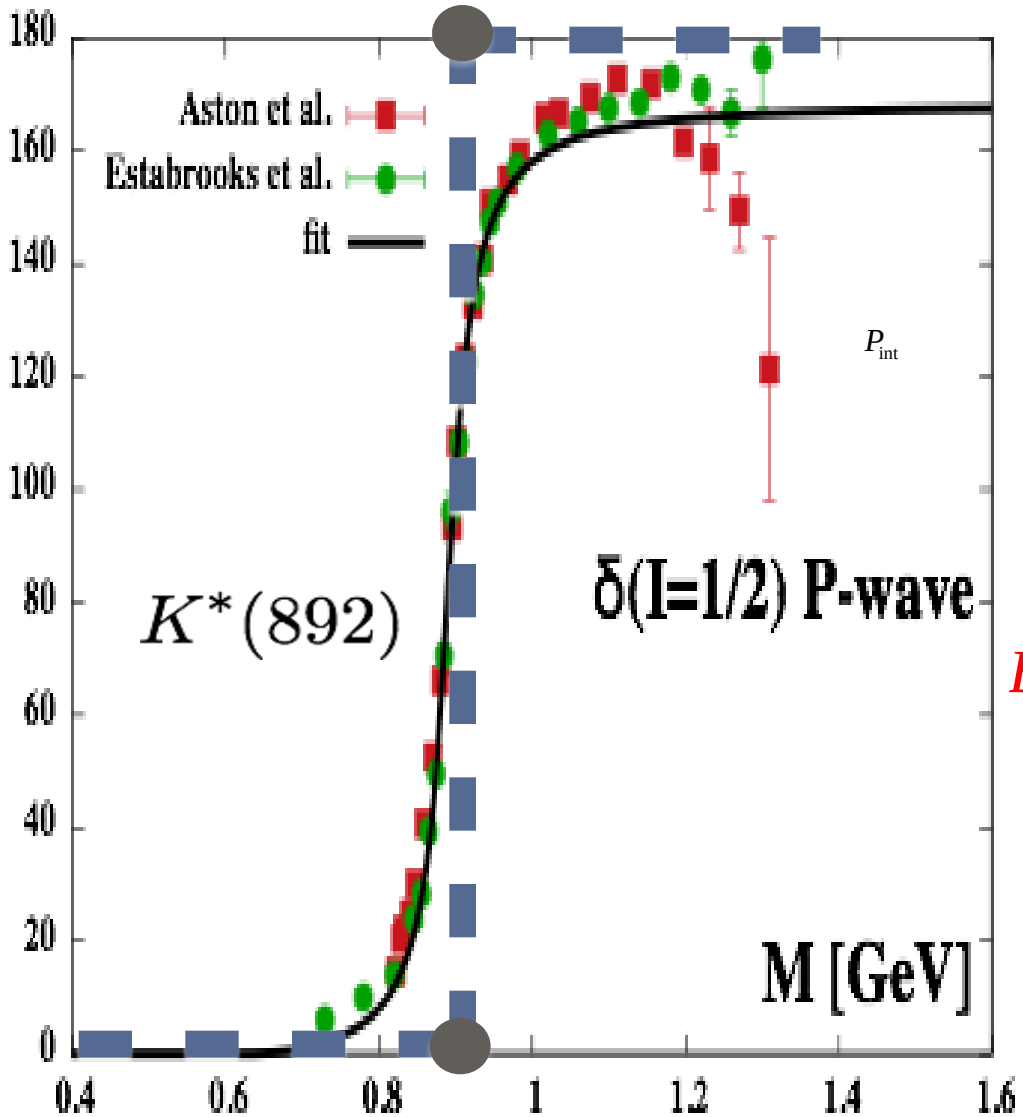
$$\int_{m_{th}}^{\infty} \frac{dM}{2\pi} B(M) = 1$$

Effective weight function *Scattering phase shift* *Normalization*

Pressure of an ideal gas of resonaces with an invariant mass M

$$P_T(M) = -2\int \frac{d^3 p}{(2\pi)^3}\left\{\ln\left[1 - e^{-\sqrt{p^2+M^2}-\mu}\right] + \ln\left[1 - e^{-\sqrt{p^2+M^2}+\mu}\right]\right\}$$

Experimental phase shift in P-wave channel



B. Friman et al, arXiv:1507.04183

For narrow resonance

$$B(M) = 2\frac{d}{dM}\delta(M)$$

very well described by the Breit-Wigner form

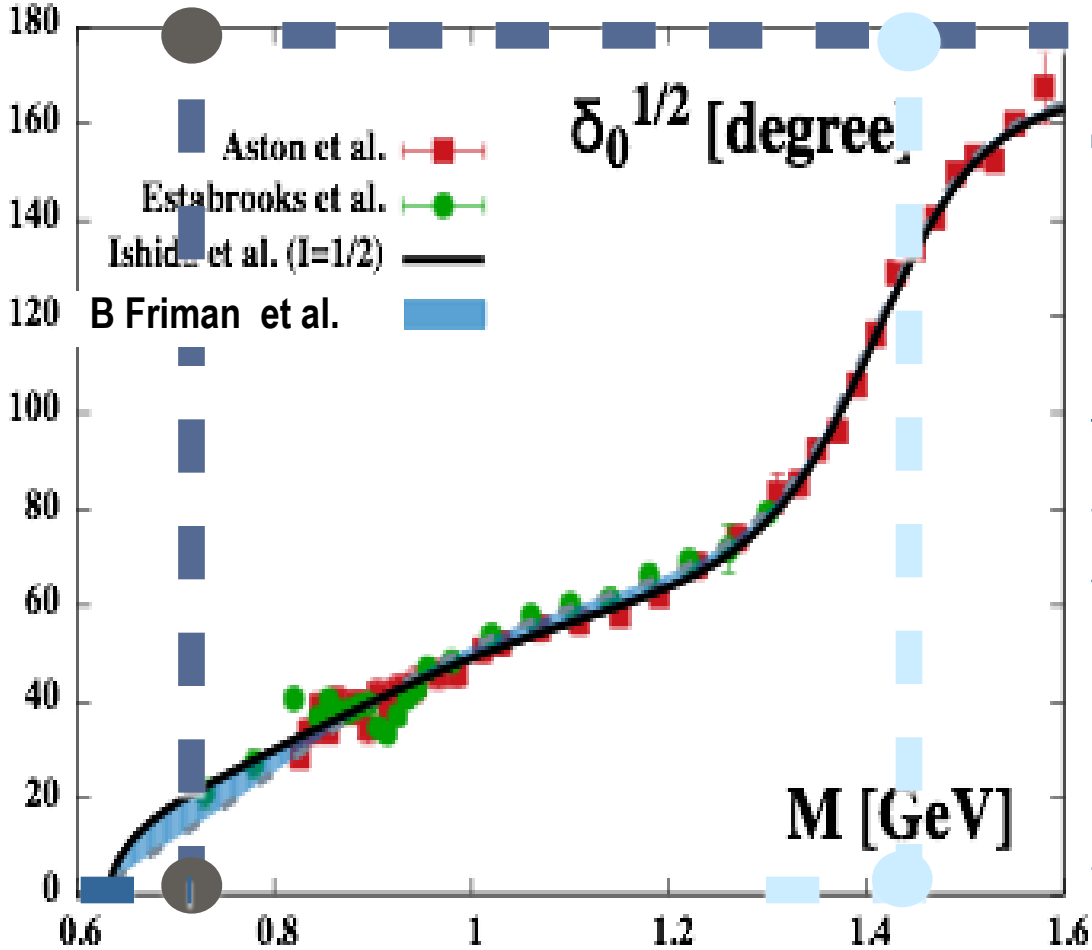
$$B(M) \approx M\frac{2M\gamma_{\mathrm{BW}}}{(M^2 - M_0^2)^2 + M^2\gamma_{\mathrm{BW}}^2}$$

for $\gamma_{BW} \to 0$

$$B(M) = \delta(M^2 - M_0^2)$$ **and**

$$P_{\pi K}^{\mathrm{int}}(T) \approx P_{K^*}^{id}(T)$$

Experimental phase shift in S channel

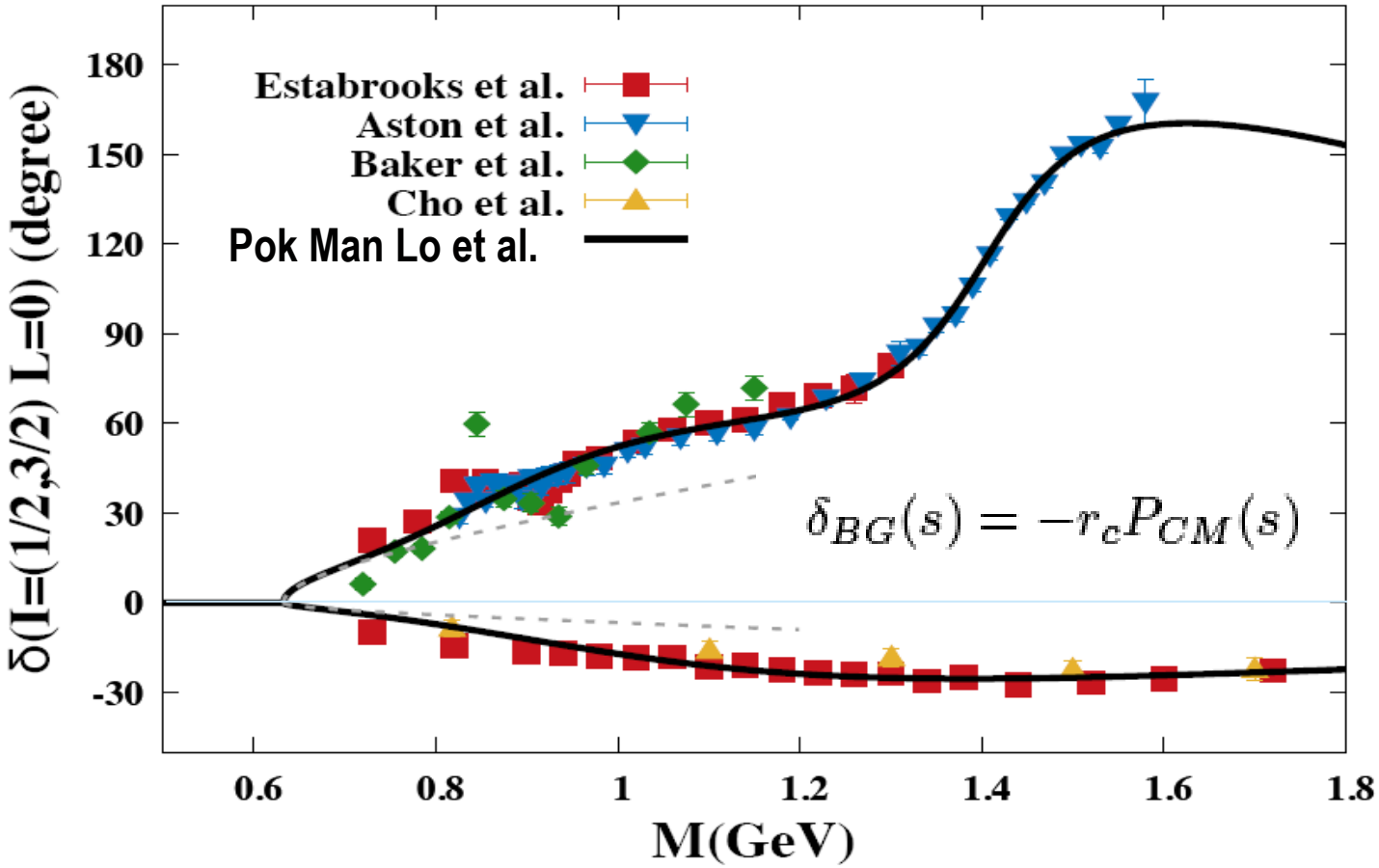


The kappa resonance is a special example that:

for broad resonances their contribution has to be taken with a special care, by considering the experimental or theoretically calculated phase shift

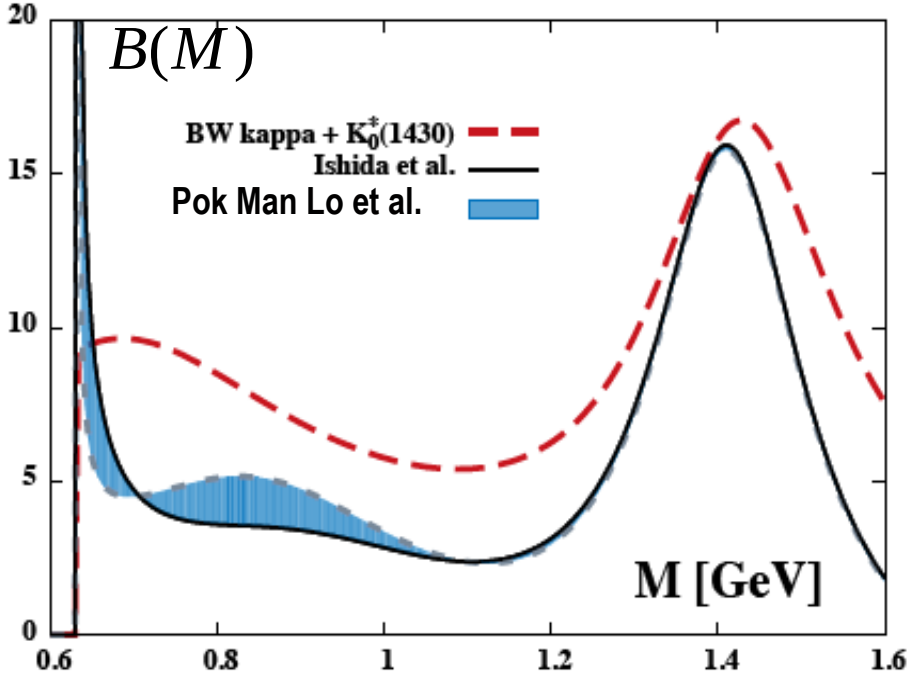
B. Friman et al, arXiv:1507.04183

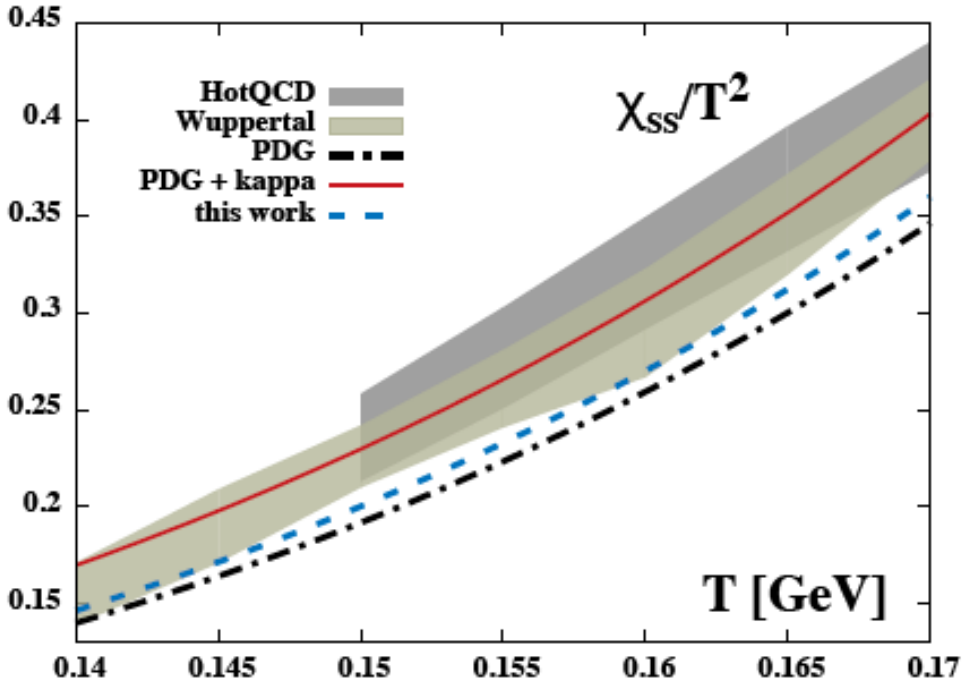
Non-resonance contribution- negative phase shift in S-wave channel



$$\delta_0^{1/2} = \delta_\kappa + \delta_{K_0^*} + \delta_{BG}. \Rightarrow B(M) = 2\frac{d}{dM}\delta(M) \Rightarrow \chi_{SS}(T)$$

S-matrix approach to strangeness fluctuations





In the S-matrix approach essential reduction of the contribution of S-wave kappa relative to naive BW approach

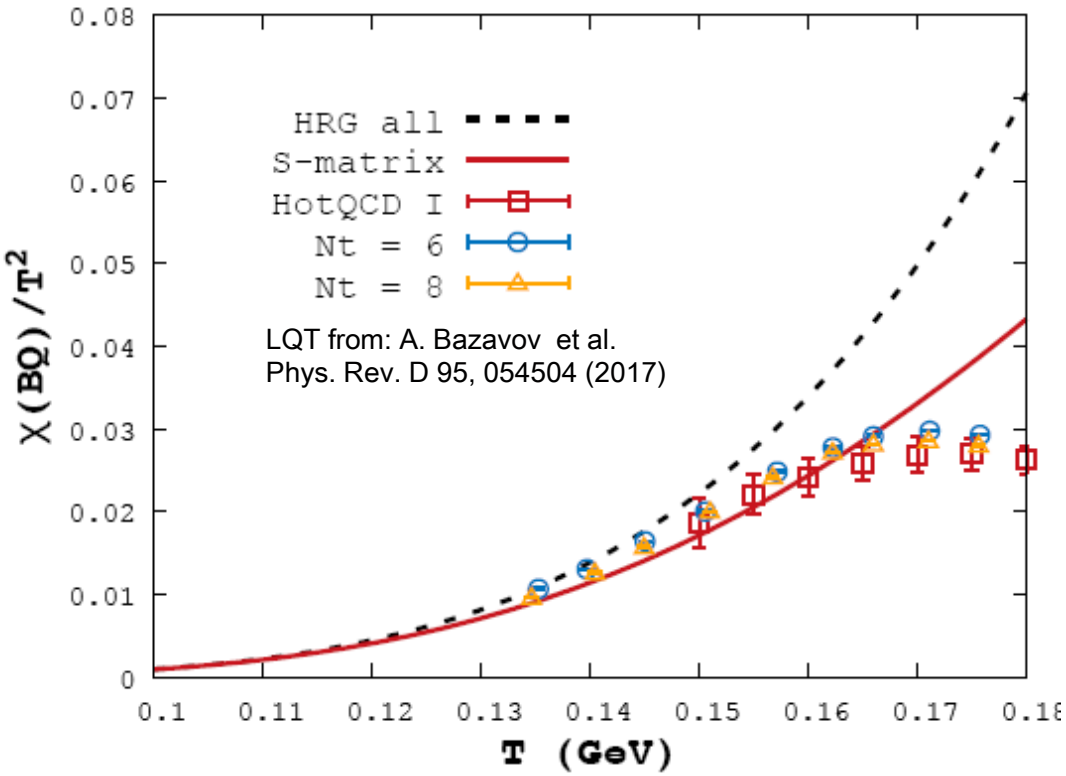
B. Friman, P. M. Lo, M. Marczenko, K. Redlich and C. Sasaki, Phys. Rev. D 92, no. 7, 074003 (2015)

Similar arguments also apply to sigma meson

V. Begun and W. Florkowski Phys.Rev. C91 (2015) 054909

Probing non-strange baryon sector

Pok Man Lo et al. (preliminary)



- Due to isospin symmetry

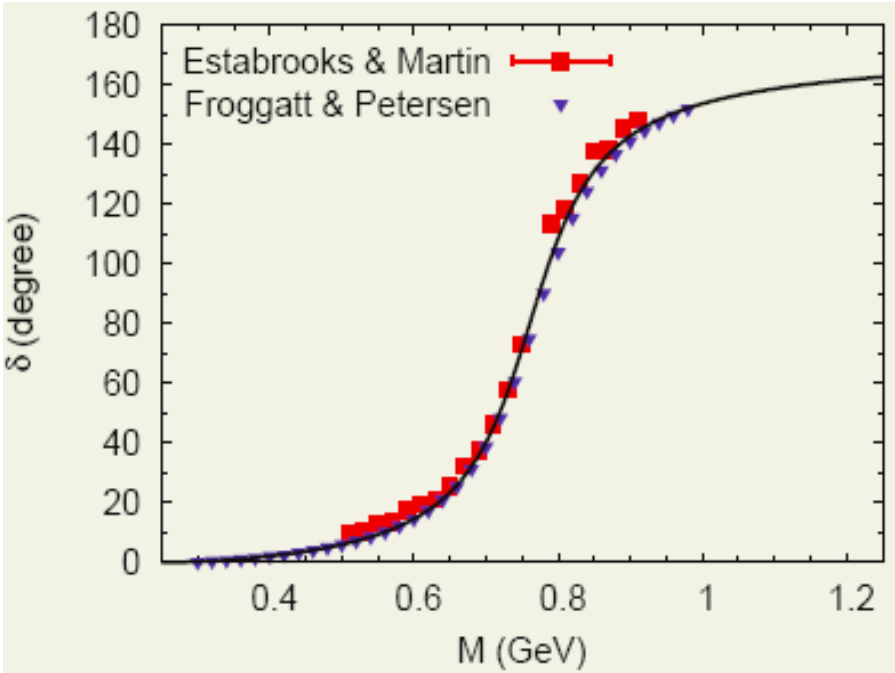
$$\chi_{BQ} = \frac{1}{2}(\chi_{BB} - |\chi_{BS}|)$$

where all $S = \pm 1$ baryon resonances are canceled out. The $S = \pm 2, \pm 3$ contribution is small, thus χ_{BQ} is governed mainly by the contribution of nucleons and $S = 0$ baryonic resonances N^*, Δ^*

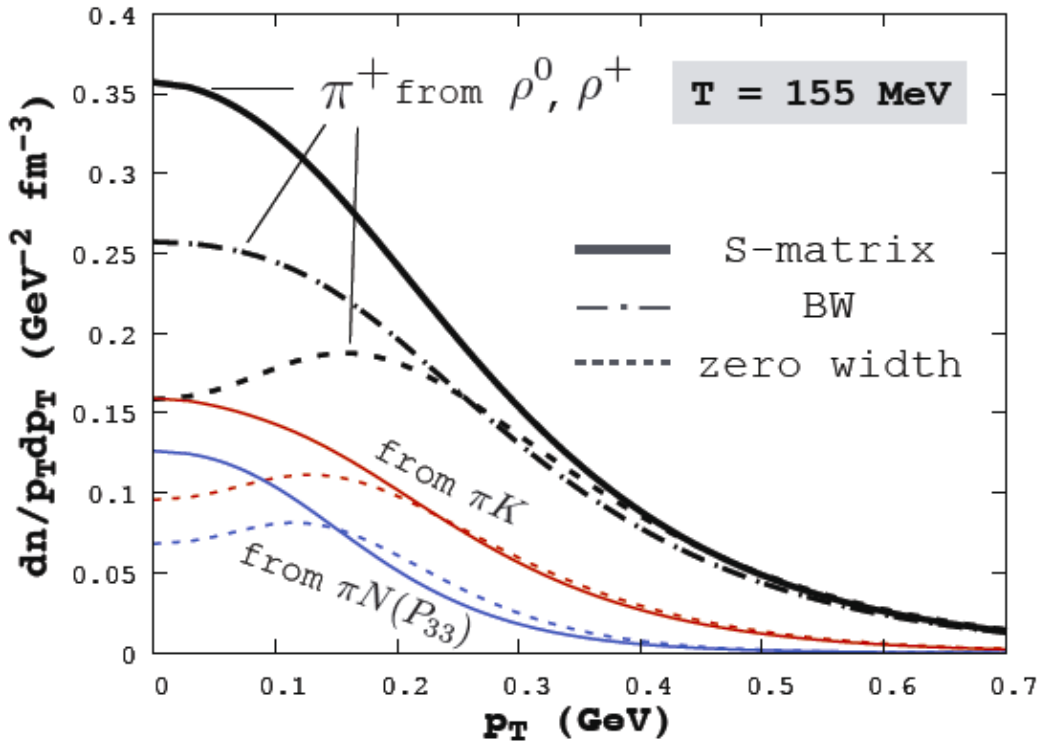
- Considering contributions of all N^*, Δ^* resonances to χ_{BQ} with correctly implemented dynamical widths within S-matrix approach imply the reduction of the HRG model results towards the LQCD data

S-matrix approach: Pion spectra

$\pi\pi$ scattering, P-wave, i.e. ρ resonance contribution



P. Huovinen, P.M. Lo, M. Marczenko, K. Morita, K. Redlich and C. Sasaki, Phys. Lett. B 769, 509 (2017)



$$\frac{dN_\pi^{de}}{dy\, p_T\, dp_T\, d\phi} = V E_\pi \int dM_\rho \frac{1}{2\pi} \mathcal{B}(M_\rho)$$

$$\times \frac{M_\rho}{2\, p_\pi p_{CM}} \int_{E_\rho^-}^{E_\rho^+} dE_\rho\, E_\rho \frac{d_\rho}{(2\pi)^3} f_\rho(E(M_\rho), T),$$

Thanks to Steffen Bass et al.:

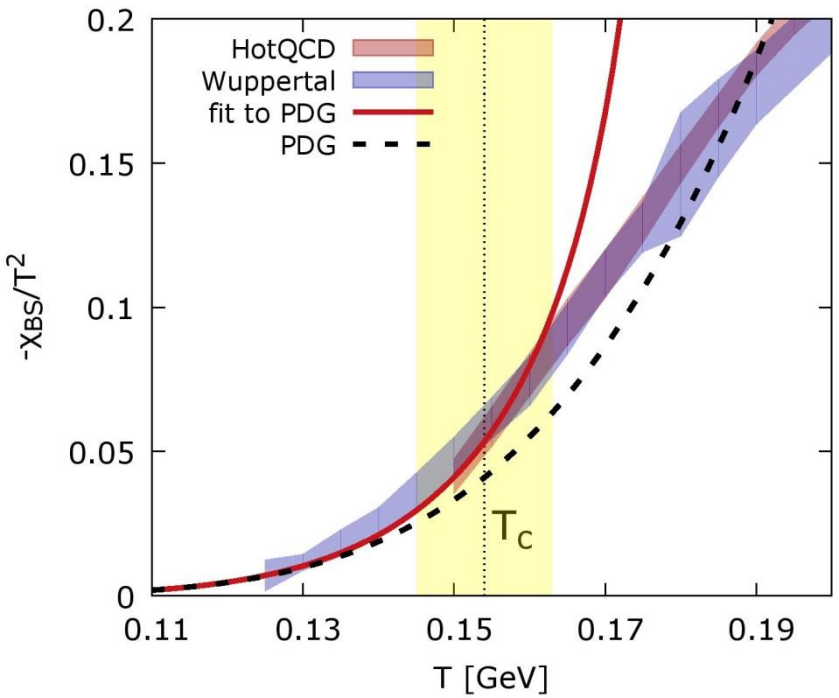
Large increase of soft pions obtained in the S-matrix approach

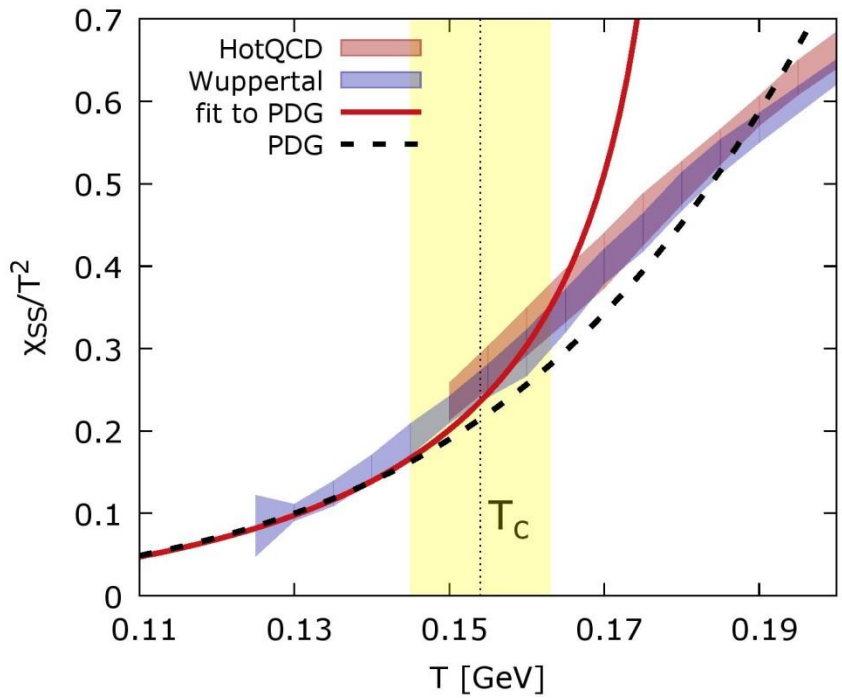
Conclusions:

The medium created in HIC at the LHC is of thermal origin and follows properties and composition expected in LQCD at the phase boundary at $148 \le T < 156\ MeV$

- The Hadron Resonance Gas is confirmed to be a very good approximation of QCD thermodynamic potential and provides quantitative description of all known particle yields in HIC at the LHC with temperature $T = 156 \pm 1.5\ (2) MeV$, consistent with the chiral crossover.
- To properly quantify fluctuation observables within HRG model the dynamical widths of broad resonances should be correctly included e.g. by using the phase shift data within S-matrix approach
- systematics of LQCD results on 2nd order fluctuations and correlations indicates that there are missing baryonic resonances in the $S = \pm 1$ strangeness sector

Hagedorn's continuum mass spectrum contribution to strangeness fluctuations





- Missing strange baryon and meson resonances in the PDG

  F. Karsch, et al., Phys. Rev. Lett. 113, no. 7, 072001 (2014)

  P.M. Lo, M. Marczenko, et al. Eur. Phys.J. A52 (2016)

- Satisfactory description of LGT with asymptotic states from Hagedorn's exponential mass spectrum $\rho^H(m) = (m^2 + m_0^2)^{-5/2} e^{m/T_H}$ fitted to PDG