LATTICE RESULTS ON FREEZE-OUT

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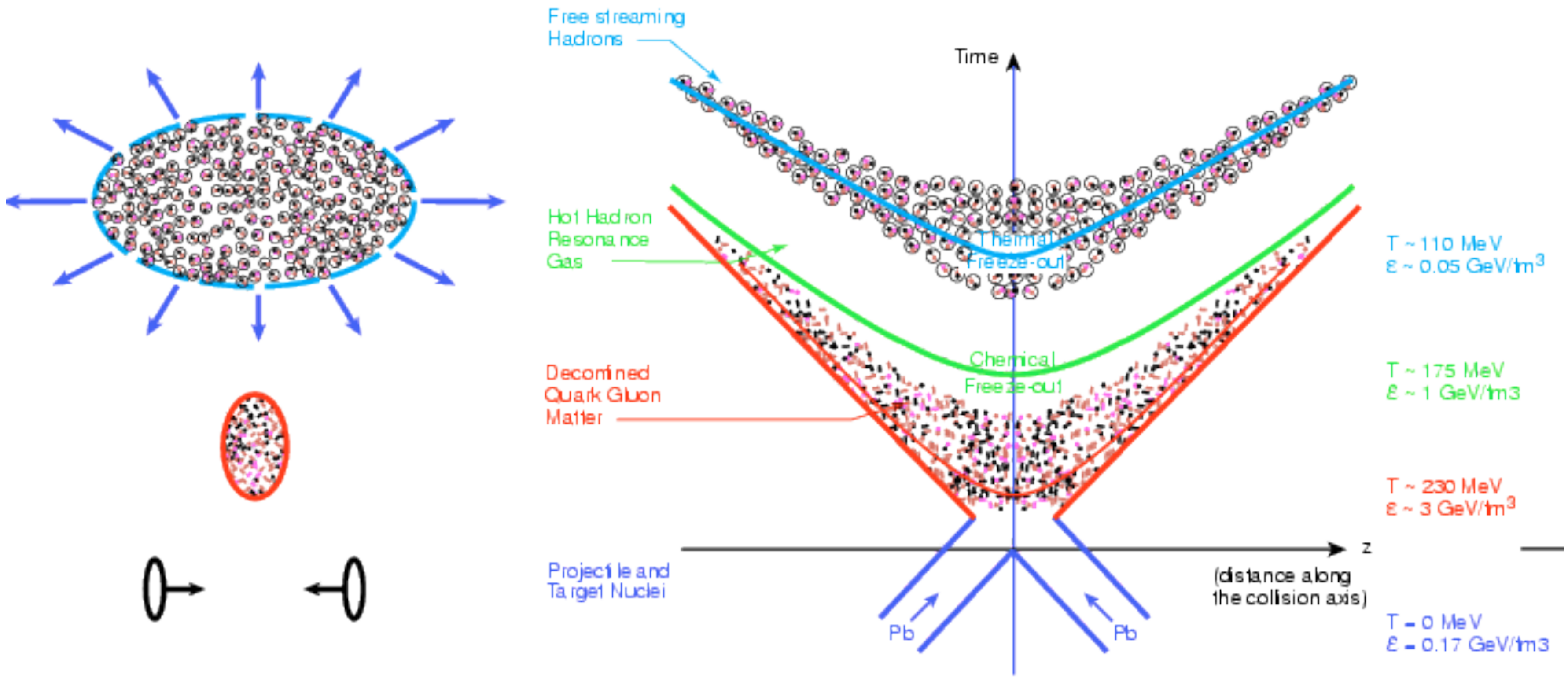
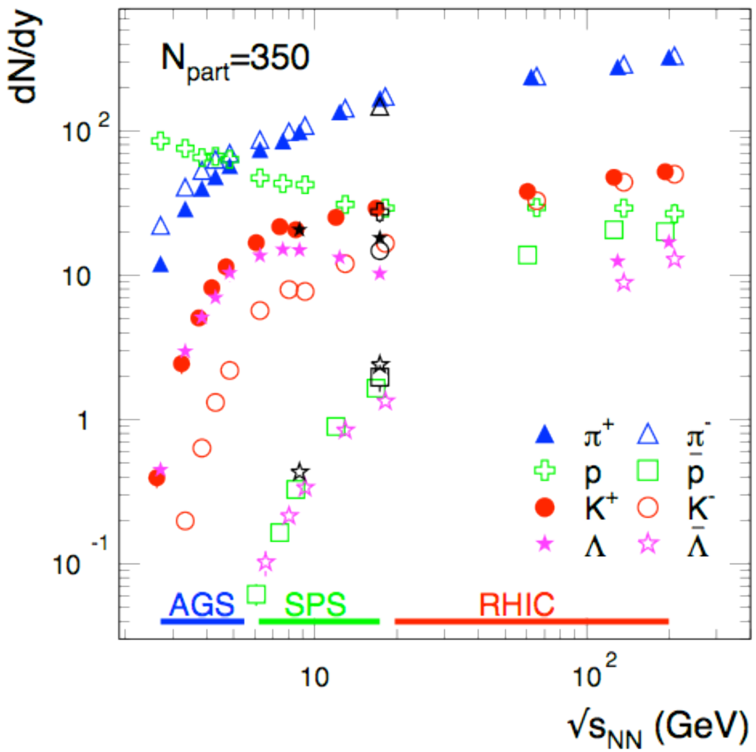
BEST COLLABORATION

Evolution of a Heavy Ion Collision



- Chemical freeze-out: inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)
- Kinetic freeze-out: elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)
- Hadrons reach the detector

Hadron yields

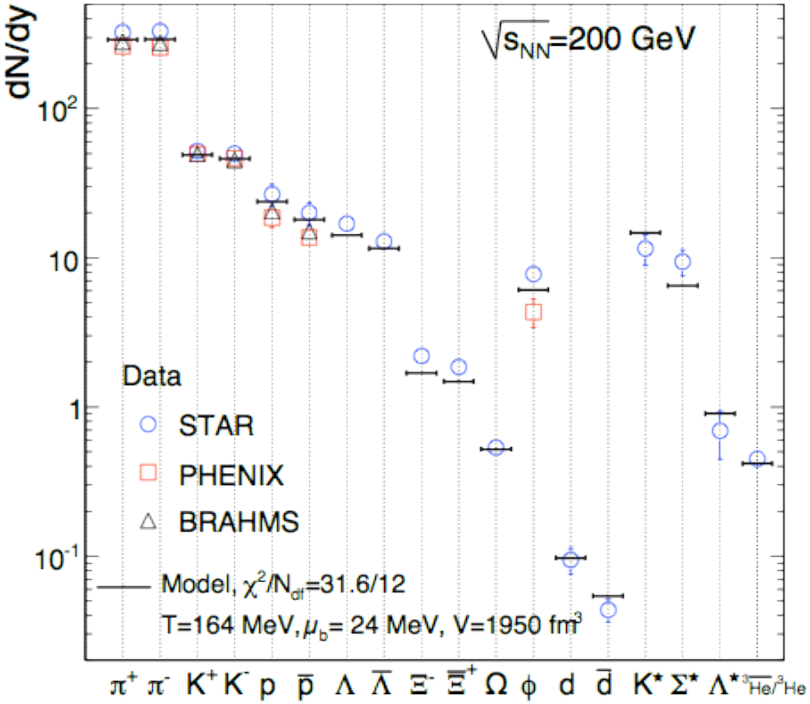


- E=mc^2: lots of particles are created
- Particle counting (average over many events)
- Take into account:
  - detector inefficiency
  - missing particles at low p_T
  - decays

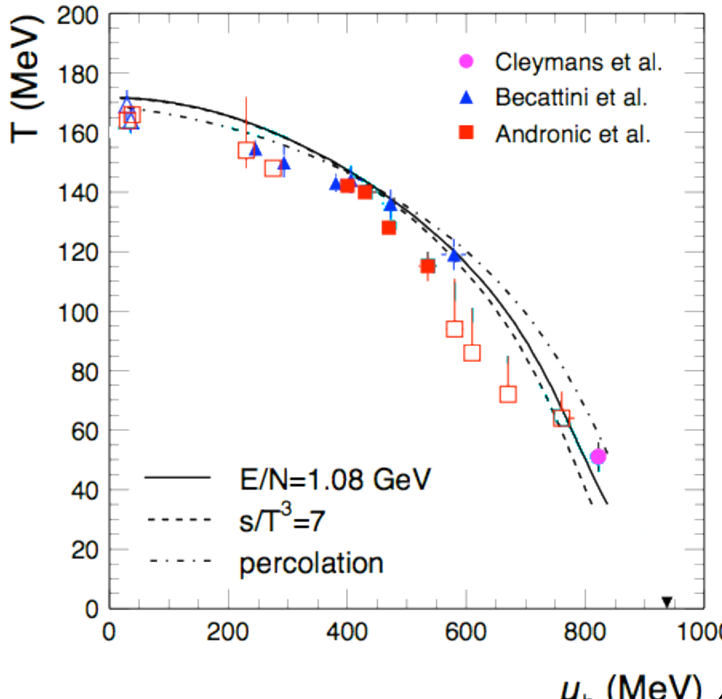
- HRG model: test hypothesis of hadron abundancies in equilibrium

$$N_i = -T\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i V}{2\pi^2}\int_0^\infty \frac{p^2 \mathrm{d}p}{\exp[(E_i-\mu_i)/T]\pm 1}$$

The thermal fits



- Fit is performed minimizing the χ^2
- Fit to yields: parameters T, μB, V
- Fit to ratios: the volume V cancels out



- Changing the collision energy, it is possible to draw the freeze-out line in the T, μB plane

Lattice QCD

- Best first principle-tool to extract predictions for the theory of strong interactions in the non-perturbative regime

- Uncertainties:
  - Statistical: finite sample, error $\sim 1/\sqrt{\text{sample size}}$
  - Systematic: finite box size, unphysical quark masses

- Given enough computer power, uncertainties can be kept under control

- Results from different groups, adopting different discretizations, converge to consistent results

- Unprecedented level of accuracy in lattice data

Fluctuations of conserved charges

- Definition:

$$\chi^{BSQ}_{lmn} = \frac{\partial^{\,l+m+n} p/T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

- Relationship between chemical potentials:

$$\begin{aligned} \mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q; \\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q; \\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S. \end{aligned}$$

- They can be calculated on the lattice and compared to experiment

Connection to experiment

- Fluctuations of conserved charges are the cumulants of their event-by-event distribution

$$\text{mean}: M = \chi_1 \qquad \text{variance}: \sigma^2 = \chi_2$$

$$\text{skewness}: S = \chi_3/\chi_2^{3/2} \qquad \text{kurtosis}: \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2 \qquad \kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2 \qquad S\sigma^3/M = \chi_3/\chi_1$$

F. Karsch: Centr. Eur. J. Phys. (2012)

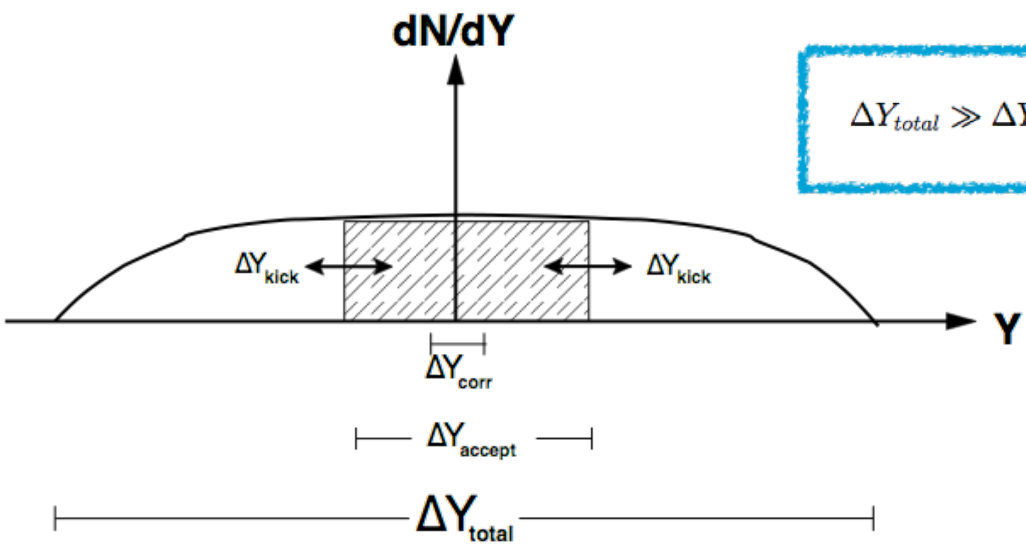
- The chemical potentials are not independent: fixed to match the experimental conditions:

$\langle n_S\rangle=0$ $\qquad$ $\langle n_Q\rangle=0.4\langle n_B\rangle$

Fluctuations of conserved charges

V. Koch (2008)

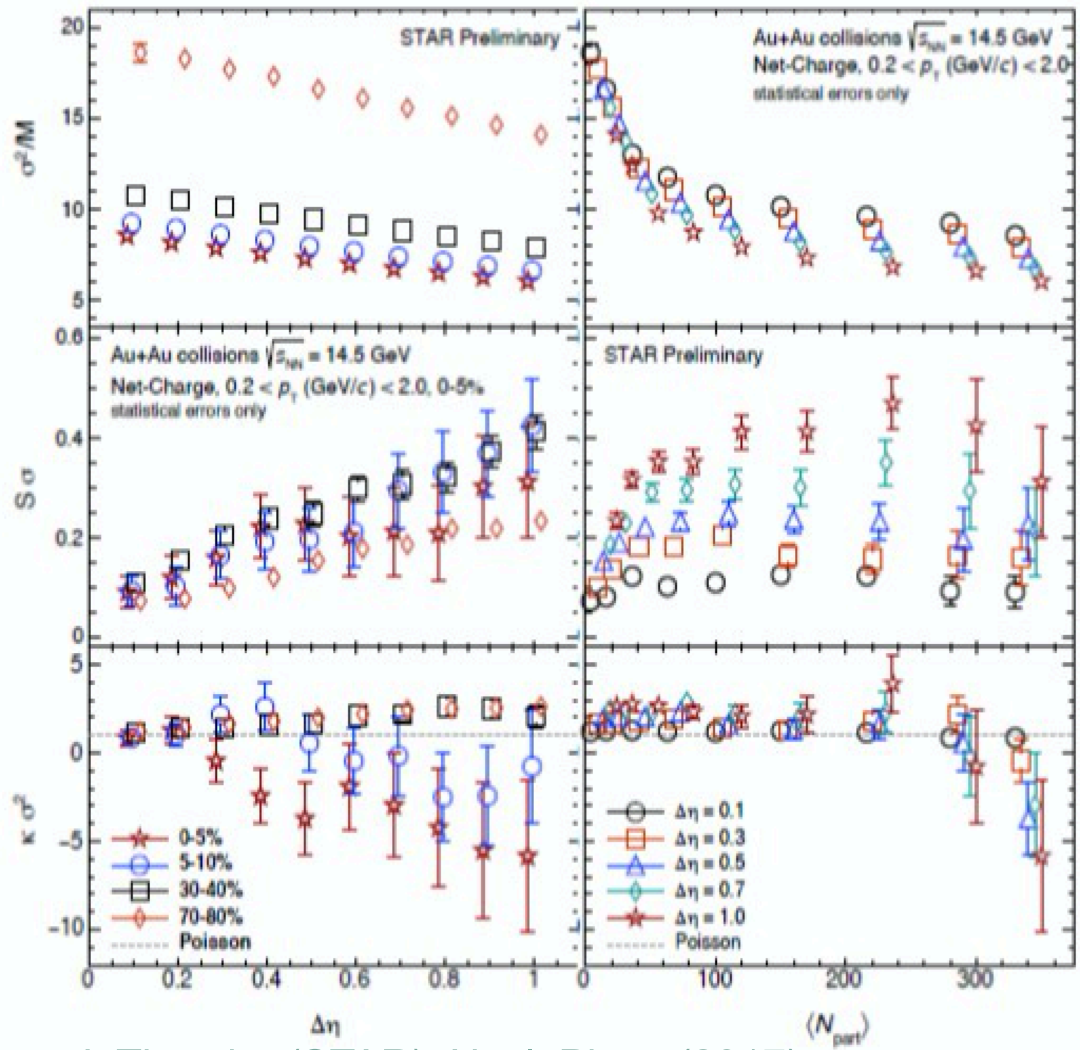
- If we look at the entire system, none of the conserved charges will fluctuate
- By studying a sufficiently small subsystem, the fluctuations of conserved quantities become meaningful



$$\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}$$

- ❑ ΔYtotal: range for total charge multiplicity distribution
- ❑ ΔYaccept: interval for the accepted charged particles
- ❑ ΔYkick: rapidity shift that charges receive during and after hadronization

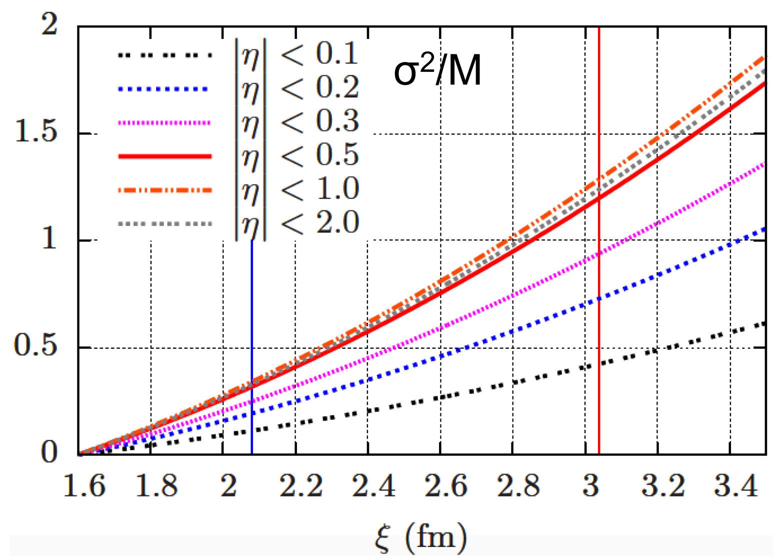
Rapidity dependence of fluctuations



J. Thaeder (STAR): Nucl. Phys. (2017).

- Mild dependence on Δη for most-central data
- Theoretical predictions: data reach saturation for Δη≥1

E. Fraga et al., (2017).



Things to keep in mind

See talk by A. Rustamov on Monday

- Effects due to volume variation because of finite centrality bin width
  - Experimentally corrected by centrality-bin-width correction method

V. Skokov et al., PRC (2013)

- Finite reconstruction efficiency
  - Experimentally corrected based on binomial distribution A.Bzdak,V.Koch, PRC (2012)
- Spallation protons
  - Experimentally removed with proper cuts in p_T
- Canonical vs Gran Canonical ensemble
  - Experimental cuts in the kinematics and acceptance V. Koch, S. Jeon, PRL (2000)
- Proton multiplicity distributions vs baryon number fluctuations
  - Recipes for treating proton fluctuations

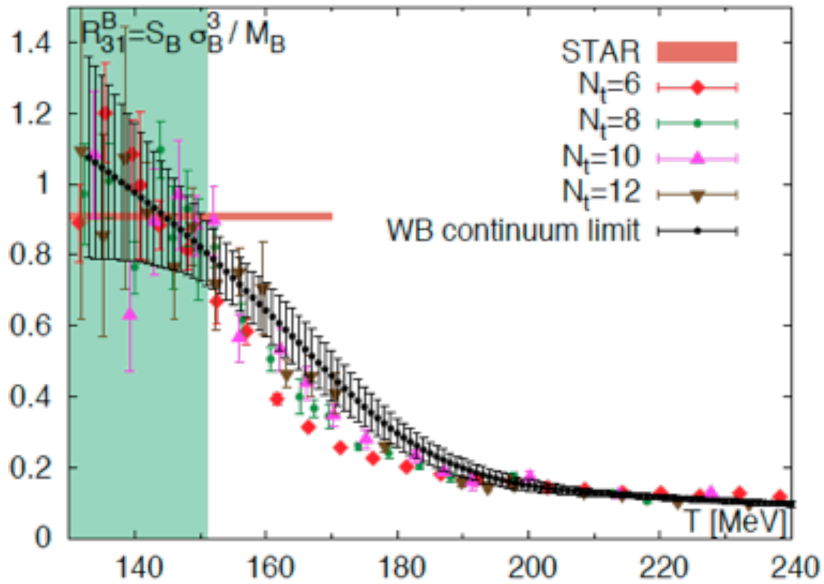
M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238

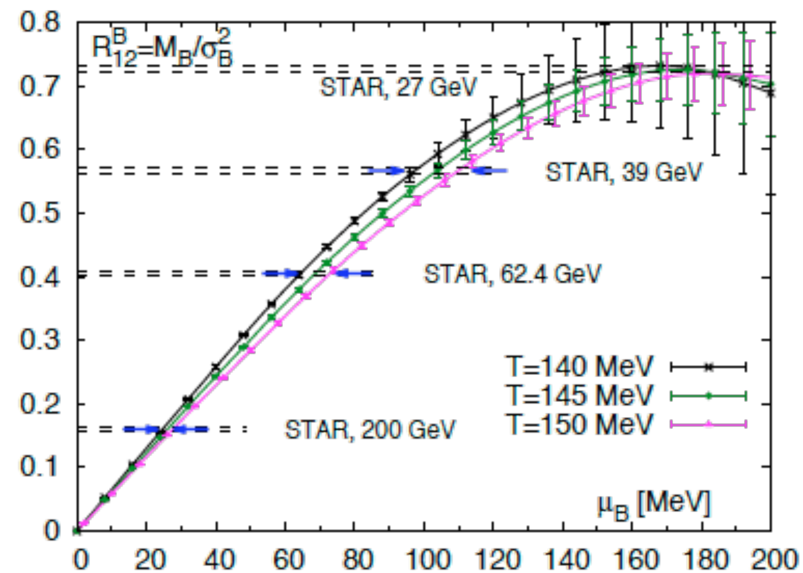
- Final-state interactions in the hadronic phase
  - Consistency between different conserved charges = fundamental test

See talk by J. Steinheimer

Freeze-out parameters from B fluctuations

- Thermometer: $\frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)}$ $=S_B\sigma_B^3/M_B$ Baryometer: $\frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)}$ $=M_B/\sigma_B^2$





WB: S. Borsanyi et al., PRL (2014)
STAR collaboration, PRL (2014)

- Upper limit: $T_f \leq 151\pm4$ MeV
- Consistency between freeze-out chemical potential from electric charge and baryon number is found.

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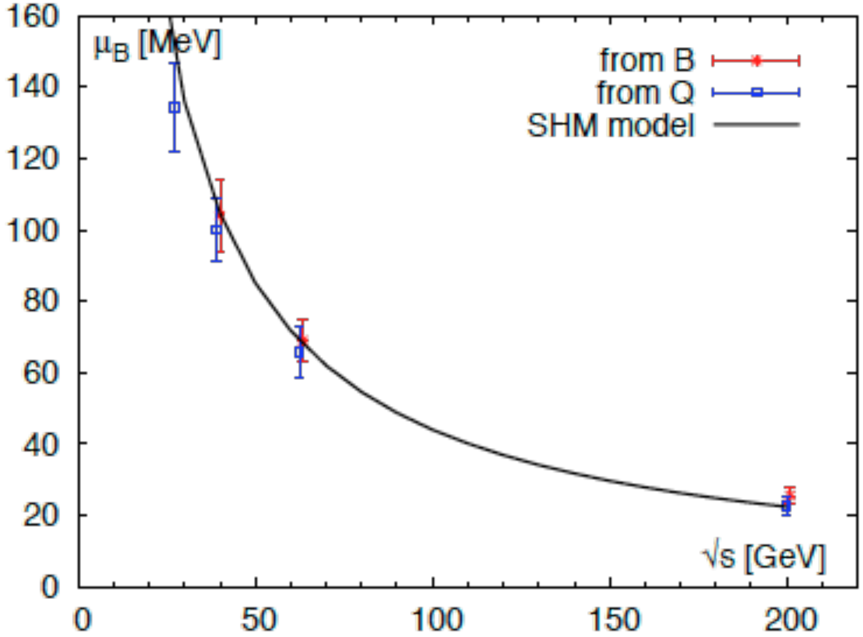

$\sqrt{s}[GeV]$	μ_B^f [MeV] (from B)	μ_B^f [MeV] (from Q)
200	25.8±2.7	22.8±2.6
62.4	69.7±6.4	66.6±7.9
39	105±11	101±10
27	-	136±13.8

WB: S. Borsanyi et al., PRL (2014)
STAR collaboration, PRL (2014)

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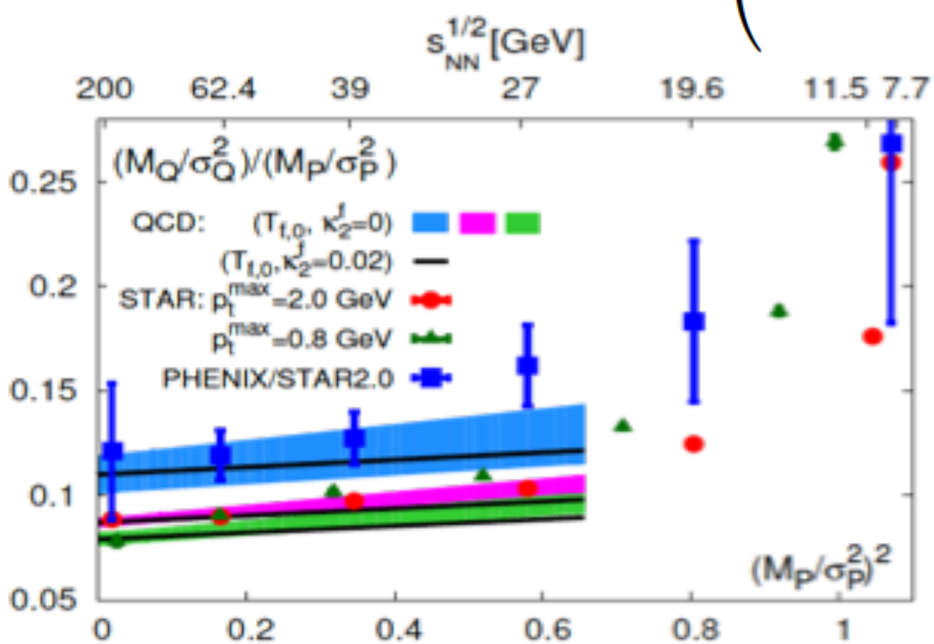
Curvature of the freeze-out line

- Parametrization of the freeze-out line:

$$T_f(\mu_B) = T_{f,0}\left(1 - \kappa_2^f \bar{\mu}_B^2 - \kappa_4^f \bar{\mu}_B^4\right)$$

- Taylor expansion of the "ratio of ratios" R_{12}^{QB}= $[M_Q/\sigma_Q^2]/[M_B/\sigma_B^2]$

$$R_{12}^{QB} = R_{12}^{QB,0} + \left(R_{12}^{QB,2} - \kappa_2^f T_{f,0} \left.\frac{\mathrm{d}R_{12}^{QB,0}}{\mathrm{d}T}\right|_{T_{f,0}}\right)\hat{\mu}_B^2$$



$$\kappa_2^f < 0.011$$
$$T_{f,0} = (147 \pm 2)\ \mathrm{MeV}$$

A. Bazavov et al., 1509.05786
STAR0.8: PRL (2013)

STAR1.2: X. Luo, PoS CPOD 2014
PHENIX: 1506.07834

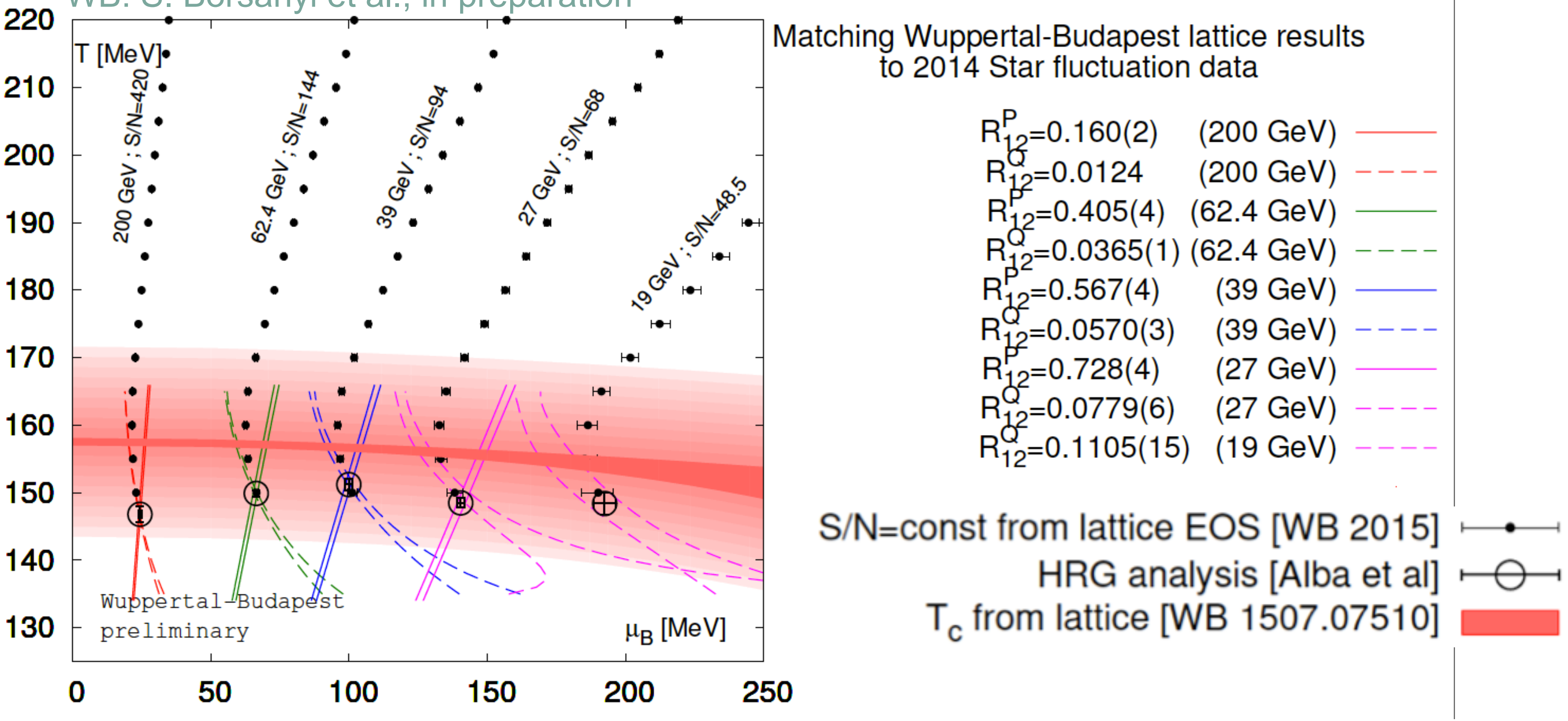
Freeze-out line from first principles

- Use T- and μ_B-dependence of R_{12}^Q and R_{12}^B for a combined fit:

$$R_{12}^Q(T,\mu_B) = \frac{\chi_1^Q(T,\mu_B)}{\chi_2^Q(T,\mu_B)} = \frac{\chi_{11}^{QB}(T,0) + \chi_2^Q(T,0)q_1(T) + \chi_{11}^{QS}(T,0)s_1(T)}{\chi_2^Q(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).$$

$$R_{12}^B(T,\mu_B) = \frac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_2^B(T,0) + \chi_{11}^{BQ}(T,0)q_1(T) + \chi_{11}^{BS}(T,0)s_1(T)}{\chi_2^B(T,0)}\frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

WB: S. Borsanyi et al., in preparation



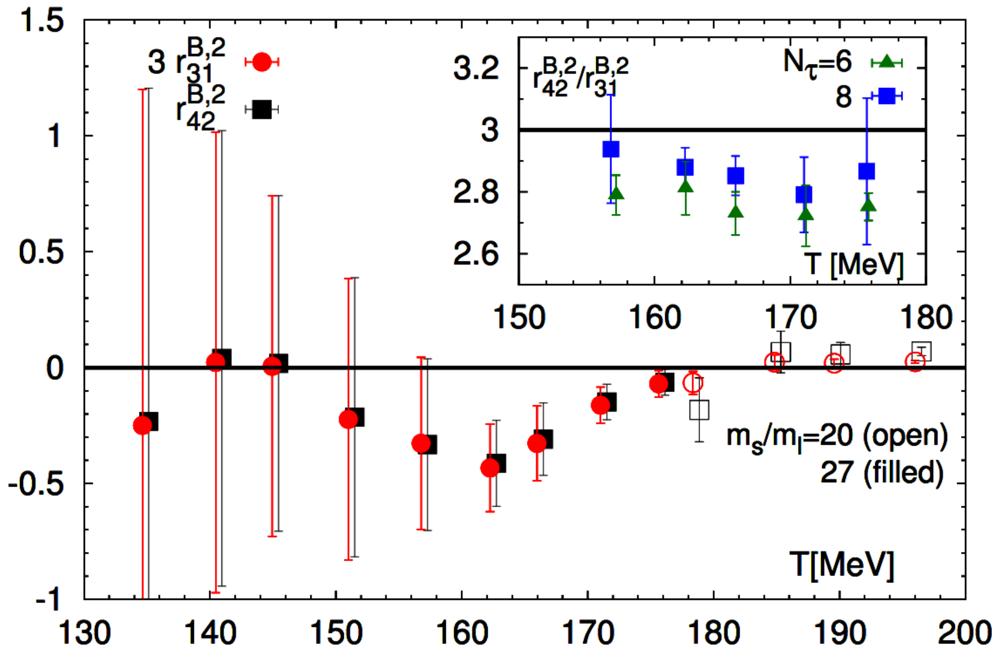
Higher order fluctuations

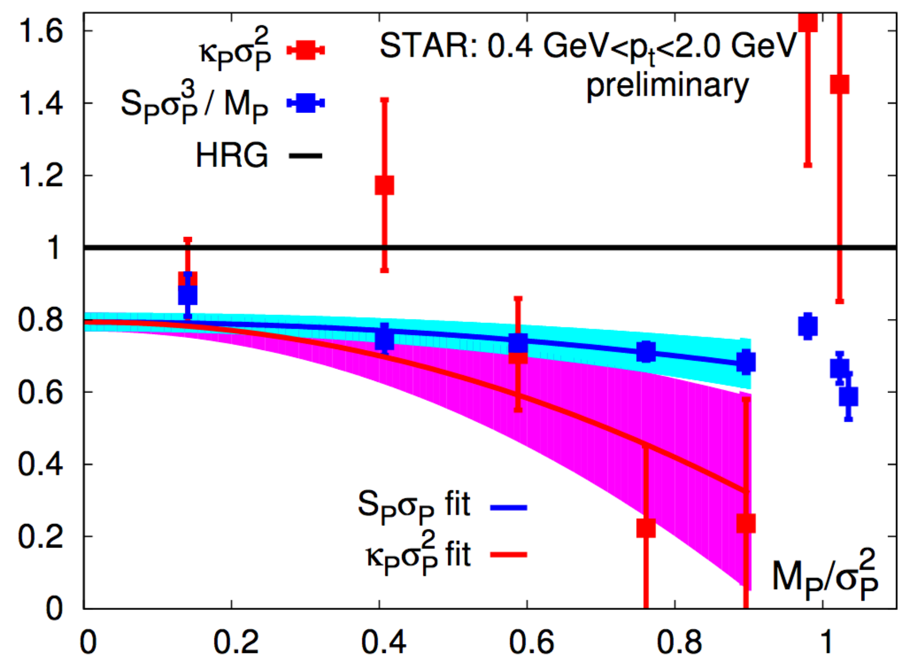
F. Karsch, J. Phys. Conf. Ser. (2017)

- Taylor expansions for skewness and kurtosis ratios

$$\frac{S_B\sigma_B^3}{M_B} = \frac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)} = \frac{\chi_4^B + s_1\chi_{31}^{BS} + q_1\chi_{31}^{BQ}}{\chi_2^B + s_1\chi_{11}^{BS} + q_1\chi_{11}^{BQ}} + \mathcal{O}(\mu_B^2) \equiv r_{31}^{B,0} + r_{31}^{B,2}\hat{\mu}_B^2 + \mathcal{O}(\mu_B^4)$$

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} = \frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^2) \equiv r_{42}^{B,0} + r_{42}^{B,2}\hat{\mu}_B^2 + \mathcal{O}(\mu_B^4)\ ,$$





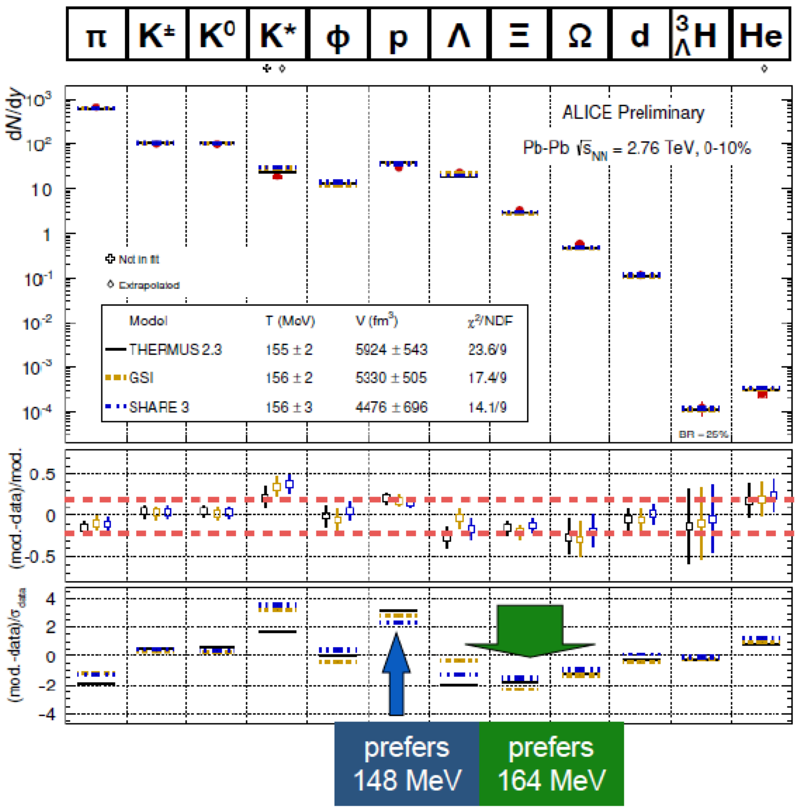
Messages so far

- Several effects still need to be understood
- Looking at the lower order fluctuations and high collision energies should be safe
- A consistent picture emerges between electric charge and baryon number
- The trend of higher order fluctuations can be understood from lattice QCD

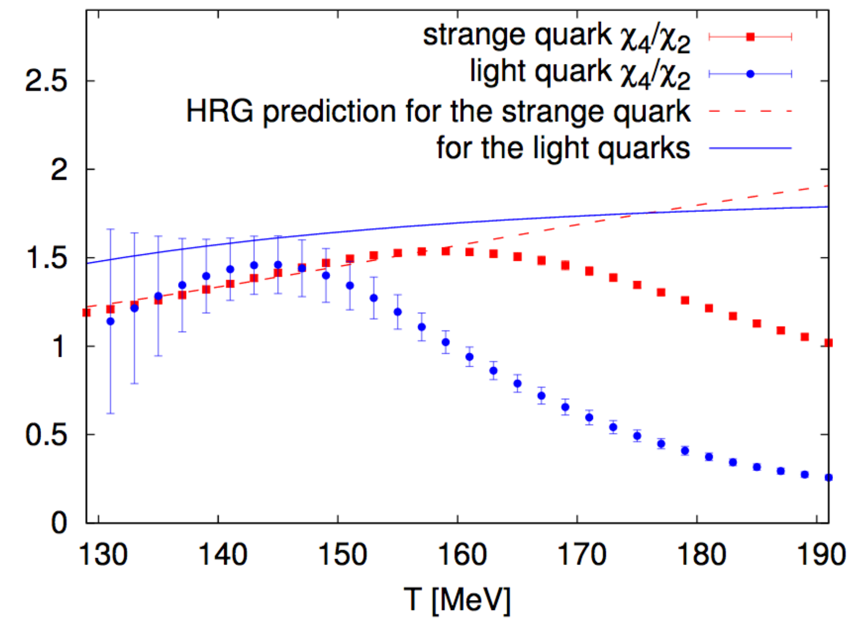
What about strangeness freeze-out?

See talk by R. Bellwied on Monday

- Yield fits seem to hint at a higher temperature for strange particles



M. Floris: QM 2014



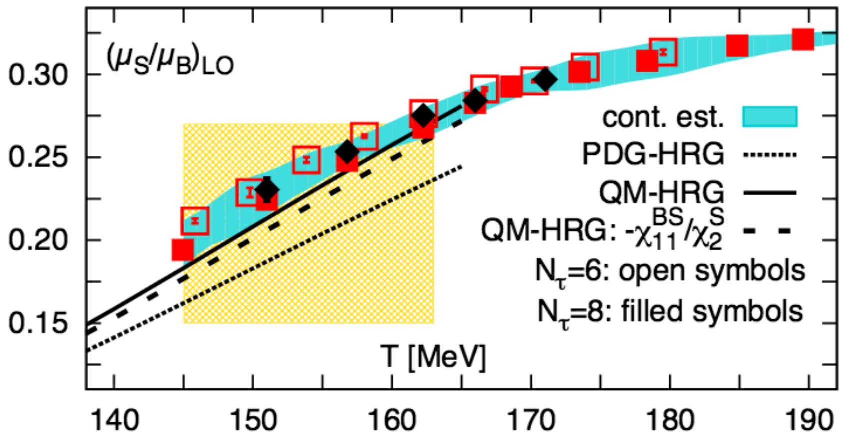
- Similar behavior found in lattice QCD results

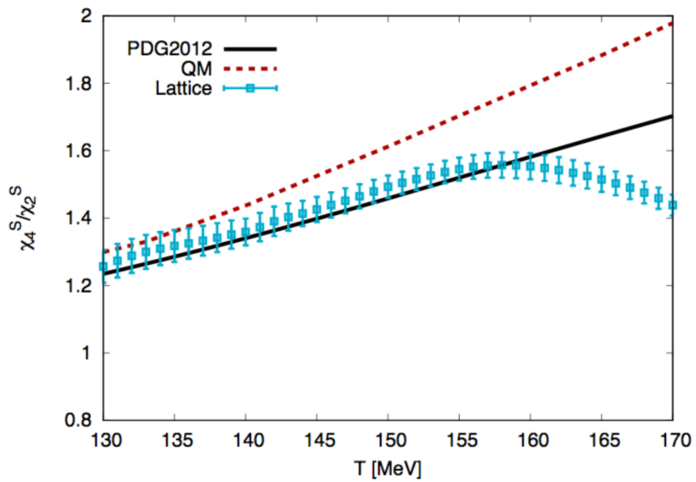
R. Bellwied et al. (WB Collaboration): PRL2013

Missing strange states?

See talk by G. Aarts on Friday afternoon

- Quark Model predicts not-yet-detected (multi-)strange hadrons





- The agreement with the HRG model improves with some observables but gets worse with others
- χ_4^S/χ_2^S is proportional to <S^2> in the system
- It seems to indicate that the quark model predicts too many multi-strange states or not enough S=1 states

A. Bazavov et al., PRL (2014); Quark Model states from Capstick and Isgur, PRD (1986) and Ebert, Faustov and Galkin, PRD (2009).

Missing strange states?

- Idea: define linear combinations of fluctuations which receive contributions only from particles with a given quantum number

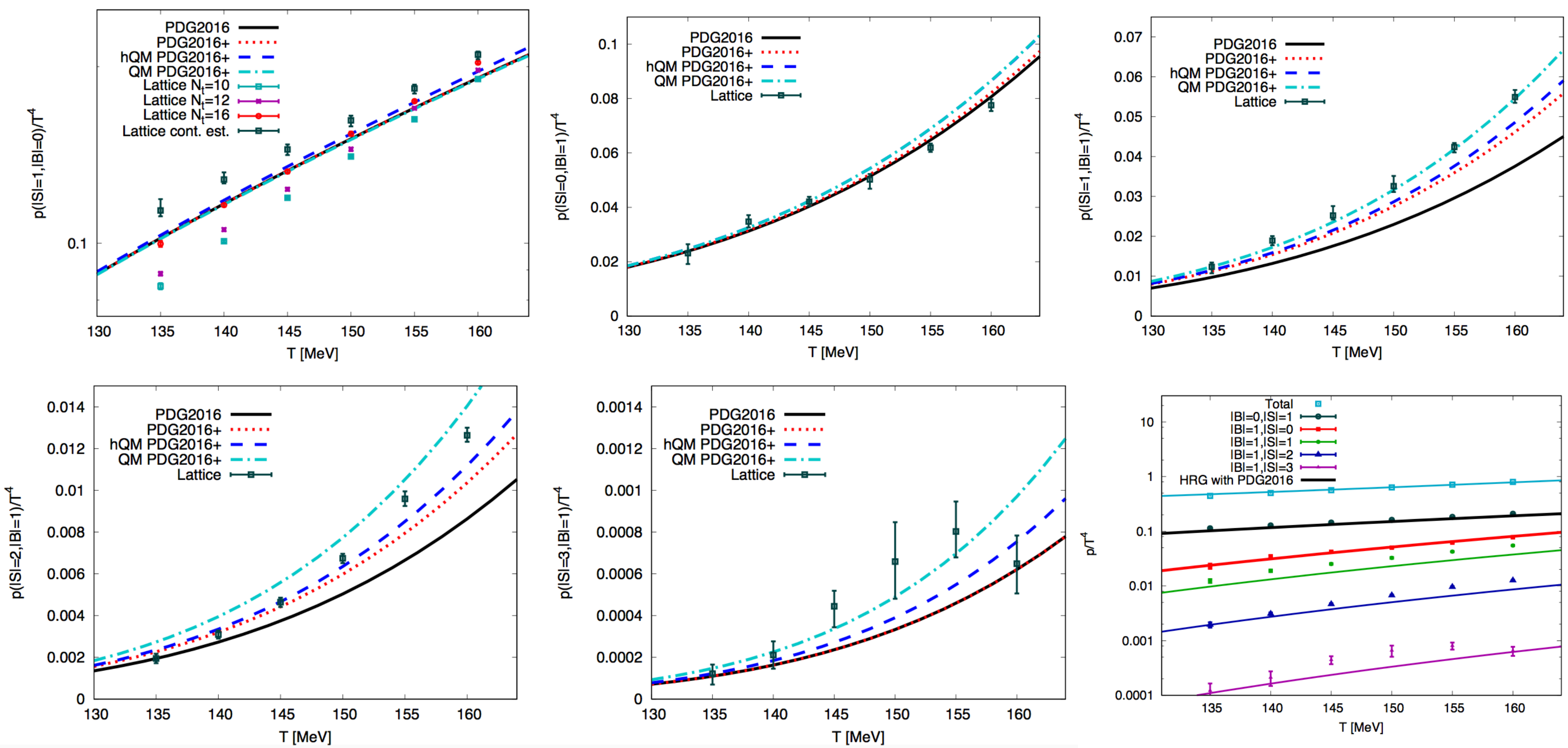
- They allow to compare PDG and QM prediction for each sector separately

$$
\begin{aligned}
P_S(\hat{\mu}_B, \hat{\mu}_S) &= P_{0|1|} \cosh(\hat{\mu}_S) \\
&+ P_{1|1|} \cosh(\hat{\mu}_B - \hat{\mu}_S) \\
&+ P_{1|2|} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) \\
&+ P_{1|3|} \cosh(\hat{\mu}_B - 3\hat{\mu}_S)
\end{aligned}
$$

$$
\begin{aligned}
P_{0|1|} &= \chi_2^S - \chi_{22}^{BS} \\
P_{1|1|} &= \frac{1}{2}\left(\chi_4^S - \chi_2^S + 5\chi_{13}^{BS} + 7\chi_{22}^{BS}\right) \\
P_{1|2|} &= -\frac{1}{4}\left(\chi_4^S - \chi_2^S + 4\chi_{13}^{BS} + 4\chi_{22}^{BS}\right) \\
P_{1|3|} &= \frac{1}{18}\left(\chi_4^S - \chi_2^S + 3\chi_{13}^{BS} + 3\chi_{22}^{BS}\right)
\end{aligned}
$$

A. Bazavov et al., PRL (2013)

Missing strange states?



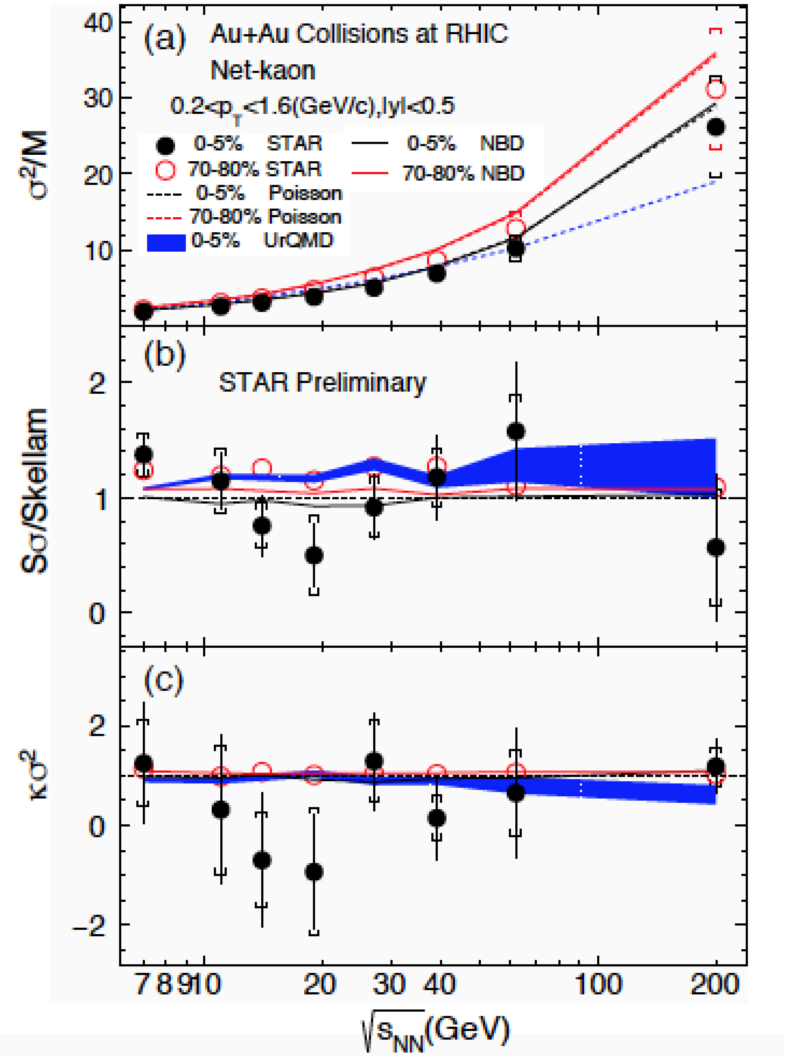
- PDG2016: ****, *** and ** states
- PDG2016+: include also * states

WB: P. Alba et al., 1702.01113

Kaon fluctuations

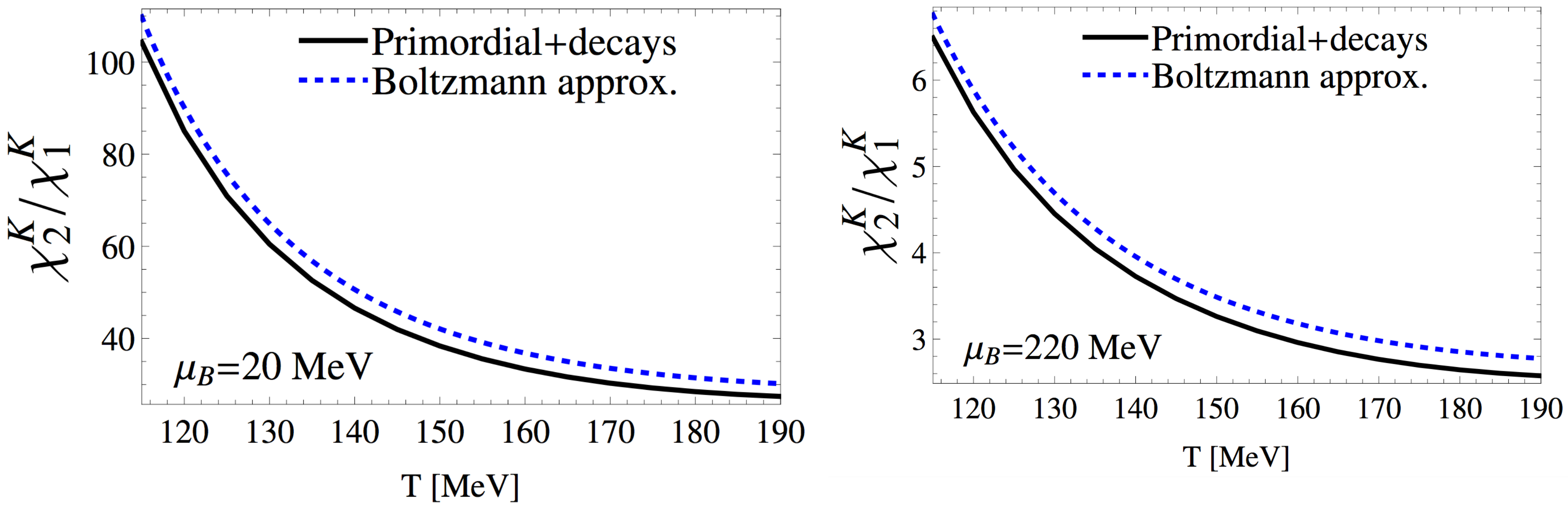
Talk by Ji XU at SQM 2016

- Experimental data are becoming available.
- Exciting result but presently hampered by systematic errors
- BES-II will help
- Kaon fluctuations from HRG model will be affected by the hadronic spectrum and decays



Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al., 1607.02527



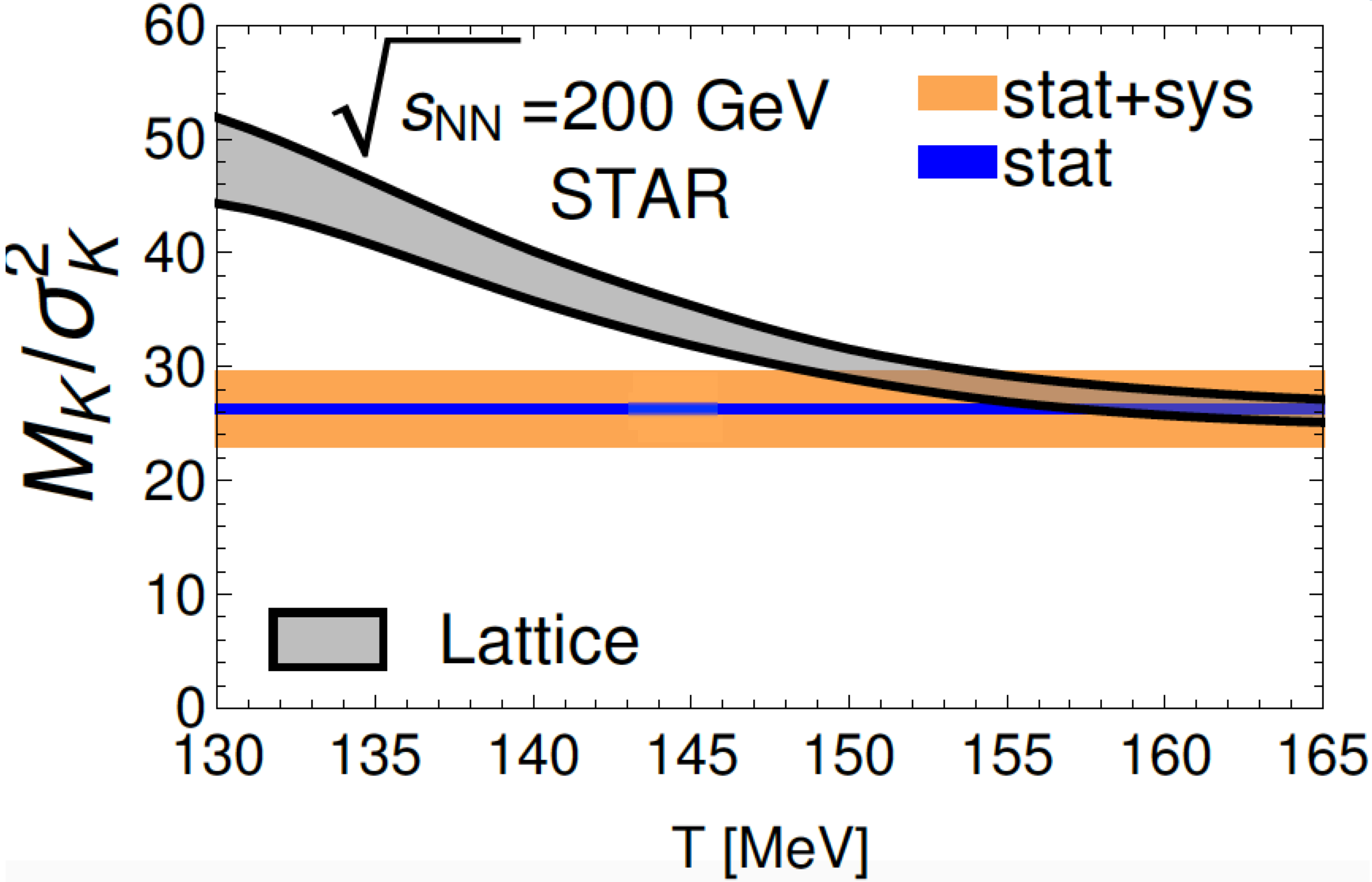
- Boltzmann approximation works well for lower order kaon fluctuations

$$\frac{\chi_2^K}{\chi_1^K} = \frac{\cosh(\hat{\mu}_S + \hat{\mu}_Q)}{\sinh(\hat{\mu}_S + \hat{\mu}_Q)}$$

- χ_2^K/χ_1^K from primordial kaons + decays is very close to the one in the Boltzmann approximation

Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al., 1607.02527



- Experimental uncertainty does not allow a precise determination of T_f^K
- It looks like T_f^K>150 MeV

Conclusions

- Unprecedented precision in lattice QCD data allows a direct comparison to experiment for the first time

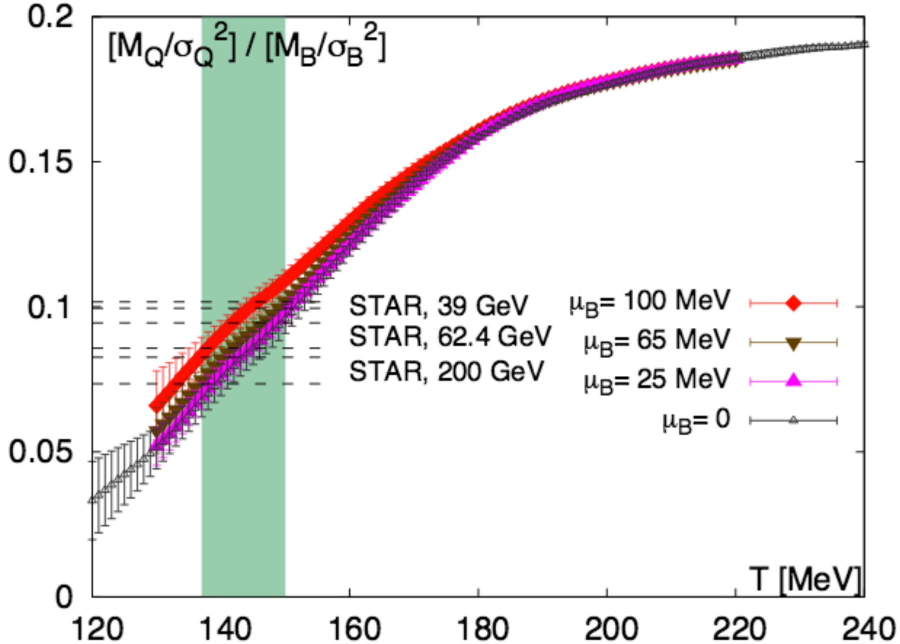
- Many effects need to be understood for a meaningful comparison

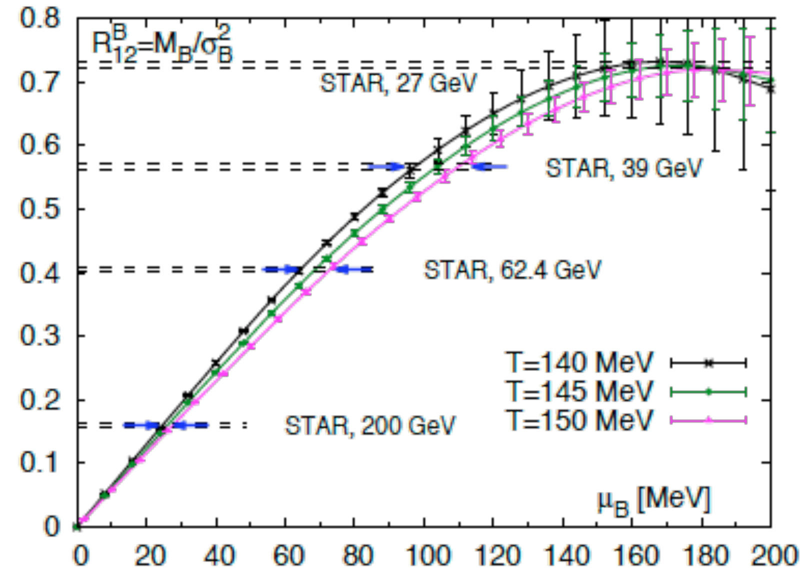
- Comparison with experiment can determine properties of strongly interacting matter from first principles

- Strangeness freeze-out still under study

Freeze-out parameters from B fluctuations

- Thermometer: $\dfrac{\chi_3^B(T,\mu_B)}{\chi_1^B(T,\mu_B)}$ $:S_B\sigma_B^3/M_B$ Baryometer: $\dfrac{\chi_1^B(T,\mu_B)}{\chi_2^B(T,\mu_B)}$ ${}_3{}^2/M_B$





WB: S. Borsanyi et al., PRL (2014)
STAR collaboration, PRL (2014)

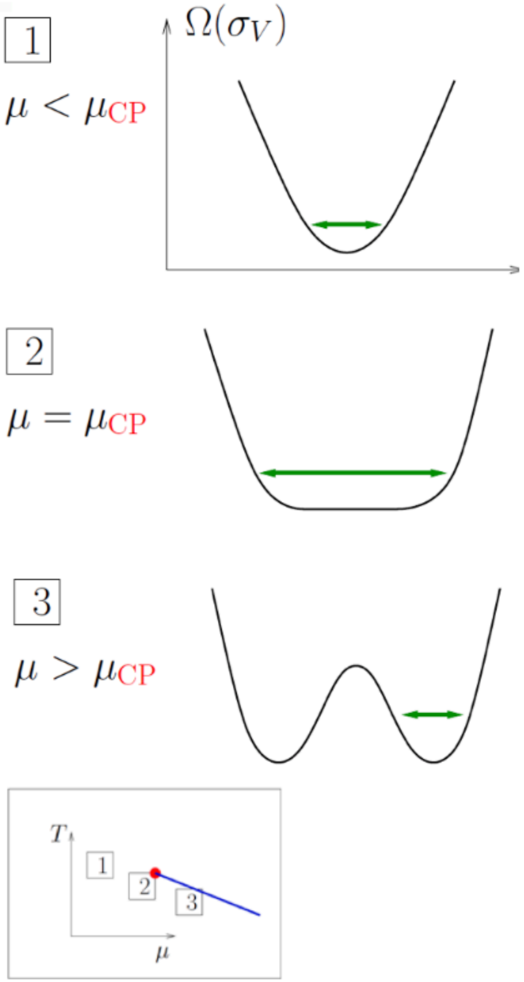
- Upper limit: $T_f \leq 151 \pm 4$ MeV
- Consistency between freeze-out chemical potential from electric charge and baryon number is found.

A different approach at large densities

- Use AdS/CFT correspondence
- Fix the parameters to reproduce everything we know from the lattice
- Calculate observables at finite density
- Fluctuations of conserved charges: they are sensitive to the critical point

Fluctuations at the critical point

M. Stephanov, PRL (2009).



The probability distribution for the order parameter

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma_2 + \frac{\lambda_3}{3}\sigma^3 + \cdots\right]$$

The correlation length ($\xi = m_\sigma^{-1}$)

$$\xi \sim |T - T_c|^{-\nu} \text{ where } \nu > 0$$

$$\chi_2 = VT\xi^2$$

$$\chi_3 = 2VT^{3/2}\hat{\lambda}_3\xi^{9/2}$$

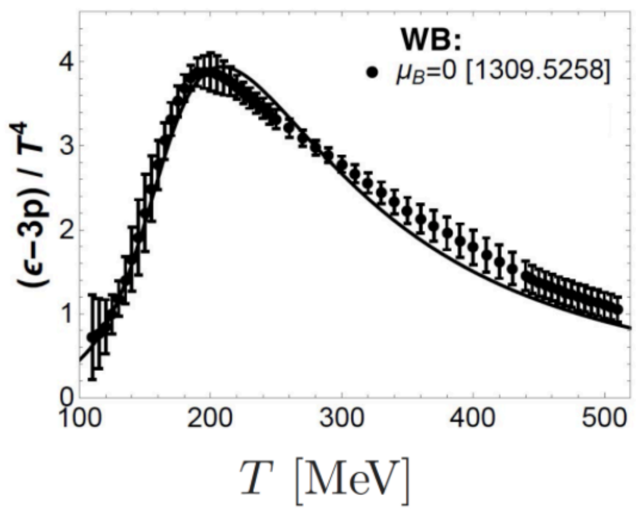
$$\chi_4 = 6VT^2[2\hat{\lambda}_3^2 - \hat{\lambda}_4]\xi^7$$

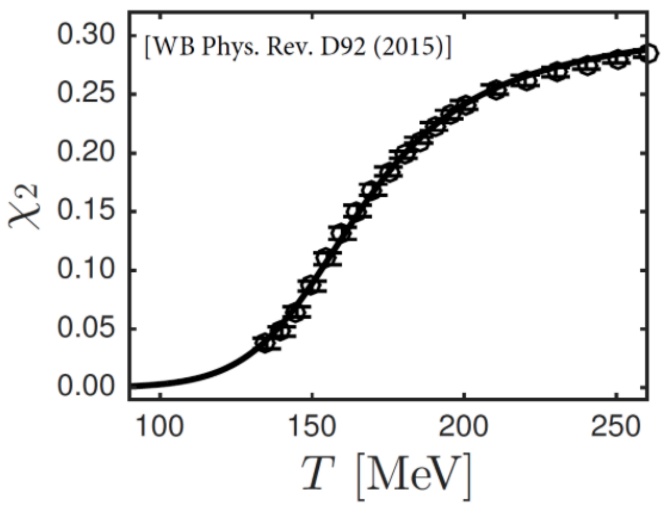
Holographic model

O DeWolfe, S S Gubser, and C Rosen, Phys. Rev. D 83, (2011)
R Rougemont, A Ficnar, S Finazzo and J Noronha, JHEP (2016) 102

Non-conformal holographic gravity dual in 5 dimensions $\Longrightarrow$ Black Hole Solution

$$\mathcal{S} = \frac{1}{16\pi G_5}\int dx^5\sqrt{-g}\,[\mathcal{R} - \tfrac{1}{2}(\partial_M\phi)^2 - \underbrace{V(\phi)}_{\text{nonconformal}} - \tfrac{1}{4}\underbrace{f(\phi)F^2_{MN}}_{\mu_B\neq 0}]$$

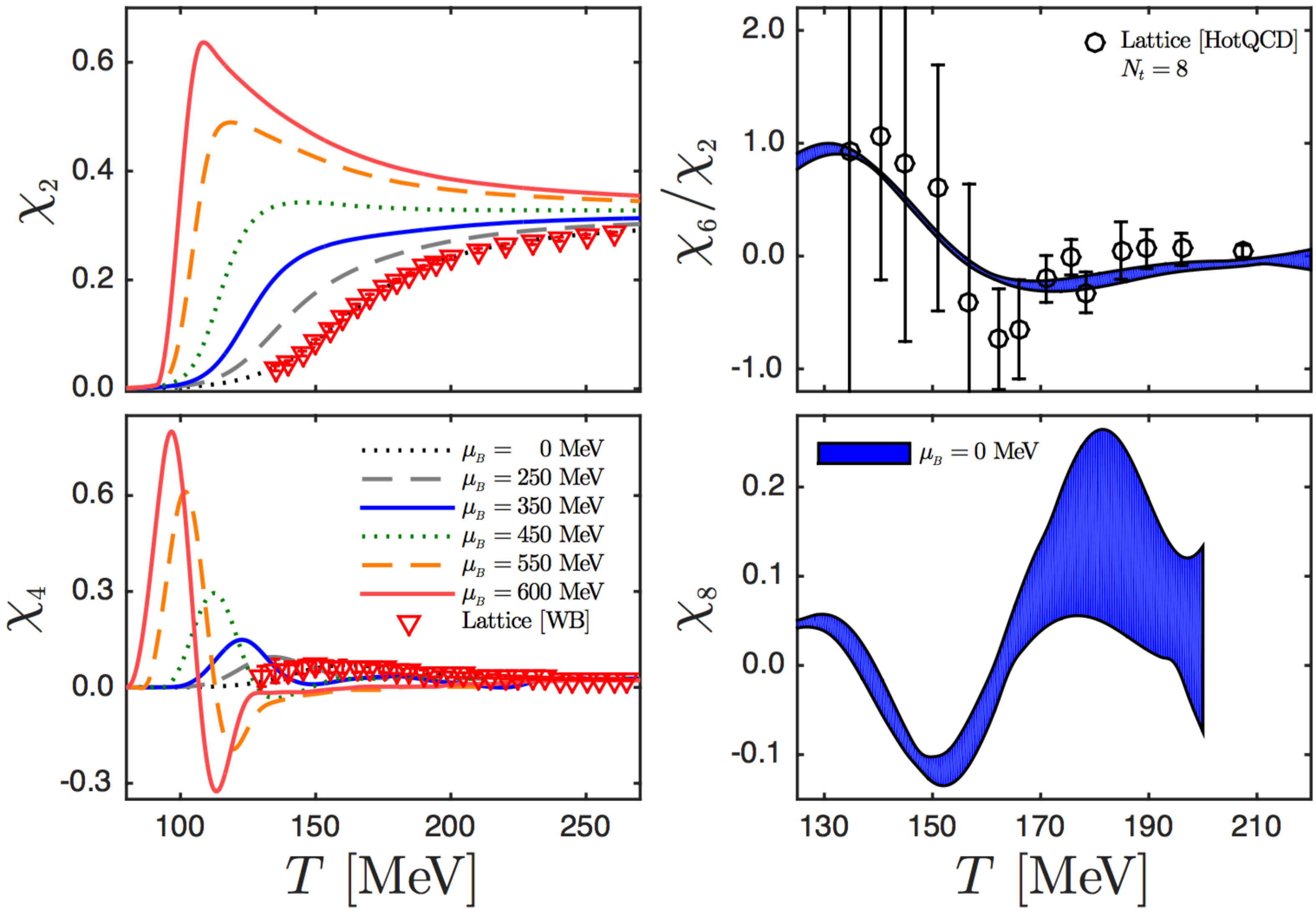




- Input parameters are fixed by lattice QCD results at $\mu_B = 0$
- Finite T and $\mu_B \rightarrow$ Predictions

Black Hole Susceptibilities

R. Critelli, C. R. et al., forthcoming



Testing the Taylor expansion

R. Critelli, C. R. et al., forthcoming

Taylor expansion of observables in terms of susceptibilities
$\chi_n = \chi_n^B(T, \mu_B = 0)$

- Pressure

$$\frac{p(T,\mu_B) - p(T,\mu_B = 0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n}$$

- Baryonic density

$$\frac{\rho_B(T,\mu_B)}{T^3} = \sum_{n=1}^{\infty} \frac{\chi_{2n}}{(2n-1)!} \left(\frac{\mu_B}{T}\right)^{2n-1}$$

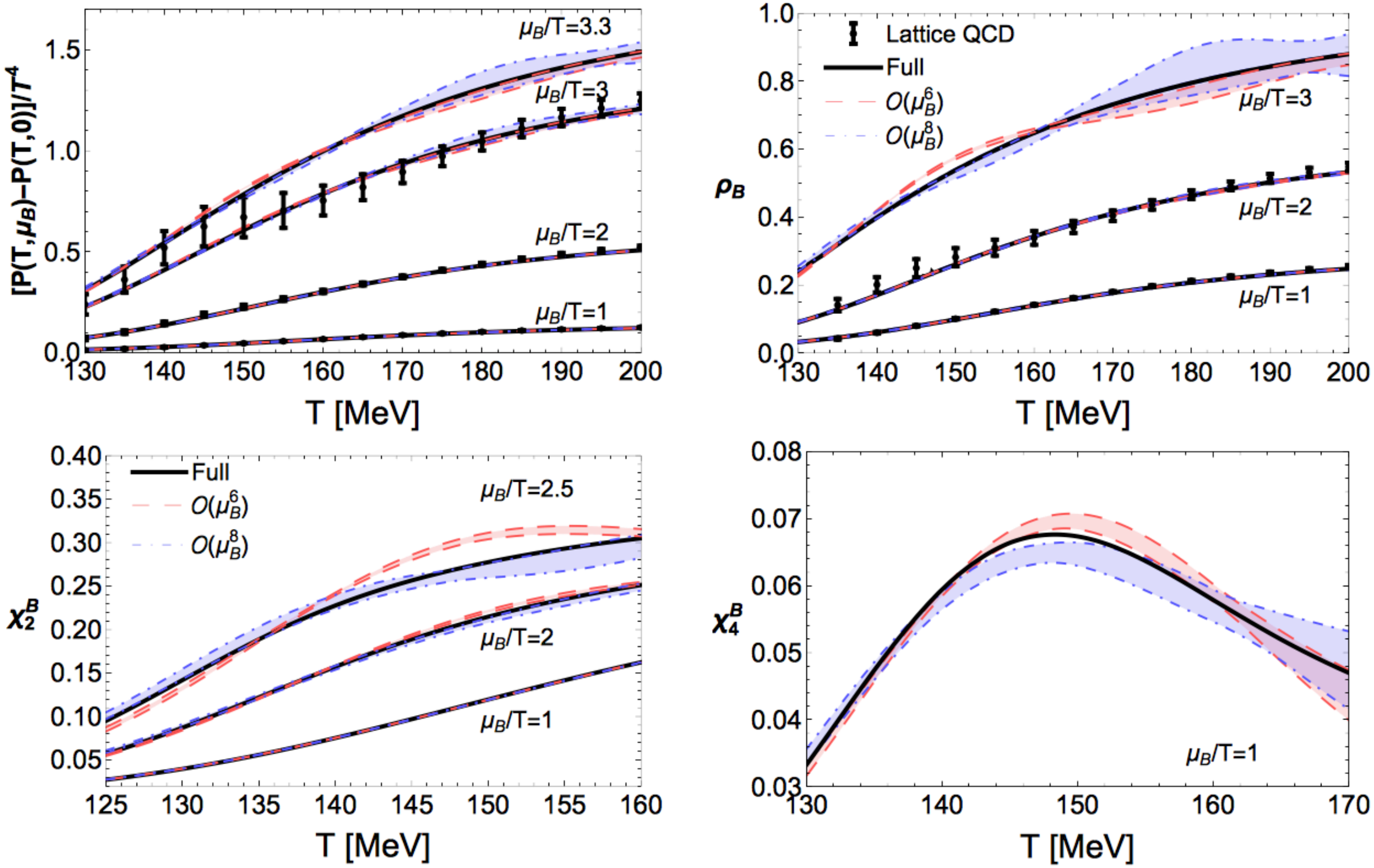
- Susceptibilities χ_2 and χ_4

$$\chi_2(T,\mu_B) = \sum_{n=0}^{\infty} \frac{\chi_{2n+2}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n} \qquad \chi_4(T,\mu_B) = \sum_{n=0}^{\infty} \frac{\chi_{2n+4}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n}$$

Testing the Taylor expansion

R. Critelli, C. R. et al., forthcoming

Reconstruction of thermodynamic quantities at different values of μ_B/T via Taylor series from calculations at $\mu_B = 0$.



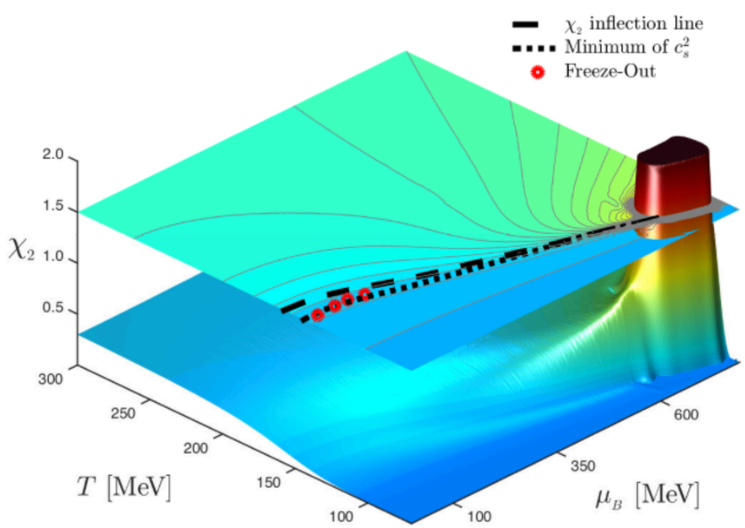
Lattice results: [HotQCD] Phys. Rev. D, **95**, 054504 (2017).

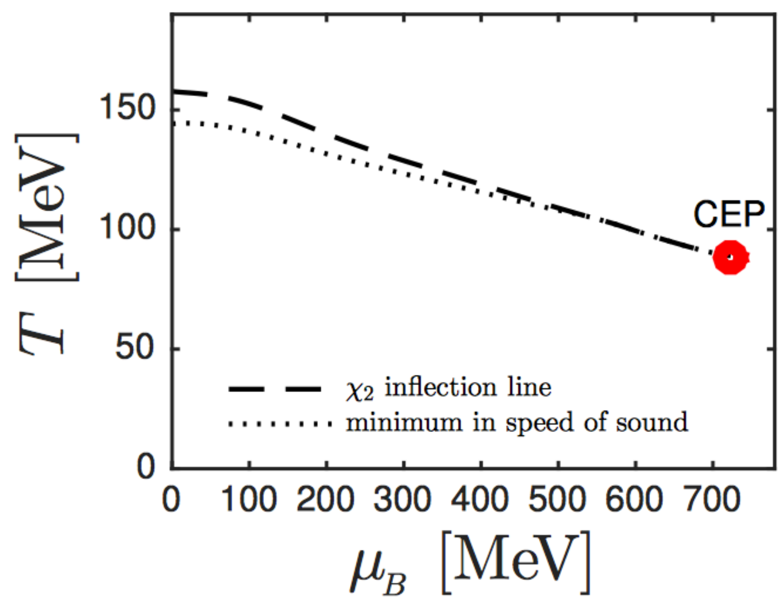
Black hole critical point

R. Critelli, C. R. et al., forthcoming

The black hole model contains a critical end point at

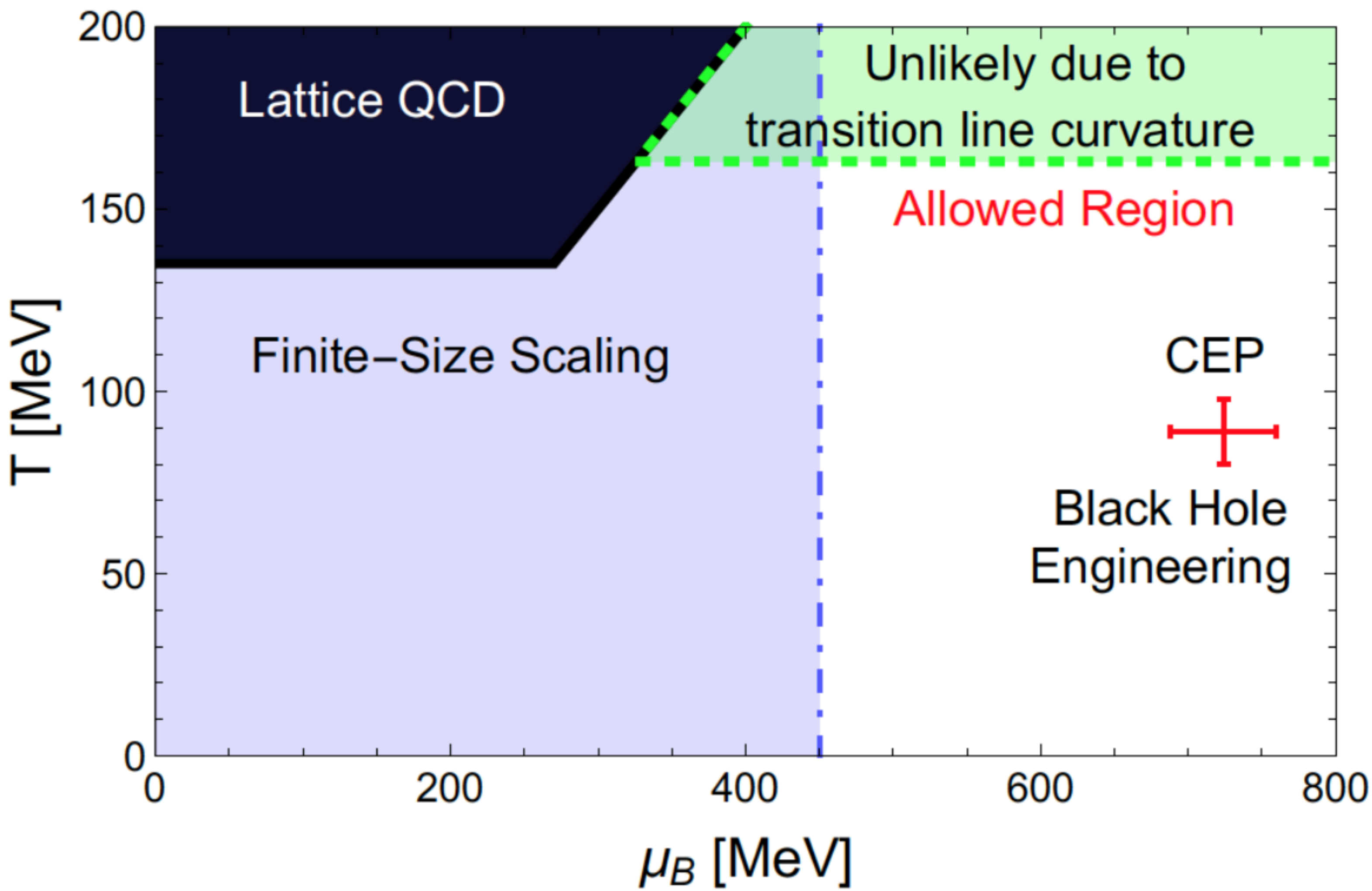
- $\mu_B = 723 \pm 36$ MeV
- $T = 89 \pm 11$ MeV





Black hole critical point

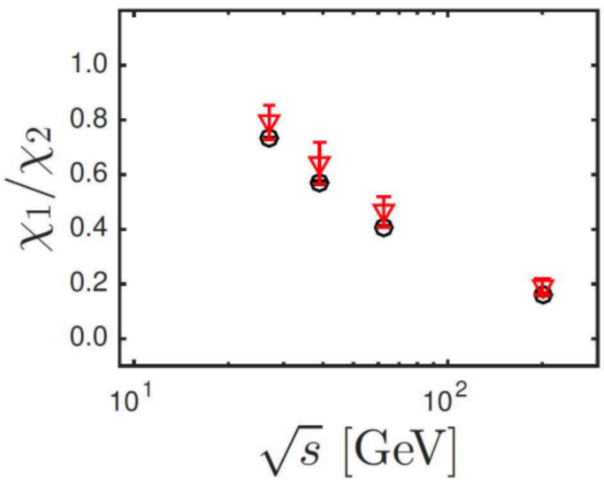
R. Critelli, C. R. et al., forthcoming

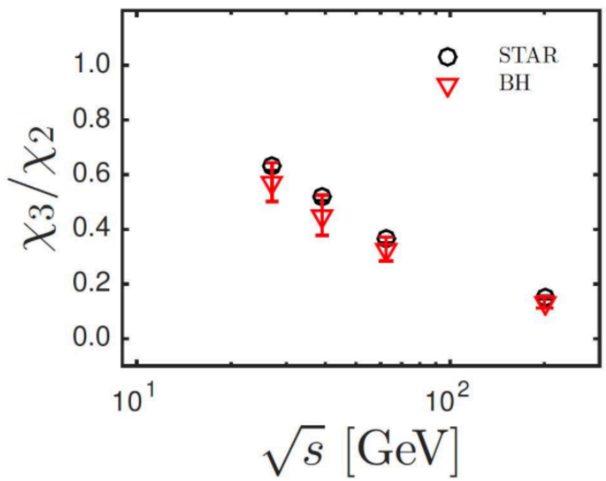


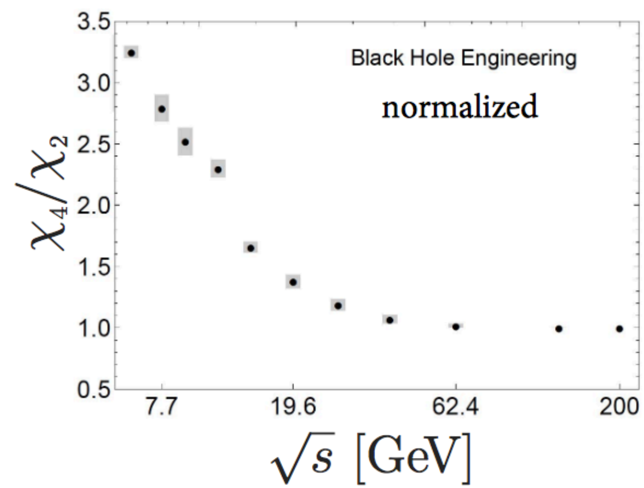
Connection to experiment

R. Critelli, C. R. et al., forthcoming

- We want to estimate the collision energy we need to find the critical point in experiments
- We compare the baryonic BH susceptibilities ratios with the net-proton moments measured at STAR
- Freeze-out parameters are extracted by fitting the experimental values for χ1/χ2 and χ3/χ2





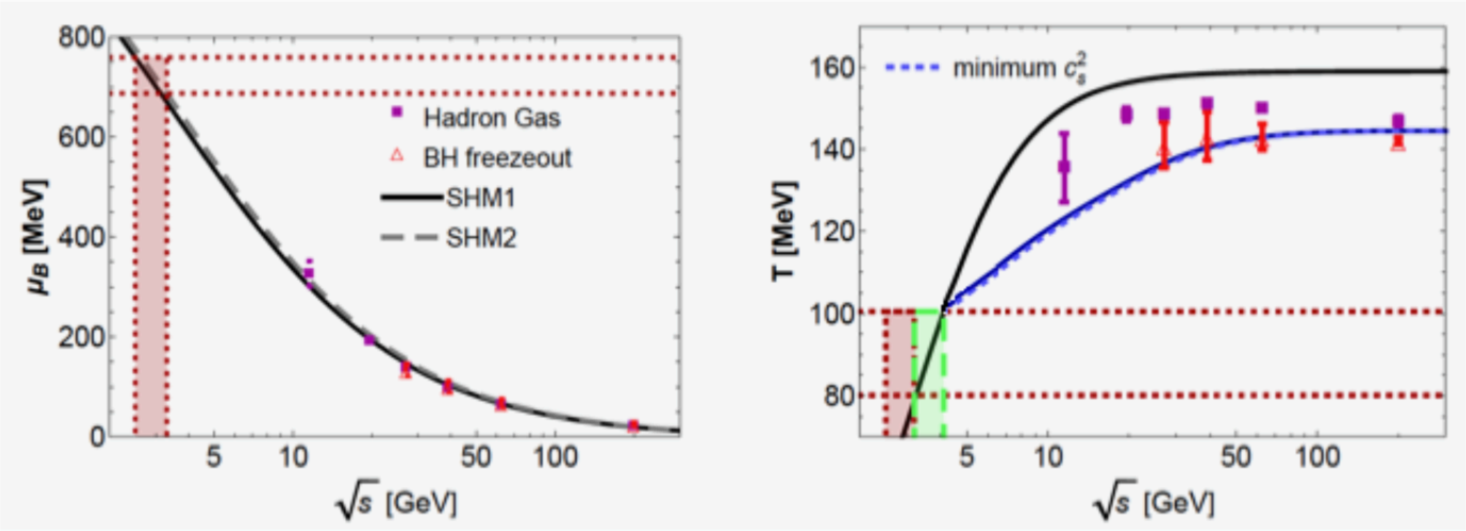


[STAR] Phys. Rev. Lett. **112** (2014)

Collision energy estimate

We estimate a collision energy needed to hit the CEP

- $\sqrt{s} = 2.5 - 4.1$ GeV



- The collision energy is reachable by the next generation of colliders

[BH] **R. Critelli, C. R. et al., forthcoming**
[HRG] Paolo Alba et al. Phys. Lett. B738 (2014),
[SHM1] A. Andronic et al. Phys. Lett. B673 (2009).
[SHM2] J. Cleymans et al. Phys. Rev. C73 (2006).

Lattice details

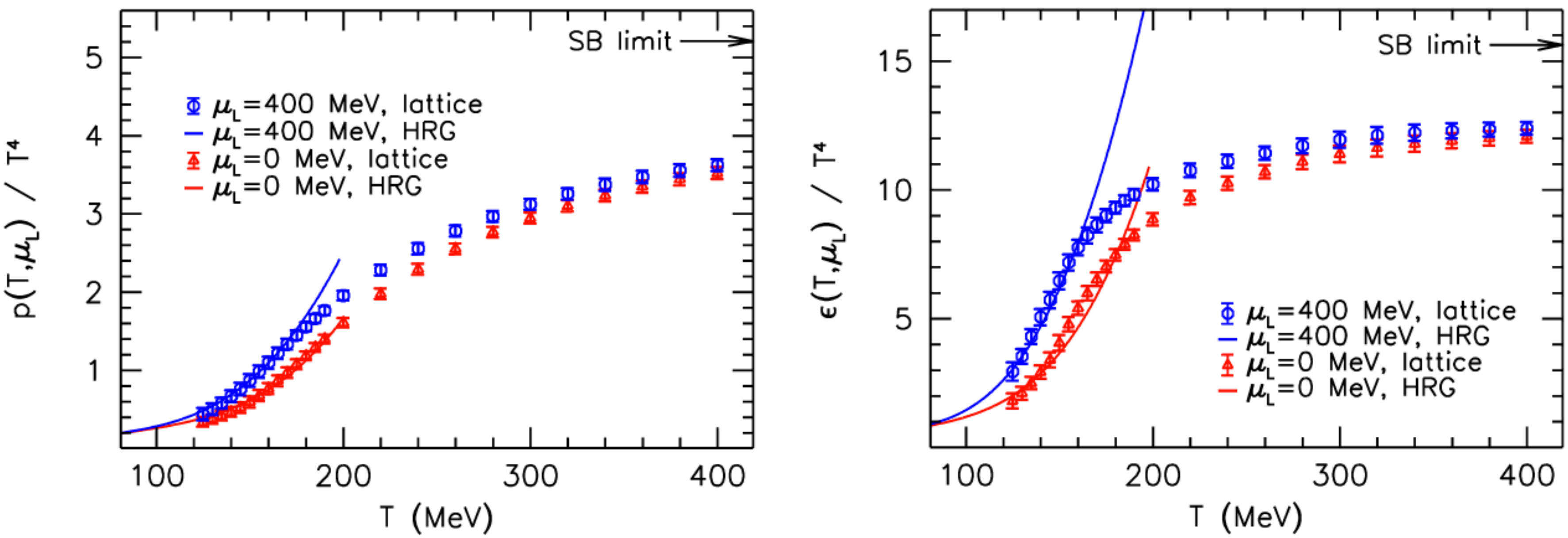
- **The 4stout staggered action**
  - 2+1+1 dynamical flavors
  - 4 levels of stout smearing in the fermionic action
  - The masses are set by bracketing both the pion and the kaon masses within a few percent, keeping m_c/m_s=11.85
  - The scale is set in two ways: f_π and w_0 (with Wilson flow). The scale setting procedure is one of the source of the systematic error in all of the plots

- **Ensembles**
  - Continuum limit from N_t=10, 12, 16
  - For imaginary μ we have $\mu_B = iT\pi j/8$, with j=3, 4, 5, 6, 6.5, 7

Equation of state at $\mu_B>0$

- Expand the pressure in powers of μ_B (or $\mu_L=3/2(\mu_u+\mu_d)$)

$$\frac{p(T,\{\mu_i\})}{T^4} = \frac{p(T,\{0\})}{T^4} + \frac{1}{2}\sum_{i,j}\frac{\mu_i\mu_j}{T^2}\chi_2^{ij} \quad \text{with} \quad \chi_2^{ij} \equiv \frac{T}{V}\frac{1}{T^2}\frac{\partial^2 \log \mathcal{Z}}{\partial\mu_i\partial\mu_j}\bigg|_{\mu_i=\mu_j=0}$$



S. Borsanyi et al., JHEP (2012)

- Continuum extrapolated results at the physical mass