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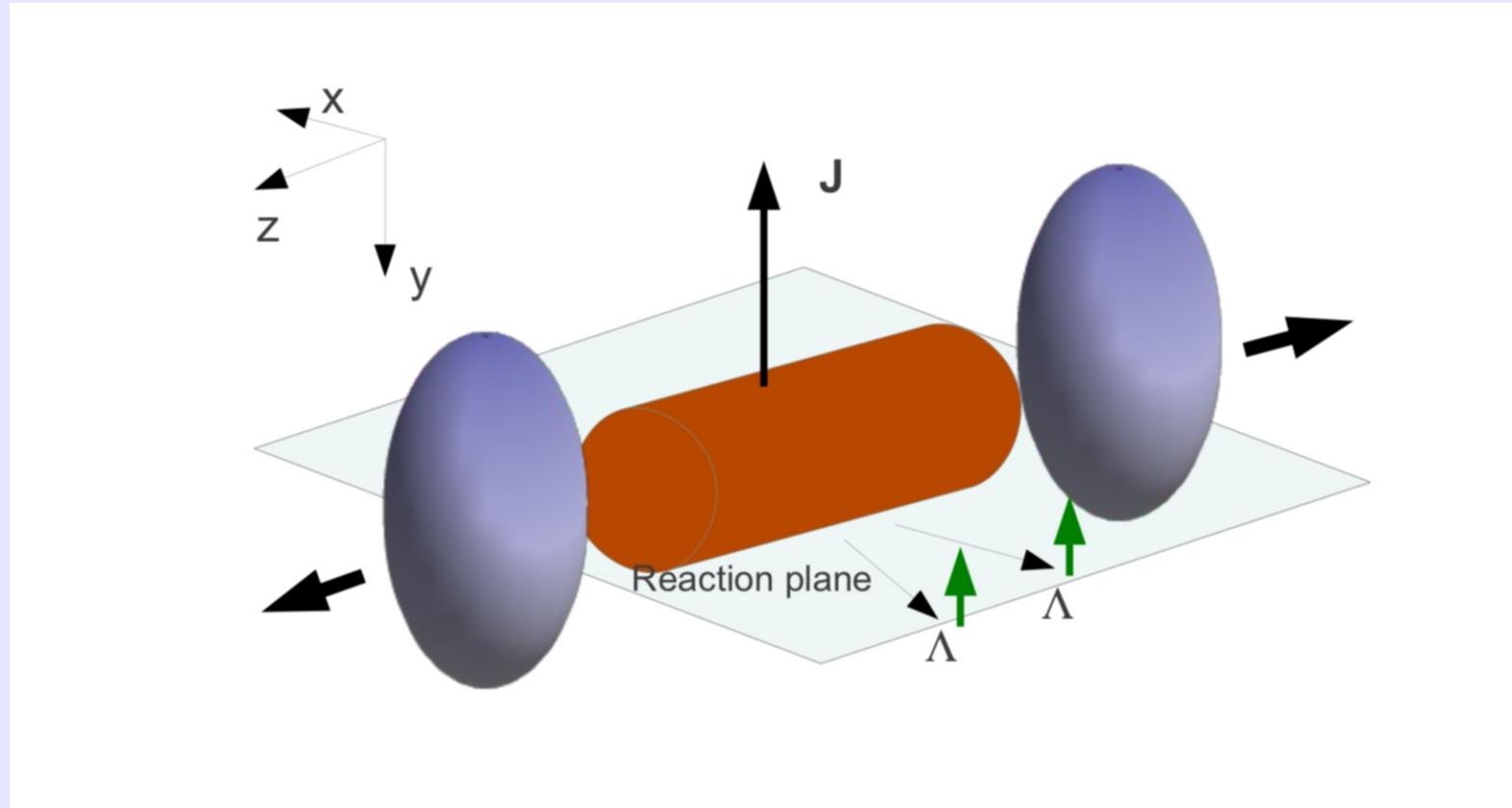
# Polarization in relativistic heavy ion collisions

## OUTLINE

- Introduction
- Theory review
- Global longitudinal polarization
- Open theoretical problems
- Conclusions

# Introduction

Peripheral collisions  $\rightarrow$  Angular momentum  $\rightarrow$  Global polarization w.r.t reaction plane

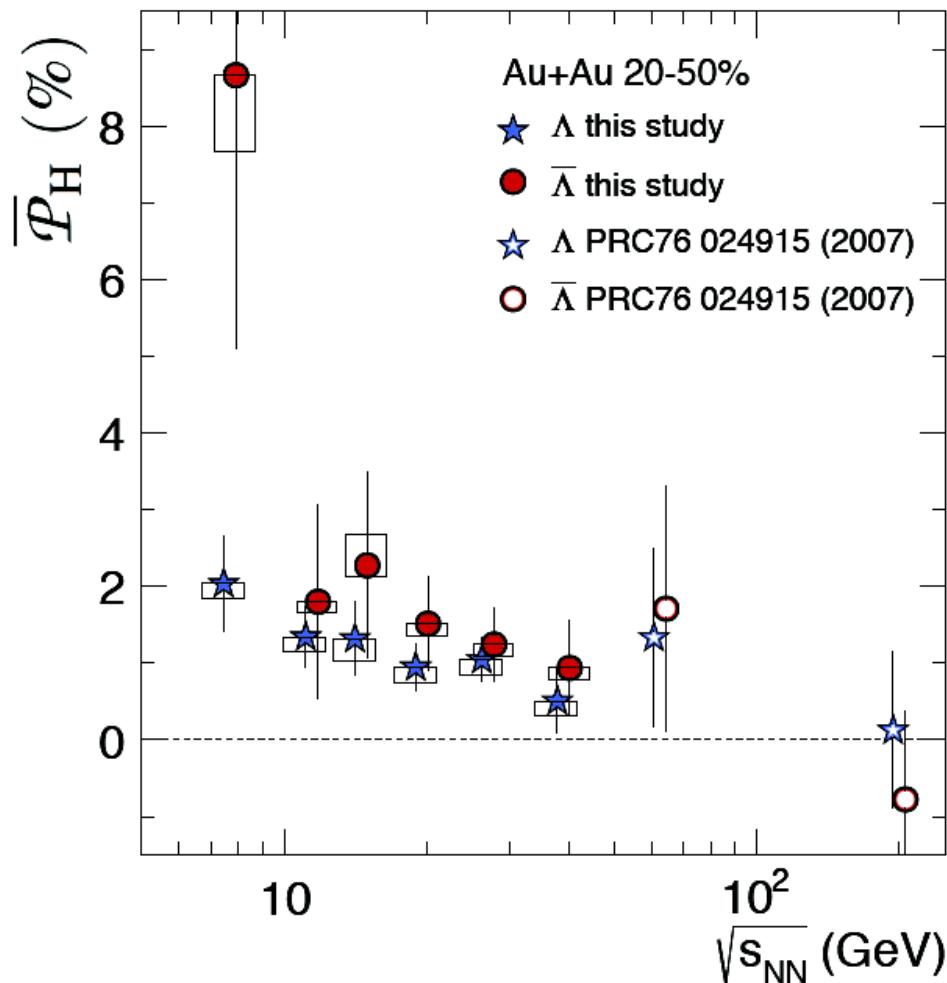


$$\frac{\hbar\omega}{KT} \approx \frac{c}{12\text{fm}200\text{MeV}} \approx 0.08$$

$$a \approx 10^{30} g \implies \frac{\hbar a}{c K T} \approx 0.06$$

# Evidence for vorticity-related polarization

STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions: evidence for the most vortical fluid*, arXiv:1701.06657 [nucl-ex].



See S. Voloshin's talk  
this conference

First evidence of a  
quantum effect in  
hydrodynamics

The measured polarization agrees with the predictions from the thermo-hydro model and is mostly determined by rotation (C-even effect)

# Theoretical approaches to global polarization

- Polarization calculated at quark level by spin-orbit coupling

Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 (2005) 102301

- By local thermodynamic equilibrium of the spin degrees of freedom

F. B., F. Piccinini, Ann. Phys. 323 (2008) 2452; F. B., F. Piccinini, J. Rizzo, Phys. Rev. C 77 (2008) 024906

Spin  $\mu$    Thermal vorticity

- Polarization in the plasma phase transferred to hadrons

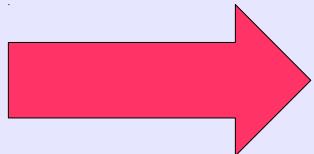
A. Sorin and O. Teryaev, Phys. Rev. C 95 (2017) 011902

Y. Sun and C. M. Ko, arXiv:1706.09467

....

# Polarization and relativistic hydrodynamics

F. B., V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338 (2013) 32



## *Spin, local equilibrium and relativity*

It is crucial to use a *quantum-relativistic* formalism from the onset

Definition of a *relativistic spin* four-vector

For a single particle

$$S^\mu = -\frac{1}{2m} \epsilon^{\mu\nu\lambda\rho} \langle \hat{J}_{\nu\lambda} \hat{P}_\rho \rangle$$

$$\langle \hat{X} \rangle = \text{tr}(\hat{\rho} \hat{X})$$

Relativistic Spin vs Pauli-Lubanski vs Polarization

$$S^\mu = \frac{1}{m} W^\mu = S P^\mu$$

# The density operator

Covariant form of the local thermodynamical equilibrium quantum density operator

$$\hat{\rho} = \frac{1}{Z} \exp \left[ - \int_{\Sigma} d\Sigma_{\mu} \left( \hat{T}^{\mu\nu} \beta_{\nu} - \zeta \hat{j}^{\mu} \right) \right]$$

Hydrodynamic limit: Taylor expansion of the  $\beta$  and  $\zeta$  fields around the point  $x$  where *Local operators* are to be calculated. Local values of  $T, u, \mu$  and their local derivatives (antisymmetric part: local thermal vorticity)

$$\begin{aligned} \hat{\rho} = \frac{1}{Z} \exp & \left[ -\beta(x) \cdot \hat{P} + \zeta(x) \hat{Q} + \frac{1}{2} \varpi_{\mu\nu}(x) \hat{J}_x^{\mu\nu} \right. \\ & \left. + \text{terms vanishing at equilibrium} \right] \end{aligned}$$

F. B., L. Bucciantini, E. Grossi, L. Tinti,  
Eur. Phys. J. C 75 (2015) 191 ( $\beta$  frame)

$$\beta^{\mu} = \frac{1}{T} u^{\mu} \quad \zeta = \mu/T$$

$$\varpi_{\nu\mu} = -\frac{1}{2} (\partial_{\nu} \beta_{\mu} - \partial_{\mu} \beta_{\nu})$$

*Thermal vorticity*  
Adimensional in natural units

# Local polarization and spin tensor

For a particle with momentum  $p$

$$S^\mu(p)N(p) = -\frac{1}{2m}\epsilon^{\mu\nu\lambda\rho} \int_{\Sigma} d\Sigma_\tau \langle \hat{S}_{\nu\lambda}^\tau \rangle_p p_\rho$$

The rank 3 operator is the SPIN TENSOR and we need *its momentum-resolved mean value*

For the Dirac field

$$\hat{\mathcal{S}}^{\lambda\mu\nu} = \frac{i}{8} \bar{\Psi} \{ \gamma^\lambda [\gamma^\mu, \gamma^\nu] \} \Psi$$

An useful tool: the covariant Wigner function

$$\begin{aligned} W(x, k)_{AB} &= -\frac{1}{(2\pi)^4} \int d^4y e^{-ik\cdot y} \langle : \Psi_A(x - y/2) \bar{\Psi}_B(x + y/2) : \rangle \\ &= \frac{1}{(2\pi)^4} \int d^4y e^{-ik\cdot y} \langle : \bar{\Psi}_B(x + y/2) \Psi_A(x - y/2) : \rangle \end{aligned}$$

$$\langle \hat{\mathcal{S}}^{\lambda\mu\nu} \rangle = \frac{i}{8} \langle \bar{\Psi} \{ \gamma^\lambda [\gamma^\mu, \gamma^\nu] \} \Psi \rangle = \frac{i}{8} \int d^4k \text{tr}_4(\{ \gamma^\lambda [\gamma^\mu, \gamma^\nu] \} W(x, k))$$

# Spin four-vector for spin $\frac{1}{2}$ particles

Approximation at first order in the gradients

$$S^\mu(x, p) = -\frac{1}{8m}(1 - n_F)\epsilon^{\mu\rho\sigma\tau}p_\tau\varpi_{\rho\sigma}$$

$$n_F = (\mathrm{e}^{\beta \cdot p - \xi} + 1)^{-1}$$

$$\varpi_{\nu\mu} = -\frac{1}{2}(\partial_\nu\beta_\mu - \partial_\mu\beta_\nu)$$

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma d\Sigma_\tau p^\tau n_F(1 - n_F)\partial_\nu\beta_\rho}{\int_\Sigma d\Sigma_\tau p^\tau n_F}$$

Same formula obtained with a perturbative expansion of the solution of the Wigner function e.o.m. in

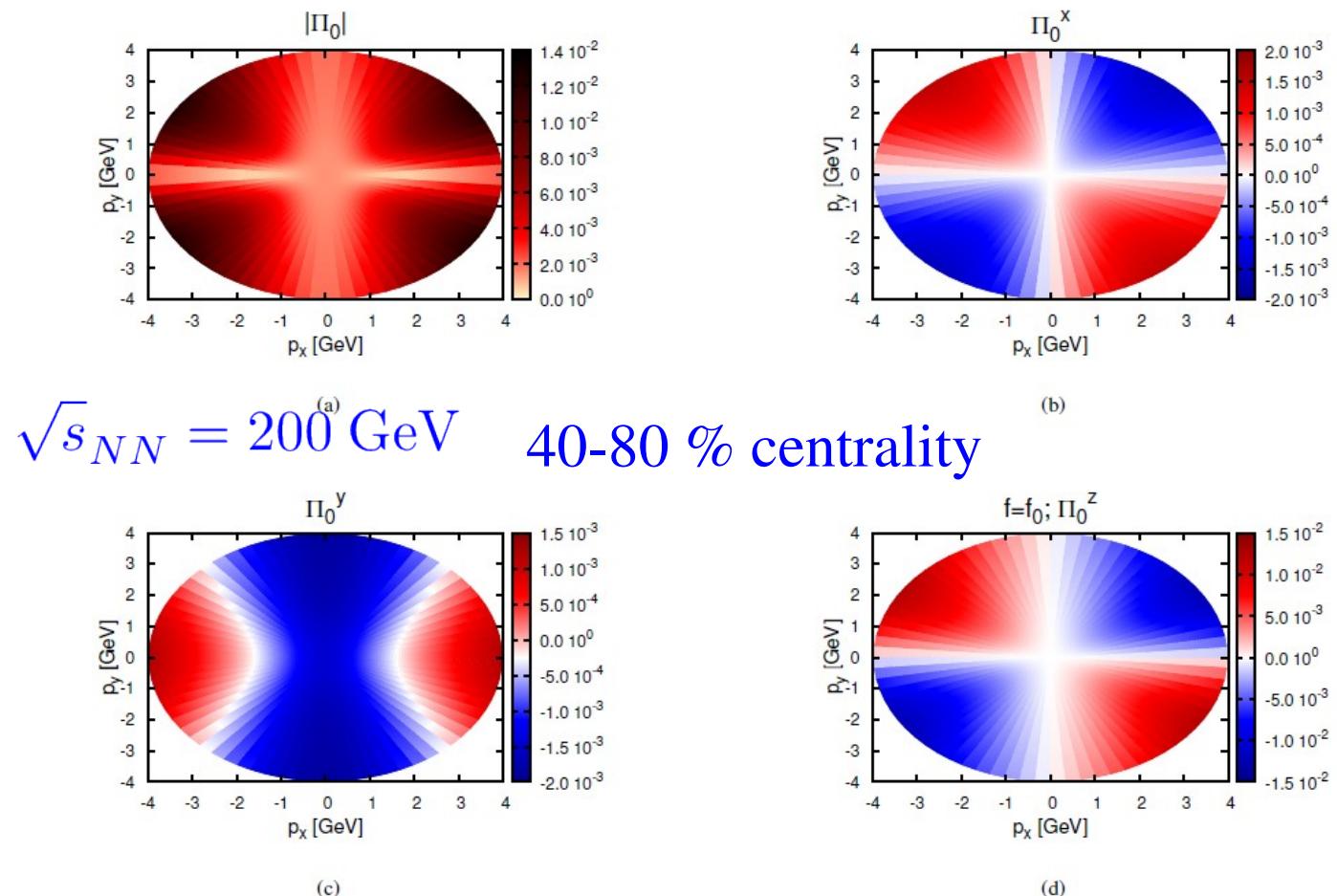
R. Fang, L.G. Pang, Q. Wang, X.N. Wang, arXiv:1604.04036

# $\Lambda$ polarization in relativistic heavy ion collisions

Distribution of protons in the  $\Lambda$  rest frame

$$\frac{1}{N} \frac{dN}{d\Omega} = \frac{1}{4\pi} (1 + \alpha \mathbf{P}_0 \cdot \hat{\mathbf{p}}^*) \quad \mathbf{P}_0(p) = \mathbf{P}(p) - \frac{\mathbf{p}}{\varepsilon(\varepsilon + m)} \mathbf{P}(p) \cdot \mathbf{p}$$

Prediction with  
“conservative”  
Bjorken-like initial  
conditions



F. B., G. Inghirami, V.  
Rolando, A. Beraudo, L.  
Del Zanna, A. De Pace, M.  
Nardi, G. Pagliara, V.  
Chandra  
Eur. Phys. J C 75 (2015) 46

Figure 14: (color online) Magnitude (panel a) and components (panels b,c,d) of the polarization vector of the  $\Lambda$  hyperon in its rest frame.

# Comparison of theoretical calculations with STAR result

I. Karpenko and F. B., Eur. Phys. J. C 77 (2017) 213

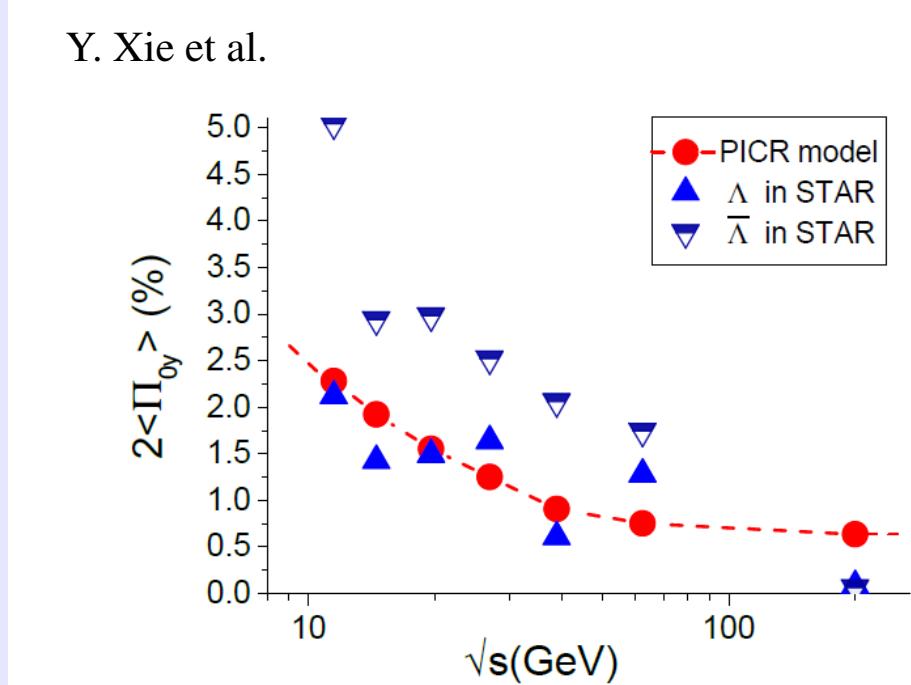
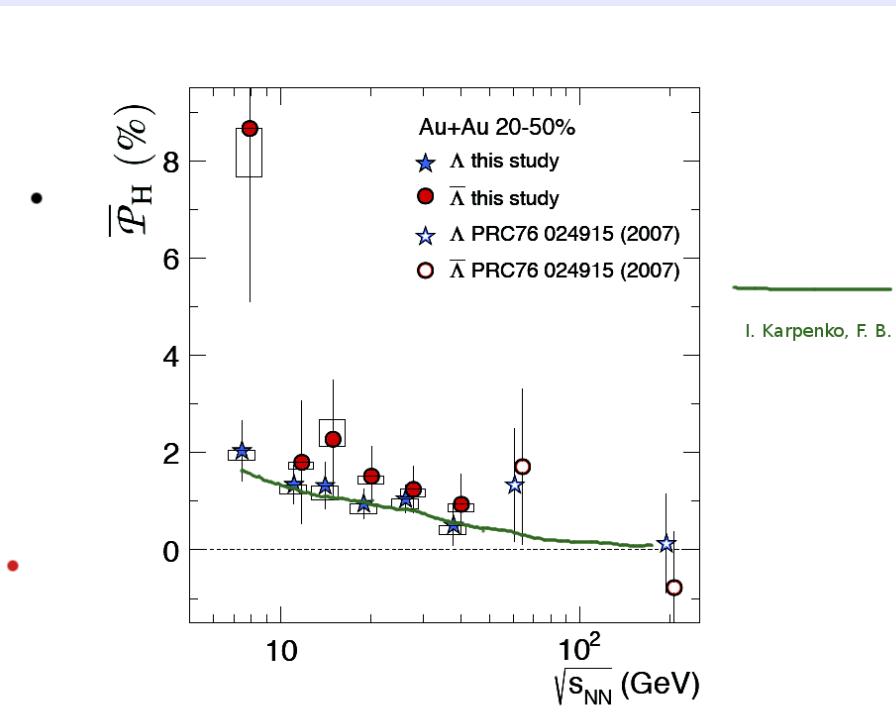
Y. Xie, D. Wang and L. P. Csernai, Phys. Rev. C 95 031901 (2017) [arXiv:1703.03770 [nucl-th]]

H. Li, H. Petersen, L. G. Pang, Q. Wang, X. L. Xia and X. N. Wang, arXiv:1704.03569 [nucl-th]

H. Li, L. G. Pang, Q. Wang and X. L. Xia, arXiv:1704.01507 [nucl-th].

Y. Sun and C. M. Ko, arXiv:1706.09467 [nucl-th].

Same thermal vorticity-related formula, but different initial conditions, evolution models as well as hadronization pictures. Good or very good description of the STAR measurement.



# Comparison of theoretical calculations with STAR result

F. B., I. Karpenko, M. Lisa, I. Uspal, S. Voloshin, Phys. Rev. C 95 054902 (2017) ; I. Karpenko and F. B., Eur. Phys. J. C 77 (2017) 213

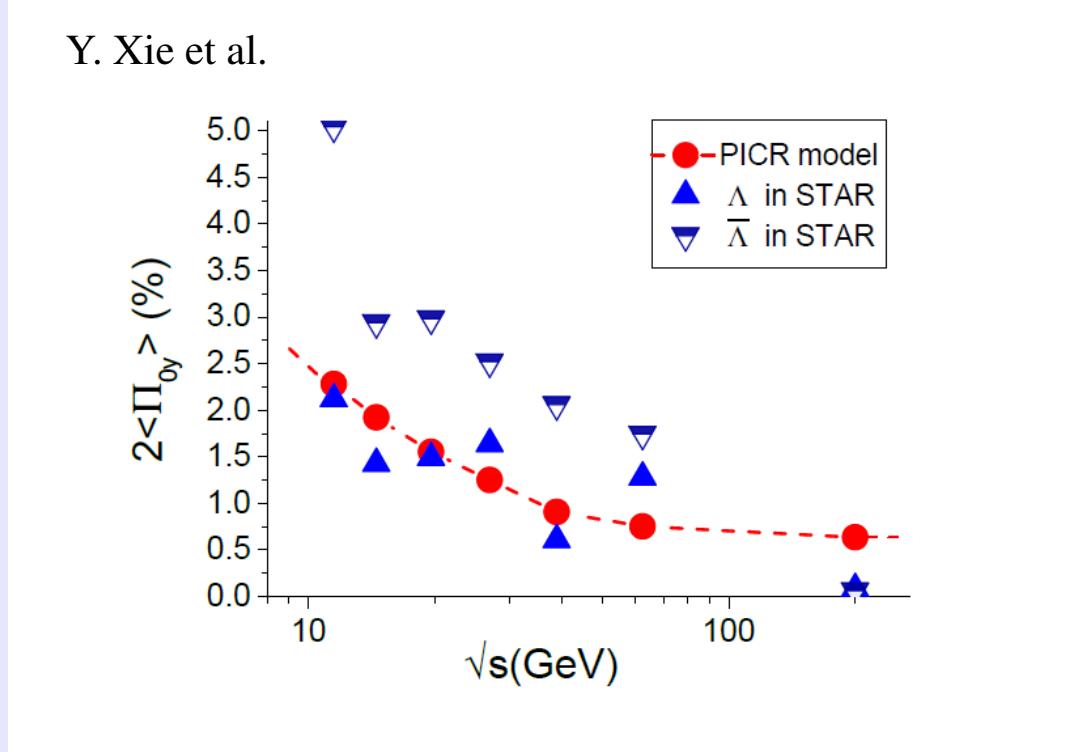
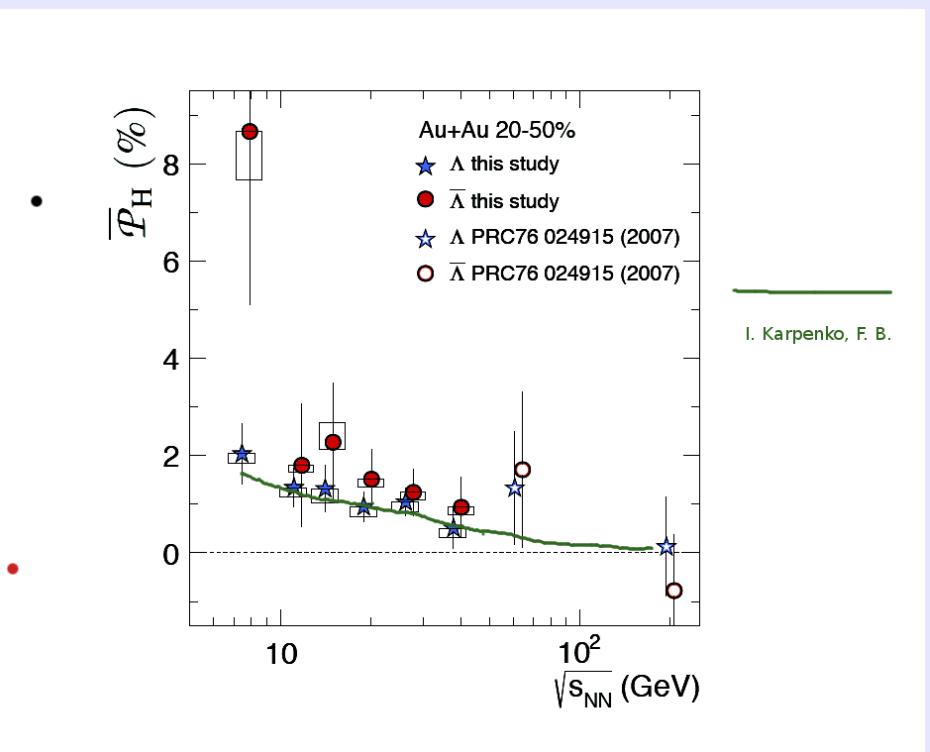
Y. Xie, D. Wang and L. P. Csernai, Phys. Rev. C 95 031901 (2017) [arXiv:1703.03770 [nucl-th]]

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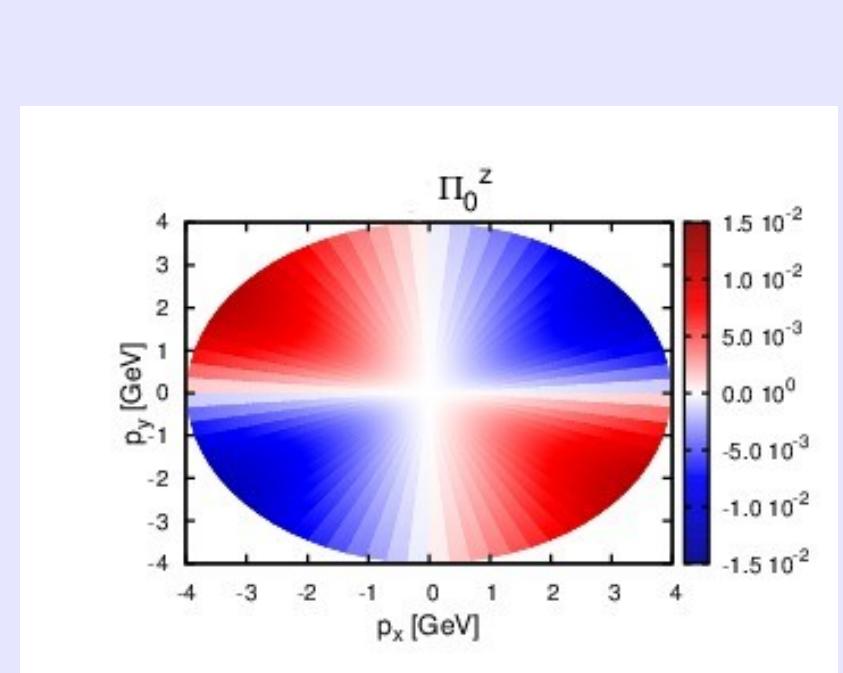
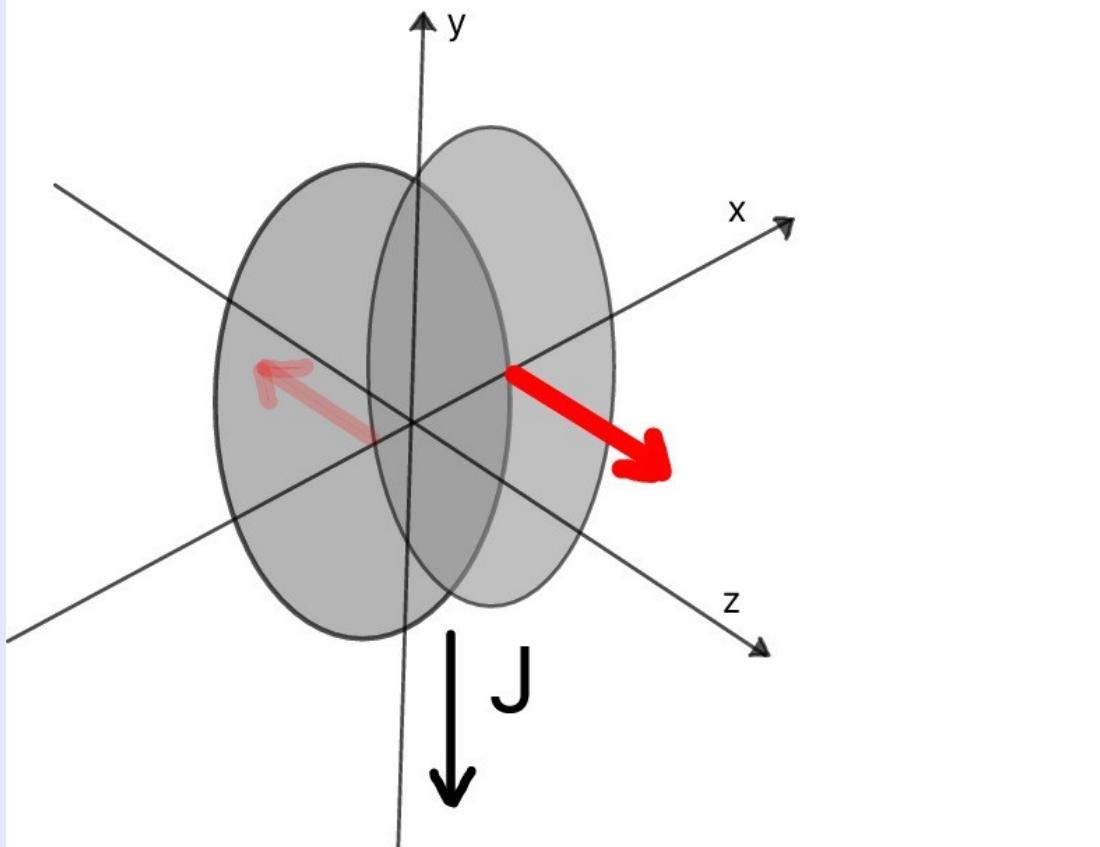
Y. Sun and C. M. Ko, arXiv:1706.09467 [nucl-th].

Same thermal vorticity-related formula, but different initial conditions, evolution models as well as hadronization pictures. Good or very good description of the STAR measurement.



# Global longitudinal polarization: quadrupole structure

F. B., I. Karpenko, arXiv:1707.xxxx



200 GeV: larger magnitude than  $S_j^z$ !

Peripheral heavy ion collisions feature two discrete symmetries: reflection w.r.t. reaction plane and rotation by 180 around its perpendicular direction. This reflects into the quadrupole pattern of the global  $\Lambda$  polarization at midrapidity



$$S^z(p_T, Y = 0) = \frac{1}{2} \sum_{k=1}^{\infty} f_{2k}(p_T) \sin 2k\varphi$$

# Longitudinal boost invariant scenario

Invariance by longitudinal boost implies

$$S^x(\mathbf{p}_T, Y = 0) = S^y(\mathbf{p}_T, Y = 0) = 0$$

For an ideal uncharged fluid with Bjorken initial conditions:

Eur. Phys. J C 75 (2015) 46

$$\varpi_{\mu\nu} = \frac{1}{T} (A_\mu u_\nu - A_\nu u_\mu)$$

$$S^\mu(p) = -\frac{1}{4m} \epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma d\Sigma_\lambda p^\lambda A_\rho \beta_\sigma n_F (1 - n_F)}{\int_\Sigma d\Sigma_\lambda p^\lambda n_F} = \frac{1}{4mT} \epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\frac{\partial}{\partial p^\sigma} \int_\Sigma d\Sigma_\lambda p^\lambda n_F \partial_\rho T}{\int_\Sigma d\Sigma_\lambda p^\lambda n_F}$$

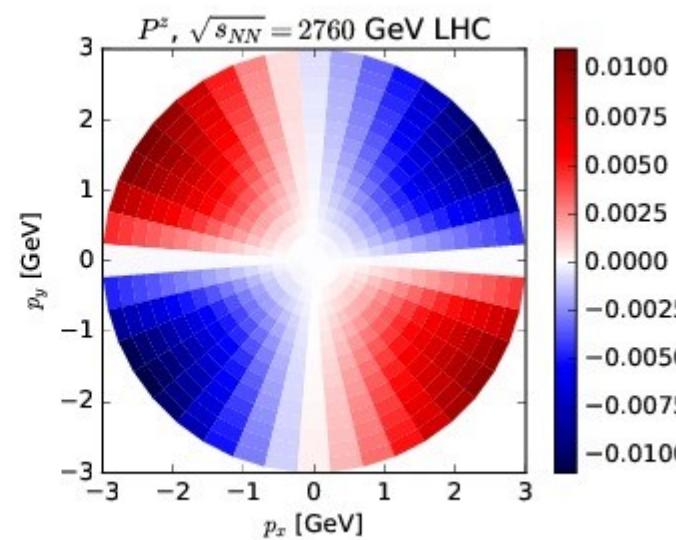
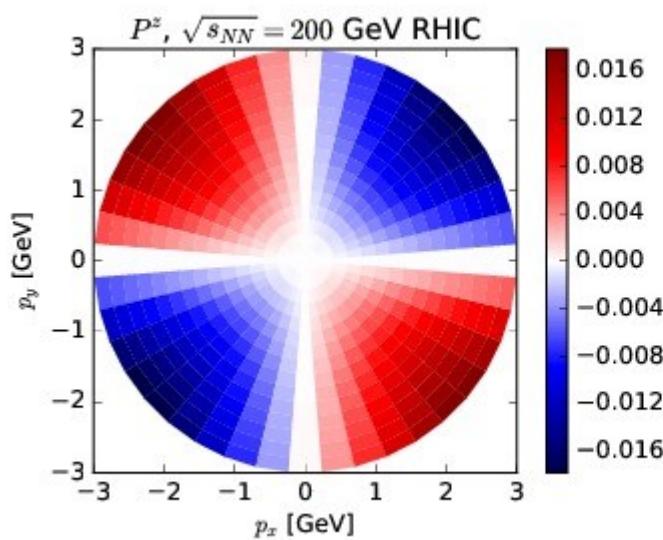
In the simple case of  $T=T(\tau)$  (Bjorken isocronous f.o.)

$$\begin{aligned} S^z(\mathbf{p}_T, Y = 0) &= -\frac{dT/d\tau}{4mT} \frac{\partial}{\partial \varphi} 2v_2(p_T) \cos 2\varphi \\ &= \frac{dT}{d\tau} \frac{1}{mT} v_2(p_T) \sin 2\varphi \end{aligned}$$



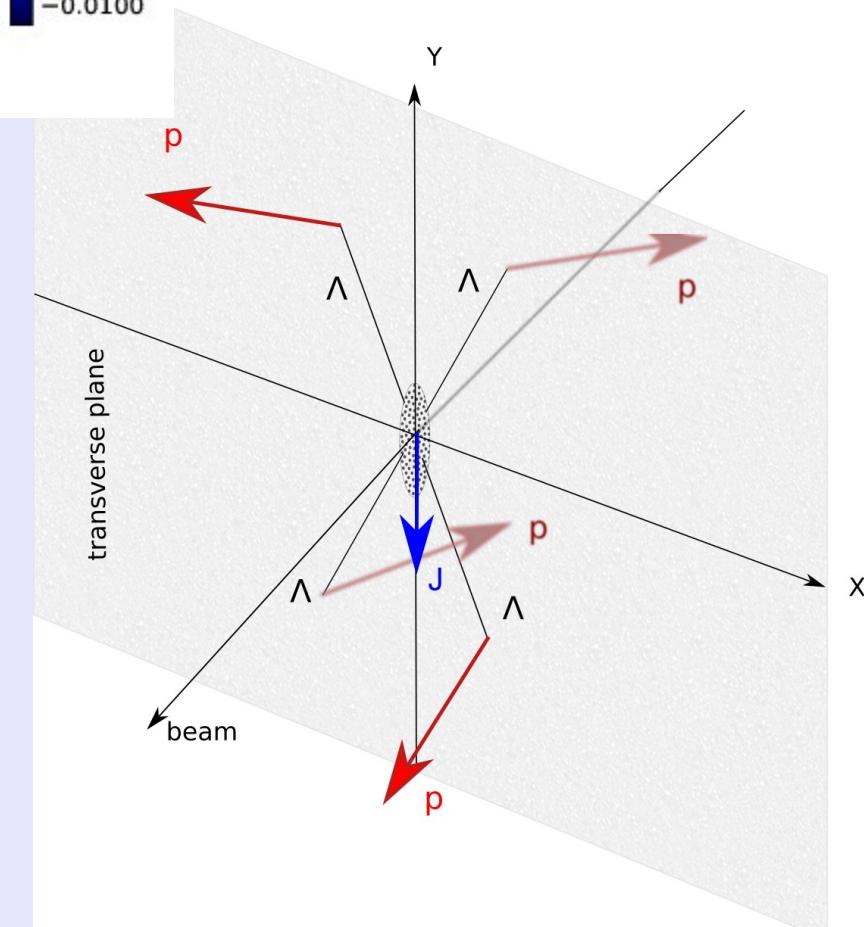
$$f_2(p_T) = 2 \frac{dT}{d\tau} \frac{1}{mT} v_2(p_T)$$

# Predictions at RHIC and LHC



Hydro calculations  
(see I. Karpenko's talk)

$f_2$  can be measured by studying the sign of the longitudinal component of the decay proton momentum as a function of the angular distance from the reaction plane



# Open theoretical problems

$$S^\mu(p) \simeq \frac{1}{8m} \epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int_{\Sigma} d\Sigma_\tau p^\tau n_F (1 - n_F) \partial_\nu \beta_\rho}{\int_{\Sigma} d\Sigma_\tau p^\tau n_F}$$

- Do we really need a spin tensor to get this formula?

Hydrodynamics should be extended: D. Montenegro et al., arXiv:1701.08263,  
W. Florkowski et al., arXiv:1705.00587

Subtle and profound theoretical issue: can the spin tensor be measured?

- Relation between anomaly and polarization

Chiral Axial Effect     $j_A^\mu \simeq \left( \frac{\mu^2}{2\pi^2} + \frac{T^2}{6} \right) \omega^\mu$     A. Sorin and O. Teryaev, Phys. Rev. C 95 (2017) 011902

The axial current is dual to the spin tensor in the Dirac theory, so an axial current implies polarization.  
But is this relation really anomalous? Recovered for a free Dirac field (M. Buzzegoli et al., arXiv:1704.02808)

- What is the exact expression of spin vector at global equilibrium with rotation?

A real theoretical challenge

# Covariant Wigner function for the free Dirac field

$$\begin{aligned} W(x, k)_{AB} &= -\frac{1}{(2\pi)^4} \int d^4y e^{-ik\cdot y} \langle : \Psi_A(x - y/2) \bar{\Psi}_B(x + y/2) : \rangle \\ &= \frac{1}{(2\pi)^4} \int d^4y e^{-ik\cdot y} \langle : \bar{\Psi}_B(x + y/2) \Psi_A(x - y/2) : \rangle \end{aligned}$$

$$(m - \not{k} - \frac{i}{2} \not{\partial}) W(x, k) = 0$$

1 – On mass shell (*De Groot et al., Relativistic kinetic theory*)

$$\begin{aligned} W^+(x, k) &\equiv \theta(k^0) W(x, k) = \frac{1}{2} \int \frac{d^3p}{\varepsilon} \delta^4(k - p) \sum_{r,s} u_r(p) f_{rs}(x, p) \bar{u}_s(p) \\ W^- (x, k) &\equiv \theta(-k^0) W(x, k) = -\frac{1}{2} \int \frac{d^3p}{\varepsilon} \delta^4(k + p) \sum_{r,s} v_s(p) \bar{f}_{rs}(x, p) \bar{v}_r(p) \end{aligned}$$

2 – Ansatz to the quantum statistics extension of the ideal Boltzmann relativistic gas with spin  
(F. B., V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338 (2013) 32)

$$\begin{aligned} f(x, p) &= \frac{1}{2m} \bar{U}(p) \left( \exp[\beta(x) \cdot p - \xi(x)] \exp[-\frac{1}{2} \varpi(x) : \Sigma] + I \right)^{-1} U(p) \\ \bar{f}(x, p) &= -\frac{1}{2m} (\bar{V}(p) \left( \exp[\beta(x) \cdot p + \xi(x)] \exp[\frac{1}{2} \varpi(x) : \Sigma] + I \right)^{-1} V(p))^T \end{aligned}$$

Still, the above  $W(x, k)$  is not an exact solution of the free spinor Wigner equation

# Summary and outlook

- Evidence for particle polarization in relativistic heavy ion collisions in agreement with predictions of relativistic hydrodynamics and local thermodynamic equilibrium. Polarization driven by acceleration, vorticity and temperature gradients.
- Evidence for a 1st order quantum effect in hydrodynamics
- Longitudinal polarization survives minimal vorticity scenarios and should be measurable at very high energy providing information on the gradients of temperature at hadronization
- Intriguing theoretical problems related to the quantum field theoretical foundation of relativistic hydrodynamics

# $\Lambda$ polarization correlations

L. G. Pang, H. Petersen, Q. Wang, X.N. Wang,  
arXiv:1605.04024 [hep-ph].

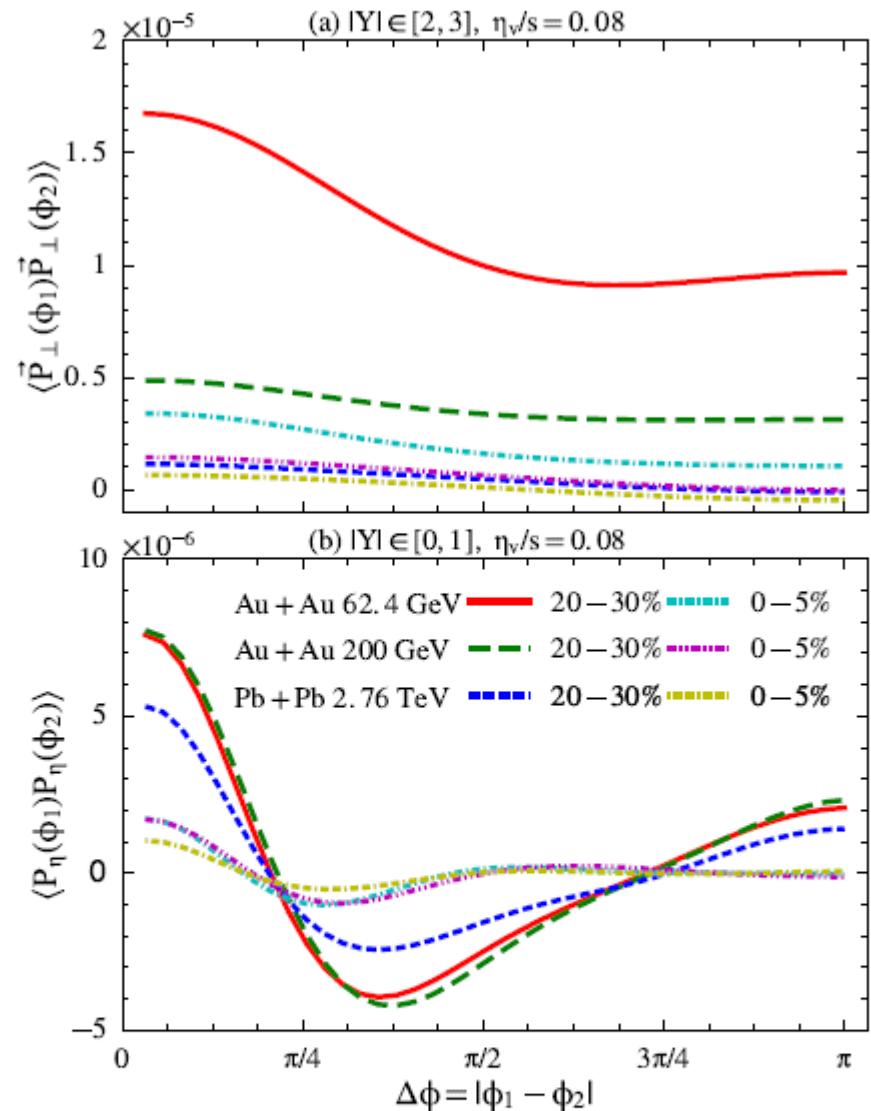


Figure 3: (color online) (a) Transverse ( $|Y| \in [2, 3]$ ) and (b) longitudinal ( $|Y| \in [0, 1]$ ) spin correlation of two  $\Lambda$ 's as a function of the azimuthal angle difference (of their momenta) in semi-peripheral (20-30%) and central (0-5%) Au+Au collisions at  $\sqrt{s_{NN}} = 62.4, 200$  GeV and Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV with  $\eta_v/s = 0.08$ .

$$\mathbf{S}^*_{\rm daughter}=C\mathbf{S}^*_{\rm parent}$$

$$C=\sum_{\lambda_A,\lambda_B,\lambda'_A}T^J(\lambda_A,\lambda_B)T^J(\lambda'_A,\lambda_B)^*\sum_{n=-1}^1\langle\lambda'_A|\widehat S_{A,-n}|\lambda_A\rangle\\ \times\frac{c_n}{\sqrt{J(J+1)}}\langle J\lambda|J1|\lambda'n\rangle\left(\sum_{\lambda_A,\lambda_B}|T^J(\lambda_A,\lambda_B)|^2\right)^{-1}$$

F. B., I. Karpenko, M. Lisa, I. Upsal, S. Voloshin,  
 Phys. Rev. C 95 054902 (2017)

# Polarization is induced by acceleration and vorticity

The thermal vorticity

$$\varpi_{\nu\mu} = -\frac{1}{2}(\partial_\nu\beta_\mu - \partial_\mu\beta_\nu)$$

It can be readily shown that at *global* equilibrium ( $\beta$  = Killing vector) the thermal vorticity has the following decomposition

$$\varpi^{\mu\nu} = \frac{1}{T} (a^\mu u^\nu - a^\nu u^\mu + \varepsilon^{\mu\nu\rho\sigma} \omega_\rho u_\sigma)$$

where  $T$  is the local proper temperature and:

$$a^\mu = u^\nu \partial_\nu u^\mu \quad \omega^\mu = -\frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \partial_\nu u_\rho u_\sigma$$

are the properly called *acceleration* and *vorticity*.

In general, in *local* thermodynamical equilibrium, there are terms involving gradients of  $T$

# Global quantum-relativistic equilibrium

General covariant expression of an equilibrium density operator

$$\hat{\rho} = \frac{1}{Z} \exp \left[ - \int_{\Sigma} d\Sigma_{\mu} \left( \hat{T}^{\mu\nu} \beta_{\nu} - \zeta \hat{j}^{\mu} \right) \right]$$

Zubarev 1979  
Weert 1982

Obtained by maximizing the entropy  $S = -\text{tr}(\hat{\rho} \log \hat{\rho})$  with respect to  $\hat{\rho}$  with the constraints of fixed energy, momentum and charge density.

Global equilibrium requires:

$$\partial_{\mu} \beta_{\nu} + \partial_{\nu} \beta_{\mu} = 0$$

$$\partial_{\mu} \zeta = 0$$

Killing equation

Solution of the Killing equation in Minkowski spacetime:

$$\beta^{\nu} = b^{\nu} + \varpi^{\nu\mu} x_{\mu}$$

Density operator becomes:  
constants

$$\hat{\rho} = \frac{1}{Z} \exp \left[ -b_{\mu} \hat{P}^{\mu} + \frac{1}{2} \varpi_{\mu\nu} \hat{J}^{\mu\nu} + \zeta \hat{Q} \right]$$

# Introduction: rotation and polarization

Take an ideal gas in a rigidly rotating vessel. At thermodynamical equilibrium (Landau) the gas will also have a velocity field

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{x}$$

For a *comoving* observer the equilibrium particle distribution function will be given by:

WARNING The potential term has a + sign as it stems from both centrifugal and Coriolis potentials

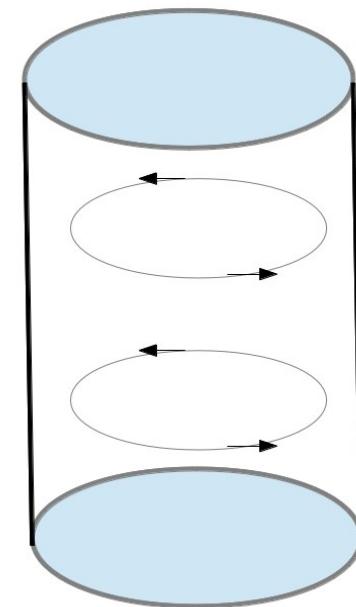


$$f(\mathbf{x}, \mathbf{p}) \propto \exp[-\mathbf{p}'^2/2mT + m(\boldsymbol{\omega} \times \mathbf{x}')^2/2T]$$

If we transform this back to the (primed) inertial observer

$$\mathbf{v} = \mathbf{v}' + \boldsymbol{\omega} \times \mathbf{x} \quad \mathbf{x} = \mathbf{x}'$$

$$\begin{aligned} f(\mathbf{x}, \mathbf{p}) &\propto \exp[-\mathbf{p}^2/2mT + \mathbf{p} \cdot (\boldsymbol{\omega} \times \mathbf{x})/T] \\ &= \exp[-\mathbf{p}^2/2mT + \boldsymbol{\omega} \cdot \mathbf{L}/T] \end{aligned}$$



It seems quite *natural* to extend this to particle with spin

$$f(\mathbf{x}, \mathbf{p}, \mathbf{S}) \propto \exp[-\mathbf{p}^2/2mT + \boldsymbol{\omega} \cdot (\mathbf{L} + \mathbf{S})/T]$$

Which implies that particles (and antiparticles) are *POLARIZED*, in a rotating ideal gas, along the direction of the angular velocity vector by an amount

$$P \simeq \frac{S+1}{3} \frac{\hbar\omega}{KT}$$

A usually tiny polarization

**WARNING:** The above formula states that the polarization states are unevenly populated in a rotating gas at thermodynamical equilibrium. It does not imply that there is a direct dynamical coupling between spin and rotation (see Mashhoon, Phys. Rev. Lett. 1988). In other words, the above formula does not imply that – for whatever reason - the hamiltonian of the inertial observer contains a term  $\boldsymbol{\omega} \cdot \mathbf{S}$  which would make the spin precessing if particle is accelerated (*not to be confused with Thomas precession, which is a purely relativistic effect*).

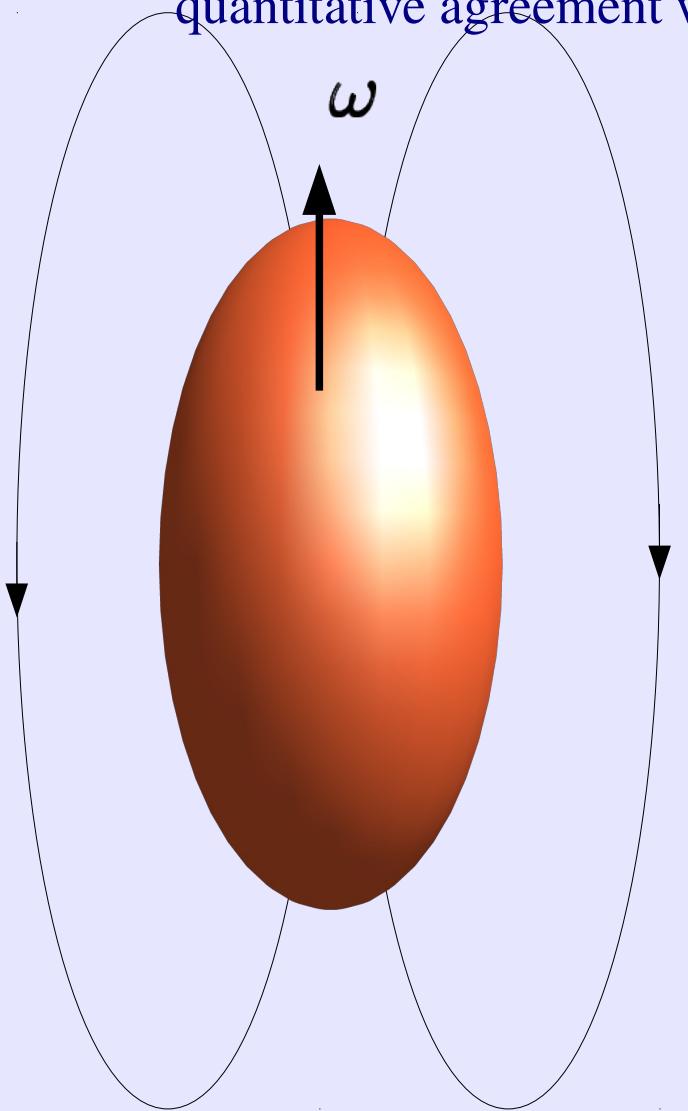
# Barnett effect

S. J. Barnett, Magnetization by Rotation, Phys. Rev.. 6, 239–270 (1915).

Spontaneous magnetization of an uncharged body when spun around its axis, in quantitative agreement with the previous polarization formula

$$M = \frac{\chi}{g} \omega$$

It is a dissipative transformation of the orbital angular momentum into spin of the constituents. The angular velocity decreases and a small magnetic field appears; this phenomenon is accompanied by a heating of the sample. Requires a spin-orbit coupling.



# Converse: Einstein-De Haas effect

*the only Einstein's non-gedanken experiment*

A. Einstein, W. J. de Haas, Koninklijke Akademie van Wetenschappen te Amsterdam, Proceedings, 18 I, 696-711 (1915)



Rotation of a ferromagnet originally at rest  
when put into an external H field

An effect of angular momentum  
conservation:  
spins get aligned with H (irreversibly) and  
this must be compensated by a on overall  
orbital angular momentum

# Ansatz for the LTE distribution function with FD statistics

The explicit calculation of  $W(x, k)$  and the extraction of  $f$  in the most general case is difficult. One can make a reasonable ansatz extending previous special cases.

The general solution must:

-  reduce to the global equilibrium solution with rotation in the Boltzmann limit
-  reduce to the known Fermi-Juttner or Bose-Juttner formulae at the LTE in the non-rotating case

$$f(x, p) = \frac{1}{2m} \bar{U}(p) \left( \exp[\beta(x) \cdot p - \xi(x)] \exp[-\frac{1}{2} \varpi(x) : \Sigma] + I \right)^{-1} U(p)$$
$$\bar{f}(x, p) = -\frac{1}{2m} (\bar{V}(p) \left( \exp[\beta(x) \cdot p + \xi(x)] \exp[\frac{1}{2} \varpi(x) : \Sigma] + I \right)^{-1} V(p))^T$$

$U, V$  4x2 Dirac spinors and  $\Sigma$  the generators of the Lorentz transformation in the fundamental representation

$$\Sigma^{\mu\nu} = \frac{i}{4} [\gamma^\mu, \gamma^\nu]$$

# Single particle distribution function at global thermodynamical equilibrium with rotation

In the Boltzmann limit, for an ideal relativistic gas, this calculation can be done without quantum field theory, just with quantum statistical mechanics and group theory (F. B., L. Tinti, Ann. Phys. 325, 1566 (2010)).

Spelled out: maximal entropy principle (equipartition), angular momentum conservation and Lorentz group representation theory.

$$f(x, p)_{rs} = e^{\xi} e^{-\beta \cdot p} \frac{1}{2} (D^S([p]^{-1} R_{\hat{\omega}}(i\omega/T)[p]) + D^S([p]^{\dagger} R_{\hat{\omega}}(i\omega/T)[p]^{\dagger -1}))_{rs}$$

$R_{\hat{\omega}}(i\omega/T) = \exp[D^S(J_3)\omega/T]$  = SL(2,C) matrix representing a rotation around  $\hat{\omega}$  axis ( $z$  or 3) by an imaginary angle  $i\omega/T$ .

Particle density in phase space

$$\text{tr}_{2S+1} f = e^{\xi} e^{-\beta \cdot p} \text{tr}_{2S+1} R_{\hat{\omega}}(i\omega/T) = e^{\xi} e^{-\beta \cdot p} \sum_{\sigma=-S}^S e^{-\sigma\omega/T} \equiv e^{\xi} e^{-\beta \cdot p} \chi\left(\frac{\omega}{T}\right)$$

As a consequence, particles with spin get polarized in a rotating gas

$$\Pi_0 = \frac{\sum_{n=-S}^S n e^{n\omega/T}}{\sum_{n=-S}^S e^{n\omega/T}} \left[ \frac{\varepsilon}{m} \hat{\omega} - \frac{\hat{\omega} \cdot \mathbf{p} \mathbf{p}}{m(\varepsilon + m)} \right]$$

F. B., F. Piccinini, Ann. Phys. 323 (2008)

