Study of non-extensive parameters in transverse momentum spectra of hadrons

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Tsallis' Entropy

In 1988 C. Tsallis **[1]** proposed the non-extensive entropy formula,

$$S_q = \frac{\sum_{i=1}^{W} p_i^q - 1}{1-q} := -\sum_{i=1}^{W} p_i^q \ln_q p_i \tag{1}$$

where $q > 0$ is the so called Tsallis q parameter. Here we introduce the deformed q-exponential function $e_q^x \equiv [1+(1-q)x]^{\frac{1}{1-q}}$ and its inverse function $\ln_q(x) \equiv \frac{x^{1-q}-1}{1-q}$ $(x > 0)$. This measures the non-extensivity by the deviance of the $q = 1$, the Boltzmann−Gibbs case.

Tsallis-Pareto distribution

We use the OLM (Optimal Lagrange Multipliers)**[2]**-Tsallis technique to give the generalized q-equilibrium probability distribution function, namely, the Tsallis−Pareto distribution,

$$p_i = \frac{1}{Z_q} e_q^{-\beta\omega_i}, \tag{2}$$

with parameters q and $\beta = 1/T$, where T is temperature-like quantity.

Applications for p_T spectra measured in high-energy collisions

One of the most important applications is the non-extensive effects on the transverse momentum (p_T) spectra of both strange and non-strange particles in high-energy collisions, where it shows a power-law tail for the large p_T range.

$$\left.\frac{d^2N}{N_{ev} 2\pi p_T dp_T dy}\right|_{y=0} \sim [1-(1-q)\frac{m_T - m}{T}]^{\frac{1}{1-q}} \tag{3}$$

where $m_T = \sqrt{p_T^2 + m^2}$ is the transverse mass.

We have already used this formula to fit and understand non-extensive statistics of the identified charged hadron production in small systems, such as electron-positron or proton-proton collisions. (See Ref. [4] and references therein). Here, we extend our investigations to the latest identified charged hadrons spectra within different centralities in $p + Pb$ collisions at $\sqrt{s} = 5.02$ TeV LHC energy. With respect to the mass dependence and strangeness dependence as well, the corresponding fitting parameters are well analysed and compared to spectra in small colliding systems.

Connection of the non-extensiv statistics to Hagedorn gas in HIC

About 30 years ago, R. Hagedorn proposed the QCD inspired empirical formula to describe experimental hadron production data:

$$E\frac{d^3\sigma}{d^3p} = C(1+\frac{p_T}{p_0})^{-n} \rightarrow \begin{cases} \exp(-\frac{np_T}{p_0}) & p_T \rightarrow 0 \\ (\frac{p_0}{p_T})^n & p_T \rightarrow \infty \end{cases} \tag{4}$$

which coincides with the above derived Tsallis−Pareto distribution taking the low momentum exponential and high momentum power-law tailed being of the measured spectra.

$$h_q(p_T) = C_q e_q^{-\beta p_T} = C_q[1-(1-q)\frac{p_T}{T}]^{\frac{1}{1-q}} \tag{5}$$

for $n = 1/(q-1)$ and $p_0 = nT$, which connects the parameters of the distribution to the QCD-like high-p_T power-law tail ($\sim p_T^{-n}$)and to the well-known exponential-like Hagedorn-gas temperature (T), respectively. It can also be seen again, that in case of $q \rightarrow 1$ the Boltzmann−Gibbs case is obtained.

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Method: the analysis of identified hadron spectra

We analysed identified hadron spectra measured by ALICE [3] at mid-rapidity in a wide energy range within different centrality regions. Seen as below, data are well described by the non-extensive parameters q and T.

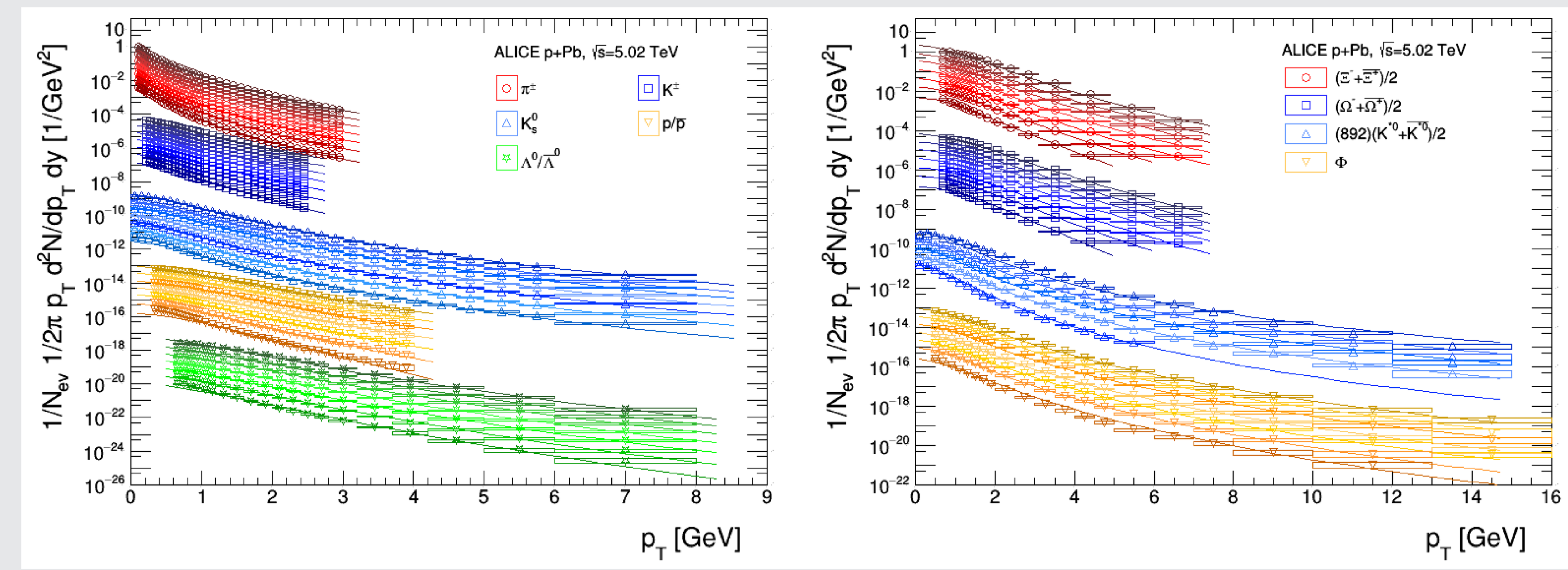
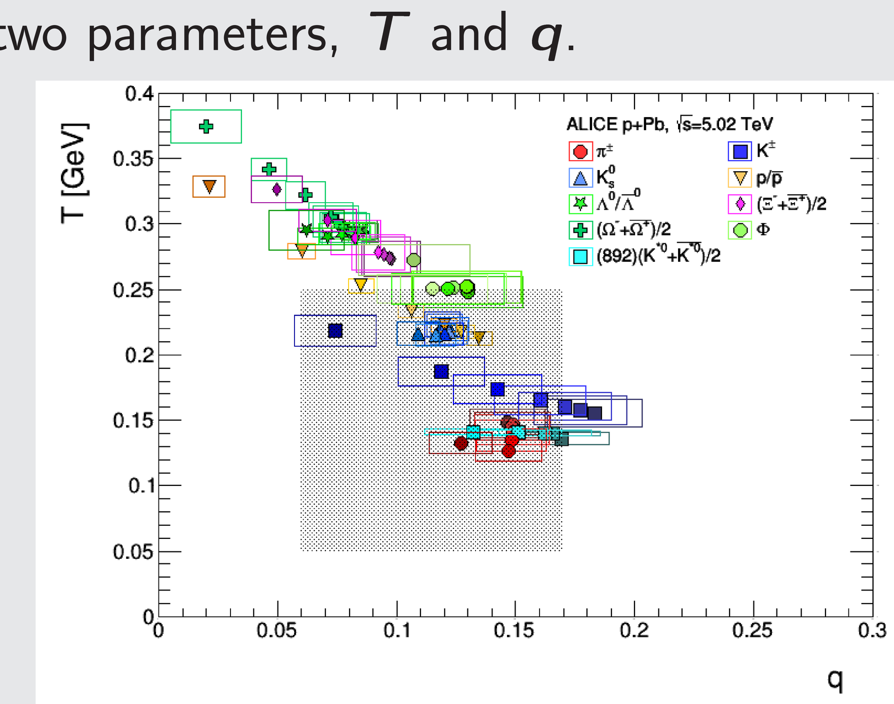


Figure 1: Fittings of the transverse momentum p_T spectra of different kinds of hadrons within the non-extensive effects for $p + Pb$ collisions at $\sqrt{s} = 5.02\,TeV$ [3].

Results: The $q - T$ parameter space for $p + Pb$ collisions

Based on the spectra fits above, we can then investigate the connection between these two parameters, T and q.



The observed $T = T_0 + C(q-1)$, with $C < 0$ means q decreases with T increasing. This proves the non-extensive parameter q indeed describes the differences of system from the thermal equilibrium statistics. Note, for high-multiplicity (central) collisions, this differs from parameter values of small systems [4]. (*See points out of the compact shaded box*).

Results: mass dependence of the parameters

Consider the fit parameters: a hadron mass hierarchy is clearly seen for parameter, T. This corresponds to our earlier result in Ref. [4], but slightly weaken at large centralities. However, the non-extensivity parameter, q decreases with the increasing hadron mass, especially for central collisions.

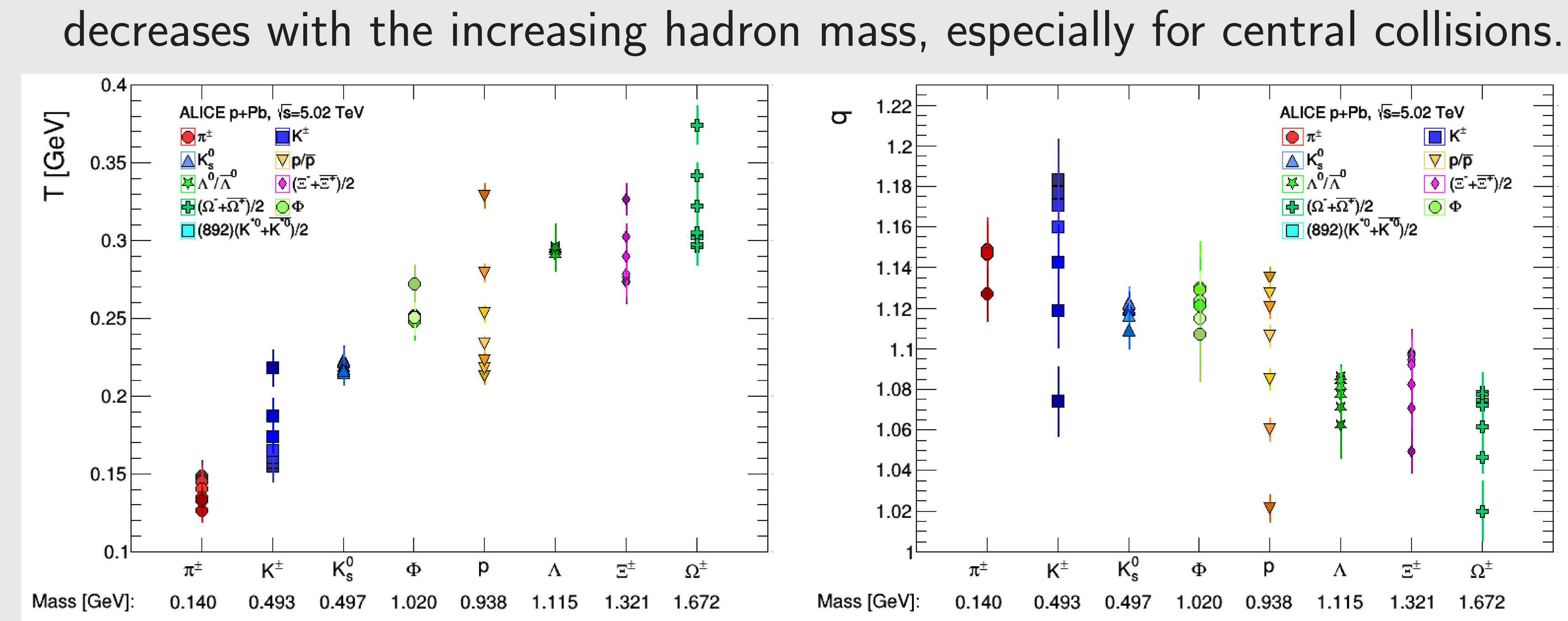


Figure 2: Fit parameters, T and q, for the p_T spectra with respect to the mass of hadron species for $p + Pb$ collisions at $\sqrt{s} = 5.02\,TeV$.

References

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