Estimation of transport coefficients in an anisotropic QGP using quasiparticle model

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Motivation

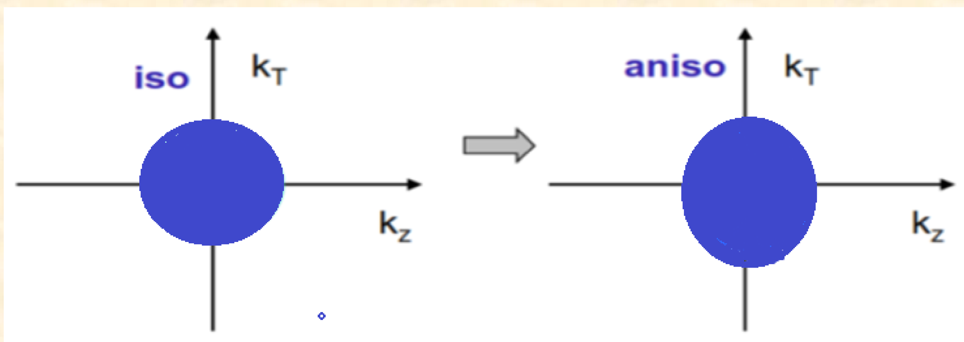
- Transport coefficients are used to quantify the properties of strongly interacting matter created at relativistic heavy-ion collisions.
- The system created after collision is a non- equilibrium system for a brief span of time due to fluctuations or external fields.
- The response of the system to such type of fluctuations or external fields is described by transport coefficients e.g. the shear and bulk viscosities, electrical conductivity etc
- The momentum anisotropy produced in the medium created in heavy-ion collisions lasts for at least $\tau \leq 2$ fm/c . Thus it is of the utmost importance to study the effect of momentum anisotropy on the transport properties.

Momentum Anisotropy in QGP

- At the early stage of ultra relativistic heavy ion collisions at RHIC or LHC, the generated parton system has an anisotropic contribution.

$$f_{aniso}(\vec{k}) = f_{iso}\left(\sqrt{k^2+\xi(\vec{k}.\hat{n})^2}\right), \qquad \xi = \frac{1}{2}\frac{\langle k_\perp^2\rangle}{\langle k_z^2\rangle} - 1$$

- The parton momentum distribution is slightly elongated along the beam direction



Shear Viscosity

Relativistic Boltzmann transport equation in Relaxation time approximation

$$k^\mu\partial_\mu f(x,k) = -\frac{k^\mu u_\mu}{\tau_f}\delta f.$$

$$k^\mu\partial_\mu \bar{f}(x,k) = -\frac{k^\mu u_\mu}{\tau_{\bar{f}}}\delta \bar{f}$$

$$k^\mu\partial_\mu b(x,k) = -\frac{k^\mu u_\mu}{\tau_b}\delta b$$

Shear viscosity for isotropic case

$$\eta_{iso} = \frac{1}{15T}\int\frac{d^3k}{(2\pi)^3}\frac{k^4}{E^2}\{2g_f\tau_f f^0(1-f^0)+g_b\tau_b b^0(1+b^0)\}$$

where $f^0(x,\mathbf{k};T) = \frac{1}{e^{\sqrt{(\mathbf{k}^2+m^2)}/T}+1}$, and $b^0(x,\mathbf{k};T) = \frac{1}{e^{\sqrt{(\mathbf{k}^2+m^2)}/T}-1}$

Distribution function in the presence of momentum anisotropy

$$f_{\rm aniso}(x,\mathbf{k};T) = \frac{1}{e^{(\sqrt{\mathbf{k}^2+\xi(\mathbf{k}.\mathbf{n})^2+m^2})/T}+1}$$

$$b_{\rm aniso}(x,\mathbf{k};T) = \frac{1}{e^{(\sqrt{\mathbf{k}^2+\xi(\mathbf{k}.\mathbf{n})^2+m^2})/T}-1}$$

For weakly anisotropic system (ξ <<1)

$$f_{\rm aniso}(x,\mathbf{k};T) = f^0 - \frac{\xi}{2E_fT}e^{E_f/T}f^{02}(\mathbf{k}\cdot\mathbf{n})^2$$

where k ≡ (k sinθcosφ, k sinθsinφ, k cosθ) and n ≡ (sinα, 0, cosα). n is the direction of anisotropy.

Shear viscosity in the presence of momentum anisotropy (ξ<<1)

$$\eta_{\rm aniso} = \frac{g_f\tau_f}{15T\pi^2}\int dk\frac{k^6}{E_f^2}\{f^0(1-f^0)\} + \frac{g_b\tau_b}{30T\pi^2}\int dk\frac{k^6}{E_b^2}\{b^0(1+b^0)\}$$
$$-\frac{g_f\tau_f}{45T\pi^2}\xi\int dk\frac{k^8}{E_f^2}\left\{f^0(1-f^0)\frac{1}{2E_fT}-\frac{(f^0)^2}{E_fT}\right\} - \frac{g_b\tau_b}{90T\pi^2}\xi$$
$$\times\int dk\frac{k^8}{E_b^2}\left\{b^0(1+b^0)\frac{1}{2E_bT}+\frac{(b^0)^2}{E_bT}\right\}$$

Entropy Density

Entropy density for isotropic medium

$$s_{iso} = -\frac{g_f}{\pi^2}\int k^2dk\left\{(1-f_f^0)\log(1-f_f^0)+f_f^0\log f_f^0\right\}$$
$$+\frac{g_b}{2\pi^2}\int k^2dk\left\{(1+b_b^0)\log(1+b_b^0)-b_b^0\log b_b^0\right\},$$

Entropy Density in anisotropic medium

$$s_{aniso} = -\frac{g_f}{\pi^2}\int k^2dk\left\{(1-f^0)\log(1-f^0)+f^0\log f^0\right\}+\frac{g_b}{2\pi^2}\int k^2dk\{(1+b^0)\log(1+b^0)$$
$$-b^0\log b^0\} - \xi\frac{g_f}{6\pi^2E_fT}\int k^4dkf^0(1-f^0)\log\frac{(1-f^0)}{f^0} - \xi\frac{g_b}{12\pi^2E_bT}\int k^4dkb^0$$
$$\times(1+b^0)\log\frac{(1+b^0)}{b^0}.$$

Electrical Conductivity

According to Ohm's law $\mathbf{J} = \sigma_{\rm el}\mathbf{E}$ — $\sigma_{\rm el}$: Electrical conductivity

Four current
$$J^\mu = \int\frac{d^3k}{(2\pi)^3E}k^\mu\{qg_ff(x,k)-\bar{q}g_{\bar{f}}\bar{f}(x,k)\}$$

Boltzmann equation in relaxation time approximation

$$k^\mu\partial_\mu f(x,k) + qF^{\alpha\beta}k_\beta\frac{\partial}{\partial k^\alpha}f(x,k) = -\frac{k^\mu u_\mu}{\tau}\delta f.$$

After considering only the electric field component, RBT equation becomes

$$q\left(k_0\mathbf{E}\cdot\frac{\partial f^0}{\partial\mathbf{k}}+\mathbf{E}\cdot\mathbf{k}\frac{\partial f^0}{\partial k^0}\right) = -\frac{k^0}{\tau}\delta f.$$

For isotropic medium

$$\sigma_{\rm el}^{\rm iso} = \frac{1}{3\pi^2T}\sum_f g_fq_f^2\int dk\frac{\mathbf{k}^4}{E_f^2}\tau_ff_f^0(1-f_f^0)$$

For anisotropic medium

$$\sigma_{\rm el}^{\rm aniso}(\mu_q=0) = \frac{1}{3\pi^2T}\sum_f g_fq_f^2\int dk\frac{\mathbf{k}^4}{E_f^2}\tau_ff^0(1-f^0)+\xi\frac{1}{6\pi^2T}\sum_f g_fq_f^2\int dk\frac{\mathbf{k}^4}{E_f^2}\tau_ff^0(1-f^0)$$
$$-\xi\frac{1}{18\pi^2T}\sum_f g_fq_f^2\int dk\frac{\mathbf{k}^6}{E_f^2}\tau_f\left[f^0(1-f^0)\left(\frac{1}{E_f^2}+\frac{1}{E_fT}\right)-\frac{2}{E_fT}(f^0)^2\right].$$

Quasiparticle Model

System of interacting massless quarks and gluons → Ideal gas of "massive" non-interacting quarks and gluons

Effective mass of quasiparticles :
$$m_f^2 = m_{0,f}^2+\sqrt{2}m_{0,f}m_{th,f}+m_{th,f}^2$$

where
$$m_{th,f}^2 = \frac{g^2(T)T^2}{6}\left(1+\frac{(\mu_f^q)^2}{\pi^2T^2}\right).$$

Relaxation times

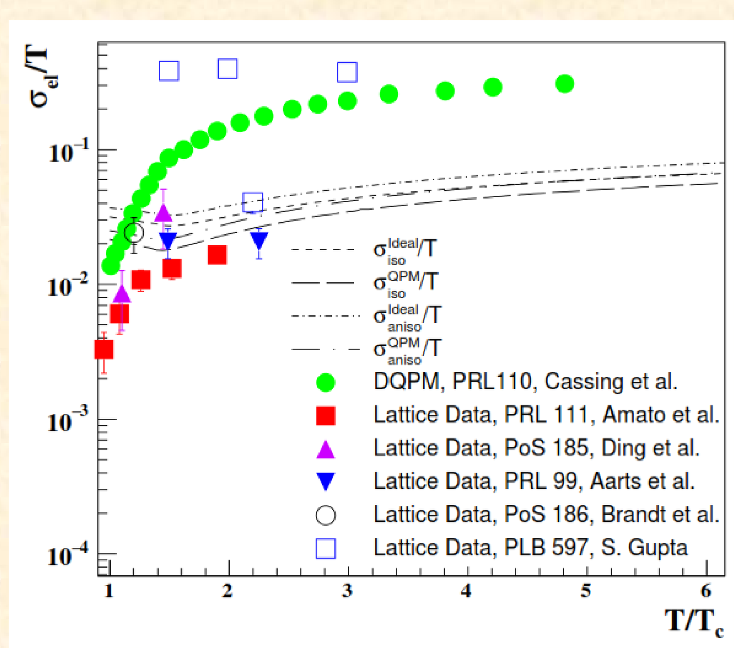
For quarks
$$\tau_{q(\bar{q})} = \frac{1}{5.1T\alpha_s^2\log\left(\frac{1}{\alpha_s}\right)(1+0.12(2N_f+1))}$$

For gluons
$$\tau_g = \frac{1}{22.5T\alpha_s^2\log\left(\frac{1}{\alpha_s}\right)(1+0.06N_f)}$$

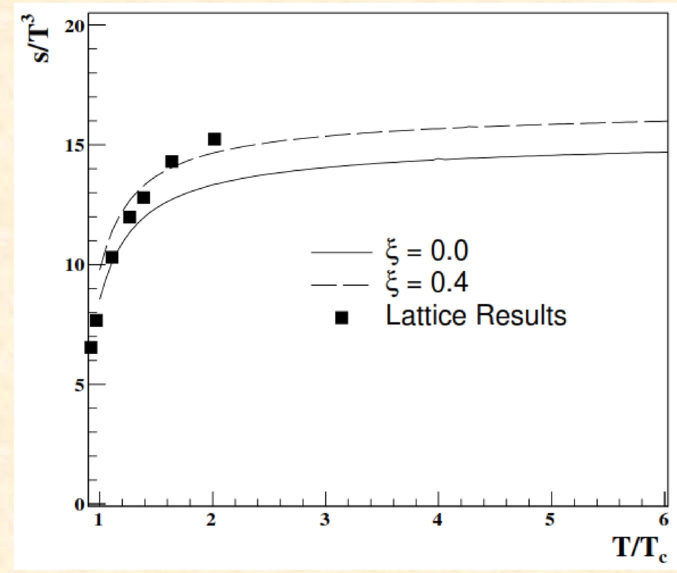
QCD running coupling constant
$$\alpha_S(T,\mu_q) = \frac{g^2(T,\mu_q)}{4\pi} = \frac{6\pi}{(33-2N_f)\ln\left(\frac{T}{\Lambda_T}\sqrt{1+a\frac{\mu_q^2}{T^2}}\right)}$$
$$\times\left(1-\frac{3(153-19N_f)}{(33-2N_f)^2}\frac{\ln\left(2\ln\frac{T}{\Lambda_T}\sqrt{1+a\frac{\mu_q^2}{T^2}}\right)}{\ln\left(\frac{T}{\Lambda_T}\sqrt{1+a\frac{\mu_q^2}{T^2}}\right)}\right),$$

Results

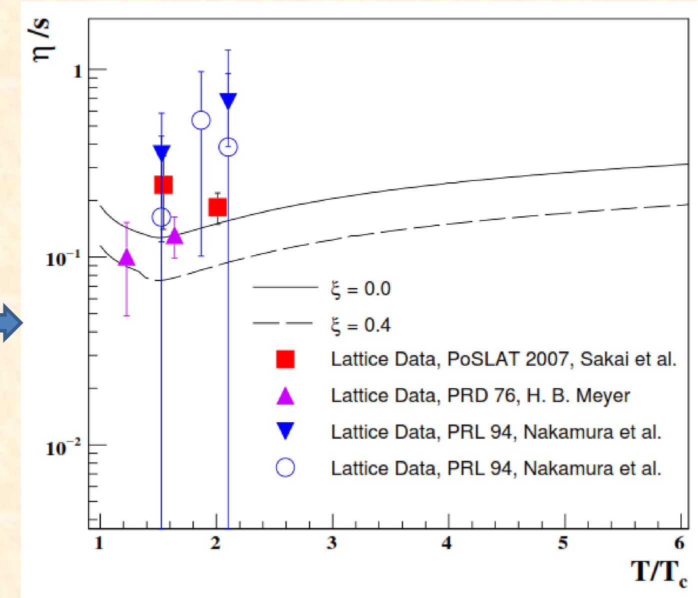
The electrical conductivity is lower with the quasiparticle model as compared to the ideal case and it increases with increase in anisotropy.



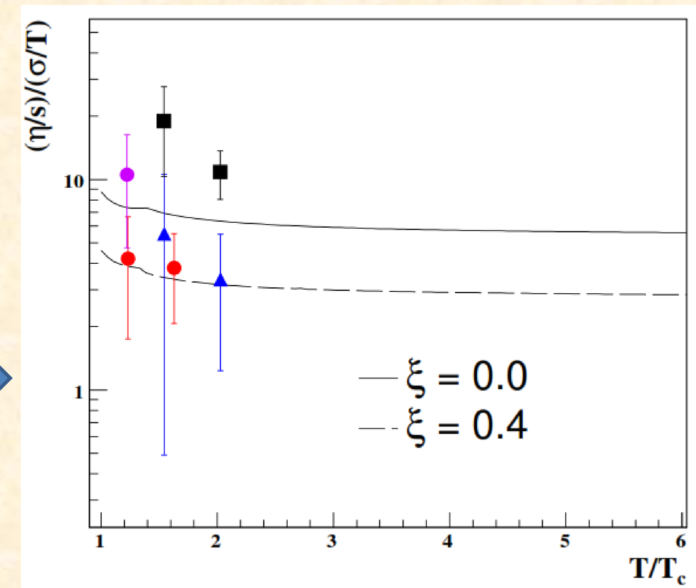
Smooth rise in entropy density in the vicinity of critical temperature



η/s decreases and then increases monotonically with increase in temperature. The ratio decreases in the presence of anisotropy.

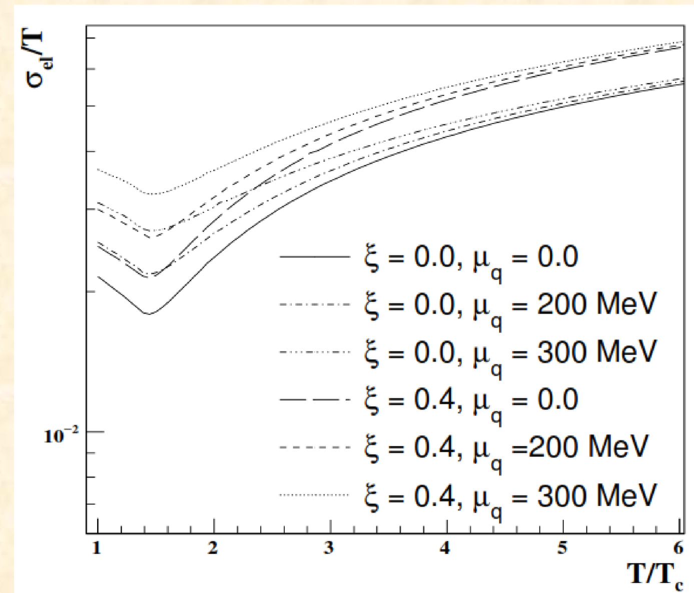


$(\eta/s)/(\sigma_{el}/T)$ starts from a large value near $T = T_c$ and then decreases with temperature and remain constant at higher temperature.

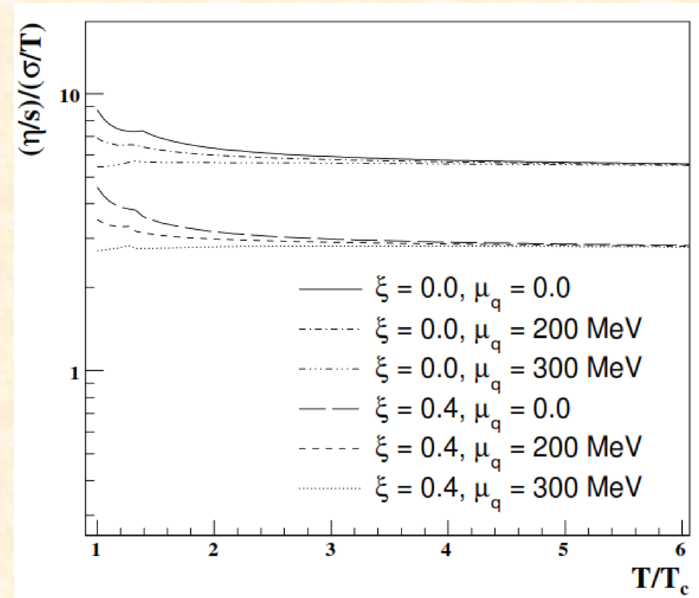


For Finite chemical potential

σ_{el}/T is large at finite μ as compared to μ=0 case and its value increases with increase in the value of μ.



The ratio $(\eta/s)/(\sigma_{el}/T)$ decreases in the presence of anisotropy and finite chemical potential.



Conclusion

- The system is electrically less conductive near critical temperature as compared to higher temperature.
- The momentum anisotropy causes the system to behave electrically more conductive.
- Smooth rise in entropy density in the vicinity of critical temperature Tc supports crossover type of transition.
- The anomalous viscosity arises due to momentum anisotropy make the system to behave as a perfect fluid.
- The gluonic contribution in total scattering cross-section is large near Tc in comparison to quark contribution.
- The effect of finite μ is more at lower temperature due to the sizable change in the distribution function of quarks at lower temperature.

References

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