fast and precise calculation of trajectories through a magnetic field

(inspired by the LHCb context)

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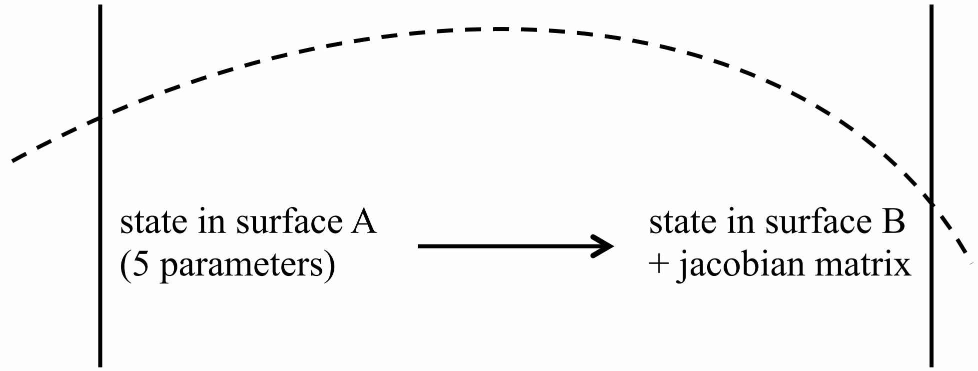
what do we need ?



wanted precision:

- pattern recognition: better than the hit separation distance
- track fit: *much better* than the combination of measurement and multiple scattering errors

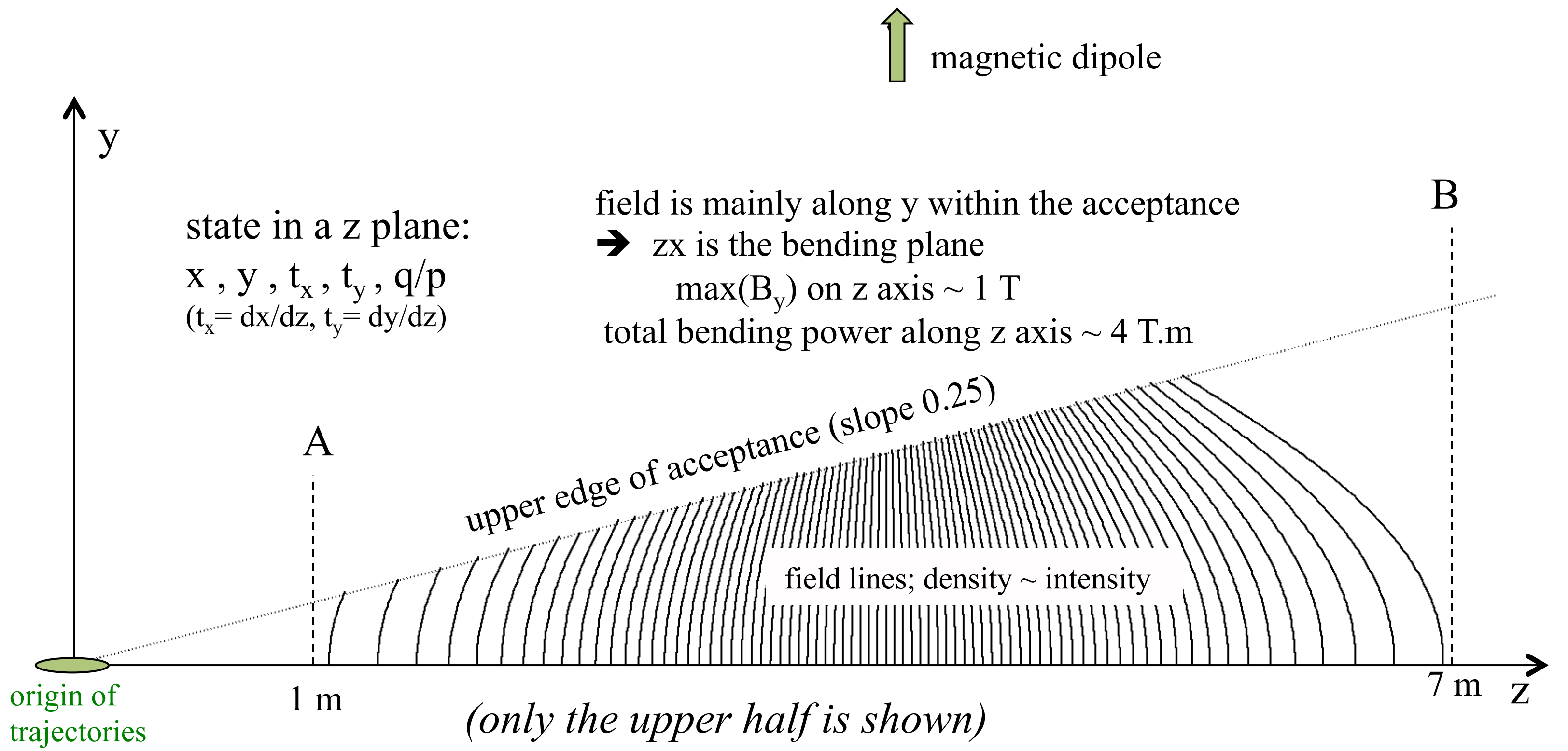
useful trajectories: from primary vertices + short lived decays + K_S/Λ decays if possible

guidelines:

- the state on surface A lies in a *small region* of the 5D-space of track parameters
- the wanted precision scales as 1/p in most cases
- in general, one needs to consider a few predefined surfaces + short range extrapolations

standard method (used as reference): stepwise propagation using the Runge-Kutta algorithm
problem: access to a field map (possibly big size) and CPU time consumption

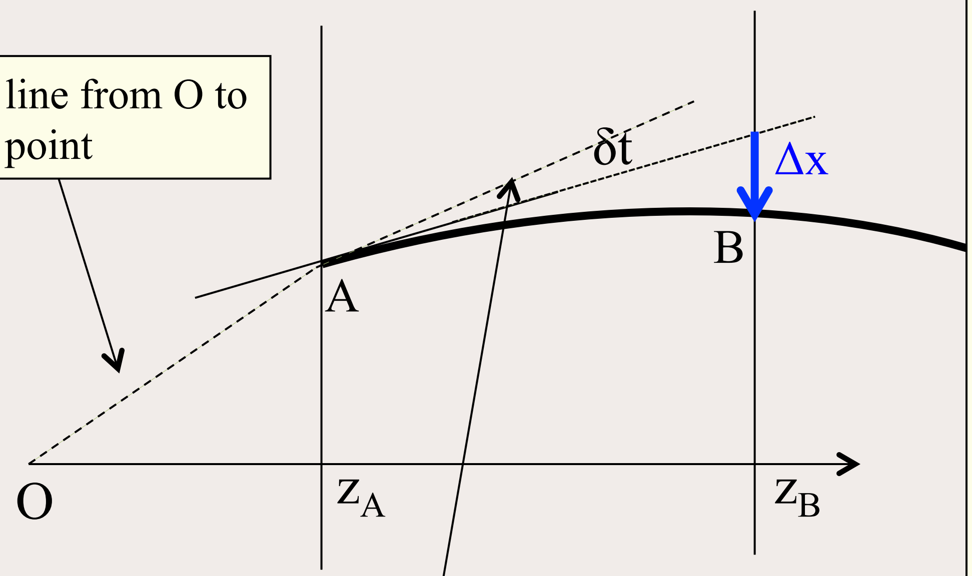
framework: forward spectrometer on a collider
(similar to LHCb)



in this study: extrapolation from the state in plane A to the state in plane B

a polynomial expansion of the deviation from the straight line

straight line from O to starting point



including the magnetic deviation from 0 to z_A as $q/p\ F(z_A)$
(F from a table)
➔ $|\delta t_x|$ and $|\delta t_x|$ are small

deviation $\Delta x = \Sigma\, C_{ijk}\, \delta t_x^{\,i}\, \delta t_y^{\,j}\, (q/p)^k$

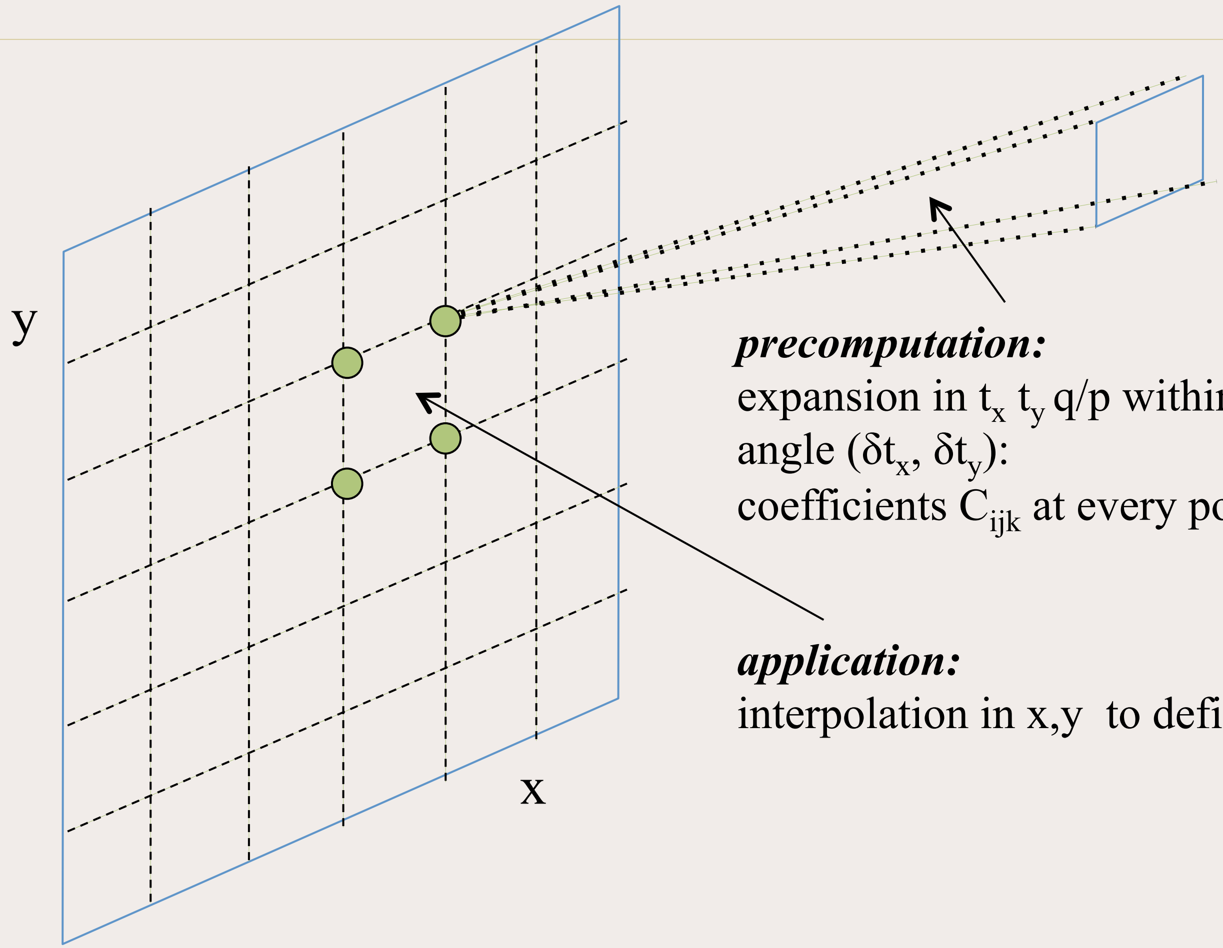
similar expressions for Δy, Δt_x, Δt_y

tabulate the coefficients as functions of x,y at z_A

+ apply global corrections in higher powers of q/p (independent of x, y, t_x, t_y)

jacobian matrix easy to obtain from this parametrization

implementation on a planar surface



precomputation:
expansion in t_x t_y q/p within a small solid angle (δt_x, δt_y):
coefficients C_{ijk} at every point of the xy grid

application:
interpolation in x,y to define C_{ijk} at any point

orders of magnitude and working conditions

typical conditions for initial plane at $z = 1$ m:
$\sigma(z_{vertex}) \sim 5$ cm ➔ $\delta t \sim 0.05 \times t$
impact parameter for K_S products ~ 2.5 cm ➔ $\delta t \sim 0.025$

domain used in initial plane to fit the coefficients

- $p > p_{min} = 3$ GeV/c (good chance to remain within the acceptance)
- $|\delta t|_{max} = 0.01$ (0.03 for K_S studies)
- $|x|$ and $|y| < 0.25$ m

computation:

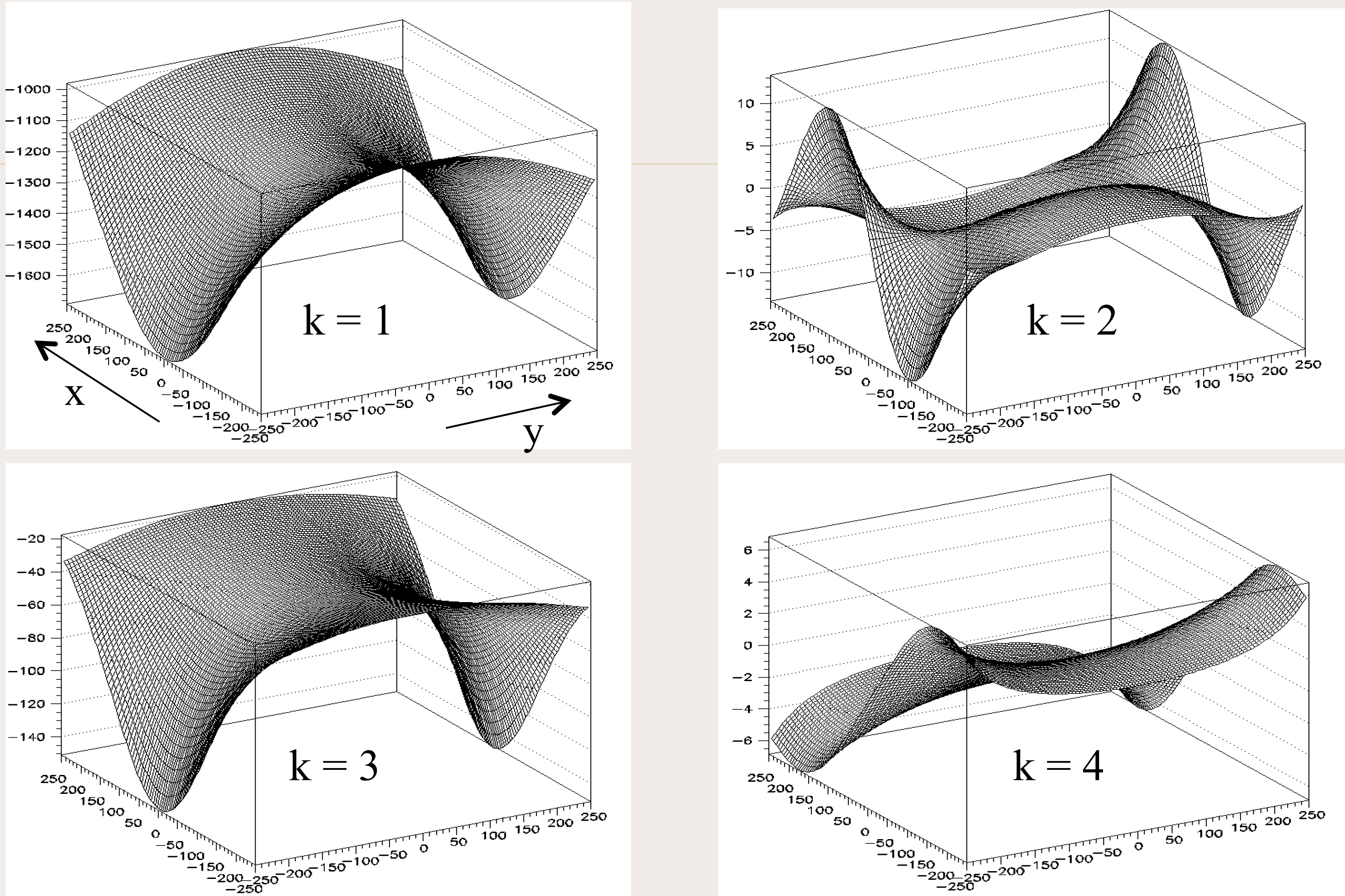
- tabulation on 100×100 or 25×25 positions in plane A within ± 0.25 m
- 20 values of q/p within $(-1/p_{min}, 1/p_{min})$
- for each one, 20×20 values of δt_x, δt_y in $(-|\delta t|_{max}, |\delta t|_{max})$
- deviations from straight line using Runge-Kutta
- l.s. fit of the coefficients up to the wanted degree
- coefficient of global dependence on q/p: fit on the test sample

test sample:

- flat distribution in q/p in $(-1/p_{min}, 1/p_{min})$
- distribution in p_t : exp $(-p_t/1$ GeV/c$)$
- vertex: $\sigma_z = 60$ mm , $\sigma_x = \sigma_y = 0.1$ mm (15 mm for K_S studies)

Remark: Δx expanded in q/p ➔ q/p expanded in Δx
(prediction of momentum with initial segment + hit assignment)

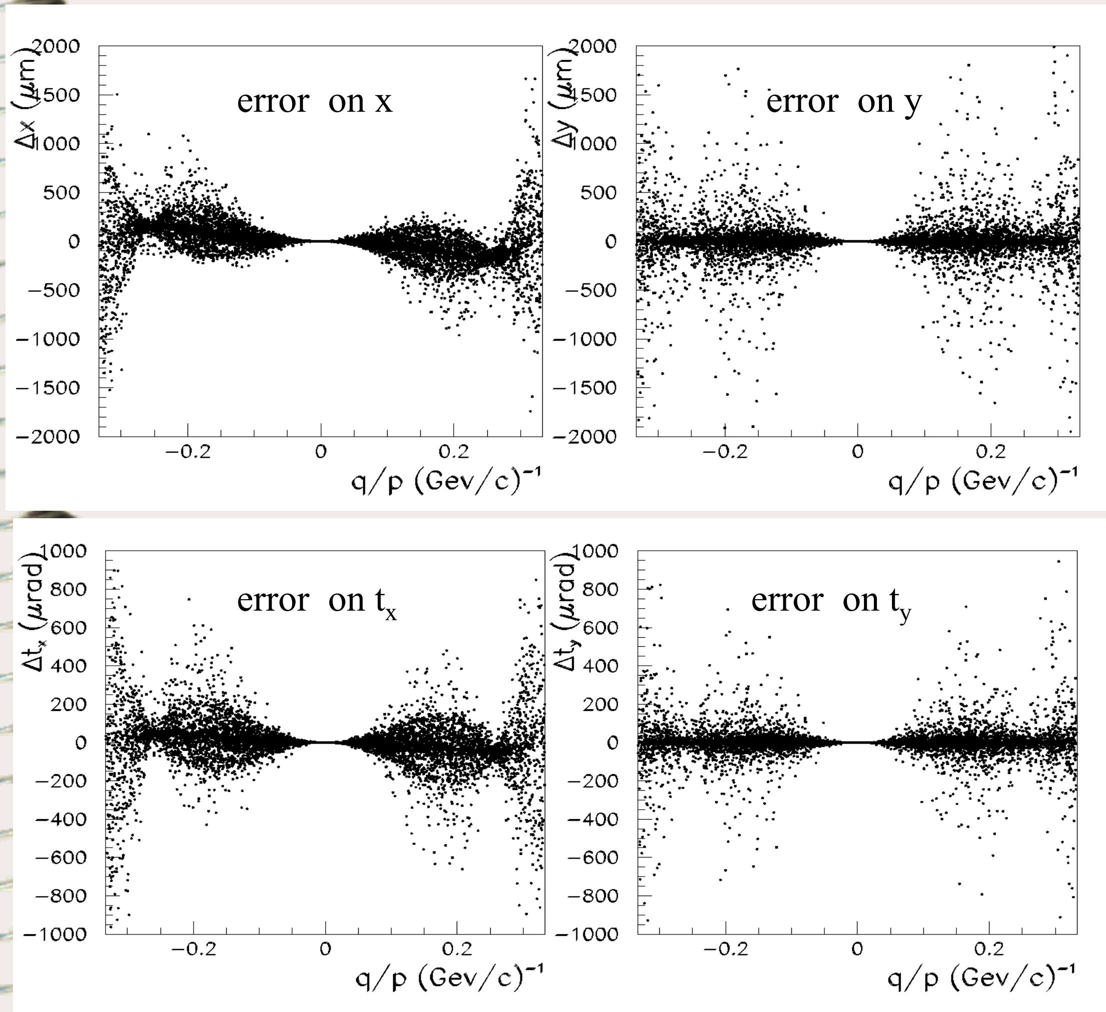
exploration of tables (here: C_{00k}/p_{min}^{k} for extrapolated x, in mm)



expectations for a submillimetric precision: we need at least k up to 4

problematic region: large y

first trial: degree 1 in $\delta t_x, \delta t_y$, 3 in q/p
100x100 tabulation in x,y (4-point interpolation) applied to the test sample

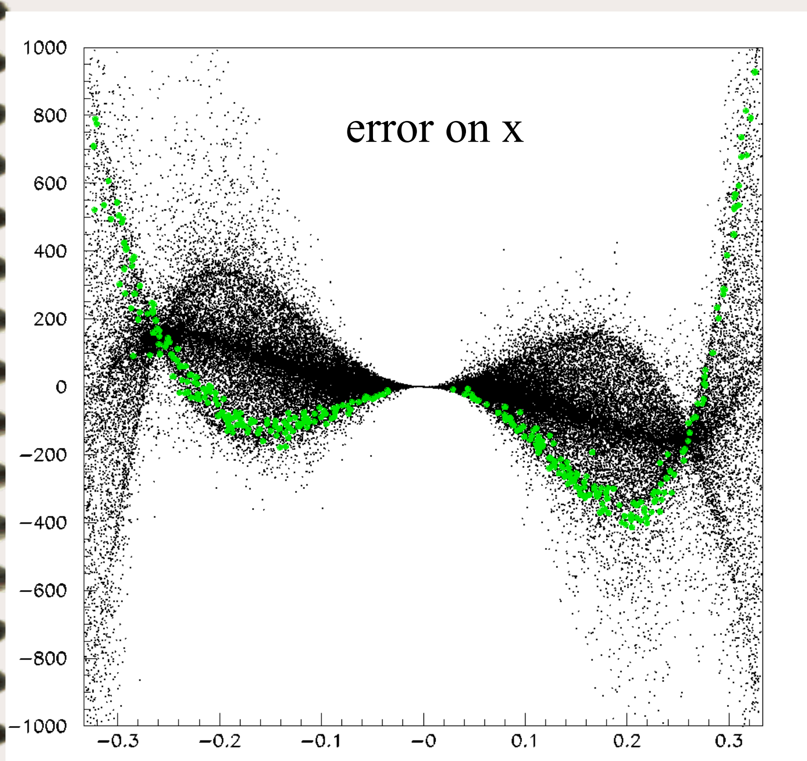


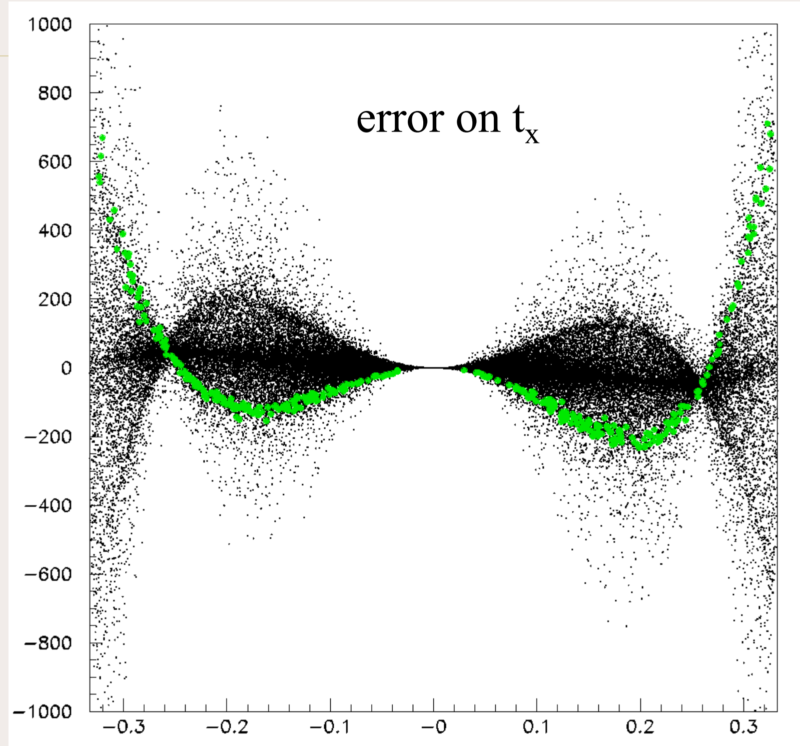
for most tracks:
error on position <~ 0.2 mm
error on direction <~ 0.2 mrad

may be good enough for some applications (especially in non-bending plane zy, where less precision is needed)

but: the structure of the plots suggests a possible improvement

need for degree 4 in q/p ?

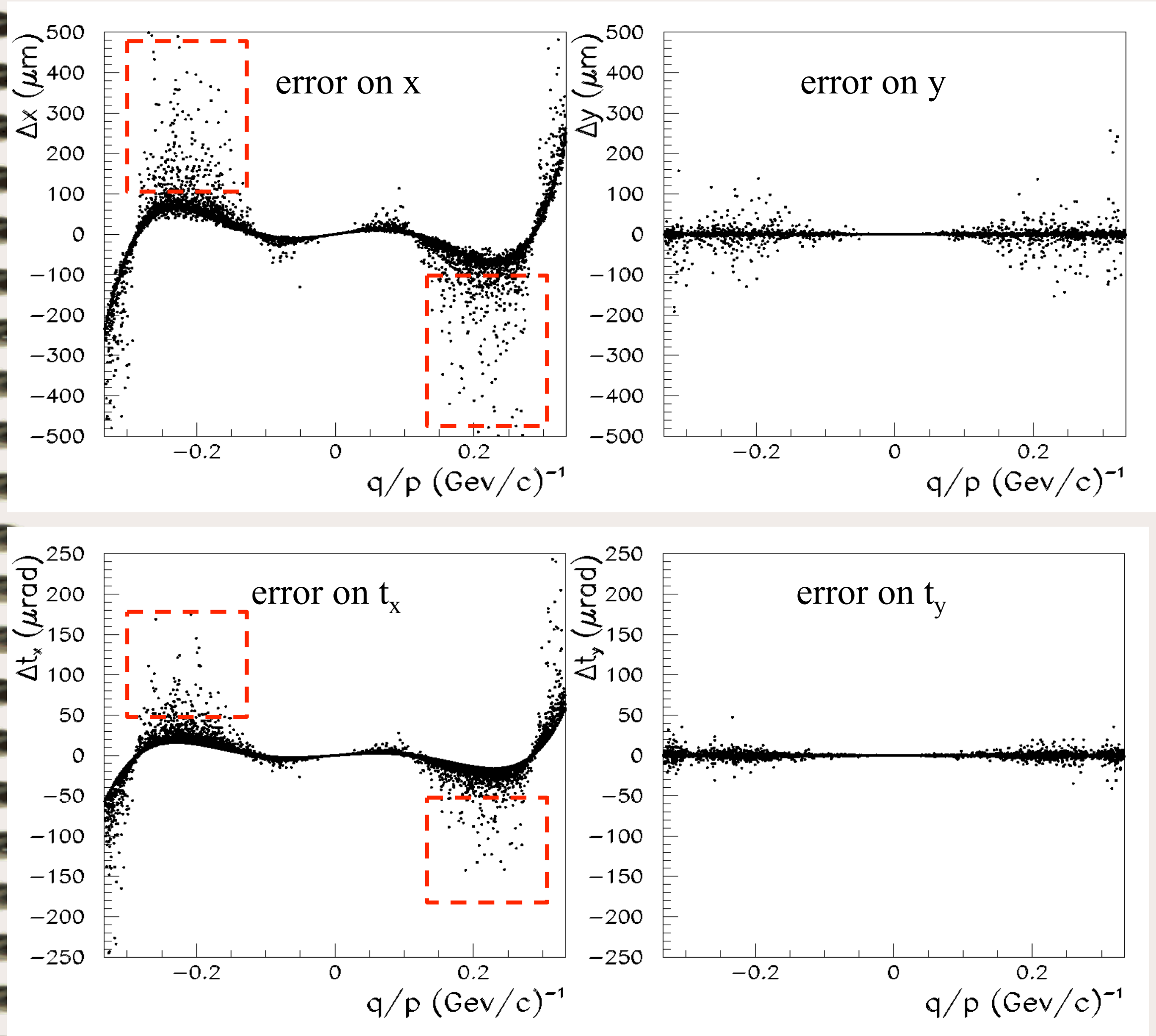




in green: x and y within (0.08, 0.12 m)

clear quartic dependence suggests a *tabulated* term in $(q/p)^4$

with degree 4 in q/p (tabulated in x,y)



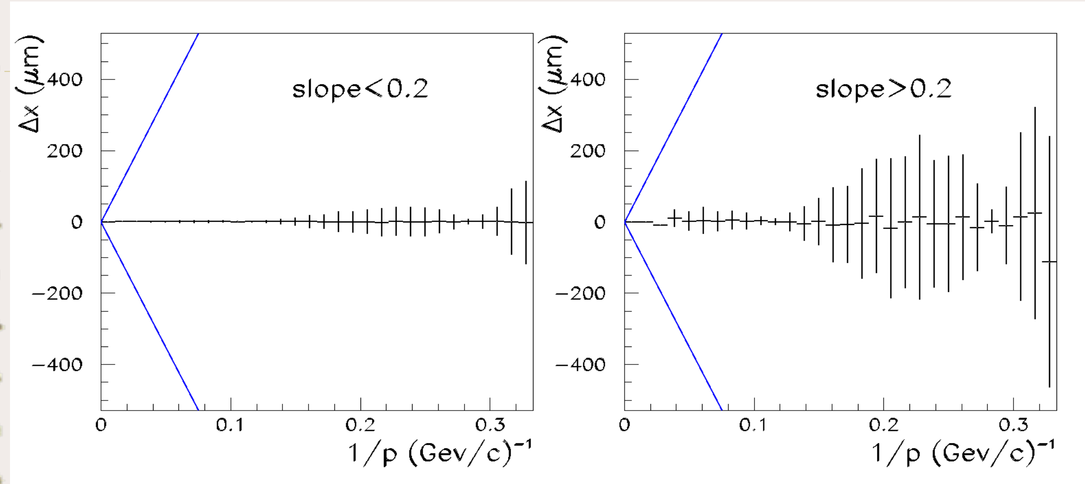
quite sufficient in zy plane

systematic dependence on q/p in zx plane suggests to add *global* corrections in $(q/p)^k$ *(here with odd k because of the symmetry of the field)*

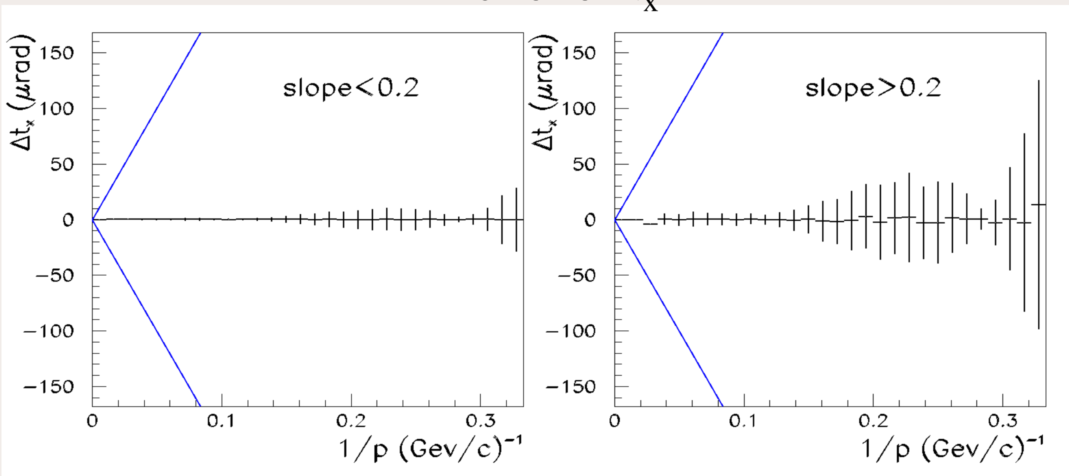
but: outliers (red rectangles) will remain far away. Where do they come from ?

with global correction (degree 7 on q/p) for x,t_x

error on x



error on t_x



blue lines: effect of multiple scattering in air *(measurement error does not matter here)*

precision is excellent in the central region

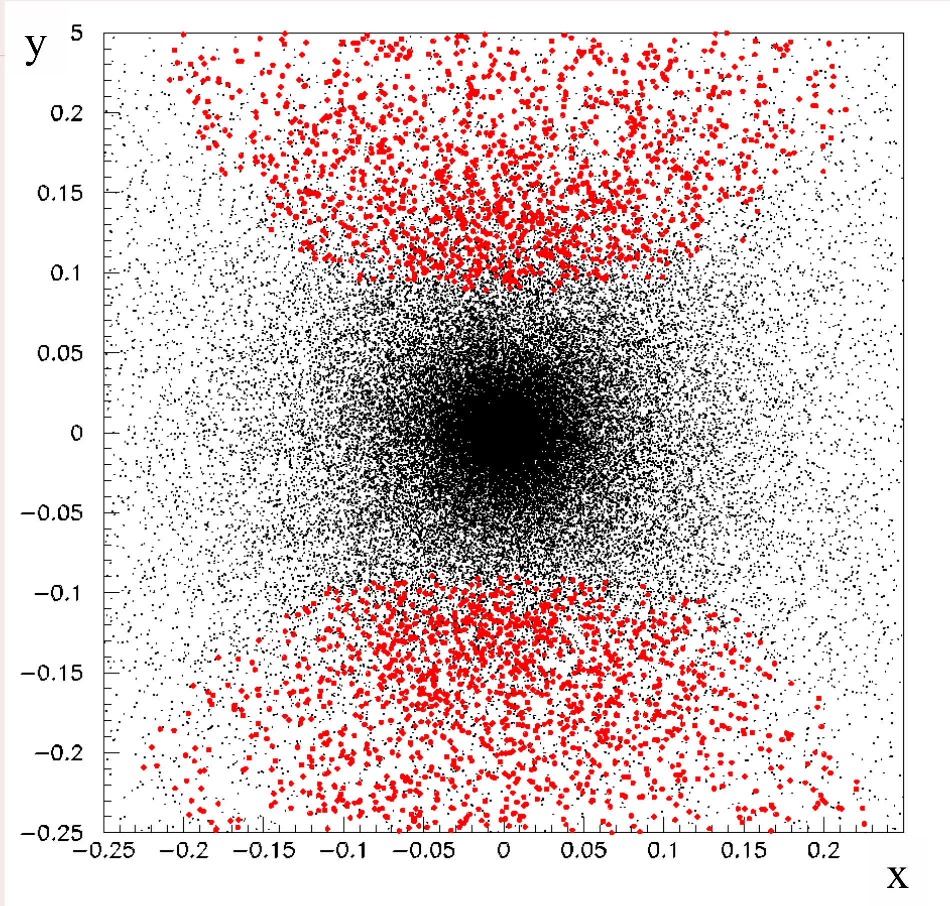
degradation for peripheral tracks is mainly due to outliers *(see possible solutions in next slide)*

warnings:

- real field
- Pmin

where are the outliers ?

(plot with more statistics)

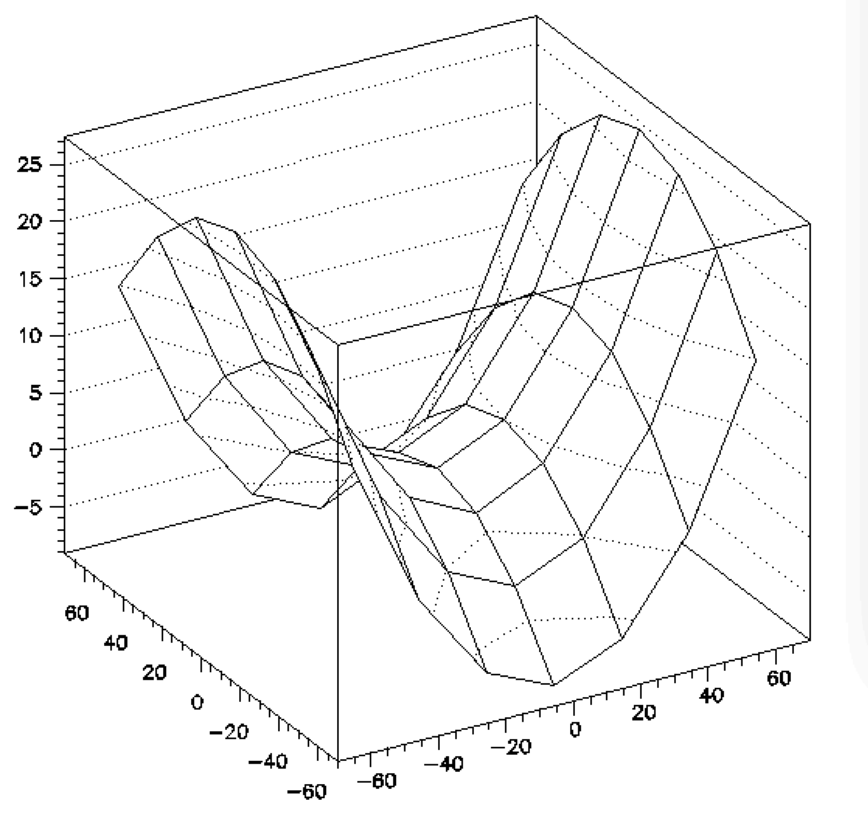


well defined region in the initial plane
(small fraction of the sample)
as expected from the C_{00k} *plots*

possible solutions:

- apply standard Runge-Kutta extrapolation in this region
- define subregions with different expansions in q/p
- find a simple parametrization in x,y for the additional corrections

errors due to interpolation
(here: trying to use a 25x25 xy table)

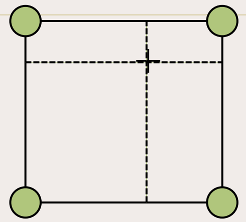


zoom on the central part of the C_{001} plot (subtracting value at 0,0):

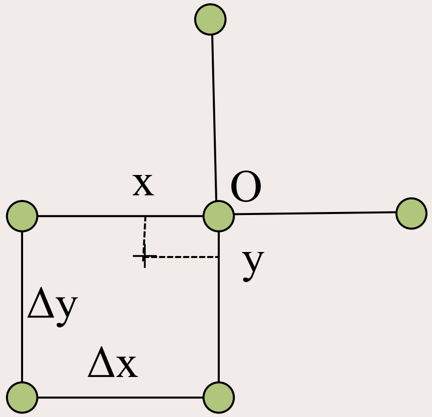
simple interpolation from 4 (x,y) points may produce a millimetric error

can we try a second degree approximation in x,y ?

interpolation from xy table: bilinear vs quadratic



bilinear interpolation using 4 neighbouring points:
amounts to define $a+bx+cy+dxy$ within the square
(exact value on the vertices of the rectangle, but biased if terms in x^2 and/or y^2 are needed to match the shape)



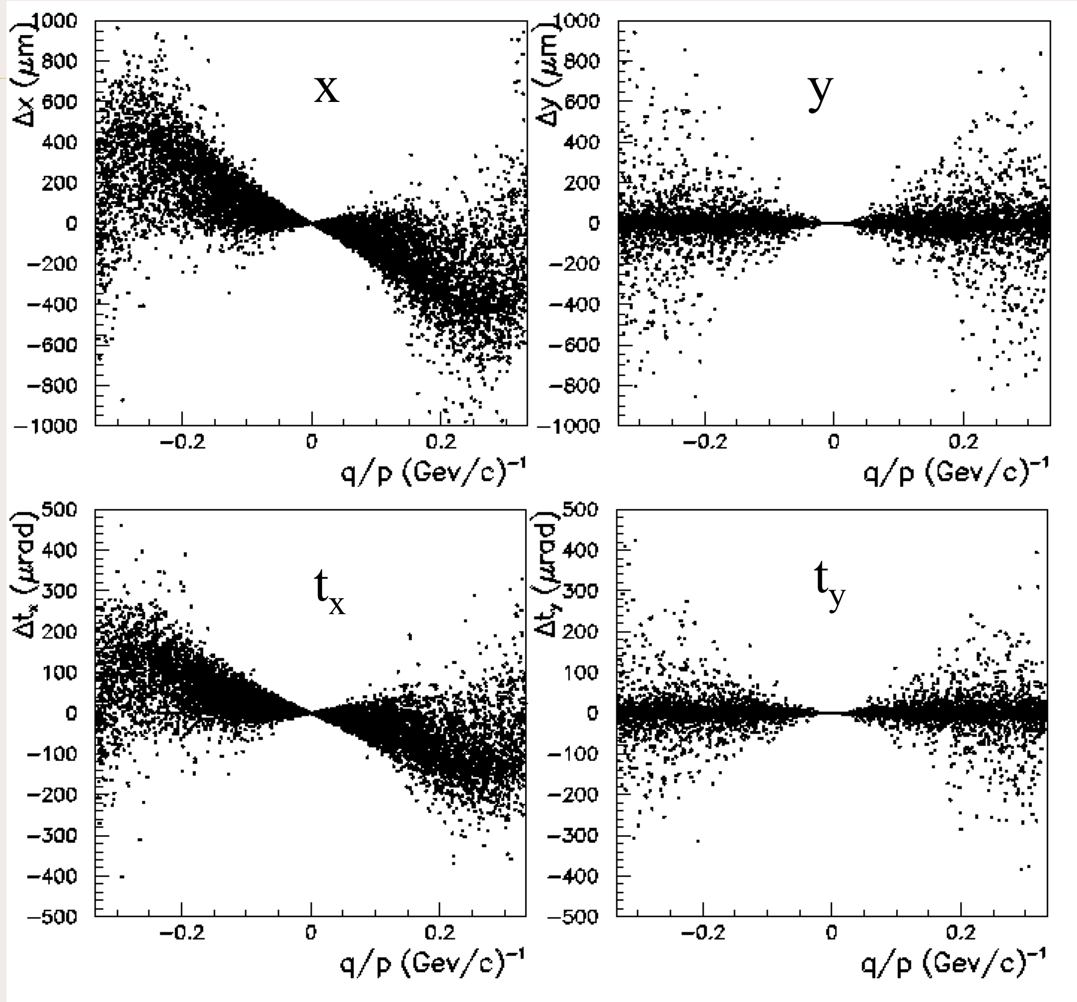
quadratic interpolation using 6 closest points to define
$a+bx+cy+dxy+ex^2+fy^2$

may be defined as a linear combination of the 6 values
with $X = x/\Delta x$, $Y = y/\Delta y$, the « matrix » of coefficients may be written in this configuration (X and Y in [-0.5,0]) as:

$$\begin{pmatrix} & 0 & \\ 0 & 1 & 0 \\ 0 & 0 & \end{pmatrix} + \frac{X}{2}\begin{pmatrix} & 0 & \\ -1 & 0 & 1 \\ 0 & 0 & \end{pmatrix} + \frac{Y}{2}\begin{pmatrix} & 1 & \\ 0 & 0 & 0 \\ 0 & -1 & \end{pmatrix} + \frac{X^2}{2}\begin{pmatrix} & 0 & \\ 1 & -2 & 1 \\ 0 & 0 & \end{pmatrix} + \frac{Y^2}{2}\begin{pmatrix} & 1 & \\ 0 & -2 & 0 \\ 0 & 1 & \end{pmatrix} + XY\begin{pmatrix} & 0 & \\ -1 & 1 & 0 \\ 1 & -1 & \end{pmatrix}$$

more computations but better precision with reduced tables

using reduced tables (25x25 instead of 100x100)
with 4-point interpolation



~ millimetric error
(as expected)

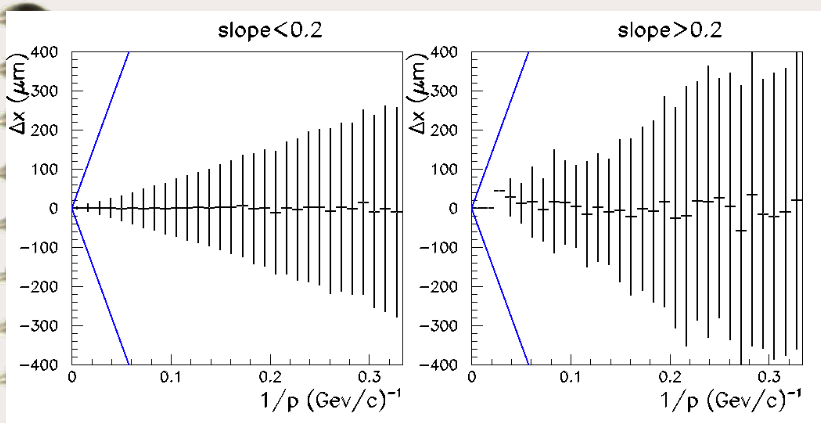
no clear substructure

systematic bias in zx plane (convexity effect): compensated in average by global coefficients in q/p expansion, but dispersion remains

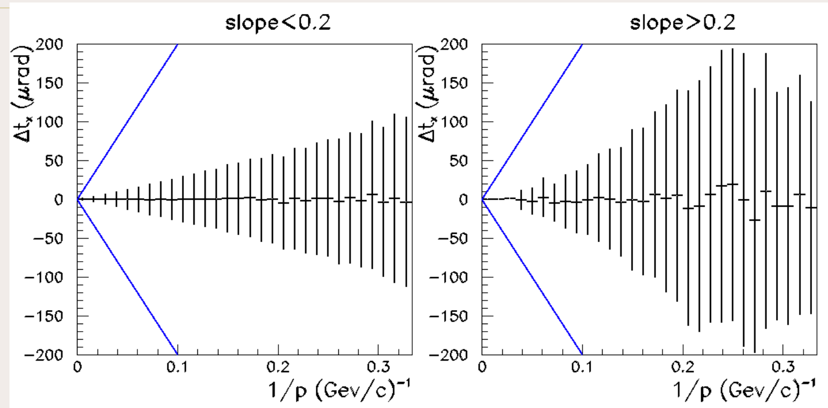
4-points/6-points results with reduced tables

bilinear interpolation

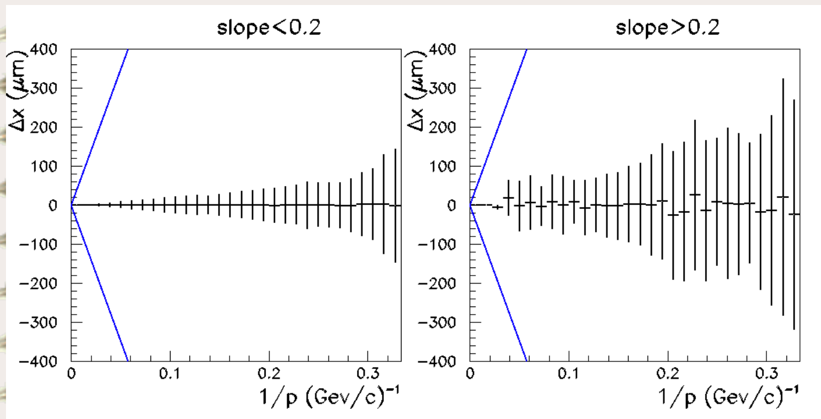
error on x

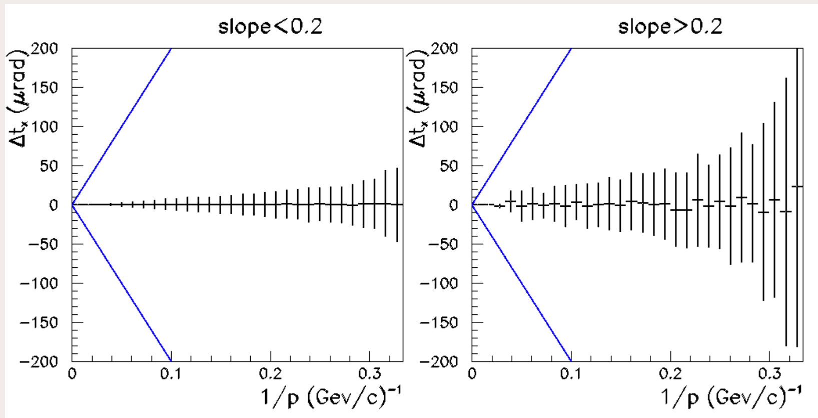


error on t_x



quadratic interpolation





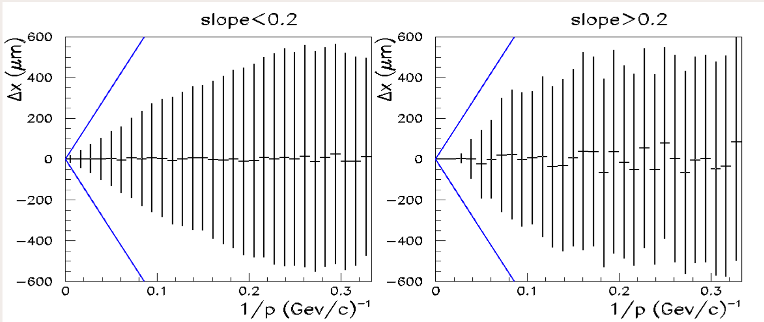
including larger impact parameters (e.g. K^0_S or Λ decay products)

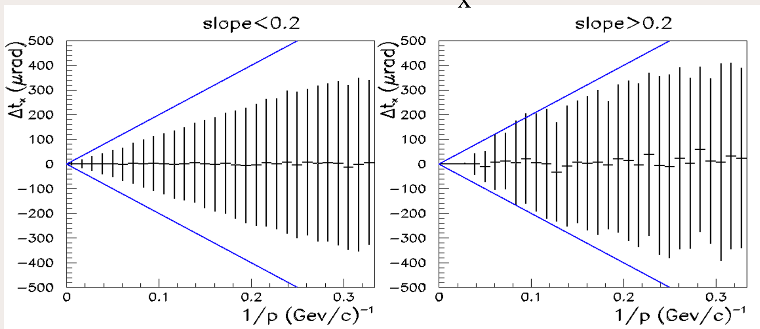
with $|\delta t|_{max} = 0.01$, the coefficients of degree 2 in δt_x , δt_y are small:
no significant difference when applied to the test sample
here we set $|\delta t|_{max} = 0.03$ and $\sigma_x = \sigma_y = 15$ mm in the test sample
table: 25×25 with quadratic interpolation

degree 1 in $\delta t_x, \delta t_y$

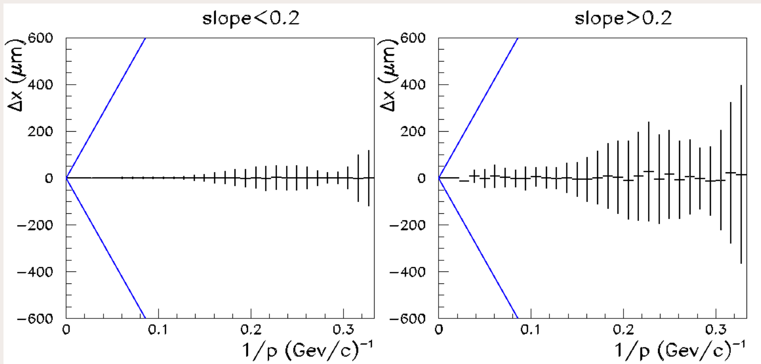
error on x

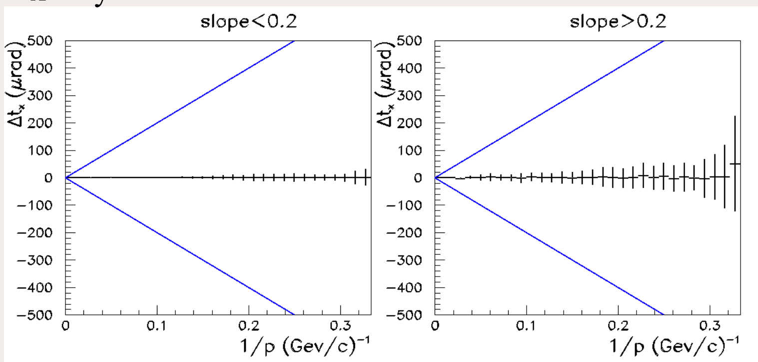
error on t_x





degree 2 in $\delta t_x, \delta t_y$





much better (and sufficient for applications)

technical issues

- **polynomial of high degree fitted on equidistant points:**

  the residuals are large on the ends (and diverge rapidly when going beyond); here the fit is done on points at (-0.9,-0.7…0.7,0.9)/p_{min} so it is actually constrained for $p > p_{min}/0.9$

  the result is often « too good » at high p compared to low p. It may be possible to compensate this effect by setting more fitting points at large 1/p; it may be advantageous to chose p_{min} lower than the value wanted for applications

- **linking extrapolations between successive planes:**

  if the trajectory is split in several steps (e.g. for a Kalman Filter), it may be easier to find separately solutions with lower degrees. However, the errors on x and t_x are tightly correlated (similarly for y and t_y), so there is a cumulative effect: even if each step fulfills the quality criteria, their combination may be unacceptable.

- **accessing big tables vs making many operations:**

  to be discussed with experts

summary and comments

- within a restricted region of the 5D phase space of trajectories (tracks of physical interest), it is possible to obtain a fast and precise extrapolation between two predefined surfaces, through a polynomial expansion with tabulated coefficients. These tables provide *also* the jacobian matrix.

- there are many tunable « handles » in the machinery (degrees of expansion, fitting ranges, region within the acceptance). It may be tailored for a specific purpose in a specific setup (done for LHCb, *see talk by S. Stemmle)*

- there may be « bad » regions in the phase space: depending on the population, one can make a local refinement, or apply the standard Runge-Kutta method. **In any case, one can know *a priori* if a state is in a bad region**

- a compromise has to be found between the size of the tables and the precision of the interpolation.

- for large p, in a central region with nearly parallel field lines, convenient parametrizations with less coefficients may be used *(see talk by S. Stemmle)*