fast and precise calculation of trajectories through a magnetic field

(inspired by the LHCb context)

Pierre Billoir
LPNHE Paris
Université Pierre et Marie Curie
what do we need?

state in surface A
(5 parameters)

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state in surface B
+ jacobian matrix

wanted precision:
- pattern recognition: better than the hit separation distance
- track fit: much better than the combination of measurement and multiple scattering errors

useful trajectories: from primary vertices + short lived decays + K_{s}/Λ decays if possible

guidelines:
- the state on surface A lies in a small region of the 5D-space of track parameters
- the wanted precision scales as 1/p in most cases
- in general, one needs to consider a few predefined surfaces + short range extrapolations

standard method (used as reference): stepwise propagation using the Runge-Kutta algorithm

problem: access to a field map (possibly big size) and CPU time consumption
framework: forward spectrometer on a collider
(similar to LHCb)

state in a z plane:
\( x, y, t_x, t_y, q/p \)
\( (t_x = dx/dz, t_y = dy/dz) \)

field is mainly along \( y \) within the acceptance
\( \Rightarrow \) zx is the bending plane
\( \max(B_y) \) on z axis \( \sim 1 \) T
total bending power along z axis \( \sim 4 \) T.m

upper edge of acceptance (slope 0.25)
field lines; density \( \sim \) intensity

in this study: extrapolation from the state in plane A to the state in plane B

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a polynomial expansion
of the deviation from the straight line

\[
\Delta x = \sum C_{ijk} \delta t_x^i \delta t_y^j (q/p)^k
\]
similar expressions for \(\Delta y\), \(\Delta t_x\), \(\Delta t_y\)

tabulate the coefficients as functions of \(x, y\) at \(z_A\)

+ apply global corrections in higher powers of \(q/p\)
  (independent of \(x, y, t_x, t_y\))

\(jacobian\) matrix easy to obtain from this parametrization

including the magnetic deviation from 0 to \(z_A\) as \(q/p \ F(z_A)\)
(F from a table)
\(\Rightarrow |\delta t_x| \ and \ |\delta t_y|\) are small
implementation on a planar surface

**precomputation:**
expansion in $t_x$, $t_y$, $q/p$ within a small solid angle ($\delta t_x$, $\delta t_y$):
coefficients $C_{ijk}$ at every point of the xy grid

**application:**
interpolation in x,y to define $C_{ijk}$ at any point
orders of magnitude and working conditions

typical conditions for initial plane at z = 1 m:
\[ \sigma(z_{\text{vertex}}) \sim 5 \text{ cm} \Rightarrow \delta t \sim 0.05 \times t \]
impact parameter for K_S products \( \sim 2.5 \text{ cm} \Rightarrow \delta t \sim 0.025 \)

domain used in initial plane to fit the coefficients
- \( p > p_{\text{min}} = 3 \text{ GeV/c} \) (good chance to remain within the acceptance)
- \( |\delta t|_{\text{max}} = 0.01 \) (0.03 for K_S studies)
- \( |x| \) and \( |y| < 0.25 \text{ m} \)

computation:
- tabulation on 100\times100 or 25\times25 positions in plane A within \( \pm 0.25 \text{ m} \)
- 20 values of \( q/p \) within (-1/p_{\text{min}}, 1/p_{\text{min}})
- for each one, 20\times20 values of \( \delta t_x, \delta t_y \) in (-|\delta t|_{\text{max}}, |\delta t|_{\text{max}})
- deviations from straight line using Runge-Kutta
- l.s. fit of the coefficients up to the wanted degree
- coefficient of global dependence on \( q/p \): fit on the test sample

test sample:
- flat distribution in \( q/p \) in (-1/p_{\text{min}}, 1/p_{\text{min}})
- distribution in \( p_t \) : \exp (-p_t/1 \text{ GeV/c})
- vertex: \( \sigma_z = 60 \text{ mm}, \sigma_x = \sigma_y = 0.1 \text{ mm} \) (15 mm for K_S studies)

Remark: \( \Delta x \) expanded in \( q/p \) \( \Rightarrow \) \( q/p \) expanded in \( \Delta x \)
(prediction of momentum with initial segment + hit assignment)
exploration of tables (here: $C_{00k}/p_{\text{min}}^k$ for extrapolated $x$, in mm)

expectations for a submillimetric precision: we need at least $k$ up to 4

problematic region: large $y$

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first trial: degree 1 in $\delta t_x, \delta t_y$, 3 in $q/p$
100x100 tabulation in $x,y$ (4-point interpolation)
applied to the test sample

for most tracks:
error on position $\sim 0.2$ mm
error on direction $\sim 0.2$ mrad

may be good enough for some applications (especially in non-bending plane $zy$, where less precision is needed)

but: the structure of the plots suggests a possible improvement
need for degree 4 in \(q/p\)?

Error on \(x\) and error on \(t_x\)

In green: \(x\) and \(y\) within \((0.08, 0.12\, \text{m})\)

Clear quartic dependence suggests a tabulated term in \((q/p)^4\)

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with degree 4 in q/p (tabulated in x,y)

quite sufficient in zy plane

systematic dependence on q/p in zx plane suggests to add global corrections in \((q/p)^k\) (here with odd \(k\) because of the symmetry of the field)

but: outliers (red rectangles) will remain far away. Where do they come from?
with global correction (degree 7 on q/p) for $x, t_x$

error on $x$

blue lines: effect of multiple scattering in air (measurement error does not matter here)

precision is excellent in the central region

degradation for peripheral tracks is mainly due to outliers (see possible solutions in next slide)

warnings:
- real field
- $P_{\text{min}}$

error on $t_x$
where are the outliers?

(plot with more statistics)

well defined region in the initial plane (small fraction of the sample) 
*as expected from the $C_{00k}$ plots*

possible solutions:
- apply standard Runge-Kutta extrapolation in this region
- define subregions with different expansions in $q/p$
- find a simple parametrization in $x,y$ for the additional corrections
errors due to interpolation
(here: trying to use a 25x25 xy table)

zoom on the central part of the $C_{001}$ plot (subtracting value at 0,0):

*simple interpolation from 4 (x,y) points may produce a millimetric error*

can we try a second degree approximation in x,y?
interpolation from xy table:
bilinear vs quadratic

bilinear interpolation using 4 neighbouring points:
amounts to define \( a + bx + cy + dxy \) within the square
(exact value on the vertices of the rectangle, but biased if
terms in \( x^2 \) and/or \( y^2 \) are needed to match the shape)

quadratic interpolation using 6 closest points to define
\( a + bx + cy + dxy + ex^2 + fy^2 \)

may be defined as a linear combination of the 6 values
with \( X = x/\Delta x, Y = y/\Delta y \), the « matrix » of coefficients may
be written in this configuration (\( X \) and \( Y \) in \([-0.5,0]\)) as:

\[
\begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
+ \frac{X^2}{2} \begin{pmatrix}
1 & 0 & 0 \\
0 & -2 & 0 \\
0 & 0 & 1
\end{pmatrix}
+ \frac{Y^2}{2} \begin{pmatrix}
1 & 0 & 1 \\
0 & -2 & 0 \\
0 & 0 & 1
\end{pmatrix}
+ XY \begin{pmatrix}
-1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{pmatrix}
\]

more computations but better precision with reduced tables
using reduced tables (25x25 instead of 100x100) with 4-point interpolation

~ millimetric error (as expected)

no clear substructure

systematic bias in zx plane (convexity effect): compensated in average by global coefficients in q/p expansion, but dispersion remains

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4-points/6-points results with reduced tables

bilinear interpolation

error on $x$

error on $t_x$

quadratic interpolation

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including larger impact parameters (e.g. $K^0_S$ or $\Lambda$ decay products)

with $|\delta t|_{\text{max}} = 0.01$, the coefficients of degree 2 in $\delta t_x$, $\delta t_y$ are small: no significant difference when applied to the test sample

here we set $|\delta t|_{\text{max}} = 0.03$ and $\sigma_x = \sigma_y = 15$ mm in the test sample

table: $25 \times 25$ with quadratic interpolation

much better (and sufficient for applications)

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technical issues

• **polynomial of high degree fitted on equidistant points:**
  - the residuals are large on the ends (and diverge rapidly when going beyond); here the fit is done on points at (-0.9,-0.7…0.7,0.9)/p_{min} so it is actually constrained for p > p_{min}/0.9
  - the result is often « too good » at high p compared to low p. It may be possible to compensate this effect by setting more fitting points at large 1/p;
  - it may be advantageous to chose p_{min} lower than the value wanted for applications

• **linking extrapolations between successive planes:**
  - if the trajectory is split in several steps (e.g. for a Kalman Filter), it may be easier to find separately solutions with lower degrees. However, the errors on x and t_x are tightly correlated (similarly for y and t_y), so there is a cumulative effect: even if each step fulfills the quality criteria, their combination may be unacceptable.

• **accessing big tables vs making many operations:**
  - to be discussed with experts
summary and comments

• within a restricted region of the 5D phase space of trajectories (tracks of physical interest), it is possible to obtain a fast and precise extrapolation between two predefined surfaces, through a polynomial expansion with tabulated coefficients. These tables provide also the jacobian matrix.

• there are many tunable « handles » in the machinery (degrees of expansion, fitting ranges, region within the acceptance). It may be tailored for a specific purpose in a specific setup (done for LHCb, see talk by S. Stemmle)

• there may be « bad » regions in the phase space: depending on the population, one can make a local refinement, or apply the standard Runge-Kutta method. In any case, one can know a priori if a state is in a bad region

• a compromise has to be found between the size of the tables and the precision of the interpolation.

• for large $p$, in a central region with nearly parallel field lines, convenient parametrizations with less coefficients may be used (see talk by S. Stemmle)