Bivariate normal distribution for finding inliers in Hough space for a Time Projection Chamber

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Connecting the Dots
March 2017
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PHYSICS AT THE TERA SCALE
Helmholtz Alliance

Bundesministerium für Bildung und Forschung

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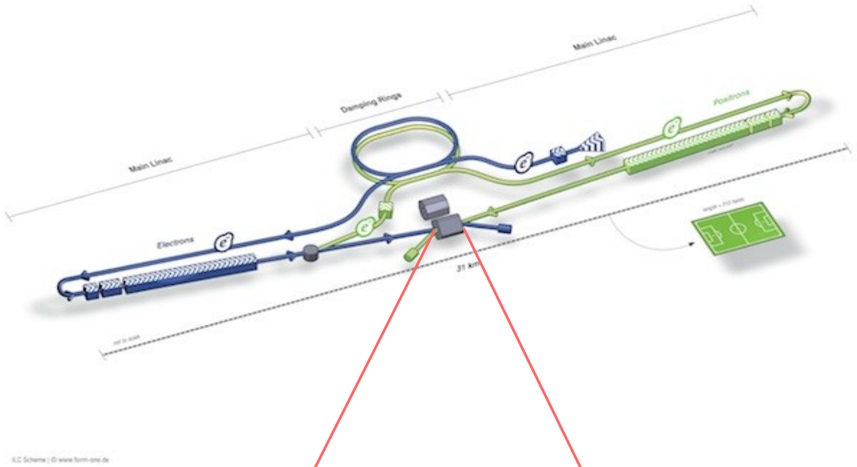
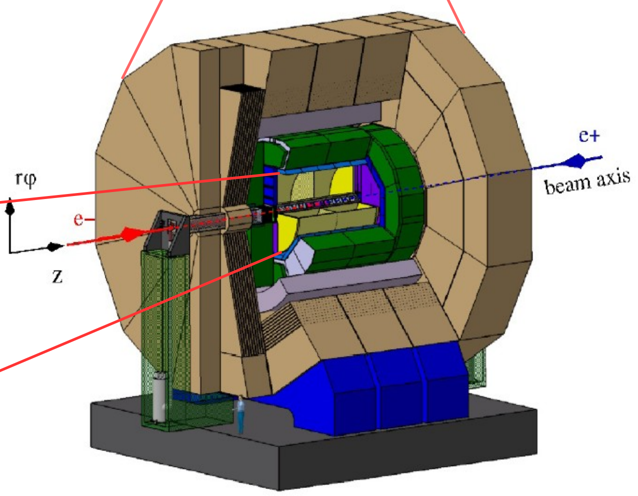
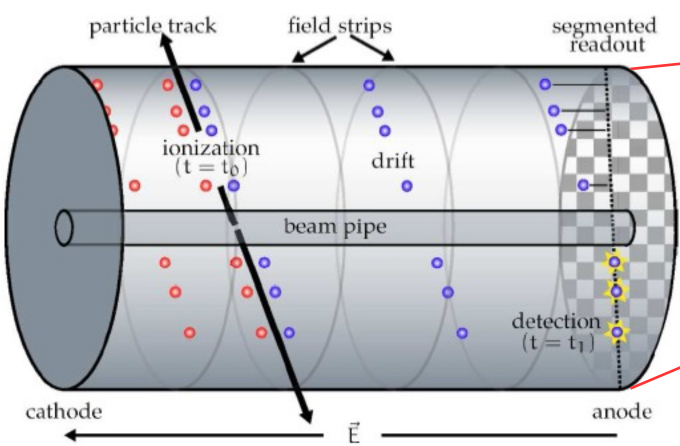
ILC – TPC: Introduction

- **International Linear Collider (ILC):** Collision between e^+e^- with cms energy 500 GeV - 1TeV
- **International Large Detector (ILD)**: A multi purpose detector fo ILC
- **Time Projection Chamber (TPC)**: The main tracker layout. A charged particle traversing the gas volume the TPC parallel to the end plate ionizes the gas and produces primary electrons.
- Drifting primary electrons toward the end plate and using gas amplification and readout system for detecting them
  - GEM or Micromegas + pad readout
  - Micromegas + pixelated readout
- Continuous tracking with many hits.





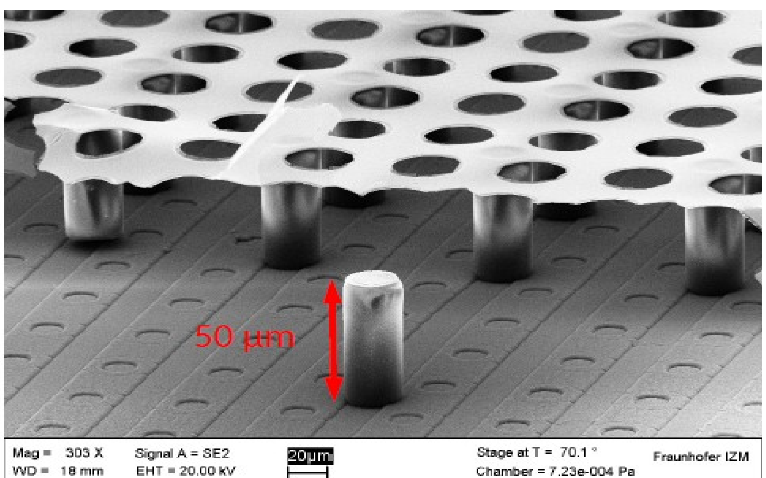
ILD (International Large Detector)

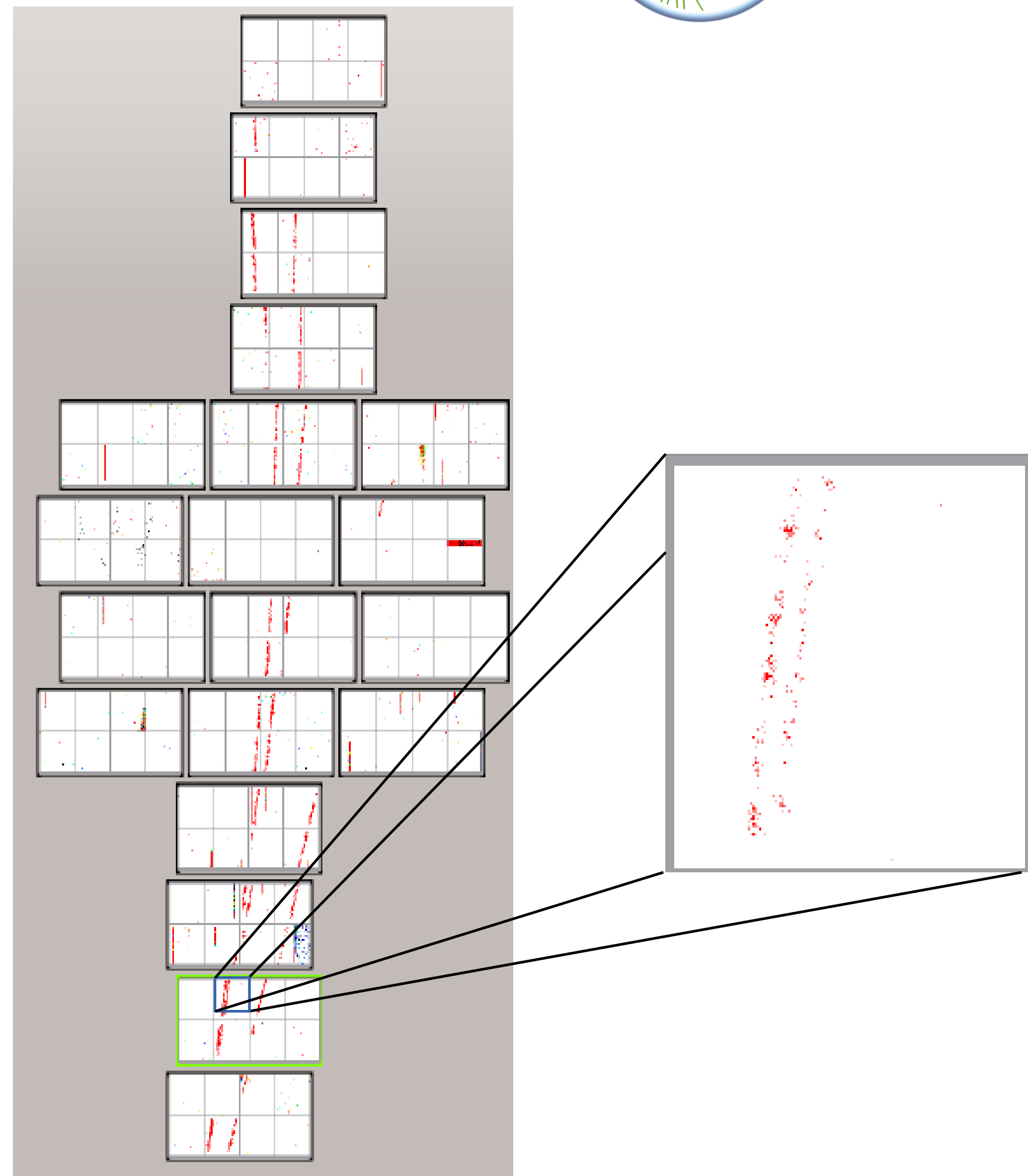


ILC – TPC: Development of a TPC readout based on GridPix

- **Timepix:**
  - Number of pixels: 256 X 256 pixels
  - Pixel size: 55 X 55 µm²
- **InGrid:**
  - Micromegas on top of a Timepix chip
  - Mesh holes aligned with pixels of the chip:

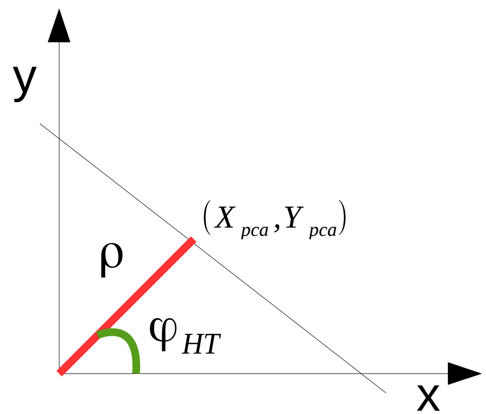
    single e$^-$ measurement
  - Many hits per track's length ≈ 10 hits per mm
  - Every 8 InGrid chips are aligned in an Octoboard
  - Using the unit of the Octoboard as a segment of the track calling tracklet

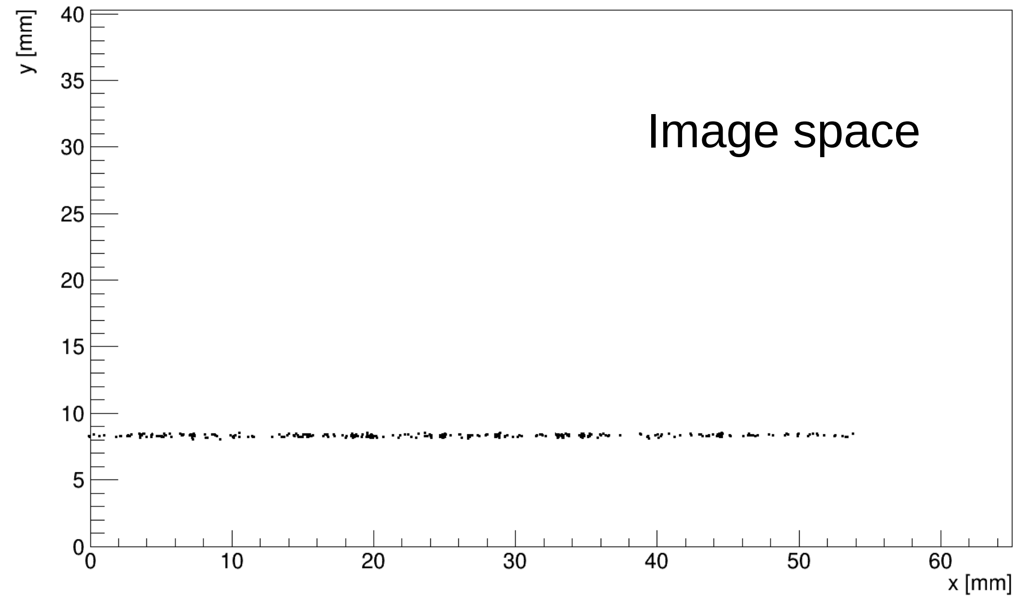


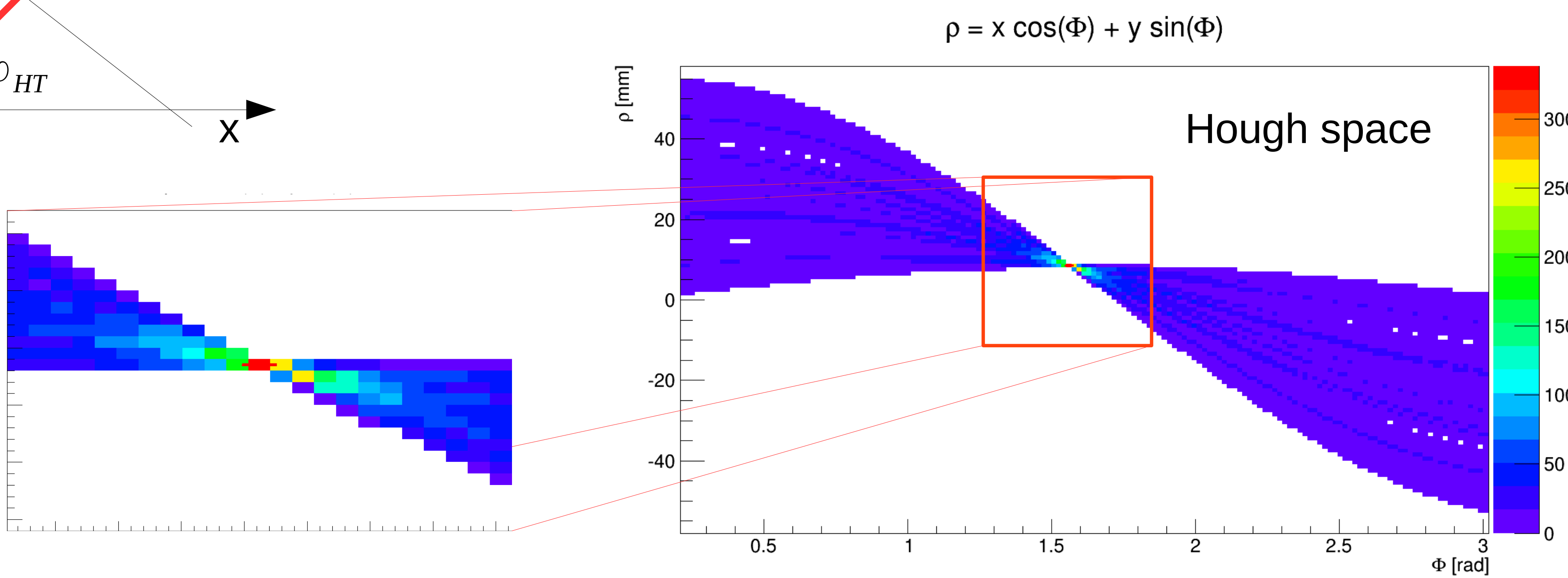


Hough Transform

- Bin with maximum entires returns φ_{HT} and ρ_{HT}
  - $\rho_{HT} = x\cos(\varphi_{HT}) + y\sin(\varphi_{HT})$
  - $d_0 = \rho_{HT}$
  - $\varphi_{track} = \varphi_{HT} - \frac{\pi}{2}$

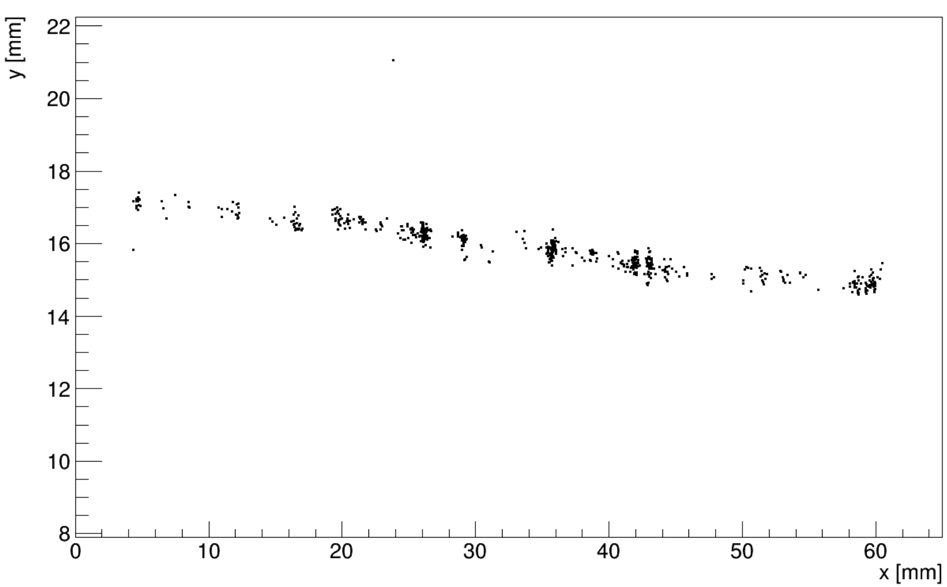


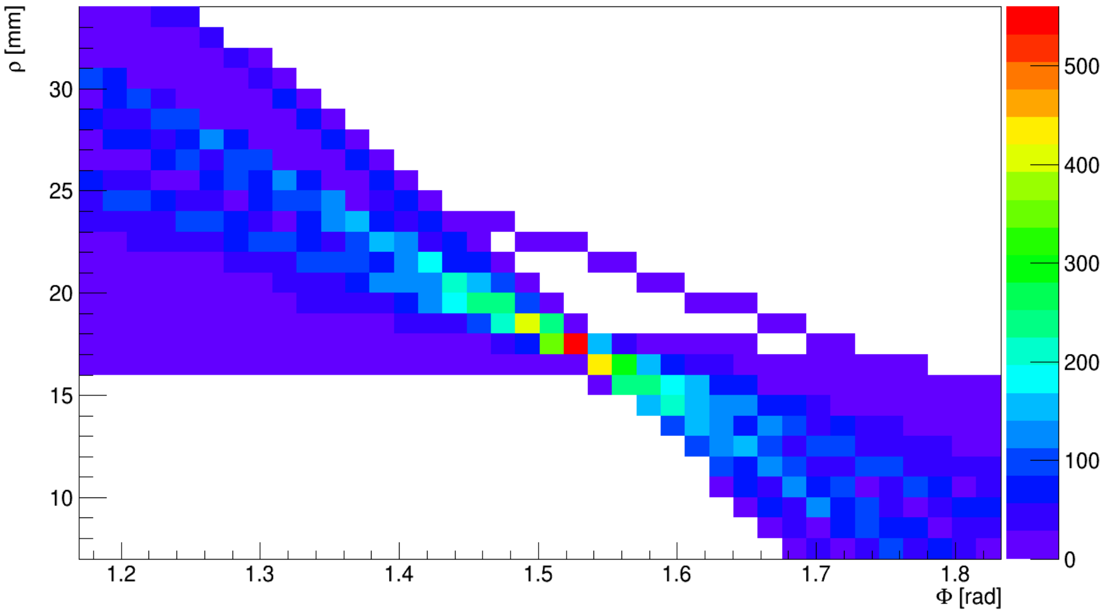




Hough Transform

- Because of the diffusion, some lines of hits pass through other bins around the bin with maximum entries.
- **Transverse resolution** (σ_D): resolution of the track in **X** and **Y** direction
- **Longitudinal resolution** (σ_L): resolution of the track in **Z** direction





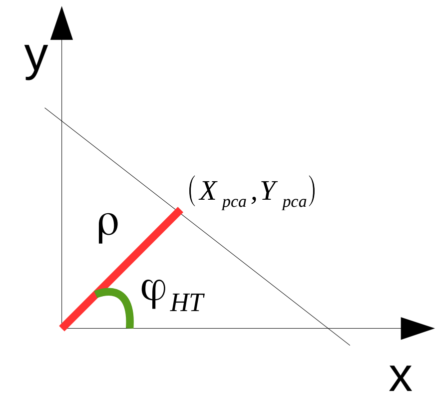
- **Calculating resolution of the track in Hough space**
- **Using Bivariate Normal Distribution for vicinity of the bin with maximum entries**

Diffusion and Covariance

Diffusion and covariance matrix:

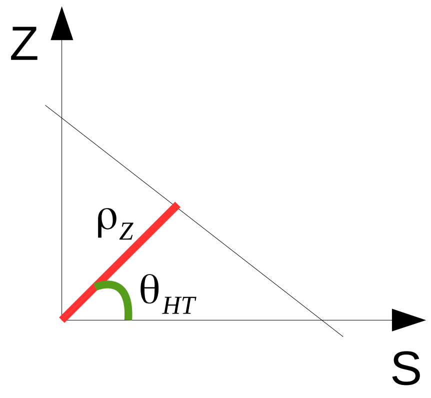
- XY_Plane: $\rho = x\cos(\varphi) + y\sin(\varphi)$
  - Transverse resolution: $\sigma_D^2 = Z D_T^2$

$$\begin{bmatrix} \sigma_\rho^2 & \sigma_{\rho\varphi} \\ \sigma_{\varphi\rho} & \sigma_\varphi^2 \end{bmatrix} = \nabla J \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_D^2 \end{bmatrix} \nabla J^T$$

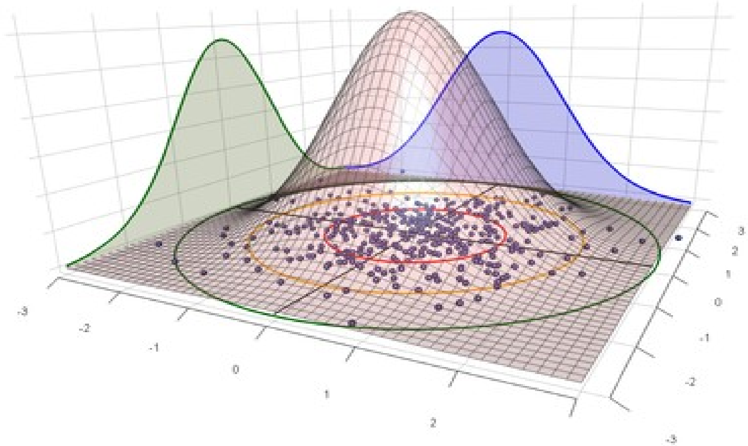


- SZ_Plane: $\rho_Z = s\cos(\theta_{HT}) + z\sin(\theta_{HT})$
  - Longitudinal resolution: $\sigma_L^2 = Z D_L^2$
  - Arc length for straight line: $S = \sqrt{(x_{hit} - x_{pca})^2 + (y_{hit} - y_{pca})^2}$

$$\begin{bmatrix} \sigma_{\rho_z}^2 & \sigma_{\rho_z\theta} \\ \sigma_{\theta\rho_z} & \sigma_\theta^2 \end{bmatrix} = \nabla J \begin{bmatrix} \sigma_S^2 & 0 \\ 0 & \sigma_L^2 \end{bmatrix} \nabla J^T$$



Bivariate Normal Distribution (BND):

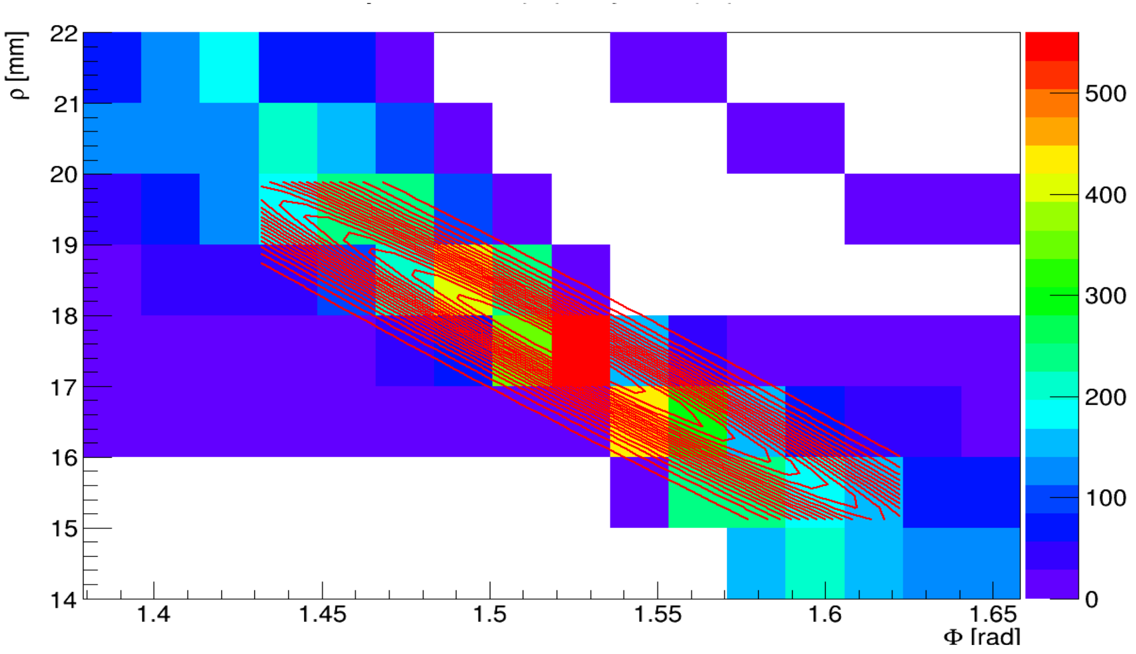


http://ballistipedia.com/index.php?title=Closed_Form_Precision

$$G(\varphi,\rho)=\frac{1}{2\pi\sigma_\rho\sigma_\varphi\sqrt{1-r^2}}\exp\left[\frac{-c}{2(1-r^2)}\right]$$

$$c\equiv\frac{(\varphi-\mu_\varphi)^2}{\sigma_\varphi^2}+\frac{(\rho-\mu_\rho)^2}{\sigma_\rho^2}-\frac{2r(\varphi-\mu_\varphi)(\rho-\mu_\rho)}{\sigma_\varphi\sigma_\rho}$$

- Correlation : $r\equiv\frac{\sigma_{\rho\varphi}}{\sigma_\rho\sigma_\varphi}$



Ellipse: minor and major axises

Ellipse equation of 1/e of Peak of BND:

- Finding ellipse equation from BND:

$$\boxed{1=\frac{(\varphi-\mu_\varphi)^2}{q\,\sigma_\varphi^2}+\frac{(\rho-\mu_\rho)^2}{q\,\sigma_\rho^2}-\frac{2r(\varphi-\mu_\varphi)(\rho-\mu_\rho)}{q\,\sigma_\varphi\sigma_\rho}} \qquad (1)$$

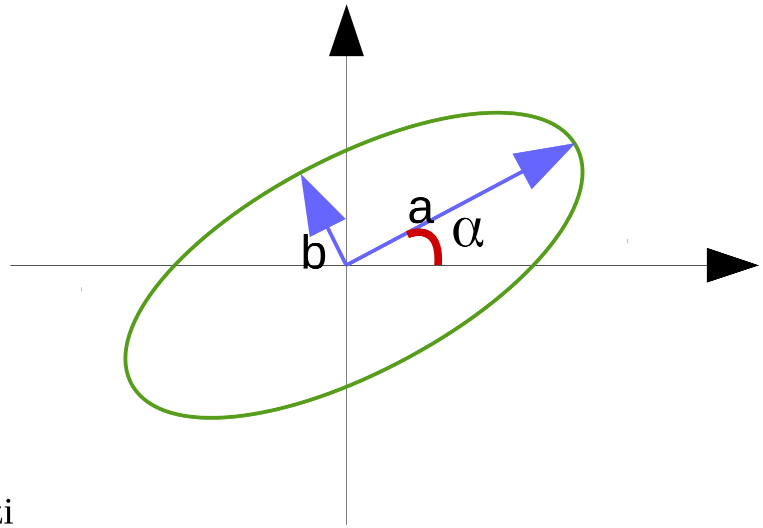
- Rotated ellipse:

$$\boxed{1=\left(\frac{\cos^2(\alpha)}{a^2}+\frac{\sin^2(\alpha)}{b^2}\right)x^2-2\cos(\alpha)\sin(\alpha)\left(\frac{1}{a^2}-\frac{1}{b^2}\right)xy+\left(\frac{\sin^2(\alpha)}{a^2}+\frac{\cos^2(\alpha)}{b^2}\right)y^2} \qquad (2)$$

- From (1) and (2):

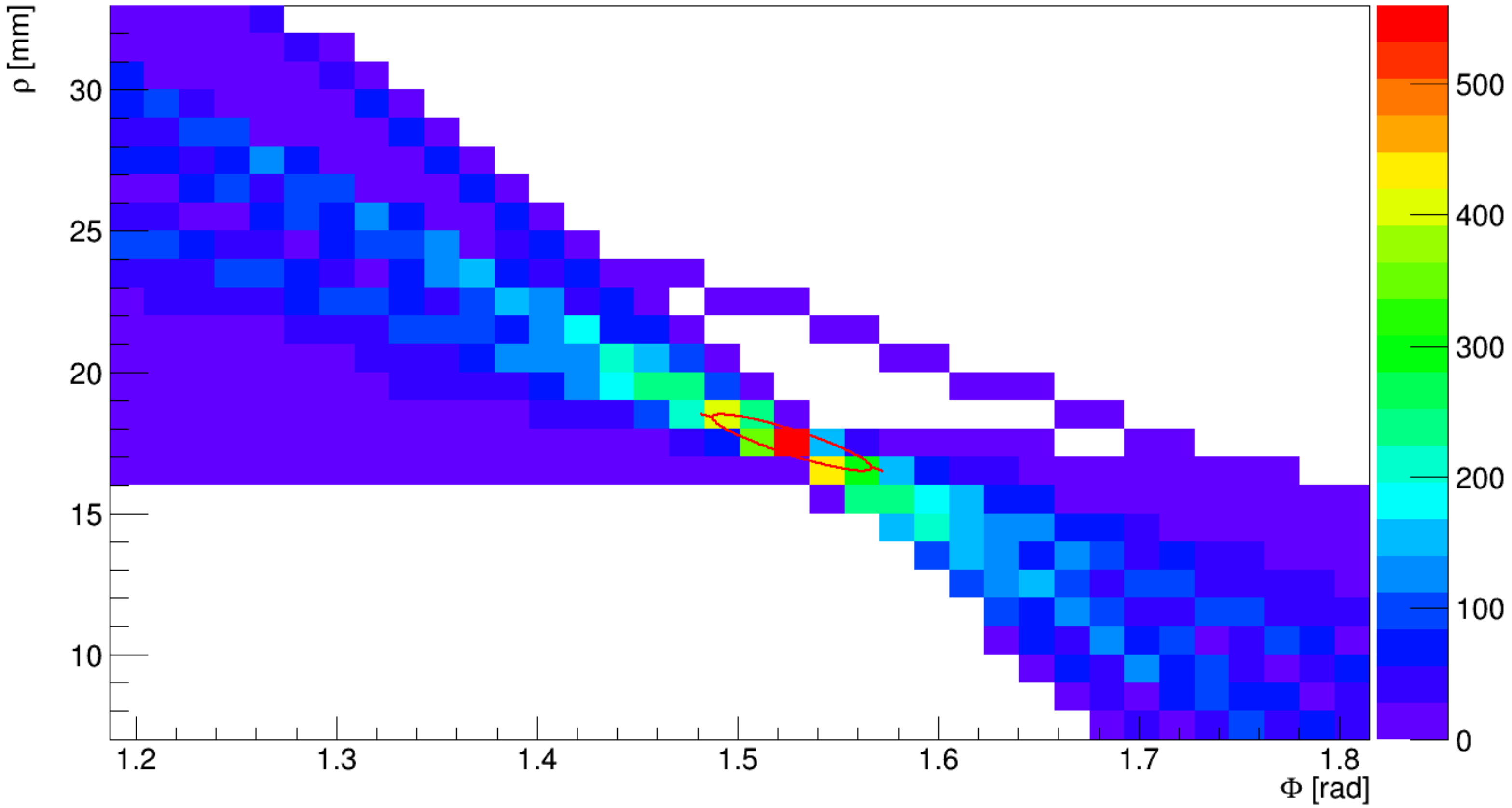
$$\boxed{a^2=\frac{q\,\sigma_\varphi^2\sigma_\rho^2\cos(2\alpha)}{\sigma_\rho^2\cos^2(\alpha)-\sigma_\varphi^2\sin^2(\alpha)}}$$

$$\boxed{b^2=\frac{-q\,\sigma_\varphi^2\sigma_\rho^2\cos(2\alpha)}{\sigma_\rho^2\sin^2(\alpha)-\sigma_\varphi^2\cos^2(\alpha)}}$$



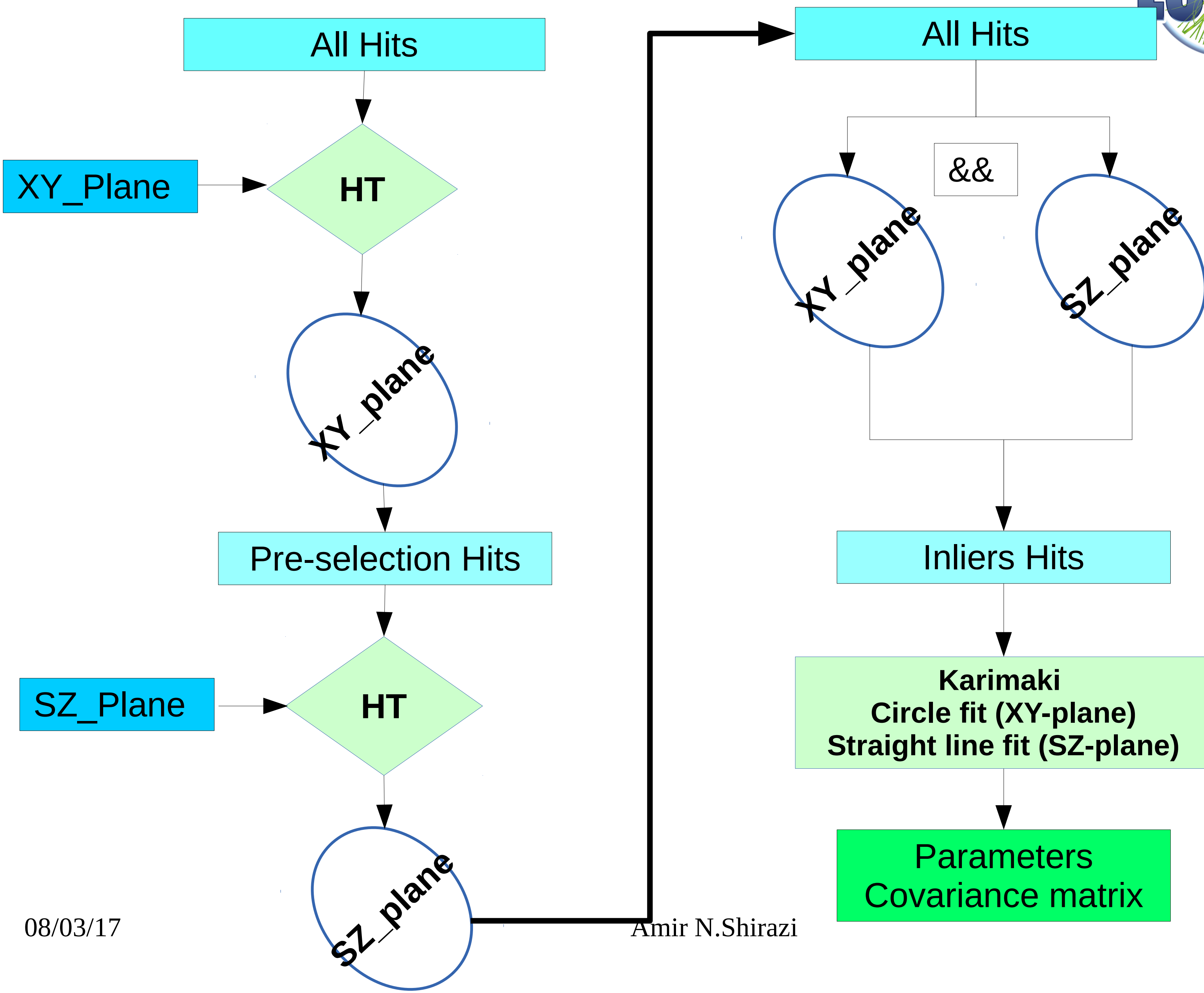
Example: XY_Plane

Inliers and 1/e of peak of BND:



- Collecting all hits inside the ellipse => **Inliers**
- Size of fitting is changeable => Number of inliers is changeable too

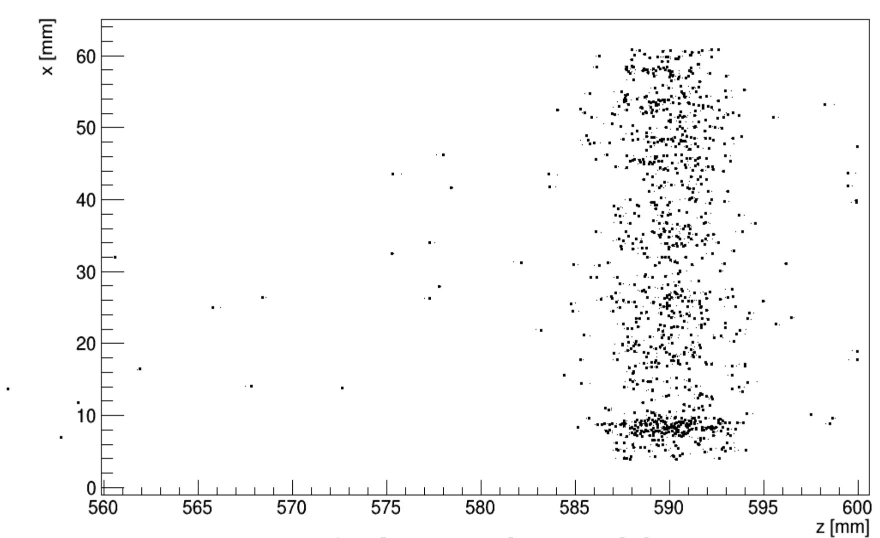
Flowchart

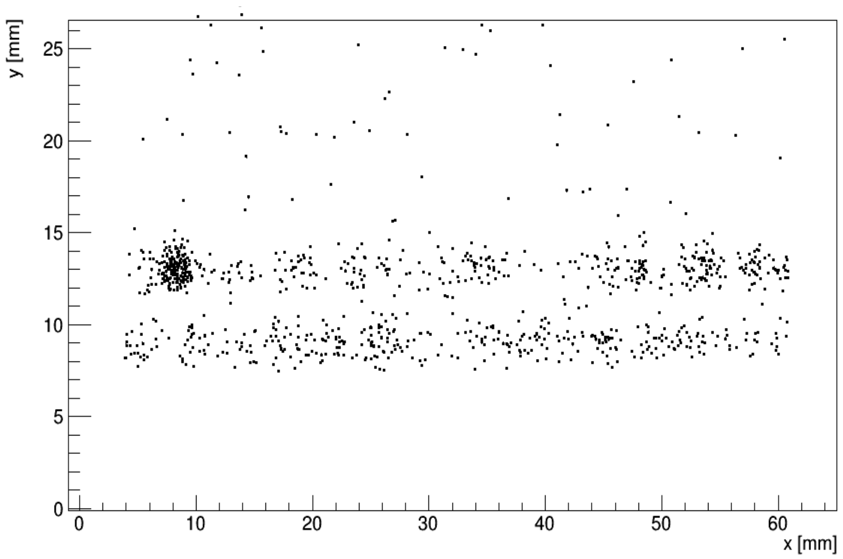


All Hits
XY_Plane
HT
XY_plane
Pre-selection Hits
SZ_Plane
HT
SZ_plane
All Hits
&&
XY_plane
SZ_plane
Inliers Hits
Karimaki
Circle fit (XY-plane)
Straight line fit (SZ-plane)
Parameters
Covariance matrix

Simulation

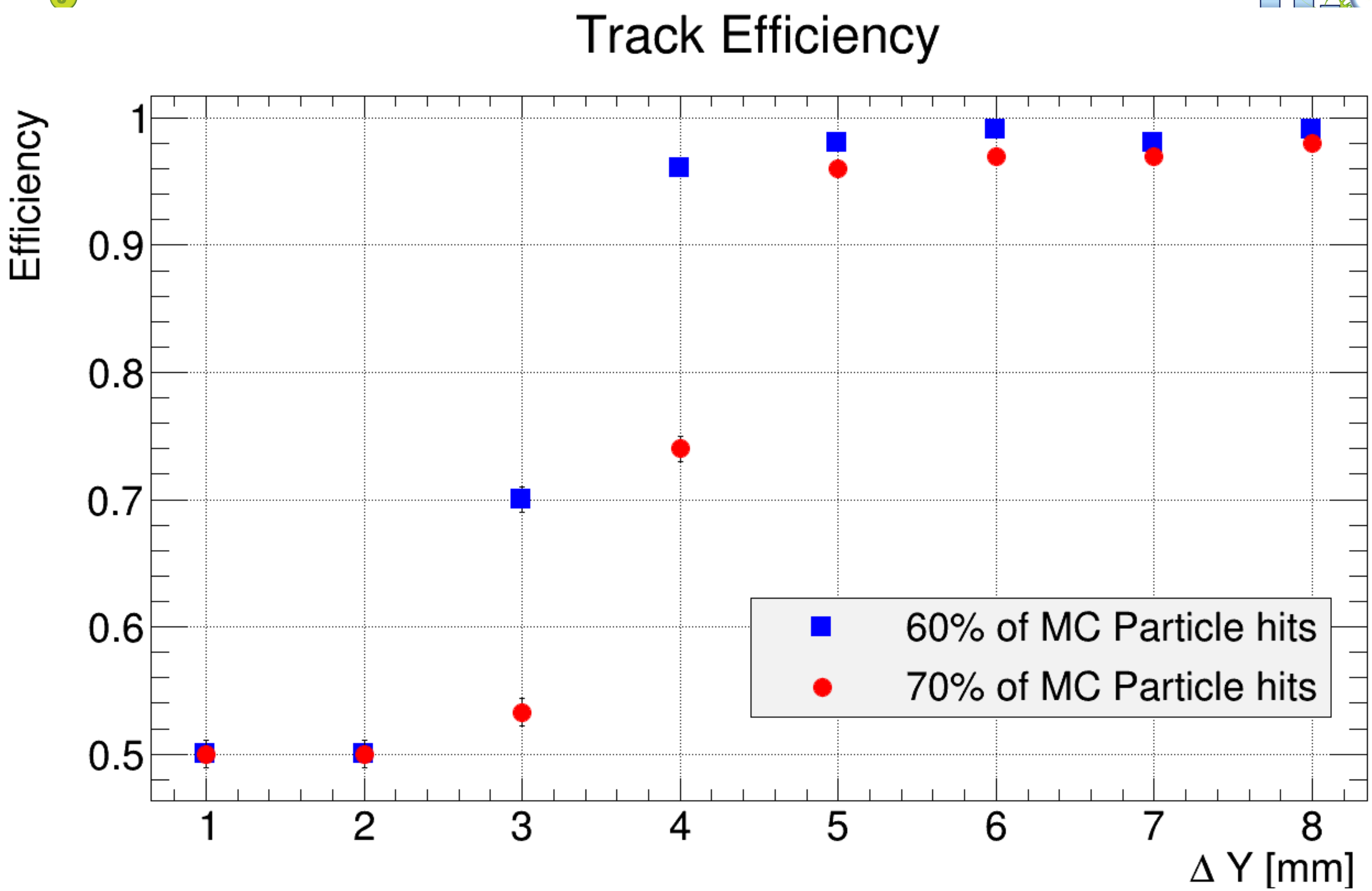
- Simulation of two tracks:
  - Simulation for Large Prototype TPC (LP-TPC) at DESY
  - Shooting two electrons 6 GeV, parallel to the X direction
  - Same Z offset
  - 590 mm from readout plane: Transverse resolution ≈ 0.70 mm in X and Y direction
  - Vary ΔY from 1mm to 8mm
  - 1000 events for each ΔY: 2000 tracks for each ΔY
  - B= 1T
  - Random Noise (Maximum 500)





Example of simulation with ΔY =4 mm

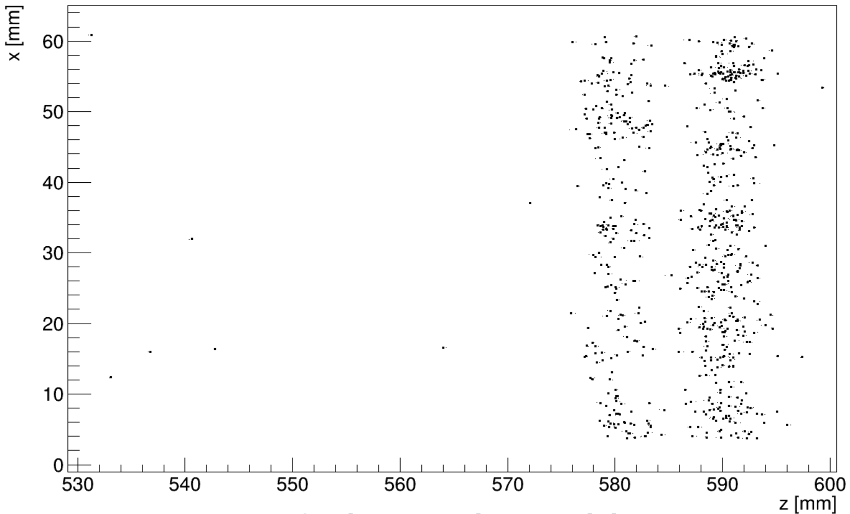
Simulation

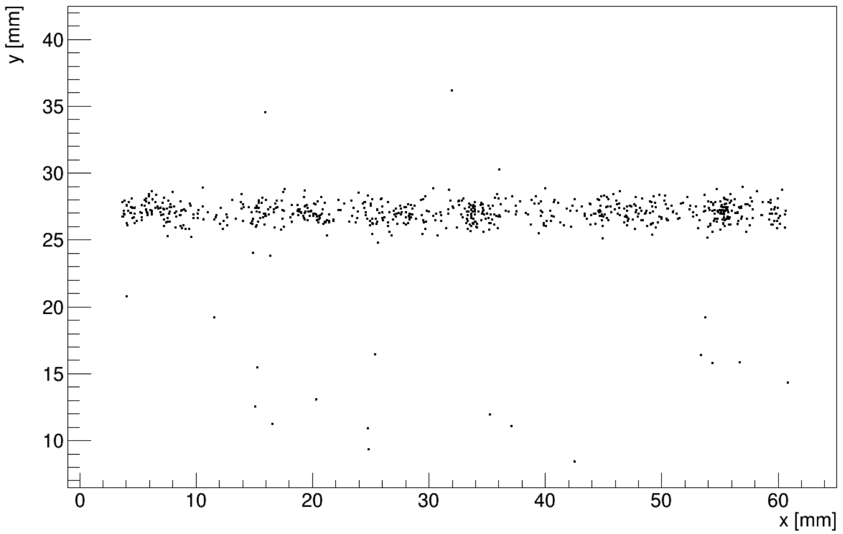


- **True Track:** is associated to the MC particle with the more than some percentages (60% , 70%) of MC particle hits .
- For track reconstruction the minor and major ellipse's axises length multiplied by 2

Simulation

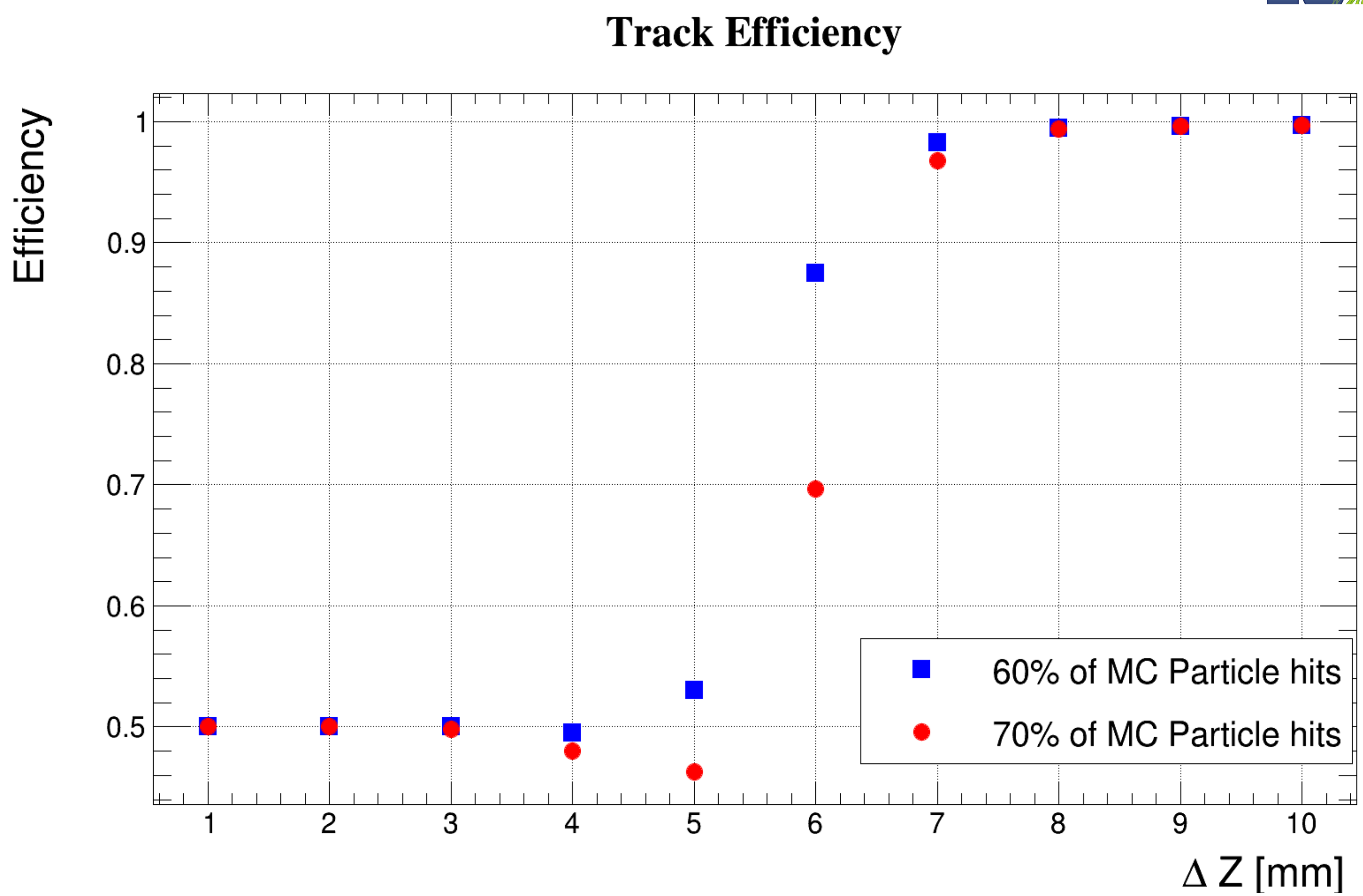
- Simulation of two tracks:
  - Simulation for Large Prototype TPC (LP-TPC) at DESY
  - Shooting two electrons 6 GeV, parallel to the X direction
  - Same Y offset
  - 590 mm from readout plane: Longitudinal resolution ≈ 1.95 mm in z direction
  - Vary ΔZ from 1mm to 10mm
  - 1000 events for each ΔZ: 2000 tracks for each ΔZ
  - B= 1T
  - Random Noise (Maximum 500)





Example of simulation with ΔZ =10 mm

Simulation



- For track reconstruction the minor and major ellipse's axises length multiplied by 3

Summery and Outlook:

- Calculating width of a track in Hough space
- Collecting inliers directly in Hough space by Bivariate normal distribution fit in Hough space
- Using Karimaki fit for XY-plane and straight line for SZ-plane calculating 5 parameters and covariance matrix.
- Preliminary analysis for one Octoboard shows this method is promising
- Tracks with 6 mm distance in Z direction and 3 mm in Y direction can be separated

- Optimization is ongoing
  - Tunning parameters to find any possibility of having better result
- Merging tracklets in order to have full track is ongoing
- Simulation for many Octoboards with two tracks

Backup

Analysis: based on simulation

Definition: Efficiency and Purity

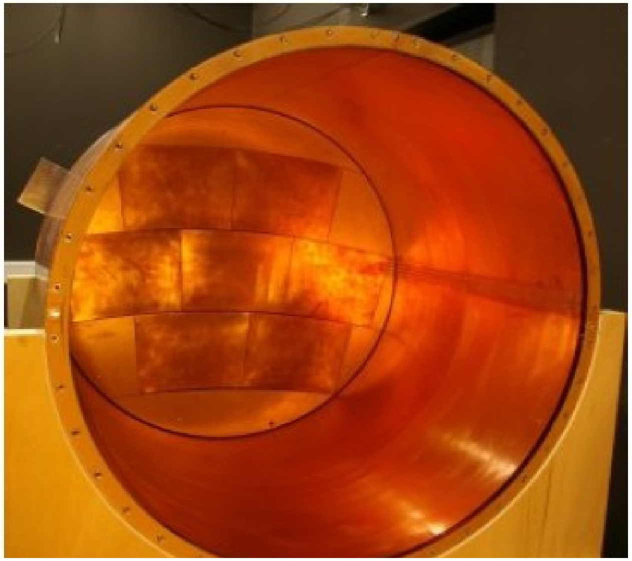
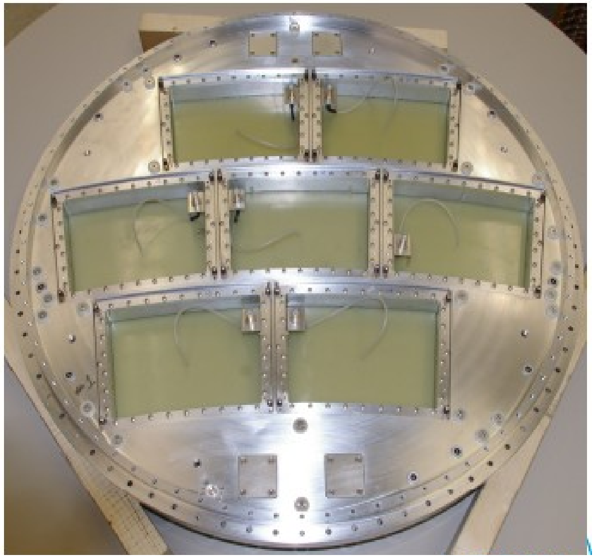
- **True Track:** is associated to the MC particle with the most hits on the track and more than some percentages (60% , 70%) of MC particle hits .
- **Track Efficiency:** Number of true track to all number of MC particle
- **Track Purity:** Number of true track to sum over number of true track, ghost and clone tracks
- **Right Hits:** Number of hits on the track from the associated MC particle
- **Hit Efficiency:** Number of right Hits to number of associated MC particle hits
- **Hit Purity**: Number of right hits to all hits of the true track

ILCTPC – DESY: Large Prototype TPC

- Test beam area at DESY(1-6 GeV e^- beams)
  - Infrastructure includes a large bore 1T magnet on a movable stage

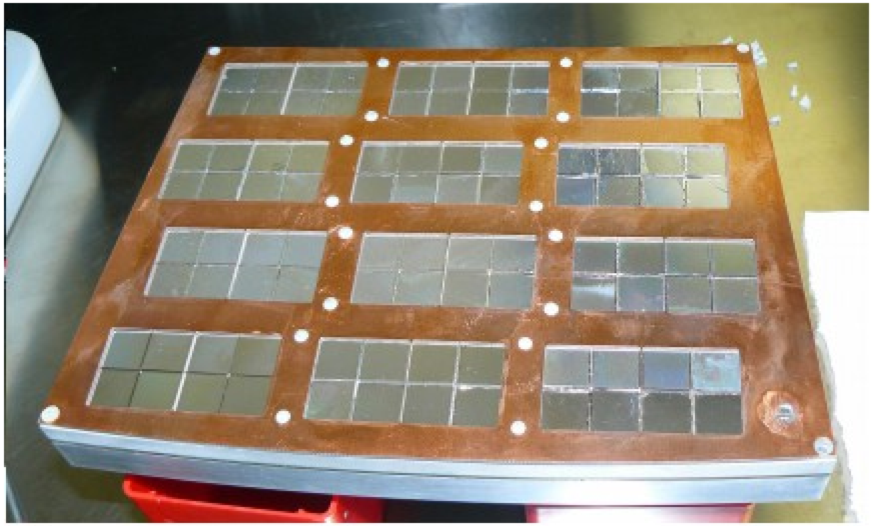
- Large Prototype (LP) built and installed to test scaling up of technologies and to compare different readout technologies on equal footing

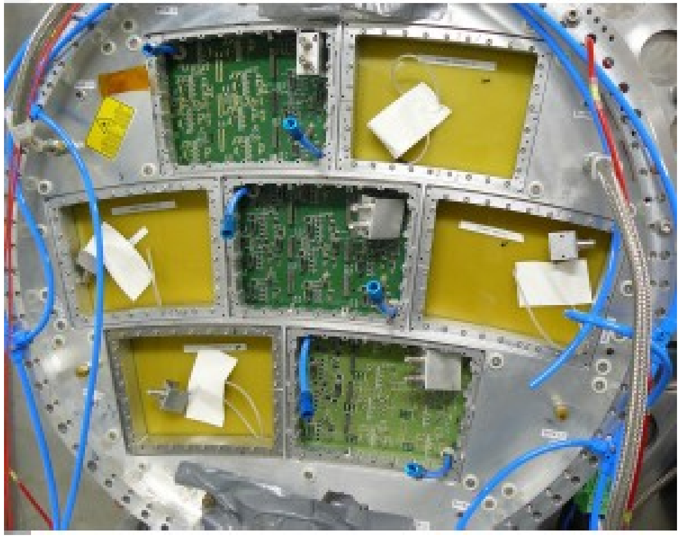
- LP field cage parameters:
  - Length: 61 cm, Diameter: 72 cm
  - Up to 25 kV => E_{drift} up to 350 V/cm
  - Wall material budget: 1.3% X_0
- Endplate is able to host 7 readout modules (dimensions ~22 X 17 cm^2)

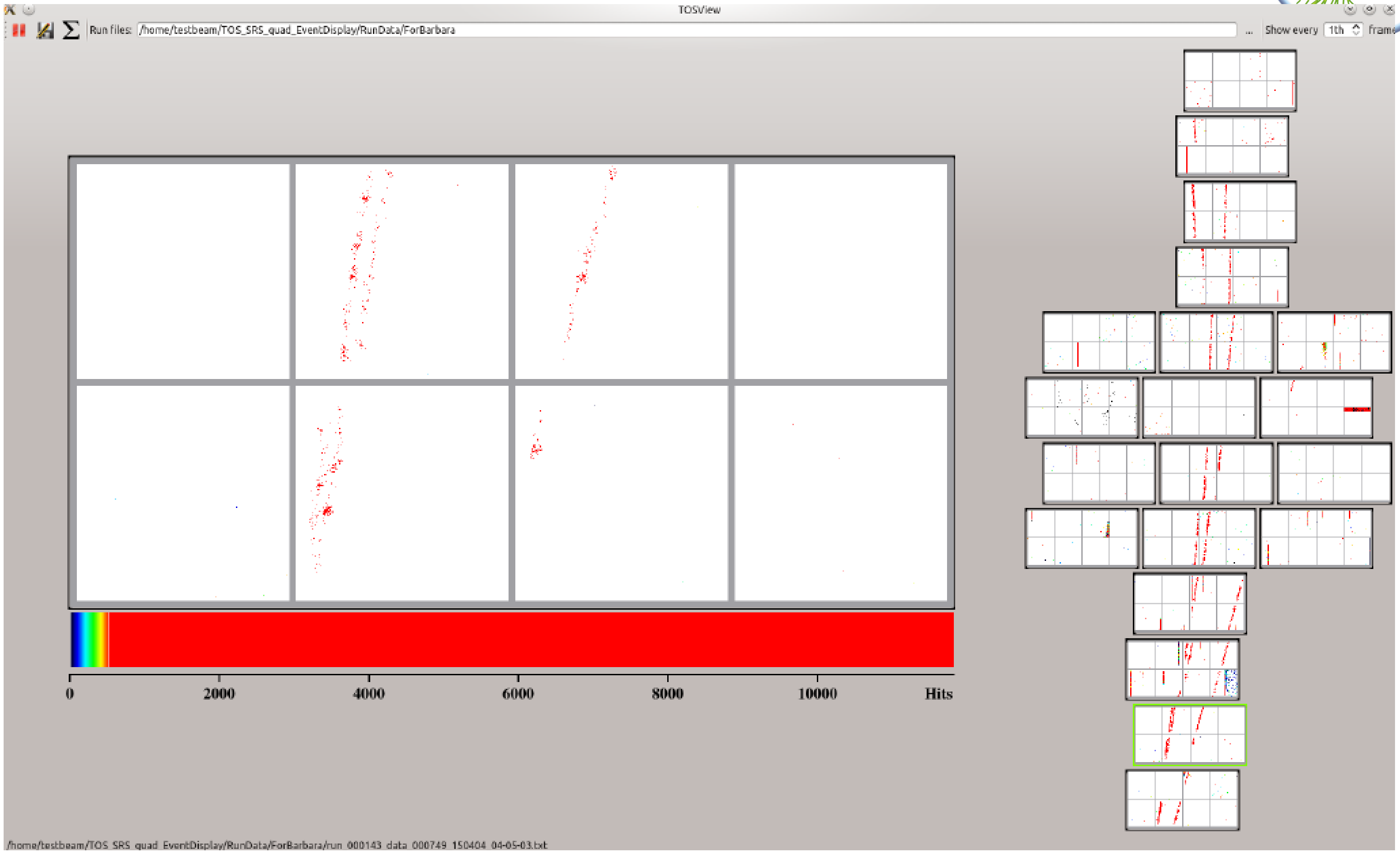
Infrastructure for test beam, TPC and Endplate from DESY group

- Test beam with 160 InGrid chips:
  - Central module with 96 chips (coverage 50 %)
  - 2 outer modules with 32 chips each
- Some Challenges:
  - InGrid production
  - Synchronized readout
  - Bonding on boards
  - LV distribution (up to 85 A @ 2.2 V)
  - Cooling
- The test beam was successful. During the test beam ~10^6 frames at a rate of around 5 Hz were collected.
- Test beam program:
  - Voltage scans (gas gain)
  - Z-scan
  - Momentum scan
  - Different angles
  - With and without magnetic field (B = 1 T)
  - Two different electrical drift fields





ILCTPC – Bonn: Preliminary results for Test beam



Further information about test beam results:

- Michael Lupberger, “Preliminary results from the 160 InGrid test beam”,University of Bonn, LCTPC-WP Meeting 10.09.2015,
- Jochen Kaminski,”Large Area Coverage of a TPC Endcap with GridPix Detectors”, University of Bonn,For the LCTPC collaboration , MPGD 2015, Trieste 12-15.10.2015,

LC-TPC parameter

Parameter			
Geometrical parameters	r_{in} 329 mm	r_{out} 1808 mm	z $\pm$ 2350 mm
Solid angle coverage	up to $\cos\theta \simeq 0.98$ (10 pad rows)		
TPC material budget	$\simeq 0.05$ X_0 including outer fieldcage in r		
	< 0.25 X_0 for readout endcaps in z		
Number of pads/timebuckets	$\simeq$ 1-2 $\times 10^6$/1000 per endcap		
Pad pitch/ no.padrows	$\simeq 1\times$ 6 mm^2 for 220 padrows		
σ_{point} in $r\phi$	$\simeq 60$ μm for zero drift, < 100 μm overall		
σ_{point} in rz	$\simeq 0.4 - 1.4$ mm (for zero – full drift)		
2-hit resolution in $r\phi$	$\simeq 2$ mm		
2-hit resolution in rz	$\simeq 6$ mm		
dE/dx resolution	$\simeq 5$ %		
Momentum resolution at B=3.5 T	$\delta(1/p_t) \simeq 10^{-4}$/GeV/c (TPC only)		