



*PRAHA -2017*

**EDS-2017**

« **Elastic and Diffractive Scattering** »

The structure and analytic properties of the scattering amplitude  
at LHC energies

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- \* Introduction
- \* Elastic hadron scattering – new data LHC
- \* Comparing the data with High Energy Generalized structure model (HEGS)
- \* The structure of the scattering amplitude in  $t$  and  $b$  - representation
- \* Simultaneously fit with the simplest phenomenological model
- \* Results and Summary

Predictions

$$\sqrt{s} = 14 TeV$$

Model	$\sigma_{tot}$	$\sigma_{el}/\sigma_{tot}$	$\sigma_{t\bar{t}}(t=0)$	$B(t=0)$
COMPET	111		0.11	
Marseilles	103	0.28	0.12	19
Dubna	128	0.33	0.19	21.
Pomer.(s+h)	150	0.29	0.24	21.4
Serpukhov	230	0.67		

Total cross sections

**TOTEM**

**ATLAS**

$$\sigma_{tot} = (98.3 \pm 2.8) mb;$$

$$(98.6 \pm 2.2) mb;$$

$$(99.1 \pm 4.3) mb;$$

$$(98.0 \pm 2.5) mb;$$

$$\sqrt{s} = 7000 GeV;$$

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$$\sigma_{tot} = (98.5 \pm 2.9) mb$$

$$\sigma_{tot} = (95.35 \pm 2.0) mb$$

$$\Delta = 3.15 mb;$$

$$\sqrt{s} = 8000 GeV;$$

$$\sigma_{tot} = (101.7 \pm 2.9) mb$$

$$\sigma_{tot} = (102.0 \pm 2.2) mb$$

$$\sigma_{tot} = (102.3 \pm 2.3) mb$$

$$\sigma_{tot} = (96.07 \pm 1.34) mb$$

$$\Delta = 5.93 mb;$$

$$F_1^{em}(t) = \alpha f_1^2(t) \frac{s - 2m^2}{t}; \quad F_3^{em}(t) = F_1^{em};$$

and for spin-flip amplitudes:

$$F_2^{em}(t) = \alpha \frac{f_2^2(t)}{4m^2} s; \quad F_4^{em}(t) = -F_2^{em}(t),$$

$$F_5^{em}(t) = \alpha \frac{s}{2m\sqrt{|t|}} f_1(t) f_2(t),$$

where the form factors are:

$$f_1(t) = \frac{4m_p^2 - (1+k)t}{4m_p^2 - t} G_d(t);$$

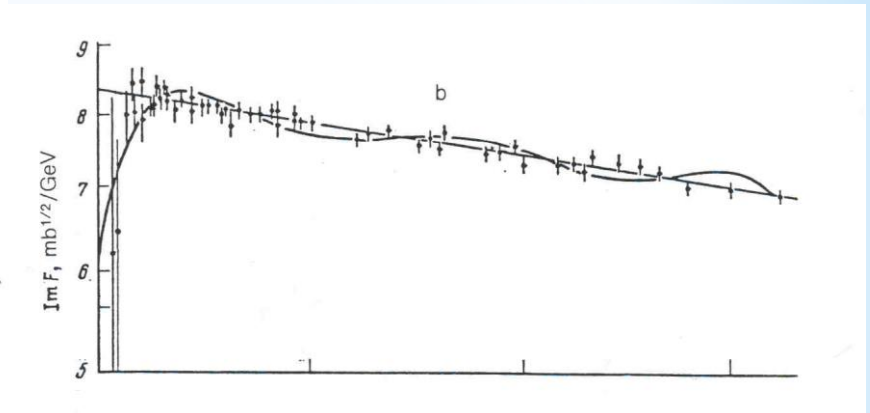
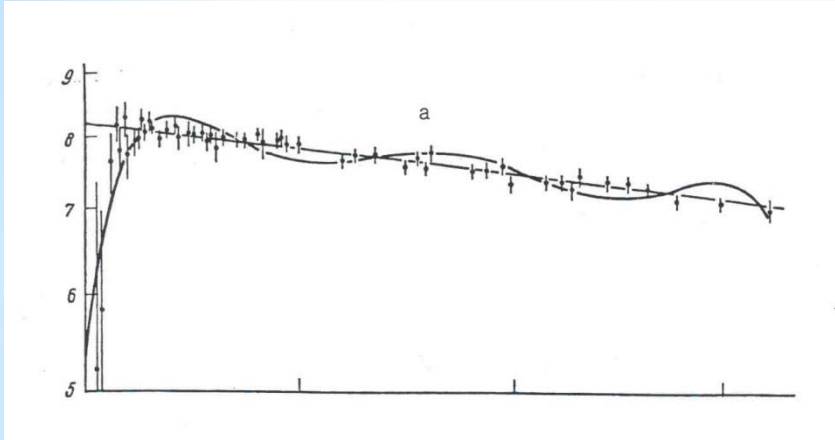
$$f_2(t) = \frac{4m_p^2 k}{4m_p^2 - t} G_d(t);$$

$$\frac{dN}{dt} = \mathcal{L} \left[ \frac{4\pi\alpha^2}{|t|^2} G^4(t) - \frac{2\alpha (\rho(s,t) + \phi_{CN}(s,t)) \sigma_{tot} G^2(t) e^{-\frac{B(s,t)|t|}{2}}}{|t|} + \frac{\sigma_{tot}^2 (1 + \rho(s,t)^2) e^{-B(s,t)|t|}}{16\pi} \right] \quad (1)$$

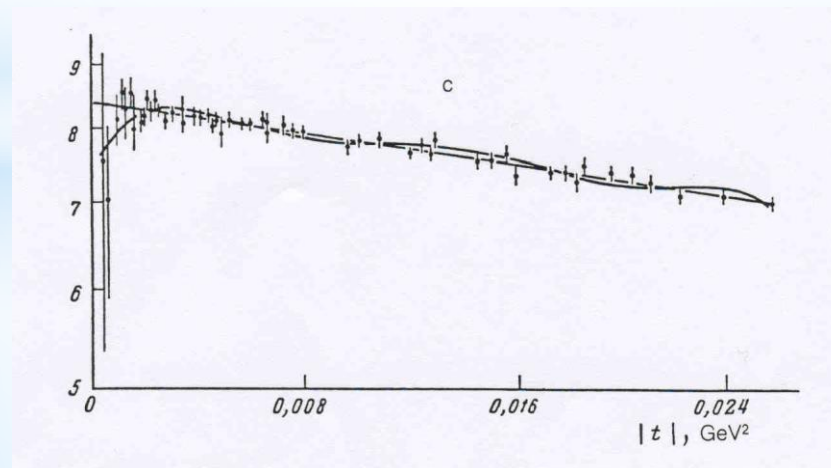
Proton-proton  $\sqrt{s} = 27.4 \text{ GeV}$ ;

*Exp.*:  $\rho = 0.012$ ,  $B = 11.95 \text{ GeV}^{-2}$ ,  $\sigma_{tot} = 40.65 \text{ mb}$ ;

*fit*:  $\rho = 0.033$ ,  $B = 14.5 \text{ GeV}^{-2}$ ,  $\sigma_{tot} = 40.9 \text{ mb}$ ;

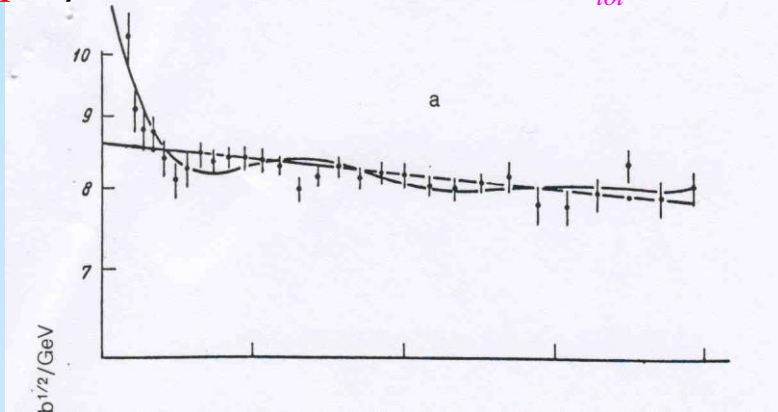


*fit*:  $\rho = 0.02$ ,  $B = 13.8 \text{ GeV}^{-2}$ ,  $\sigma_{tot} = 41.34 \text{ mb}$ ,  $n = 1.02$ ;

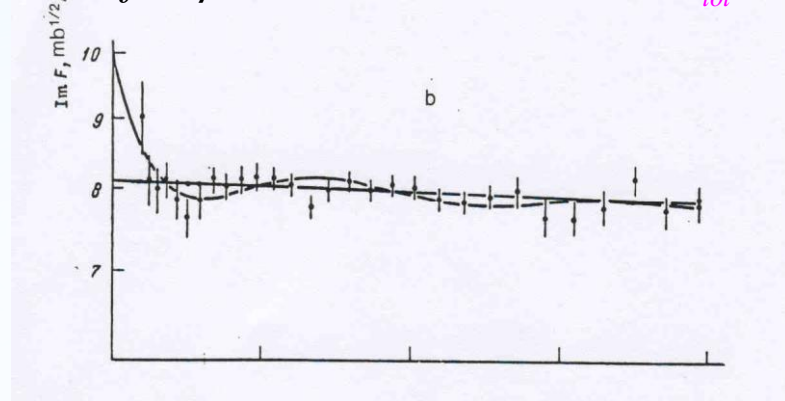


Proton-antiproton  $\sqrt{s} = 30.4 \text{ GeV}$ ;

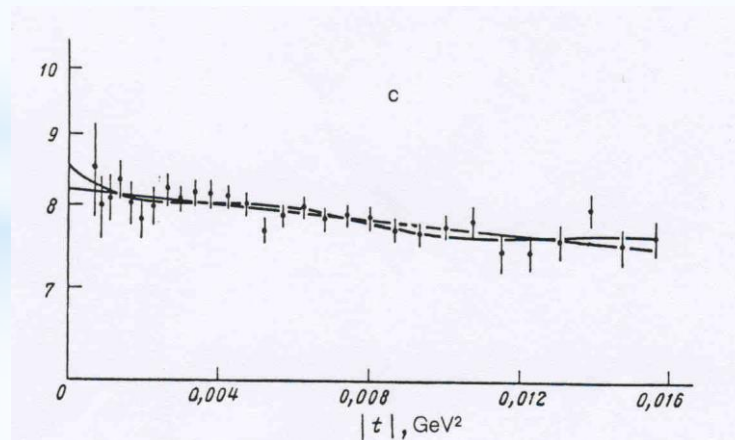
*Exp.*:  $\rho = 0.055$ ,  $B = 12.7 \text{ GeV}^{-2}$ ,  $\sigma_{tot} = 42.13 \text{ mb}$ ;



*fit*:  $\rho = 0.12$ ,  $B = 14.1 \text{ GeV}^{-2}$ ,  $\sigma_{tot} = 39.65 \text{ mb}$ ;



*fit*:  $\rho = -0.01$ ,  $B = 11.75 \text{ GeV}^{-2}$ ,  $\sigma_{tot} = 40.22 \text{ mb}$ ,  $n = 0.9$ ;



**Table 4:** Fit results with  $N_b = 1$ . Each column corresponds to a fit with different interference formula and/or nuclear phase.

	SWY, constant	KL, constant	KL, peripheral
step 1: $\chi^2/\text{ndf}$	48.0/27 = 1.78	48.1/27 = 1.78	27.7/27 = 1.03
step 2: $\chi^2/\text{ndf}$	180.8/58 = 3.12	181.2/58 = 3.12	64.3/58 = 1.11
$a$ [mb/GeV <sup>2</sup> ]	533 ± 23	533 ± 23	551 ± 23
$b_1$ [GeV <sup>-2</sup> ]	19.42 ± 0.05	19.42 ± 0.05	19.74 ± 0.05
$\rho$	0.05 ± 0.02	0.05 ± 0.02	0.10 ± 0.02
$\zeta_1$			800
$\kappa$			2.311
$\nu$ [GeV <sup>-2</sup> ]			8.161
$\sigma_{\text{tot}}$ [mb]	102.0 ± 2.2	102.0 ± 2.2	103.4 ± 2.3

In each case, the fit results are used to calculate the total cross-section via the optical theorem:

$$\sigma_{\text{tot}}^2 = \frac{16\pi (\hbar c)^2}{1 + \rho^2} a. \quad (27)$$



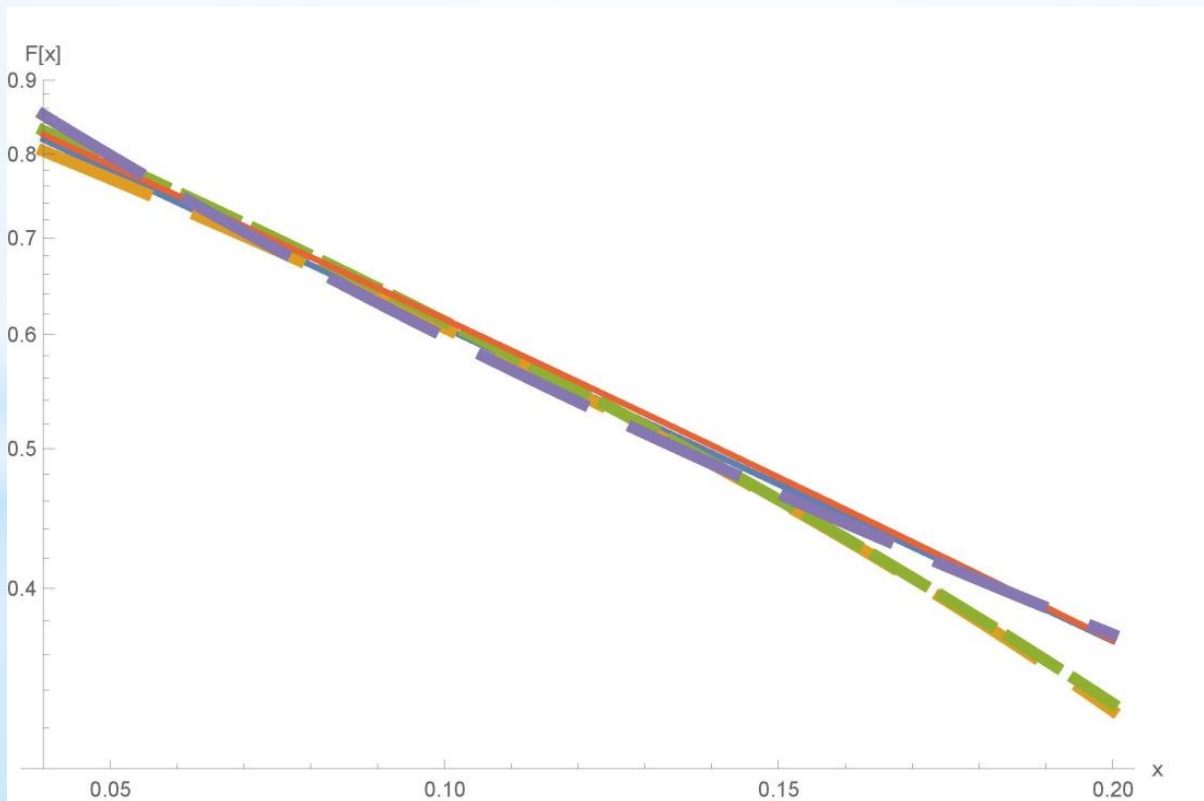
$$\Gamma(b) = e^{-b^2/R^2}; \quad \rightarrow F_1(t) \square e^{5t};$$

$$\Gamma(b) = \delta(R_0); \quad \rightarrow F_1(t) \square J_0(R\sqrt{|t|});$$

$$\Gamma(b) = e^{-(b-R_0)^2/R_0^2}; \quad \rightarrow F_1(t) \square e^{2.8t} J_0(2.8\sqrt{|t|});$$

$$\Gamma(b) = C(0-R_0); \quad \rightarrow F_1(t) \square J_1(R_0\sqrt{|t|}) / R_0\sqrt{|t|};$$

$$\Gamma(b) = e^{-\mu(\sqrt{4R_0^2-t})}; \quad \rightarrow F_1(t) \square e^{-5R_0(\sqrt{\mu^2-t})};$$



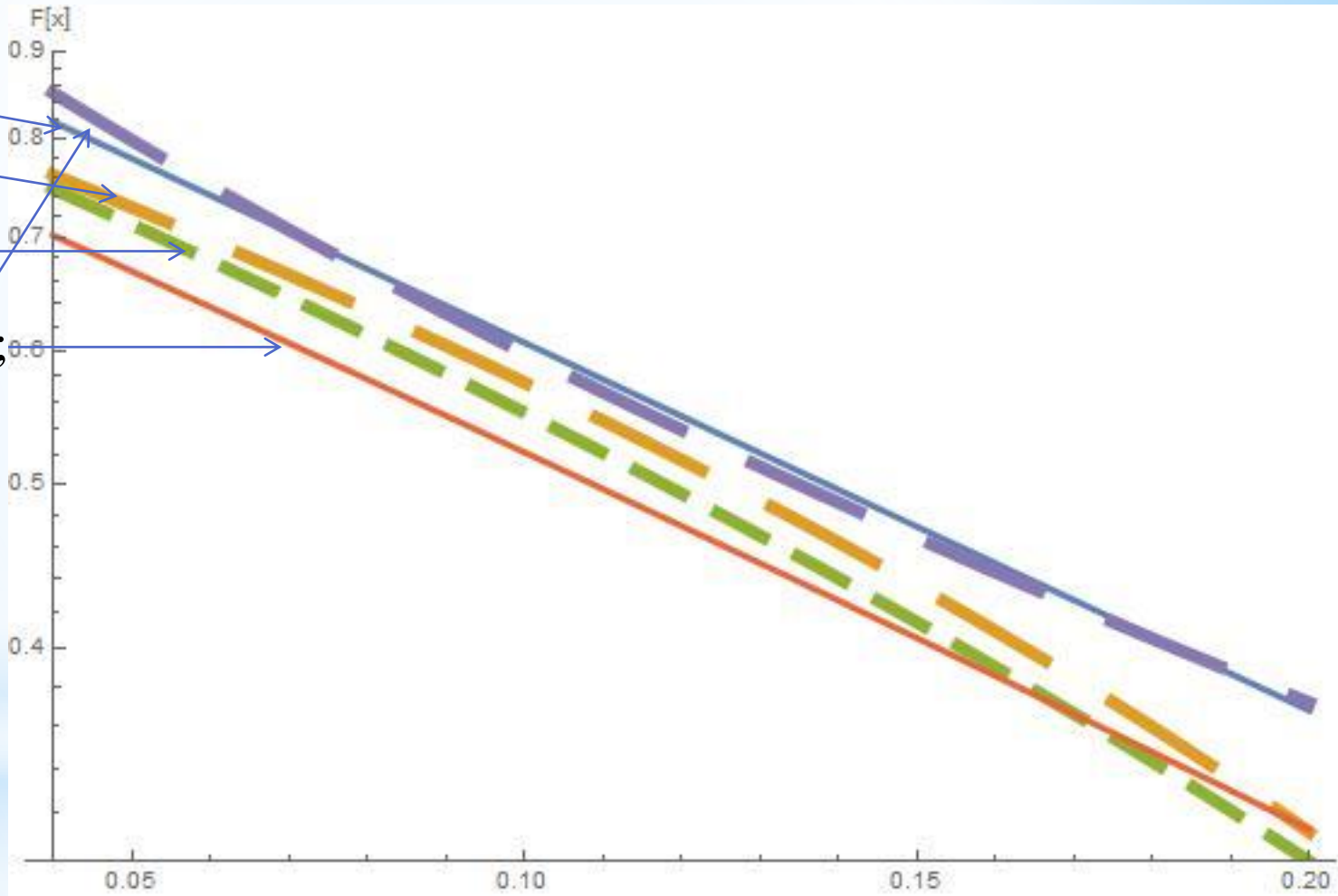
$$e^{5t};$$

$$J_0(4\sqrt{|t|});$$

$$e^{2.8t} J_0(2.8\sqrt{|t|});$$

$$J_1(6\sqrt{|t|}) / 6\sqrt{|t|};$$

$$e^{4.5R_0(\sqrt{\mu^2-t})};$$



BSW<sub>1</sub> - C. Bourrely, J. Soffer, T.T. Wu - ( )

BSW<sub>2</sub> - C. Bourrely, J. Soffer, T.T. Wu - ( )

HEGS<sub>0</sub> – O.V.S. -

HEGS<sub>1</sub> – O.V.S. -

	BSW <sub>1</sub>	BSW <sub>2</sub>	HEGS <sub>0</sub>	HEGS <sub>1</sub>
N <sub>exp</sub>	369	955	980	3416
N <sub>par</sub>	7+Regge	11+Regge	3+2	5+4
$\sqrt{s}$ , <u>GeV</u>	24 ÷ 630	13.4 ÷ 1800	52 ÷ 1800	9 ÷ 8000
$\Delta t$ , <u>GeV<sup>2</sup></u>	0.1 ÷ 2.6	0.1 ÷ 5	$8.7 \cdot 10^{-4} \div 10$	$3.7 \cdot 10^{-4} \div 15$
( $\Sigma x^2$ )/N	4.45	1.95	1.8	1.28

# GPDs

limit  $Q^2 = 0$ , and  $\xi = 0$

X.Ji [Sum Rules \(1997\)](#)

$$\Phi_{\xi=0}(\mathbf{x};t) = \Phi(\mathbf{x};t)$$

$$F_1^q(t) = \int_{-1}^1 dx H^q(x, \xi, t);$$

$$F_2^q(t) = \int_{-1}^1 dx E^q(x, \xi, t);$$

$$H^q(\mathbf{x};t) = H^q(\mathbf{x},0,t) + H^q(-\mathbf{x},0,t)$$

$$E^q(\mathbf{x};t) = E^q(\mathbf{x},0,t) + E^q(-\mathbf{x},0,t)$$

$$F_1^q(t) = \int_0^1 dx \square^q(x, \xi, t);$$

$$F_2^q(t) = \int_0^1 dx \mathcal{E}^q(x, \xi, t);$$

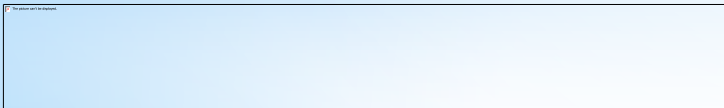
$$\int_{-1}^1 dx x [H^q(x, \xi, t) + E^q(x, \xi, t)] = A_q(\Delta^2) + B_q(\Delta^2);$$

$$\hat{s} = s / s_0 e^{-i\pi/2}; \quad s_0 = 4m_p^2.$$

$$F_1^B(s, t) = h_2 G_{em}(t) (\hat{s})^{\Delta_1} e^{\alpha_1 t \ln(\hat{s})}; \quad F_3^B(s, t) = h_3 G_A(t)^2 (\hat{s})^{\Delta_1} e^{\alpha_1/4 t \ln(\hat{s})};$$

$$F^B(\hat{s}, t) = F_2^B(\hat{s}, t) (1 + R_1 / \sqrt{\hat{s}}) + F_3^B(\hat{s}, t) (1 + R_2 / \sqrt{\hat{s}}) + F_{odd}^B(s, t);$$

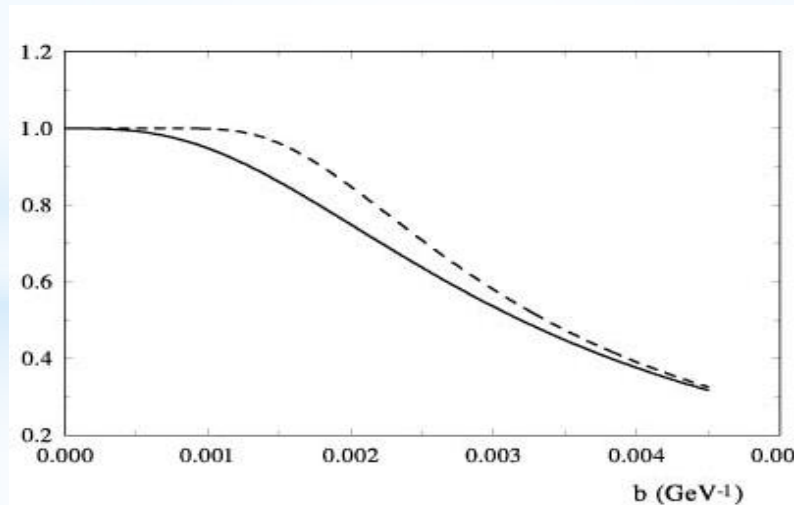
$$F_{Odd}^B(s, t) = h_{Odd} G_A(t)^2 (\hat{s})^{\Delta_1} \frac{t}{1 - r_o^2 t} e^{\alpha_1/4 t \ln(\hat{s})};$$

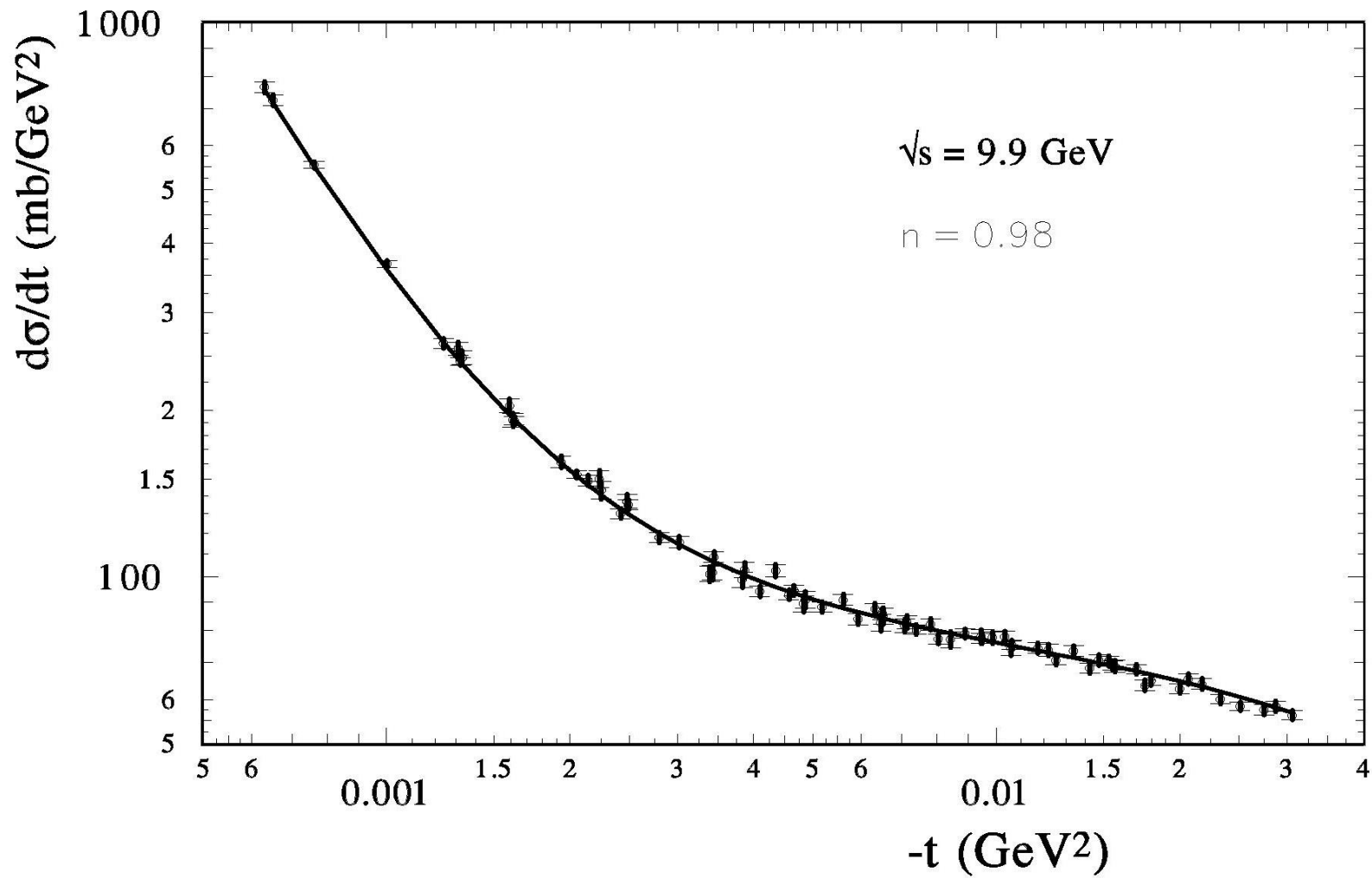


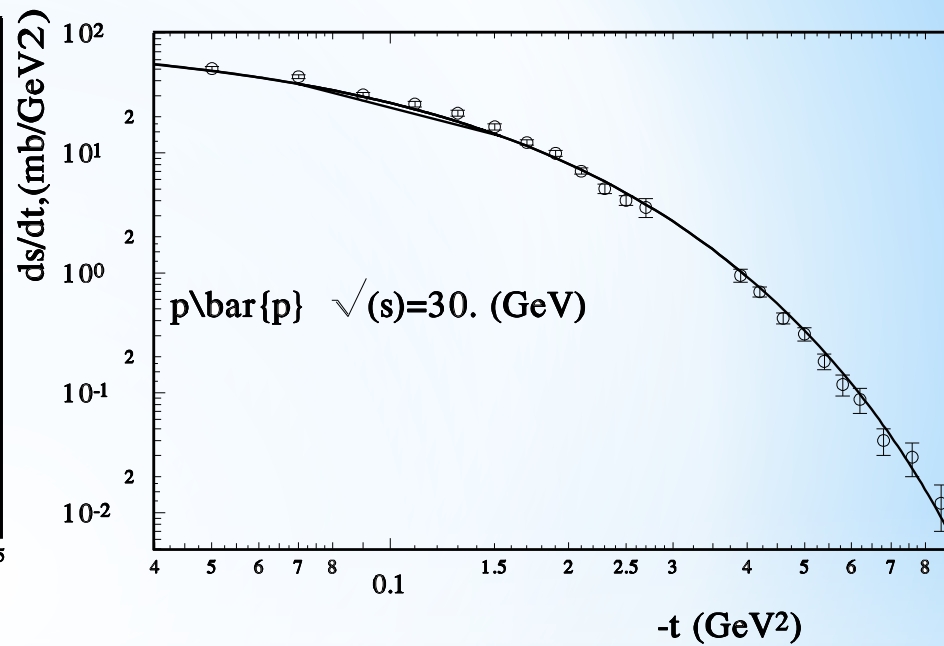
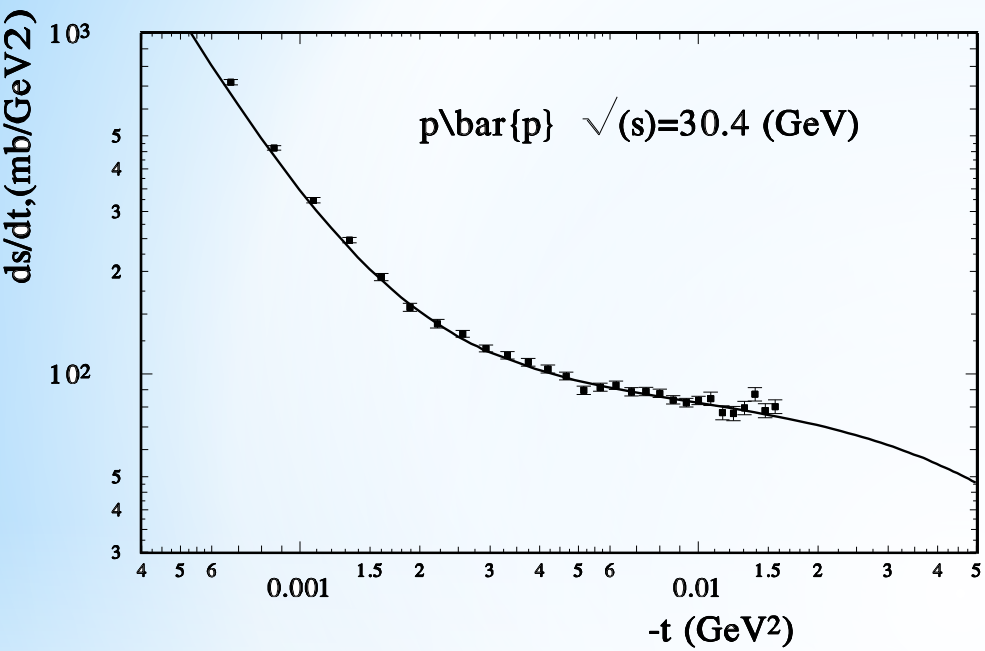
UNITARIZATION → eikonal representation

$$\chi(s,b) = 2\pi \int_0^\infty q J_0(bq) F_B^h(s,q) dq \qquad \chi(s,b) = -\frac{1}{2k} \int_{-\infty}^{\infty} dz V[\sqrt{z^2 + b^2}]$$

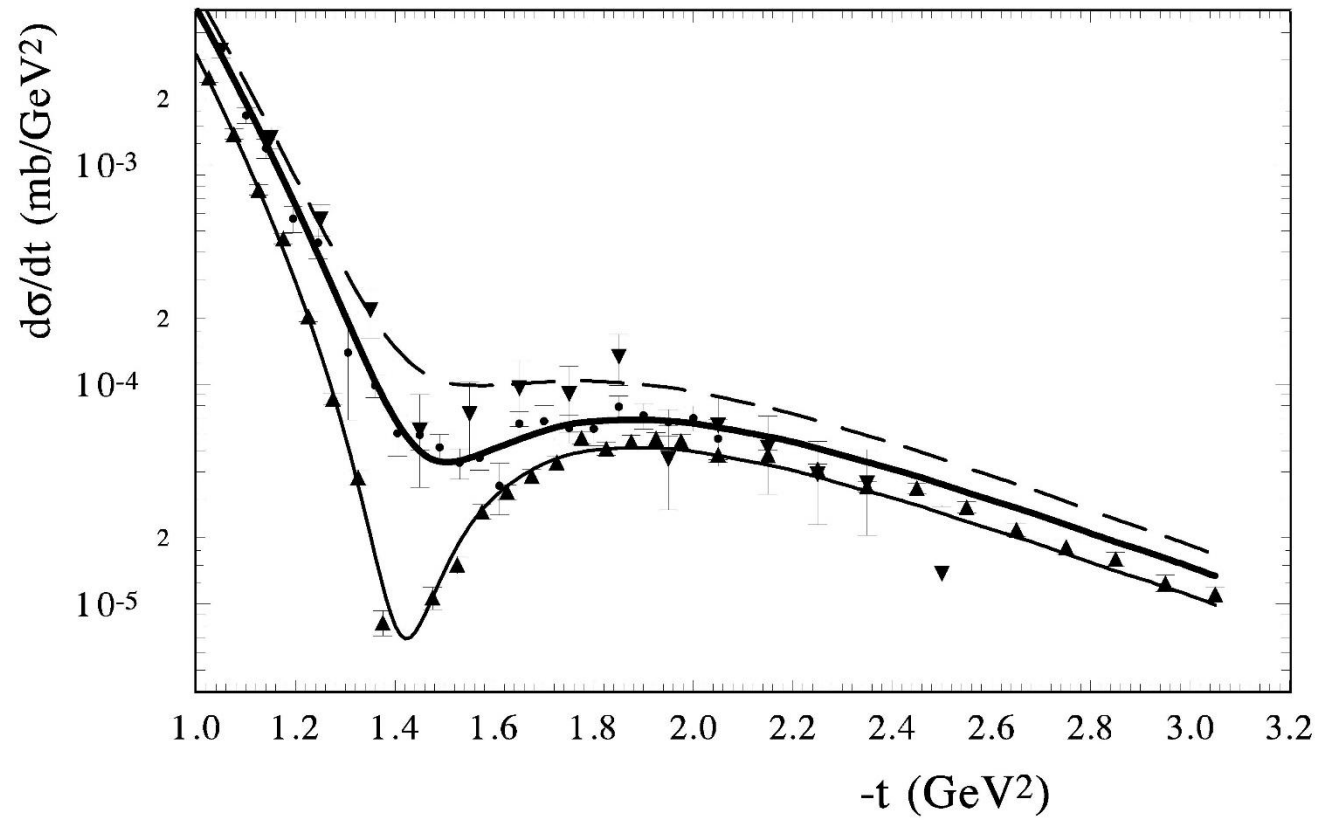
$$F^h(s,t) = \frac{1}{2\pi} \int_0^\infty b J_0(bq) [1 - e^{-\chi(s,b)}] db$$



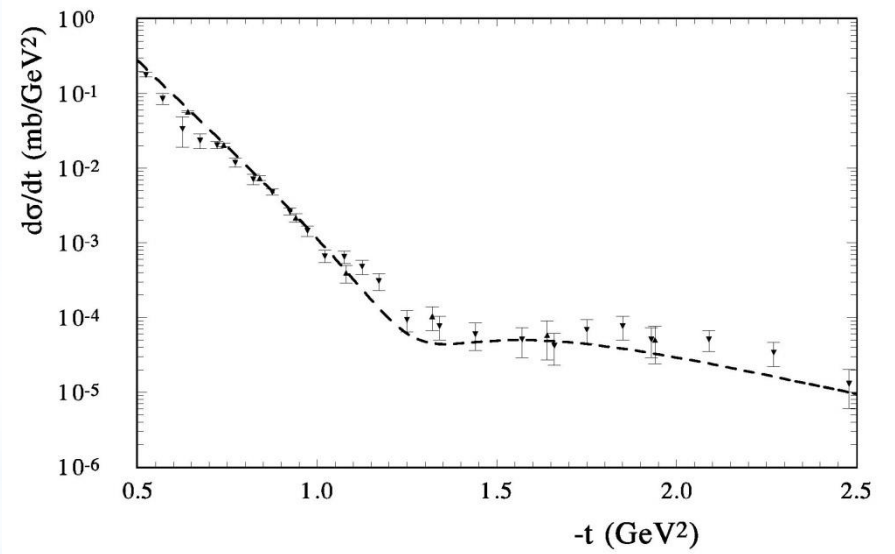
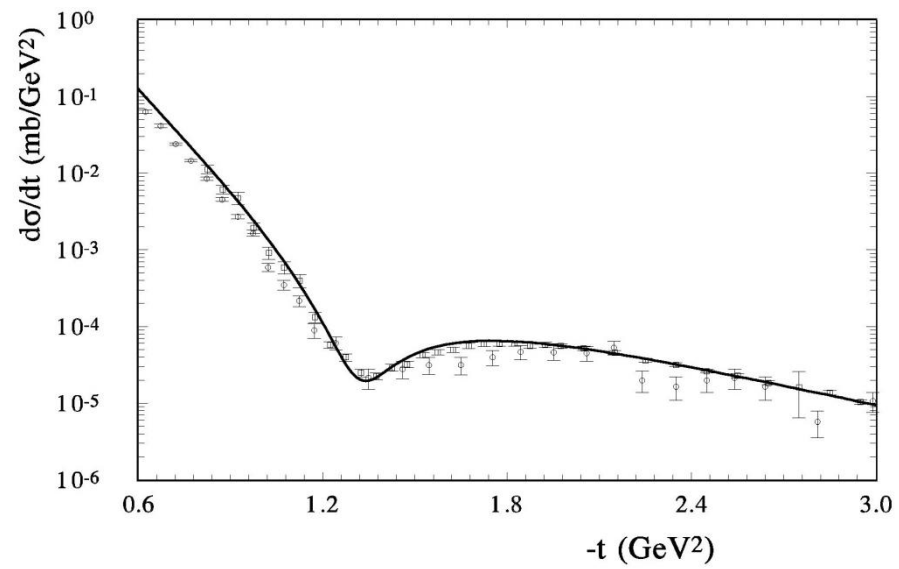


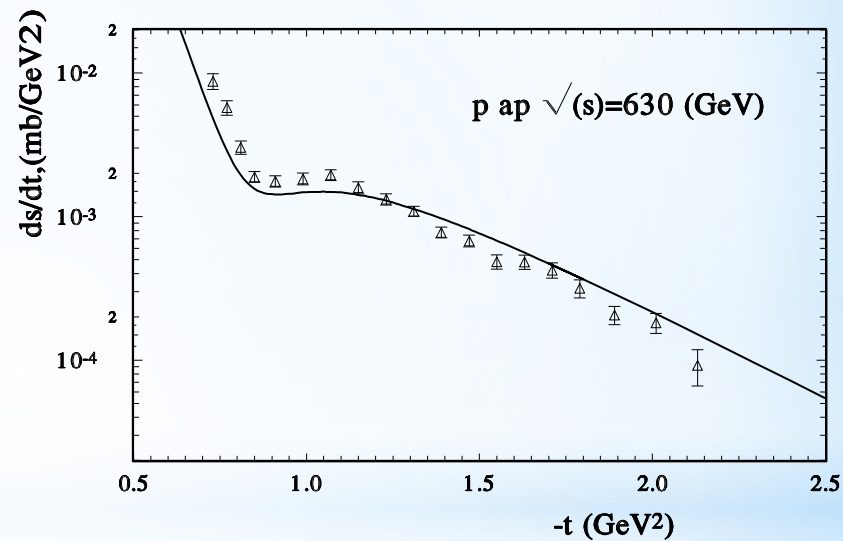
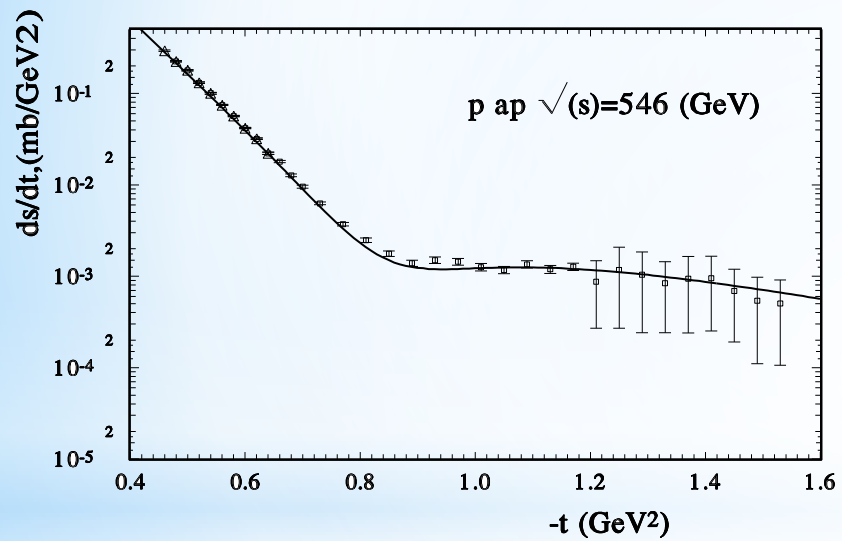






$$\sqrt{s} = 52.8 \text{ GeV}$$

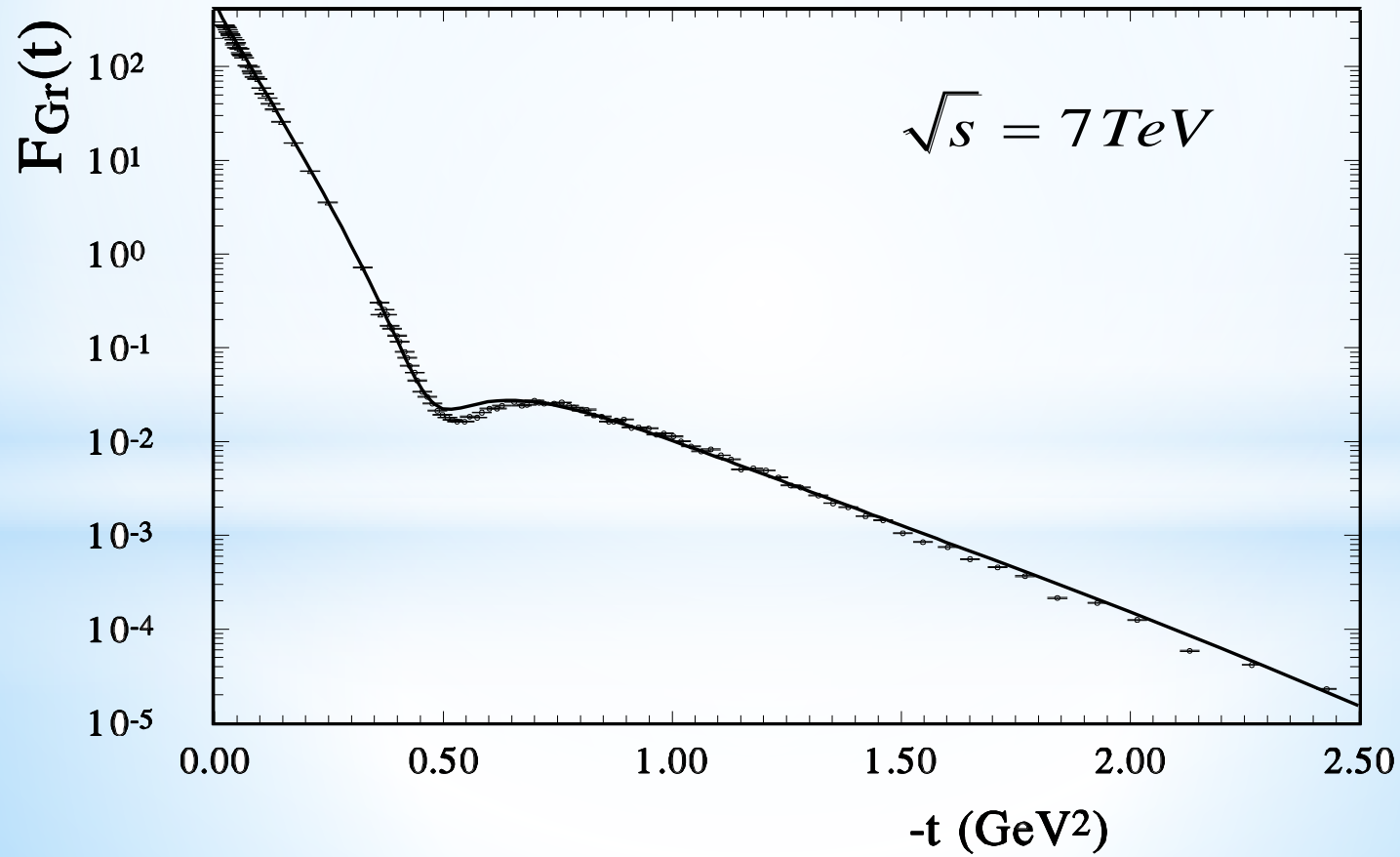




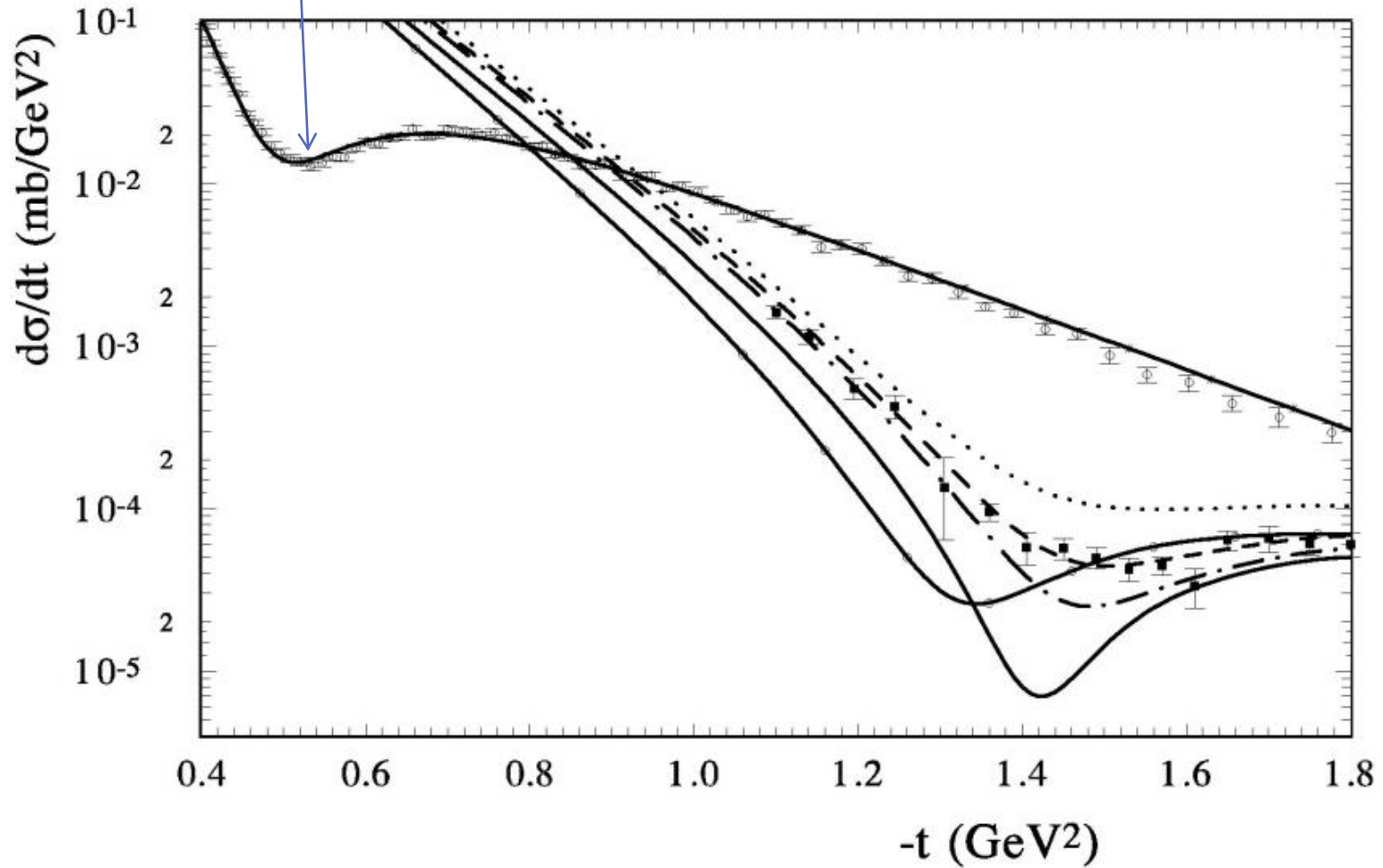
$$52.8 \text{ GeV} \leq \sqrt{s} \leq 1.8 \text{ TeV}$$

O.V.S. Eur. Phys. J. C (2012) 72:2073(2012);

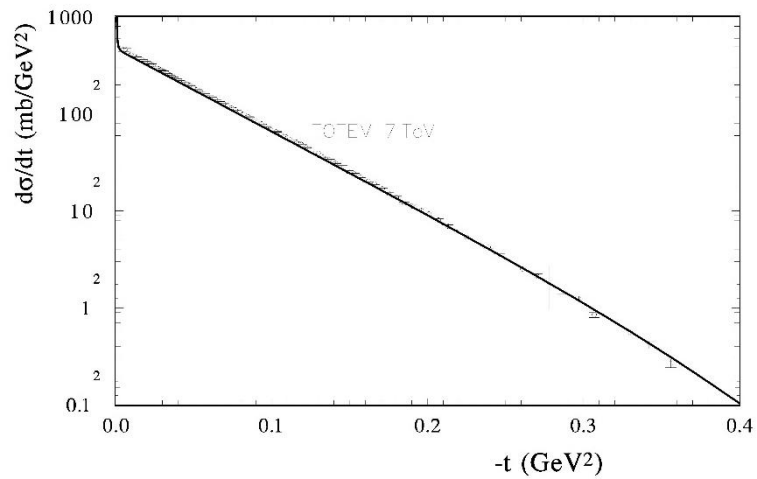
predictions



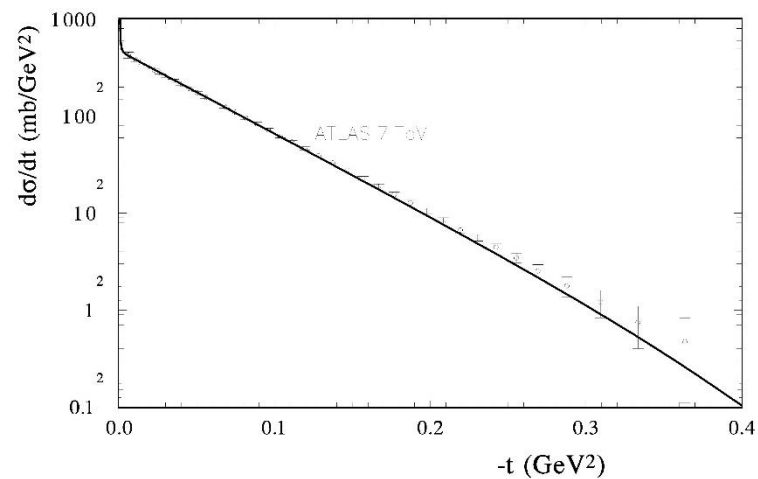
TOTEM data – 7 TeV



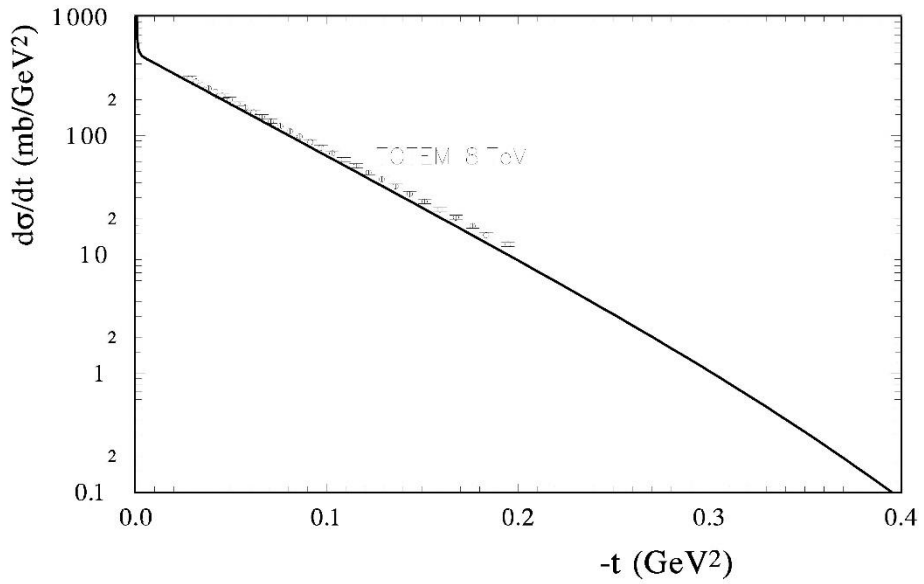
7 TeV (TOTEM) -  $t$  [0.00515 – 0.371]  
 $n = 0.94$



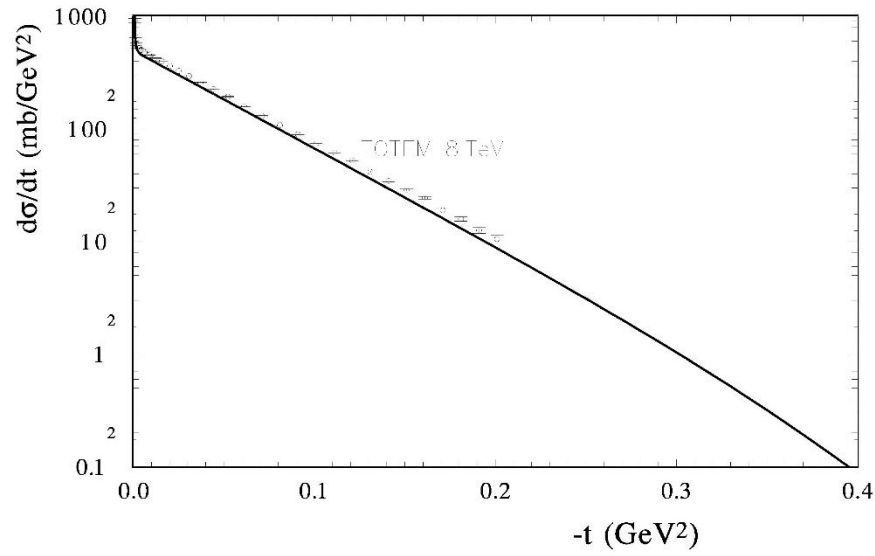
7 TeV (ATLAS) -  $t$  [0.0062 – 0.3636]  
 $n = 1.0$



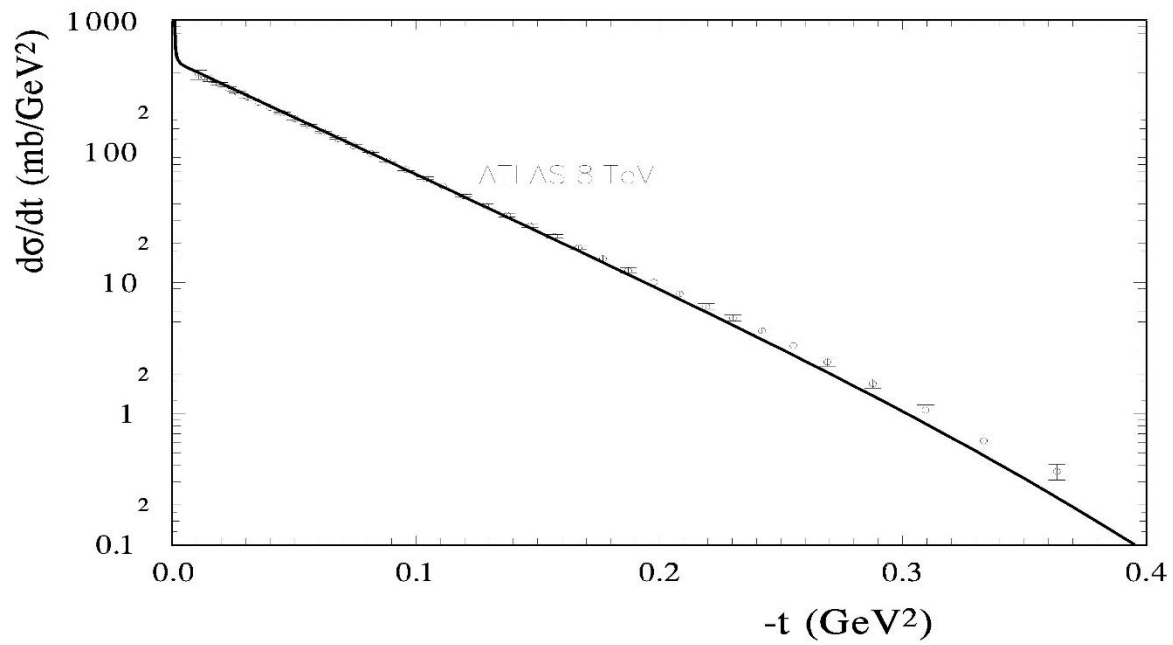
8 TeV (TOTEM) - t [0.0285 – 0.1947]  
n=0.9



8 TeV (TOTEM) - t [0.000741 – 0.201]  
n=0.9

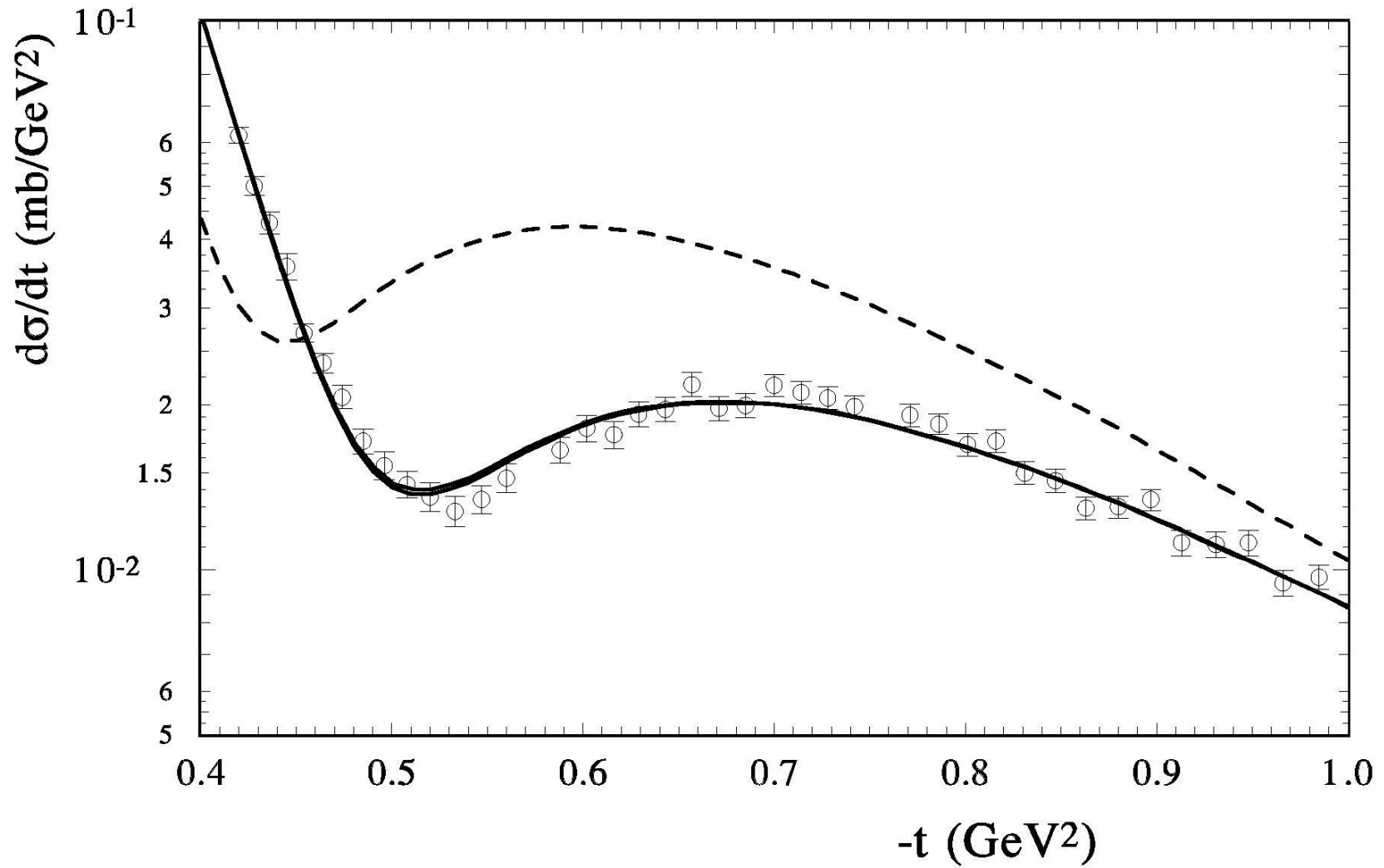


8 TeV (ATLAS) -  $t$  [0.01050 – 0.3635]  
 $n=1.0$

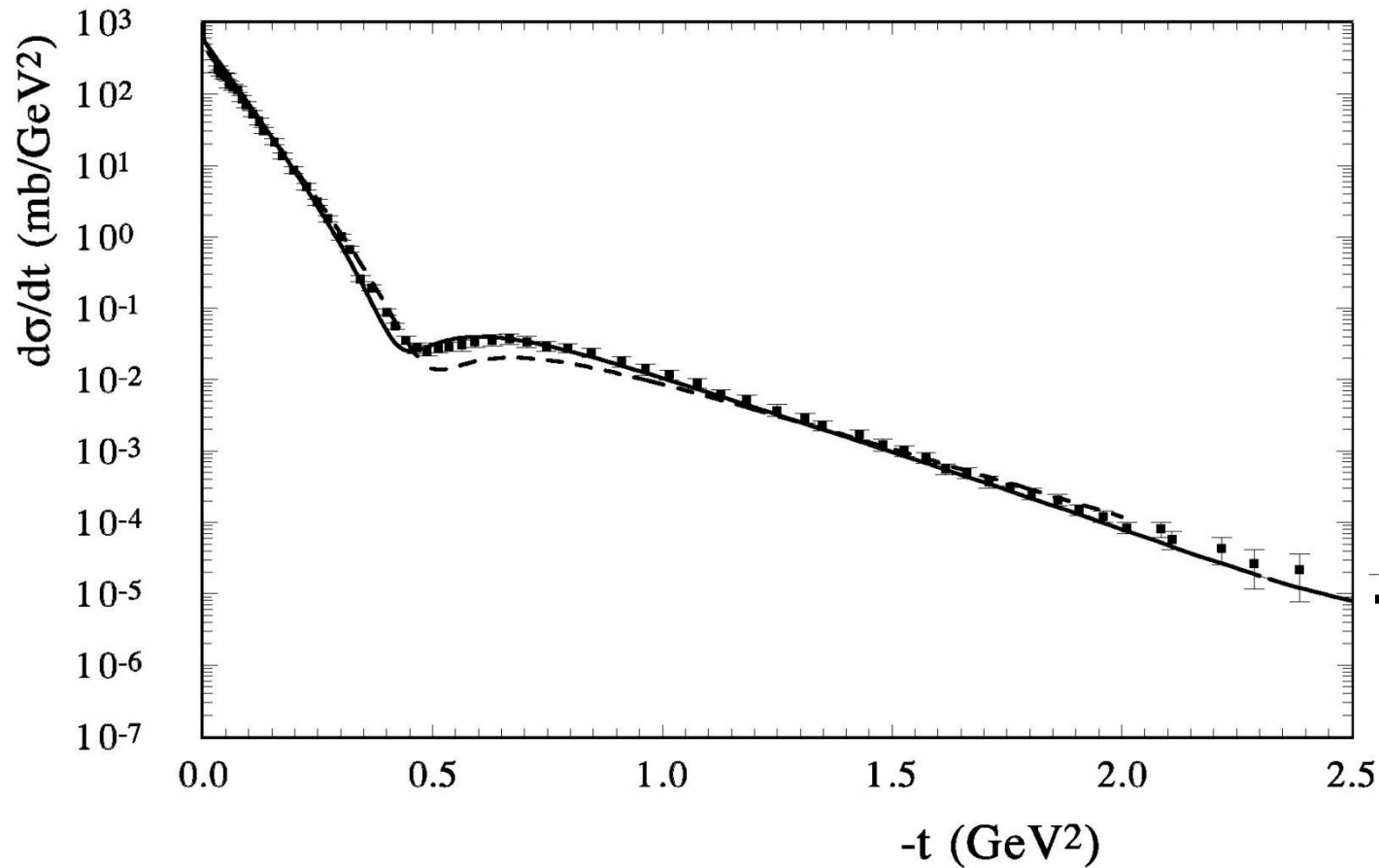




$$\sqrt{s} = 7 \text{ TeV}; \quad 14 \text{ TeV}.$$

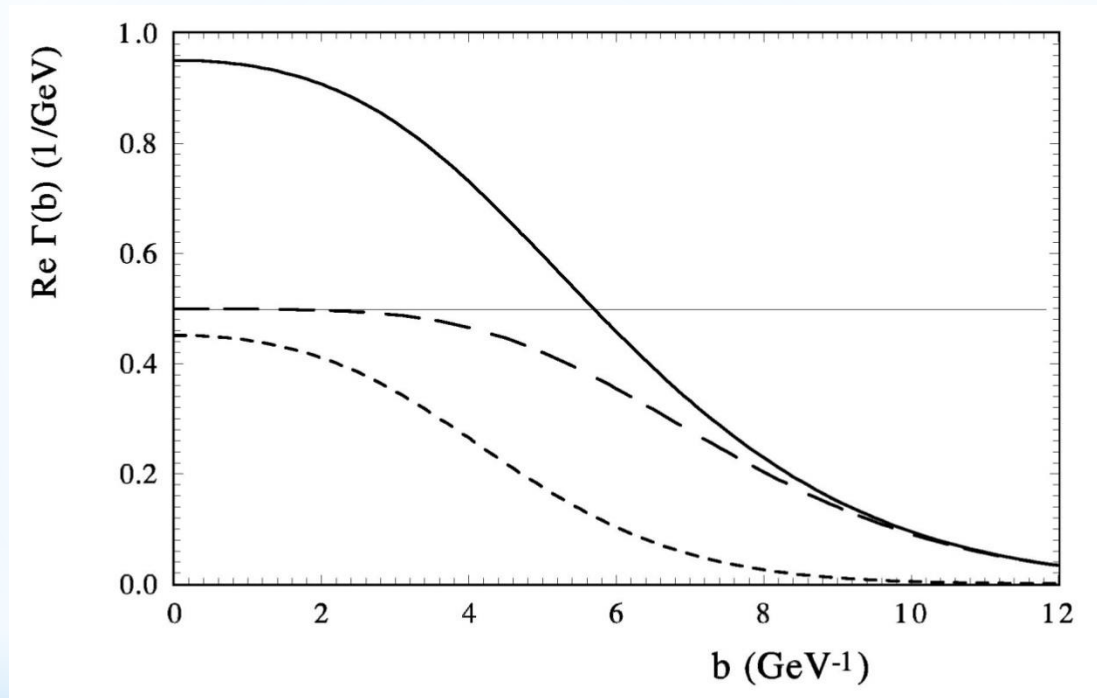


13 TeV (line and points) and 7 TeV – dashed line



( The normalization of the 13 TeV data on the model calculations)

$$\sqrt{s} = 14 \text{ TeV}$$

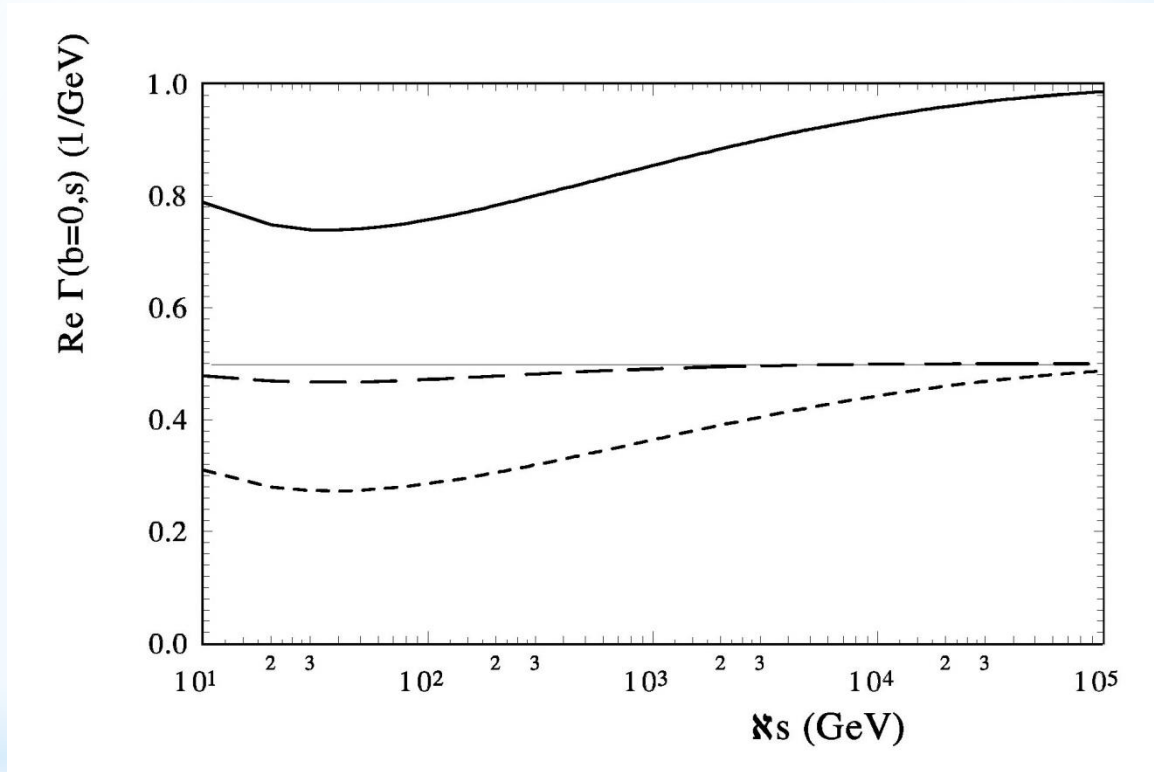


$\Gamma(s, b)_{tot}$  (hard line)

$\Gamma(s, b)_{elast.}$  (dashed line);

$\Gamma(s, b)_{inel.}$  (long dashed line)

# The energy dependence of

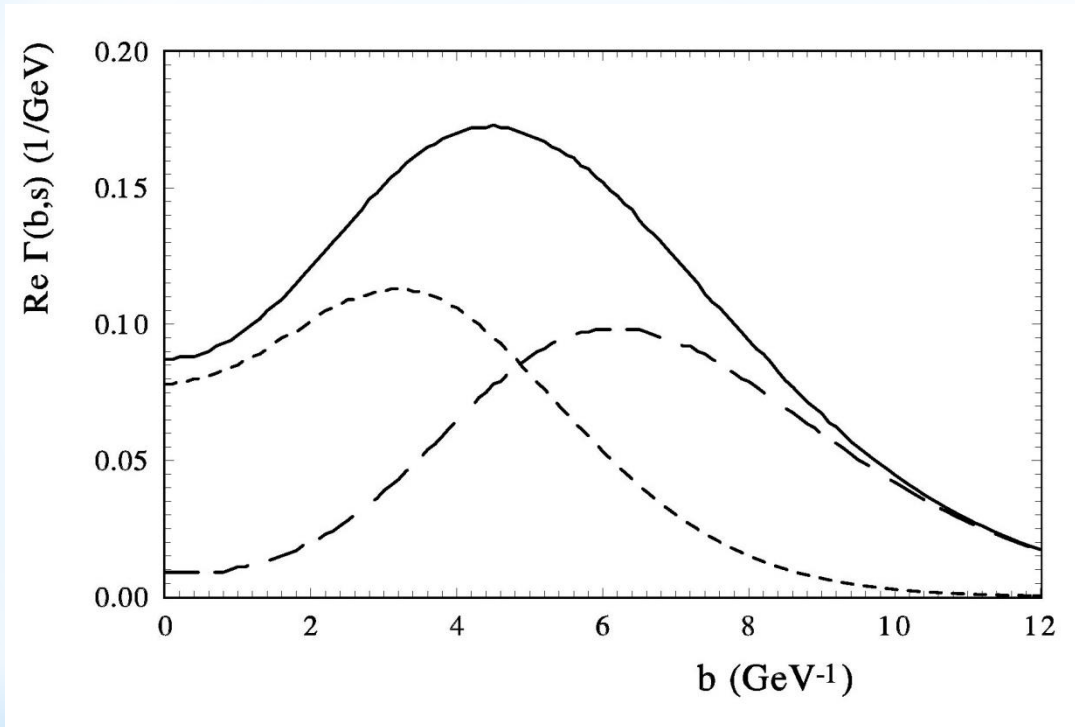


$\Gamma(s, b = 0)_{tot}$  (hard line)

$\Gamma(s, b = 0)_{elast.}$  (dashed line);

$\Gamma(s, b = 0)_{inel.}$  (long dashed line)

The difference between  $\sqrt{s} = 14\text{TeV}$  and  $\sqrt{s} = 7\text{TeV}$

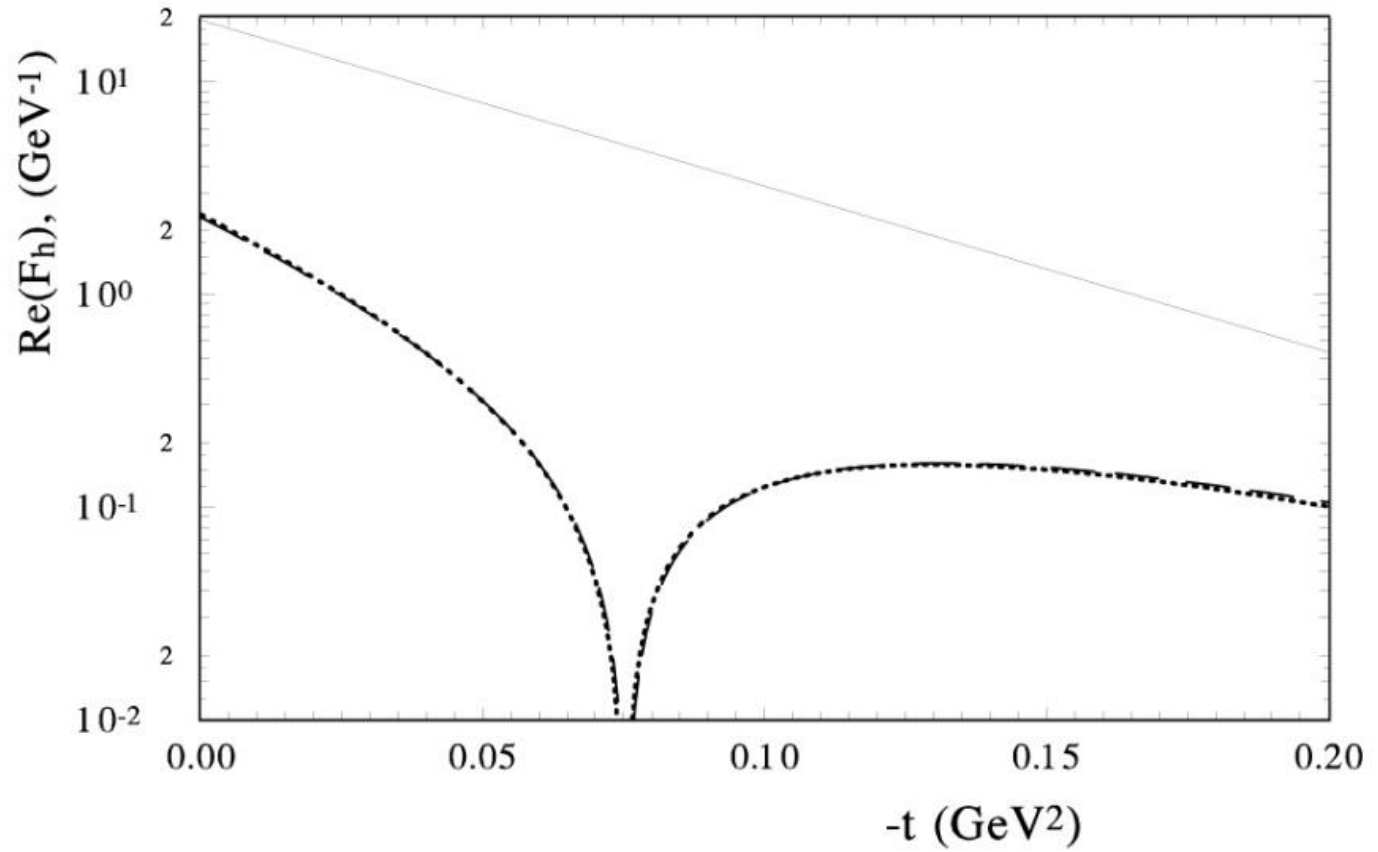


$\Gamma(s,b)_{tot}$  (hard line)

$\Gamma(s,b)_{elast.}$  (dashed line);

$\Gamma(s,b)_{inel.}$  (long dashed line)

$$\text{Im } F(s, t) = (C + a \ln(s)^2) e^{-Bt \ln(s)};$$



The description of the differential cross sections at LHC energies

$$F_h(s, t) = ih \ln(s)^2 (1. - i\rho) e^{B_1/2t + B_2/2t^2} G^2(t);$$

with the form factor

$$G(t) = \frac{4m_p^2 - \mu t}{4m_p^2 - t} \frac{\Lambda^2}{(\Lambda - t)^2}.$$

with  $m_p$  being the proton mass,  $\Lambda = 0.71 \text{ GeV}^2$  and  $\mu = 2.79$ .

# The description of the differential cross sections at LHC energies

$\sum_N \chi^2; (\text{err.})$	$h$	$B_1$	$B_2$	$\rho$	$\sigma_{tot}$ 7TeV/8TeV	$n_i$ T;A;—T;T;A
48337 ( $\sigma_{st.}^2$ )	0.30	0.55	-0.39	$0_b$	95.3/98.2	1.; 1.;  1.; 1.; 1.
421 ( $\sigma_{st.+syst.}^2$ )	0.30	0.55	-0.45	$0_b$	95.1/98.0	1.; 1.;  1.; 1.; 1.
1812 (7 TeV)	0.31	0.58	-0.26	$0_b$	96.7	1.03; 0.98.;
( $\sigma_{st.}^2$ ) (8TeV)					99.7	1.06; 1.06; 0.94



# LHC

- \* The new data bounded essentially the limits of the models.

The elastic scattering reflects the generalized structure of the hadron and the scattering amplitude is satisfied the basic analytic properties. GPDs open the new way to connections of the elastic and inelastic interactions

The standard eikonal approximation works perfectly from  $\sqrt{s}=9$  GeV up to 8 TeV.

But: where is the hard Pomeron?;  $t$  and  $s$  dependence of the Odderon?  
Unitarization – eikonal?

- \* The problems of the determination of  $\rho(s,t)$  and  $\sigma_{tot}(s)$ 
    - The thin structure of the slope -  $B(s,t)$ ,
    - (non-exponential, oscillations)
- Asymptotic of the scattering amplitude - wide region

Wait the high precision of the new data at small  $t$  and 13 TeV

TOTEM;

ATLAS

**THANKS**  
**FOR YOUR ATTENTION**

END

