Eikonal model analysis of elastic proton-proton collisions at 53 GeV and 8 TeV

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Contemporary descriptions of elastic collisions of (charged) hadrons

- measured elastic $\mathrm{d}\sigma/\mathrm{d}t$ given by

$$\frac{\mathrm{d}\sigma(s,t)}{\mathrm{d}t} = \frac{\pi}{sp^2}\left|F^{\mathrm{C+N}}(s,t)\right|^2 \tag{1}$$

  - $F^{\mathrm{C+N}}(s,t)$ - complete elastic scattering amplitude of Coulomb-hadronic interaction depending on both Coulomb $F^{\mathrm{C}}(s,t)$ and hadronic $F^{\mathrm{N}}(s,t)$ amplitudes
  - eq. (1) allows "separation" of Coulomb interaction and to study less known hadron (nuclear) scattering; Coulomb interaction is assumed to be well known from QED
  - Coulomb-hadronic interference used to constrain t-dependence of phase of $F^{\mathrm{N}}(s,t)$

- two approaches for description of elastic collisions of charged hadrons (amplitude $F^{\mathrm{C+N}}(s,t)$)
  - West and Yennie (Feynman diagram technique)
  - eikonal model

Approach of West and Yennie

- H. A. Bethe, "Scattering and polarization of protons by nuclei", Ann. Phys. **3**, 190–240 (1958)

$$F^{\mathrm{C+N}}(s,t) = F^{\mathrm{C}}(s,t)\,\mathrm{e}^{\mathrm{i}\alpha\phi(s,t)} + F^{\mathrm{N}}(s,t) \tag{2}$$

- G. B. West and D. R. Yennie, "Coulomb interference in high-energy scattering", Phys. Rev. **172**, 1413–1422 (1968)
  integral formula for relative phase

$$\alpha\phi(s,t) = \mp\alpha\left[\ln\left(\frac{-t}{s}\right) + \int_{-4p^2}^{0} \frac{\mathrm{d}t'}{|t-t'|}\left(1 - \frac{F^{\mathrm{N}}(s,t')}{F^{\mathrm{N}}(s,t)}\right)\right]. \tag{3}$$

- simplified interference formula of WY (1968)

$$F^{\mathrm{C+N}}_{\mathrm{WY}}(s,t) = \pm\frac{\alpha s}{t}G_1(t)G_2(t)\,\mathrm{e}^{\mathrm{i}\alpha\phi(s,t)} + \frac{\sigma^{\mathrm{tot,N}}(s)}{4\pi}p\sqrt{s}\,(\rho(s)+\mathrm{i})\,\mathrm{e}^{B(s)t/2} \tag{4}$$

  where (see also Locher 1967)

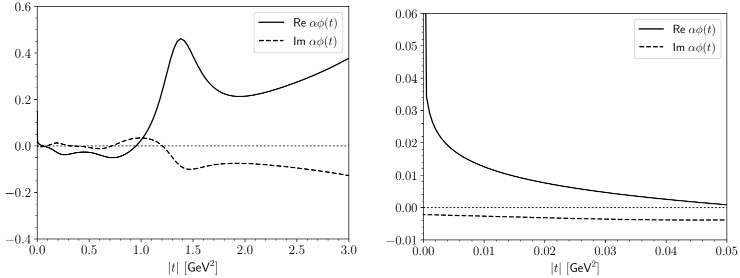
$$\alpha\phi(s,t) = \mp\alpha\left[\ln\left(\frac{-B(s)t}{2}\right) + \gamma\right] \tag{5}$$

  assuming for all kinematically allowed values of t
  - t-independence of phase of $F^{\mathrm{N}}(s,t)$, i.e., quantity $\rho(t) = \frac{\mathrm{Re}\,F^{\mathrm{N}}(t)}{\mathrm{Im}\,F^{\mathrm{N}}(t)}$ = const
  - purely exponential $\left|F^{\mathrm{N}}(s,t)\right|$ in t, i.e., diffractive slope $B(t) = \frac{2}{\left|F^{\mathrm{N}}(t)\right|}\frac{\mathrm{d}}{\mathrm{d}t}\left|F^{\mathrm{N}}(t)\right|$ = const
- used widely in the era of ISR for determination of $\sigma^{\mathrm{tot,N}}$, quantity $\rho(t=0)$ and $B(t=0)$

- relative phase $\alpha\phi(s,t)$ defined as real (defined as imaginary part of a complex function)
- Does imaginary part of $\alpha\phi(s,t)$ given by eq. (3) equal to zero (as it should) for all values of t if $\rho(t) \neq$ const?
  - No, analytical arguments: V. Kundrát, M. Lokajíček, and I. Vrkoč, "Limited validity of West and Yennie integral formula for elastic scattering of hadrons", Phys. Lett. **B656**, 182–185 (2007)
  - No, numerical calculations: see the following plots at 53 GeV and 8 TeV (corresponding hadronic amplitudes $F^{\mathrm{N}}(s,t)$ determined with the help of the eikonal model fitted to experimental data)

Relative phase $\alpha\phi(s,t)$ at $\sqrt{s} = 53$ GeV

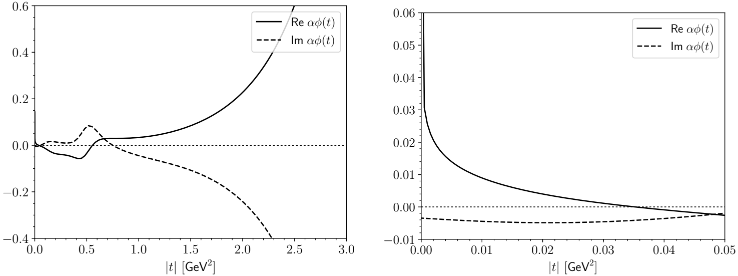
- example $F^{\mathrm{N}}(s,t)$ with strongly t-dependent hadronic phase (Fit VII at 53 GeV, see next slides)
- $\mathrm{Im}\,\alpha\phi(t) \neq 0$ **already at low values** of $|t| \Rightarrow$ **contradiction** in the model



(a) peripheral case **(b)** peripheral case (zoom)

Relative phase $\alpha\phi(s,t)$ at $\sqrt{s} = 8$ TeV

- example $F^{\mathrm{N}}(s,t)$ with strongly t-dependent hadronic phase (Fit VII at 8 TeV, see next slides)
- again $\mathrm{Im}\,\alpha\phi(t) \neq 0$ **already at low values** of $|t| \Rightarrow$ **contradiction** in the model



(a) peripheral case **(b)** peripheral case (zoom)

- $\Rightarrow$ WY approach cannot be used for data analysis with arbitrary hadronic amplitude
- several other problems and limitations exist, too
- see detailed discusion in J. Procházka and V. Kundrát, "Eikonal model analysis of elastic hadron collisions at high energies", arXiv: hep-th/1606.09479 (2016)
- one should look for different and more general description of Coulomb-hadronic intereference

Eikonal model approach

- introduces dependence of elastic collisions on impact parameter
- several authors start from it or have been developing it (Glauber, van Hove, Miettinen, Islam, Cahn,...); results on various level of sophistication
- Coulomb-hadronic interference formula derived by Kundrát and Lokajíček (1994)

$$F^{C+N}_{eik}(s,t) = \pm\frac{\alpha s}{t} G_1(t)G_2(t) + F^{N}(s,t)[1 \mp i\alpha \bar{G}(s,t)] \tag{6}$$

where

$$\bar{G}(s,t) = \int_{t_{min}}^{0} dt' \left\{ \ln\left(\frac{t'}{t}\right) \frac{d}{dt'}[G_1(t')G_2(t')] - \frac{1}{2\pi}\left[\frac{F^{N}(s,t')}{F^{N}(s,t)} - 1\right] I(t,t') \right\} \tag{7}$$

and

$$I(t,t') = \int_{0}^{2\pi} d\Phi'' \frac{G_1(t'')G_2(t'')}{t''}. \tag{8}$$

- derived for *any* s and t, no a priory restriction imposed on $F^{N}(s,t)$
- allows description of data in the whole measured t-range and taking into account both the Coulomb and hadronic interactions simultaneously
  $\Rightarrow$ consistent description of data (no duality)

Hadronic quantities - I

Several physically interesting quantities derived from the hadronic amplitude $F^{\mathrm{N}}(s,t)$

- total cross section (optical theorem)

$$\sigma^{\mathrm{tot,N}}(s) = \frac{4\pi}{p\sqrt{s}} \, \mathrm{Im}\, F^{\mathrm{N}}(s, t=0) \tag{9}$$

- elastic and inelastic cross section

$$\sigma^{\mathrm{el,N}} = \int \frac{\mathrm{d}\sigma^{\mathrm{el,N}}}{\mathrm{d}t}; \qquad \sigma^{\mathrm{inel}} = \sigma^{\mathrm{tot,N}} - \sigma^{\mathrm{el,N}} \tag{10}$$

- elastic hadronic amplitude in b-space - Fourier-Bessel transformation (Adachi, Kotani, Takeda, Islam, ...)

$$\begin{aligned} h_{\mathrm{el}}(s,b) &= h_1(s,b) + h_2(s,b) \\ &= \frac{1}{4p\sqrt{s}} \int_{t_{\min}}^{0} F^{\mathrm{N}}(s,t) J_0(b\sqrt{-t})\mathrm{d}t + \frac{1}{4p\sqrt{s}} \int_{-\infty}^{t_{\min}} \lambda(s,t) J_0(b\sqrt{-t})\mathrm{d}t \end{aligned} \tag{11}$$

- unitarity condition at *finite* energies

$$\mathrm{Im}\, h_1(s,b) + c(s,b) = |h_1(s,b)|^2 + g_1(s,b) + K(s,b) + c(s,b) \tag{12}$$

Hadronic quantities - II

- profile functions
  - main b-dependent characteristics of collisions, introduced in analogy to description of some optics phenomena (light meeting an obstacle of a given profile which describes its absorptive properties)
  - sometimes interpreted as probability or corresponding distribution functions of impact parameter
  -

$$D^{\text{el}}(s,b) \equiv 4\,|h_1(s,b)|^2, \tag{13}$$

$$D^{\text{tot}}(s,b) \equiv 4\,(\text{Im}\, h_1(s,b) + c(s,b)), \tag{14}$$

$$D^{\text{inel}}(s,b) \equiv 4\,(g_1(s,b) + K(s,b) + c(s,b)) \tag{15}$$

- cross sections on b-space (X=tot, el, inel)

$$\sigma^{\text{X}}(s) = 2\pi \int_0^\infty b\text{d}b\; D^{\text{X}}(s,b). \tag{16}$$

mean-square values of impact parameter b

- definition ($n = 2$ and $w(b) = 2\pi b$)

$$\langle b^n \rangle^{\mathrm{X}} = \frac{\int\limits_0^\infty b^n\ w(b) D^{\mathrm{X}}(s,b) \mathrm{d}b}{\int\limits_0^\infty w(b) D^{\mathrm{X}}(s,b) \mathrm{d}b} \tag{17}$$

- expressions for the mean-square values as functions of $F^{\mathrm{N}}(s,t)$

V. Kundrát, M. V. Lokajíček, and D. Krupa, "Impact parameter structure derived from elastic collisions", Phys. Lett. B **544**, 132–138 (2002)

$$\begin{aligned}
\langle b^2 \rangle^{\mathrm{el}} =& \langle b^2 \rangle^{\mathrm{mod}} + \langle b^2 \rangle^{\mathrm{ph}} \\
=& \frac{4 \int\limits_{t_{\min}}^{0} \mathrm{d}t |t| \left(\frac{\mathrm{d}}{\mathrm{d}t} \left| F^{\mathrm{N}}(s,t) \right| \right)^2}{\int\limits_{t_{\min}}^{0} \mathrm{d}t \left| F^{\mathrm{N}}(s,t) \right|^2} + \frac{4 \int\limits_{t_{\min}}^{0} \mathrm{d}t \left| F^{\mathrm{N}}(s,t) \right|^2 |t| \left(\frac{\mathrm{d}}{\mathrm{d}t} \zeta^{\mathrm{N}}(s,t) \right)^2}{\int\limits_{t_{\min}}^{0} \mathrm{d}t \left| F^{\mathrm{N}}(s,t) \right|^2} \\
\langle b^2 \rangle^{\mathrm{tot}} =& 4 \left(\frac{\frac{\mathrm{d}}{\mathrm{d}t} \left| F^{\mathrm{N}}(s,t) \right|}{\left| F^{\mathrm{N}}(s,t) \right|} - \tan \zeta^{\mathrm{N}}(s,t) \frac{\mathrm{d}}{\mathrm{d}t} \zeta^{\mathrm{N}}(s,t) \right) \Bigg|_{t=0} \\
\langle b^2 \rangle^{\mathrm{inel}} =& \frac{\sigma^{\mathrm{tot,N}}(s) \langle b^2 \rangle^{\mathrm{tot}} - \sigma^{\mathrm{el,N}}(s) \langle b^2 \rangle^{\mathrm{el}}}{\sigma^{\mathrm{inel}}(s)}
\end{aligned} \tag{18}$$

Definition: central vs. peripheral behaviour of elastic collisions

- Definition: two basic types of behaviour (models) of elastic hadron collisions in dependence on impact parameter may be distinguished
  1. peripheral: $\sqrt{\langle b^2 \rangle^{\mathrm{el}}} > \sqrt{\langle b^2 \rangle^{\mathrm{inel}}}$
     i.e., if elastic collisions correspond in average to higher impact parameter b then the inelastic ones; corresponds to usual ideas of collision of two matter objects
  2. central: $\sqrt{\langle b^2 \rangle^{\mathrm{el}}} < \sqrt{\langle b^2 \rangle^{\mathrm{inel}}}$
     the opposite; anti-ontological behaviour; some kind of transparency of colliding particles

Application of the eikonal model to 53 GeV and 8 TeV pp data

Fitting procedure

- eikonal Coulomb-hadronic model approach

$$\frac{\mathrm{d}\sigma(s,t)}{\mathrm{d}t} = \frac{\pi}{sp^2}\left|F^{C+N}_{eik}(s,t)\right|^2 \tag{19}$$

- parameterization of hadronic amplitude $F^{N}(s,t) = \mathrm{i}\left|F^{N}(s,t)\right| \mathrm{e}^{-\mathrm{i}\zeta^{N}(s,t)}$
- modulus

$$\left|F^{N}(s,t)\right| = (a_1 + a_2 t)\,\mathrm{e}^{b_1 t + b_2 t^2 + b_3 t^3} + (c_1 + c_2 t)\,\mathrm{e}^{d_1 t + d_2 t^2 + d_3 t^3} \tag{20}$$

- phase corresponding to widely used assumptions - leads to central behaviour of el. collisions

$$\zeta^{N}(s,t) = \arctan \frac{\rho_0}{1 - \left|\frac{t}{t_{\mathrm{dip}}}\right|} \tag{21}$$

- much more general parameterization of the phase

$$\zeta^{N}(s,t) = \zeta_0 + \zeta_1 \left|\frac{t}{t_0}\right|^{\kappa} \mathrm{e}^{\nu t} \tag{22}$$

which may lead to peripheral behaviour of el. collisions

Performed fits of 53 GeV and 8 TeV data

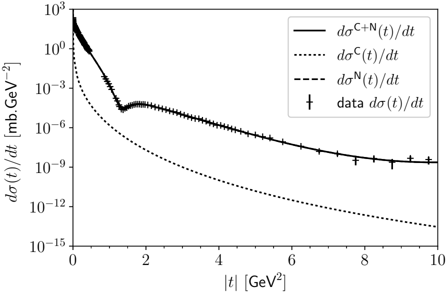
53 GeV

- central case
  - Fit I corresponding to effective electric form factor
  - Fit II corresponding to effective electromagnetic form factor
  - both fits quite straightforward
- peripheral case
  - no unique solution (parameterization of the phase too general)
  - $\Rightarrow$ fits performed with additional constrain $\sqrt{\langle b^2 \rangle^{\mathrm{el}}} \approx 1.6$, 1.75 and 1.9 fm (for each of the two form factors)
  - i.e., need to solve very complicated problem of bounded extremes
  - used advanced fitting method of penalty functions to find corresponding solution (highly non-trivial fitting)
- 8 fits in total (Fit I-VIII) of the same 53 GeV data under different assumptions

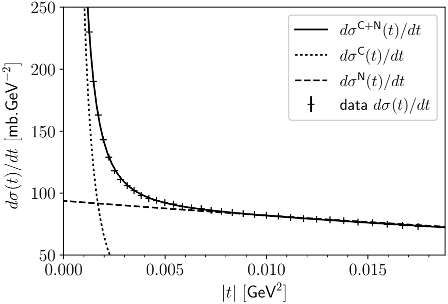
8 TeV

- similar fitting approach as at 53 GeV
- peripheral case - fits performed with additional constrain $\sqrt{\langle b^2 \rangle^{\mathrm{el}}} \approx 1.8$, 1.85 and 1.9 fm (for each of the two form factors)
- 8 fits in total (Fit I-VIII) of the same 8 TeV data under different assumptions

Results at 53 GeV - fitted $d\sigma/dt$



(a) full available $|t|$-range of measured data

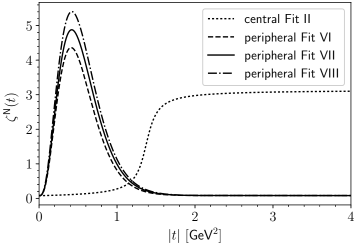


(b) region of very low values of $|t|$

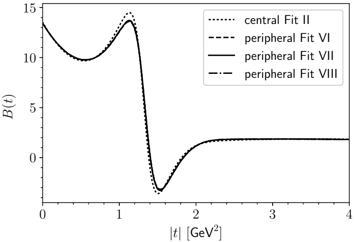
- fit in the whole measured range $|t| \in \langle 0.00126, 9.75 \rangle$ GeV^2
- all the Fits I - VIII at 53 GeV give similar figure in terms of $d\sigma/dt$

$\sqrt{s}$	[GeV]	53	53	53	53
Fit		II	VI	VII	VIII
Case		central	peripheral	peripheral	peripheral
Form factor		effective electromagnetic	effective electromagnetic	effective electromagnetic	effective electromagnetic
ρ_0		0.0766 $\pm$ 0.0017	-	-	-
ζ_0		-	0.0798 $\pm$ 0.0018	0.0800 $\pm$ 0.0018	0.0804 $\pm$ 0.0018
ζ_1		-	2000 $\pm$ 12	2000 $\pm$ 12	2000 $\pm$ 12
κ		-	3.254 $\pm$ 0.073	3.225 $\pm$ 0.066	3.175 $\pm$ 0.058
ν	[GeV^{-2}]	-	7.92 $\pm$ 0.34	7.70 $\pm$ 0.29	7.56 $\pm$ 0.23
a_1		12237 $\pm$ 24	12189 $\pm$ 15	12190 $\pm$ 20	12194 $\pm$ 15
a_2	[GeV^{-2}]	10440 $\pm$ 110	10704 $\pm$ 91	10720 $\pm$ 140	10727 $\pm$ 88
b_1	[GeV^{-2}]	5.909 $\pm$ 0.022	5.864 $\pm$ 0.020	5.865 $\pm$ 0.023	5.864 $\pm$ 0.020
b_2	[GeV^{-4}]	3.757 $\pm$ 0.087	3.477 $\pm$ 0.073	3.474 $\pm$ 0.084	3.468 $\pm$ 0.071
b_3	[GeV^{-6}]	1.757 $\pm$ 0.069	1.538 $\pm$ 0.051	1.535 $\pm$ 0.063	1.533 $\pm$ 0.050
c_1		-24 $\pm$ 20	53 $\pm$ 11	53 $\pm$ 16	52 $\pm$ 10
c_2	[GeV^{-2}]	-90 $\pm$ 11	-1.4 $\pm$ 7.6	-1 $\pm$ 10	-3.1 $\pm$ 7.3
d_1	[GeV^{-2}]	1.425 $\pm$ 0.025	0.77 $\pm$ 0.14	0.76 $\pm$ 0.35	0.79 $\pm$ 0.14
d_2	[GeV^{-4}]	0.0038 $\pm$ 0.0086	-0.061 $\pm$ 0.037	-0.062 $\pm$ 0.062	-0.061 $\pm$ 0.034
d_3	[GeV^{-6}]	-0.00430 $\pm$ 0.00081	-0.0068 $\pm$ 0.0029	-0.0068 $\pm$ 0.0045	-0.0068 $\pm$ 0.0027
χ^2/ndf		323.268/205	259.69/202	260.16/202	263.37/202
$\Delta\chi^2$		0	1.29	0.31	2.67
$\rho(t=0)$		0.0766 $\pm$ 0.0017	0.0799 $\pm$ 0.0018	0.0802 $\pm$ 0.0018	0.0806 $\pm$ 0.0018
$B(t=0)$	[GeV^{-2}]	13.514 $\pm$ 0.050	13.417 $\pm$ 0.041	13.420 $\pm$ 0.048	13.421 $\pm$ 0.040
$\sigma^{\mathrm{tot,N}}$	[mb]	42.71 $\pm$ 0.15	42.795 $\pm$ 0.090	42.80 $\pm$ 0.13	42.809 $\pm$ 0.087
$\sigma^{\mathrm{el,N}}$	[mb]	7.472	7.525	7.525	7.527
σ^{inel}	[mb]	35.23	35.27	35.28	35.28
$\sigma^{\mathrm{el,N}}/\sigma^{\mathrm{tot,N}}$		0.1750	0.1758	0.1758	0.1758
$\mathrm{d}\sigma^{\mathrm{N}}/\mathrm{d}t(t=0)$	[mb.GeV^{-2}]	93.74	94.18	94.21	94.26
$\sqrt{\langle b^2\rangle^{\mathrm{tot}}}$	[fm]	1.027	1.023	1.023	1.023
$\sqrt{\langle b^2\rangle^{\mathrm{el}}}$	[fm]	0.6764	1.612	1.746	1.908
$\sqrt{\langle b^2\rangle^{\mathrm{inel}}}$	[fm]	1.086	0.8456	0.7868	0.7023
$D^{\mathrm{tot}}(b=0)$		1.29	1.30	1.30	1.30
$D^{\mathrm{el}}(b=0)$		0.536	0.0606	0.0625	0.0783
$D^{\mathrm{inel}}(b=0)$		0.753	1.24	1.24	1.22

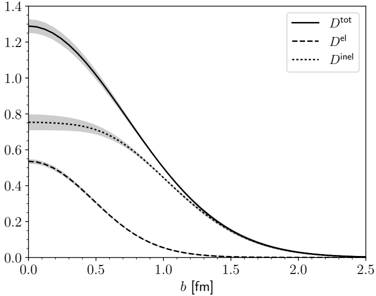
Results at 53 GeV (effective electromagnetic form factors)



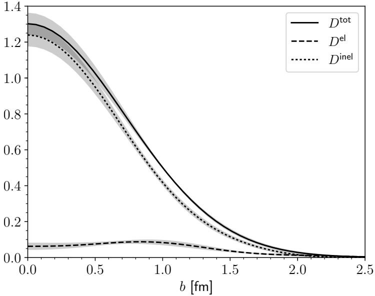
(a) hadronic phases



(b) diffractive slopes

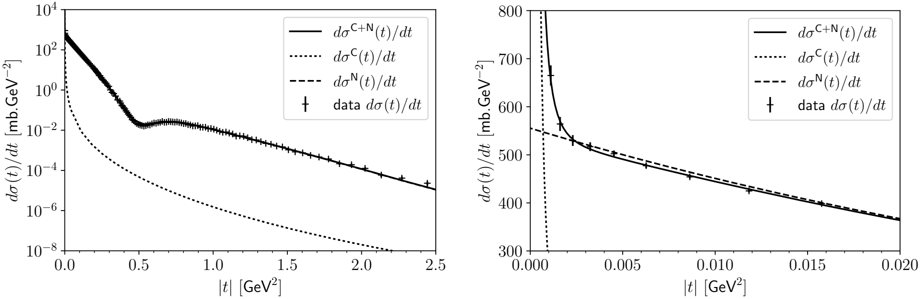


(c) profile functions: central case, Fit II



(d) profile functions: peripheral case, Fit VII

Results at 8 TeV - fitted dσ/dt



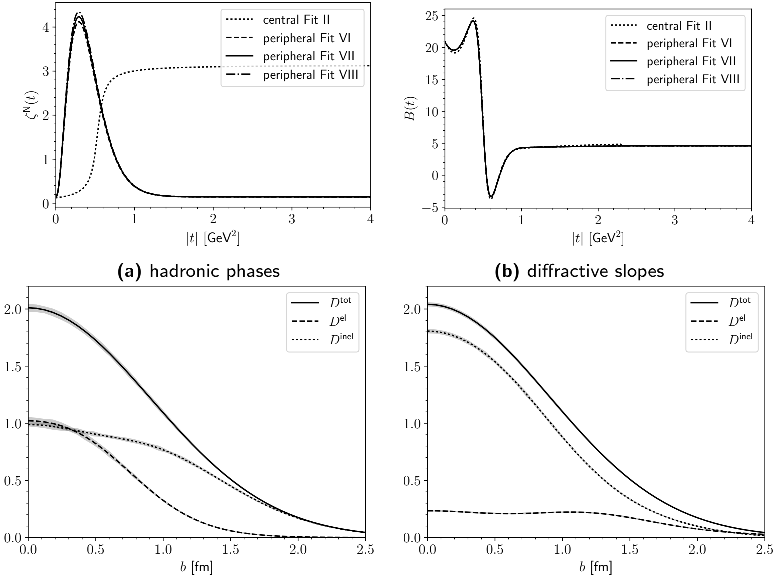
(a) full available $|t|$-range of measured data

(b) region of very low values of $|t|$

- TOTEM 8 TeV data (1000m and 90m optics data up to $|t| = 0.2$ GeV^2, see [6, 7]); extended by renormalized TOTEM 7 TeV data [8] up to 2.5 GeV^2 to obtain data in wider t-region $\Rightarrow$ approximate data denoted as "8 TeV data" (only statistical errors taken into account)
- all the Fits I - VIII at 8 TeV give similar figure in terms of dσ/dt

$\sqrt{s}$	[GeV]	8000	8000	8000	8000
Fit		II	VI	VII	VIII
Case		central	peripheral	peripheral	peripheral
Form factor		effective electromagnetic	effective electromagnetic	effective electromagnetic	effective electromagnetic
ρ_0		0.122 ± 0.016	-	-	-
ζ_0		-	0.1413 ± 0.0154	0.1422 ± 0.0157	0.144 ± 0.0157
ζ_1		-	800.0	800.0	800.0
κ		-	2.3825 ± 0.0129	2.3653 ± 0.0124	2.347 ± 0.0121
ν	[GeV^{-2}]	-	8.121 ± 0.105	8.0948 ± 0.0971	8.0819 ± 0.0907
a_1	[10^{-7}]	66.675 ± 0.325	66.262 ± 0.295	66.321 ± 0.317	66.388 ± 0.254
a_2	[$10^{-7}\mathrm{GeV}^{-2}$]	163.729 ± 0.939	169.97 ± 1.07	169.98 ± 1.24	169.996 ± 0.928
b_1	[GeV^{-2}]	8.2596 ± 0.0267	8.1540 ± 0.0288	8.1671 ± 0.0291	8.1829 ± 0.0299
b_2	[GeV^{-4}]	9.171 ± 0.169	7.585 ± 0.173	7.618 ± 0.171	7.663 ± 0.175
b_3	[GeV^{-6}]	14.777 ± 0.296	12.186 ± 0.292	12.211 ± 0.281	12.249 ± 0.284
c_1	[10^{-7}]	1.507 ± 0.310	2.421 ± 0.271	2.407 ± 0.309	2.392 ± 0.228
c_2	[$10^{-7}\mathrm{GeV}^{-2}$]	-3.381 ± 0.662	-1.412 ± 0.784	-1.446 ± 0.888	-1.479 ± 0.640
d_1	[GeV^{-2}]	2.7910 ± 0.0714	2.537 ± 0.134	2.542 ± 0.150	2.548 ± 0.107
d_2	[GeV^{-4}]	0	0	0	0
d_3	[GeV^{-6}]	0	0	0	0
χ^2/ndf		235.0 / 131	358.6 / 129	367.2 / 129	376.3 / 129
$\Delta\chi^2$		0	8.5	8.6	8.6
$\rho(t=0)$		0.122 ± 0.016	0.142 ± 0.016	0.143 ± 0.016	0.145 ± 0.016
$B(t=0)$	[GeV^{-2}]	20.981 ± 0.096	20.82 ± 0.095	20.84 ± 0.10	20.874 ± 0.087
$\sigma^{\mathrm{tot,N}}$	[mb]	103.48 ± 0.98	103.98 ± 0.88	104.0 ± 0.97	104.1 ± 0.76
$\sigma^{\mathrm{el,N}}$	[mb]	27.6	27.9	27.9	28.0
σ^{inel}	[mb]	75.9	76.1	76.1	76.1
$\sigma^{\mathrm{el,N}}/\sigma^{\mathrm{tot,N}}$		0.267	0.270	0.269	0.269
$\mathrm{d}\sigma^{\mathrm{N}}/\mathrm{d}t(t=0)$	[$\mathrm{mb.GeV}^{-2}$]	555	564	564	565
$\sqrt{\langle b^2\rangle^{\mathrm{tot}}}$	[fm]	1.28	1.27	1.27	1.28
$\sqrt{\langle b^2\rangle^{\mathrm{el}}}$	[fm]	0.879	1.81	1.86	1.90
$\sqrt{\langle b^2\rangle^{\mathrm{inel}}}$	[fm]	1.40	1.01	0.978	0.945
$D^{\mathrm{tot}}(b=0)$		2.01	2.04	2.04	2.04
$D^{\mathrm{el}}(b=0)$		1.02	0.234	0.214	0.196
$D^{\mathrm{inel}}(b=0)$		0.989	1.81	1.83	1.84

Results at 8 TeV (effective electromagnetic form factors)



(a) hadronic phases

(b) diffractive slopes

(c) profile functions: central case, Fit II

(d) profile functions: peripheral case, Fit VII

Eikonal model - summary of the fitted results at 53 GeV and 8 TeV

FZÚ

1. one may compare numerical values of several physicaly interesting quantitites at 53 GeV and 8 TeV under different assumptions
2. impact of choice of:
   - form factor (electric vs. electromagnetic, see pages 25 and 26) - small or negligible
   - t-dependence of hadronic phase - very strong, completely different character of collisions in b-space
3. t-dependence of hadronic phase constrained only weakly by the eikonal interference formula

Summary and conclusion

- WY approach used widely at ISR for determination of $\sigma^{\mathrm{tot,N}}$, $B(t=0)$ and $\rho(t=0)$
- many problems and limitations identified later (several papers exist)
- even the more generally looking integral formula (3) for relative phase $\alpha\phi(t)$ is consistent only with t-independent hadronic phase
- $\Rightarrow$ WY approach should be abandoned in the era of LHC as it may lead to completely wrong physical conclusions (it should not be used for constraining hadronic models based on inconsistent assumptions, see also page 24)
- $\Rightarrow$ one should look for other descriptions of el. scattering of (charged) hadrons
- eikonal model more general and relevant for analysis of el. data in the era of LHC, see recent results
  - 8 TeV: TOTEM Collaboration, "Measurement of elastic pp scattering at $\sqrt{s} = 8$ TeV in the Coulomb-nuclear interference region – determination of the ρ-parameter and the total cross-section", Eur. Phys. J. C **76**, 661 (2016), see also CERN-PH-EP-2015-325, arXiv:1610.00603
  - 53 GeV: J. Procházka and V. Kundrát, "Eikonal model analysis of elastic hadron collisions at high energies", arXiv: hep-th/1606.09479 (2016) (see also appendix B concerning discussion of the relative phase in the WY approach)
- eikonal model allows to study t-dependence of elastic hadronic amplitude and characteristics of protons in dependence on impact parameter; allows to study spatial structure of proton, see further comments and open problems in J. Procházka, M. V. Lokajíček, and V. Kundrát, "Dependence of elastic hadron collisions on impact parameter", Eur. Phys. J. Plus **131**, 147 (2016), see also arXiv: hep-ph/1509.05343 (2015) (see also recent papers of V. Petrov [10, 11])

Backup

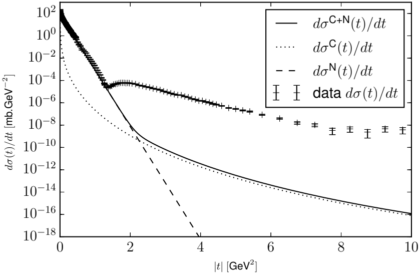
Several other limiting assumptions in WY approach

- e.g., derived only at small $|t|$; form factors $G_{1,2}(t)$ added by hand; dependence of elastic hadronic collisions on impact parameter not considered, ...

- if $B(t)$ is t-independent $\Rightarrow$ contradiction to existence of observed dip-bump structure

  J. Procházka, "Elastic hadron scattering at high energies", Bachelor thesis (Charles University, Prague, 2007)

  J. Kašpar, V. Kundrát, M. Lokajíček, and J. Procházka, "Phenomenological models of elastic nucleon scattering and predictions for LHC", Nucl.Phys. **B843**, 84–106 (2011)



- relative phase $\phi(s,t)$ is real (defined as imaginary part of a complex function) $\Rightarrow$ the integral WY formula (3) consistent only with $\rho(t) = \text{const}$

  V. Kundrát, M. Lokajíček, and I. Vrkoč, "Limited validity of West and Yennie integral formula for elastic scattering of hadrons", Phys. Lett. **B656**, 182–185 (2007)

- measured $\mathrm{d}\sigma/\mathrm{d}t$ divided commonly into two parts
  1. region of very low values of $|t|$ (e.g., $|t| \lesssim 0.01$ GeV^2 at 53 GeV)
  2. higher values of $|t|$ (containing dip-bump structure)

  first region analyzed with the help of WY interference formula and the second with the help of hadronic models based on inconsistent assumptions $\Rightarrow$ inconsistent dual description of data

Proton electromagnetic form factors

- proton form factors $G_{1,2}(t)$ entering into both the interference formulas (WY and eikonal)
- WY formula used in past mainly with electric dipole form factors: $G_{\mathrm{E}}^{\mathrm{D}}(t) = \left(1 - \frac{t}{\Lambda^2}\right)^{-2}$
- the eikonal interference formula originally used with electric form factors of Borkowski

$$G_{\mathrm{E}}^{\mathrm{B}}(t) = \sum_{j=1}^{4} \frac{g_k^{\mathrm{E}}}{w_k^{\mathrm{E}} - t}, \qquad G_{\mathrm{M}}^{\mathrm{B}}(t) = \mu_p \sum_{j=1}^{4} \frac{g_k^{\mathrm{M}}}{w_k^{\mathrm{M}} - t} \tag{23}$$

- effective electromagnetic form factor squared ($\tau = -t/(4m^2)$)

$$G_{\mathit{eff}}^2(t) = \frac{1}{1+\tau} \left[G_{\mathrm{E}}^2(t) + \tau\; G_{\mathrm{M}}^2(t)\right] \tag{24}$$

- difference between proton electric and effective electromagnetic form factors?

Comparison of proton electromagnetic form factors

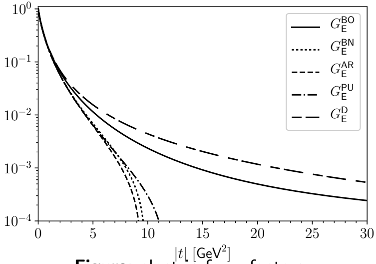


Figure: electric form factors

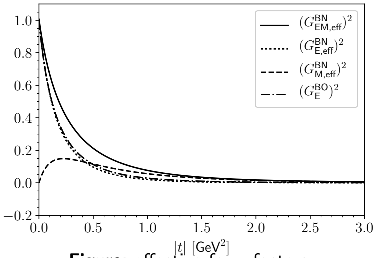


Figure: effective form factors

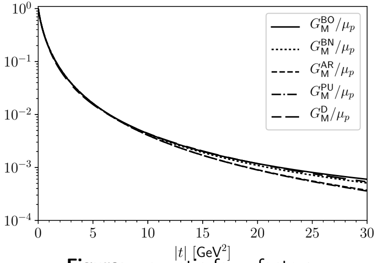


Figure: magnetic form factors

- different authors/analyses
  - different shapes of form factors
- electric form factors
  - differences visible at $|t| > 2.5$ GeV^2
- electric vs. effective electromagnetic form factors
  - very significant differences already at lower $|t|$ values
- impact on calculations in the eikonal interference formula?

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