One-loop amplitudes with an off-shell gluon

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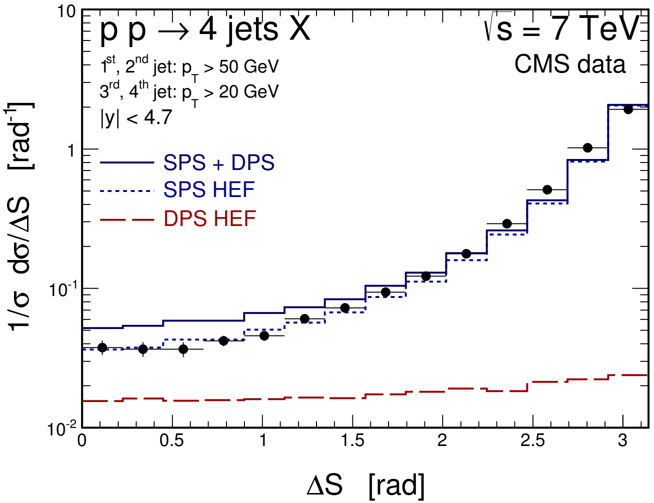
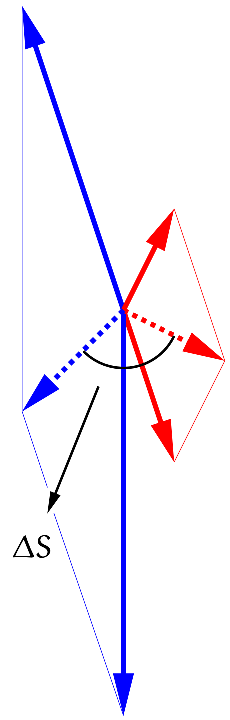
Outline

- Factorized cross section calculation
- Off-shell amplitudes
- KaTie: for parton-level event generation with k_T-dependent initial states
- Off-shell one-loop amplitudes

Four jets with k_T-factorization

Maciuła, Szczurek, Kutak, Serino, AvH 2016





- ΔS is the azimutal angle between the sum of the two hardest jets and the sum of the two softest jets.
- This variable has no distribution at LO in collinear factorization: pairs would have to be back-to-back.
- k_T-factorization allows for the necessary momentum inbalance.

Factorization for hadron scattering

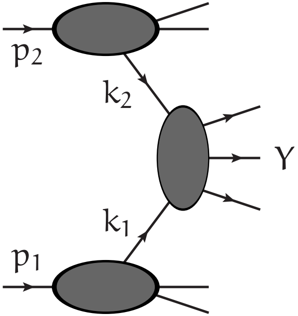
General formula for cross section with $\pi^* \in \{g^*, q^*, \bar{q}^*\}$:

$$d\sigma\big(h_1(p_1)h_2(p_2) \to Y\big) = \sum_{a,b} \int d^4k_1\, \mathcal{P}_{1,a}(k_1) \int d^4k_2\, \mathcal{P}_{2,b}(k_2)\, d\hat{\sigma}\big(\pi_a^*(k_1)\pi_b^*(k_2) \to Y\big)$$

Collinear factorization: $\mathcal{P}_{i,a}(k) = \int_0^1 \frac{dx}{x}\, f_{i,a}(x,\mu)\, \delta^4(k - x\,p_i)$

k_T-factorization: $\mathcal{P}_{i,a}(k) = \int \frac{d^2\mathbf{k}_T}{\pi} \int_0^1 \frac{dx}{x}\, \mathcal{F}_{i,a}(x, |\mathbf{k}_T|, \mu)\, \delta^4(k - x\,p_i - k_T)$

- The *parton level* cross section $d\hat{\sigma}\big(\pi_a^*(k_1)\pi_b^*(k_2) \to Y\big)$ can be calculated within perturbative QCD.
- The *parton distribution functions* $f_{i,a}$ and $\mathcal{F}_{i,a}$ must be modelled and fit against data.
- Unphysical scale μ is a price to pay, but its dependence is calculable within perturbative QCD via *evolution equations*.



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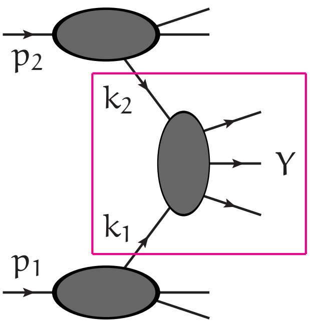
$$\hat{\sigma} = \int d\Phi(1,2 \to 3,4,\ldots,n)\, \big|\mathcal{M}(1,2,\ldots,n)\big|^2\, \mathcal{O}(p_3,p_4,\ldots,p_n)$$

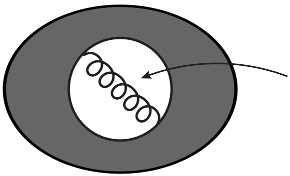


phase space includes summation over color and spin
squared amplitude calculated perturbatively
observable includes phase space cuts, or jet algorithm

Gauge invariance

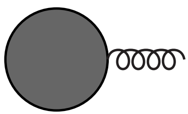
In order to be physically relevant, any scattering amplitude following the constructive definition given before must satisfy the following

Freedom in choice of gluon propagator:

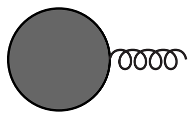


$$\begin{cases} \dfrac{-i}{k^2}\left[g^{\mu\nu} - (1-\xi)\dfrac{k^\mu k^\nu}{k^2}\right] \\[2ex] \dfrac{-i}{k^2}\left[g^{\mu\nu} - \dfrac{k^\mu n^\nu + n^\mu k^\nu}{k\cdot n} + (n^2 + \xi k^2)\dfrac{k^\mu k^\nu}{(k\cdot n)^2}\right] \end{cases}$$

Ward identity:



$${}_\mu \varepsilon^\mu(k) \quad \rightarrow$$



$${}_\mu k^\mu \; = \; 0$$

- Only holds if all external particles are on-shell.
- k_T-factorization requires off-shell initial-state momenta $k^\mu = p^\mu + k_T^\mu$.
- How to define amplitudes with off-shell intial-state momenta?

Amplitudes with off-shell gluons

Amplitudes with off-shell gluons

n-parton amplitude is a function of n momenta $k_1, k_2, \ldots, k_n$
and n *directions* $p_1, p_2, \ldots, p_n$

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$$k_1^\mu + k_2^\mu + \cdots + k_n^\mu = 0 \qquad \text{momentum conservation}$$

$$p_1^2 = p_2^2 = \cdots = p_n^2 = 0 \qquad \text{light-likeness}$$

$$p_1 \cdot k_1 = p_2 \cdot k_2 = \cdots = p_n \cdot k_n = 0 \qquad \text{eikonal condition}$$

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With the help of an auxiliary four-vector q^μ with $q^2 = 0$, we define

$$k_T^\mu(q) = k^\mu - x(q) p^\mu \quad \text{with} \quad x(q) \equiv \frac{q \cdot k}{q \cdot p}$$

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Construct k_T^μ explicitly in terms of p^μ and q^μ:

$$k_T^\mu(q) = -\frac{\kappa}{2}\,\varepsilon^\mu - \frac{\kappa^*}{2}\,\varepsilon^{*\mu} \quad \text{with} \quad \begin{cases} \varepsilon^\mu = \dfrac{\langle p|\gamma^\mu|q]}{[pq]} \quad , \quad \kappa = \dfrac{\langle q|\not{k}|p]}{\langle qp\rangle} \\[2ex] \varepsilon^{*\mu} = \dfrac{\langle q|\gamma^\mu|p]}{\langle qp\rangle} \quad , \quad \kappa^* = \dfrac{\langle p|\not{k}|q]}{[pq]} \end{cases}$$

$k^2 = -\kappa\kappa^*$ is independent of q^μ, but also individually κ and κ^* are independent of q^μ.

Example of a 4-gluon amplitude

$$\mathcal{A}(1^*, 2^-, 3^*, 4^+) = \frac{\langle 13\rangle^3 [13]^3}{\langle 34\rangle\langle 41\rangle\langle 1|\not{k}_3 + \not{p}_4|3]\langle 3|\not{k}_1 + \not{p}_4|1][32][21]}$$
$$+ \frac{1}{\kappa_1^* \kappa_3} \frac{\langle 12\rangle^3 [43]^3}{\langle 2|\not{k}_3|4]\langle 1|\not{k}_3 + \not{p}_4|3](k_3 + p_4)^2} + \frac{1}{\kappa_1 \kappa_3^*} \frac{\langle 23\rangle^3 [14]^3}{\langle 2|\not{k}_1|4]\langle 3|\not{k}_1 + \not{p}_4|1](k_1 + p_4)^2}$$

- Eventual matrix element needs factor $k_1^2 k_3^2 = |\kappa_1|^2 |\kappa_3|^2$.
  This *must not* be included at the amplitude level not to spoil analytic structure.

- Last two terms dominate for $|k_1| \to 0$ and $|k_3| \to 0$, and give the on-shell helicity amplitudes in that limit.

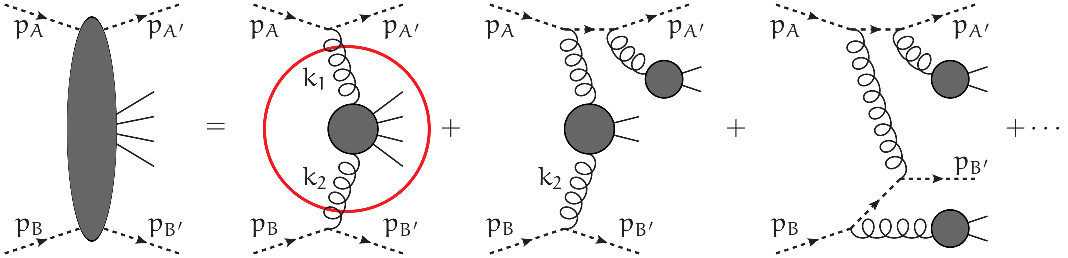
$$\mathcal{A}(1^*, 2^-, 3^*, 4^+) \xrightarrow{|k_1|,|k_3|\to 0} \frac{1}{\kappa_1^* \kappa_3} \mathcal{A}(1^-, 2^-, 3^+, 4^+) + \frac{1}{\kappa_1 \kappa_3^*} \mathcal{A}(1^+, 2^-, 3^-, 4^+)$$

- Coherent sum of amplitudes becomes incoherent sum of squared amplitudes via angular integrations for $\vec{k}_{1T}$ and $\vec{k}_{3T}$.

Amplitudes with off-shell gluons

AvH, Kutak, Kotko 2013:

Embed the process in an on-shell process with auxiliary partons

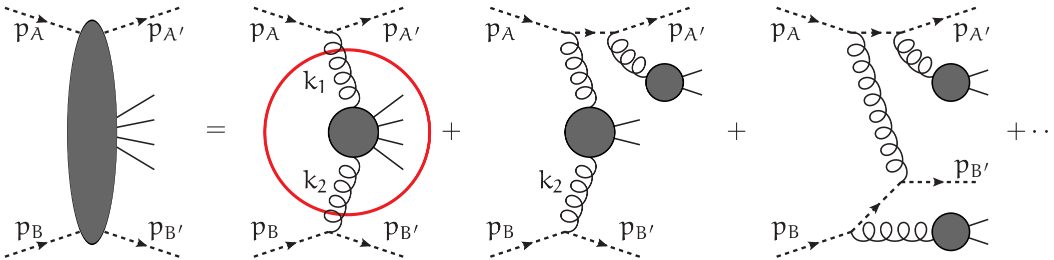


$$p_A^\mu = \Lambda p_1^\mu - \frac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu}$$

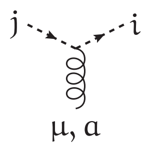
$$p_{A'}^\mu = -(\Lambda - x_1)p_1^\mu - \frac{\kappa_1}{2}\,\varepsilon_1^\mu$$

Amplitudes with off-shell gluons

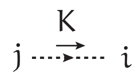
AvH, Kutak, Kotko 2013:
Embed the process in an on-shell process with auxiliary partons and eikonal Feynman rules.



$$
\left.
\begin{array}{c}
p_A^\mu = \Lambda p_1^\mu - \dfrac{\kappa_1^*}{2}\,\varepsilon_1^{*\mu} \\
p_{A'}^\mu = -(\Lambda - x_1)p_1^\mu - \dfrac{\kappa_1}{2}\,\varepsilon_1^\mu \\
\Lambda \to \infty
\end{array}
\right\} \Rightarrow
$$



$= -\mathrm{i}\, T^a_{i,j}\, p_1^\mu$



$= \delta_{i,j}\,\dfrac{\mathrm{i}}{p_1\cdot K}$

Amplitudes with off-shell partons

AvH, Kutak, Kotko 2013, AvH, Kutak, Salwa 2013:
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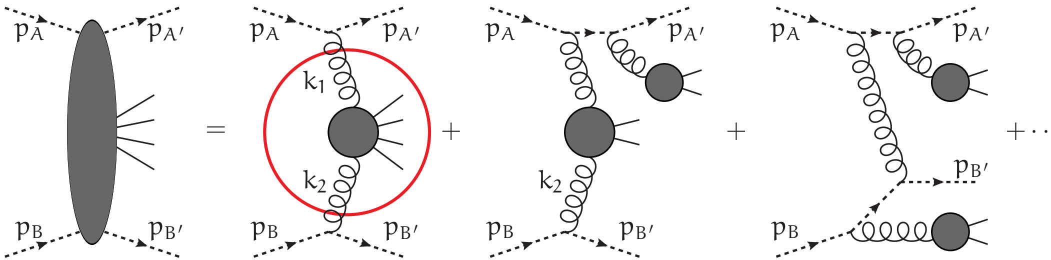

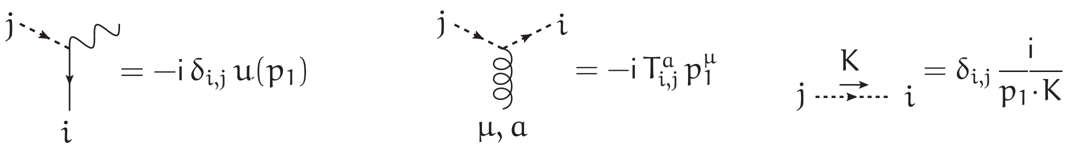

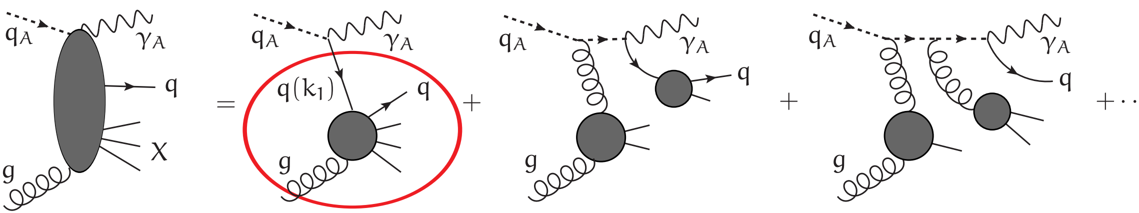

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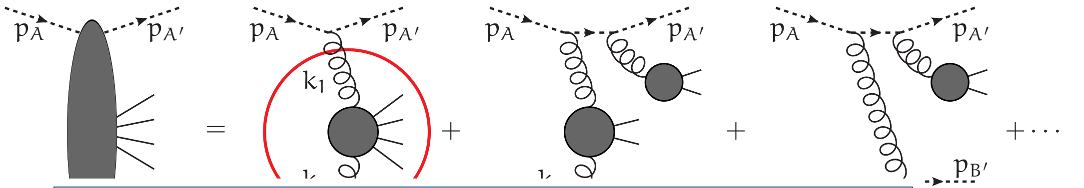
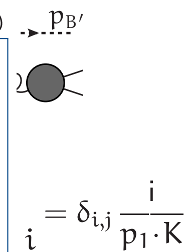
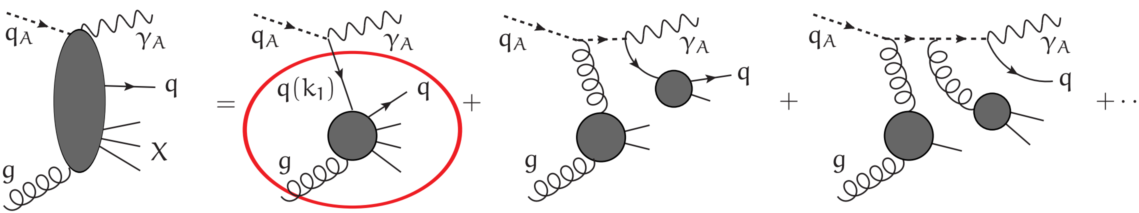




In agreement with the effective action approach of
Lipatov 1995, Antonov, Lipatov, Kuraev, Cherednikov 2005
Lipatov, Vyazovsky 2000, Nefedov, Saleev, Shipilova 2013
and the Wilson-line approach of
Kotko 2014



KaTie https://bitbucket.org/hameren/katie

- parton level event generator, like Alpgen, Helac, MadGraph, etc.
- arbitrary processes within the standard model (including effective Hg) with several final-state particles.
- 0, 1, or 2 off-shell intial states.
- produces (partially un)weighted event files, for example in the LHEF format.
- requires LHAPDF. TMD PDFs can be provided as files containing rectangular grids.
- a calculation is steered by a single input file.
- employs an optimization phase in which the pre-samplers for all channels are optimized.
- during the generation phase several event files can be created in parallel.
- can generate (naively factorized) MPI events.
- event files can be processed further by parton-shower program like CASCADE (next talk by Mirko Serino).

```
Ngroup = 1
Nfinst = 3
process = g u  -> mu+ mu- u   factor = 1   groups = 1   pNonQCD = 2 0 0
process = g u~ -> mu+ mu- u~  factor = 1   groups = 1   pNonQCD = 2 0 0
process = g d  -> mu+ mu- d   factor = 1   groups = 1   pNonQCD = 2 0 0
process = g d~ -> mu+ mu- d~  factor = 1   groups = 1   pNonQCD = 2 0 0
lhaSet = MSTW2008nlo68cl
offshell = 1 0
tmdTableDir = /home/user0/kTfac/tables/krzysztof02/
tmdpdf = g  KMR_gluon.dat
tmdpdf = u  KMR_u.dat
tmdpdf = u~ KMR_ubar.dat
tmdpdf = d  KMR_d.dat
tmdpdf = d~ KMR_dbar.dat
tmdpdf = s  KMR_s.dat
tmdpdf = s~ KMR_sbar.dat
tmdpdf = c  KMR_c.dat
tmdpdf = c~ KMR_cbar.dat
tmdpdf = b  KMR_b.dat
tmdpdf = b~ KMR_bbar.dat
Nflavors = 5
helicity = sampling
Noptim = 1,000,000
Ecm = 7000
Esoft = 20
```

```
cut = deltaR|1,3| > 0.4
cut = deltaR|2,3| > 0.4
cut = pT|1| > 20
cut = pT|2| > 20
cut = pseudoRap|1| > 2.0
cut = pseudoRap|2| > 2.0
cut = pseudoRap|1| < 4.5
cut = pseudoRap|2| < 4.5
cut = mass|1+2| > 60
cut = mass|1+2| < 120
cut = pT|3| > 20
cut = rapidity|3| > 2.0
cut = rapidity|3| < 4.5
scale = (pT|3|+pT|1+2|+91.1882D0)/3
mass = Z 91.1882 2.4952
mass = W 80.419 2.21
mass = H 125.0 0.00429
mass = t 173.5
switch = withQCD Yes
switch = withQED Yes
switch = withWeak Yes
switch = withHiggs No
switch = withHG No
coupling = Gfermi 1.16639d-5
```

$p\,p \to Z + j$ in the forward direction

Off-shell one-loop amplitudes

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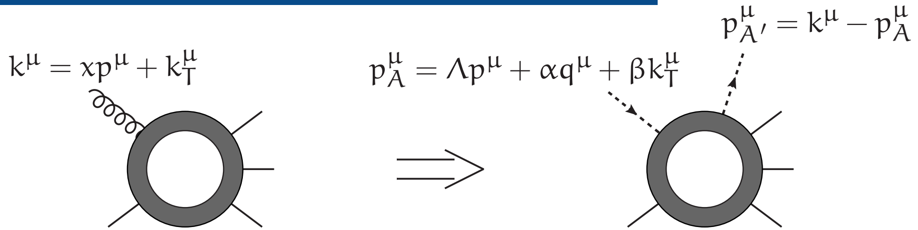


where p, q are light-like with $p\cdot q > 0$, where $p\cdot k_T = q\cdot k_T = 0$, and where

$$\alpha = \frac{-\beta^2 k_T^2}{\Lambda(p+q)^2} \quad , \quad \beta = \frac{1}{1+\sqrt{1-x/\Lambda}} \; .$$

With this choice, the momenta $p_A, p_{A'}$ satisfy the relations

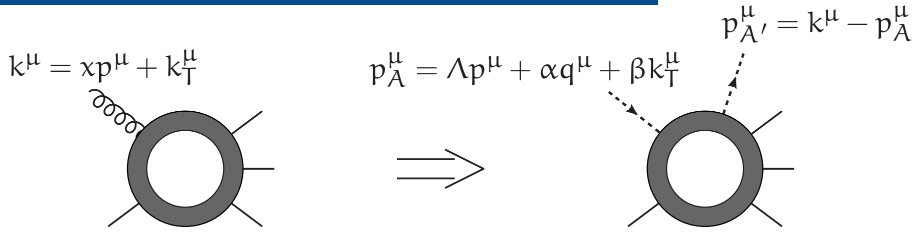
$$p_A^2 = p_{A'}^2 = 0 \quad , \quad p_A^\mu + p_{A'}^\mu = xp^\mu + k_T^\mu$$

for any value of the parameter Λ. Auxiliary quark propagators become eikonal for $\Lambda \to \infty$.

$$i\,\frac{\not{p}_A + \not{K}}{(p_A+K)^2} = \frac{i\,\not{p}}{2p\cdot K} + \mathcal{O}(\Lambda^{-1})$$

Taking this limit after loop integration will lead to singularities $\log\Lambda$.

Off-shell one-loop amplitudes



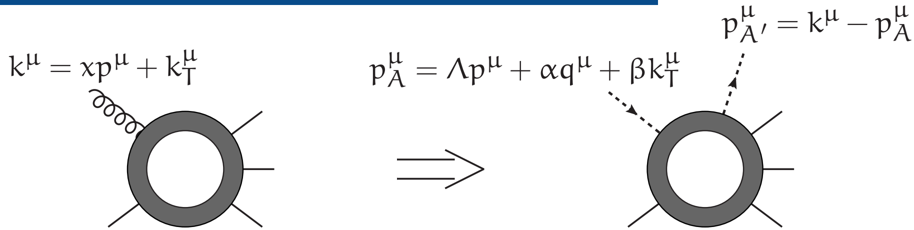
Well-known decomposition for on-shell one-loop amplitudes in terms of master integrals still holds for finite Λ.

$$\mathcal{A}^{(1)} = \int d^{4-2\varepsilon}\ell \, \frac{\mathcal{N}(\ell)}{\prod_i \mathcal{D}_i(\ell)} = \sum_{i,j,k,l} c_4(i,j,k,l)\, I_4(i,j,k,l) + \sum_{i,j,k} c_3(i,j,k)\, I_3(i,j,k) + \sum_{i,j} c_2(i,j)\, I_2(i,j) + \sum_i c_1(i)\, I_1(i) + \mathcal{R} + \mathcal{O}(\varepsilon)$$

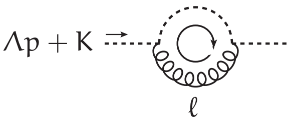
$$I_4(i,j,k,l) = \int d^{4-2\varepsilon}\ell \, \frac{1}{\mathcal{D}_i(\ell)\mathcal{D}_j(\ell)\mathcal{D}_k(\ell)\mathcal{D}_l(\ell)} \quad , \quad \mathcal{D}_i(\ell) = (\ell + K_i)^2 - m_i^2 + i\eta$$

The coefficients $c_4, c_3, c_2.c_1$ are rational functions of the external momenta and Λ, the master integrals I_4, I_3, I_2 contain powers of $\log\Lambda$.

Off-shell one-loop amplitudes



Integrand-based reduction methods cannot be applied with naïve limit $\Lambda \to \infty$ on integrand. For example, the integrand of the following graph (Feynman gauge) vanishes in that limit, but the integral does not:



$$= \int d^{4-2\varepsilon}\ell \, \frac{\langle p|\gamma^\mu(\not{\ell} + \Lambda \not{p} + \not{K})\gamma_\mu|p]}{\ell^2(\ell + \Lambda p + K)^2}$$

$$= 2p\cdot K \left[\log\Lambda - \frac{1}{\varepsilon} - 1 + \log\left(-\frac{2p\cdot K}{\mu^2}\right) + \mathcal{O}(\varepsilon) \right]$$

Naïve power counting in Λ does not work.

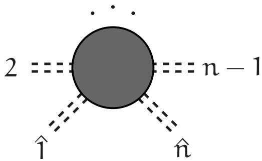
Conclusions

- k_T-factorization allows for the parton-level description of kinematical situations inaccessible with LO collinear factorization, eg. ΔS for four jets.
- Factorization prescriptions with explicit k_T dependence in the pdfs ask for hard matrix elements with off-shell initial-state partons.
- The necessary amplitudes can be defined in a manifestly gauge invariang manner
- KaTie generates parton-level events with k_T-dependent initial states.
- One-loop amplitudes necessary for NLO doable but non-trivial

Backup

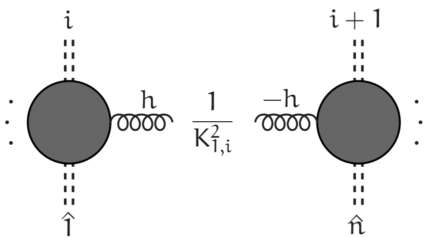
BCFW recursion for off-shell amplitudes

The BCFW recursion formula becomes

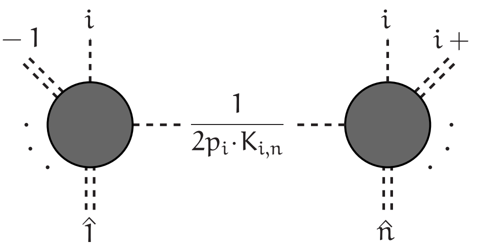


$$= \sum_{i=2}^{n-2} \sum_{h=+,-} A_{i,h} + \sum_{i=2}^{n-1} B_i + C + D \,,$$

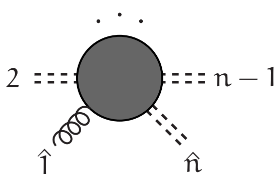
$A_{i,h} =$



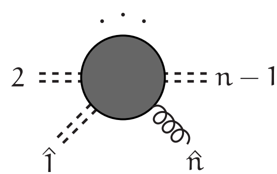
$B_i =$



$C = \frac{1}{\kappa_1}$



$D = \frac{1}{\kappa_1^*}$



The hatted numbers label the shifted external gluons.