Model Independent Extesion of the Glauber-Velasco Model of Elastic PP Scattering

T. Csörgő[1,2], R. J. Glauber[3]

[1] Wigner RCP, Budapest, Hungary
[2] EKE KRC, Gyöngyös, Hungary
[3] Harvard University, Cambridge, MA, USA

Introduction to Diffraction

Glauber-Velasco model:
Non-exponential behaviour of $d\sigma/dt$ at low-t

Analysis of TOTEM/LHC p+p @ 7 TeV
Imaging with shadow profile functions

Model-independent shape analysis:
Levy expansions
Application in elastic pp scattering

Parton level:
Model-independent extension of
Glauber-Velasco model

Summary

arxiv:1306.4217
arxiv:1311.2308
arXiv:1412.0813
+ manuscripts in preparation

Diffraction – Hofstadter, Nobel (1961)

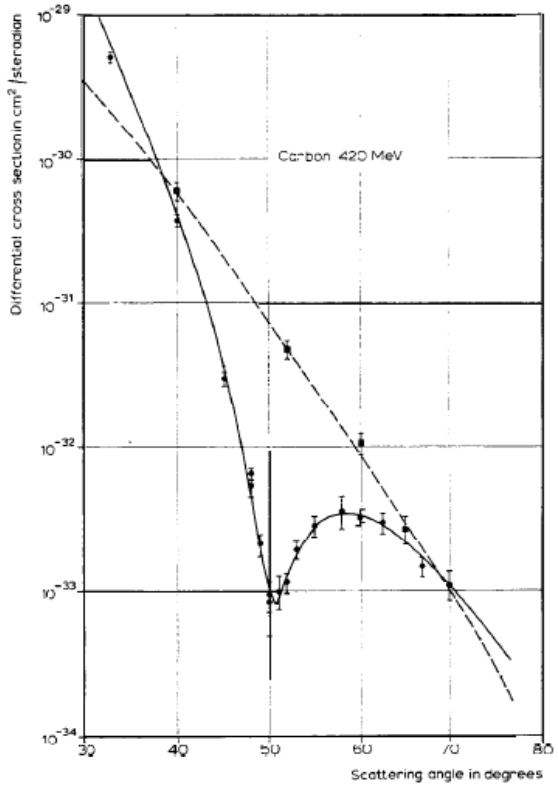
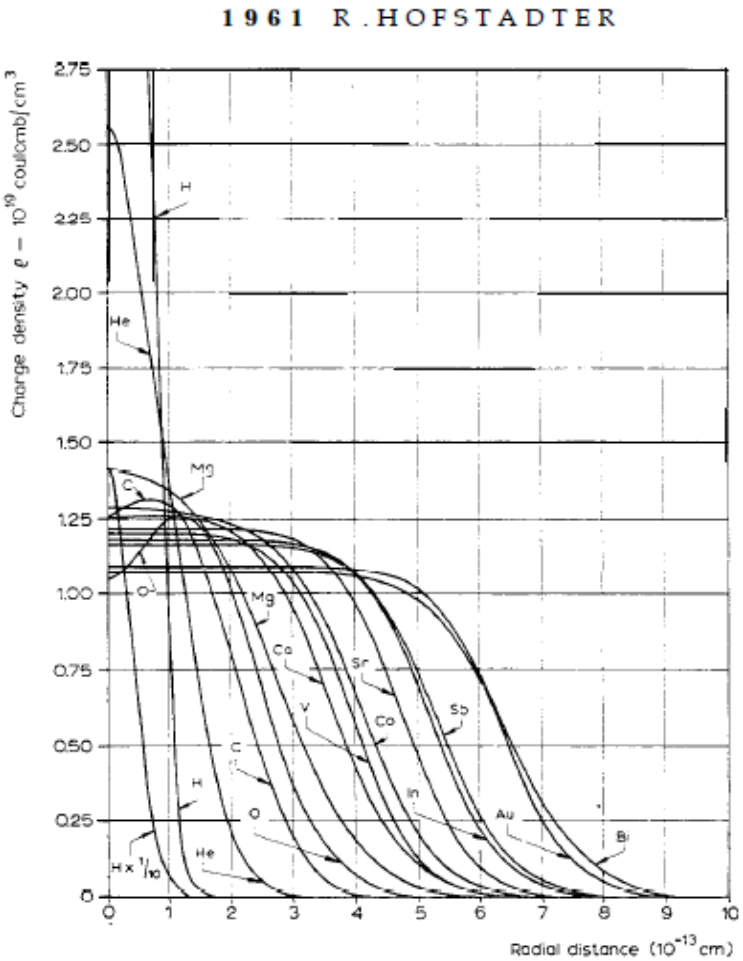
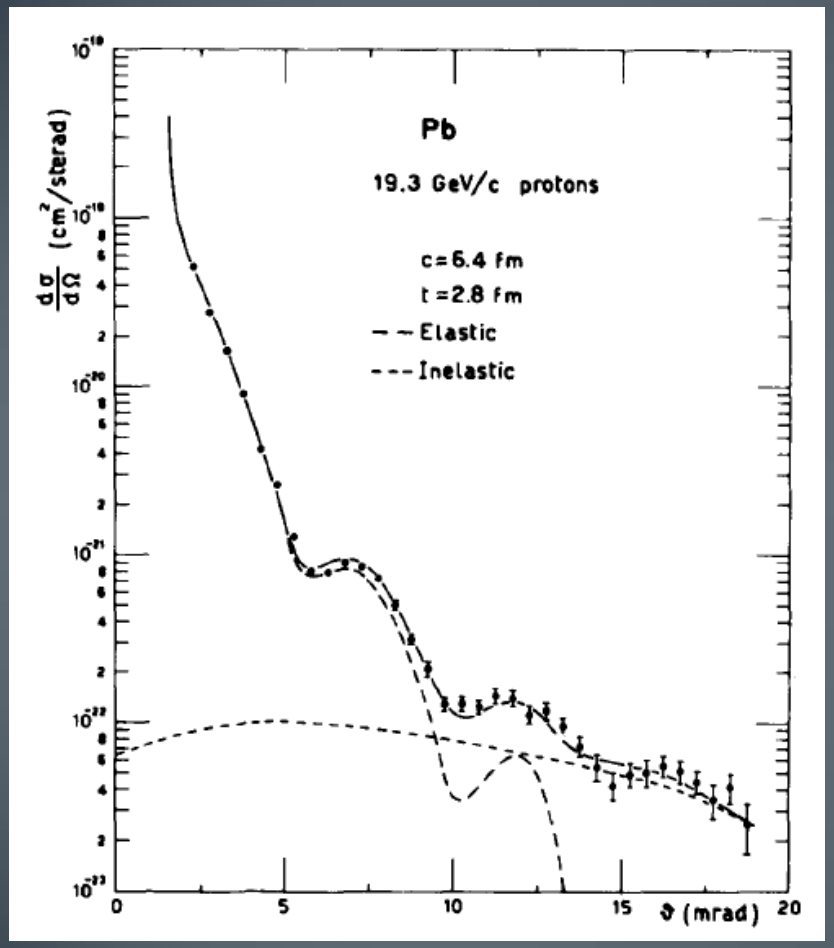


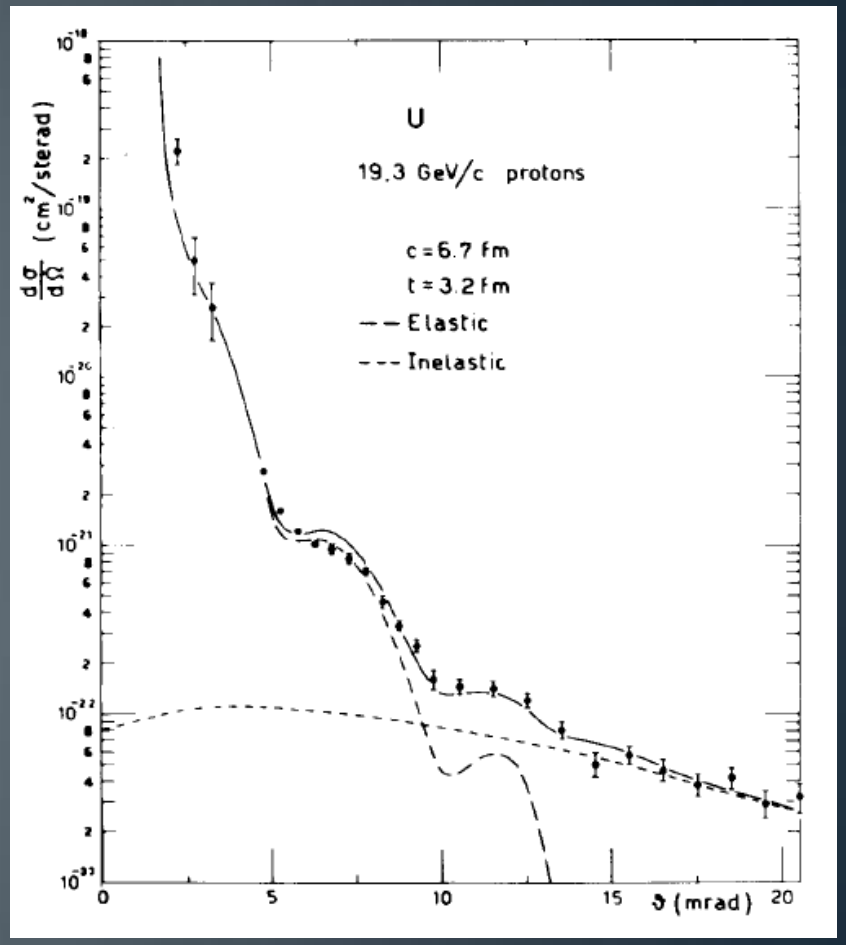
Fig. 5. This figure shows the elastic and inelastic curves corresponding to the scattering of 420-MeV electrons by ^{12}C. The *solid circles*, representing experimental points, show the elastic-scattering behavior while the *solid squares* show the inelastic-scattering curve for the 4.43-MeV level in carbon. The *solid line* through the elastic data shows the type of fit that can be calculated by phase-shift theory for the model of carbon shown in Fig. 8.



Diffractive electron scattering on nuclei and the resulting charge density distributions, images of spherical nuclei

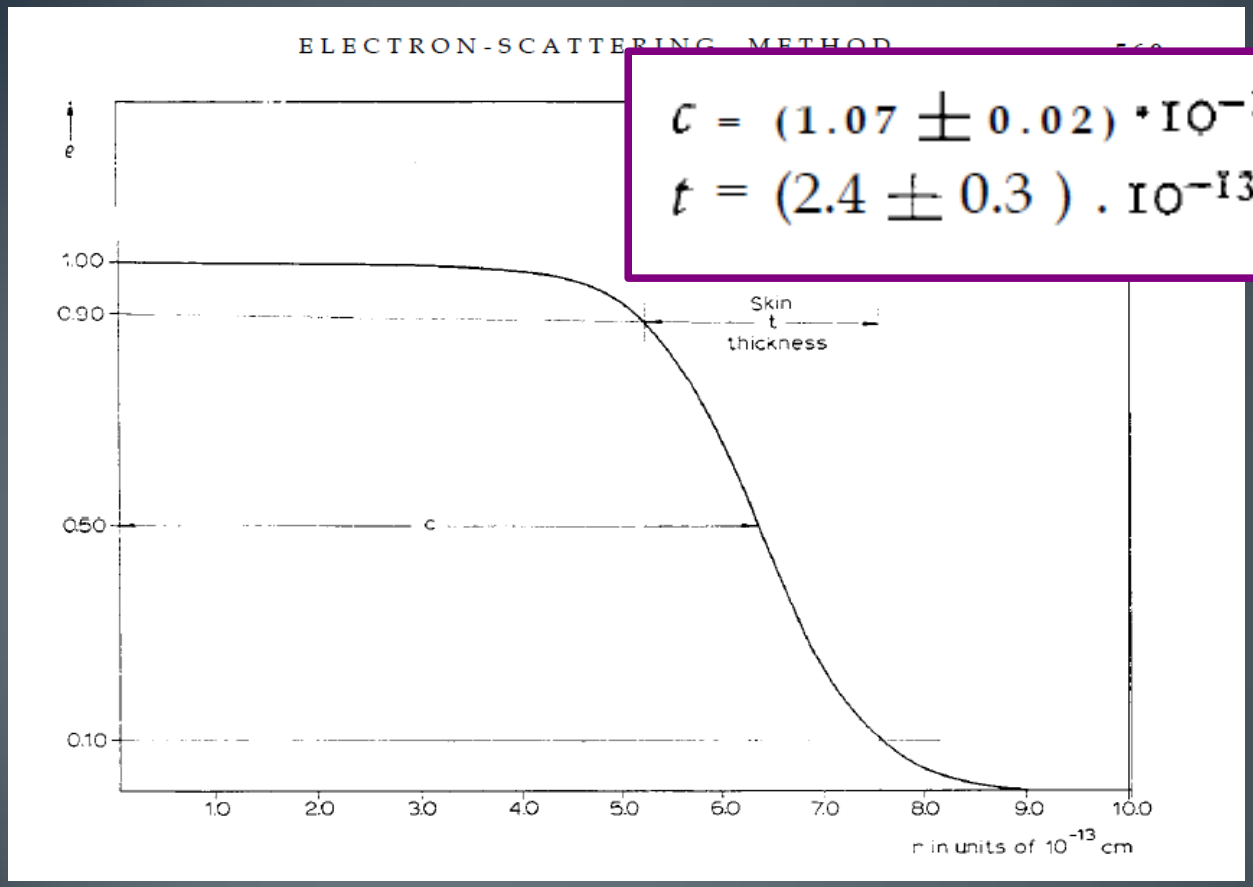
Diffraction in pA, Glauber and Matthiae, NPB21 (1970) 135





Diffractive proton scattering on nuclei confirms the charge density distributions of spherical nuclei

Diffraction – What have we learned?



$$c = (1.07 \pm 0.02) \cdot 10^{-13} A^{\frac{1}{3}} \text{ cm}$$

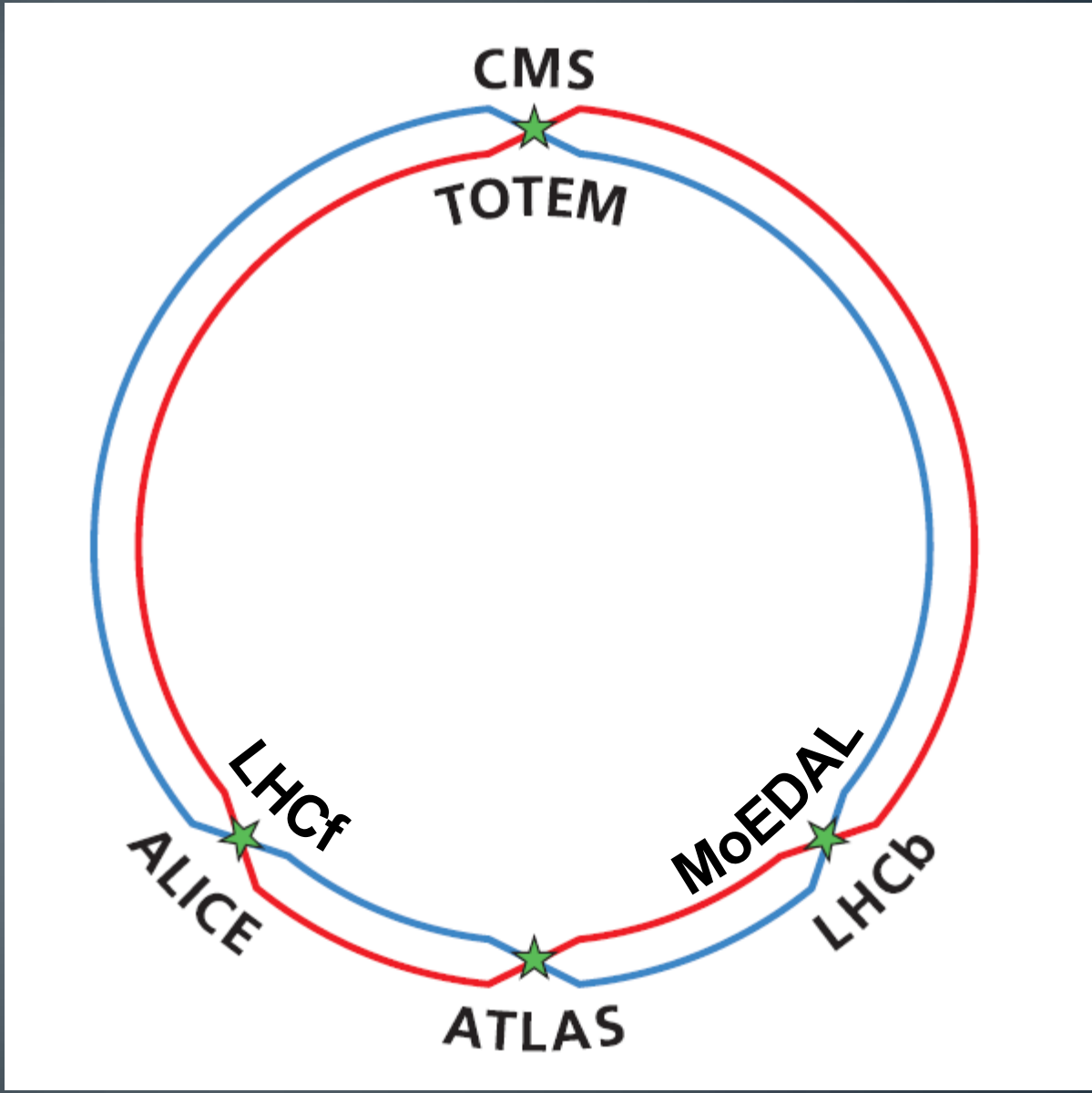
$$t = (2.4 \pm 0.3) \cdot 10^{-13} \text{ cm} = \text{constant}$$

- The volume of spherical nuclei is proportional to A
- The surface thickness is constant, independent of A

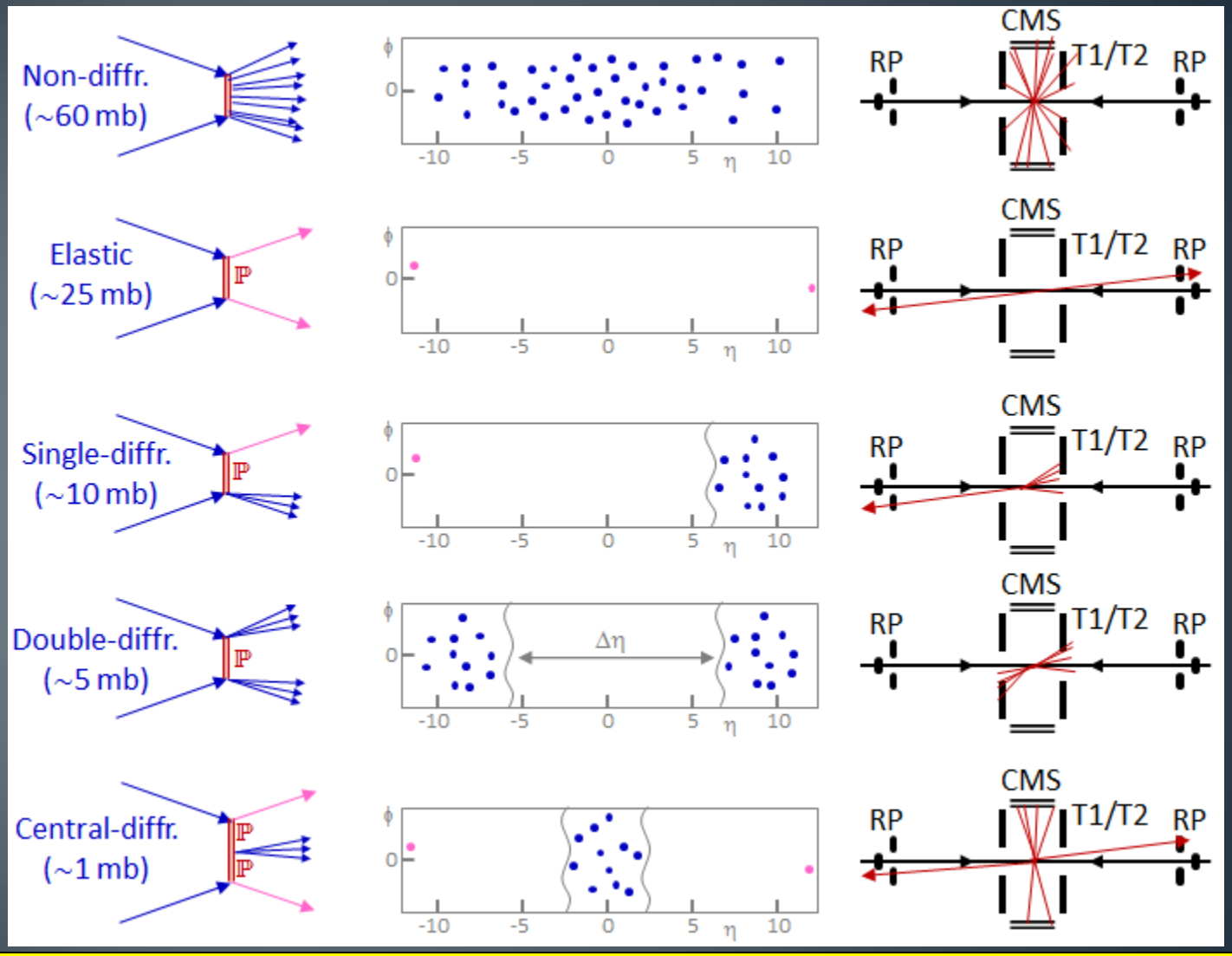
→ Central charge density of large nuclei is approx constant

R. Hofstadter, Nobel Lecture (1961)

LHC experiments reporting to RRB/LHCC



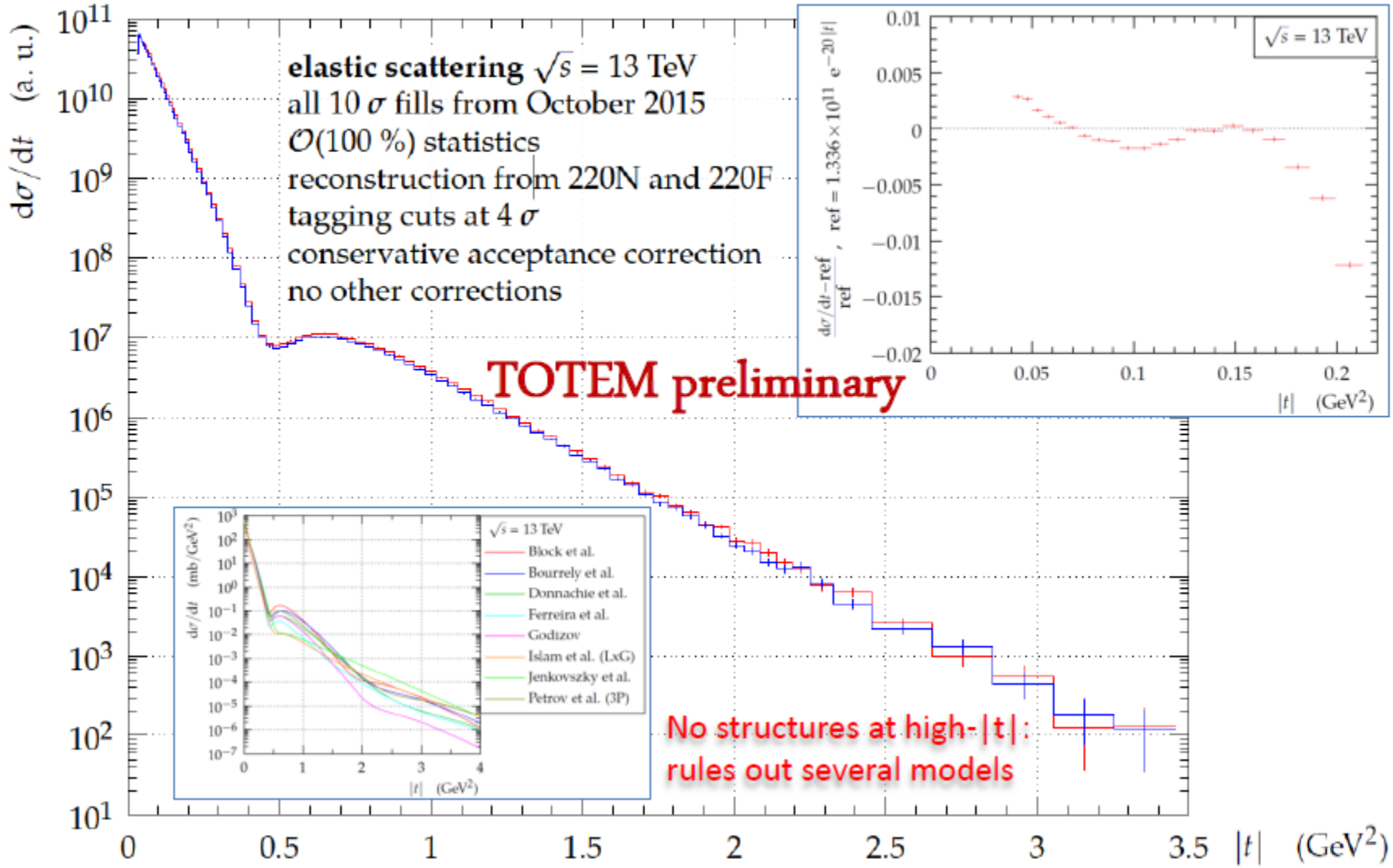
TOTEM physics at LHC



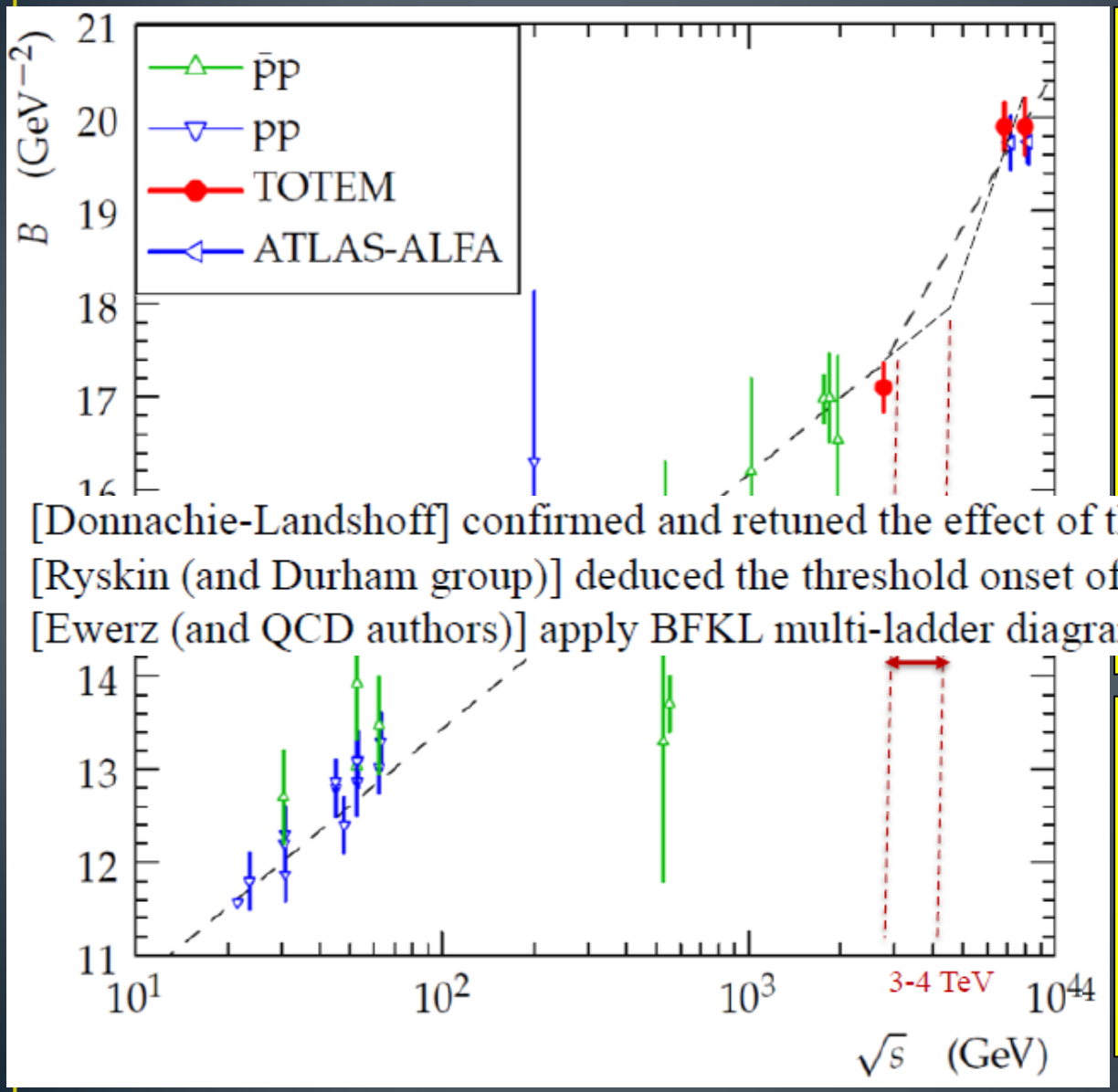
Elastic and diffractive scattering: colorless exchange

TOTEM preliminary at √s = 13 TeV

- Large amount of data (trigger rate 50× w.r.t. Run I)



TOTEM preliminary at √s = 13 TeV



[Donnachie-Landshoff] confirmed and retuned the effect of the hard-Pomeron;
[Ryskin (and Durham group)] deduced the threshold onset of the multi-Pomeron exchange;
[Ewerz (and QCD authors)] apply BFKL multi-ladder diagrams in non-perturbative domain.

Growth of B:
Universal properties of Pomeron

Acceleration of B:
Opening of an additional physics channel

threshold ≤ 3-4 TeV
followed by
very sharp growth

Change of low-|t| trend at LHC

Multiple Diffraction Theory of Elastic Proton-Proton Scattering

Glauber and Velasco, Phys. Lett. B147 (1984) 380
Glauber and Velasco, IFIC preprint, 1996, unpublished

$$\sigma_{tot} = 4\pi ImF(s,0).$$

$$\frac{d\sigma}{dt} = \pi |F(s,t)|^2,$$

$$\rho = \frac{ReF(0)}{ImF(0)}$$

$$s = (p_a + p_b)^2,$$

$$t = (p_a - p_b)^2.$$

$$q = \sqrt{-t}.$$

$$(|t|/s \ll 1)$$

$$F(s,t) = i \int J_0(qb)(1 - e^{-\Omega(s,b)})bdb,$$

Multiple Diffraction Theory of Elastic Proton-Proton Scattering

Glauber and Velasco, Phys. Lett. B147 (1984) 380
Glauber and Velasco, IFIC preprint, 1996, unpublished

$$f_{ij} = \frac{i}{2\pi} \int e^{i\vec{q}\,\vec{b}} \Gamma_{ij}(s, \vec{b}) d^2 b$$

$$\Gamma_{ij}(s, \vec{b}) = \frac{1}{2\pi i} \int e^{-i\vec{q}\,\vec{b}} f_{ij}(t) d^2 q,$$

$$\Omega(\vec{b}) = AB \int \rho_a(\vec{r}_1) \langle \Gamma_{ij}(\vec{b} + \vec{s}_1 - \vec{s}_2) \rangle \rho_b(\vec{r}_2) d^3 r_1 d^3 r_2,$$

$$\Omega(s, \vec{b}) = \frac{AB}{2\pi i} \int e^{-i\vec{q}\,\vec{b}} G_a G_b f(t) d^2 q,$$

Glauber-Velasco Model of Elastic Proton-Proton Scattering:

Glauber and Velasco, Phys. Lett. B147 (1984) 380
Glauber and Velasco, IFIC preprint, 1996, unpublished

$$G_a(t) = G_b(t) = G_p(t),$$

$$\frac{f(t)}{f(0)} = \frac{e^{i(b_1|t|+b_2|t|^2)}}{(1+a|t|)^{1/2}}$$

$$A^2 f(0) = \frac{\kappa}{4\pi}(i + \alpha)$$

Glabuer-Velasco Model: a parametrization of f(t)

$$\Omega(s,\vec{b}) = \frac{\kappa}{4\pi}(1 - i\alpha)\int J_0(qb)G_p^2(t)\frac{f(t)}{f(0)}qdq,$$

$$G_E = (1.0 - 3.04t + 1.54t^2 - 0.068t^3)^{-1}$$

Felst EM form factor G_E

Assumptions: pp scattering, angular symmetry, nuclear and electromagnetic form factors: the same (data driven G_E)

Glauber – Velasco model summary

$$F(t) = i \int_0^\infty J_0\left(b\sqrt{-t}\right)\left\{1 - \exp\left[-\Omega\left(b\right)\right]\right\} b db$$

$$\Omega\left(b\right) = \frac{\kappa}{4\pi}\left(1 - i\alpha\right)\int_0^\infty J_0\left(q\,b\right) G^2_{p,E}\left(-t\right)\frac{f(t)}{f(0)} q dq$$

$$\frac{f(t)}{f(0)} = \frac{e^{i\left(b_1|t| + b_2\,t^2\right)}}{\sqrt{1 + a\,|t|}}$$

$$G_{p,E}\left(q^2\right) = \sum_{i=1}^{n} \frac{a_i^E\left(m_i^E\right)^2}{\left(m_i^E\right)^2 + q^2}\,,\ \sum_{i=1}^{n} a_i^E = 1\,,\ G_{p.E}(0) = 1$$

$$d\sigma_{el}/d\,|t| = \pi\,|F(t)|^2$$

a_i^E	$(m_i^E)^2$ (fm^{-2})
0.219	3.53
1.371	15.02
-0.634	44.08
0.044	154.20

F(t): f. sc. amplitude
Ω(b): opacity, complex

f(t): cluster averaged parton-parton scattering amplitude

$-t = q^2$: momentum transfer
b: impact parameter

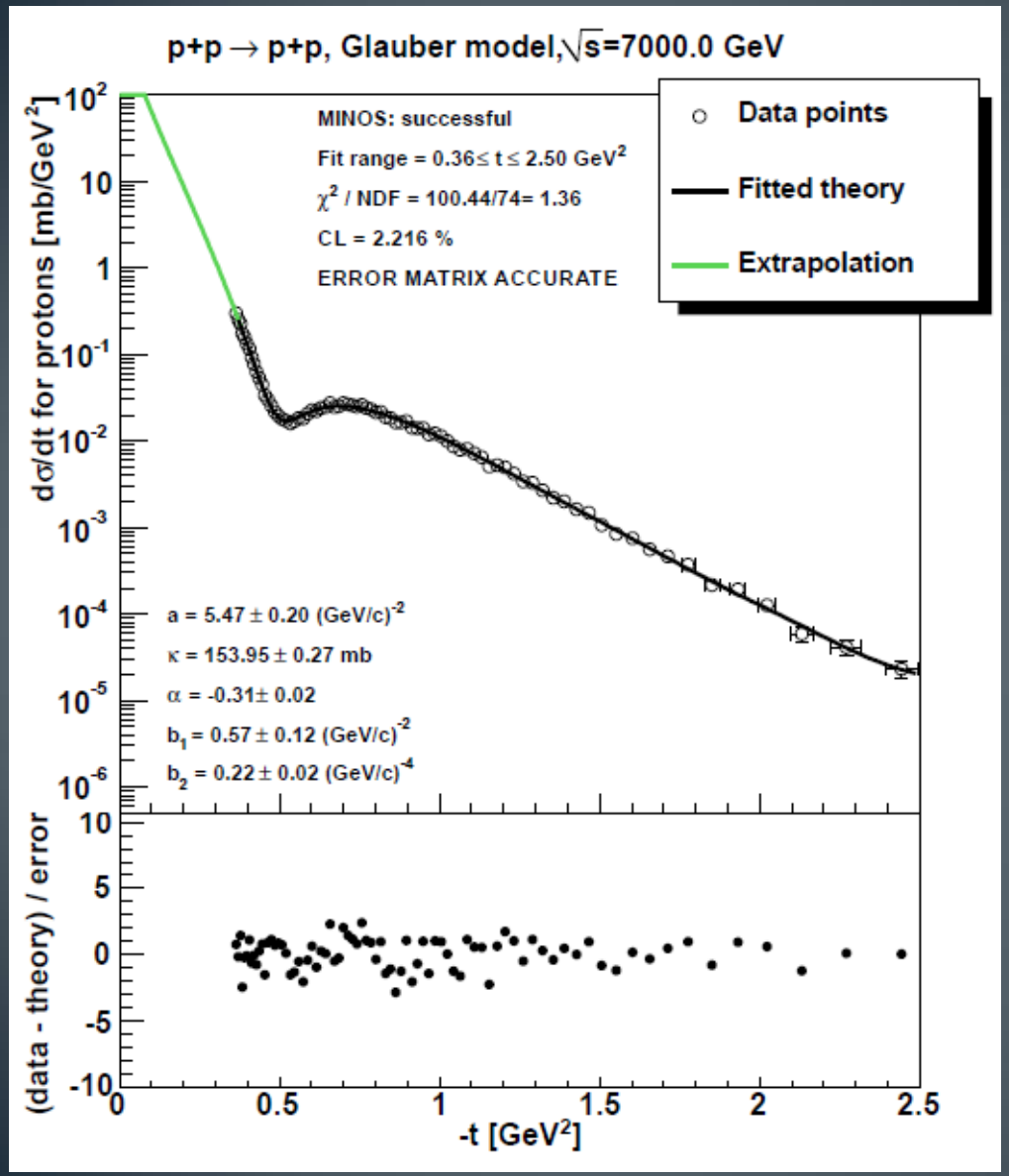
dσ/dt: diff. cross-section elastic pp scattering

R.J. Glauber and J.Velasco
Phys. Lett. B147 (1987) 380

BSWW EM form factors G_E

Fcross-check with Felst G_E

First results @ Low-X 2013: GV works for dσ/dt dip



Glauber-Velasco (GV)
(original)

describes dσ/dt data
Both at ISR and
TOTEM@LHC
in the dip region

arxiv:1311.2308

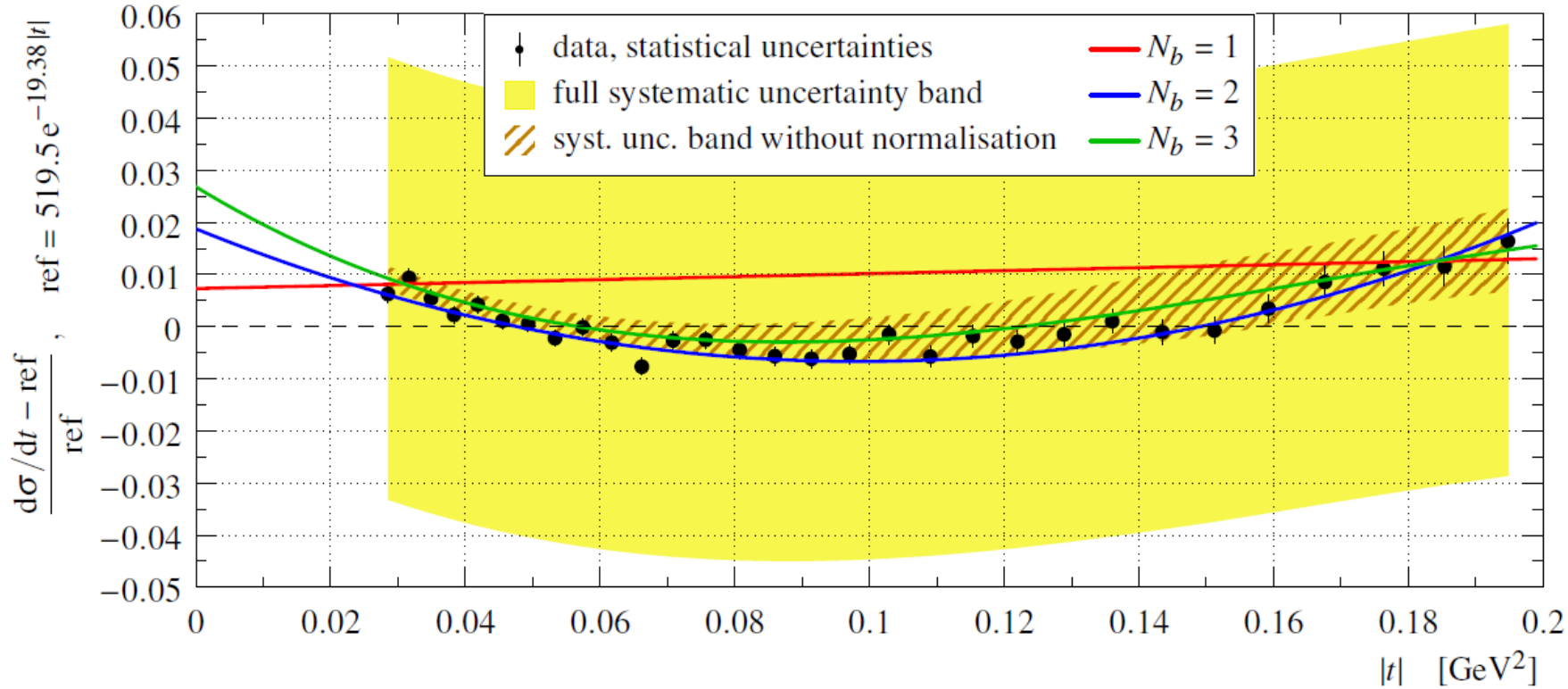
Note: at low-t
GV is ~ exponential

Really?
Lower energies?

TOTEM results @ 8 TeV, arxiv:1503.08111: dσ/dt non-exponential at low-t

Table 4: Fit quality measures for fits in Figure 11.

N_b	χ^2/ndf	p-value	significance
1	117.5/28 = 4.20	$6.1 \cdot 10^{-13}$	$7.2\,\sigma$
2	29.3/27 = 1.09	0.35	$0.94\,\sigma$
3	25.5/26 = 0.98	0.49	$0.69\,\sigma$



Non-exponential $\bar{p}p$ in GV model

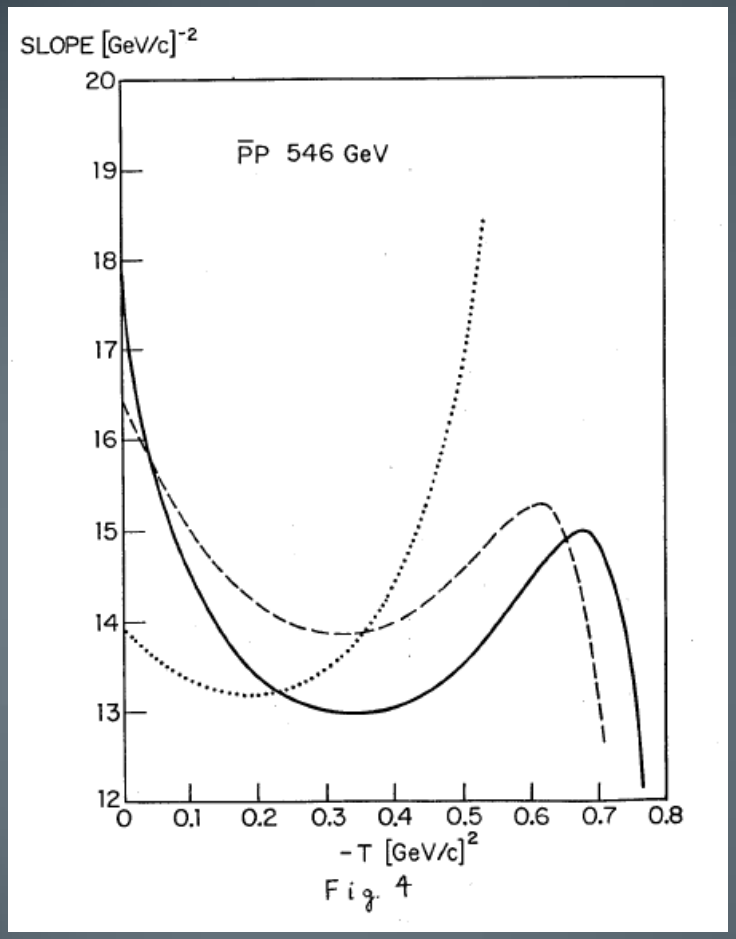


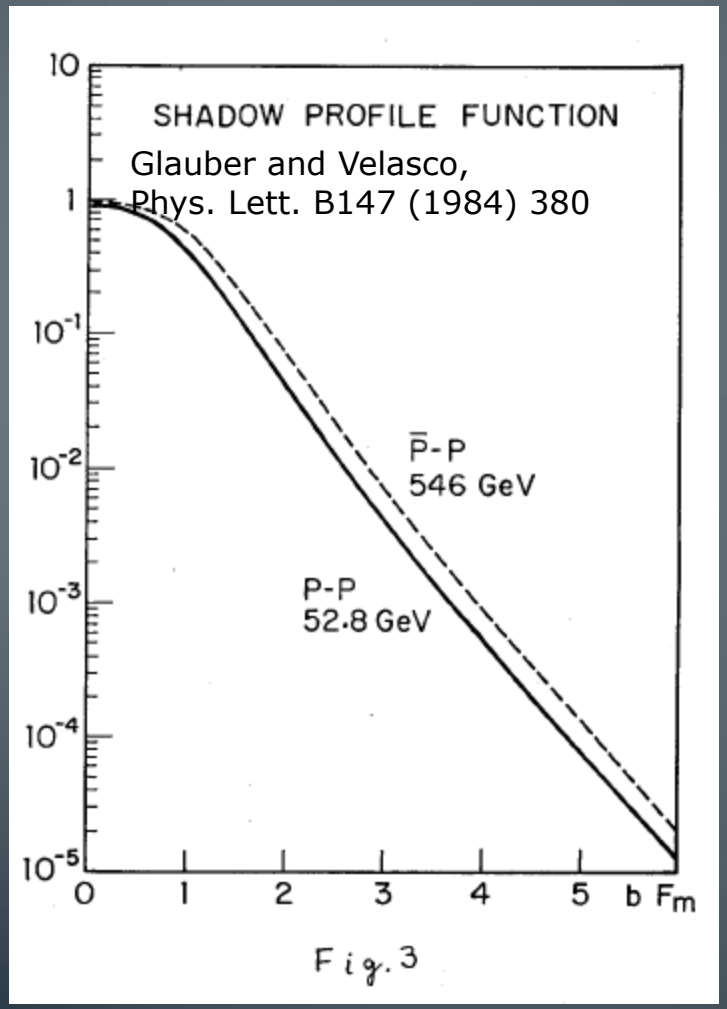
Figure 4 from
Glauber-Velasco
PLB 147 (1984) 380
Slope is not quite
exponential:

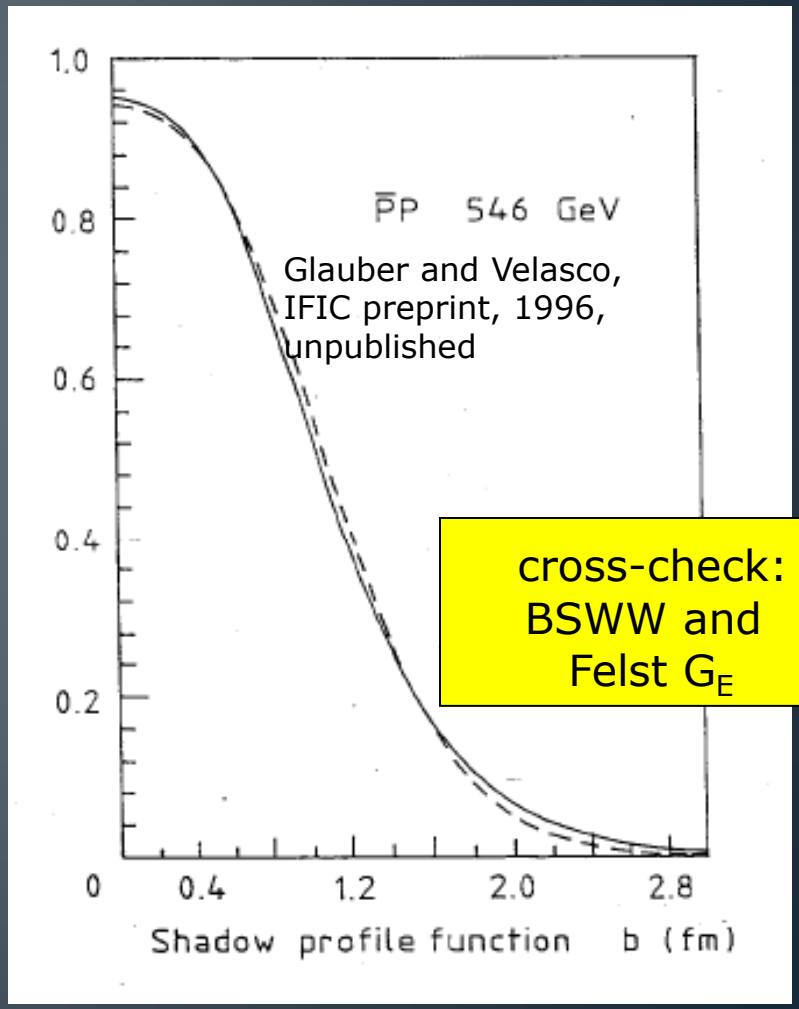
A non-Gaussian behaviour
of proton scattering
in coordinate space

Fig. 4: Logarithmic slopes of the $\bar{p} - p$ differential cross-section at 546 GeV calculated according to: the *BSWW* form factor, which is accurate at small momentum transfers (solid wave), the Felst form factor, which accounts only for the data at larger momentum transfers (dashed curve) and the dipole form factor together with the Chou-yang scattering amplitude (dotted curve).

Imaging with shadow profile

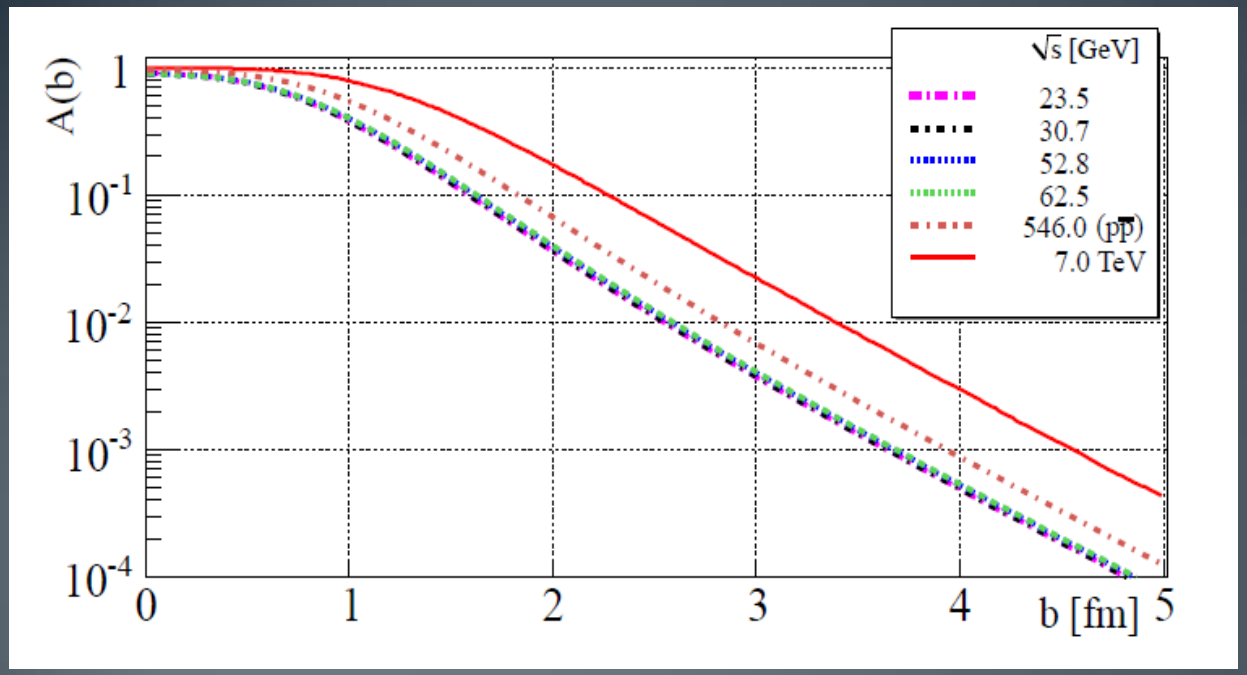
$$A(b) = 1 - |e^{-\Omega(b)}|^2$$





cross-check:
BSWW and
Felst G_E

Saturation from shadow profiles



at 7 TeV
proton becomes

Blacker, but
not **E**dgier,
and **L**arger

BEL ➡ **BnEL** effect
arxiv:1311.2308

$$A(b) = 1 - |e^{-\Omega(b)}|^2$$

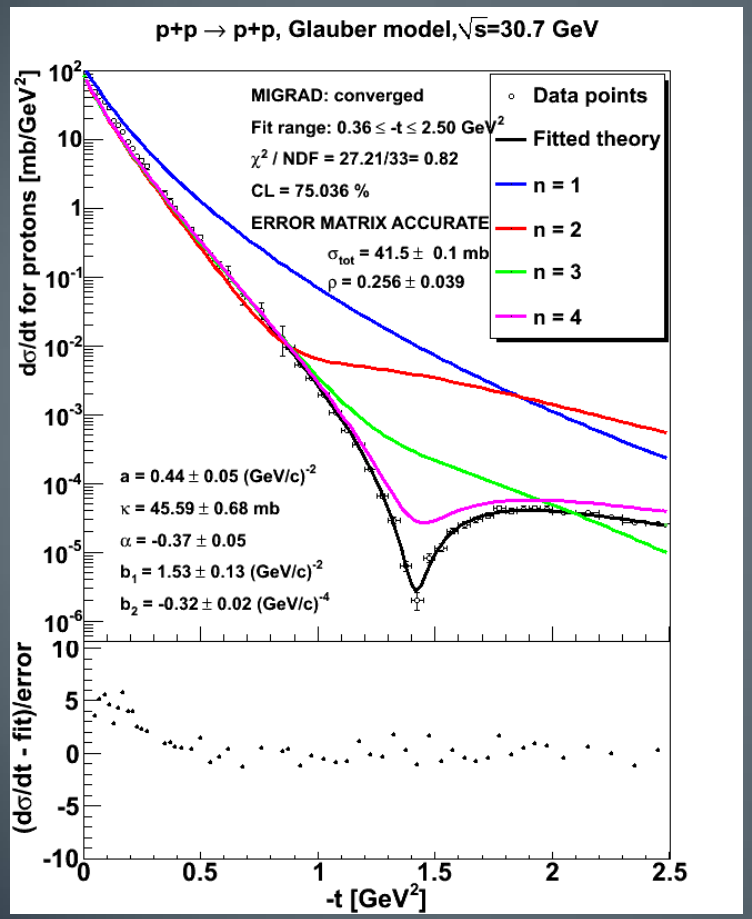
ISR and SppS:
R.J. Glauber and J.Velasco
Phys. Lett. B147 (1987) 380
b_1,b_2 fixed

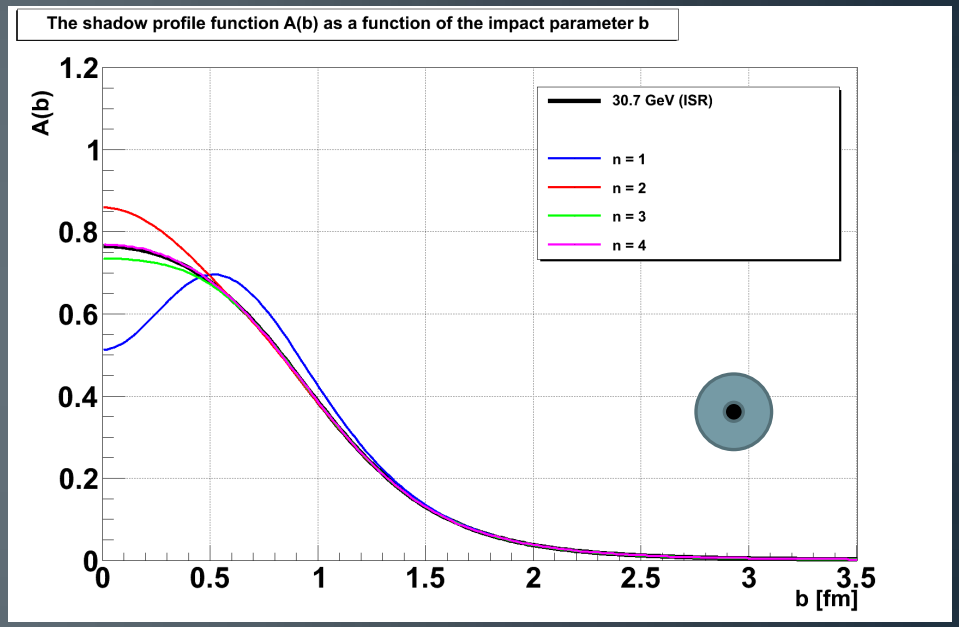
apparent saturation:
center of proton ~ black
at LHC, up to
r ~ 0.7 fm

see also Lipari and Lusignoli,
arXiv:1305.7216

Shadow imaging in p+p at ISR

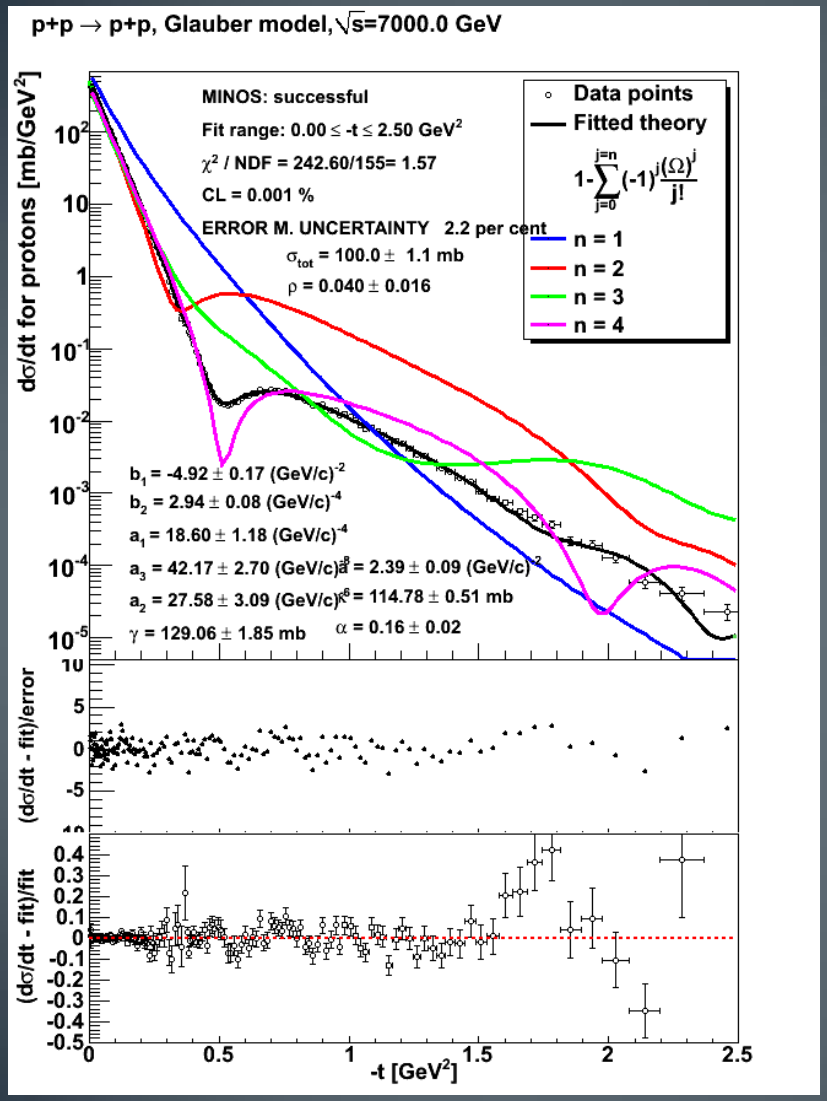
Opacity expansion of order n: $\exp(-\Omega) = 1 - \Omega + \Omega^2/2! - \ldots$





Proton image: gray (n=1) edge, dark (n~3) in very center

Shadow imaging in p+p at LHC



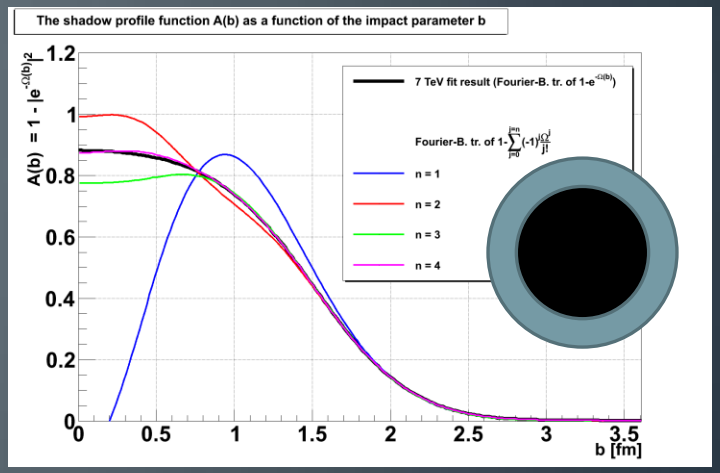
Simple choice for the generalization of GV:

$f(t)/f(0) = \exp(i\, b_1\, |t| + i\, b_2\, |t|^2) / \mathrm{sqrt}(1 + a\, t)$

→

$f(t)/f(0) = \exp(i\, b_1\, |t| + i\, b_2\, |t|^2) / \mathrm{sqrt}(1 + a|t| + a_1\, |t|^2 + a_2\, |t|^3 + \ldots)$

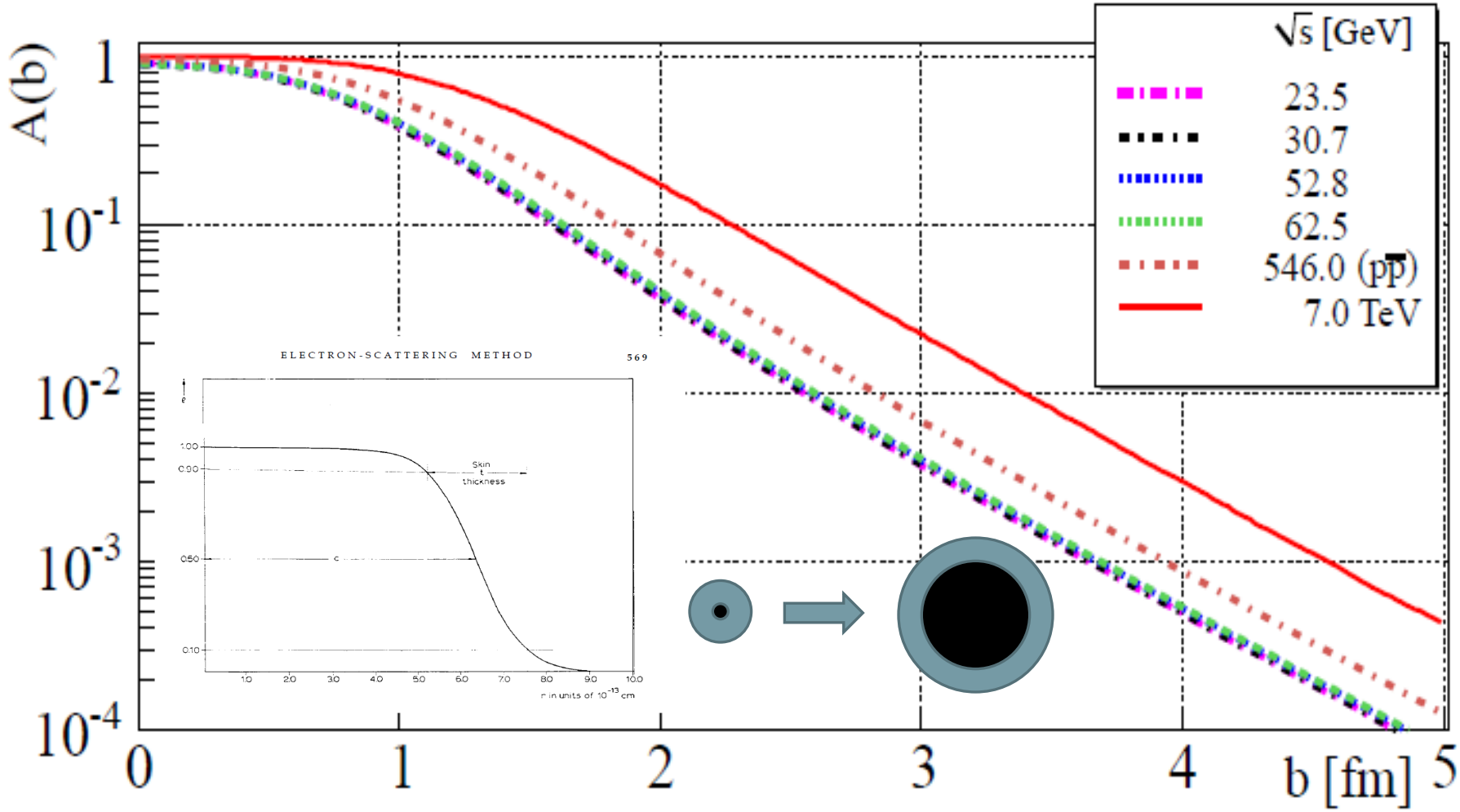
A more systematic expansions and further improvements are needed!



Opacity expansion of order n:
$\exp(-\Omega) = 1 - \Omega + \Omega^2/2! - \ldots$

Proton at 7 TeV: gray edge, very dark region (n ≥ 4) in center

Shadow imaging in p+p at LHC



The **BnEL** effect.

Can it explain TOTEM data, new trends of B at LHC?

MODEL-INDEPENDENT SHAPE ANALYSIS

Model-independent method, originally to analyze Bose-Einstein correlations IF experimental data satisfy

- **The data *tend to a constant* for *large va*lues of the observable Q.**
- **There is a *non-trivial structure* at some value of Q, shift it to Q = 0.**

Model-independent, but experimentally testable:

- **t = Q R**
- **dimensionless scaling variable**
- **approximate form of the correlations *w(t)***
- ***Consider w(t)* as an abstract measure in a Hilbert-space**

$$\int dt w(t) h_n(t) h_m(t) = \delta_{n,m},$$

$$f(t) = \sum_{n=0}^{\infty} f_n \, h_n(t),$$

$$f_n = \int dt w(t) \, f(t) \, h_n(t).$$

e.g. $t = Q_I R_I$

T. Csörgő and S: Hegyi, hep-ph/9912220, T. Csörgő, hep-ph/001233

GAUSSIAN w(t|α=2): EDGEWORTH EXP.

$$t = \sqrt{2} Q R_E,$$
$$w(t) = \exp(-t^2/2),$$
$$\int_{-\infty}^{\infty} dt \, \exp(-t^2/2) H_n(t) H_m(t) \propto \delta_{n,m},$$

$$H_n(t) = \exp(t^2/2) \left(-\frac{d}{dt}\right)^n \exp(-t^2/2).$$

$$H_1(t) = t,$$
$$H_2(t) = t^2 - 1,$$
$$H_3(t) = t^3 - 3t,$$
$$H_4(t) = t^4 - 6t^2 + 3, \ \ldots$$

$$C_2(Q) = \mathcal{N} \left\{ 1 + \lambda_E \exp(-Q^2 R_E^2) \times \left[1 + \frac{\kappa_3}{3!} H_3(\sqrt{2} Q R_E) + \frac{\kappa_4}{4!} H_4(\sqrt{2} Q R_E) + \ldots \right] \right\}.$$

3d generalization straightforward

- **Applied by NA22, L3, STAR, PHENIX, ALICE, CMS (LHCb)**

EXPONENTIAL w(t|α=1): LAGUERRE

Model-independent but experimentally tested:

- $w(t)$ exponential
- t = QR dimensionless
- $L_n(t)$ **Laguerre** polynomials

$$t = QR_L,$$
$$w(t) = \exp(-t)$$
$$\int_0^\infty dt\, \exp(-t) L_n(t) L_m(t) \propto \delta_{n,m},$$

$$L_n(t) = \exp(t) \frac{d^n}{dt^n} (-t)^n \exp(-t).$$

$$L_0(t) = 1,$$
$$L_1(t) = t - 1,$$

$$C_2(Q) = \mathcal{N} \left\{ 1 + \lambda_L \exp(-QR_L) \left[1 + c_1 L_1(QR_L) + \frac{c_2}{2!} L_2(QR_L) + \ldots \right] \right\}$$

Laguerre expansion successful on

- NA22, UA1 data
- convergence criteria: satisfied
- intercept $\lambda_* \sim 1$

$$\int_0^\infty dt\, R_2^2(t) \exp(+t) < \infty,$$

$$\lambda_* = \lambda_L [1 - c_1 + c_2 - \ldots],$$
$$\delta^2 \lambda_* = \delta^2 \lambda_L \left[1 + c_1^2 + c_2^2 + \ldots \right] + \lambda_L^2 \left[\delta^2 c_1 + \delta^2 c_2 + \ldots \right]$$

STRETCHED EXPONENTIAL w(t): LEVY EXPANSIONS

$$w(t) = \exp(-t^\alpha)$$

Model-independent but:

- Levy: stretched exponential
- generalizes exponentials and Gaussians
- ubiquoutous in nature
- How far from a Levy?
- New set of polynomials orthonormal to Levy weight

$$\begin{aligned} L_0(t\,|\,\alpha) &= 1, \\ L_1(t\,|\,\alpha) &= \det\begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} \\ 1 & t \end{pmatrix}, \\ L_2(t\,|\,\alpha) &= \det\begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} \\ 1 & t & t^2 \end{pmatrix} \end{aligned}$$

$$\mu_{n,\alpha} = \int_0^\infty dt\, t^n \exp(-t^\alpha) = \frac{1}{\alpha}\Gamma\left(\frac{n+1}{\alpha}\right)$$

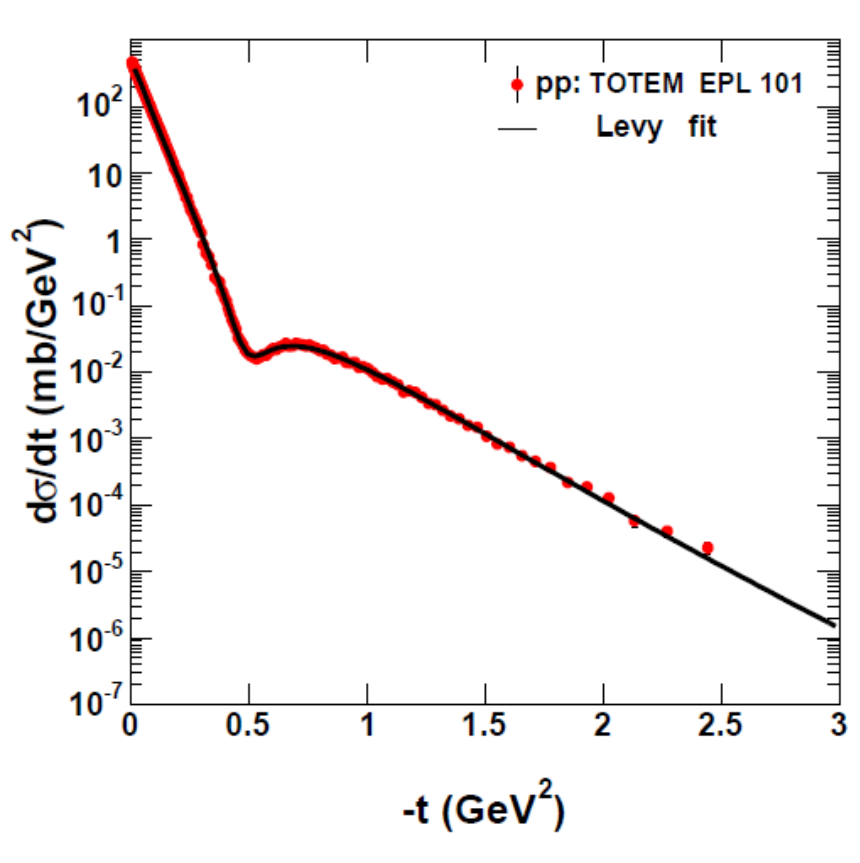
$$\Gamma(z) = \int_0^\infty dt\, t^{z-1} e^{-t}$$

T. Cs, T. Novák, H.C. Eggers and M. de Kock, arXiv:1604.05513 [physics.data-an]

MODEL INDEPENDENT LEVY EXPANSION

$$z = \sqrt{|t|}\,R$$

$$\frac{d\sigma}{dt} = \left.\frac{d\sigma}{dt}\right|_{t=0} \exp(-z^{\alpha})\,|1 + c_1 L_1(z|\alpha) + c_2 L_2(z|\alpha) + \ldots|^2$$



J. Chwastowski, Trento, 2016:
What about imaging the proton?

T. Csörgő, T. Novák and A. Ster
in preparation
arXiv:1604.05513 [physics.data-an]

GENERALIZED GLAUBER-VELASCO MODEL

Old: Model Dependent Phenomenology

$$\frac{f(t)}{f(0)} = \frac{e^{i\left(b_1|t|+b_2\,t^2\right)}}{\sqrt{1+a\,|t|}}$$

New: Model Independent Levy Expansion of f(t)

$$\frac{f(t)}{f(0)} = \exp\left(-\frac{1}{2}z^{\alpha}\right)\left[1+c_1L_1(z|\alpha)+c_2L_2(z|\alpha)+\ldots\right],$$

$$z=\sqrt{|t|}R, \qquad 0<\alpha\leq 2,$$

$$\begin{aligned}
L_0(t\,|\,\alpha) &= 1,\\
L_1(t\,|\,\alpha) &= \frac{1}{\alpha}\left\{\Gamma\left(\frac{1}{\alpha}\right)t-\Gamma\left(\frac{2}{\alpha}\right)\right\},\\
L_2(t\,|\,\alpha) &= \frac{1}{\alpha^2}\left\{\left[\Gamma\left(\frac{1}{\alpha}\right)\Gamma\left(\frac{3}{\alpha}\right)-\Gamma^2\left(\frac{2}{\alpha}\right)\right]t^2-\right.\\
&\quad -\left[\Gamma\left(\frac{1}{\alpha}\right)\Gamma\left(\frac{4}{\alpha}\right)-\Gamma\left(\frac{3}{\alpha}\right)\Gamma\left(\frac{2}{\alpha}\right)\right]t+\\
&\quad \left.+\left[\Gamma\left(\frac{2}{\alpha}\right)\Gamma\left(\frac{4}{\alpha}\right)-\Gamma^2\left(\frac{3}{\alpha}\right)\right]\right\}.
\end{aligned}$$

Fit parameters:
Levy scale R
Levy exponent α
Complex values of c_i

SUMMARY AND CONCLUSIONS

MODEL INDEPENDENT LEVY EXPANSION

at LHC energies
proton becomes

Blacker, but
not **E**dgier,
and **L**arger:

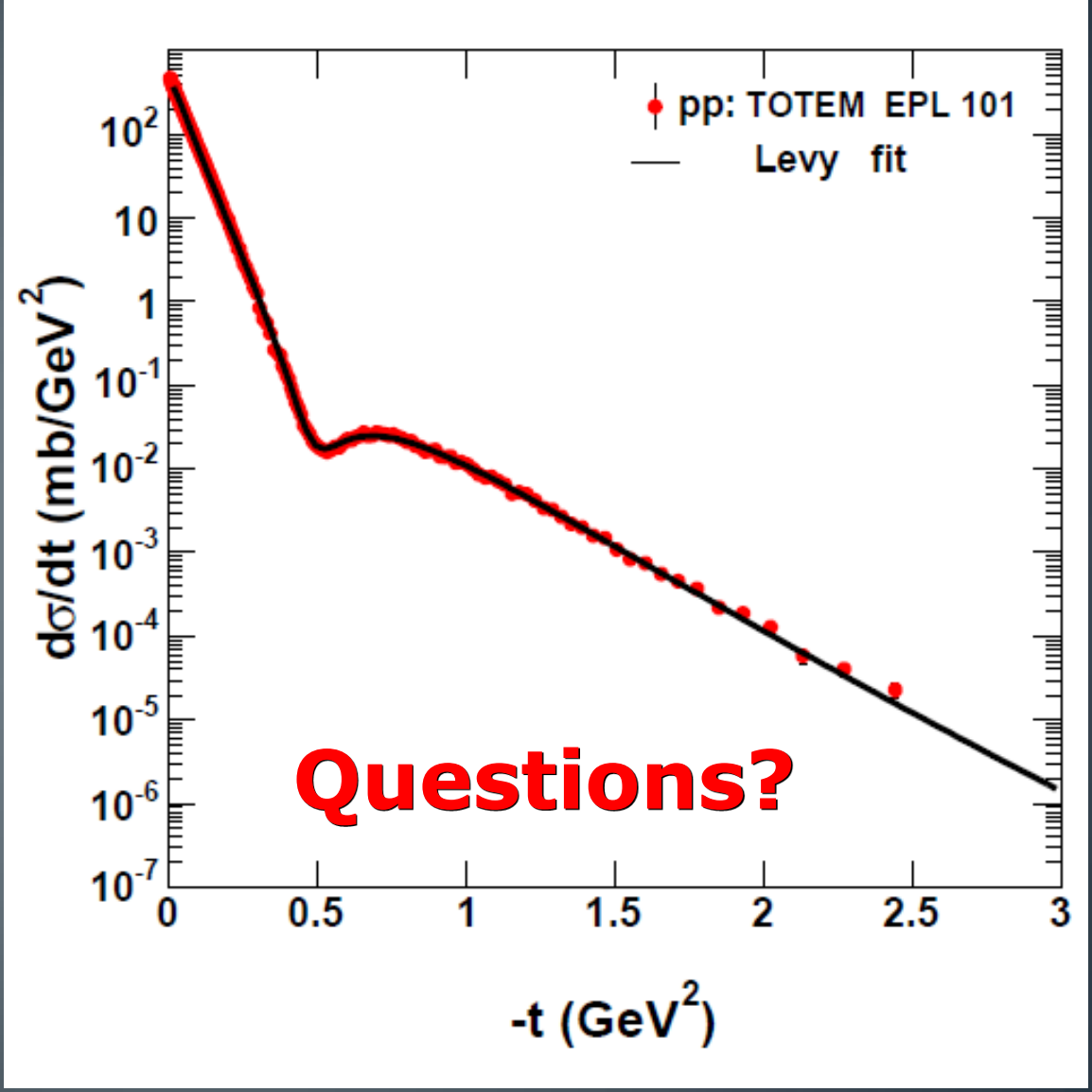
BnEL effect

Glauber-Velasco model:
Non-exponential, dip regions fine, but…

Model-independent Levy expansion
Glauber-Velasco model's f(t)

Promising, but needs further testing

Thank you for your attention!



Backup slides

STRETCHED EXPONENTIAL w(t): LEVY EXPANSIONS

In case of $\alpha = 1$,
in 1 dimension
Laguerre expansion is recovered

$$
\begin{aligned}
L_0(t\,|\,\alpha = 1) &= 1, \\
L_1(t\,|\,\alpha = 1) &= t - 1, \\
L_2(t\,|\,\alpha = 1) &= t^2 - 4t + 2.
\end{aligned}
$$

This case reduces to the Laguerre polynomials.

arXiv:1604.05513 [physics.data-an]

STRETCHED EXPONENTIAL w(t): LEVY EXPANSIONS

In case of α = 2 ,
a new formulae for one-sided (t ≥ 0) Gaussians:

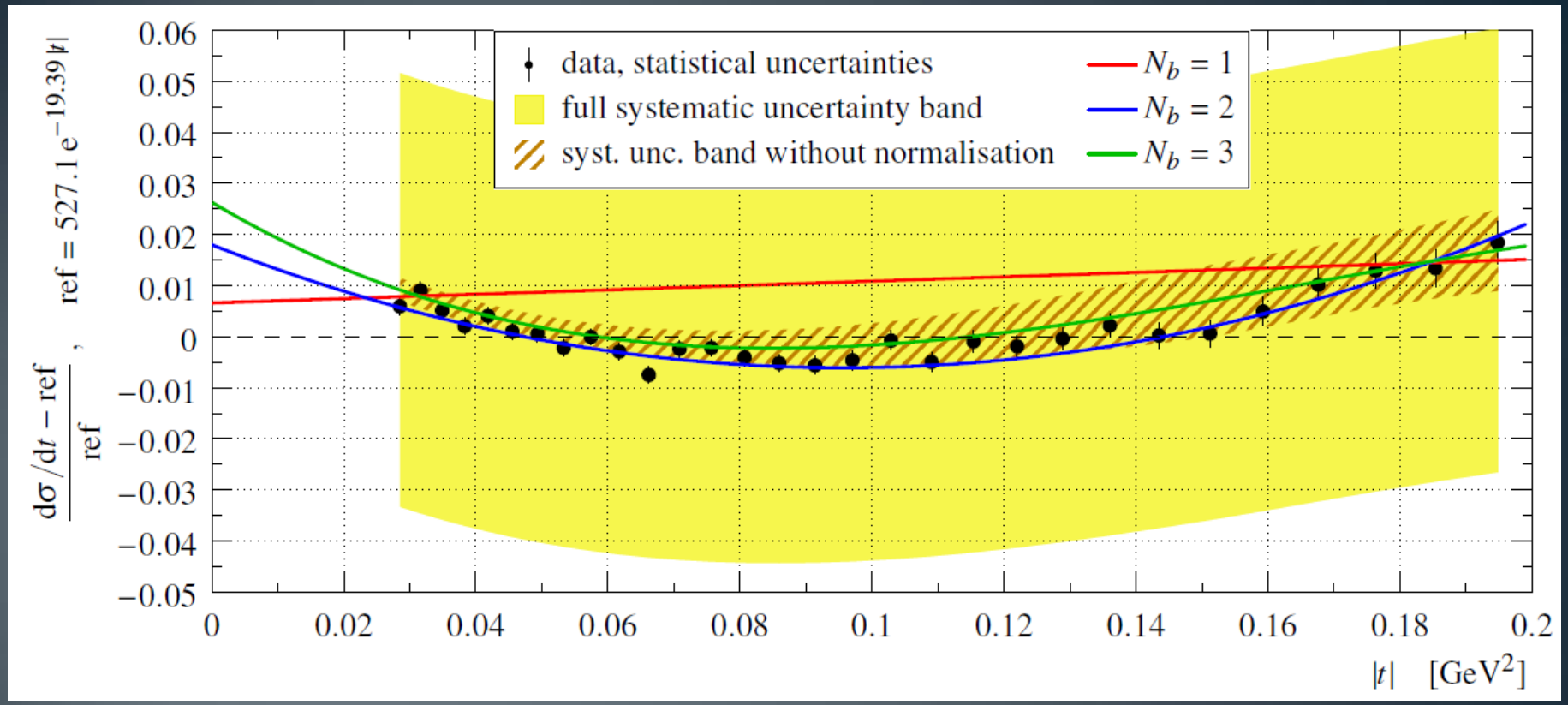
$$
\begin{aligned}
L_0(t\,|\,\alpha=2) &= \frac{\sqrt{\pi}}{2}, \\
L_1(t\,|\,\alpha=2) &= \frac{1}{2}\left\{\sqrt{\pi}t-1\right\}, \\
L_2(t\,|\,\alpha=2) &= \frac{1}{32}\left\{(\pi-2)t^2-\sqrt{\pi}t+2-\frac{\pi}{2}\right\}.
\end{aligned}
$$

Provides a new expansion around a Gaussian shape that is defined for the non-negative values of *t* only.

Edgeworth expansion is different, it is around two-sided Gaussian, including the non-negative values of *t* also.

arXiv:1604.05513 [physics.data-an]

Differential cross-section @ 8 TeV



$$\frac{d\sigma}{dt}(t) = \left.\frac{d\sigma}{dt}\right|_{t=0} \exp\left(\sum_{i=1}^{N_b} b_i\, t^i\right),$$

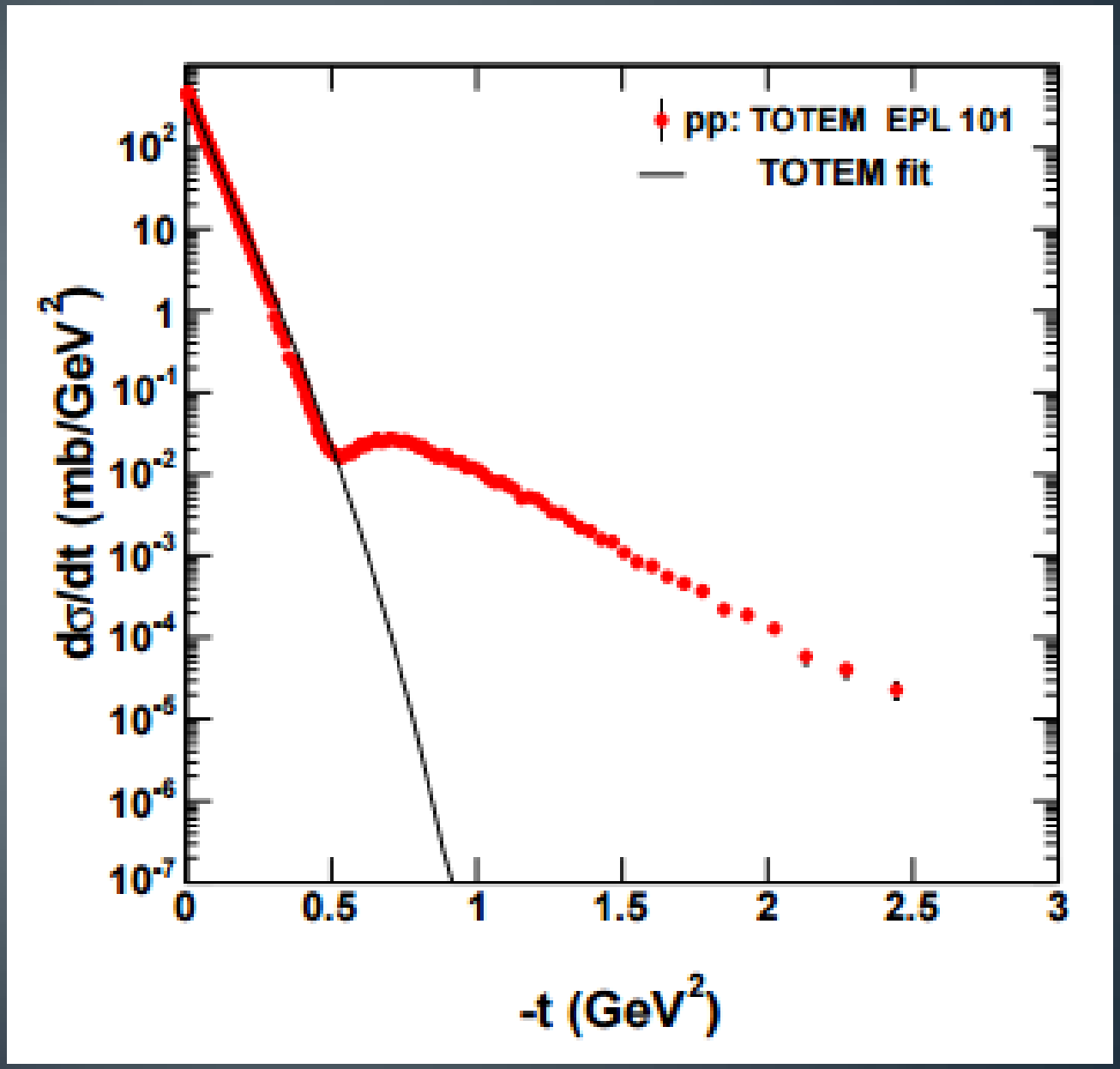
$$\chi^2 = \Delta^T V^{-1} \Delta\,,$$

$$V = V_{stat} + V_{syst}$$

$$\Delta_i = \left.\frac{d\sigma}{dt}\right|_{\text{bin } i} - \frac{1}{\Delta t_i}\int_{\text{bin } i} f(t)\, dt\,,$$

TOTEM, arxiv:1503.08111, cumulant expansion: $N_b = 1$ fits excluded.
Relative to best exponential, a significant (7σ) deviation found.

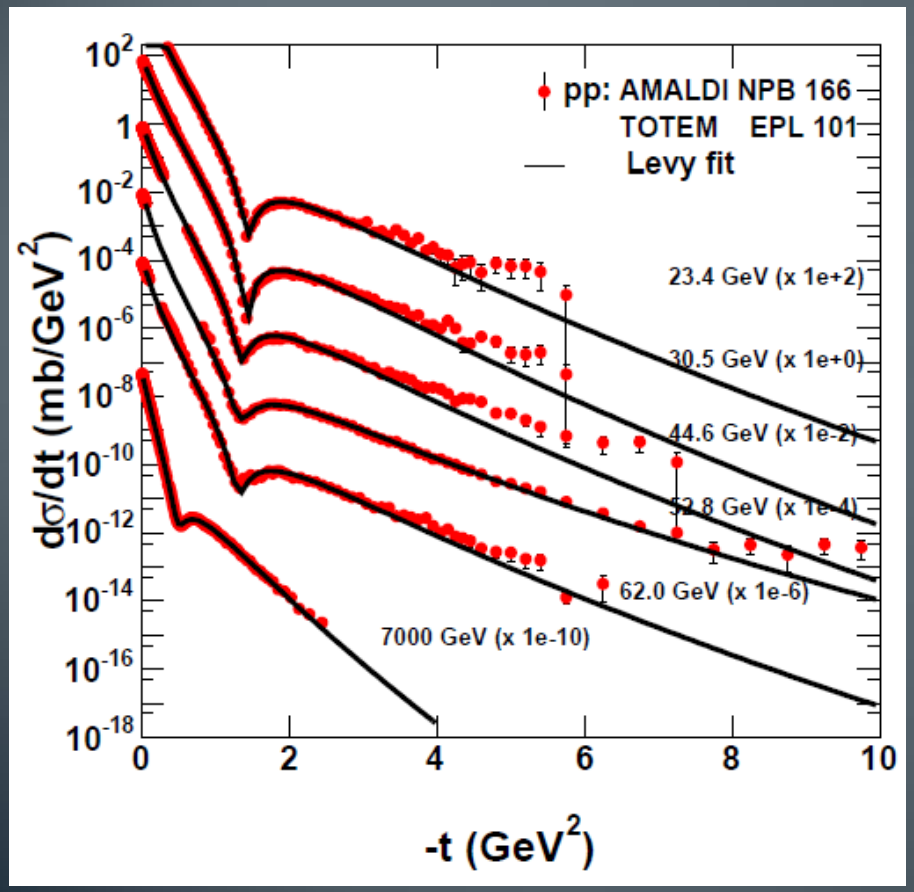
CUMULANT EXPANSION AT LARGE -t

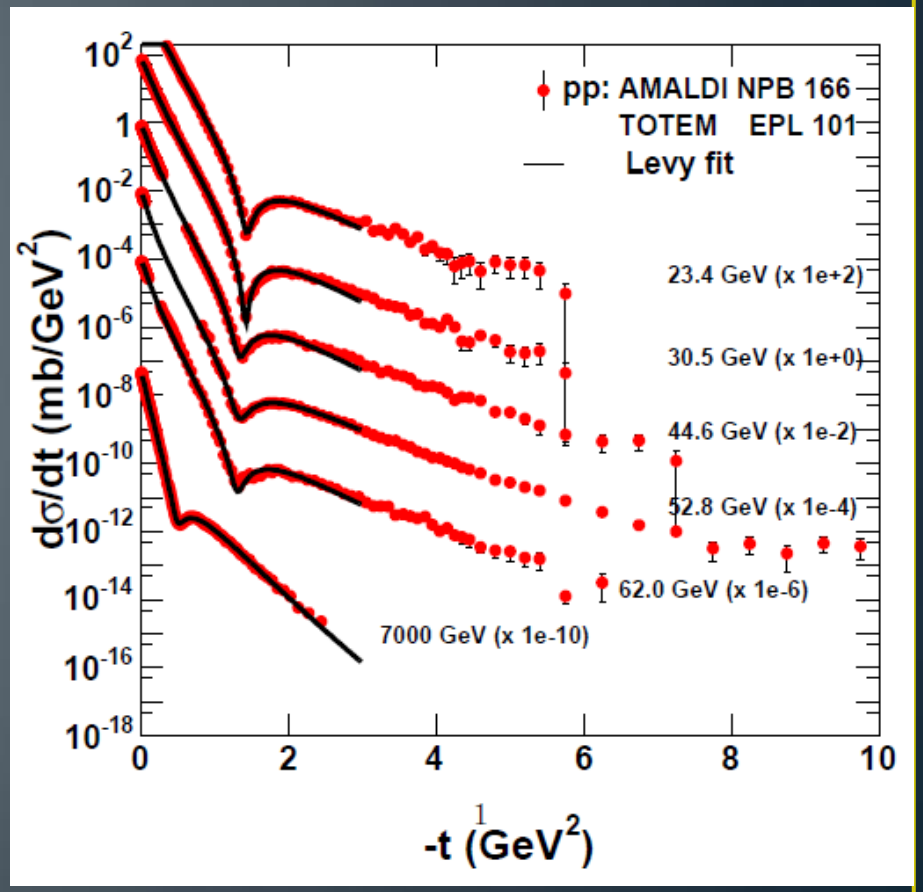


LEVY EXPANSION AT ISR AND AT 7 TEV

$$z = \sqrt{|t|}\, R$$

$$\frac{d\sigma}{dt} = \left.\frac{d\sigma}{dt}\right|_{t=0} \exp(-z^{\alpha})\, |1 + c_1 L_1(z|\alpha) + c_2 L_2(z|\alpha) + \ldots|^2$$





T. Csörgő, T. Novák and A. Ster, in preparation.

LEVY POLYNOMIALS EXPLICITELY

$$
\begin{aligned}
L_0(t\,|\,\alpha) &= 1, \\
L_1(t\,|\,\alpha) &= \det\begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} \\ 1 & t \end{pmatrix}, \\
L_2(t\,|\,\alpha) &= \det\begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} \\ 1 & t & t^2 \end{pmatrix}
\end{aligned}
$$

$$
\mu_{n,\alpha} = \int_0^\infty dt\, t^n \exp(-t^\alpha) = \frac{1}{\alpha}\,\Gamma\left(\frac{n+1}{\alpha}\right)
$$

$$
\Gamma(z) = \int_0^\infty dt\, t^{z-1} e^{-t}
$$

$$
\begin{aligned}
L_0(t\,|\,\alpha) &= 1, \\
L_1(t\,|\,\alpha) &= \frac{1}{\alpha}\left\{\Gamma\left(\frac{1}{\alpha}\right) t - \Gamma\left(\frac{2}{\alpha}\right)\right\}, \\
L_2(t\,|\,\alpha) &= \frac{1}{\alpha^2}\left\{\left[\Gamma\left(\frac{1}{\alpha}\right)\Gamma\left(\frac{3}{\alpha}\right) - \Gamma^2\left(\frac{2}{\alpha}\right)\right] t^2 - \right. \\
&\quad - \left[\Gamma\left(\frac{1}{\alpha}\right)\Gamma\left(\frac{4}{\alpha}\right) - \Gamma\left(\frac{3}{\alpha}\right)\Gamma\left(\frac{2}{\alpha}\right)\right] t + \\
&\quad \left. + \left[\Gamma\left(\frac{2}{\alpha}\right)\Gamma\left(\frac{4}{\alpha}\right) - \Gamma^2\left(\frac{3}{\alpha}\right)\right]\right\}.
\end{aligned}
$$

arXiv:1604.05513 [physics.data-an]