Inclusive dihadron production at the LHC in the NLA BFKL

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Outline

Outline

Motivation

So far, search for BFKL effects had these general drawbacks:

- ◇ too low $\sqrt{s}$ or rapidity intervals among tagged particles in the final state
- ◇ too inclusive observables, other approaches can fit them

Advent of LHC:

- → higher energies ↔ larger rapidity gaps
- → unique opportunity to **test pQCD in the high-energy limit**
- → disentangle applicability region of energy-log resummation (**BFKL approach**)

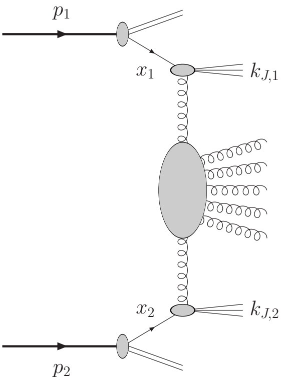
[V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977)]
[Y.Y. Balitskii, L.N. Lipatov (1978)]

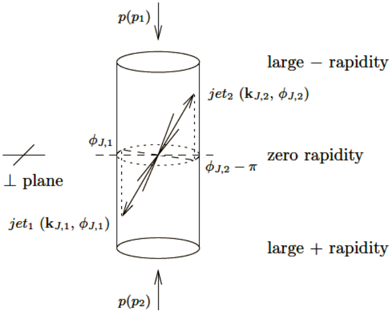
Last years:
hadroproduction of two jets featuring high transverse momenta and well separed in rapidity, so called **Mueller–Navelet jets**...

- ◇ ...possibility to define *infrared-safe* observables...
- ◇ ...and constrain the PDFs...
- ◇ ...theory vs experiment

[B. Ducloué, L. Szymanowski, S. Wallon (2014)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

Mueller–Navelet jets





Pictures from
[D. Colferai, F. Schwennsen, L. Szymanowski, S. Wallon (2010)]

...large jet transverse momenta: $\vec{k}_{J,1}^{\,2} \sim \vec{k}_{J,2}^{\,2} \gg \Lambda_{\rm QCD}^2$

...large rapidity gap between jets (high energies) $\Rightarrow$ $\Delta y = \ln \frac{x_{J,1} x_{J,2} s}{|\vec{k}_{J,1}||\vec{k}_{J,2}|}$

How could we further and deeply probe BFKL?

1. **Study a less inclusive two-body final state...**

Dihadron production

- inclusive production of a pair of charged light hadrons well separed in rapidity
- hadrons can be detected at the LHC at much smaller values of the transverse momentum than jets!
- possibility to constrain not only the PDFs, but also the FFs!

2. **Study three- and four-body final state processes...**

Multi-jet production

[F. Caporale, G. Chachamis, B. Murdaca, A. Sabio Vera (2015)]
[F. Caporale, F.G. C., G. Chachamis, A. Sabio Vera (2016)]
[F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016, 2017, 2017, in progress)]

- demand the tagging of one or/and two further jets in more central regions of the detectors with a relative separation in rapidity from each other
- definition of new, **suitable BFKL observables**...
- ...in order to further investigate the azimutal distribution of the final state

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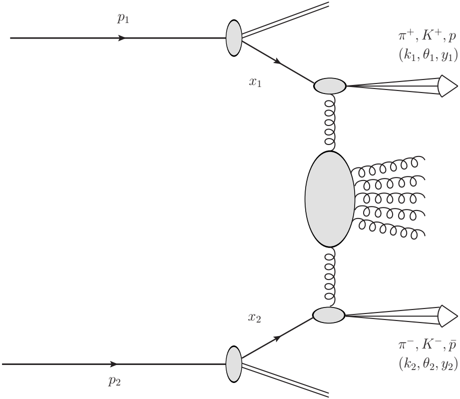
Dihadron production

Process: $\mathrm{proton}(p_1) + \mathrm{proton}(p_2) \rightarrow h_1(k_1) + h_2(k_2) + X$ *...LHC physics!*



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$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2} = \sum_{i,j=q,g} \int_0^1 \int_0^1 dx_1 dx_2 f_i(x_1,\mu) f_j(x_2,\mu) \frac{d\hat{\sigma}(x_1 x_2 s,\mu)}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2}$$

- ◇ large hadron transverse momenta: $\vec{k}_1^{\,2} \sim \vec{k}_2^{\,2} \gg \Lambda^2_{\rm QCD}$ $\Rightarrow$ *pQCD allowed*
- ◇ QCD collinear factorization
- ◇ large rapidity gap between hadrons (high energies) $\Rightarrow$ $\Delta y = \ln \frac{x_1 x_2 s}{|\vec{k}_1||\vec{k}_2|}$
  $\Rightarrow$ BFKL resummation: $\sum_n \left(a_n^{(0)} \ \alpha_s^n \ln^n s + a_n^{(1)} \ \alpha_s^n \ln^{n-1} s \right)$
- ◇ Collinear fragmentation of the parton i into a hadron h
  $\Rightarrow$ convolution of D_i^h with a coefficient function C_i^h

$$d\sigma_i = C_i^h(z) dz \to d\sigma^h = d\alpha_h \int_{\alpha_h}^1 \frac{dz}{z} D_i^h\left(\frac{\alpha_h}{z},\mu\right) C_i^h(z,\mu)$$

where α_h is the momentum fraction carried by the hadron

Outline

The BFKL resummation

pQCD, **semi-hard processes**: $s \gg Q^2 \gg \Lambda^2_{\rm QCD}$

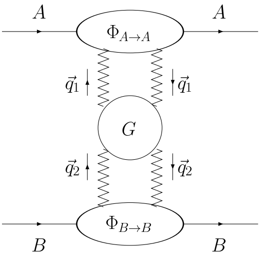
total cross section for $A + B \to X$: $\sigma_{AB}(s) = \frac{\mathcal{I}m_s\left(\mathcal{A}_{AB}^{AB}\right)}{s}$ $\Leftarrow$ **optical theorem**

- Pomeron channel: $t = 0$ + singlet colour representation in the t-channel
- Regge limit: $s \simeq -u \to \infty$, t not growing with s

- **BFKL resummation**:

  leading logarithmic approximation (LLA): $\alpha_s^n (\ln s)^n$

  next-to-leading logarithmic approximation (NLA): $\alpha_s^{n+1} (\ln s)^n$



- $\mathrm{Im}_s\left(\mathcal{A}_{AB}^{AB}\right)$ factorization:

  convolution of the **Green's function** of two interacting Reggeized gluons with the **impact factors** of the colliding particles

$$\mathrm{Im}_s\left(\mathcal{A}\right) = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}} \Phi_A(\vec{q}_1, s_0) \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}} \Phi_B(-\vec{q}_2, s_0) \int\limits_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

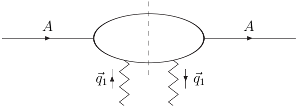
- **Green's function** is **process-independent**

  $\longrightarrow$ determined through the **BFKL equation**

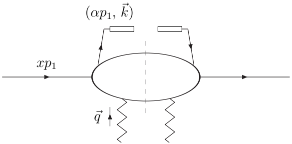
  [Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

- **Impact factors** are **process-dependent**

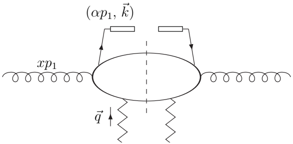
  $\longrightarrow$ known in the NLA just for few processes



* forward identified hadron production



quark vertex

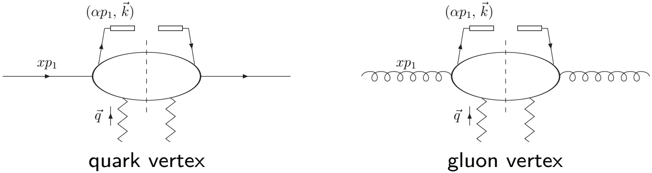


gluon vertex

[D.Yu. Ivanov, A. Papa (2012)]

Forward hadron impact factor

- "open" one of the integrations over the phase space of the intermediate state to allow one parton to fragment into a given hadron



- use QCD collinear factorization

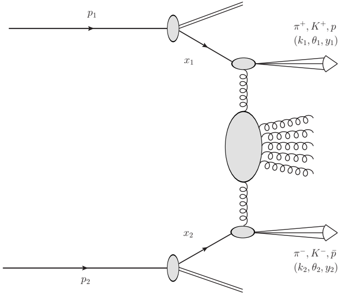
$$\sum_{s=q,\bar{q}} f_s \otimes [\text{quark vertex}] \otimes D_s^h \quad + \quad f_g \otimes [\text{gluon vertex}] \otimes D_g^h$$

- project onto the eigenfunctions of the LO BFKL kernel, i.e. transfer to the (ν, n)-representation

Outline

BFKL cross section...

$$\frac{d\sigma}{dx_1 dx_2 d^2\vec{k}_1 d^2\vec{k}_2} = \sum_{i,j=q,\bar{q},g} \int_0^1 dx_1 \int_0^1 dx_2 \; f_i(x_1,\mu) f_j(x_2,\mu) \frac{d\hat{\sigma}_{i,j}(x_1 x_2 s,\mu)}{dx_1 dx_2 d^2\vec{k}_1 d^2\vec{k}_2}$$



- slight change of variable in the final state
- project onto the eigenfunctions of the LO BFKL kernel, i.e. transfer from the reggeized gluon momenta to the (n,ν)-representation
- suitable definition of the **azimuthal coefficients**

$$\frac{d\sigma}{dx_1 dx_2 \, d|\vec{k}_1| \, d|\vec{k}_2| d\phi_1 d\phi_2} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\,\mathcal{C}_n\right]$$

with $\phi = \phi_1 - \phi_2 - \pi$

...useful definitions: $Y = \ln \frac{x_1 x_2 s}{|\vec{k}_1||\vec{k}_2|}$, $Y_0 = \ln \frac{s_0}{|\vec{k}_1||\vec{k}_2|}$

...and azimuthal coefficients

$$
\begin{aligned}
\mathcal{C}_n \equiv \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \, \cos[n(\phi_1 - \phi_2 - \pi)] \, \frac{d\sigma}{dy_1 dy_2 \, d|\vec{k}_1| \, d|\vec{k}_2| d\phi_1 d\phi_2} \\
= \frac{e^Y}{s} \int_{-\infty}^{+\infty} d\nu \left(\frac{\alpha_1 \alpha_2 s}{s_0} \right)^{\bar{\alpha}_s(\mu_R)\left[\chi(n,\nu) + \bar{\alpha}_s(\mu_R) K^{(1)}(n,\nu)\right]} \\
\times \alpha_s^2(\mu_R) c_1(n,\nu,|\vec{k}_1|,\alpha_1) c_2(n,\nu,|\vec{k}_2|,\alpha_2) \\
\times \left[1 + \alpha_s(\mu_R) \left(\frac{c_1^{(1)}(n,\nu,|\vec{k}_1|,\alpha_1)}{c_1(n,\nu,|\vec{k}_1|,\alpha_1)} + \frac{c_2^{(1)}(n,\nu,|\vec{k}_2|,\alpha_2)}{c_2(n,\nu,|\vec{k}_2|,\alpha_2)} \right) \right] .
\end{aligned}
$$

where

$$
\chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2} + \frac{1}{2} + i\nu\right) - \psi\left(\frac{n}{2} + \frac{1}{2} - i\nu\right)
$$

$$
K^{(1)}(n,\nu) = \bar{\chi}(n,\nu) + \frac{\beta_0}{8N_c}\chi(n,\nu)\left(-\chi(n,\nu) + \frac{10}{3} + i\frac{d}{d\nu}\ln\left(\frac{c_1(n,\nu)}{c_2(n,\nu)}\right) + 2\ln\left(\mu_R^2\right) \right)
$$

...several NLA-equivalent expressions can be adopted for $\mathcal{C}_n$!

$\longrightarrow$ ...we are using the *exponentiated* one

[F. Caporale, D.Yu Ivanov, B. Murdaca, A. Papa (2014)]

Outline

BLM method

NLO BFKL corrections to C_0 with opposite sign with respect to the leading order (LO) result and large in absolute value...

- ...call for some optimization procedure...
- ...choose scales to mimic the most relevant subleading terms

- **BLM** optimization procedure

[S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]

  - ✓ preserve the conformal invariance of an observable...
  - ✓ ...by making vanish its β_0-dependent part

* "Exact" BLM:

suppress NLO IFs $+$ NLO Kernel β_0-dependent factors

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]

Outline

Observables and kinematics

- **Observables:**

  ϕ-averaged cross section $\mathcal{C}_0$, $\langle \cos(n\phi) \rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0} \equiv R_{n0}$, with $n = 1, 2, 3$

  $$\frac{\langle \cos(2\phi) \rangle}{\langle \cos(\phi) \rangle} \equiv \frac{\mathcal{C}_2}{\mathcal{C}_1} \equiv R_{21} \, , \quad \frac{\langle \cos(3\phi) \rangle}{\langle \cos(2\phi) \rangle} \equiv \frac{\mathcal{C}_3}{\mathcal{C}_2} \equiv R_{32} \, .$$

◇ *Integrated coefficients:*

$$C_n = \int_{y_{1,\text{min}}}^{y_{1,\text{max}}} dy_1 \int_{y_{2,\text{min}}}^{y_{2,\text{max}}} dy_2 \int_{k_{1,\text{min}}}^{k_{1,\text{max}}} dk_1 \int_{k_{2,\text{min}}}^{k_{2,\text{max}}} dk_2 \delta(y_1 - y_2 - Y) \, \mathcal{C}_n(y_1, y_2, k_1, k_2)$$

- **Kinematic settings:**
  - ◇ $\sqrt{s} = 7, \; 13$ TeV
  - ◇ $|y_i| \leq 2.4, 4.7$, with $i = 1, 2$
  - ◇ $k_{1,2} \geq$ **5 GeV** ...vs $k_{J_{1,2}}^{\text{MN}-\text{jets}} \geq 35$ GeV! $\rightarrow$ more secondary gluon emissions!
- **Phenomenological analysis:**
  - ◇ full **NLA** BFKL
  - ◇ $\mu_R = \mu_R^{\text{BLM}}$
  - ◇ $(\mu_F)_{1,2} = \mu_R^{\text{BLM}}, \; |\vec{k}_{1,2}|, \; \text{r}\sqrt{|\vec{k}_1||\vec{k}_2|}$

Numerical specifics

- **Numerical tools:**

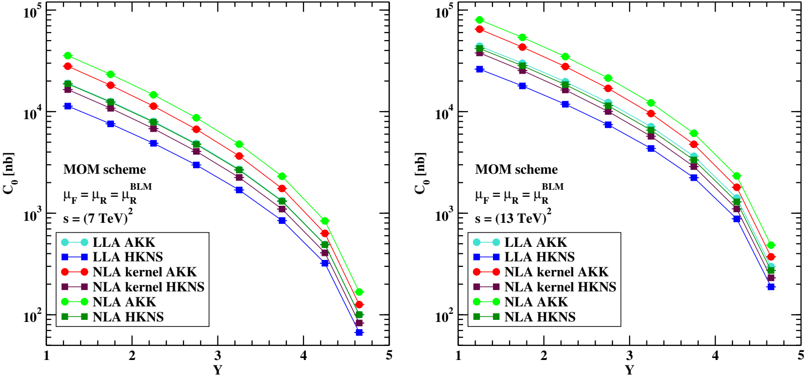
  FORTRAN → weak time dependence on multidim. integration ranges

  + NLO **MSTW08** PDFs (comparison with **MMHT14** and **CTEQ14**)
    [A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt, (2009)]

  + **three** different FF parameterizations!
    - **AKK**
      [S. Albino, B.A. Kniehl, G. Kramer, (2008)]
    - **DSS**
      [D. de Florian, R. Sassot, M. Stratmann, (2007)]
    - **HKNS**
      [M. Hirai, S. Kumano, T.-H. Nagai, K. Sudoh, (2007)]

  + CERNLIB
    http://cernlib.web.cern.ch/cernlib

C_0 at $\sqrt{s} = 7, 13$ TeV, $Y \leq 4.8$, $\mu_F = \mu_R^{\rm BLM}$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

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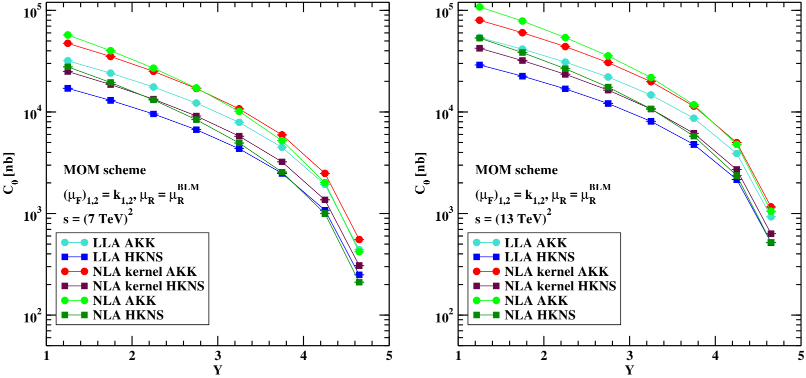

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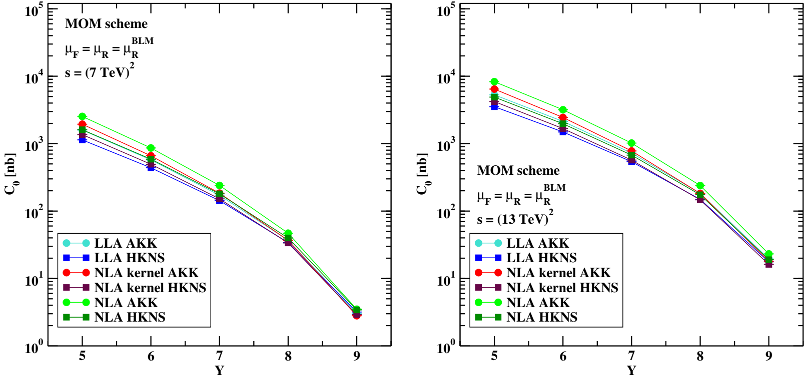
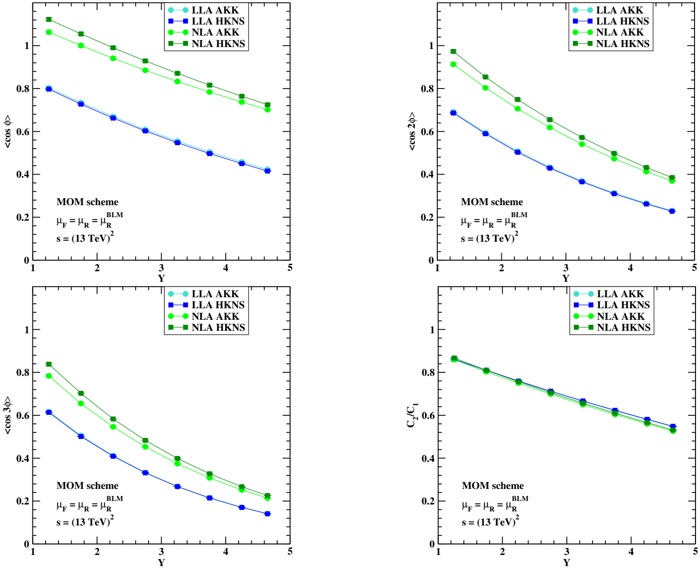
C_0 at $\sqrt{s} = 7, 13$ TeV, $Y \leq 9.4$, $\mu_F = \mu_R^{\rm BLM}$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

R_{nm} at $\sqrt{s} = 13$ TeV, $Y \leq 4.8$, $\mu_F = \mu_R^{\rm BLM}$



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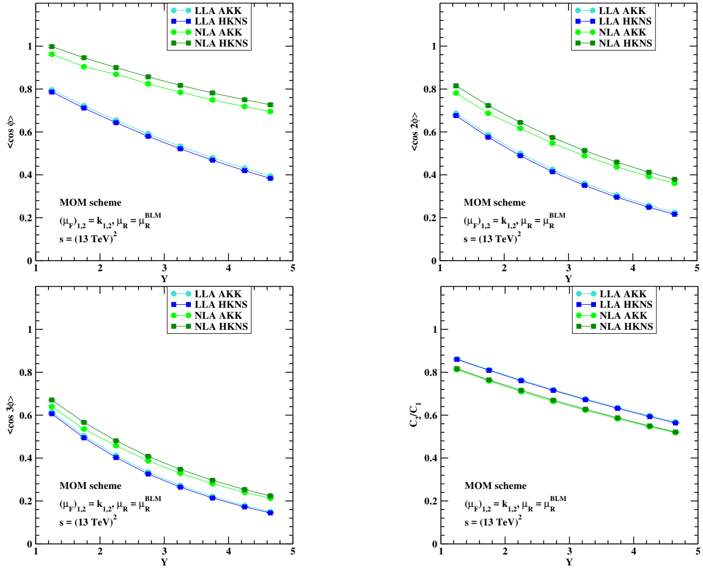

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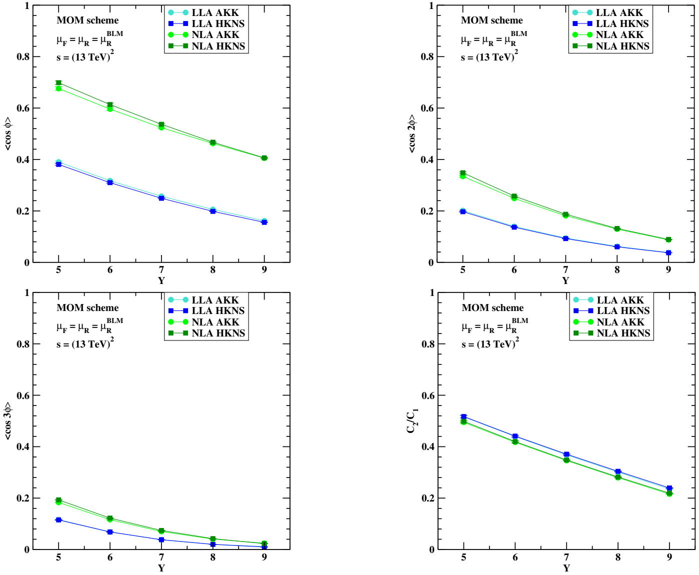

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Conclusions...

Comparison of predictions for C_0 and several R_{nm} in the full NLA BFKL approach

- ◇ lower transwerse momenta than jets, PDFs + FFs at work...
- ◇ ...new and complementary study of strong interactions at high energies

- implementation of <u>*exact*</u> **BLM method** for μ_R
- two different ways to optimize μ_F value
- four different kinematical ranges: $\sqrt{s} = 7, 13$ TeV; $Y \leq 4.8, 9.4$

- ▶ NLA corrections to hadron vertices make C_0 bigger: they are positive (**!**) and partially compensate the negative effect of NLA BFKL kernel corrections
- ▶ comparison with NLO DGLAP calculations needed

$\Rightarrow$ New suitable channel to improve our knowledge about the dynamics of strong interactions in the Regge limit

...Outlook

- ◇ enrich the final-state exclusiveness: **hadron-jet** correlations
  (FF dependence + asymmetric rapidity and transverse momenta ranges)

  [F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (in progress)]

- ◇ probe BFKL through other processes: **heavy-quark pair** production

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Thanks for your attention!!

BACKUP slides

Forward hadron impact factor: (ν, n)-projection

$$c_1(n,\nu,|\vec{k}_1|,\alpha_1) = 2\sqrt{\frac{C_F}{C_A}}(\vec{k}_1^2)^{i\nu-1/2}\int_{\alpha_1}^1 \frac{dx}{x}\left(\frac{x}{\alpha_1}\right)^{2i\nu-1} \times \left[\frac{C_A}{C_F}f_g(x)D_g^h\left(\frac{\alpha_1}{x}\right)+\sum_{a=q,\bar{q}} f_a(x)D_a^h\left(\frac{\alpha_1}{x}\right)\right]$$

$$c_1^{(1)}(n,\nu,|\vec{k}_1|,\alpha_1) = 2\sqrt{\frac{C_F}{C_A}}\left(\vec{k}_1^2\right)^{i\nu-\frac{1}{2}}\frac{1}{2\pi}\int_{\alpha_1}^1 \frac{dx}{x}\int_{\frac{\alpha_1}{x}}^1 \frac{d\zeta}{\zeta}\left(\frac{x\zeta}{\alpha_1}\right)^{2i\nu-1} \times \left[\frac{C_A}{C_F}f_g(x)D_g^h\left(\frac{\alpha_1}{x\zeta}\right)C_{gg}(x,\zeta)+\sum_{a=q\bar{q}} f_a(x)D_a^h\left(\frac{\alpha_1}{x\zeta}\right)C_{qq}(x,\zeta) \times D_g^h\left(\frac{\alpha_1}{x\zeta}\right)\sum_{a=q\bar{q}} f_a(x)C_{qg}(x,\zeta)+\frac{C_A}{C_F}f_g(x)\sum_{a=q\bar{q}} D_a^h\left(\frac{\alpha_1}{x\zeta}\right)C_{gq}(x,\zeta)\right]$$

The BFKL BLM cross section

$$C_n^{\rm BLM} = \frac{e^Y}{s} \int_{y_{\rm min}}^{y_{\rm max}} dy_1 \int_{k_{1,\rm min}}^{\infty} dk_1 \int_{k_{2,\rm min}}^{\infty} dk_2 \int_{-\infty}^{+\infty} d\nu \, \exp\left[(Y - Y_0)\, \bar{\alpha}_s^{\rm MOM}(\mu_R^*) \left\{ \chi(n,\nu) \right.\right.$$

$$\left.\left. + \, \bar{\alpha}_s^{\rm MOM}(\mu_R^*) \left(\bar{\chi}(n,\nu) + \frac{T^{\rm conf}}{C_A} \chi(n,\nu) \right) \right\} \right] 4 (\alpha_s^{\rm MOM}(\mu_R^*))^2 \frac{C_F}{C_A} \frac{1}{|\vec{k}_1||\vec{k}_2|} \left(\frac{\vec{k}_1^2}{\vec{k}_2^2} \right)^{i\nu}$$

$$\times \int_{\alpha_1}^{1} \frac{dx}{x} \left(\frac{x}{\alpha_1} \right)^{2i\nu - 1} \left[\frac{C_A}{C_F} f_g(x) D_g^h\left(\frac{\alpha_1}{x}\right) + \sum_{a=q,\bar{q}} f_a(x) D_a^h\left(\frac{\alpha_1}{x}\right) \right]$$

$$\times \int_{\alpha_2}^{1} \frac{dz}{z} \left(\frac{z}{\alpha_2} \right)^{-2i\nu - 1} \left[\frac{C_A}{C_F} f_g(z) D_g^h\left(\frac{\alpha_2}{z}\right) + \sum_{a=q,\bar{q}} f_a(z) D_a^h\left(\frac{\alpha_2}{z}\right) \right]$$

$$\times \left[1 + \bar{\alpha}_s^{\rm MOM}(\mu_R^*) \left(\frac{\bar{c}_1^{(1)}(n,\nu)}{c_1(n,\nu)} + \frac{\bar{c}_2^{(1)}(n,\nu)}{c_2(n,\nu)} + 2 \frac{T^{\rm conf}}{C_A} \right) \right] ,$$

with the μ_R^* scale chosen as the solution of the following integral equation...

...choosing the $\mu_R^{\rm BLM}$ scale

$$
\begin{aligned}
C_n^\beta = \frac{e^Y}{s} \int_{y_{\rm min}}^{y_{\rm max}} dy_1 \int_{k_{1,\rm min}}^{\infty} dk_1 \int_{k_{2,\rm min}}^{\infty} dk_2 \int_{-\infty}^{+\infty} d\nu \, \exp\left[(Y - Y_0)\, \bar\alpha_s^{\rm MOM}(\mu_R) \chi(n,\nu) \right] \\
\times 4 \left(\alpha_s^{\rm MOM}(\mu_R) \right)^3 \frac{C_F}{C_A} \frac{1}{|\vec k_1||\vec k_2|} \left(\frac{\vec k_1^2}{\vec k_2^2} \right)^{i\nu} \\
\times \int_{\alpha_1}^{1} \frac{dx}{x} \left(\frac{x}{\alpha_1} \right)^{2i\nu - 1} \left[\frac{C_A}{C_F} f_g(x) D_g^h\left(\frac{\alpha_1}{x}\right) + \sum_{a=q,\bar q} f_a(x) D_a^h\left(\frac{\alpha_1}{x}\right) \right] \\
\times \int_{\alpha_2}^{1} \frac{dz}{z} \left(\frac{z}{\alpha_2} \right)^{-2i\nu - 1} \left[\frac{C_A}{C_F} f_g(z) D_g^h\left(\frac{\alpha_2}{z}\right) + \sum_{a=q,\bar q} f_a(z) D_a^h\left(\frac{\alpha_2}{z}\right) \right] \\
\times \frac{\beta_0}{2C_A} \left[\frac{5}{3} + \ln \frac{\mu_R^2}{k_1 k_2} + f(\nu) - 2\left(1 + \frac{2}{3} I\right) \right. \\
\left. + \bar\alpha_s^{\rm MOM}(\mu_R)\,(Y - Y_0)\, \frac{\chi(n,\nu)}{2} \left(-\frac{\chi(n,\nu)}{2} + \frac{5}{3} + \ln \frac{\mu_R^2}{k_1 k_2} + f(\nu) - 2\left(1 + \frac{2}{3} I\right) \right) \right] \stackrel{!}{=} 0
\end{aligned}
$$

...choosing the μ_R^{BLM} scale

...which represents the condition that terms proportional to β_0 in C_n disappear

$$\alpha^{\text{MOM}} = -\frac{\pi}{2T}\left[1 - \sqrt{1 + 4\alpha_s\left(\mu_R\right)\frac{T}{\pi}}\right] ,$$

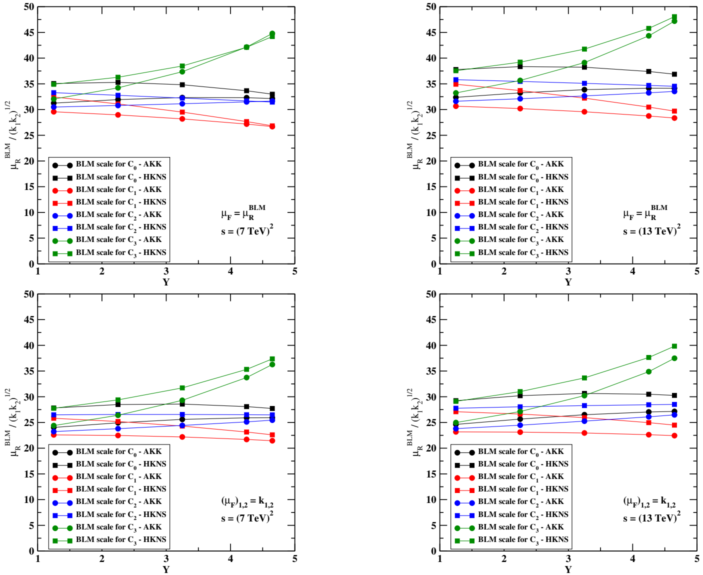
with $T = T^{\beta} + T^{\text{conf}}$,

$$T^{\beta} = -\frac{\beta_0}{2}\left(1 + \frac{2}{3}I\right) ,$$

$$T^{\text{conf}} = \frac{C_A}{8}\left[\frac{17}{2}I + \frac{3}{2}\left(I - 1\right)\xi + \left(1 - \frac{1}{3}I\right)\xi^2 - \frac{1}{6}\xi^3\right] ,$$

where $I = -2\int_0^1 dx \frac{\ln(x)}{x^2 - x + 1} \simeq 2.3439$ and ξ is a gauge parameter.

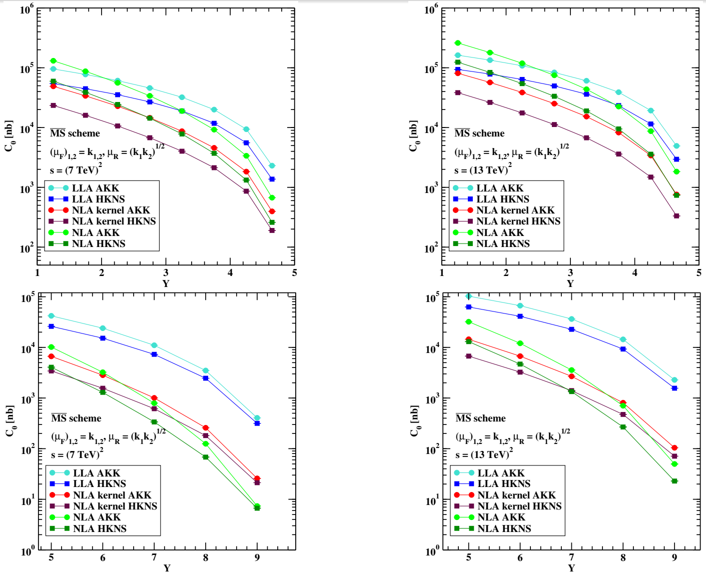
BLM vaules for μ_R



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

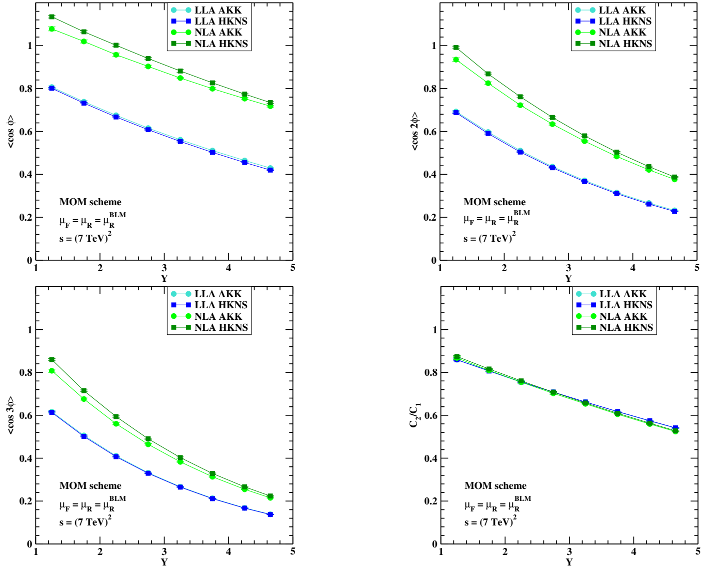
BACKUP slides

C_0 at $\sqrt{s} = 7, 13$ TeV, $\mu_R = \sqrt{|\vec{k}_1||\vec{k}_2|}$, $(\mu_F)_{1,2} = |\vec{k}_{1,2}|$



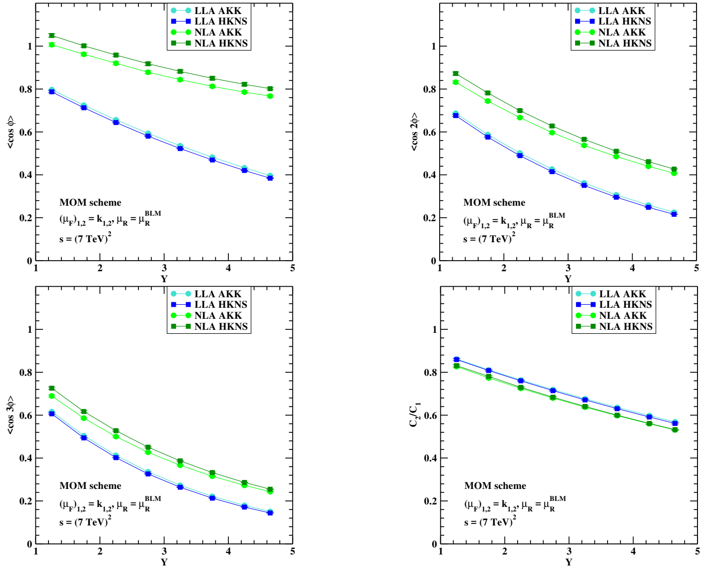
[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

R_{nm} at $\sqrt{s} = 7$ TeV, $Y \leq 4.8$, $\mu_F = \mu_R^{\rm BLM}$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

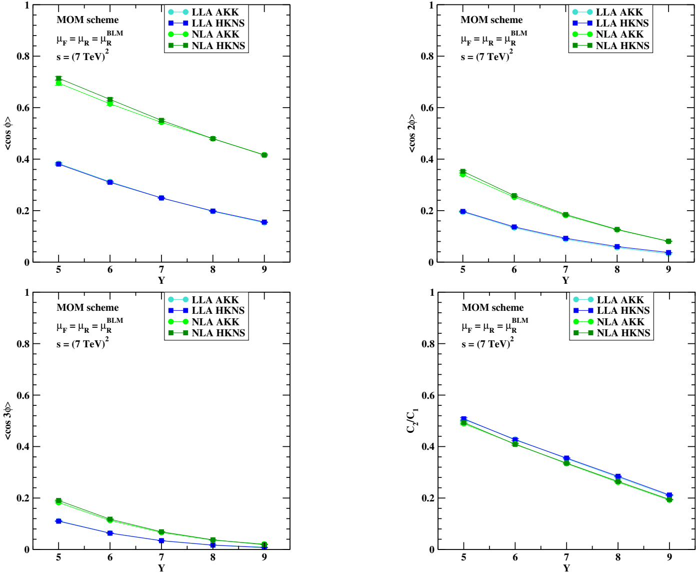
R_{nm} at $\sqrt{s} = 7$ TeV, $Y \leq 4.8$, $(\mu_F)_{1,2} = |\vec{k}_{1,2}|$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

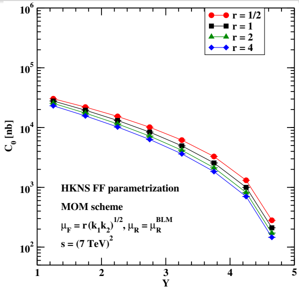
R_{nm} at $\sqrt{s} = 7$ TeV, $Y \leq 9.4$, $\mu_F = \mu_R^{\rm BLM}$

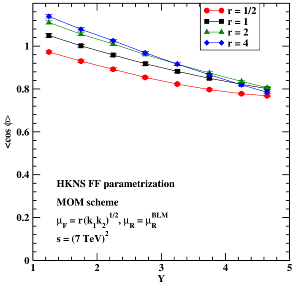


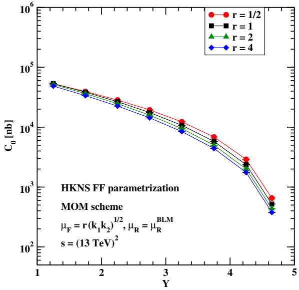
[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

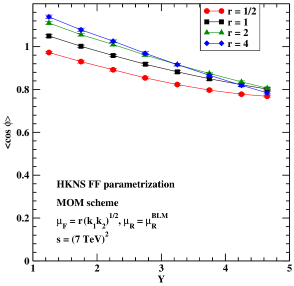
BACKUP slides

C_0, R_{10} at $\sqrt{s} = 7, 13$ TeV, $Y \leq 4.8$, $\mu_F = r\sqrt{|\vec{k}_1||\vec{k}_2|}$









[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]