Some aspects of Theoretical Cosmology
How Inflation begins and a connection to Differential Geometry
The Problem

- If we have the inflaton on top of his potential
- and the space is homogeneous on a patch
- then inflation starts $a(t) \sim e^{Ht}$

- Question: how likely is to have an homogenous patch of this size?

- If we imagine random initial condition, since overdensities lead to collapse, every point will tend to collapse into a black hole by the time start inflation, compelling argument (at least to me) implies that if

$$H_I \ll M_{Pl} \quad \Rightarrow \quad \text{Prob} \sim (H_I/M_{Pl})^\#$$

- this is the so-called `initial patch problem'

- Particularly relevant for low energy models
- and we will show that this argument is not really correct
How did inflation start?

• Hard to make progress: evolution in completely inhomogenous universe leads to singularities, difficult to describe

• However, recently progress was made in two unrelated aspects of science
  • numerical codes that can handle singularities Pretorius 2005
  • the Thorston Geometrization Classification Hypothesis, has been proven Thorston, Hamilton, Perelman Fields Medal 2006

(i.e. the Poincarè)
• We start with an highly inhomogenous configuration:

\[ \phi(t = 0, x) = \phi_0 + \delta \phi \left[ \sum_{1 \leq |kL/2\pi| \leq N} \cos(k \cdot x + \theta_k) \right], \]

• And find that inflation starts
– Inflation starts

– BH emit GW
Analytically: A no Big-Crunch theorem

• Consider a cosmological spacetime with an initial surface expanding everywhere.

• Can the universe globally collapse?

• Since \( \mathcal{L}_n \log \sqrt{h} = K \), \( K_{\mu\nu} = \nabla_\mu n_\nu \)

• \( \Rightarrow \) there must exist a maximal surface, where

\[ K = 0 \quad \text{everywhere} \]
- Take a surface, and deform it forward or backward according to sign of $K$

- The change of volume:
  \[
  \frac{\partial V}{\partial \lambda} = \int d^3 x K^2 \sqrt{h} \equiv \langle K^2 \rangle \geq 0
  \]

- So this procedure either converges to an extremal surface, if it can exist, with $K = 0$ everywhere

- Or it gives a surface of larger volume indefinitely

- The surface will never its a spacetime singularity nor become singular.

- This is a field of Mathematics, and now it has a new connection to Physics
• To have a global recollapse,

• there must exist a maximal surface, where

\[ K = 0 \quad \text{everywhere} \]

• Consider the following linear combination of Einstein Equations

\[ n^\mu n^\nu \left( 8\pi G_N T_{\mu\nu} = G_{\mu\nu} \right) n^\mu n^\nu \]

• Using Gauss-Codazzi

\[ 16\pi G_N T_{\mu\nu} n^\mu n^\nu = R^{(3)} + \frac{2}{3} K^2 - \sigma_{\mu\nu} \sigma^{\mu\nu} \]

- where \( K = \nabla_\mu n^\mu \), \( \sigma_{\mu\nu} = K_{\mu\nu} - \frac{1}{3} K h_{\mu\nu} \)

• On Extremal surface

\[ 16\pi G_N T_{\mu\nu} n^\mu n^\nu = R^{(3)} - \sigma^{\mu\nu} \sigma_{\mu\nu} \]
A no Big-Crunch theorem

• If we impose Weak Energy Condition

$$16\pi G_N T_{\mu\nu} n^\mu n^\nu = R^{(3)} - \sigma^{\mu\nu} \sigma_{\mu\nu} \geq 0 \text{ by WEC} \leq 0$$

$$T_{\mu\nu} t^\mu t^\nu \geq 0 \text{ (i.e. } \rho \geq 0, \rho + p > 0\text{)} \text{, for any } t^\mu \text{ timelike}$$

• If there is a topological condition such that $$R^{(3)} \leq 0$$ at least at one point

• ⇒ The equation cannot be satisfied, ⇒ extremal surface does not exists

• The Thorston Geometrization Classification, indeed shows that “most” of 3-manifold must have $$R^{(3)} \leq 0$$ at least at one point

– for these topologies, some regions of the universe keep expanding, notwithstanding the development of singularities

with Kleban, JCAP 2016
see also Barrow and Tippler 1985
with Creminelli and Vasy 2019
To determine which manifolds must have $R^{(3)} \leq 0$ at least at one point, consider that all compact oriented 3-manifolds fall into one of these three classes

(i) "Closed": any function on $M_t$ can be the $R^{(3)}$ of a smooth metric on $M_t$

  - ex: $S^3$, $S^2 \times S^1$, $S^3/\Gamma$ (with $\Gamma \in SO(4)$), $RP^3$
  - and connected sums

(ii) "Flat": a function on $M_t$ can be the $R^{(3)}$ of a smooth metric on $M_t$ if it is negative somewhere or zero everywhere

  - ex: $R^3/\Gamma$ (with $\Gamma$ an isometry of $R^3$)
  - and connected sums

(iii) "Open": a function on $M_t$ can be the $R^{(3)}$ of a smooth metric on $M_t$ if it is negative somewhere

  - ex: $H^3/\Gamma$, $H^2 \times R$, nil, sol, $\widetilde{SL}(2, R)$
  - Any connected sum of (i) and (ii) with a factor of (iii) is of kind (iii)
Therefore, in most manifolds (fast) expansion has to continue.

- this, and some additional results using mean curvature flow, suggests that it is extremely likely that expansion continues, volume reaches infinity, inhomogenous energy dilutes, potential energy (which does not dilutes) dominates, inflation starts notwithstanding the formation of singularities.

- partial, stronger, results in 3+1d with Creminelli, Kleban, Vasy in progress

In 2+1d, this statement as recently been rigorously proved by

- using the expertise of the Math Department Professor with Creminelli, Vasy 2019
The Cow

– Pictorial representation of the late time manifold

Regions of no convergence with finite physical volume.

Regions of no convergence with physical volume going to zero.
Summary on starting Inflation

- For ~ 30 years, widely believed: to start inflation, need an homogenous $H_I$ patch.
- This seemed to require fine tuning: `initial patch problem`
- By using numerical GR simulations that are able to handle singularities and horizons
- & less-usual mathematical techniques such as mean curvature flow and topology
- we have shown that inflation starts very often, and actually seems always to start in some topologies:
  - 2+1d proved
  - 3+1d partial-statements proved

- these results are apparently appealing also for mathematicians

- So, we can take that inflation starts….
… but we mentioned Black Holes…
… we cannot think to LIGO/VIRGO…
Lessons for Fundamental Physics from LIGO/VIRGO
An EFT for testing extensions of GR with Gravitational Waves

with Endlich, Huang and Gorbenko

2017

with Sennett, Brito, Buonanno and Gorbenko

in completion
An EFT for probing extensions of GR at LIGO

• Gravity waves have just been discovered.
  – An amazing lesson of perseverance for whole mankind

• They will teach us a lot about astrophysics compact objects

• Can these observations teach us something about the fundamental nature of gravity?

• The most general parametrization (i.e. EFT) for doing that with the following requirements:
  – testable at LIGO/VIRGO
  – no additional degree of freedom
  – no violation of locality (superluminality, etc.)
  – in no conflict to other GR experiments
An EFT for probing extensions of GR at LIGO/VIRGO

- The most general such a Lagrangian is

\[ S_{\text{eff}} = 2M_{\text{pl}}^2 \int d^4x \sqrt{-g} \left( -R + \frac{C^2}{\Lambda^6} + \frac{\tilde{C}^2}{\tilde{\Lambda}^6} + \frac{\tilde{C}C}{\Lambda^6} + \ldots \right) \]

\[ \mathcal{C} \equiv R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}, \quad \tilde{\mathcal{C}} \equiv R_{\alpha\beta\gamma\delta} \epsilon^{\alpha\beta}_{\mu\nu} R^{\mu\nu\gamma\delta}; \]

- No superluminality \( \sim \Rightarrow \) No \( R^{3}_{\mu\nu\rho\sigma} \)

- Testable at LIGO \( \Rightarrow \) \( \Lambda \sim 10^{-1} \text{ Km}^{-1} \)

- Not ruled out by GR tests

\[ \Rightarrow \delta g^{\mu\nu} T_{\mu\nu} = \text{as in GR} \quad \& \quad \text{amplitudes saturate when UV enters} \]

- this `soft' UV completion is the same that happens in \( WW \) scattering when the Higgs enters: the amplitude soft growing

- in all test prior LIGO/VIRGO, at UV scale \( R_{\mu\nu\rho\sigma} \Delta x^2 \ll 1 \)
An EFT for probing extensions of GR at LIGO/VIRGO

• The most general such a Lagrangian is

\[ S_{\text{eff}} = 2M_{\text{pl}}^2 \int d^4x \sqrt{-g} \left( -R + \frac{C^2}{\Lambda^6} + \frac{\tilde{C}^2}{\tilde{\Lambda}^6} + \frac{\tilde{C}C}{\Lambda_0^6} + \ldots \right) \]

\[ C \equiv R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}, \quad \tilde{C} \equiv R_{\alpha\beta\gamma\delta} \epsilon^{\alpha\beta}_{\mu\nu} R^{\mu\nu\gamma\delta}, \]

• In this way, information from LIGO/VIRGO can be mapped into parameters of a fundamental physics Lagrangian $\Lambda, \Lambda_-, \tilde{\Lambda}$ – similar to what we do in particle physics at colliders (Precision EW tests, or the LHC Brazilian-flag plots)

• instead of into some arbitrary and potentially-unphysical rescaling of the post-Newtonian parameters of the templates $\omega(r) = \sum_n c_n v^n$

  • this was suboptimal

  • explored physically uninteresting regions such as superluminal ones.
An EFT for probing extensions of GR at LIGO/VIRGO

- Predictions

\[ S_{\text{eff}} = 2M_{\text{pl}}^2 \int d^4x \sqrt{-g} \left( -R + \frac{c^2}{\Lambda^6} + \frac{\tilde{c}^2}{\tilde{\Lambda}^6} + \frac{\tilde{c}c}{\Lambda^6} + \ldots \right) \]

- For \( r_s \lesssim 1/\Lambda \), leading signal is in the insparalling phase:

  - Using the EFT of Goldberger and Rothstein 2004, change the potential energy of the two bodies, and their single-body effective radiation multipoles

- For \( r_s \gtrsim 1/\Lambda \), leading signal is in the modification of Black Hole geometry and quasi normal modes spectrum with Cardoso, Kimura, Maselli 2018

- Similar considerations apply to the exchange of axions in neutron-neutron merger

Huang, Johnson, Samunski, Sakellariadu, Zhang 2018
First bounds on fundamental Lagrangian from LIGO data

- LIGO members, preliminary

\[ S_{\text{eff}} = 2M_{\text{pl}}^2 \int d^4x \sqrt{-g} \left( -R + \frac{C^2}{\Lambda^6} + \frac{\tilde{C}^2}{\tilde{\Lambda}^6} + \frac{\tilde{C}C}{\Lambda^6} + \ldots \right) \]
Lessons on Dark Energy from LIGO/VIRGO

Creminelli, Vernizzi, 2017
Sakstein, Jain, 2017
Ezquiaga, Zumalacarregui, 2017
Baker etal, 2017
Copeland etal 2018
Creminelli, Lewandowski, Tambalo, Vernizzi 2018
The universe is Accelerating

• Many people, especially outside of Europe, and including me, have a favorite explanation:
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- it agrees with data to ~4%
Weinberg probably got it right (again)

• S. Weinberg, prior the detection of the acceleration of the universe, predicted, based on Anthropic reasoning, the value of the observed cosmological constant. It worked.
  – In a universe dominated by cosmological constant, structures do not form any longer
  – \( \Rightarrow \) for experimentalists to be there, cosmological constant better dominate after structure form
  – If there is a landscape with lots of local values of \( \Lambda \), observes will be where structures are, and so where \( \Lambda \) is small enough.
  – But since it is hard to make \( \Lambda \) small, most likely \( \Lambda \) will be closed to threshold: it will dominate just after structure formed
• This is exactly where we found \( \Lambda \)
• This interpretation finds support in the presence of the Landscape, and in the Higgs tuning.
• There are issues of, to me, detail, and, also, this is truly revolutionary point of view.
Observational side

• Due to such a revolutionary change of point of view, and due to the fact that, observationally, we are probing with unprecedented precision the low-redshift universe, many people have started focusing on theories that tend to modify gravity.
  – it is unclear if any of them actually addresses the Cosmological Constant problem

• These are just theories where one adds a degree of freedom that, due to FRW spacetime, spontaneously breaks time translation.

• There is an extremely large field, especially in Europe.

• And tremendous progress in this field has come from the following experimental fact
Black-Hole Neutron-Star merger

GW170817 = GRB170817A

Lightcurve from Fermi/GBM (50 – 300 keV)

Gravitational-wave time-frequency map
Black-Hole Neutron-Star merger

\[ GW170817 = \text{GRB170817A} \]

\[-3 \cdot 10^{-15} \leq \frac{c_g}{c} - 1 \leq 7 \cdot 10^{-16} \]
The EFT of Dark Energy

• It is possible to write down the most general theory for the fluctuations

$$\phi(t) \neq 0$$

$$ds^2 = -N^2 dt^2 + h_{ij} (N^i dt + dx^i)(N^j dt + dx^j)$$

• Assume that time-translation are spontaneously broken, there is preferred slicing, write the most general Lagrangian

$$S = \int d^4x \sqrt{-g} L[t; N, K^i_j, (^{(3)}R^i_j, \ldots]$$

with Creminelli, Luty and Nicolis, 2006
Vernizzi, Piazza, + many others

• Action contains all possible scalar under spatial diffs, oder by number of perturbations and derivatives
The EFT of Dark Energy

- At level of fluctuations

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - c g^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right. \\
\left. - \frac{m_3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathcal{H}_2 + \frac{\bar{m}_4}{2} \delta g^{00} R - \frac{m_5}{2} \delta g^{00} \delta \mathcal{H}_2 \\
\right. \\
\left. - \frac{m_6}{3} \delta \mathcal{H}_3 - \bar{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7}{3} \delta g^{00} \delta \mathcal{H}_3 \right] . \]

\[ \delta \mathcal{H}_2 \equiv \delta K^2 - \delta K^\nu_\mu \delta K^\mu_\nu , \quad \delta \mathcal{G}_2 \equiv \delta K^\nu_\mu R^\mu_\nu - \delta KR/2 , \]
\[ \delta \mathcal{H}_3 \equiv \delta K^3 - 3 \delta K \delta K^\nu_\mu \delta K^\mu_\nu + 2 \delta K^\nu_\mu \delta K^\mu_\rho \delta K^\rho_\nu . \]

- \[ m_2^4 = \alpha_K H^2 M_*^2 \] for LSS, we are interested in \[ \alpha \sim 0.1 \]
The EFT of Dark Energy

- At level of fluctuations
  \[ S = \int d^4 x \sqrt{-g} \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - c g^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right] \]

- Dark Energy speed of sound
  \[ \frac{m_5^2}{2} g^{00} \delta \mathcal{K}_2 \]

- Quintessence and Brans-Dicke
  \[ - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathcal{K}_2 + \frac{m_4^2}{2} \delta g^{00} R - \frac{m_4^2}{2} \delta g^{00} \delta \mathcal{K}_2 \]

- DGP and brading
  \[ - \frac{m_6^3}{3} \delta \mathcal{K}_3 - \tilde{m}_6^3 \delta g^{00} \delta \mathcal{K}_2 - \frac{m_7^3}{3} \delta g^{00} \delta \mathcal{K}_3 \]

- Non-linear terms, screening
  \[
  \delta \mathcal{K}_2 \equiv \delta K^2 - \delta K^\nu \delta K^\nu, \quad \delta \mathcal{G}_2 \equiv \delta K^\nu R^\mu_\nu - \delta K R/2, \quad \delta \mathcal{K}_3 \equiv \delta K^3 - 3 \delta K \delta K^\nu \delta K^\nu + 2 \delta K^\nu \delta K^\mu_\nu \delta K^\rho_\nu.
  \]

- \[ m_2^4 = \alpha_k H^2 M_*^2 \] for LSS, we are interested in \( \alpha \sim 0.1 \)
- All models unified in one Lagrangian
The EFT of Dark Energy

- At level of fluctuations

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_*^4}{2} f^{(4)} R - \Lambda - cg^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right. \]

\[ - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathcal{H}_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{H}_2 \]

\[ - \frac{m_6^3}{3} \delta \mathcal{H}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7^3}{3} \delta g^{00} \delta \mathcal{H}_3 \right]. \]

Changes the speed of sound of gravity waves

\[ \dot{\gamma}_{ij}^2 \subset \delta \mathcal{H}_2 \equiv \delta K^2 - \delta K_\mu^\nu \delta K^\mu_\nu, \quad \delta \mathcal{G}_2 \equiv \delta K_\mu^\nu R^\mu_\nu - \delta K R/2, \]

\[ \delta \mathcal{H}_3 \equiv \delta K^3 - 3 \delta K \delta K_\mu^\nu \delta K^\mu_\nu + 2 \delta K_\mu^\nu \delta K^\mu_\nu \delta K^\rho_\nu. \]

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- All models unified in one Lagrangian
The EFT of Dark Energy

- At level of fluctuations

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - c g^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \right. \]

\[ \left. - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathcal{H}_2 + \tilde{m}_4^2 \delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{H}_2 \right. \]

\[ \left. - \frac{m_6^3}{3} \delta \mathcal{H}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{G}_2 - \frac{m_7^3}{3} \delta g^{00} \delta \mathcal{H}_3 \right] . \]

\[ \dot{\gamma}_{i,j} \subset \delta \mathcal{H}_2 \equiv \delta K^2 - \delta K^\nu_{\mu} \delta K^\mu_{\nu} , \quad \delta \mathcal{G}_2 \equiv \delta K^\nu_{\mu} R^\mu_{\nu} - \delta K R / 2 , \]

\[ \delta \mathcal{H}_3 \equiv \delta K^3 - 3 \delta K \delta K^\nu_{\mu} \delta K^\mu_{\nu} + 2 \delta K^\nu_{\mu} \delta K^\mu_{\rho} \delta K^\rho_{\nu} . \]

- \[ m_2^4 = \alpha_K H^2 M_*^2 \] for LSS, we are interested in \( \alpha \sim 0.1 \)

- All models unified in one Lagrangian
The EFT of Dark Energy

- The non-relativistic dynamics allows also gravitational waves to decay into dark energy.

\[ \frac{\tilde{m}_4^2}{2} \delta g^{00} \left( (3) R - \delta \mathcal{K}_2 \right) \]

\[ \alpha_H \equiv \frac{2 \tilde{m}_4^2}{M_{Pl}^2} \lesssim 10^{-10} \]

- Irrelevant for LSS observations
The EFT of Dark Energy

• The non-relativistic dynamics allows also gravitational waves to decay into dark energy.

\[ \frac{\tilde{m}_4^2}{2} \delta g^{00} \left( ^{(3)}R - \delta \mathcal{K}_2 \right) \]

\[ \alpha_H \equiv \frac{2\tilde{m}_4^2}{M_{P1}^2} \lesssim 10^{-10} \]

• Irrelevant for LSS observations
The EFT of Dark Energy

• Somewhat unexpectedly, the discovery of gravitational waves has hit very heavily the theories of dark energy.

• Some theories are surviving (at least so far), but, to me, we are talking of corners.

• Plus, we already have a theory that works very well: \( \Lambda \)

  – and which goes well together in what we are seeing in the standard model Higgs.

• So, we go back to the early universe, where, with Inflation, we definitely have something to better understand.
The way ahead
Cosmology is a luminosity experiment

- Tremendous progress has been made through observation of the primordial fluctuations

- We are probing a statistical distribution:
  - In order to increase our knowledge of Inflation, apart for one exception, we need more modes:
    \[ \Delta(\text{everything}) \propto \frac{1}{\sqrt{N_{\text{modes}}}} \]

- Planck has just observed almost all the modes from the primordial CMB

- Large-Scale Structure offer the only medium-term place for hunting for more modes
  - but we are compelled to understand them better
  - Lots of applications for
    - astrophysics, inflation, dark energy, neutrinos, dark matter, etc.
What is next?

• LSST, Euclid and Chime are the next big missions: this is our next chance
  – we need to understand how many modes are available

Number modes $\sim V_{\text{survey}} k_{\text{max}}^3$

– Need to understand short distances
Massive Neutrinos

- Not only inflation
- Neutrinos have a mass, their energy affects the gravitational clustering
- By understanding the clustering, we can measure their mass

![Graph showing clustering curves with labels from Wong 2011](image)

- We need to understand these curves
Some marvelous results already achieved

- Baryon Acoustic Oscillations in Galaxies distribution

- But most new information is just about low-z universe $k \ (h \ Mpc^{-1})$

- Wherever CMB was not degenerate, it dominates
The Effective Field Theory of Large-Scale Structure
The EFTofLSS: A well defined perturbation theory

- Non-linearities at short scale

\[ k^3 P(k) \sim \left( \frac{\delta \rho}{\rho} \right)^2 \]
A long, long journey

- Dark Matter & Baryons
- Galaxies
- Redshift space
- IR-resummation

- Of course, none of this would have been possible without the precedent work of people like Bernardeau, Bond, Boucher, Kaiser, Matsubara, MacDonald, Peebles, Refregier, Scheth, Scoccimarro, Seljak, Wechsler, White, and Zeldovich…

- But the EFTofLSS provides the first (and only) rigorous, convergent formalism to the true answer for $k \ll k_{NL}$
  
  - to do fundamental physics, we have to be very accurate, and therefore rigorous.
The EFTofLSS and Dielectric Materials

– The theory of dielectric materials is the theory of a massless spin-one object (light) interacting with composite objects (atoms)

– Very similarly, the EFTofLSS is the theory of massless spin-two object (gravity), interacting with composite objects (galaxies)

• so it is conceptually quite easy

– It is also similar to the Chiral Lagrangian
The Effective ~Fluid

- In history of universe Dark Matter moves about $1/k_{NL} \sim 10\,\text{Mpc}$
  - it is an effective fluid-like system with mean free path $\sim 1/k_{NL} \sim 10\,\text{Mpc}$

- Skipping many subtleties, the resulting equations are equivalent to fluid-like equations

$$
\nabla^2 \Phi_l = H^2 \frac{\delta \rho_l}{\rho}
$$

$$
\partial_t \rho_l + H \rho_l + \partial_i \left( \rho_l v^i_l \right) = 0
$$

$$
\dot{v}^i_l + Hv^i_l + v^j_l \partial_j v^i_l = \frac{1}{\rho} \partial_j \tau_{ij}
$$

- short distance physics appears as a non trivial stress tensor for the long-distance fluid

$$
\tau_{ij} \sim \delta_{ij} \rho_{\text{short}} \left( v^2_{\text{short}} + \Phi_{\text{short}} \right)
$$
Dealing with the Effective Stress Tensor

- For dealing with long dist., expectation value over short modes (integrate them out)
  
  \[ \langle \tau_{ij}(\vec{x}, t) \rangle_{\text{long fixed}} = \int f_{\text{very complicated}} \left\{ H(t'), \Omega_{\text{dm}}(t'), \ldots, \rho_{\text{dm}}(x', t'), \ldots, m_{\text{dm}}, \ldots \right\}_{\text{on past light cone}} \]

- At long-wavelengths, the only fluctuating fields have small fluctuations: Taylor expand
  
  \[ \Rightarrow \langle \tau_{ij}(\vec{x}, t) \rangle_{\text{long fixed}} \sim c_s^2 \delta \rho(\vec{x}, t) + \mathcal{O}(\delta \rho^2) \]

- We obtain equations containing only long-modes

  \[ \nabla^2 \Phi_i = H^2 \frac{\delta \rho_i}{\rho} \]
  
  \[ \partial_t \rho_i + H \rho_i + \partial_i \left( \rho_i v_i^j \right) = 0 \]
  
  \[ \dot{v}_i^i + H v_i^i + v_i^j \partial_j v_i^i = \frac{1}{\rho} \partial_j \tau_{ij} \]

  \[ \langle \tau_{ij} \rangle_{\text{long-fixed}} \sim \delta_{ij} \left[ p_0 + c_s \delta \rho_i + \mathcal{O} \left( \frac{\partial}{k_{\text{NL}}}, \partial_i v_i^j, \delta \rho_i^2, \ldots \right) + \Delta \tau \right] \]

- How many terms to keep?

  - each term contributes as an extra factor of

    \[ \frac{\delta \rho_i}{\rho} \sim \frac{k}{k_{\text{NL}}} \ll 1 \]
Perturbation Theory with the EFT
Perturbation Theory within the EFT

- In the EFT we can solve iteratively \( \delta_\ell, \nu_\ell, \Phi_\ell \ll 1 \), where \( \delta = \frac{\delta \rho}{\rho} \)

\[
\nabla^2 \Phi_\ell = H^2 \frac{\delta \rho_\ell}{\rho} \\
\partial_t \rho_\ell + H \rho_\ell + \partial_i \left( \rho_\ell \nu^i_\ell \right) = 0 \\
\dot{\nu}^i_\ell + H \nu^i_\ell + \nu^i_\ell \partial_j \nu^j_\ell = \frac{1}{\rho} \partial_j \tau_{ij} \\

\langle \tau_{ij} \rangle_{\text{long-fixed}} \sim \delta_{ij} \left[ p_0 + c_s \delta \rho_\ell + \mathcal{O} \left( \frac{\partial}{k_{\text{NL}}}, \partial_i \nu^i_\ell, \delta \rho^2_\ell, \ldots \right) + \Delta \tau \right]
\]
Perturbation Theory within the EFT

• Solve iteratively in \[ \delta = \frac{\delta \rho}{\rho} \]

• Since equations are non-linear, we obtain convolution integrals (loops)

\[ \delta^{(n)} \sim \int \text{GreenFunction} \times \text{Source}^{(n)} [\delta^{(1)}, \delta^{(2)}, \ldots, \delta^{(n-1)}] \]

\[ \Rightarrow \delta^{(2)}(k_l) \sim \int d^3k_s \delta^{(1)}(k_s) \delta^{(1)}(k_l - k_s), \Rightarrow \langle \delta_l^2 \rangle \sim \int d^3k_s \langle \delta_s^{(1)2} \rangle^2 \]

\[ \overset{\rightarrow}{k}_s \quad \overset{\rightarrow}{k}_l \quad \overset{\rightarrow}{k}_s' \]

• Integrand has support at high wavenumber where expressions do not make sense

• Need to add counterterms from \[ \tau_{ij} \supset c^2_s \delta \rho \] to make the result finite and correct

• we just found loops and renormalization applied to galaxies
EFT of Large Scale Structures at Two Loops

- Order by order improvement \( \left( \frac{k}{k_{NL}} \right)^L \)
- Theory error estimated
- \( k \)-reach pushed to \( k \sim 0.34 \, h \, \text{Mpc}^{-1} \)
- Huge gain wrt former theories

All Former Theories fail at \( k \sim 0.03 \)

Estimated Theory error

3 parameters

with Carrasco, Foreman and Green \textbf{JCAP1407}
with Zaldarriaga \textbf{JCAP1502}
with Foreman and Perrier \textbf{1507}
see also Baldauf, Shaan, Mercolli and Zaldarriaga \textbf{1507, 1507}
Perturbation theory is extremely slow to converge due to the effect of IR-displacements. They affect the feature in real space named BAO peak.

The first, and in a sense unique, consistent way to resum the IR-displacements was obtained in with Zaldarriaga.\cite{JCAP2015}

\[ P_{\text{IR-resummed}}(k) \sim \int dq \, M(k, q) \cdot P_{\text{non-resummed}}(q) \]

~Similar to soft photons.

\[ r^2 \xi \]
• It works very well

with Zaldarriaga JCAP2015
with Trevisan JCAP1805

• Similarly well in redshift space

with Lewandowski et al PRD2018
Analysis of the BOSS/SDSS data

Jerome Gleyzes, Nickolas Kockron, Dida Markovic, Leonardo Senatore, Pierre Zhang, Florian Beutler, Hector Gill-Marin

in completion
Analysis of the SDSS/BOSS data

- Preliminary results of the power spectrum analysis of the CMASS&low-z samples

\[ \left\langle \ln(10^{10} A_s) \right\rangle = 2.92 \pm 0.16 \ (^{+0.16}_{-0.16}) \]
\[ \left\langle \Omega_m \right\rangle = 0.295 \pm 0.012 \ (^{+0.012}_{-0.012}) \]
\[ \left\langle h \right\rangle = 0.650 \pm 0.024 \ (^{+0.025}_{-0.023}) \]

\[ \ln(10^{10} A_s) \quad \Omega_m \quad h \]

-some results are preliminary

-more checks, plots, runs, still to do

• We assume flat $\Lambda$CDM and Planck’s $n_s \& \Omega_b / \Omega_m$

• and measure $A_s, \Omega_m, H_0, b_1 \leftrightarrow f, \sigma_8, H_0, b_1$

• These preliminary results, if confirmed, tell us that there is the potentiality of much improving the whole legacy of SDSS.
Analysis of the SDSS/BOSS data

- Preliminary results of the power spectrum analysis of the CMASS&low-z samples

\[
\langle \ln(10^{10} A_s) \rangle = 2.92 \\
\pm 0.16 \ (^{+0.16}_{-0.16})
\]

\[
\langle \Omega_m \rangle = 0.295 \\
\pm 0.012 \ (^{+0.012}_{-0.012})
\]

\[
\langle h \rangle = 0.650 \\
\pm 0.024 \ (^{+0.025}_{-0.023})
\]

- Major improvement with respect to what done before
- Error bars comparable to CMB
- New way to measure $H_0$, and a further data point in the ‘tension’. 
Summary

- Several aspects of physics in cosmology:
  - Numerical GR and unusual Math
    - to explain how the universe started

  - LIGO/VIRGO
    - the nature of gravity
    - hitting hard Dark Energy

- The Effective Field Theory of Large Scale Structure
  - The ‘Chiral Lagrangian’ for galaxies
  - applied to data: it works!
Proof in 2+1 dimensions

- We would like to prove that an initial expanding topologically-non-closed 2+1dim cosmology, with a positive cosmological constant and matter satisfying suitable energy conditions, reaches de Sitter space almost everywhere.

- In 2+1 dimensions, the usual Einstein equation:

  \[ n^\mu n^\nu G_{\mu\nu} = 8\pi G (T_{\mu\nu} + \Lambda g_{\mu\nu}) n^\mu n^\nu \]

  \[ \Rightarrow (2)R + \frac{1}{2}K^2 - \sigma_{\mu\nu}^2 = \frac{1}{2}K_\Lambda^2 + 16\pi G T_{\mu\nu} n^\mu n^\nu \quad \text{where} \quad K_\Lambda^2 = 32\pi G \Lambda \]

- On MCF-surfaces: \( d/d\lambda = H \cdot d/dt \)

  \[ \frac{dV}{d\lambda} = \int d^2x \sqrt{h} K^2 = \int d^2x \sqrt{h} \left( 32\pi G T_{\mu\nu} n^\mu n^\nu + K_\Lambda^2 + 2\sigma_{\mu\nu}^2 - 2 \cdot (2)R \right) \geq \]

  \[ K_\Lambda^2 \int d^2x \sqrt{h} - 2 \int d^2x \sqrt{h} (2)R = K_\Lambda^2 V - 8\pi \chi . \]

  - here we assumed WEC

- The solution reads \( V(\lambda) \geq \frac{8\pi \chi}{K_\Lambda^2} + e^{K_\Lambda^2 \lambda} \left( V(0) - \frac{8\pi \chi}{K_\Lambda^2} \right) \chi < 0 \)

  \[ \Rightarrow \quad V(\lambda) \geq V(0) e^{K_\Lambda^2 \lambda} \to \infty \]

  - for all topologies but the sphere, the volume goes to infini